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A STUDY OF THE COUPLING RESONANCES IN THE AGS BOOSTER*

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Abstract

Analytic method is used to analyze the tracking results for the AGS-BOOSTER lattice. We found the amplitude dependence of the tune is very important in understanding characteristic of the tracking result. With the perturbed tune, the effective second order perturbation theory works very well. The method can be used to analyze the optimized operational condition for the lattice. For the Booster, the analysis suggests that chromaticity of −2 to −5 and a minimum unperturbed tune split $|\Delta\nu^0| > 0.01$. At the zero chromaticity point, the required unperturbed tune split is considerably larger in order to obtain a similar performance.

Introduction

The beam size of the particles is important to the performance of the accelerators. For instance, when the beam size is increased, the effect of the nonlinearities increases. These nonlinearities can be described by nonlinear resonances in the Hamiltonian. We study the effect of one of these resonances. A special kind of resonance is the nonlinear coupling resonance caused by sextupoles due to eddy current, saturation and the chromatic corrections. Because of these nonlinear magnet elements, the beam emittance is not a constant of motion. The emittance may grow without bound, if the important resonance is a sum resonance. Likewise the sum of emittances of horizontal and vertical planes remains constant, if the only important resonance is a difference resonance.

The Hamiltonian of the system near to a resonance line can be expressed as,[1-3]

$$H = H_o(I_x,I_y) + b_{m,n}(I_x,I_y)\cos(m\phi_x+n\phi_y-p\theta+\psi) \tag{1}$$

where I_x and I_y are the action variables or the emittances of the beam and ϕ_x, ϕ_y are the conjugate angular variables. The resonance line is described by $m\nu_x+n\nu_y=p$. When resonance strength $b_{m,n}$ of Eq. 1 is zero, the ϕ_x, ϕ_y are cyclic variables and the actions becomes constant of motion. The equation of motion of the particles becomes

$$\nu_x = \dot{\phi}_x = \partial H/\partial I_x \cong \nu_x^o + \alpha_{xx}I_x + \alpha_{xy}I_y + \ldots \tag{2}$$

$$\nu_y = \dot{\phi}_y = \partial H/\partial I_y \cong \nu_y^o + \alpha_{xy}I_x + \alpha_{yy}I_y + \ldots \tag{3}$$

$$\dot{I}_x = -\partial H/\partial\phi_x = m\, b_{m,n}\sin(m\phi_x+n\phi_y-p\theta+\psi) \tag{4}$$

$$\dot{I}_y = -\partial H/\partial\phi_y = n\, b_{m,n}\sin(m\phi_x+n\phi_y-p\theta+\psi) \tag{5}$$

where the time derivative is the differential with respect to θ, the rotation angle. From Eqs. 4 and 5, one obtains the condition,

$$n\, I_x - m\, I_y = \text{constant} \tag{6}$$

Note here that the invariant of the Hamiltonian becomes the linear combination of action variables. For a sum resonance, where m and n are positive, the action variables of each plane may become unbounded while Eq. (6) remain satisfied. For a difference resonance, the emittances of x and y plane are bounded while the magnitudes oscillate around its mean value. We are interested mainly in the difference resonance. To take into account the contribution of tune versus amplitude modulation, Eqs. (1) and (2) would include the amplitude dependence. It is interesting to note that the condition,

$$\partial(m\phi_x+n\phi_y)/\partial\theta = m\nu_x+n\nu_y \cong \text{constant} \tag{7}$$

can be approximately satisfied. Eqs. (4) and (5) can be solved as,

$$I_x = I_x^o + \frac{m\, b_{m,n}}{m\nu_x+n\nu_y-p}\cos(m\phi_x+n\phi_y-p\upsilon+\psi) \tag{8}$$

$$I_y = I_y^o + \frac{n\, b_{m,n}}{m\nu_x+n\nu_y-p}\cos(m\phi_x+n\phi_y-p\upsilon+\psi) \tag{9}$$

Note here that the actions or the emittances of the system would oscillate around the average values. The amplitude of the oscillation is proportional to $b_{m,n}/(m\nu_x+n\nu_y-p)$. The smear defined by,

$$S_x = \frac{\left(I_x^{max} - I_x^{min}\right)}{\sqrt{3}\left(I_x^{max} + I_x^{min}\right)} = \frac{1}{\sqrt{3}\, I_x^o}\left|\frac{m\, b_{m,n}}{m\nu_x+n\nu_y-p}\right| \tag{10}$$

is a measure of the amplitude of oscillations(i.e. the deviation of the emittance from the constant of motion). Here $\sqrt{3}$ in Eq. 10 is used to define the rms. value for a Gaussian distribution. When the smear is large, the particle is strongly influenced by the resonance. It is noted that the smear can be large when the width of the resonance $b_{m,n}$ is large or when the particle is very near to the resonance line, i.e. $m\nu_x+n\nu_y-p \cong 0$. It is important to note that the tunes of the machine ν_x and ν_y depend on the emittance of machine.

The Coupling Resonance of the AGS Booster Due To Eddy Current

AGS Booster is a rapid cycling machine (10 Hz) when operating with protons. The fast acceleration rate creates a sizable eddy current sextupole contribution to the accelerator.[4] In this section, we shall apply the theory in Section 1 to the coupling resonance in the booster. The important coupling resonance is $2\nu_x-2\nu_y = 0$ systematic resonance excited by the sextupoles. The tracking study of Dell and Parzen[5] shows that there exist a window in the chromaticity space where the coupling is fairly small. When the machine is corrected to a zero chromaticity, the coupling becomes large. This effect is important in choosing the operating point of the machine. In this section, we shall use this analytic theory to study this effect.

The upper part of Fig. 1 shows the amplitudes α_{xx}, α_{xy}, and α_{yy} of the booster lattice as a function of the chromaticity of the machine. The lower part shows the nonlinear tune difference defined as, $\Delta\nu_{NL}$

$$\Delta\nu_{NL} = \alpha_{xx}\varepsilon_x+\alpha_{xy}\varepsilon_y-\alpha_{yx}\varepsilon_x-\alpha_{yy}\varepsilon_y \tag{11}$$

as a function of the chromaticity where ε_x and ε_y are the vertical and horizontal emittance respectively. Let us define the enhancement factor due to the tune difference for the $2(\nu_x-\nu_y) = 0$ resonance as,

$$f = \left|\frac{1}{\nu_x - \nu_y}\right| = \left|\frac{1}{\nu_x^o - \nu_y^o + \Delta\nu_{NL}}\right|$$

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Note here that the amplitude dependence of Eqs. (2) and (3) has been included in Eq. (11). Figure 2 shows the enhancement factor for the unperturbed tune $\Delta\nu^o = \nu_x^o-\nu_y^o = 0.01$. We observe that the enhancement of the coupling resonance peaks at the chromaticity 5 due mainly to cancellation between the unperturbed tune and the amplitude dependence tune shifts. Indeed the tracking result of

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Parzen shows a peak in the smear at the chromaticity 5. Figure 2 also shows the similar tracking result by using the TEAPOT program.[6] The peak of the smear at the chromaticity 5 is clearly seen for the vertical emittance. Using Eq. (10), we can deduce the effective coupling resonance strength $b_{2,-2}$ from the tracking result. Figure 3 compares the results of an analytic calculation obtained from the second order perturbation theory[2] with that obtained from the tracking analysis. The agreement is good.

This procedure of integrating the amplitude dependent tune into the perturbation expansion is to some sense similar to the superconvergent perturbation expansion.[3] At or near a resonance, the effective Hamiltonian of Eq. (1) can be used to describe the motion of the particles. It is natural to use the perturbed tune in the calculation. When the residue of the coupling is small, the tune split required to control the coupling resonance is small. When the coupling strength is large, the tune split needed becomes large.

We have found that the perturbation expansion of the nonlinear theory should take the tune vs amplitude modulation into account in the second order calculation to explain the behavior of the peak in the smear function at the chromaticity 5 for the initial emittances 50 π mm-mrad for both planes. There exists a window in the chromaticity degree of freedom where the smear is relatively small. This is due to the cancellation of sextupole contributions to the coupling resonances. To maintain a small effect to the coupling resonances, i.e. smear of 15% or less, the tune should stay away from the resonance line by $2\sqrt{3}\,|b_{2,-2}|$ for all particles in the bunch.

Resonance Locking

When the particle is approaching a resonance, the tunes of the particle satisfies $m\nu_x+n\nu_y-p \cong 0$. In fact, the emittances of the particles would be modulated to change the tunes of the machine so that the resonance can be avoided. In reality, the resonance may be locked on for long time before the tune vs amplitude modulation can be used for avoiding the resonance. This can be understood from Eqs. (8) and (9), when $m\nu_x+n\nu_y-p \cong 0$ is satisfied, the emittance would not change for a long period of time. Figure 4 shows a particle tracking example of the resonance locking mechanism. We set up a condition that when a given particle reached the emittances of $\varepsilon_y = 100\ \pi$mm-mrad and $\varepsilon_x = 0$, this particle is near to the coupling resonance line. Those particles approaching the resonance line will take a long time to get out of the resonance condition [see Eqs. (8) and (9)]. The important resonance in this example is the coupling resonance, thus the emittance are bounded oscillations. When a sum resonance is encountered, the particle can lock on the resonance and grow without bound.

Let us assume that Eqs. (2) and (3) can be truncated at the linear order and the coupling strength $b_{m,n}$ is independent of I_x and I_y. We obtain, from Eqs. (2)-(4),

$$\ddot{\Phi} = \left(m^2\alpha_{xx} + 2mn\alpha_{xy} + n^2\alpha_{yy}\right)b_{m,n}\sin\Phi$$

where $\Phi \equiv m\phi_x+n\phi_y-p\theta+\psi$. This equation can be integrated to

$$\frac{1}{2}\dot{\Phi}^2 + \left(m^2\alpha_{xx} + 2mn\alpha_{xy} + n^2\alpha_{yy}\right)b_{mn}\cos\Phi = \text{constant} \tag{12}$$

Since $\dot{\Phi} = \Delta\nu = m\nu_x+n\nu_y-p$ would oscillate according to Eq. (12) around the stable or unstable fixed points of the separatrix, the particle, once near to the resonance, cannot escape the resonance condition. The action variables, I_x and I_y can be solved analytically from Eqs. (6) and (12).

Particle Distribution in the Nonlinear Field

The amplitude of a particle in the beam of an accelerator is usually described in terms of the emittance. For a linear machine, the emittance is a constant of motion. The distribution, Gaussian in the coordinate space can be described by,

$$f(\varepsilon) = \frac{1}{\varepsilon_o}\exp(-\varepsilon/\varepsilon_o)\ , \tag{13}$$

where ε_o is the emittance of the beam.

In the presence of the nonlinear field, the bunch distribution function is also a function of time. If the particle satisfies the Hamiltonian motion, then the distribution function satisfies the Vlasov equation,

$$\frac{\partial}{\partial\theta}f + \dot{\phi}_1\frac{\partial}{\partial\phi_1}f + \dot{\varepsilon}_1\frac{\partial}{\partial\varepsilon_1}f = 0\ . \tag{14}$$

where ε and ϕ are conjugate variables of the Hamiltonian system. In the case of a single resonance shown in Eq. (1), the solution of the Vlasov equation becomes,

$$f(\varepsilon,\ \phi,\ \upsilon) = f_o(\hat{\varepsilon}(\varepsilon,\phi,\theta))\ . \tag{15}$$

provided that the tune is not modulated by the emittance growth. The smear of the particle bunch can be obtained from the time or phase average of the disturbed distribution functions.

Conclusion

We have used the analytic theory to understand the tracking result. The existance of the operational window in the chromaticity degree of freedom is useful in controlling the coupling resonance. Based on the analytic study, one can minimize the effect of resonance by properly choosing the tunes of the machine. We have found that the effective second order perturbation method agrees very well with the tracking calculation. The derived phenomenological residue from the tracking calculation agrees well with the analytic second order resonance strength.

For the AGS Booster, the best operational chromaticity, from the beam dynamic point of view, is around −2 to −5, where the tune spread of the particles in the bunch is small due to the mutual cancellation of sextupoles. Similar cancellation leads to smaller resonance width.

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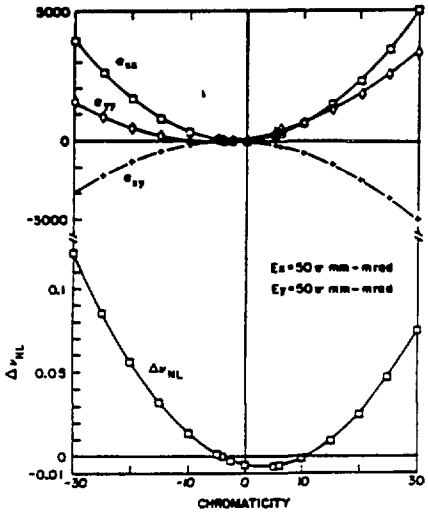

Fig. 1 The coefficients of tune vs. amplitude is shown as a function of chromaticity in the upper part of the graph. $\Delta\nu_{NL}$ of Eq. 11 is shown for $\varepsilon_x = \varepsilon_y = 50$ πmmmrad as a function of the chromaticity. Note here that if $\Delta\nu^o = -.01$, exist two resonances at chromaticities of −10 and +15 respectively. If the unperturbed tune is $\Delta\nu^o = 0.01$, the resonance will be most important at the chromaticity of 5.

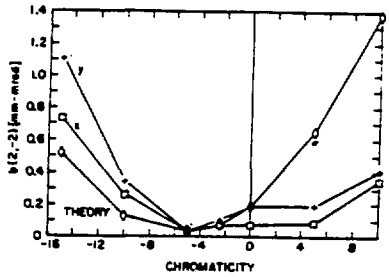

Fig. 3 The reduced coupling resonance strength b(2,−2) is compared with the analytic calculation of Ref. 2.

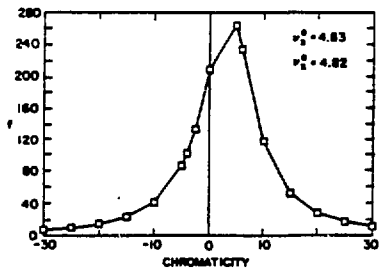

Fig. 2 The enhancement factor f is shown for $\Delta\nu^o = \nu_x^o - \nu_y^o = 0.01$. The tracking result of the smear factor defined in Eq. 10 is also shown as a function of chromaticity. The smear peaks at chromaticity equal to 5.

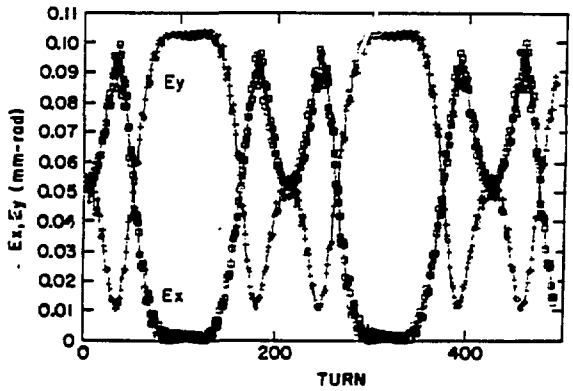

Fig. 4 The emittances of the particle is shown as a function of the tracking turns. At the specific chosen condition, i. e. $\varepsilon_y = 100$ πmm-mrad and $\varepsilon_x = 0$, the particle would experience the encounter the coupling resonance $2\nu_x - 2\nu_y = 0$. The particle would be locked on the resonance and stay in that condition for a long time. Note that $\hat{\varepsilon}(\varepsilon,\phi,\theta)$ must be an invariant of the motion for the particle in the nonlinear field. Using Eqs.(1-5), we can find the invariant for a particle near a single resonance and study the corresponding distribution function.