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ANALYSIS OF CENTER-NOTCHED, UNIDIRECTIONAL COMPOSITES*

E. David Reedy, Jr.
Sandia National Laboratories
Albuquerque, New Mexico 87185

ABSTRACT

A method for calculating the stresses in a notched, unidirectional monolayer is briefly described. This numerical formulation, based upon the familiar shear-lag assumptions, permits the modeling of a finite-dimensioned monolayer which contains a centered notch transverse to the fibers. Elastic-work hardening constitutive relationships may be specified for the fibers and/or matrix. Notch growth under increasing load can be analyzed. Illustrative calculations are presented to demonstrate the utility of this analysis in determining how constituent properties affect notch-tip fiber stress concentrations. Calculations for unidirectional boron/aluminum indicate that stress concentrations are reduced by a matrix with (1) sufficiently high yield strength to prevent large-scale yielding (uniform traction loading can cause large fiber stress concentrations when global yielding occurs), and (2) a low rate of work-hardening (to reduce stress concentrations). The analysis has also been applied to Kevlar 49 plain weave fabric/epoxy monolayers. Predicted matrix and fiber stress concentrations are quite localized in this material with nonlinear material response limited to the notch-tip region. This is quite different from the widespread yielding predicted and observed in as-fabricated, unidirectional boron/ aluminum.

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INTRODUCTION

Composite fracture can be studied from several different perspectives. For example, the primary interest may be in methods aimed at insuring the integrity of a specific composite structure under a prescribed load history. Alternatively, the aim may be to gain a fundamental understanding of the material parameters which affect flaw resistance. The analyses used in such diverse studies will often differ. When analyzing structures, it is often convenient to model a laminate, or sometimes each lamina, as a homogeneous, orthotropic material. The analysis is done on a scale on which only average composite stresses, not individual fiber and matrix stresses, are considered. When studying the details of the fracture process it is at times necessary to perform analyses in which the heterogeneous nature of the composite is treated. The analysis described in this paper falls into this latter category. While such analyses are too detailed to use for complex structures, they can provide insights into processing or material modifications that will improve the flaw resistance of a composite.

A monolayer, a planar array of aligned, continuous fibers, may be considered the fundamental building block from which laminates are constructed. Notched monolayers can be analyzed within the context of a shear-lag theory. This approximate analysis, in which fibers are assumed to carry only axial loads while the matrix carries only shear, is particularly convenient when analyzing monolayers with a large number of fibers. Unless fiber breaks occur in a regular manner and a repeat cell can be identified, a more detailed finite element solution would be prohibitive. Hedgepeth (1961) was the first to apply a shear-lag analysis to monolayers. In his analysis, (1) the fibers were of infinite length, (2) the monolayer was of infinite width, (3) tensile loads were uniformly applied parallel to the fibers at infinity, (4) both fibers and matrix were linear elastic, and (5) both constituents were fully bonded. Hedgepeth's original analysis has undergone several direct extensions as reviewed by Reedy (1980). These extensions are rather limited in scope. In general, an infinite monolayer is analyzed in which any matrix damage or nonlinear material response must occur between the last broken fiber and the first intact fiber.

Reported below is a numerical formulation of the center-notched monolayer analysis which permits arbitrary monolayer dimensions, either traction or displacement boundary loads, notches composed of multiple fiber breaks, flaws of arbitrary geometry, nonuniform fiber spacing, monolayers containing more than one type of fiber, and constituents which can work-harden. The analysis is formulated to model slow notch growth and/or the growth of matrix shear cracks. A number of illustrative examples are presented to demonstrate the utility of the analysis and comparison is made between experimentally determined behavior and theoretical predictions.

ANALYSIS

Method

A previously developed method for analyzing center-notched monolayers is employed. This method allows one to calculate fiber and matrix stresses in a finite-dimensioned monolayer whose matrix hardens kinematically. A detailed description of the monolayer model and the method of solution is given by Reedy (1980). In essence, the monolayer (Fig. 1) is discretized axially into fiber and matrix elements (axial boundaries indicated by dashed lines) and an analog of the three-dimensional Principle of Stationary Complementary Energy for a deformation theory of plasticity is applied to the structure. The principle is suitably modified to allow unloading during notch growth. The analysis is performed within the context of a shear-lag theory. Fibers are considered to carry only axial loads while the matrix carries only shear loads. Matrix shear stress is assumed constant within a matrix element while the stresses in a fiber element vary linearly along its length. Equilibrium is enforced with Lagrangian multipliers. The resulting set of nonlinear, simultaneous equations is solved numerically by a multi-dimensional Newton-Raphson method. Slow notch growth is modeled in the following manner. When the notch-tip fiber stress reaches a prescribed fracture stress, applied boundary loads are held fixed and the fiber is "broken" by unloading it in several steps. If after this process the fiber stress at the extended notch is below the fracture stress, the notch is said to have grown stably. In this case the applied boundary loads are then increased until the notch-tip fiber breaks. This process is continued as long as stable notch growth is possible. Stable notch growth under increasing load is a consequence of the residual matrix shear strains left behind the advancing notch.

Illustrative Calculations

To date, most of the calculations have been for unidirectional, 5.6 mil diameter boron/6061 aluminum. This material, with its large diameter fibers, ductile matrix, and strong fiber-matrix bond can be considered something of a model system. In the analysis, the boron fibers were treated as linear elastic to failure with a 58 msi modulus and a 520 ksi strength. These are typical values for boron fibers and are consistent with four-point bend data for the two 14-ply thick plates from which the notched specimens used in this study were taken. Rail shear test data for these plates were used to infer a plate's in-situ matrix shear stress-strain relation, Reedy (1982a). Both plates were nominally identical (fabricated by DWA Composite Specialties, Inc.) and were tested in the as-fabricated condition. Shear data for each plate was fit to a modified Ramberg-Osgood relationship:

$$\gamma/\gamma_y = \begin{cases} \tau/\tau_y & \tau/\tau_y \leq 1 \\ (\tau/\tau_y)^N & \tau/\tau_y > 1 \end{cases} \qquad (1)$$

where τ is shear stress, γ is shear strain, τ_y and γ_y are their values at yield and N is the hardening exponent. Unidirectional boron/aluminum has a shear modulus of approximately 7 Msi so γ_y was assumed to equal τ_y/7 Msi. The shear data for the one plate (hereafter called Plate 1) were best fit with τ_y = 8.5 ksi and N = 4.88. The shear data of the other plates (Plate 2) were fit with τ_y = 5.0 ksi and N = 5.38. These results indicate, for the plates tested, that the as-fabricated state of boron/6061 aluminum can vary significantly.

Predicted and measured Plate 1 load vs notch opening displacement relations for center-notched tensile specimens with three different initial notch lengths (0.1, 0.25, 0.50 in) are displayed in Fig. 2. The specimens were all 1.0 in wide with a 3.0 in gage length. Both uniform traction and uniform edge displacement boundary conditions were considered. Prior to first fiber breakage, the predicted relations (solid curves) for both types of boundary conditions are identical. The saw-toothed portion of the curves is a consequence of the manner in which stable notch growth is modeled. Both types of boundary conditions predict similar response during the initial phase of stable notch growth. However, for the two longer notches, a traction condition causes failure at maximum load while the rigid-grip condition allows notch growth under decreasing load until a limiting displacement is reached. The experimental records are in excellent agreement with prediction. These results demonstrate the ability of the monolayer analysis to predict the response of the 14 ply, unidirectional composite. The composite appears to behave much like an assemblage of independent monolayers. It should be emphasized that the predicted response is based entirely upon independently measured constituent properties.

The predicted response of a Plate 2 specimen with a 0.5 in notch differs from that of a similar Plate 1 specimen in one important aspect. Predicted Plate 2 response is strongly influenced by the type of boundary condition (Fig. 3). The predicted curves are initially coincident, but diverge as load increases. This is a result of extensive matrix yielding. The magnitude of the yielding is dependent on the method of loading. When the specimen yields over the entire gage length, a uniform traction condition will pull the broken fibers away from the intact ligament and the loaded boundaries do not remain straight. This results in a much greater notch-tip fiber stress concentration than if the boundary is uniformly displaced. The experimental data for the Plate 2 specimens (solid curves) are bounded by the predicted curves.

Figure 4 provides further information on how matrix shear yield strength and hardening exponent affect fiber stress concentrations and also indicates when the type of boundary load influences results. These calculations are all for a 1.0 in wide monolayer containing a 0.5 in notch. The specimen has a 3.0 in gage length and both uniform traction and displacement loads are considered. The notch-tip stress concentration is defined as the ratio of notch-tip fiber stress to the average applied fiber stress. Note that with this definition and for the specimen geometry considered, the notch-tip stress concentration equals 2 when the specimen is completely notch insensitive (i.e., when the broken fibers carry no load and the intact fibers all carry the same load). The value of the notch-tip fiber stress concentration just prior to first fiber failure is plotted as a function of shear yield strength for a range of hardening exponents. Figure 4 indicates that for sufficiently high values of yield strength there is no dependence on the method of loading. In this regime (e.g., $\tau_y > 12$ ksi, $N < 9$ in Fig. 4) fiber stress concentration always decreases as the hardening exponent increases. Therefore, for a given yield strength, the stress concentration is reduced as the matrix becomes more like an elastic perfectly-plastic material. The stress concentration can be roughly cut in half by increasing the hardening exponent from 1 to 9 when $\tau_y = 15$ ksi. Stress concentration is not very sensitive to yield strength as long as the yielding is sufficiently localized. However, once large-scale yielding commences, the fiber stress concentration is strongly dependent on yield strength, and a uniform traction loading can cause extremely high stress concentrations as yield strength decreases. In contrast, fiber stress concentration always monotonically decreases with decreasing yield strength when a uniform displacement condition is applied.

A similar parametric study was carried out for a specimen with a single broken fiber. In this case a notch insensitive material has a fiber stress concentration of approximately 1. These calculations indicate that the fiber stress concentration for this case is independent of the type of boundary load for shear yield strengths in the range of interest ($2.5 < \tau_y < 20$ ksi). However, the same sort of boundary condition dependence will show up if the shear yield strength is very low. For example, for N = 9 and a shear yield strength of only 200 psi, the stress concentration for a uniform traction loading is 1.16 while that for a uniform displacement loading is 1.08. The results for a single broken fiber are qualitatively similar to the 0.5 in notch (69 breaks). For a given hardening exponent, provided that the shear yield strength is high enough to prevent large-scale yielding, fiber stress concentration increases with yield strength and decreases as the hardening exponent increases. At sufficiently high yield strengths, no yielding will occur and the fiber stress concentration is equal to the linear elastic (N = 1) value. As expected, the stress concentration for a single broken fiber in the 1.0 inch wide strip (141 fibers) equals that calculated by Hedgepeth (1961) for an infinite monolayer--1.33. The rate of reduction in fiber stress concentration with hardening exponent decreases as the hardening exponent increases. For example, when τ_y = 20 ksi, the fiber stress concentration equals 1.2414 for a hardening exponent of 9. This is reduced to 1.2205 when the hardening exponent is 100. It appears that the limit corresponding to an elastic-perfectly plastic matrix is quickly approached as N increases.

Although not presented in detail here, other calculations for unidirectional boron/aluminum [Reedy (1980c)] have shown that 1) the details of flaw geometry (holes vs notches) do not greatly influence flaw sensitivity, and 2) fiber spacing has little effect on fiber stresses, although it can increase matrix stresses significantly.

In addition to the calculations for boron/aluminum, some preliminary modeling of notched Kevlar 49 plain weave fabric/epoxy monolayers has been completed, Reedy (1982b). Fabric monolayers stiffen in tension and yield in shear. The fabric examined (style 328) has 17 yarns per inch. In the analysis each yarn was treated as a single unit. The fabric monolayer was abstracted as a unidirectional composite with yarn and matrix stress-strain relations chosen in such a way that when there are no broken yarns the calculated response to tension or shear is identical to that actually measured for the monolayer. Predicted strength of a 1.0 in wide monolayer with a 3.0 in gage length for varying notch lengths is displayed in Fig. 6. The experimental results (as indicated by the various symbols) are in reasonable agreement with prediction. Matrix shear yielding was quite localized in these calculations and a fully elastic analysis predicts similar fiber stress concentrations.

CONCLUSIONS

The utility of a shear-lag monolayer analysis which permits the analysis of finite-dimensioned monolayers with work-hardening constituents has been demonstrated. Calculations for unidirectional boron/aluminum have shown that for this ductile matrix composite

1. Matrix yield strength and rate of work-hardening substantially affect the mechanical response of notched specimens.

2. Fiber stress concentrations are effectively controlled by a matrix with (a) sufficiently high-shear yield strength to prevent large-scale yielding (uniform traction loading can cause large fiber stress concentrations when global yielding occurs), and (b) a low rate of work-hardening (to reduce stress concentrations).

Initial calculations for Kevlar 49 plain weave (style 328) fabric epoxy monolayers indicate that shear yielding is quite localized and a fully elastic analysis is adequate.

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4. Reedy, Jr., E. D., (1982b), "Notched Kevlar Fabric/Epoxy Monolayers," Fibre Sci. and Tech., 16, p. 261.

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LIST OF FIGURES

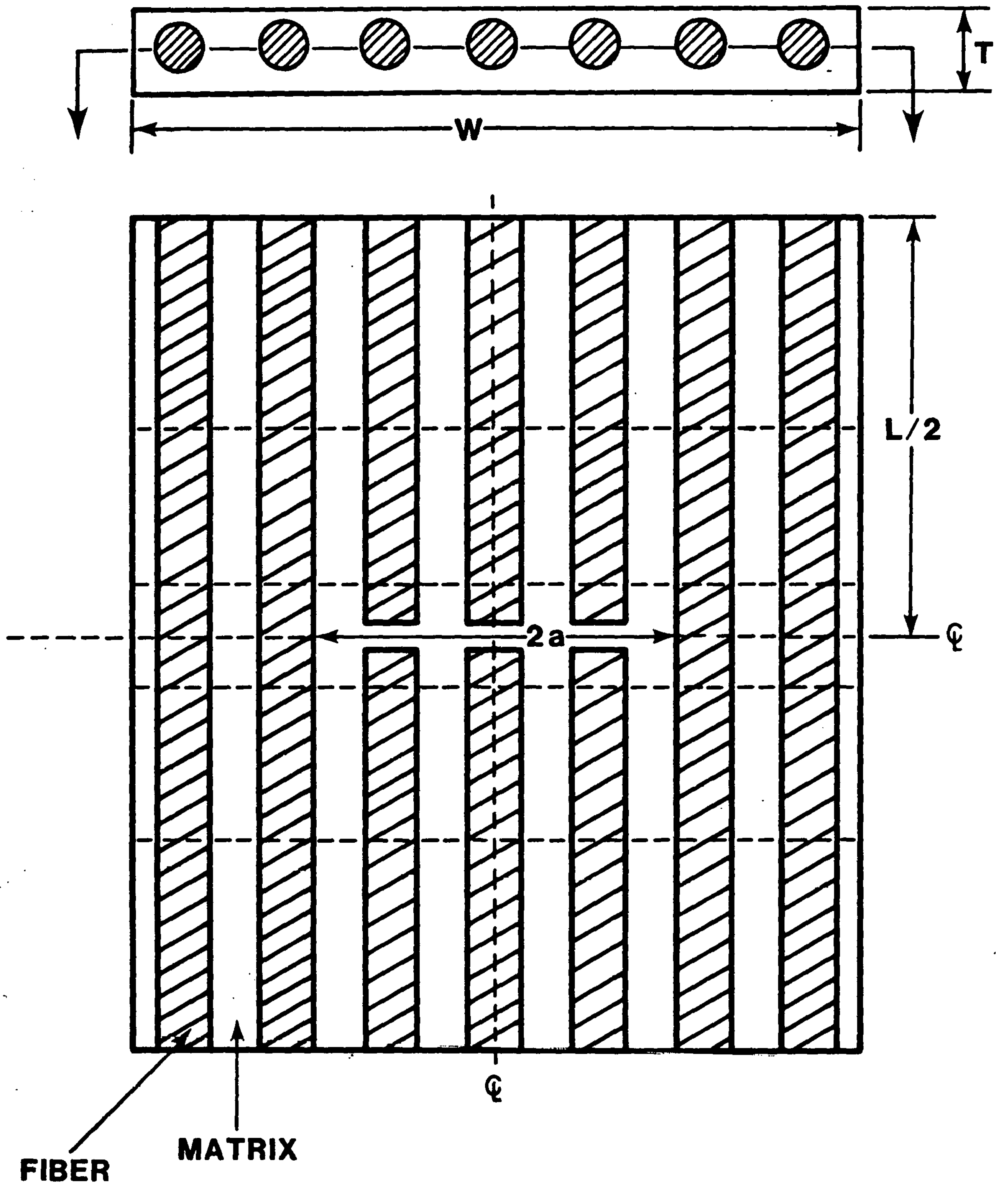
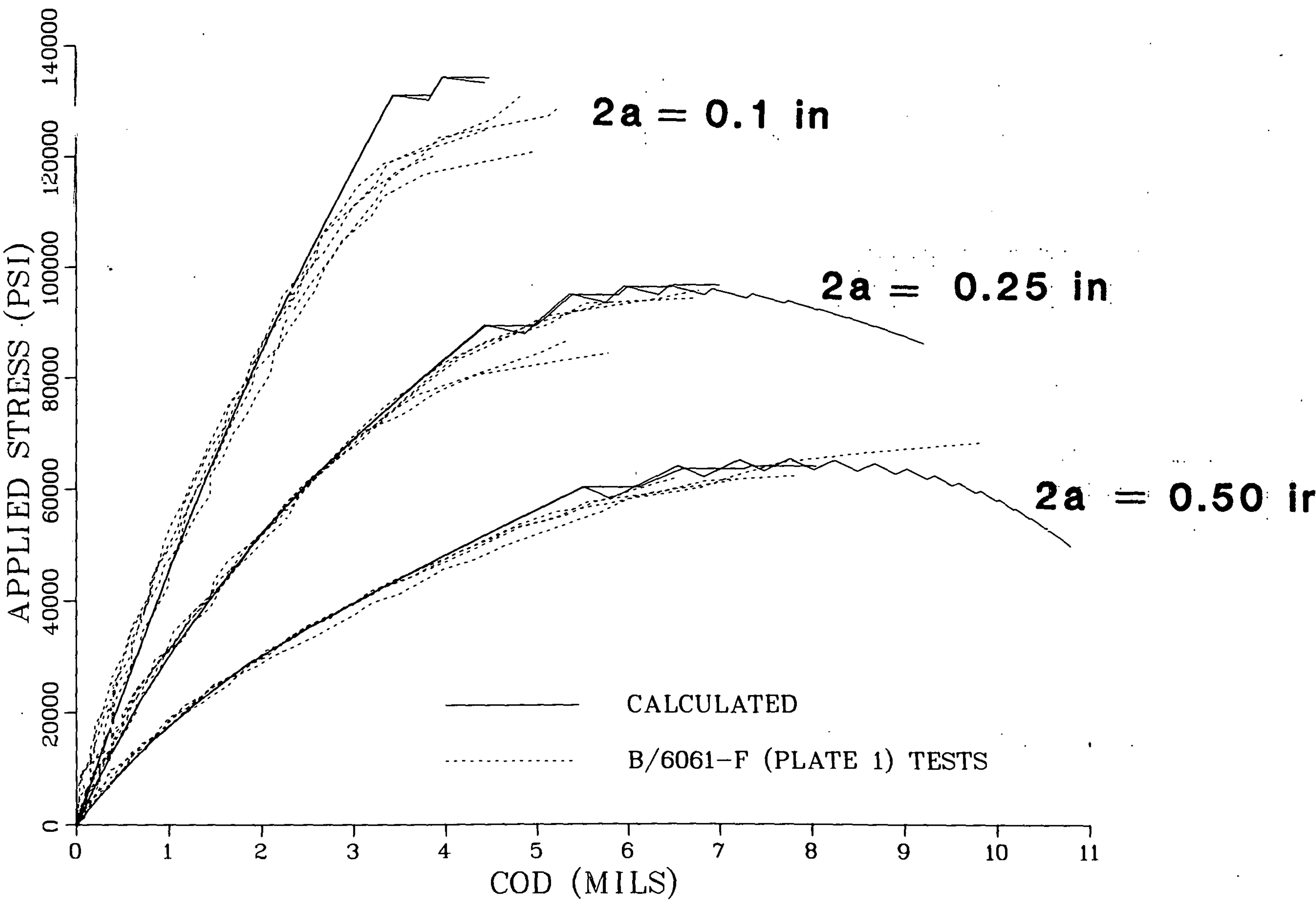


Fig. 1


2a = 0.1 in
2a = 0.25 in
2a = 0.50 in
APPLIED STRESS (PSI)
COD (MILS)
CALCULATED
B/6061-F (PLATE 1) TESTS
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20000
40000
60000
80000
100000
120000
140000
0 1 2 3 4 5 6 7 8 9 10 11

Fig. 2

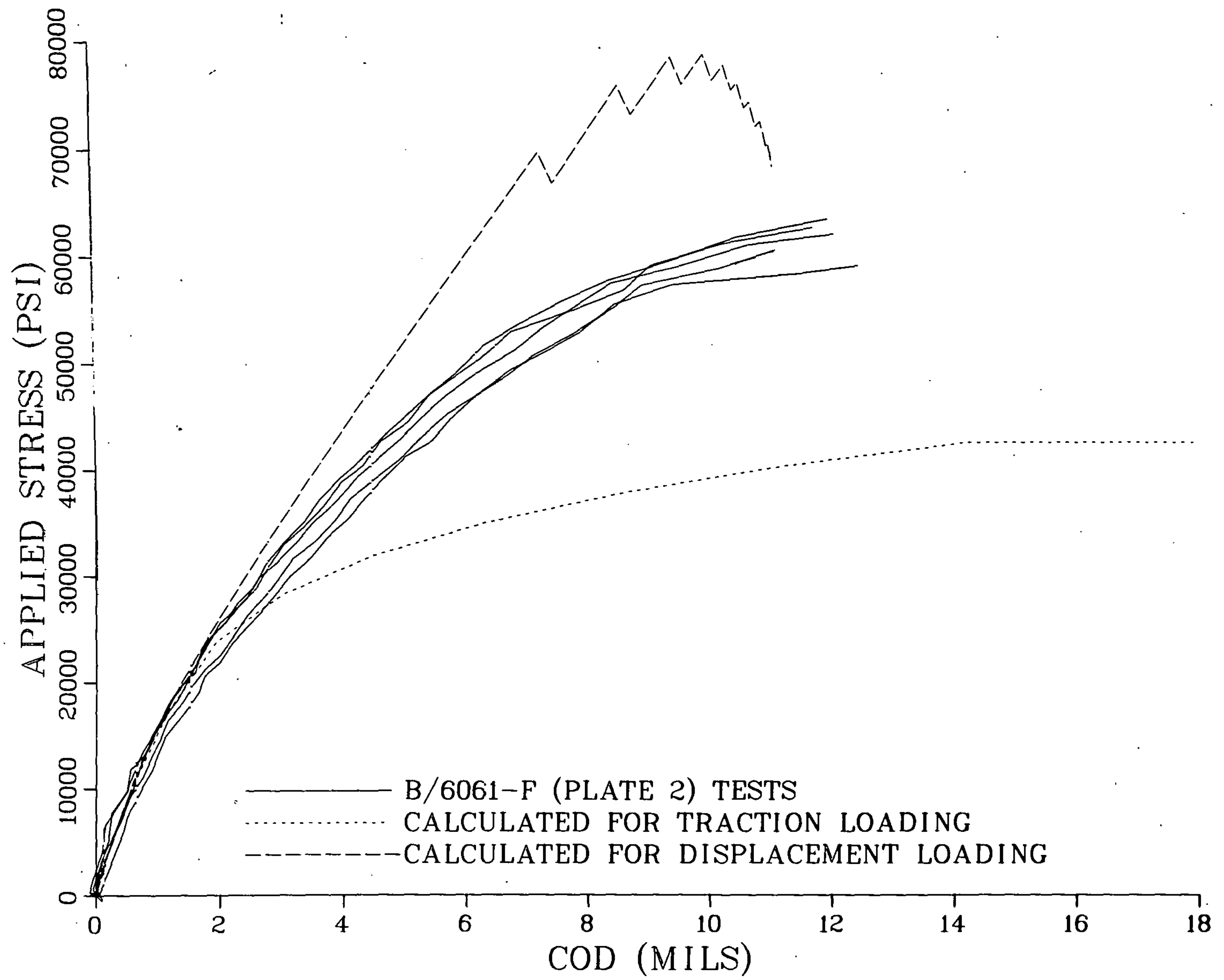
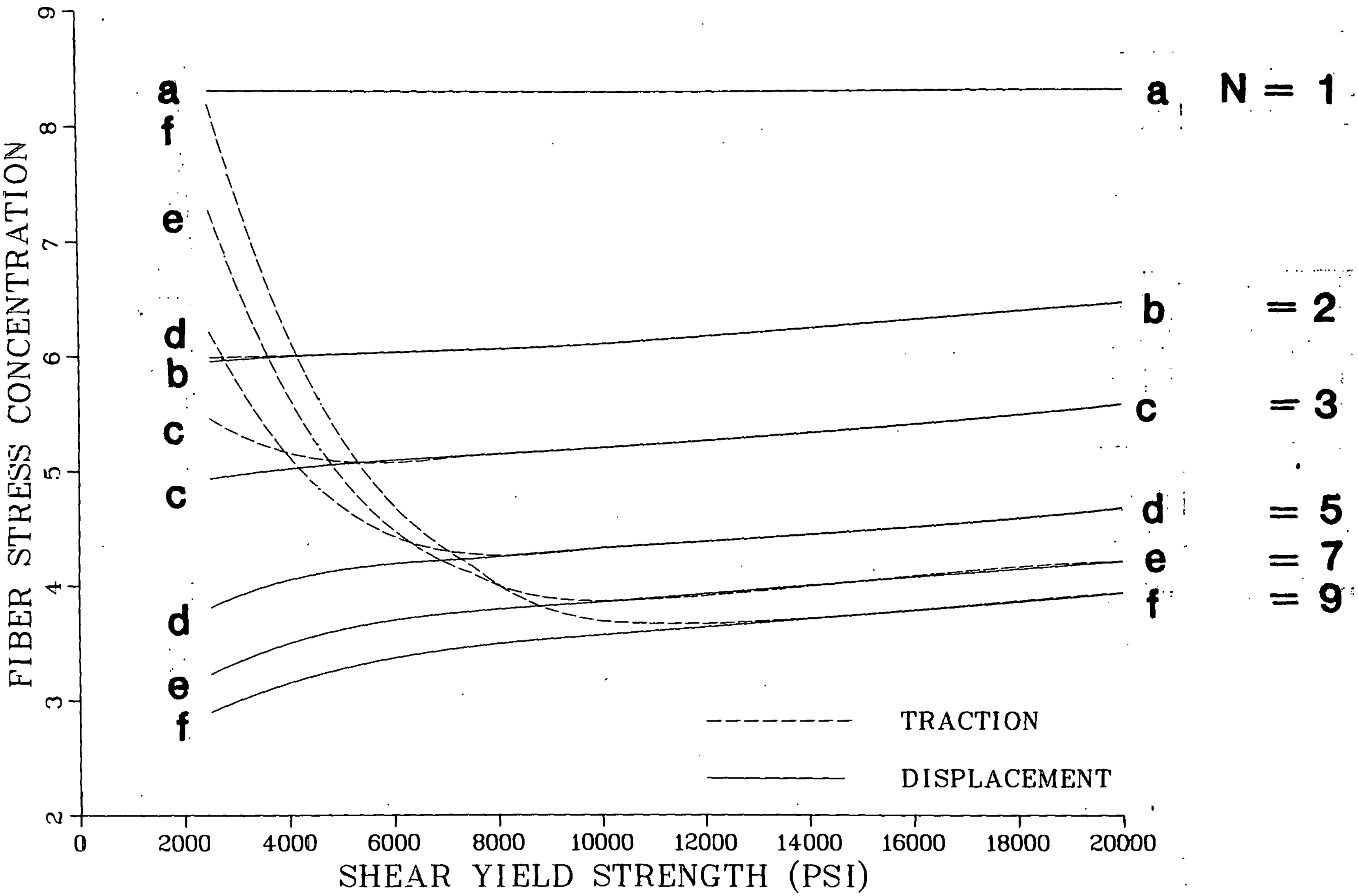

80000
70000
60000
50000
40000
30000
20000
10000
0
APPLIED STRESS (PSI)
0
2
4
6
8
10
12
14
16
18
COD (MILS)
B/6061-F (PLATE 2) TESTS
CALCULATED FOR TRACTION LOADING
CALCULATED FOR DISPLACEMENT LOADING

Fig. 3



FIBER STRESS CONCENTRATION
SHEAR YIELD STRENGTH (PSI)
TRACTION
DISPLACEMENT
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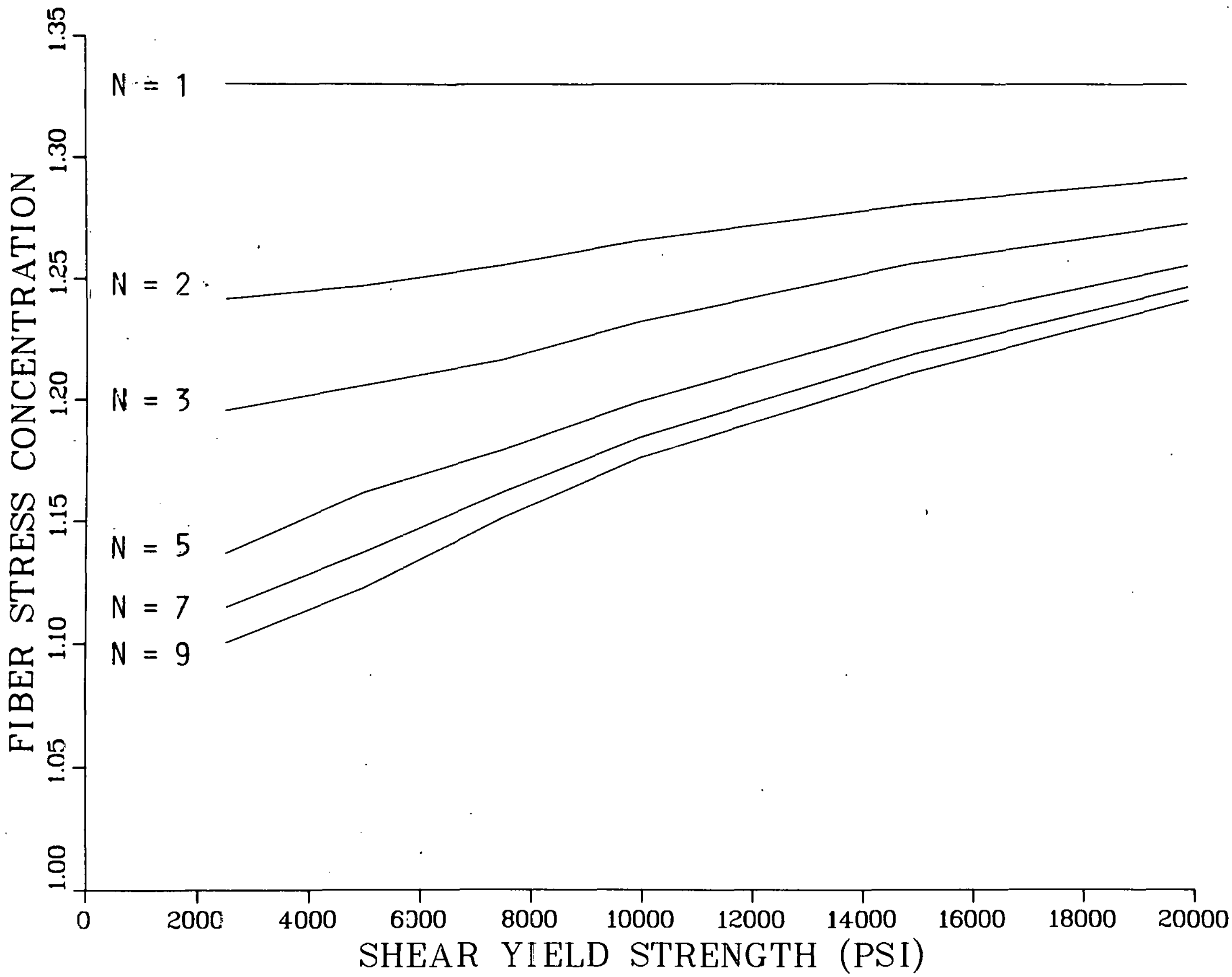
b = 2
c = 3
d = 5
e = 7
f = 9

Fig. 4



FIBER STRESS CONCENTRATION
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
N = 1
N = 2
N = 3
N = 5
N = 7
N = 9
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
SHEAR YIELD STRENGTH (PSI)

Fig. 5

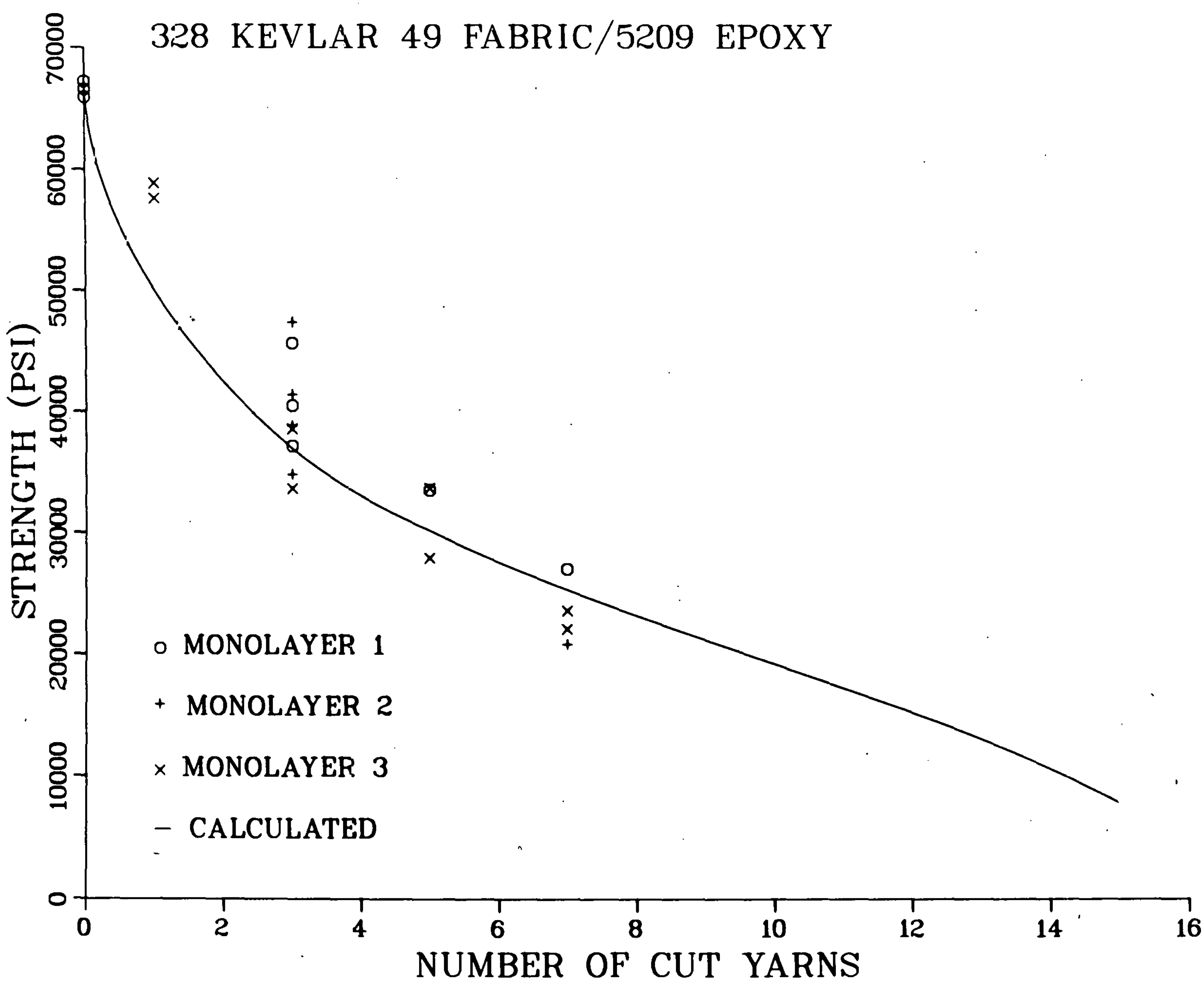

328 KEVLAR 49 FABRIC/5209 EPOXY
STRENGTH (PSI)
70000
60000
50000
40000
30000
20000
10000
0
NUMBER OF CUT YARNS
0
2
4
6
8
10
12
14
16
o MONOLAYER 1
+ MONOLAYER 2
× MONOLAYER 3
— CALCULATED

Fig. 6