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Rare Events—A State of the Art

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RARE EVENTS - A STATE OF THE ART

V. R. R. Uppuluri

ABSTRACT

The study of rare events has become increasingly important in the context of nuclear safety. In section 1 of this paper, some philosophical considerations, such as, (1) the framework for the definition of a rare event, (2) rare events and science, (3) rare events and trans-science, and (4) rare events and public perception are discussed. In section 3, the technical work of the Task Force on Problems of Rare Events in the Reliability Analysis of Nuclear Plants (1976-1978), sponsored by OECD is reviewed. Some recent technical considerations are discussed in section 4, and Conclusions are presented in section 5. The appendix contains an essay written by Anne E. Beachey, under the title: A Study of Rare Events - Problems and Promises.

1. Introduction

There are several questions that come to mind when one thinks about rare events. (1) What is a rare event? (2) If one wishes to define a rare event as an event with low probability, then the question is, what is the associated probability space? (3) Can experiments be performed in laboratories, to study rare events? (4) Are questions concerning rare events, trans-scientific questions? (5) Does the public perception of rare events change, after a rare events is observed? We will discuss these questions in section 2.

In section 3, we review some of the work of the Task Force on Rare Events, appointed by the Committee on Safety of Nuclear Installations (CSNI), Office of the Economic Cooperation and Development (OECD), and performed during 1976-1978. Only highlights of some of the technical contributions are reviewed.

In section 4, we review some of the recent technical work performed in the area of rare events. In section 4.1, we discuss the axiomatic approaches useful in the study of rare events. In section 4.2, we discuss a basic theorem about the superposition of a large number of rare processes. In the remaining section, we discuss about inference problems, use of Bayesian methods, and the current research in the area of risk assessment and rare events. Section 5, presents the conclusions reached in this paper.

2. Philosophical Considerations

2.1 Events of Low Probabilities

Events whose probabilities are very small are generally considered to be rare events. A student of probability normally thinks of a probability space defined by the triplet (Ω, F, P), where Ω is an arbitrary set, F an algebra of subsets of Ω which may be considered to be the set of all possible events that could be observed, and P a probability measure associated with every event that belongs to F. Thus given an experiment with an associated probability space, one can compute the probability of any desired event; though at times this may turn out to be a difficult computational problem. One may call events whose associated probabilities are less than a given number ε (such as 10^{-6}) as rare events. After imbedding the problem in an acceptable probability space, one can decide whether an event is a rare event or not. The association of a probability space with the experiment under consideration may not be an easy problem.

Suppose a pack of 52 playing cards is shuffled well, and a bridge hand of 13 cards is dealt. There are exactly h = 635,013,559,600 different hands that can appear; therefore the probability of any specified set of 13 cards appears in a hand is equal to $1/h$, which is a very very small number, and the event may be considered as an improbable event. But everytime a hand is dealt, one of the h possiblities is absolutely certain to occur. Warren Weaver [21] discusses this example and suggests that smallness of probabilities is not enough to characterize rare events.

2.2 Rare Events and Science

It is not clear whether one can perform experiments in a laboratory, on rare events. In the Presidential Lecture at the AAAS annual meeting in 1980, Boulding [3] says that a field of knowledge is likely to be insecure if the available data only covers a small part of the field and if the actual structures and relationships in it are extremely complex. Two examples of insecure fields are, (i) knowledge of human behavior and (ii) cosmology. Further, Boulding says that any field of knowledge which also deals with rare events is also likely to be insecure. Improbable events in a small field cannot be studied in laboratories; only repeatable events can be studied in laboratories, and that is why, the field of experimental sciences is secure. But as in the case of evolutionary processes, rare events in the unfamiliar part of the field are of import in explaining the overall pattern of time, and where the usual scientific studies are not of help. In such cases one has to resort to theoretical approaches. At times, rare and sudden changes, such as "mutations" are of great interest to scientists; see Muller [9].

2.3 Rare Events and Trans-Science

Weinberg [22], goes a little further and says that several problems involving rare events are trans-scientific issues. A question which transcends the proficiency of science is called a trans-scientific question. He cites the following two examples, which involve rare events, that fall in the category of trans-scientific questions. These are, (i) the effect of extremely low levels of insult on the biosphere, and (ii) the probability of catastrophic events that have never occured in reactor accidents. Weinberg also refers to the behavior of an individual in a specific situation to be "rare", in the sense that each individual's action is unique, and it is usually influenced by seemingly chance mechanisms. He also points out that in some cases the prediction of rare events transcends the proficiency of science not in principle but as a practical matter, because of the prohibitive cost to get an answer or because of the lack of advances in scientific progress. Thus to get statistics on catastrophic reactor accidents one has to build more reactors or wait for a long time; to observe a genetic effect at extremely low doses, one would require billions of mice. In the light of this trans-scientific arena, how can rare events be tackled?

2.4 Rare Events and Public Perceptions

While studying the safety of fussion reactors, Lewis [7] says that the very business of quantitative risk assessment is to calculate the probability of an accident. He says, "No one will calculate a probability if he believes it is zero. What is at stake here is the widespread misunderstanding of the probability of infrequent events, a misunderstanding that is by no means confined to the nonscientific community." He also says, " People have a tendency to think that anything that actually occurs cannot have had a small probability of occurrence, because their view of the world is inevitably influenced by those things that do occur." Thus the public perceptions of rare events, if ever they happen, complicates further the study of rare events.

3. CSNI Task Force on Rare Events

3.1 Scope of the Task Force

The Organization for Economic Cooperation and Development (OECD) has a Nuclear Safety Division, which has a Committee on the Safety of Nuclear Installations (CSNI). This committee appointed a Task Force on problems of rare events in the reliability analysis of nuclear plants (1976-1978). The task force addressed the following questions: (1) What are rare events? (2) What events should be treated like rare events? (3) What are the correct methods for analyzing rare events from a statistical viewpoint, and (4) Methods for handling problems in reliability analysis which involve rare events?

The Task Force reported its findings [11] in the following areas:

(i) rare event data collection and analysis,

(ii) common mode failure analysis,

(iii) human factor analysis and quantification,

(iv) decision theories and statistics applicable to rare events, and

(v) interdisciplinary communications and tutorial programmes.

3.2 Statistically Rare Events

In this section, we briefly summarize the ideas of Vesely [20] in the context of the reactor safety study, and the ideas of Bastl [10] in the context of systems analysis.

According to Vesely [20], in the reactor safety study (or WASH-1400, 1975), two types of rare events were specifically handled, the probabilistically rare events and statistically rare events. A probabilistically rare event is an event which has a frequency of occurrence per interval of time which is smaller than some criterion, eg., smaller than 10^{-6} per reactor year. A statistically rare event is an event which has a small frequency of occurrence, not with regard to time, but with regard to the total possible data sample which could be collected for that problem.

In the reactor safety study, four techniques were used to handle statistically rare events:

(1) aggregating data samples

(2) discretizing continuous events

(3) extrapolating from minor to catastrophic severeties, and

(4) decomposing events using event trees and fault trees.

The details of these techniques and problems for further work may be found in the paper by Vesely [20].

Bastl [10] suggests that in the context of system analysis one has to take into account two key events: (i) the initiating event (failure of the operational system) and, (ii) the failure event (failure of the protective system to operate on demand). He suggests that probabilities of source events and initiating events are needed to compute the probability of the failure of a system.

3.3 Extreme Value Theory and Stochastic Processes

The distributions of the maximum and minimum of independent identically distributed random variables were suggested as possible tools in the context of realiability problems by Tiago de Oliveira [10]. It is also well known that the maximum of independent identically distributed random variables, properly normalized converges either to the Gumbel distribution or to the Fréchet distribution or to the Weibull distribution, as the sample size goes to infinity. This asymptotic result helps one to study only few distributions from the view point of applications.

More generally, the maximum value of a stochastic process in a given time has obvious relevance to the study of catastrophes. The distribution of the maximum can be directly related to failure probability. The problem is to evaluate or approximate this distribution under realistic assumptions. A quantity closely related to the maximum is the number of times the stochastic process crosses a given level x, during time t. The statistical properties of maxima and the level crossings are discussed by Leadbetter [10].

The theory of Point Processes is an useful approach to study the power plant operation in many situations. A point process is simply a series of events occurring in time according to some statistical law. If a point process has intensity λ, the quantity λt may be interpreted as the probability of an event in time t, if t is small. When it is said that the failure rate is equal to, $\lambda = 10^{-4}$/year, the implication is that either the times between failures are exponential with this parameter, or more generally these rates represent the expected number of events per unit time in a point process, i.e., its intensity.

3.4 Decision Theory Applied to Rare Events

Morlat [8] proposes the following characteristics for a rare event:

(1) A rare event has to be an event.

(2) An event is a fact or a set of facts whose probability can be modified by observations or new knowledge, but not by the choice of a decision.

(3) The quality of an event depends on the decision one has to take.

(4) One is only concerned with rare events which have catastrophic consequences.

(5) The degree of scarcity of an event is important.

Further, Morlat [8] gives the following suggestions about the appropriateness of an applicable theory, for a given size of observations:

Size of observations	Adequate Theory
None	Decisions under uncertainly
Rare	Bayesian Methods
Moderate	Inductive Statistics
Many	Data Analysis

In conclusion, if one has a rare amount of data, Morlat [8] suggests to use Bayesian Methods. This seems to be one of the conclusions that is accepted by the Task Force.

3.5 Miscellaneous Observations

According to Freudenthal [10] there are three basic types of rare events:

(1) Those arising from combinations of simple, not necessarily rare, events of reliably observable recurrence periods.

(2) Those that are themselves simple events, presumably of recurrence periods far beyond any practical range of observation, and therefore predictable only by circumstantial evidence or by combined physical-probabilistic modeling.

(3) Those simple events the prediction of which can be based on extrapolation from extensive, systematic records of their past occurrences.

Freudenthal, says that the interaction between the quality of the knowledge of the performance parameters and the modelling of their distributions represents a serious problem in reliability analysis to which insufficient attention has so far been paid.

To demonstrate communication techniques and their application, the Task Force prepared an audiovisual package of slides in order to highlight various aspects of the rare events problem (see Carnino, Royen and Stephens [4]).

4. Some Recent Technical Considerations

4.1 Axiomatic Approaches

One way to develop the mathematical foundations of rare events is through the axiomatic approach. At times, the Poisson process is referred to as the phenomenon associated with rare events. The Poisson process can be shown to be the only process which is stationary with independent increments, and where the occurrence of more than one event in a small interval of time is impossible. In 1950, Jansossy, Renyi and Aczel [5], introduced an axiom of rarity and showed how the Poisson process can be obtained as the solution of a functional equation.

Let N(t) denote the number of events observed during the interval [0,t], and let $P_k(t) = P[N(t) = k]$. According to Janossy, Renyi and Aczel, the events said to be rare, whenever

$$\underset{t \to 0}{\text{line}} \quad \frac{P_1(t)}{1 - p_0(t)} = 1.$$

This condition can be generalized in several ways. If E[N(t)] denotes the average number of events in the interval [0, t], one may define a condition of rarity by defining

$$\lim_{t \to 0} E[N(t) \mid N(t) \geq 1] = 1,$$

where $E[N(t) \mid N(t) \geq 1]$ denotes the conditional expectation of N(t) given $N(t) \geq 1$. One may say that the process has "unprecedented events", whenever the interval between the occurrence of two events has an infinite first moment. The characterization of processes with unprecedented events, and generalized definitions of rarity, seems to be an open problem. Some of these problems, and the contributions of Kotlarski and Leipnik are given in the paper by Uppuluri [18]. A report by Uppuluri and Chernick [19], giving a review of different axiomatic approaches leading to compound Poisson processes is in preparation.

4.2 Theorem of Grigelionis

The Central limit theorem is one of the basic results in the theory of probability. In essence this theorem says that the sum of a large number of independent random variables, properly normalized, behaves like a normal variable. This is an extremely useful result for statistical applications.

Similarly there is a corresponding result for the superposition of rarely occurring discrete phenomena. This result is due to Grigelionis. In essence this theorem says that the superposition of a large number of rare processes, leads to a Poisson process. Details of this result may be found in the report by Thompson [16]. The corresponding generalization to a compound Poisson process is an open problem.

4.3 Inferences about Rare Events

Amongst stochastic processes, the so called Bernoulli process is a simple process. In a discrete time situation, at any instant of time, in a Bernoulli process, the system is either on or off. Suppose that p is the probability that the system is on, and $1-p$ is the probability that the system is off. In the case of a rare event process, the system will be observed to be working most of the time. Given that we observed that the system did not fail in n consecutive units of time, the problem is to give confidence limits on p. This problem can be solved if we consider the conjugate problem of the inter arrival time between events, and use the properties of this conjugate variable. Details of the solution of this problem may be found in the report by Uppuluri and Patil [17]. The idea of using the properties of the conjugate random variable, characterizing the distribution of the inter arrival time between events, is helpful in the context of rare event phenomena.

4.4 Use of Bayesian Methods

In 1978, Apostolakis and Mosleh [1], studied risk assessment problems, in the presence of rare events, and the lack of available data. They use the subjectivistic interpretation of probabilities, axioms of coherence, Bayesian methods, and expert opinions to evaluate the probabilities of rare events.

4.5 Risk Assessment and Rare Events

In a July 1980 report, Sampson and Smith [13] consider the problem in risk assessment of evaluating the probability of occurrence of rare but potentially catastrophic events. In order to find the likelihood of one or more such catastrophic events to occur, the authors provide an information theoretic model for merging a decision maker's opinion with expert judgment. There is also a methodology provided for the reconciliation of conflicting expert judgments. It was shown that this merging approach is invariant to the decisions maker's viewpoint in the limiting case of exceptionally rare events. These methods were applied to case studies in likelihood assessment of Liquid Natural Gas Tanker Spills and seismic induced light water nuclear reactor meltdowns.

5. Conclusions

There are several difficulties associated with problems involving rare events. If one wants to study rare events in the context of probability theory, it is not too easy to find the appropriate probability space. Any field of endeavor, which involves rare events, and where experiments cannot be performed in laboratories is bound to be insecure. Some of the questions involving rare events are trans-scientific questions. The public perception of rare events, such as the eruption of Mount St. Helens or the accident at Three Mile Island, seems to be at odds with principles of logic.

There is a need for a better understanding of various aspects of rare events. This may be accomplished by using audio-visual techniques, where rare event phenomena are discussed. The use of Bayesian methods seems to be a viable approach in some situations of systems analysis.

In the areas of research, axiomatic approaches leading to functions appropriate to the study of rare events should be explored. This will lead to a better understanding of the mechanisms which cause rare events. At present, the tools of Point Processes seem to lead to the best available methods to study rare events. There is some good work going on in bringing together subjective theory of probability and expert opinion, in risk assessment of technological systems where the rare events are the main source for catastrophies. New methods involving information theory and the merging of opinions, are also under consideration for the problem of rare events in risk assessment.

We hope that this report will contribute to a little more understanding of rare events, which in turn may help avoid catastrophies.

REFERENCES

1. Apostolakis, G., and Mosleh, A. (1978), "On the Use of Expert Opinion in the Evaluation of Probabilities of Rare Events", UCLA - School of Engineering and Applied Science Report # UCLA-34P252-01.

2. Boulding, K. E. (1976), "The Importance of Improbable Events", Technology Review, (MIT), 79, Number 4, 5-6.

3. Boulding, K. E. (1980), "Science: Our Common Heritage", Science, Vol. 207, 831-836.

4. Carnino, A., Royen, J. and Stephens, M. (1978), "The Problems of Rare Events in the Reliability Analysis of Nuclear Power Plants", Proc. of the Topical Meeting on Probabilistic Analysis of Nuclear Reactor Safety (May 8-10, 1978 New Port Beach, Calif.), Am. Nuclear Society ISBN: 0-89448-101-0, pp VIII. 5.1 to 5.24.

5. Janossy, L., Renyi, A. and Aczel, J. (1950), "On Composed Poisson Distribution I", Acta Mathematica Academia Scientiarum Hungaricae 2, 83-98.

6. Keilson, J. (1979), "Markov Chain Models - Rarity and Exponentiality", Springer - Verlag New York (Vol. 28 Applied Mathematics Series).

7. Lewis, H. W. (1980), "The Safety of Fission Reactors", Scientific American, Vol. 242, Number 3, pp. 53-65.

8. Morlat, G. (1976), "On Decision Theory Applied to Rare Events", Proc. Meeting of a Task Force on Problems of Rare Events, JRC Ispra Meeting, OECD Nuclear Energy Agency, Paris, France 158/1 to 6.

9. Muller, H. J. (1958), "Evolution by Mutation". Bull. Am. Math. Soc. 64, 137-160.

10. OECD Nuclear Energy Agency (1976). Proceedings of a Meeting of a Task Force on Problems of Rare Events in the Reliability Analysis of Nuclear Power Plants, June 8-10, 1976. CSNI Report No. 10, OECD, Paris.

11. OECD Nuclear Energy Agency (1978), "Final Report of the CSNI Group of Experts on Statistics and Decision Theories Applicable to Rare Events. SINDOC (78) 85, OECD, Paris.

12. Okrent, D. (1975), "A Survey of Expert Opinion on Low Probability Earthquakes". Annals of Nuclear Energy, 2, 601-614.

13. Sampson, A. R. and Smith, R. L. (1980), "Assessing Risks Through the Determination of Rare Event Probabilities". University of Pittsburgh, Department of Mathematics and Statistics, Technical Report # 80-9.

14. Starr, C. and Whipple, C. (1980), "Risks of Risk Decisions". Science, Vol. 208, 1114-1119.

15. Starr, C., Rudman, R. and Whipple, C. (1976), "Philosophical Basis for Risk Analysis", Annual Review of Energy, 1, 629-662.

16. Thompson, Jr. W. A. (1979), "Observations on Risk Analysis", Oak Ridge National Laboratory Report No. ORNL/CSD-45 (Nov. 1979).

17. Uppuluri, V. R. R. and Patil, S. A. (1976), "Inferences About Rare Events", Oak Ridge National Laboratory Report Number ORNL/CSD-12 (June 1976).

18. Uppuluri, V. R. R. (1979), "Risk Analysis and Reliability". Proceedings 1979 DOE Statistical Symposium, held at Gatlinburg, Tenn., (October 24-26, 1979). pp. 187-191, CONF-791016, published September 1980.

19. Uppuluri, V. R. R. and Chernick, M. (1980), "Axiomatic Approaches to Poisson Processes - A Review". ORNL/CSD report (in preparation).

20. Veseley, W. E. (1976), "Rare Event Techniques Applied in the Rasmussen Study". Proc. Second ERDA Statistical Symposium, held at Oak Ridge (Oct. 25-27, 1976), LA-6758 C. pp. 17-36.

21. Weaver, Warren (1963), "Lady Luck", Anchor Books, Doubleday & Co., New York, NY.

22. Weinberg, A. M. (1976), "Trans-Science and the Limits of Science". Lecture Presented at the Lustrum Symposium of the Twente University of Technology, Enchede, The Netherlands (Nov. 29, 1976).

APPENDIX

A Study of Rare Events: Problems and Promises*

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I. Introduction

In recent years, the study of rare events has become increasingly important in the context of nuclear safety. Why should rare events be considered so significant? Why be so concerned about events which may never even occur within our lifetime? These are only two of the many questions that have been raised in trying to understand and analyze rare events. This discussion will attempt to clarify what rare events are, why there is a need to study them, some of the problems involved in their sutdy, some of the methods used thus far to analyze them, and what direction the study of rare events is currently taking in the context of nuclear risk and safety.

II. Definition and Examples of Rare Events

First, what exactly is a rare event? Nearly everyone has an idea of what the word "rare" means--unique or distinctive in one sense, infrequently occurring in another. It is the second of these senses with which we will be concerned for most of this discussion. Thus, a rare event can be logically defined as one which seldom or never occurs; or, in more mathematical terms, one with an extremely low probability of occurrence, usually on the order of 10^{-5} to 10^{-8} or smaller [9].

Using this rather simplistic definition, we can find examples of rare events in many areas. One example, which many can easily understand, involves the probability of one player obtaining all thirteen spades on the deal of a bridge hand. Using the methods of combinatorial analysis, this probability can be calculated as being on the order of 10^{-12}, which is generally agreed to be extremely low. Another example from the same area involves the tossing of a coin and stopping rules. Here, the coin is tossed until a certain number of heads (previously agreed upon) is obtained as a run; at this point, tossing stops. The probability of stopping after a certain number of tosses can also be calculated using methods in probability theory, although calculations become much more difficult than those in the above example. It is found that as the number of tosses increases, the probability of stopping becomes increasingly smaller, on the order of 10^{-6}, and once again we are in the realm of rare events.

Rare events can be observed in other areas as well. For example, in the field of genetics, the birth of identical twins or of a child with a very rare deformity would be considered rare events. The occurrence of certain karyotypes, namely XYY and XXY (superman phenomenon), has also been found to be a rare event [5]. H. J. Muller has further pointed out that, in an evolutionary sense, we and our fellow creatures can all be considered improbable occurrences; since we could have come about only as the result of a particular sequence of mutations over a long period of time. However, Muller goes on to show that because of external factors, our existence may not be as rare as it might at first appear [10].

Going somewhat beyond the definition given earlier, we might also consider a rare event as one which cannot be easily predicted because we have so little information concerning it--one for which the complexity of the system defies examination. For instance, in the area of psychology, the reaction of an individual in a given situation would fit into this category. The human brain is a complex organism, and human actions are often unique and unpredictable. Another example which falls into this category is a natural diaster. The recent eruption of Mount St. Helens was not thought to be a very likely event at the time it occurred. Scientists were not able to agree on a prediction of when it would happen because they were unable to obtain enough information about it. Nevertheless, it did occur and resulted in both loss of life and property damage.

III. Why study rare events?

The definition of a rare event given earlier is not really a satisfying one, because although the previous examples are interesting, there seems to be no real reason to be concerned with calculating their associated probabilities. However, the last example given above, concerning natural disasters, provides a clue to the category of events with which the remainder of this discussion will be concerned. This category includes only those low-probability events which, if and when they do occur, have potentially serious, even catastrophic, consequences. Such events are the focus of what Talbot Page has aptly termed "zero-infinity dilemmas", where zero refers to the extremely low probabilities involved, and infinity refers to the extremely great consequences [12]. This narrower meaning of a rare event differs from the sense in which one might normally think of many rare things. A rare bird or flower, for example, is something one might have a desire to observe; on the other hand, a rare event, in the above sense, like a rare, deadly disease, is something one would hope never to come in contact with during the course of his life. George Morlat made an interesting observation when he wrote that the fact that an event has catastrophic consequences necessarily implies that it is rare because "otherwise we should not even have conceived to come into a situation which makes it possible to happen [7]. For instance, in the nuclear field, a serious core meltdown would have to be rare or nuclear power plants would not even be allowed to exist. One of the primary reasons for the study of rare events now becomes clear. Perhaps if we could understand and analyze them, we could take preventive measures which might cause such events to be even more rare than we already believe them to be--maybe even to the point of nonexistence, whatever we may decide that to be.

Returning now the eruption of Mount St. Helens, it is obvious that this falls into the above category, even thought the consequences cannot be necessarily considered catastrophic. However, this is not the area where concern lies at present. An event which is presently considered much more serious by the general public is that which was briefly mentioned above--nuclear reactor failure, such as the well-publicized accident at Three-Mile Island. This being the case, from this point on in the discussion, we will consider rare event to be synonymous with nuclear accident. At this point, a question comes to mind. Why do people tend to have different attitudes toward the risk concerned with natural disaster and the risk involved in nuclear reactor failure? The potential harm is great in each case, yet society tends to view nuclear reactor failure as the more serious situation of the two. Starr and Whipple contend that the difference in people's reactions or attitudes is primarily due to the fact that one of the situations above is a risk to which they expose themselves voluntarily, while the other falls into the category of risks to which they discover themselves exposed whether they want to be or not [10]. For instance, many people choose to live on or near the San Andreas fault in California, where the chance of a devastating earthquake within the next ten years has been estimated to be quite high. Certainly, these people must realize the danger they are in, yet they refuse to move because they believe that the risk has been overestimated. Many of these, are people, who have already suffered large losses from previous earthquakes. On the other hand, people have exactly the opposite attitude where nuclear reactor failure is involved. As soon as even one accident occurs, everyone is up in arms about the risks involved if one lives in the vicinity of a nuclear power plant. Another difference can be seen between these two cases. People understand what an earthquake is and what its consequences are because it is already within their experience; since reactor accidents are much more rare, people have very little knowledge about them, and for that reason, fear them. In looking at how the public views nuclear power, it becomes quite obvious that people's attitudes are strongly dependent on how they individually perceive the probability or risk assoicated with the occurrence of an accident. This is not only true in the case of rare events where virtually no data exists, but also in common, everyday situations. For example, more people have a fear of flying than of driving, even though the latter has a much greater accident probability associated with it. What are the reasons for what seems to be somewhat irrational behavior? Starr and Whipple attempt to explain such behavior in several ways, the most important of which involve: (1) The control an individual has over his situation, (2) the conditional probability of survival given that the accident does occur, and (3) the catastrophic nature of the accident [10].

From the above discussion, other important reasons for studying rare events come out. Society demands to know the risk involved; people would like to be reassured about their fears of the unknown. Those involved with the implementation of nuclear energy also want the public to be reassured. A single isolated event such as Three-Mile Island can do a great deal to sway public opinion to the extent that there may be no future whatsoever for nuclear energy. People may accept natural disaster because it is inevitable; there is little that can be done to prevent nature from taking its course. In the case of man-made disaster, however, most people believe that if man can create a disaster, then man can also prevent that disaster.

IV. Problems Encountered in the Study of Rare Events

Now that what rare events are and why they are being studied have been clarified to a certain degree, the discussion will turn to some of the many problems encountered in their study.

One of the first problems that arises involves the treatment of the extremely low probabilities under consideration. Can the frequency of an event that has never or only very seldom occurred be meaningfully defined? When probability is so low and data is nonexistent, the concept of frequency, especially in a statistical context, becomes meaningless. It has often been observed that as probabilities, and hence frequencies, become very low, most people tend to rely almost completely on intuition in an attempt to understand them. This, of course, usually leads to incorrect estimation of the probabilities involved. Many people see the probability 10^{-12} (as in the bridge hand example mentioned earlier) and immediately assume that this is close enough to zero to essentially be zero. This, however, is an incorrect assumption to make. Any event with some probability of occurrence, no matter how small, can occur at any time. It must also be remembered that when we begin to consider time periods far beyond the scope of our lifetime, events which are considered rare at present assume a greater probabilistic significance [3]. An event may be estimated to have a 10^{-5} chance per year of occurring; for any given year this is indeed a rare event. However, considered over a period of 100,00 years, the probability that it will occur at least once has risen to 63%; over a period of 1,000,000 years, the improbable has, as far as we're concerned, become a virtual certainty. The same thing happens as the number of systems, or, to be specific, nuclear power plants, increases. The estimate of the probability of a nuclear accident is made relative to reactor years; when 100 reactors are operating in any one year, this is equivalent to one reactor operating for 100 years, or 100 reactor years. Thus, the probability of any one reactor failing in a given year may be extremely small; but, as in the previous example, as the number of plants increases, so also does the probability of an accident. This important principle has applications in the field concerning the history of this planet, in which a million years corresponds to perhaps a second of the time we are familiar with. In such a time context, it seems that low-probability events are bound to happen [6]. Herein lies the previously stated reason for trying to understand and study rare events. The tendency is to focus too heavily on the very short run, simply because it is of immediate concern and we are apt to be directly affected by any catastrophic event occurring during the short run. In doing this, we tend to ignore whatever consequences there may be for future generations. An event which may be considered rare now may, as we have seen above, no longer be rare in the future, especially considering that different conditions may exist. Certainly our hope is that by studying rare events that do occur, we can learn enough about them to cause them to be virtually nonexistent in the future.

Another problem that has arisen in the study of rare events has to do with the statistical methods used to analyze them. Statistics and rare events? Isn't that a contradiction in terms? Statistics, in its classical sense, tends to imply a large data base; this is no wonder when we look at its dictionary definition: "The mathematics of the collection, organization, and interpretation of numerical data." The problem is that rare events have virtually no data to offer. Also, there is no past experience upon which to base our estimates of present or future probabilities. Where, then, do we turn? Fortunately, there exists an area of statistical methods, namely Bayesian, in which very little information is needed to analyze the event. Bayesian methods have only recently begun gaining general acceptance, however, because of their non-objective, almost non-mathematical character. What they involve and how they have proved useful in the study of rare events will be discussed in a later section.

Now we turn to the final and possibly most important problem to be considered in this discussion. That is, how should the structure of a rare event be approached? And, once the event has occurred, how do we go about determining and analyzing the mechanism which led to it? This mechanism is what is really at the heart of the whole issue of rare events. It is a mechanism which could lead to catastrophe under the proper conditions, yet which is extremely difficult to prevent because of our lack of knowledge about it. Such ignorance might even cause us to unknowingly create the circumstances under which the mechansim might be activated and the occurence of the one rare event would no longer be rare at all. This idea was vividly illustrated in the true account of a train accident that occurred in 1927 [11]. Upon examination, this accident can be seen to fit into the category of rare events with which we are concerned, namely low-probability, serious-consequence events, since the accident resulted in death and injury.

First, why was this accident labelled a rare event? This conclusion was reached from the observation that never before had anyone seen anything like it--not necessarily the accident itself, but rather the particular sequence of events leading to its occurrence. It was literally months before the mechanism behind it was fully discovered, and when it was discovered, it was generally agreed that the accident could have been prevented by a simple modification of the system. Was it negligence on the safety system designer's part that such a small detail was overlooked? If the particular mechanism leading to the event never even occurred to the designer, the answer to this question would have to be no. In such a case, since no such accident had previously occurred in his experience, he would have had no idea that such a thing might occur; thus, he would have had no reason to take steps to prevent it. In fact, in his design of the system, he may have unwittingly created conditions which made the occurrence of the accident not quite so rare as we might have originally supposed.

Another factor which could not have been accounted for beforehand was human error, which played a significant part in this accident. At the precise moment when it was crucial to pull the right lever to switch a train from one track to another, the signal operator unknowingly pulled the one beside it instead. This event at any other time may have been trivial; however, in this case it happened in the split-second in which it mattered a great deal, and perpetuated the sequence of subevents leading to the accident.

It can be seen from this example how difficult it can be to delineate the particular sequence of subevents leading to a rare event. The task becomes many more times difficult as the complexity of the system increases, as in the case of a nuclear reactor. And, even if we do finally discover mechanisms which might lead to the occurrence of a rare event, how are we to know if all possibilities have been exhausted? Based on the mechanism(s) we have discovered, we will be able to make some estimate of the probability. However, there may be several mechanisms which we might have absolutely no conception of until the accident occurs (as the result of some mechansim we have not taken into consideration). Because of this factor, the probability of occurence of such an accident could be grossly underestimated.

V. Methods Used Thus Far in the Study of Rare Events

How has the structure of the mechanism leading to the occurence of a rare event been previously approached? There are several models which have been used in their analysis; the experts still remain undecided as to which is the best approach concerning the evaluation of nuclear risk.

The first model is that which treats a rare event as the conjuction of more frequent subevents. What does this mean? Consider the following example. We wish to evaluate the probability of obtaining ten sixes on ten throws of one die. The rare event, obtaining ten sixes in a row, could be considered as the conjunction of the ten more frequent subevents, namely obtaining one six on a single throw of the die. Hence, the probability of the rare event occurring would be the product of the individual probabilities of each of the subevents. Assuming an unbiased die, this would give the result $(1/6)^{10} = 1.65 \times 10^{-8}$. It can be seen at a glance that the probability of the rare event is much, much smaller than the probability of any one of the subevents.

The conjunctive model was used extensively in the well-known Reactor Safety Study done under the direction of Professor Norman Rasmussen of the Massachusetts Institute of Technology. In this study extensive use was made of fault or event trees in an attempt to find as many sequences of less rare subevents that could ultimately lead to catastrophes. Since in a complex system this becomes extremely painstaking because of the many variables to be taken into consideration, many possibilities were naturally left out--possibilities which were discovered a few years later when the accident at Three-Mile Island occurred. These will be discussed in a later section.

Another model which has been used is that which treats rare events as the disjunction of less frequent subevents. In this situation, probabilities are added rather than multiplied. This might be represented in a system in which all of the individual components are connected to one another in series; each of the components is considered essential to the system in the sense that if any of them fails, the entire system fails. In this case, the probability of failure of any one component might be extremely low, yet the probability of system failure could be much higher, although still small enough to be considered a rare event. According to Tversky and Kahneman (15), whereas the probability associated with a conjunctive event tends to be overestimated, that of a disjunctive event tends to be underestimated. The reason for this is that a certain type of bias exists, namely anchoring, in which people tend to estimate the final probability as being too close to the original probability. Since the probabilities of conjunctive subevents are usually quite high, the probability of the rare event is also judged to be quite high when, in reality, it is quite low. By the same token, probabilities of disjunctive subevents tend to be low; thus, the final probability is also judged to be low when, in reality, it is much higher. In the case of conjunctive events, this bias may tend to be evened out somewhat by the fact mentioned earlier that becomes the final probability of the rare event may be underestimated.

The third model which has been used in trying to understand rare events is that for which the event is considered as the extrapolation of more frequent events. Extrapolation refers to the process of estimating an unknown quantity of extending or projecting information from quantities which are already known. In this situation the theory of extreme values begins to enter the picture. This theory deals with events which are extremely intense realizations of events that may occur quite often, but which may not usually be considered as very significant. For instance, a tornado or hurricane might be thought of as an extreme case of a windy or breezy day. This theory is extremely important for the study of rare events, since classical statistics tends to dismiss extreme cases as deviations from some pre-established standard. Kenneth Boulding [4] has observed that the field concerning rare events is an insecure one since little available data exists. Repeatable experiments dealing with rare events cannot be performed in a laboratory setting; thus, close attention must be paid to a rare event if and when it does occur in order to gain as much knowledge as possible about that small part of the total field. The goal of this theory is to study the distribution of the maximum (or minimum) value of a set of random variables [13]. It can be applied to the study of nuclear safety where either the event of nuclear reactor failure is treated as the rare event, or extreme case, or external "aggressions", such as earthquakes and floods, which threaten the safety of the nuclear power plant are treated as such [2]. The application of this theory is discussed in greater detail by both Bernier [2] and Saporta [13].

Now, returning to an important method mentioned earlier, we will consider Bayesian statistical methods in the analysis of rare events. What do these methods involve? Although Bayesian statistics often means different things to different people, it almost always implies the use of subjective, or personal, probabilities, as opposed to the objective probabilities of classical statistics. What is the distinction here? Objective probabilities are those we calculate according to specific laws of combinatorial analysis and probability theory. The same final answer is obtained regardless of who does the calculation. On the other hand, subjective probability is a measure of an individual's degree of belief about the occurrence of an event. As may be obvious, the final answer will depend on the biases of whoever is doing the evaluation, and, as a result, will differ from individual to individual. The study of rare events almost necessarily implies the use of subjective probabilities, since so little is known about such events to begin with. Perhaps the best we can hope for at present is an estimate of probability that experts feel best reflects the actual situation.

Subjective probabilities are the starting point for the application of Bayesian methods, in which prior probability distributions are taken into account in the calculation of posterior probability distributions. In other words, additional knowledge about an event can change the probability distribution of the occurrence of that event. This is accomplished by using Bayes' theorem, which says that if we formulate an hypothesis H that a rare event will occur, where P(H) is our prior degree of belief in H, and D represents new information in the form of an actual occurrence of the event, then the posterior probability is given by:

$$P(H|D) = \frac{P(H)P(D|H)}{P(H)P(D|H) + P(\overline{H})P(D|\overline{H})}$$

where:

$P(H|D)$ = posterior probability that H is true given D

$P(D|H)$ = probability of D given H is true

$P(D|\overline{H})$ = probability of D given H is false

$P(\overline{H}) = 1 - P(H)$.

One might tend to question the validity of the final result obtained in such an evaluation, since its basis is prone to bias and uncertainty. However, since so little is known about rare events, this seems so far to be one of the best methods available for analyzing those for which some prior probability distribution can be formulated. To see how this might work, an example will be considered.

Let a rare event be produced in accordance with a Poisson process of intensity λ, where, in the case of a reactor accident, λ is on the order of 10^{-6} to 10^{-9} per reactor year. Such a value is arrived at based on factors such as the analysis included in the Rasmussen Report, and from the fact that we have observed several reactor-years without the occurrence of a serious accident. Supposing a certain prior distribution of λ, we can use Bayes' formula to find how the original distribution will be changed when, after N reactor-years, there is only one major accident. It is found that if one accident is observed, even for very large N, the estimated posterior risk is several times greater than the prior risk. The main implication of such a result is that the way in which the risk or probability was evaluated before the occurrence of the accident must be reconsidered [7].

In the case for which there exist no observations of the event upon which to base a prior evaluation of the probability involved, analysis becomes even more difficult. Bayes' theorem can be used in such a case, but it may not provide much information. The theory which is usually applied under these circumstances is the theory of decision under uncertainty. One of the basic hypotheses of this theory is that the decisionmaker's choice cannot affect the occurrence of the event being studied. Thus, in order to use this theory, the decision to employ must be chosen carefully to avoid confusion and comply with the above hypothesis. When the decision is such that it exerts no influence upon the occurrence of the accident, then the probability can be evaluated in some subjective manner, using expert judgment, previous analytical studies, and analogies with observations in related fields. A more detailed treatment of all the applications of this theory can be found in a report by Morlat [7].

VI. Where does the study of rare events go from here?

Certainly the methods discussed above do not exhaust all of the approaches that have been taken up to the present. However, they are some of the most significant. The problem is that none of them seem to be conclusive concerning how rare event probabilities should be evaluated. Morlat has observed that Bayesian methods or decision theory under uncertainty would be best suited to the study of rare events because of the peculiarities of such events. However, the majority of the studies that have been done concerning rare events have concentrated instead on methods which stem from classical statistics. Extreme values theory falls into this category, along with reliability methodology and the use of random processes, two methods which have not been discussed here. Why has the concentration been in these areas rather than those where the study of rare events seems to fit more closely? The first reason is that Bayesian statistics and subjective probabilities have yet to be widely accepted. Most of the present working statisticians learned and are more familiar with classical statistics; they use these methods because they feel comfortable with them. Another reason is that when it is possible to treat rare events as either the conjunction or extrapolation of more frequent events, this is much easier to deal with using methods of classical statistics, since we have moved from a situation with few observations to one with numerous observations. This is appropriate, however, only provided that the assumptions made in going from one situation to the other are valid. In the case that such assumptions cannot be made, there is really nowhere to turn, except in the direction of Bayesian methodology. This indicates that perhaps more effort should be put toward trying to understand and justify the use of such methods, especially since they seem much more applicable [9].

Nevertheless, it may be the case that none of the approaches used thus far is appropriate. We may have yet to develop the proper tools. We cannot even be completely sure that we are looking in the right area of mathematics; we may be in the wrong field altogether! Perhaps, as Alvin Weinberg has observed, the rare event question cannot be answered within the realm of science; it falls into the category of what he terms "trans-science." His proposal for a solution to the problem concerning the risks of nuclear power plants is a realistic one--to refine and improve technology so as to minimize possible effluents of nuclear power plants as best we can. If this is still insufficient, then we must begin looking at ways to cure any diseases or problems caused by radiation residuals [14].

None of all this is to say that no advances have been made in the analysis of rare events in the context of nuclear risk. The occurrence of the accident at Three-Mile Island, although unfortunate, has been quite beneficial as far as the study of nuclear accidents is concerned. Much has been learned from this accident about factors that had not been taken into account before. One such factor is that concerning human error. Although studies of the incident are not complete, improvements have already begun in the area of operations. Previous to the accident, safety concerns had been primarily focused on system performance rather than the performance of those responsible for proper operation of the system. Improvements that have been made are in the areas of personnel selection and qualification, training, licensing, operating environment, man-machine interface, and preparation for the unusual, especially in panic situations. Also, as a result of the accident, the scope of possible future accidents has been greatly increased. Prior to the TMI incident, too much emphasis was being placed on design-based loss-of-coolant accidents, and not enough on other types of possible accidents, such as those involving severe core damage. Among the goals that have been formulated are: significant reduction of the probability of a core-damaging accident, overall risk improvement on the order of 10^{-1} to 10^{-2}, improvement in systems reliability, and reduction of catastrophic accident consequences [19].

The occurrence of the accident at Three-Mile Island is by no means conclusive as far as the issue of rare events is concerned. In fact, additional questions have arisen. Most people had quite a bit of faith in Rasmussen's estimates of the risk of a nuclear accident until the Three-Mile Island incident. Now there is a great deal of doubt about the methods of risk assessment used in that evaluation, as well as doubt about any type of risk assessment that might be carried out in the future. Will we ever be able to completely understand rare events and discover solutions to the many problems associated with them? Possibly not, but it is an issue that cannot be ignored, because of the possible consequences involved. This was perhaps best summed up in the following statement made by experts in the field: Even if we have not found the complete solution to the problem, it is perhaps worth remembering that an event is "rare" when thinking of its estimated low probability in an extended space and time scale. When dealing with the safety of nuclear power plants, events of big importance in terms of consequences are rare precisely due to the fact that our knowledge in design, fabrication, and operation has resulted in good overall technical reliability. But, in terms of risks, in the nuclear field we are always asked to do "better" with respect to probabilities which are already low [18].

REFERENCES

1. Apostolakis, G. (1978). Probability and Risk Assessment: The Subjectivistic Viewpoint and Some Suggestions. Nuclear Safety 19 305-315.
2. Bernier, J. (1978). Bayesian Methods and Theories of Extreme Values in Application to Rare Events. In OECD Task Force Report on Problems of Rare Events: Statistics and Decision Theories Applicable to Rare Events. 123-135.
3. Boulding, K. E. (1976). The Importance of Improbable Events. M.I.T. Review 79 5, 71.
4. Boulding, K. E. (1980). Science: Our Common Heritage. Science 207. 831-836.
5. Boucher, D. H. (1976). Statistics and Rare Phenomena. Science 193 634.
6. Gretener, P. E. (1966). The Role of the Rare Event in Geology. Geophysics 31 1201.
7. Morlat, G. (1978). Decision Theory Applied to Rare Events. OECD Task Force Report on Problems of Rare Events: Statistics and Decision Theories Applicable to Rare Events. 102-108.
8. Morlat, G. (1978). Introduction to the Use of Bayesian Methods for the Study of Rare Events. In OECD Task Force Report on Problems of Rare Events: Statistics and Decision Theories Applicable to Rare Events. 89-101.

9. Morlat, G. (1978). Probabilistic Approach to Rare Events: Some Theoretical Thoughts. In OECD Task Force Report on Problems of Rare Events: Statistics and Decision Theories Applicable to Rare Events. 82-88.

10. Muller, H. J. (1958). Evolution by Mutation. Bulletin of the American Mathematical Society 64 137-160.

11. OECD Nuclear Energy Agency. (1978). Final Report of a Task Force on Problems of Rare Events in the Reliability Analysis of Nuclear Power Plants: Interdisciplinary Communication Techniques and Tutorial Programmes on Rare Event Problems and Their Solution.

12. Page, T. (1979). Keeping Score: An Actuarial Approach to Zero-Infinity Dilemmas. In Energy Risk Management (G. T. Goodman and W. D. Rowe, eds.), 177-186. Academic Press, New York.

13. Saporta, G. (1978). Extreme Value Distributions: A Tool for Studying Some Kinds of Rare Events. In OECD Task Force Report on Problems of Rare Events: Statistics and Decision Theories Applicable to Rare Events. 172-186.

14. Starr, C. and Whipple, C. (1980). Risks of Risk Decisions. Science 208 1114-1119.

15. Tversky, A. and Kahneman, D. (1974). Judgment Under Uncertainty: Heuristics and Biases. Science 185 1124-1131.

16. Weaver, W. (1963). Rare Events, Coincidences, and Surprising Occurrences. In Lady Luck, 278-303. Doubleday and Co., Inc., Garden City, New York.

17. Weinberg. A. M. (1977). The Limits of Science and Trans-Science. Interdisciplinary Science Reviews 2 337-342.

18. Carnino, A., Royen, J. and Stephens, M. (1978). The Problems of Rare Events in the Reliability Analysis of Nuclear Power Plants. On Behalf of the CSNI Task Force on Problems of Rare Events in the Reliability Analysis of Nuclear Power Plants.

19. Mynatt, F. (1980). The Big Surprises at Three-Mile Island--Impacts on Reactor Safety Assessments. ORNL Summer Seminar Series.