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CRACK BRIDGING PROCESSES IN TOUGHENED CERAMICS.*

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ABSTRACT. The fracture toughness of ceramics can be improved by the incorporation of a variety brittle discontinuous reinforcing phases. Observations of crack paths in these systems indicate that these reinforcing phases bridge the crack in the region behind the crack tip. Recent developments in toughening models based on crack bridging processes in such systems are discussed and compared to the experimentally observed toughening responses with second phase whisker and self (matrix grain) reinforcement. The bridging model then can be used to optimize the toughening effects based modification of the pertinent material characteristics (e.g., microstructure and physical properties).

Introduction

The brittle nature of ceramics has, over the years, prompted us to explore a variety of approaches to increasing their fracture toughness/resistance. Initially the concern was to toughen these materials to to improve their fracture strength and/or reduce the flaw size sensitivity of the fracture strengths. Then it was recognized that resistance to damage in

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service was a further issue and that toughening these materials could enhance their damage resistance. While many issues still need to be addressed, e.g., cyclic fatigue resistance, crack size effects-R-curve behavior, improving the fracture toughness has been deemed, in general, to be quite beneficial.

One approach to toughening ceramics has been the incorporation of strong discontinuous brittle phases, e.g., whiskers [1], which is the subject of this paper. The mechanisms contributing to the increased fracture toughness are described herein in terms of crack bridging by the reinforcement. A crack bridging model is discussed which is found to accurately predict the observed toughening response in SiC whisker reinforced ceramics [2]. The results reveal that debonding of the interface between the reinforcing phase and the matrix is required to achieve significant toughening. The bridging model also illustrates how some of the properties of the matrix, interface, and reinforcing phase influence the fracture resistance of the composite. The predictive capability of the whisker bridging model then allows us to develop other approaches to achieving toughness by crack bridging. These include crack bridging by other types of second phases (platelets) and by matrix grains (self-reinforced).

Crack Bridging By Discontinuous Reinforcements

Bridging of the crack surfaces behind the crack tip by a strong discontinuous reinforcing phase which imposes a closure force on the crack is, at times, accompanied by pull out of the reinforcement [1-6]. The extent of pull out, i.e. the pull out length, brittle discontinuous reinforcing phases is generally quite limited due both to the short length of such phases and the fact that bonding and clamping stresses often discourage pull-out. However, pull-out cannot be ignored as even short pull-out lengths contribute to the toughness achieved. Crack deflection by such reinforcements has also been suggested to contribute to the fracture resistance. Often, out of plane (non mode I) crack deflections are limited in length and angle and are probably best considered as means of debonding the reinforcement-matrix interface. Such interfacial debonding is important in achieving frictional bridging (bridging by elastic ligaments which are partially debonded from the matrix) and pull-out processes. Frictional bridging elastic ligaments can contribute significantly to the fracture toughness as is described herein.

ANALYSIS OF TOUGHENING BY DISCONTINUOUS BRIDGING PHASES

Here we will concentrate on the toughening due to crack bridging by various brittle reinforcing phases where the reinforcement simply bridges the crack surfaces and effectively pins the crack and increases the resistance to crack extension. The bridging contribution to the toughness for is:

$$\Delta K^{wr} = (E^{c}\, \Delta J)^{1/2} = K_{IC}{}^{c} - K_{IC}{}^{m} \qquad 1$$

where $K_{IC}{}^{c}$ is the overall toughness of the composite, $K_{IC}{}^{m}$ is the matrix toughness, and the term ΔJ corresponds to the energy change due to the bridging process.

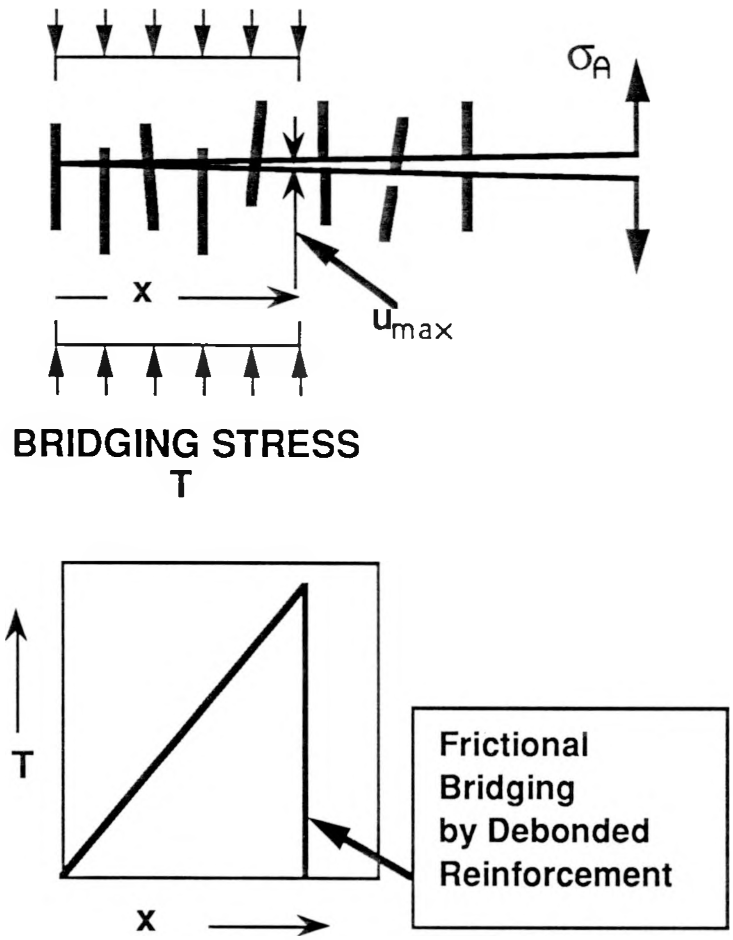
The energy change associated with the bridging process is a function of the bridging stress/traction, T_u, and the crack opening displacement, u and is defined as:

$$\Delta J = \int_{0}^{u_{max}} T_u \, du \qquad 2$$

where u_{max} is the maximum displacement at the end of the zone [7], Figure 1.

One can equate the maximum crack opening displacement at the end of the bridging zone, u_{max}, to the tensile displacement in the bridging brittle ligament at the point of failure:

$$u_{max} = \varepsilon_f{}^{l}\, l_{db} \qquad 3$$



$$\Delta K^{\omega r} = (2E^{c}\Delta J)^{1/2}$$

$$\Delta J = \int_{0}^{u_{max}} T\,du$$

Figure 1. Crack bridging by discontinuous brittle reinforcing phases impose a closure or bridging stress in the wake of the crack tip and enhance the fracture resistance of the brittle matrix.

where ε_f^l represents the strain to failure of the whisker and l_{db} is the length of the debonded matrix-whisker interface, Figure 2. The strain to failure of the whisker can be defined as:

$$\varepsilon_f^l = (\sigma_f^l \; / \; E^l) \qquad 4$$

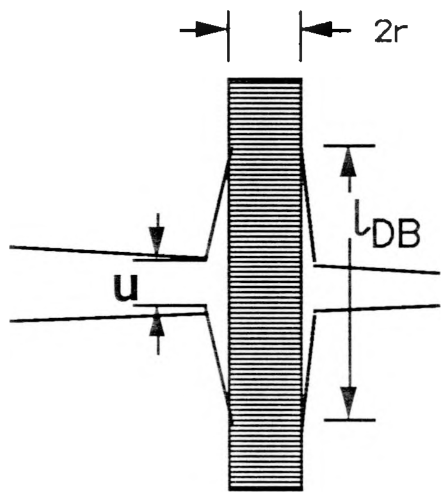
where E^l is the Young's modulus of the reinforcing phase. The interfacial debond length depends on the fracture criteria for the reinforcing phase versus that of the interface and can be defined in terms of fracture stress or fracture energy. The analysis of Budiansky et al [8] yields:

$$l_{db} = (r \, \gamma^l / 6 \, \gamma^i) \qquad 5$$

where γ^l / γ^i represents the ratio of the fracture energy of the bridging ligament to that of the reinforcement-matrix interface.

From equation 3, one quickly notices that the tensile strain displacement achieved in the bridging reinforcement and hence the maximum crack opening displacement at the end of the bridging zone increases as the debonded length/the gage length of the reinforcing ligament increases. Consideration of equations 4 and 5 show that increasing the reinforcing phase strength and/or enhancing interface debonding will contribute to greater tensile displacement within the reinforcing ligament. Increases in the crack opening displacement supported by the bridging zone will enhance the toughening achieved by such reinforcements. Therefore debonding of the matrix-reinforcement interface can be a key factor in the attainment of increased fracture toughness in these elastic systems. In fact in ceramics reinforced by strong ceramic whiskers, debonding is observed only in those systems which exhibit substantial toughening. An example of interfacial debonding associated with a bridging whisker in the wake of the crack tip is seen in Figure 3. In this case debonding is evidenced by the interfacial offsets at the leading and trailing sides of the bridging whisker.

For the case of a bridging stress which increases linearly from zero at the crack tip to a maximum at the end of the bridging zone and immediately decreases to zero, equation 2 can be reduced to $T_{max}(u_{max})/2$. The maximum closure stress T_{max} imposed by the reinforcing ligaments in the crack tip wake



CRACK OPENING u (=u_{max}) AT THE END OF BRIDGING ZONE EQUATED TO MAXIMUM WHISKER TENSILE DISPLACEMENT ($l_{DB}\,\varepsilon_f^{\,w}$).

Figure 2. The formation of the bridging zone behind the crack tip requires that the reinforcing phase-matrix interface separate/debond (a) during fracture. The crack opening displacement associated with the bridging zone then is related to the tensile displacement in the bridging ligaments (b). At the end of the bridging zone the maximum crack opening is equivalent to the displacement in the ligament corresponding to its fracture stress.

Figure 3. Debonded whisker-matrix interfaces are associated with whisker bridging in region immediately behind the crack tip in a polycrystalline aluminum oxide matrix.

is the product of the fracture strength of the ligaments, σ_f^l, and the areal fraction of ligaments intercepting the crack plane, A^l:

$$T_{max} = \sigma_f^l A^l \approx \sigma_f^l V^l \qquad 6$$

where A^l is approximated by the volume fraction, V^l, for ligaments which have large aspect ratios (e.g., $l/r \geq 30$ for whiskers). Reinforcement by frictional bridging introduces an change in energy equal to:

$$\Delta J^{flb} = [\, \sigma_f^l V^l (\sigma_f^l / E^l)(r \gamma^l / \gamma)]/12 \qquad 7.$$

From these results, the resultant toughness contribution from frictional bridging by the reinforcing phase in the crack tip wake is:

$$\Delta K^{flb} = \sigma_f^l [\, (r V^l / 36) (E^c / E^l)(\gamma^l / \gamma)]^{1/2} \qquad 8.$$

The overall toughness of the composite then includes both the bridging contribution, equation 8, and that the fracture resistance of the matrix per equation 1.

MATERIAL CHARACTERISTICS INFLUENCING TOUGHNESS

The toughening contribution then can be enhanced by utilizing matrix-reinforcing phase combinations with comparable Young's moduli and by improving the strength of reinforcing phase and increasing the reinforcement content and diameter. There are obvious limits as to how large a diameter reinforcing phase can be used in systems employing a matrix with a thermal expansion coefficient greater than that of the reinforcement as the thermal contraction mismatch tensile stress intensity scales with increase in inclusion/reinforcing phase diameter. In the alumina-SiC whisker system, the larger thermal expansion coefficient of the matrix versus the whisker and the high elastic property values result in substantial hoop and longitudinal tensile strains in the matrix [3,9]. Larger diameter reinforcements can generate matrix crack during post-fabrication cooling and degrade the properties of such composite [10]. The maximum reinforcement diameter employed will depend on the elastic and thermal expansion properties of the matrix versus those of the reinforcing phase.

A critical factor in such toughening processes is interfacial debonding which can be achieved if the interfacial failure conditions are much less than those required to fracture the reinforcement. In fact, substantial toughening by such crack bridging is obtained only when the reinforcement-matrix interface debonds before or just as the main crack tip reaches the interface. The formation of a debonded interface spreads the strain displacement imposed on the bridging reinforcement ligament over a longer gage section generating a larger crack opening displacement per unit of stress supported by the ligament. As a result, the bridging traction/stress supported by the reinforcement increase more slowly with distance behind the crack tip, and a longer bridging zone is developed behind the crack tip. The resultant increase in crack opening displacement with distance behind the crack tip due to interfacial debonding, equations 3-5, significantly enhances the fracture resistance/toughness of the composite.

At this point, this model of the frictional bridging contribution by discontinuous brittle reinforcing ligaments provides a very useful means of designing such composites and analyzing their response. One can, at least, characterize those properties which are most important when selecting materials, and then systematically dissect the toughening response of composites to either uncover problem areas or to develop advanced systems. The bridging ligament model can be further refined by including a pull-out contribution and by addressing the response and contribution of whiskers which are inclined to the crack plane. In fact, the simple crack bridging model describe here and the effects of reinforcement by brittle whiskers have been successfully applied to a variety of oxide (including glasses) and nonoxide matrix ceramics.

Observed Toughening By Crack Bridging Processes

Several types of discontinuous brittle reinforcements have been successfully employed to form toughened ceramics including second phase whiskers [1-6] and platelets [11-13] and both elongated [14-17], plate-like [18] and large [3, 19-22] matrix grains. Studies of cracks in such materials reveal that, within the wake of the crack tip, the reinforcement does bridge the crack. The following sections will describe the observed toughening response in whisker reinforced ceramics, ceramics with both elongated grains and larger grains, and when such bridging processes are combined in a composite.

Crack Bridging by Brittle Whiskers

The experimental fracture toughness results obtained to date confirm the various features of the model for crack bridging by these discontinuous brittle reinforcements [2] as shown in Figure 4 which compares experimental data with predicted curves based on equation 8. These results are based on a specific SiC whisker of a given strength and diameter. Thus Figure 4 reveals several features. First that the whisker bridging toughening contribution, $\Delta K^{wr} = \Delta K^{flb}$, does increase with volume/areal content of the reinforcing phase as predicted. Second, the toughening contribution also increases as the ratio of the composite's Young's modulus to that of the whisker increases. This best illustrated by the increase in ΔK^{wr} with increase in E^c at a given whisker content. For the examples here, E^c values were obtained by rule of mixtures [$E^c = E^m(1-V_f) + E^w V_f$]; thus at a constant volume fraction of whiskers, E^c increases in the order from glass (E^m = 80 GPa) to mullite (E^m = 210 GPa) to alumina (E^m = 400 GPa) vs SiC (E^w = 500 GPa).

These same experimental observations [2] also show that the whisker bridging toughening contribution, ΔK^{flb}, increases as (r, the whisker radius)$^{1/2}$ increases as predicted by equation 8. For example, the toughness of alumina composites containing 20 vol % SiC whiskers, increased from ≈6.5 to ≈9 to ≈12 MPa $\sqrt{m}$ when the mean diameter of the SiC whiskers increased from 0.4 to 0.75 to 1-1.5 microns, respectively. From the toughening model, we also expect the toughness to increase as the matrix-whisker interface fracture energy (strength) decreases with respect to that of the whisker (γ^w substituted for γ^f). While, values of the ratio of the whisker to interface fracture energy (γ^w / γ^i) are not available there are two observations which support the predicted behavior. First, whisker-matrix interfacial debonding and crack bridging by the whiskers are only observed in the composites exhibiting significant toughening. Second, the length of the whiskers protruding above the fracture surface increases with increased toughening

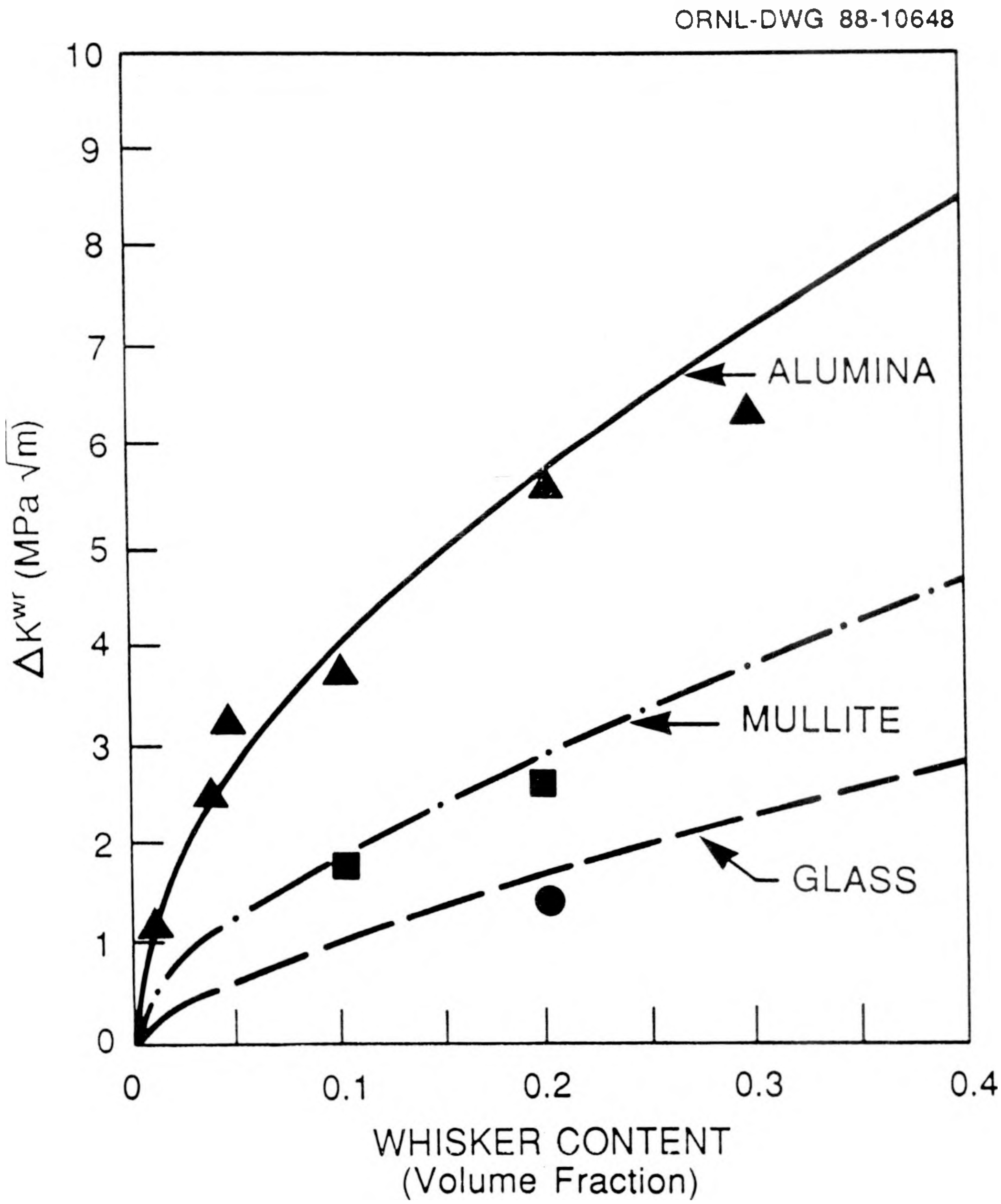


Figure 4. The Fracture Toughness Contribution from Whisker Bridging Increases with Increase in SiC Whisker Content. Curves represent predicted behavior(equation 8) for strong (10 GPa), 0.8 μm diameter SiC whisker while symbols represent experimental results.[5]

and this length can be related to the interfacial debond length. These findings indicate that the bridging contribution does indeed increase with increasing whisker diameter and when the fracture energy (and) of the interface decreases with respect to that of the whisker.

Matrix Grain Bridging: Grain Shape Effects

Crack bridging phenomena and toughening effects which are very comparable to those observed in whisker reinforced ceramics are also found in ceramics containing other reinforcing phase geometries. For example in the development of more thermal shock resistant electrical insulator, alumina ceramics which had microstructures which contain large (~100-200 μm across by ~10 μm thick) plate-like alumina grains in a medium sized (~5 micron) equiaxed grained matrix. These materials had excellent thermal shock resistance; in fact, their thermal shock resistance was much greater than any of the variety of ceramics tested including zirconias, various other oxides, silicon nitrides, and aluminas with equiaxed grains. Further examination showed that fracture toughness values were ~7 MPa √m for samples containing ~ 25 vol % of these large single crystal alumina plates [18]. Aluminas prepared at the same time but with only equiaxed grains which were~ 5μm in size had toughness values of only 4-4.5 MPa √m. Observations of the crack paths in the alumina containing the plate-like grains revealed that cracks deflected along the interface between the matrix and the large plate-like grains. This produced plates which bridged the main crack and contributed to the high toughness in much the same manner as SiC whiskers do.

The logical extension of this is to consider whether or not crack bridging by second phase platelets contributes to fracture toughness. Composites consisting of an equiaxed polycrystalline matrix of TiO_2 in which alumina platelets are dispersed also exhibit increased fracture resistance as described by Hori et al. [11]. This work shows that under conditions where the platelet dimensions remained fairly similar that toughness increased with platelet content leading to nearly a three-fold increase at 30 vol % of alumina platelets. Initial studies also reveal that SiC platelets can produce similar increases in toughness in alumina as do SiC whiskers [12]. Each of these composites give evidence for crack bridging by the reinforcement.

In this same vein, reinforcement of Si_3N_4 [14-16] and SiAlON [17] ceramics by the in situ growth of elongated or whisker-like grains is also a potent toughening approach resulting in toughness values of ≥10 MPa √m. Such materials have been labeled as self-reinforced and from the crack observations of Li and Yamanis [15] crack bridging by these grains contributes to the improved toughness. Sufficient additional experimental results exist to begin to test how well the current crack bridging model describes the toughening effects of such elongated grains. First, Tajima et al. results show that the toughening contribution, ΔK^{flb}, increases with increase in volume content of the elongated grains [23].

More recent observations also reveal that ΔK^{flb} increases with increase in the cross section of the elongated grains, Table 1. In fact the authors plotted the data in the form of ΔK^{flb} versus the square root of the diameter of the elongated grains [24]. The resulting plot exhibit excellent fit to the behavior predicted by equation 8. The diverse sources of observations then would support crack bridging by the elongated grains as the toughening process in these silicon nitride ceramics.

Matrix Grain Bridging: Grain Size Effects

In the present discussion, grain size effects on toughness are related to bridging ligaments formed by matrix grains which are left intact behind the crack tip [3, 19, 21, 25]. The toughening analysis is analogous to that for the whisker reinforcement described above. However here the bridging stress supported by ligaments formed by microcracking along grain boundaries is product of the frictional stress required to pull out each bridging grain times the fraction of bridging grains, $f_{gb}\,\tau_{gb}$. The grain bridging zone length is dictated by equating the crack opening displacement at the end of the zone u to that required to completely pull out the bridging grains. Assuming that half the grain must be pull out to disrupt a ligament, u will be equal to one half the grain size (d), and the incremental increase in fracture toughness due to grain bridging ΔK^{gb} is:

$$\Delta K^{gb} = [f_{gb}\, \tau_{gb}\, E^{c}\, (d/2)]^{1/2} \qquad 9$$

yielding a grain bridging toughening contribution consistent with experimental observations [3, 18, 21] at grain sizes below those resulting in spontaneous microcracking [18]. As noted in Figure 5, the grain size dependence of the fracture toughness of alumina ceramics is consistent with this behavior.

Table 1. Fracture Toughness of Silicon Nitride Ceramics With Elongated Grain Structures.+

Diameter of Elongated Grains, μm	Fracture Toughness, MPa √m
2.8	5.7
3.5	6.4
4.5	7.0
7	8.3
8.7	9.0
10-11	10-11

+ Data taken from results of H. Okamoto and T. Kawashima, NKK Corporation, Kawasaki, Japan.

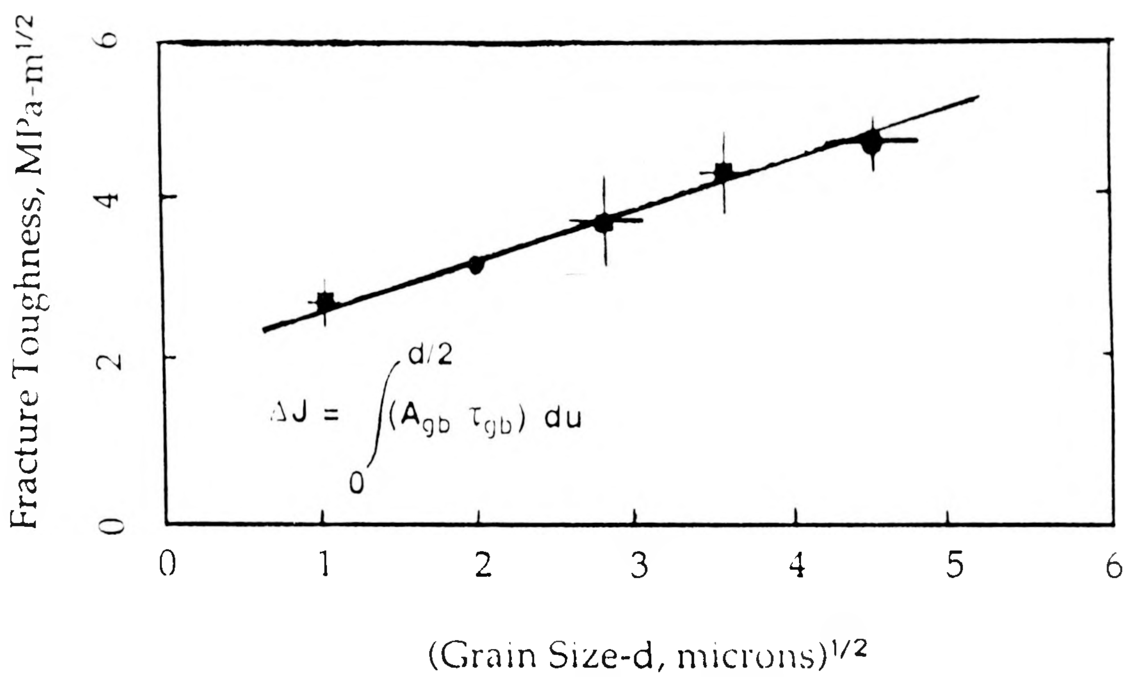


Figure 5. The fracture toughness of alumina ceramics is enhanced by increase in the matrix grain size.

Conclusions

Reinforced ceramics including reinforcement by strong whiskers initiate crack bridging processes to achieve improved fracture resistance. Similar toughening processes and effects are achieved by changes in grain size in noncubic ceramics, and/or by altering grain shape, e.g., formation of elongated grains in Si_3N_4 and SiAlON and plate-like grains in alumina ceramics. These reinforcing phases can contribute considerable toughening to brittle ceramics; factors of three increases in the fracture toughness are not uncommon.

The bridging processes involve frictional bridging where the matrix-reinforcement interface debonds which allows the reinforcement to elastically stretch over some finite gage length hindered only by frictional sliding against the matrix. The contribution of pull-out of these reinforcements to the toughness is rather limited; in part, due to their limited pull-out dimension. Enhanced interfacial debonding leads to greater toughening effects in these systems by promoting the crack opening displacement supported by the bridging zone. The amount of toughness realized is dependent upon the properties and characteristics of the reinforcing phase and the interface as described by the micromechanics models developed for these systems. The model for frictional crack bridging reveals that the bridging contribution to the toughness is a function of the whisker strength, diameter, and content, as well as the ratio of the whisker to interface fracture resistance, and the ratio of the composite to whisker Young's moduli. The predicted effect of these parameters are supported by experimental observations for SiC whisker reinforced ceramics.

Extension of the micromechanics model of toughening by crack bridging reinforcements illustrate the importance of considering other reinforcements including second phases and changes in matrix microstructure. Experimental results confirm various aspects of the toughening response due to crack bridging resulting from grain size and grain shape changes in alumina and silicon nitride ceramics. These finding suggest a variety of approaches may be possible to obtain improved fracture toughness in ceramic and other brittle systems by incorporating reinforcing phases which can generate crack bridging mechanisms. Such processes can be combined with each other or with other toughening mechanisms to develop synergistic toughening effects. The approach described here offers a means of developing these materials by considering the material characteristics/parameters which control the crack bridging contribution.

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REFERENCES

1. P. F. Becher and G. C. Wei, "Toughening Behavior in SiC-Whisker Reinforced Alumina," J. Am. Ceram. Soc. 67(12) C267-69 (1984).
2. P. F. Becher, C. H. Hsueh, P. Angelini, and T. N. Tiegs, "Toughening Behavior in Whisker Reinforced Ceramic Matrix Composites," J. Am. Ceram. Soc. 71(12) 1050-61 (1988).
3. P. F. Becher, "Recent Advances in Whisker Reinforced Ceramics," in Annual Review of Materials Science, R. A. Huggins (editor), in press.
4. N. Claussen and G. Petzow, pp. 649-62, Tailoring of Multiphase and Composite Ceramics, Vol. 20, Materials Science Research, R. E. Tressler, G. L. Messing, C. G. Pantano, R. E. Newnham (eds.), Plenum, New York 1986.
5. P. F. Becher, T. N. Tiegs, and P. Angelini, "Whisker Toughened Ceramic Composites," in Fiber Reinforced Ceramics, K. S. Mazdiyasni (editor), Noyes Publications, Park Ridge, N. J., in press.
6. R. Warren, and V. K. Sarin, "Fracture of Whisker Reinforced Ceramics," in Applications of Fracture Mechanics to Composite Materials, K. Friedrich (ed.), Elsevier, Amsterdam, in press.
7. J. R. Rice, "Mathematical Analysis in the Mechanics of Fracture," pp 191-311 in Fracture, Vol. II, H. Liebowitz (editor), Academic Press, New York, 1968.

8. B. Budiansky, J. W. Hutchinson, and A. G. Evans, "Matrix Fracture in Fiber-Reinforced Ceramics," J. Mech. Phys. Solids 34(2) 167-89 (1986).
9. P. Angelini, W. Mader, and P. F. Becher, "Strain and Fracture in Whisker Reinforced Ceramics," pp. 241-57 in Mater. Res. Soc. Symp. Proc., Vol. 78, P. F. Becher, M. V. Swain, and S. Somiya (editors), Materials Research Society, Pittsburg, PA, 1987.
10. T. N. Tiegs, "Tailoring of Properties of SiC Whisker-Oxide Matrix Composites," pp. 937-49 in Ceramic Materials and Components for Engines, V. J. Tennery (editor), Am. Ceram. Soc., Westerville, OH, 1989.
11. S. Hori, H. Kaji, M. Yoshimura, and S. Somiya, "Deflection-Toughened Corundum-Rutile Composites," pp. 283-88 in ref. 8.
12. P. F. Becher, "Microstructural Design of Toughened Ceramics," presented at the Am. Ceram. Soc. annual meeting, Dallas, Tx, April, 1990, to be published.
13. F. J. Cambier, "Processing and Properties of SiC-Platelet Silicon Nitride Ceramics," Proc. Int'l Workshop for Fine Ceramics: New Processing and Properties of Fine Ceramics," Nagoya, Japan, March 15-16, 1990 (to be published).
14. Lange, F. F., "Fracture Toughness of Si3N4 as a Function of the Initial a-Phase Content," J. Am. Ceram. Soc. 62 (7-8) 428-30 (1979).
15. C. W. Li and J. Yamanis, "Super-Tough Silicon Nitride with R-Curve Behavior," Ceram. Sci. and Engr Proc. (1989).
16. G. Himsolt, H. Knoch, H. Huebner, and F. W. Kleinlein, "Mechanical Properties of Hot-Pressed Silicon Nitride with Different Grain Structures," J. Am. Ceram. Soc. 62 (1) 29-32 (1979).
17. M. H. Lewis, pp. 713-30 in Fracture Mechanics of Ceramics, R. C. Bradt, A. G. Evans, D. P. H. Hasselman, and F. F. Lange (eds.), Plenum, New York, 1986.
18. T. N. Tiegs and P. F. Becher, "Particulate and Whisker Toughened Alumina Composites," pp. 479-85 in Proc. 22nd Auto. Tech. Dev. Contractors' Coord. Meeting, Vol. P-155, Society of Automotive Engineers, Warrendale, Pa., March, 1985.
19. R. W. Rice, S. W. Freiman, and P. F. Becher, "Grain-Size Dependence of Fracture Energy in Ceramics: I, Experiment," J. Am. Ceram. Soc. 64 (6) 345-50 (1981).

20. a. P. L. Swanson, C. J. Fairbanks, B. R. Lawn, and Y. W. Mai, J., "Crack-Interface Grain Bridging as a Fracture Resistance Mechanisms in Ceramics: I, Experimental Study of Alumina," J. Am. Ceram. Soc. 70 (4) 279-89 (1987).
b. Y. W. Mai and B. R. Lawn, "II, Theoretical Fracture Mechanics Model," ibid. 70 (4) 289-94 (1987).
21. a. B. Mussler, M. V. Swain and N. Claussen, "Dependence of Fracture Toughness of Alumina on Grain Size and Test Technique," J. Am. Ceram. Soc. 65(11) 566-72 (1982).
22. P. F. Becher, E. R. Fuller, Jr., and P. Angelini, "Matrix Grain Bridging in Whisker Reinforced Ceramics," to be published.
23. Y. Tajima, K. Urashima, M. Watanabe, and Y. Matsuo, "Fracture Toughness and Microstructure Evaluation of Silicon Nitride Ceramics," pp. 1034-41 in Ceramic Transactions, Vol. 1: Ceramic Powder Science-IIB, G. L. Messing, E. R. Fuller, Jr., and H. Hausner (editors), Am. Ceram. Soc., Westerville, OH, 1988.
24. H. Okamoto and T. Kawashima, unpublished results.
25. C. Cm. Wu, S. W. Freiman, R. W. Rice, and J. J. Mecholsky, "Microstructural Aspects of Crack Propagation in Ceramics," J. Mater. Sci. 13:2659-70 (1978).