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SINGLE PASS COLLIDER MEMO

CN- 234-R1

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TITLE: 1ST ORDER COMPARISON OF NUMERICAL CALCULATION AND TWO DIFFERENT TURTLE INPUT SCHEMES TO REPRESENT A SLC DEFOCUSING MAGNET

Correcting the dispersion function in the SLC north arc it turned out that backleg-windings (BLW) acting horizontally as well as BLW acting vertically have to be used. In the latter case the question arised what is the best representation of a defocusing magnet with excited BLW acting in the vertical plane for the computer code TURTLE /r1/ ?

Two different schemes, the 14.-scheme and the 20.-scheme were studied and the TURTLE output for one ray through such a magnet compared with the numerical solution of the equation of motion; only terms of first order have been taken into account.

Numerical solution

From the expansions for the magnetic field components to first order in the vertical and horizontal plane /r2/

$$B_x(x,y,s) = a_2 y + \dots$$

$$B_y(x,y,s) = a_1 + a_2 x + \dots$$

one finds the field expansions for the rotated (the dipole field by 90° and the quadrupole field by 45°) system

$$B_x(x,y,s) = b_1 + b_2 x + \dots$$

$$B_y(x,y,s) = -b_2 y + \dots$$

where a_1, b_1 represent the dipole and a_2, b_2 the quadrupole components. To first order the expansions for the magnetic field components of a combined function magnet with normal and skew components can now be expressed in the form

$$B_x(x,y,s) = b_1 + b_2 x + a_2 y + \dots$$

$$B_y(x,y,s) = a_1 + a_2 x - b_2 y + \dots$$

Introducing these expansions into the equation of motion to first order in x and y one finds

$$x'' = h(1 + hx) - \frac{a_1}{B\rho(1+\delta)}(1 + 2hx) - \frac{a_2}{B\rho(1+\delta)}\, x + \frac{b_2}{B\rho(1+\delta)}\, y$$

$$y'' = \frac{b_1}{B\rho(1+\delta)}(1 + 2hx) + \frac{b_2}{B\rho(1+\delta)}\, x + \frac{a_2}{B\rho(1+\delta)}\, y$$

where $\delta = \frac{\Delta p}{p_o}$ represents the change in momentum.

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For a defocusing magnet with the parameters /r3/

$$\ell = 2.5462 \text{ m}, \quad h = 3.5186893 \cdot 10^{-3} \text{ m}^{-1}$$

$a_1 = 5.86847$ kG $\qquad b_1 = 0.11474$ kG

$a_2 = -700.510464$ kGm^{-1} $\qquad b_2 = -14.05$ kGm^{-1}

the numerical solution of the equation above by the NAPLUG-routine ODE using 2500 steps gives the following results for the orbit coordinates ($\delta \equiv 0$) and the dispersion functions:

initial condition				values at the magnet exit			
x[μm]	x'[μrad]	y[μm]	y'[μrad]	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	0	0	0	-1.03445	-1.62292	176.78346	105.82378
1	0	0	0	1.66556	0.00259	176.75560	105.80101
100	0	0	0	268.96642	160.92845	173.99651	103.54604
0	1	0	0	2.83520	1.07709	176.76008	105.79591
0	100	0	0	355.93807	208.37795	174.44542	103.03683
0	0	1	0	-1.06232	-1.64570	176.70429	105.17791
0	0	100	0	-3.82156	-3.90090	168.86616	41.23601
0	0	0	1	-1.05733	-1.65079	178.32166	105.74461
0	0	0	100	-3.37263	-4.41004	330.60320	97.9064[illegible]

The dispersion functions at the end of this magnet are estimated using e. g.

$$D_x = \frac{x(\delta)-x(-\delta)}{2\delta} .$$

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.03445	-1.62292	176.78346	105.82378
10^{-3}	13.19136	11.97782	176.59680	105.69570
-10^{-3}	-15.29493	-15.26148	176.97053	105.95219
	D_x [mm]	D'_x [mrad]	D_y [mm]	D'_y [mrad]
	14.24315	13.61965	-0.18687	-0.12825

14. - scheme

A scheme using a combination of 7.0 and 14.0 instructions in TURTLE was developed by S. Kheifets:

7.0	0	0	0	$\Delta y'$	0	0;		
14.0	0	1	$-\psi$	0	0	0	2;	
14.0	$-\psi$	0	0	1	0	$-\Delta y' * 10^{-3}$	4;	
4.0	ℓ/N	5.86347	33924.13774			'DFOC';		(with ℓ = 2.5462 m)
7.0	0	0	0	$\Delta y'$	0	0;		
14.0	0	1	$-\psi$	0	0	0	2;	
14.0	$-\psi$	0	0	0	0	$-\Delta y' * 10^{-3}$	4;	

where ψ and $\Delta y'$ are related to each other by

$$\psi = 1.23 * 10^{-4} \, \Delta y' [\mu rad]$$

When slicing the magnet into N equal parts this series of instructions have to be repeated N-times. The following table shows for the initial conditions $x = y = 0$, $x' = y' = 0$ the convergency of $N * \Delta y'$ versus N when the goal value y' at the end of the magnet was y' = 105.82378 μrad (this value was taken from the numerical calculation).

N	$\Delta y'$	$N * \Delta y'$
2	46.458570	92.917140
4	22.212320	88.840280
6	14.690331	88.141986
8	10.987200	87.897600
10	8.7785030	87.785038
12	7.310335	87.724020

Using 10 slices for the defocusing magnet 1st order TURTLE produces the following results:

initial condition				values at the magnet exit			
x[μm]	x'[μrad]	y[μm]	y'[μrad]	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	0	0	0	-1.91825	-1.62433	176.78323	105.82378
1	0	0	0	1.68176	0.00119	176.75545	105.80081
100	0	0	0	268.98263	160.92735	173.99012	103.52599
0	1	0	0	2.85147	1.07568	176.76009	105.79560
0	100	0	0	385.95404	268.37655	174.45414	103.00505
0	0	1	0	-1.04644	-1.64730	176.70421	105.17791
0	0	100	0	-3.83698	-3.92212	168.86615	41.23643
0	0	0	1	-1.04154	-1.65226	178.32158	105.74461
0	0	0	100	-3.34750	-4.41759	330.60291	97.90655

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.01825	-1.62433	176.78338	105.82378
10^{-3}	13.20652	11.97481	176.59719	105.69457
-10^{-3}	-15.27769	-15.26126	176.96952	105.95294
	D_x[mm]	D_x'[mrad]	D_y[mm]	D_y'[mrad]
	14.24211	13.61804	-0.10617	-0.12919

Comparing these values with the results from the numerical calculation gives a satisfactory agreement for the orbit coordinates as well as the dispersion functions.

20. - scheme

The scheme using the combination of 7.0 and 20.0 instructions in TURTLE was suggested by J. Murray:

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 7.0   0     0        0           Δy'   0    0;
20.0  -φ;
 4.0   ℓ/N   5.86047  33924.13774 'DFUC';         (with ℓ = 3.5462 m)
20.0   φ;
 7.0   0     0        0           Δy'   0    0;
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where the relation between φ and Δy' is given by

$$\phi = 1.27902^{\circ} * 10^{-2}\ \Delta y'[\mu rad] * N/2$$

The number of magnet slices is again N. The convergence behaviour of N*Δy' versus N for this scheme is demonstrated in the table below for the same initial conditions and the same goal value for y' at the end of the magnet as they were noted for the 14. - scheme.

N	Δy'	N * Δy'
2	46.451637	92.903274
4	22.209767	88.839068
6	14.688719	88.132314
8	10.986013	87.888104
10	8.7775622	87.775622
12	7.3095553	87.7146636

In order to compare the two schemes, also for the 20. - scheme the defocusing magnet was sliced into 10 pieces. First order TURTLE then produces the following results:

initial condition				values at the magnet exit			
x[μm]	x'[μrad]	y[μm]	y'[μrad]	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	0	0	0	-1.00813	-1.58307	176.77260	105.82378
1	0	0	0	1.69143	0.04200	176.74537	105.80153
100	0	0	0	268.95236	160.92396	174.04999	103.59866
0	1	0	0	2.36125	1.11653	176.74976	105.79655
0	100	0	0	385.93522	268.37747	174.48852	103.10117
0	0	1	0	-1.03541	-1.60532	176.69358	105.17795
0	0	100	0	-3.73078	-3.80819	168.87056	41.24061
0	0	0	1	-1.03102	-1.61030	178.31096	105.74476
0	0	0	100	-3.29225	-4.30567	330.60877	97.92174

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.00918	-1.50307	176.77260	105.32378
10^{-3}	13.21580	12.91513	176.67566	105.75059
-10^{-3}	-15.26663	-15.71008	[illegible]	[illegible]
	D_x[mm]	D'_x[mrad]	D_y[mm]	D'_y[mrad]
	14.24132	13.61711	-0.09707	-0.07232

The orbit coordinates at the magnet exit as well as the dispersion function in the horizontal plane are well represented by this scheme. The small difference in the vertical dispersion could be corrected by multiplying ϕ by N rather then by $N/2$. This would improve the dispersion function but the orbit coordinates in the horizontal plane would then differ by few micrometers. Nevertheless this scheme approximates the vertical BLW well enough.

An additional comparison of these two schemes with the numerically obtained results leads to an agreement of the same accuracy. At the entrance of the last defocusing magnet before the "INDOOR" point in the corrected SLC north arc the dispersion functions are:

$$D_x = 22.2724 \text{ mm} \qquad D_y = 3.2400 \text{ mm}$$

$$D'_x = -12.9216 \text{ mrad} \qquad D'_y = -2.3951 \text{ mrad}$$

With these [illegible] ues at the entrance of the magnet the 3 methods give the following dispersion functions at the end of the defocusing magnet:

Numerical calculation

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.03445	-1.62292	176.78346	105.82378
10^{-3}	22.99231	13.07410	172.34009	103.64657
-10^{-3}	-25.14895	-16.40449	181.22908	107.99957
	D_x[mm]	D'_x[mrad]	D_y[mm]	D'_y[mrad]
	24.07063	14.73930	-4.44450	-2.17650

14. - scheme

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.01825	-1.62433	176.78338	105.82378
10^{-3}	23.00623	13.07065	172.33762	103.64496
-10^{-3}	-25.13053	-16.40384	181.23029	108.00051
	D_x[mm]	D'_x[mrad]	D_y[mm]	D'_y[mrad]
	24.06838	14.73725	-4.44634	-2.17777

20. - scheme

δ	x[μm]	x'[μrad]	y[μm]	y'[μrad]
0	-1.00813	-1.58307	176.77260	105.82378
10^{-3}	23.01235	13.10695	172.42636	103.70460
-10^{-3}	-25.11644	-16.35761	181.12091	107.94147
	D_x[mm]	D'_x[mrad]	D_y[mm]	D'_y[mrad]
	24.06440	14.73228	-4.34729	-2.11644

References

/r1/ D. C. Carey, K. L. Brown, Ch. Iselin, SLAC Report No. 246 (1982)

/r2/ K. L. Brown, SLAC Report No. 75 (1982)

/r3/ R. Early, Steering effects in the SLC main E bend magnet. CN-216 (1983)

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