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METHODS FOR POWER SYSTEM PLANNING

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ABSTRACT

This report describes new results in power system planning, notably the development of efficient computer programs for finding the optimal transmission system expansion policy and methods for further extension and refinements of these first prototype programs. The computational procedures are based on linear-programming and dynamic-programming techniques, and they accommodate system reliability requirements as well as the discrete nature of the future additions. The report also introduces approaches to take account of uncertainties in load forecasts, generation plans, imposed constraint values, costs and possibly other input data. Methods for establishing the line-loading constraints are discussed in detail. Finally, the report suggests future approaches to integrated system-subsystem planning, and summarizes the related recent work done by other utilities.

The prototype planning program is demonstrated on a real-world transmission-system planning problem, and associated parametric studies.

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PART ONE

RESULTS AND CONCLUSIONS

Chapter I

INTRODUCTION

A. Background

The Branch of System Engineering of the Bonneville Power Administration has been undertaking a continuous effort in the last years to improve the efficiency of present procedures for long-term power system planning by making increasing use of their available digital computer facilities. In January 1969, the Bonneville Power Administration (BPA) contracted with Wolf Management Services (WMS), Palo Alto, California, to develop analytical and computational methods to aid in long-term transmission-system planning (Contract 14-03-79948). This study was a logical extension of the work performed by WMS under the previous Contract 14-03-79378.

This final report summarizes the results and accomplishment of the work performed under the present contract for the period of 23 January, 1969 to 30 December, 1969.

B. Objective and Scope

The objective of this work was to guide the development of practical planning tools (computer programs) which add the necessary engineering considerations and constraints to the body of analytical methods previously studied.[1,2*] The two main tasks of the study were specified as follows:

(1) Analysis of the effects of generation location upon planning of future transmission facilities;

(2) Development of models and approximations for transmission system planning.

To take account of the effects of generation facilities upon long-term transmission planning involves determining the transmission reserves required to compensate for generation outages, and the effects of contemplated alternative generation plans upon the optimum expansion schedule. Development of models and approximations for transmission-system planning procedures involves the following: the study and modeling of the transient-stability and other constraints imposed on transmission-line power flows; the study of the effects of future changes in load and generation upon the present investment decisions; the study of

*References are listed at the end of the report.

possible integration of the overall system planning and subsystem planning procedures handled separately at the present time; and the study and evaluation of the numerous rules of thumb, approximations and other time-saving methods used currently in system planning.

A more detailed breakdown of these two tasks was spelled out in Exhibit A of the contract, as envisioned at the initiation of the study. During the course of the project, some of the tasks were enlarged in scope while others were deemphasized, as required or mutually agreed upon. Specifically, the study of alternative generation plans (using the prototype transmission system planning program) has been deemphasized; instead, detailed information on a practical, real-life planning problem was supplied by BPA to WMS, with the request to implement and demonstrate the planning program on this detailed planning example, at BPA facilities in Portland and in close cooperation with designated BPA staff members.

The main conclusions and recommendations are presented in Chapter II. The main results of the work performed under Task (1) of the study are discussed in Chapter III, while the various work items of Task (2) and the corresponding findings are summarized in Chapters IV and V.

Chapter II

SUMMARY AND CONCLUSIONS

A. General

The problem of long term transmission system planning has been formulated in two earlier reports,[1,2] in two recent joint publications by WMS and BPA staff members,[3,4] and in two recent doctoral dissertations[5,6]. The aim of a systematic planning procedure is to minimize the total capital expenditures over a given period of time, with given economic, technological, geographic and reliability constraints. The salient points of related previous work[1-4] were:

- Application of mathematical optimization techniques to provide a systematic search for the least costly expansion schedule;
- Definition of the reliability constraints and efficient procedures for testing planned network configuration for prescribed reliability criteria;
- Concepts and procedures for incorporating the effects of uncertainties about the future into present planning decisions.

The analytical and computational techniques developed earlier for transmission system planning[1-4] were not intended for solving large-scale, real-world problems. Also, the many approximations, shortcuts and empirical rules extensively used by the BPA planning staff were not incorporated in these earlier planning techniques.

The work reported in this report overcomes many of the difficulties and limitations of the procedures and computer programs developed earlier. The main new outputs are:

(1) Operational computer programs, refined and augmented with many new ideas, tested on a real-world planning problem supplied by the BPA staff, and implemented at the BPA computer facilities.

(2) Methods for including the effects of uncertainties in future generation and load in the developed planning procedures.

(3) Methods for incorporating various present-day planning techniques (or improved versions of them) in the computer program.

B. Summary of Results

The results of the study were reported in several oral presentations, five published technical memoranda and a prototype computer program, submitted to the Branch of System Engineering during 1969. The titles of the technical memoranda are as follows:

(1) Application of Overall System Planning Program[7]

(2) Alternative Methods for the Planning of Transmission Systems in the Presence of Uncertainty[8]

(3) Network Reduction in System Planning with Application to the Study of Generation Outages[9]

(4) Methods of Network Reduction[10]

(5) Implications of the Doctoral Thesis: "On the Application of Dynamic Programming to Long-Range Planning with an Uncertain Future", by P. Henault, Stanford University, to Power System Planning[11]

Many valuable suggestions of the BPA Staff have been incorporated directly into this final report, without any need to revise the above memoranda.

The prototype computer program was developed at WMS facilities, and was subsequently implemented in Portland on the CDC 6400 general-purpose computer of the BPA. Designated staff members of the Branch of System Engineering have assisted in the implementation and have been trained in the use of the program. Since its first implementation in September 1969, this program has been further developed and refined by the BPA staff*.

The main results of the study are summarized in the body of this report; these include:

- Development of a prototype computer program for transmission-system planning, and illustration of the use of this program on a real-world problem;
- A procedure for overcoming a fundamental difficulty associated with the testing for line outages to assess the reliability of the planned transmission system;
- Inclusion of the effects of generator outages in the testing of the reliability of the planned transmission systems;
- A better understanding of the transient-stability and other constraints on transmission-line power flows;

*In particular, Dr. E. C. Ogbuobiri of the Branch of System Engineering has introduced sparsity techniques into both the linear-programming and the power-flow contingency-testing subprograms, achieving a substantial reduction in running time and a better utilization of storage, that has resulted in the ability of the prototype program to handle networks of larger size.

- Use of known methods of Production Costing in conjunction with a dynamic-programming procedure to simulate in detail the operating costs and constraints associated with each proposed configuration;
- Concepts and computational tools for coping with the effects of the various uncertainties which arise in power system planning;
- Methods for network reduction and decomposition in the context of power system planning;
- Methods for the integration of computational procedures used for the planning of the overall transmission system and for the more detailed planning of subsystems;
- Summary and brief evaluation of applicable planning methods used by other utilities.

C. Conclusions and Recommendations

(1) The application of the theory of optimum long-term transmission system planning to the two following situations has been developed in great detail:

(a) Deterministic expansion of a relatively coarse model of the network with an optimization procedure that is predominantly a linear program; the continuous output of the linear program is converted into a usable plan by an approximate approach that uses dynamic programming. This linear-programming-based procedure is referred to as the continuous and static optimization method (Chapter III).

(b) Expansion in the presence of uncertainty of part or all of the system; the planning alternatives, each represented with considerable detail, are given as an input and the optimum decision policy is found by stochastic dynamic programming. This dynamic-programming-based procedure is referred to as planning under uncertainty (Chapter IV).

(2) Computer studies performed at BPA and recent refinements of the programs by BPA personnel indicate that the BPA staff is thoroughly familiar with the capabilities and limitations of the linear-programming-based procedure and is in the process of implementing and significantly extending the program supplied.

The application and extension of the dynamic-programming-based procedure by BPA has received a lower priority for several very valid reasons, one being the significant additional complication introduced by the concepts and methods of stochastic optimization. However, this second procedure constitutes a very significant advance beyond the state of the art and therefore should be one of several advanced planning methods at the disposal of BPA's planning section. In its present form, it comprises the following two elements:

(a) An effective algorithm of optimally planning systems in the presence of uncertainty. This algorithm is sufficiently general to take into account various uncertainties (e.g., demand, generation, equipment age, technological improvements) and it is also computationally very efficient. Furthermore, it can accommodate construction lead times.

(b) A simulation algorithm for production costing. This algorithm provides the cost of operating the system in the presence of all the assumed future uncertainties.

(3) The above two planning procedures constitute very acceptable approximations to the optimization of an exceedingly complex mathematical problem, for which no exact solution methods are known at present. In addition to these two approximate procedures, each of which is particularly well-suited to a certain class of planning problems, several other optimum planning approaches and computational tools are available. (Chapter V-C). Foremost among these are: the "Best Investment Policy" Code, Mixed Integer Programming, and Dynamic Programming with Successive Approximations. These methods constitute suitable approximations for planning problems of concern to BPA and, therefore, their future developments should be monitored.

(4) The linear-programming-based procedure (Chapter III) can be used to analyze the effects of alternative generation plans upon transmission planning, and to determine the transmission reserves required to compensate for generation outages. It thus provides an effective tool to analyze the total facility (generation and transmission) and operating costs associated with selected generation expansion plans, and to find the best of the proposed plans by an exhaustive search. It is possible to improve this trial-and-error procedure by carefully screening the generation sites to be considered. With a minor effort, the existing transmission planning program could be extended to perform this screening function and generate reasonable siting alternatives for new generation; the total capacity, type, size, and year of construction would have been determined by a separate computation, possibly involving the concept of n-best trajectories.

(5) Experiments with real-world transmission planning examples indicate the need for computerized data preparation, network reduction, and an efficient scheme of man/machine interaction. With such a scheme, simplifications can be introduced easily and their effects on the validity and accuracy of the results

can be evaluated. The program should not necessarily be considered as a means for one-step direct optimization, but rather as a tool to aid the planning engineer in a step-by-step search directed by the computer.

(6) The linear-programming-based optimization procedure developed for long-term expansion studies (nominally 25 years) does not use detailed network representations; for instance, voltage considerations are omitted. There is a need for medium or near-term (normally five years) optimization procedures in which the network is modeled with much greater accuracy. The main principles developed for optimizing the long-term expansion can be adapted to the case of medium or near-term expansion of detailed network models. The schemes for effective man/machine interaction and computerized data preparation found to be important for long-term expansion would be equally important for the medium- and short-term optimization. Also, extensive use of reduction and decomposition techniques should be made to direct the attention of the planning engineer toward those parts of the system being studied in detail without neglecting the expansion of the remainder of the system. (Chapter V-A).

(7) With the exception of the production-costing subroutine (Chapter IV-B) developed in conjunction with the dynamic-programming expansion procedure, only peak loading conditions are considered. The argument is that a network capable of handling selected peaks is capable of handling all other load and generation patterns. This is not necessarily true, and efficient techniques for checking the performance of a proposed system under conditions other than peak are needed.

(8) The linear-programming-based expansion procedure makes extensive use of the line-loading limits to determine the need for increasing transmission capacity (Chapter V-B). In many cases, these line-loading limits depend on the stability properties of the system. Precise modeling of the stability-related line-loading constraints remains a difficult (and, to date, largely unresolved) problem. Improved modeling of these constraints could be achieved by various simplified, approximate, transient-stability analysis methods studied in a preliminary way in the course of the project. Also, the expected future increase in line-loading limits that may result from improved methods of transient control needs to be taken into account.

PART TWO

TECHNICAL DISCUSSION

Chapter III

A PROTOTYPE COMPUTER PROGRAM FOR TRANSMISSION SYSTEM PLANNING

A. Introduction

This chapter gives a description of the present status of the computer programs that have been developed for transmission system planning. Many of the key ideas were already discussed in earlier reports and technical memoranda,[1,2,7,9] and they will not be repeated here. In particular, the formulation of the overall planning problems is not repeated here (see Ref. 2, pp. 7-34, and Ref. 5, pp. 18-44). But many new concepts were recently introduced, with computer programs developed and tested, that have not been documented yet. This chapter aims primarily at putting all these elements in perspective. It presents a general computational procedure that could very soon be ready for actual implementation and everyday use as an aid in transmission planning; the corresponding prototype computer program is operational and available for experimentation.

This chapter will first survey the main new ideas recently developed; then describe briefly the computational procedure, with a flow chart. Its application to a real-life example is then discussed in detail. Finally, the limits on the validity of the solution obtained are stressed, and several improvements are suggested. For brevity, frequent references are made to earlier work in place of detailed repetition of material already known to the BPA.

B. Survey of the Main New Ideas

1. Man-Machine Concept

In the present time, a fully automatized mathematical optimization procedure for EHV transmission system planning seems to be an unrealistic goal.

No automatic model could take care of all the intricacies, and yet provide a true optimum.

There is a major conflict between:

(a) The need for a simple mathematical model that would permit a true optimization with reasonable computation times;

(b) The ever-increasing complexity of the real-life constraints and cost data.

The man-machine scheme is less ambitious, but

(i) It would represent a very substantial improvement over current practice, in terms of both dollar cost and system reliability.

(ii) It could be extended very soon for actual implementation within the Branch of System Engineering of BPA.

The principles of man-machine interaction for optimization are the following:

(a) A computer program generates a "reasonable" first sketch of the system expansion by combining long-term cost minimization and reliability testing, using the simplifying assumptions and decomposition hypotheses that make the model mathematically and computationally workable. Such a program is discussed in this chapter.

(b) The inaccuracies overlooked by the above model are resolved via a set of local optimizations, taking benefit of the flexibility of the present manual techniques and of human judgment (supplemented whenever possible by computer runs).

The advantage of such an approach over the use of the present manual procedures alone is to remove some of their myopic character by starting with a "globally optimized" first sketch. The optimization program will thus provide the system planner with guidelines over which he can exercise his judgment and experience. The idea can easily be extended to a back-and-forth iterative process between the engineer and the computer.

2. The Idea of Decomposition

We decompose the entire system under study into two components:

(i) An active system, the expansion of which is currently under study (dimension: n_a nodes; typically $n_a = 50$);

(ii) A passive system, the expansion of which is given throughout the planning period (dimension: n_p nodes; typically $n_p = 1000$).

We want to perform a network reduction, i.e., to obtain an n_a-node "equivalent" of the n_p-node passive subsystem, and to superimpose it on the active system.

Several network reduction techniques have already been discussed in earlier reports,[9,10,13] and the subject is also treated elsewhere in this report (Chapter V, Sec. A). We briefly recall the reduction technique for linear systems (Ref. 13, pp. 55-56):

(a) Decomposition of

$$A\theta = I \tag{III-1}$$

into

$$\begin{cases} A_{aa}\,\theta_a + A_{ap}\,\theta_p = I_a \\ A_{pa}\,\theta_a + A_{pp}\,\theta_p = I_p \end{cases} \tag{III-2}$$

(b) Equivalent equation:

$$\left[A_{aa} - A_{ap}(A_{pp})^{-1}\,A_{pa}\right]\,\theta_a = I_a - A_{ap}(A_{pp})^{-1}\,I_p . \tag{III-3}$$

This reduction can be made off-line, i.e., before the actual planning computations are made and separately from them. For the linear case above, it consists of computing the matrices $A_{ap}(A_{pp})^{-1}\,A_{pa}$ and the vectors $A_{ap}(A_{pp})^{-1}\,I_p$ for each given set of injections and the given states of the passive network.

The decomposition method has the following advantages:

- It allows planning the expansion of a subsystem while taking into account the expansion plans already made for the system it is embedded in;
- It provides a way of taking into account the influence of the neighboring systems on BPA's planning;
- It fits very well the man-machine concept discussed above. The planning engineer can manipulate the active/passive partition in a way that he optimizes each subsystem in sequence, each time assigning the plans he has just obtained to some other part of the system.

3. The Continuous and Static Optimization in Perspective

(a) Continuous Optimization Aspects

The word "continuous" in the title of this section refers to the idea of first solving a continuous version of the problem, and then finding a discrete expansion schedule through a series of successive approximations. This approach has been discussed in earlier reports[1,2] and in recent work.[5] Here we shall be primarily interested in the latest developments of that continuous model. This does not mean that the discrete optimization is left out, but since a detailed discussion of it is to be found in Ref. 5, duplication of that text seemed undesirable. There, a branch-by-branch dynamic programming scheme is explained (Ref. 5, pp. 60-80); it has been programmed and tested. Typical results are shown in Section D of this chapter. The successive approximations in dynamic programming must be viewed in the spirit of the man-machine concept discussed above.

The purpose of the continuous and static optimization is to determine the set of capacity additions $\Delta\gamma^{(p)}$ that must be performed on each branch of the initial system configuration at the beginning of the planning period, to ensure that all reliability constraints are observed when the system is subjected to a given set of injection forecasts $I^{(p)}$. The superscript (p) refers to the time interval for which those injections are considered. The variables of this optimization are the branch capacities γ_k, which are defined in Ref. 1, p. 15, and also in Ref. 2, p. 19. We thus assume (temporarily) that the state of Branch k can be represented by a continuous variable γ_k.

The objective is to minimize a linear function of the required capacity additions $\Delta\gamma_k$, which approximates the cost of these additions. This is by itself a very questionable assumption, since economies of scale are involved (see Ref. 2, p. 45). But, as pointed out earlier, this continuous and static optimization may not be the final procedure, and the economy-of-scale phenomena are presently taken into account in the dynamic programming optimizations which come next.* The possible use of a more accurate concave cost function $c(\gamma)$ instead of this linear model is studied in Ref. 5, pp. 94-96, but it is shown to lead to computational difficulties (large mixed-integer programming problems).

(b) Static Optimization Aspects

The word "static" in the title of this section refers to the fact that each set of injections $I^{(p)}$ is treated separately; i.e., the computation of the addition vector $\Delta\gamma^{(p)}$ is independent from the computation of $\Delta\gamma^{(p-1)}$, $\Delta\gamma^{(p-2)}$, etc., with each of these addition vectors being defined with respect to the same initial state $\gamma^{(0)}$. Thus, the results obtained for the final years are very significant in that they represent the minimum-cost requirements for these years independent of what may have been required earlier. In particular,

*These dynamic programming optimizations use the actual costs of lines and of terminal equipment, with all required detail, instead of the continuous cost model, which is no longer necessary (see Ref. 5, pp. 66-74).

the minimum-cost system for the last year of planning could be determined first, and then the requirements for the earlier years could be calculated backward.

Indeed, only the simultaneous global optimization of all the early requirements would be optimal. But such an optimization is contingent upon the use of a nonlinear (concave) cost model, to take into account economies of scale. As mentioned before, and as explained in Ref. 5, (pp. 94-96,) this leads to mixed-integer programming problems, of a size for which efficient codes do not yet exist (although they might become available). In any case, the dynamic programming optimization which follows the "continuous and static" approach will take those economies of scale into account when deciding upon a discrete expansion schedule.

The man-machine concept can once again be introduced here. After having run the continuous and static optimization for all time periods p , the system engineer might wish to recompute some of the $\Delta\gamma^{(p)}$ using different assumptions, and compare the results obtained from the standpoint of cost. For example, some capacity additions suggested for period p by the continuous and static procedure might in fact be unacceptable; if so, then $\Delta\gamma^{(p)}$ could be computed in a new run with those additions forbidden, and the new cost could be compared to the previous cost. Specific examples of this "lock" feature will be discussed later.

In conclusion, the continuous and static optimization method gives as an answer the set of vectors $\gamma^{(p)}$, which may be too conservative due to temporary disregard of economies of scale. Vectors $\gamma^{(p)}$ nevertheless form a starting solution that is worthy of consideration because:

- It guarantees feasibility with respect to the reliability constraints;
- It minimizes cost by comparing the effect upon transmission-system reliability of various possible investments in different parts of the system;
- It can be subjected to further cost minimization via successive approximations (dynamic programming; see Ref. 5, pp. 74-80).

The continuous and static optimization should thus be the key machine element of the man-machine planning scheme; the other important machine element is the dynamic-programming cost minimization for the expansion of a single branch, which is discussed in Ref. 5, pp. 66-74.

4. Line-Outage Tests as a Fundamental Difficulty

The choice of a criterion for system reliability has been discussed in Ref. 2, p. 25. The system is considered to be reliable if the loss of one line, possibly the strongest, in any branch will not cause any overload elsewhere in the system that would trigger the loss of additional equipment. An overload is defined here as a load exceeding the maximum permissible value of the voltage phase-angle difference ψ_k between the end nodes of a given branch (the question of line loading limits is discussed later in Chapter V).

Such a reliability criterion is simple to formulate. However, problems arise when it becomes desirable to translate this criterion into a set of mathematical constraints for the continuous and static case:

(a) The magnitude of the capacity to be removed for a line-outage test in a given branch is not known in advance; the capacity to be subtracted is that of the strongest line in the branch, which may not yet exist and for whose capacity no value may have yet been determined.

(b) The constraint that the angular difference ψ_k must not exceed in magnitude a maximum permissible value $\bar{\bar{\psi}}_k$ holds only for the lines that physically exist.

Therefore,

- If there is only one line in Branch k, the constraint on ψ_k vanishes whenever we test for an outage in Branch k;
- If there is initially* no line in Branch k, then no constraint exists on ψ_k; but if the continuous optimization then calls for some capacity addition $\Delta\gamma_k > 0$, this introduces constraints on ψ_k, which now must not exceed its prescribed maximum value $\bar{\bar{\psi}}_k$ during any of the outage tests.

The above remarks show that the outage criterion introduces not only numerical but also logical constraints. These logical constraints are inherent in the nature of the criterion; i.e., they accompany whatever solution method we want to use, whether integer, mixed-integer, or dynamic programming and with either the discrete or the continuous approach.

A way of satisfying these logical constraints when the "continuous and static" method is used has been developed. The details of it are explained in Sections C and D below, with the help of a specific example.

*To allow for the possibility of creating new links, we put dummy branches in the initial network--i.e., branches with zero initial capacity, in which capacity additions are possible.

5. Testing for Generation Outages

The last of the main results of this most recent research is concerned with the effects of generation location upon transmission requirements. This is not merely a question of making parametric runs with different sets of node injections; it involves the difficult problem of generation outages, which often place heavy strains on the tie-lines with the neighboring systems.

The problem is stated as follows:

Given:

- A certain injection pattern; and
- The set of possible generation outages;

Find:

- Whether any overload results from the generation outages (especially in the tie-lines); and
- What are the minimum-cost branch capacity additions (if any) that will eliminate those overloads.

A solution method is explained below in Sec. C-2-e Case (iii) and Sec. C-3 of this Chapter for the "continuous and static" optimization approach. It is essentially similar to the method used for line-outage tests, and the two optimizations can be combined into a single procedure.

The fundamental difficulty encountered with line-outage tests (Sec. B-4 above) now vanishes, but a new problem arises. The energy-balance equation

$$\sum P_i = \sum C_i + \text{losses} \qquad \text{(III-4)}$$

must always be satisfied; thus a generator outage of magnitude $-\Delta P_i$ triggers a dynamic change in some other terms of Eq. (III-4), and consequently in some elements of the injection vector **I**. This may cause temporary overloads, especially in the tie-lines.

The dynamic problem is a very difficult one to deal with, because of the large amounts of information* needed, especially for long-range planning. A detailed study of this problem has been made in the context of the security of an operating system.[12] In this report, we propose a simple linear model that permits, under

* Principally on the system's own response characteristics and its external control (governor action).

certain assumptions, the optimization defined above. It is discussed in Sec. C.3, and illustrated with computational examples.

C. Description of the Computational Procedure

1. General Flow Chart

A general flow-chart of the man-machine procedure is shown in Fig. III-1. The detailed flow-chart of the continuous and static optimization is given in Fig. III-2. The dynamic programming expansion of Branch k is explained in Ref. 5, pp. 66-80.

2. Continuous and Static Optimization

The main blocks of the flow-chart for this optimization (given in Fig. III-2), are explained in the paragraphs below. General comments on the procedure can be found in Ref. 5, pp. 46-61.

a. Block 1-"Set Capacities γ_k to Their Initial Values at Time 0"

These capacities are given for the entire system (active and passive, see Section B-2). In some branches, a zero initial capacity indicates that no line exists yet, but that line additions may occur in the future.

The index r , in the flow-chart, represents the number of branches in the active network. N is the number of time periods considered.

b. Block 2-"Compute Maximum Angles $\bar{\bar{\psi}}_k$"

A discussion of line loading limits is given in Chapter V, Section B. We use here the criterion already defined in Ref. 1, pp. 20 and 22. A positive angular limit $\bar{\bar{\psi}}_k$ is assigned to every branch, taking into account the branch mileage and the various types of lines it may contain. The thermal constraints, if considered alone, yield angle limits per mile of line which are close to one another for the types of lines usually available. To witness, the ratio (max. flow/ψ per mile) has been computed for different types of lines, and entered in Table III-1. In the computational examples discussed in Sec. D, $\bar{\bar{\psi}}_k$ has been chosen to be the smallest of these ratios (among the types of lines available in Branch k) multiplied by the length, in miles, of Branch k .

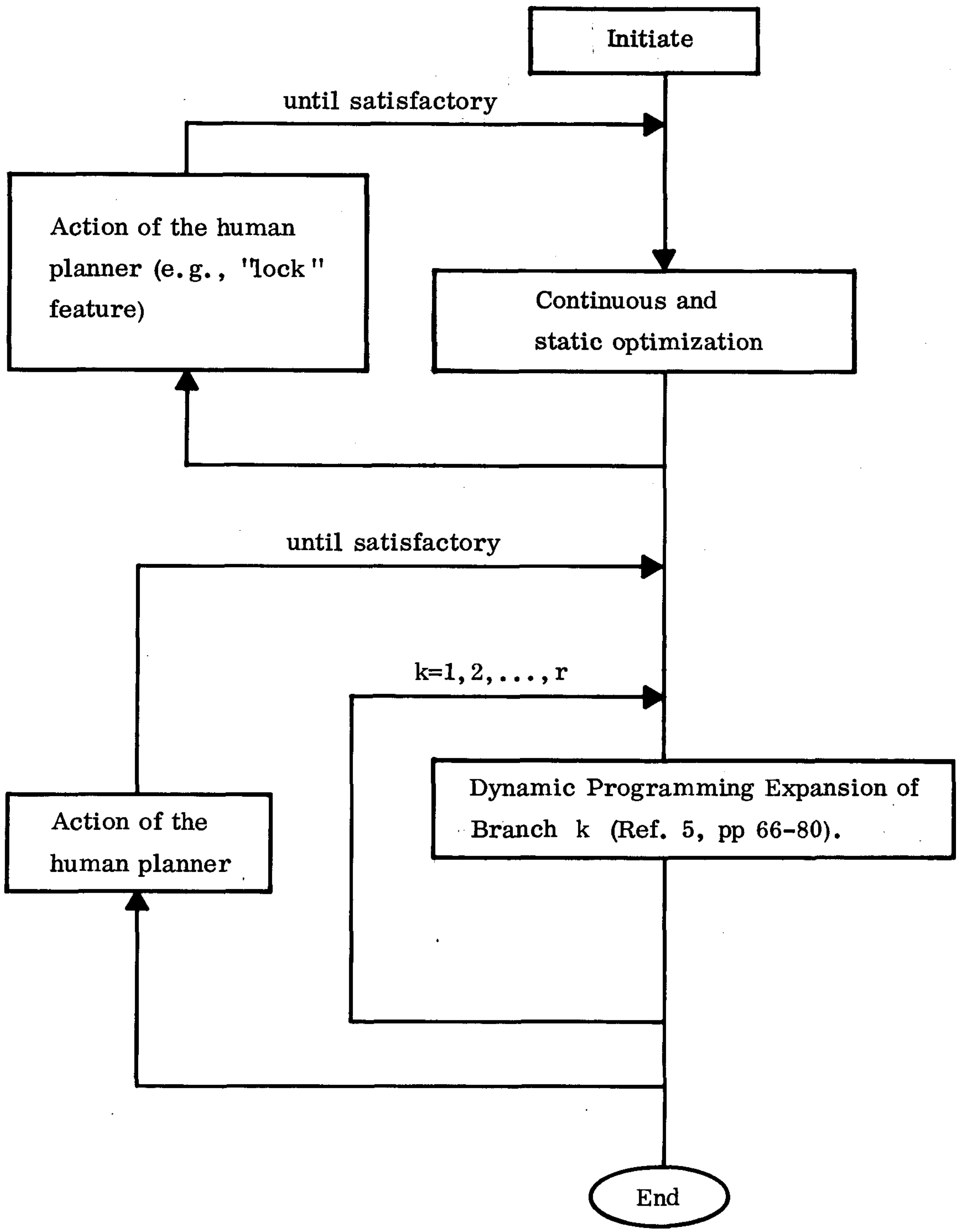


Fig. III-1. General Flow Chart

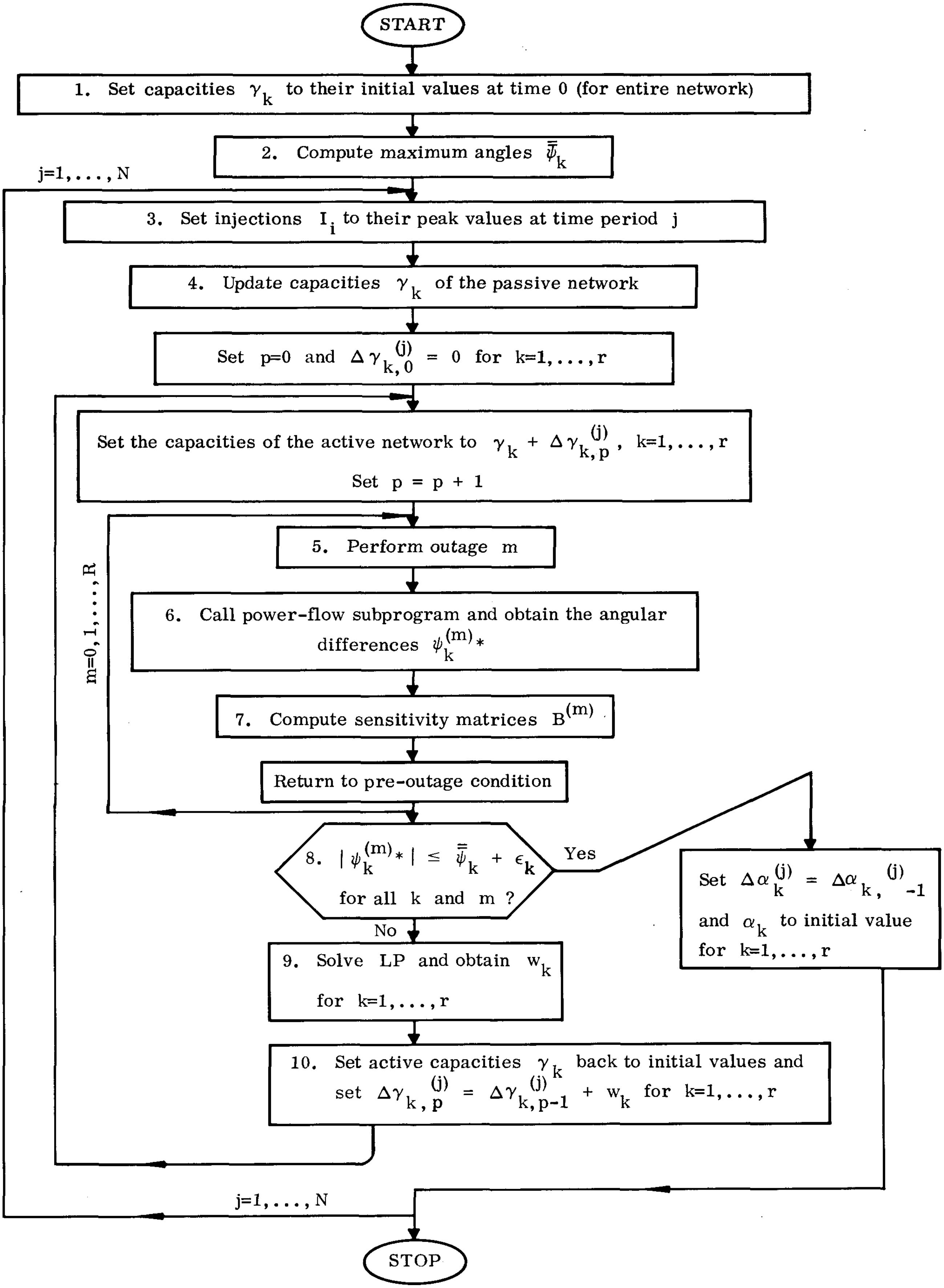


Fig. III-2. Flow Chart of the Continuous and Static Optimization Procedure

Table III-1. Thermal Limits for Various Types of Lines

Type of line	69 kV	115 kV	230 kV	525 kV
Max. Flow (thermal constraint)	36 MW	125 MW	480 MW	2700 MW
$\left[\gamma = \frac{V^2 X}{Z^2}\right]$ (MW)	0.428×10^4	0.162×10^5	0.635×10^5	0.482×10^6
$\left[\frac{\text{(max. flow)}}{(\psi \text{ (per mile)}}\right]$ (MW-mile/degrees)	0.842×10^{-2}	0.773×10^{-2}	0.755×10^{-2}	0.561×10^{-2}

c. Block 3 - "Set Injections I_i to Their Peak Values at Time Period J"

These injections are given for the entire system (active and passive).

d. Block 4 - "Update Capacities γ_k of the Passive Network"

Indeed, the expansion plans are known for the passive network (see Sec. B-2), but no updating is made for the capacities γ_k of the active network, which are the quantities to be determined (see Sec. B-3-b).

In the course of the man-machine optimization procedure, it might be of interest to compare the results obtained in this way with those obtained when also updating the branch capacities of the active network, to the values $\gamma_k + \Delta\gamma_k^{(p)}$. (See Ref. 5, pp. 63-65 for the "look-ahead factor" method).

e. Block 5 - "Perform Outage m"

This may be a line outage, a branch outage, or a generation outage. The total number of outages considered is R . The case m = 0 corresponds to no outage at all.

(i) Line Outage* -- The fundamental difficulty discussed in Sec. B-4 must be remembered. The magnitude of the line outage to test in Branch k in Period j depends on what lines have been added before Period j . We do not know this information, since we are still in the continuous optimization. This difficulty is approximately resolved by the following approach:

- The strongest line of Branch k in its initial state is always known;
- The addition $\Delta\gamma_k^{(j)}$ is determined recursively, via a series of Newton-Raphson-type iterations involving linear programming:

*Line outages, as well as branch outages, are only tested in the "active" subsystem.

Let $\Delta\gamma_{k,p}^{(j)}$ = the value at the p^{th} iteration. Take $\Delta\gamma_{k,o}^{(j)} = 0$ to determine $\Delta\gamma_{k,1}^{(j)}$, and so on.

- The magnitude of the line outage to consider when computing $\Delta\gamma_{k,p}^{(j)}$ is given by

$$\left\{ \inf \ \sup(\lambda, \mu) \ , \ \upsilon \right\}$$

where

λ = capacity of the biggest line existing initially in Branch k

$\mu = \Delta\gamma_{k,p-1}^{(j)}$;

υ = capacity of the biggest type of line that may ever be added in Branch k during the planning period.

It is easy to verify that the above rule can only be <u>too conservative,</u> for testing outages that might sometimes be bigger than the actual outage. The man-machine iterations that follow the determination of $\Delta\gamma_k^{(j)}$ will take care of that possible excess capacity.

(ii) <u>Branch Outage</u> -- An easy way to avoid the fundamental difficulty of line outage tests is to slightly modify the security criterion, to allow outages of an entire branch instead of only its strongest line. This viewpoint can be justified from engineering considerations, since in many cases the first lines to overload will be those parallel to the one lost. Then, a line outage would be allowed to cause cascading outages in the same branch, but not in any other branch.

Computationally, the test for a branch outage consists of setting the corresponding capacity to zero.

(iii) <u>Generator Outage</u> -- If a generator is lost, no change in branch capacities occurs, but the node injections I_i are modified according to the dynamic response of the system (see Sec. B-5). This response is very difficult to know in advance, especially in a planning context. However, it is imperative to have an idea of how these injections will behave, in order to test one (or possibly several) critical injection vectors from the standpoint of the line overloads they may cause. An approximate approach to this problem is suggested in Sec. C-3.

f. Block 6 - "Call Power-Flow Subprogram and Obtain the Angular Differences $\psi_k^{(m)*}$"

$\psi_k^{(m)*}$ is the voltage phase angle difference in Branch k when the m^{th} outage is tested. Normally, those angular differences will be monitored for all the branches of the active subsystem, or for only a subset of them.

Several load-flow programs are available, ranging from the complete ac load flow to the dc power flow. In the examples discussed in Sec. D, a dc load flow (see Ref. 2, p. 19) has been used, combined with the network reduction of the type explained in Sec. B-2 to process only matrices of small size.† For actual implementation, it might be preferable to use the so-called "ac real power only" load flow, combined with a preliminary dc network reduction. This would represent a trade-off between accuracy of the angle computations and computer time needed.

g. Block 7 - "Compute Sensitivity Matrix $B^{(m)}$"

$B^{(m)}$ is the matrix expressing the first-order sensitivity of angular differences ψ to variations of the branch capacities γ, for given injections and outage condition (m). Its computation is explained in Ref. 1, p. 17, and also in Ref. 5, p. 49. Note that, contrary to what was explained in Ref. 1, the sensitivity matrices are computed and stored for each outage (m) ; this will lead to a linear programming formulation which will be longer, but much more accurate than the one explained in Ref. 1.

h. Block 8 - "Overload Tests $|\psi_k^{(m)*}| \leq \bar{\bar{\psi}}_k + \epsilon_k$"

These tests are performed only in the branches with nonzero capacity (see Sec. B-4-b above). This excludes all the branches with initial zero capacity, except for those in which the result of a preceding Newton-Raphson iteration (in time period j) has yielded a nonzero $\Delta\gamma_{k,p}^{(j)}$ (see Sec. B-4-j). Also, when a branch outage is tested in Branch k , no overload must be looked for in that branch. This does not hold for line outage tests, in which some parallel lines may remain.

The quantity ϵ_k is a margin which prescribes the accuracy sought in the convergence of the Newton-Raphson iterations. For instance, it might be taken as 5 percent of the value of $\bar{\bar{\psi}}_k$.

†The "inverse matrix" of the passive network does not change when different line outages (or branch outages are tested, since those may only occur in the active network (see footnote on page 19).

i. Block 9 - "Solve Linear Program and Obtain w_k for $k = 1, \ldots, r$"

The idea of using a linear-programming approach in continuous optimization was introduced in Ref. 1, pp. 23 - 29 and in Ref. 5, p. 53 and 61. This formulation is used again here, but in a slightly different manner from that explained in Ref. 1. Instead of combining and summarizing all the information obtained from the outage tests (see Ref. 1, p. 23), all those outage tests are treated in parallel by the linear-programming algorithm. We shall show that this new approach is very beneficial for accuracy, and that it does not increase substantially the computation time needed.

The linear program is formulated as follows:

Find: $w_1, \ldots, w_r$

that minimize $\sum_{k=1}^{r} c_k w_k$

subject to

$$\left.\begin{aligned} \sum_{k=1}^{r} B_{ik}^{(m)} w_k &\geq -\bar{\bar{\psi}}_i - \psi_i^{(m)*} \\ \sum_{k=1}^{r} B_{ik}^{(m)} w_k &\geq -\bar{\bar{\psi}}_i + \psi_i^{(m)*} \end{aligned}\right\} \begin{aligned} & i=1,\ldots,r \\ & m=1,\ldots R \end{aligned} \qquad \text{(III-5)}$$

$$w_k \geq 0 \; ; \; k = 1, \ldots, r$$

which expresses how the capacity changes w_k cause changes in the angular differences ψ_i that eliminate all overloads (using first-order sensitivity of ψ to γ, and a linear cost model).

The following points are to be noted:

- As explained in Sec. C-2-h, the overload tests must be performed only in those branches with nonzero capacity during the current iteration; also tests must not be performed in branches where branch outages are currently tested. Thus, some of the inequality constraints written in Eqs. (III-5) must be suppressed.

- Let m_i be the value of the index m that corresponds to a branch outage or a line outage in Branch i. We make

$$B_{ik}^{(m_i)} = 0 \; ; \; k = 1,\ldots,r \; ; \; i = 1,\ldots,r \, . \qquad \text{(III-6)}$$

  The reason for this change is that we do not want the linear program to make additions in Branch i to cure overloads created by a branch or line outage in Branch i.

- The number r in Eqs. (III-5) corresponds to the number of branches in the active system, i.e., to only those branches in which capacity additions are permitted.

  The linear programming problem (III-5) is solved by the dual method, as explained in detail in Ref. 5, pp. 54-57. Solving the dual instead of the primal problem has the following three advantages:

  - A feasible solution to the dual is available by inspection;
  - The dual problem has r rows, and up to possibly Rr columns. Since the computation time is primarily dependent upon the number of rows, and not so much on the number of columns, it will not be very sensitive to the number of outage tests required. This is the reason why we can afford to treat all these outage tests in parallel in the linear program.
  - The number of pivotings to obtain the LP optimal solution has reliably proved to be much smaller than expected for problems of this size, when using the dual method.

j. Block 10 -"Iterate"

As explained in Ref. 1, p. 28, and Ref. 5, pp. 57-59, the solution to the linear program will not usually remove every one of the overloads, because of the nonlinearities which were discarded in the first-order sensitivity approach. It is necessary to repeat the linear programming optimization with the active capacities temporarily modified to the values $\gamma_k = \Delta\gamma_{k,p}^{(j)}$, where $\Delta\gamma_{k,1}^{(j)}, \ldots, \Delta\gamma_{k,p}^{(j)}$ are the successive approximations of the required capacity addition.

The formulation of the linear programming problem as given in Eqs. (III-5) is valid only for the first one of these iterations. When $p \geq 2$, it must be replaced by the following formulation:

Find: $w_1, \ldots, w_r$

that minimize $\sum_{k=1}^{r} c_k w_k$

subject to

$$\left.\begin{aligned} \sum_{k=1}^{r} B_{ik}^{(m)} w_k &\geq -\bar{\bar{\psi}}_i - \psi_i^{(m)*} - \sum_{k=1}^{r} B_{ik}^{(m)} \Delta\gamma_{k,p-1}^{(j)} \\ \sum_{k=1}^{r} B_{ik}^{(m)} w_k &\geq -\bar{\bar{\psi}}_i + \psi_i^{(m)*} + \sum_{k=1}^{r} B_{ik}^{(m)} \Delta\gamma_{k,p-1}^{(j)} \end{aligned}\right\}$$

$$i = 1, \ldots, r$$

$$m = 1, \ldots, R$$

$$w_k \geq 0 \; ; \; k = 1, \ldots, r \; . \tag{III-7}$$

This successive approximation approach is close to the Newton-Raphson method. In all the experimental cases treated so far, it showed a very fast monotonous convergence. This will be illustrated in the examples of Sec. D.

3. Approximation of System Behavior after the Loss of a Generator

a. Discussion

As already explained in Sec. B-5 and C-2-e(iii), if we want to consider the possibility of generator outages in our planning procedures, it is necessary first to build a model of the behavior of the various loads and generators following the loss of one of them. So-called "energy-balance" models have been proposed to that effect[45, 46] and are currently under study.[12] This is essentially a very complex problem, an accurate model of which requires knowing a large number of parameters, such as individual rotor inertias, prime-mover response characteristics, and governor settings. Although these parameters might conceivably be obtainable with good accuracy on a real-time basis (e.g., for control purposes), their prediction on a long-range basis might be very unreliable, thus preventing in most cases the use of the same "energy-balance" models for planning.

In the present study, we nevertheless designed and used a very simplified model of the system behavior in the event of the loss of a generator. This model, explained below, is by no means an accurate representation of the real world; its only purpose is to illustrate how, when the postfault behavior of the injections can be approximated, the linear programming optimization of Sec. C-2 can be used with generator outages as well as with line (or branch) outages. This model has been incorporated into the program to permit generator-outage tests.

b. Linear Reallocation Model

Let

P_i = amount of generation in Node i

C_i = amount of load in Node i

j = node in which a generator outage takes place

G_j = magnitude of the generator outage

K_i = a coefficient attached to Node i .

The model is based on two assumptions:

(i) The loads do not change after the outage; i.e.,

$$\Delta C_i = 0 \; ; \; i = 1, \ldots, n \, . \tag{III-8}$$

(ii) The remaining generators pick up power in proportion to the K_i coefficients:

$$\Delta P_i = \frac{K_i G_j}{\sigma} \; ; \; i = 1, \ldots, j-1, j+1, \ldots, n \tag{III-9}$$

$$\Delta P_j = -G_j + \frac{K_i G_j}{\sigma}\left(1 - \frac{G_j}{P_j}\right) \tag{III-10}$$

$$\sigma = \sum_{i \neq j} K_i + K_j \left(1 - \frac{G_j}{P_j}\right). \qquad \text{(III-11)}$$

The K_i coefficients may correspond to the prime-mover inertias, or to some rule of power reallocation (new steady-state). Equation (III-10) reflects the fact that only the fraction $(1 - G_j/P_j)$ of the initial generation remains in Node j after the outage G_j.

This model, which is very crude indeed, can be refined by setting upper bounds to the ΔP_i, or by using, simultaneously, several sets of K_i coefficients, each set corresponding to one critical point in the postfault behavior of the system.* Equations (III-8) to (III-10) yield the new injections, to use in the linear programming formulation of Sec. C-2.

D. Computational Examples

Computer programs have been written along the lines of the procedures described in this chapter. These test programs have been consolidated into a first prototype transmission planning program, written in FORTRAN IV. This experimental program was run most of the times on a UNIVAC 1108 computer, but it has also been adapted to the equipment presently used by BPA (CDC 6400).

The computational examples presented here have been based on real-life data provided by the Branch of System Engineering of BPA. It must be pointed out that the few results which follow are given for illustration only; they are not the final answers to the actual planning problems from which these examples were taken. The "man" part of the man-machine scheme discussed in Sec. B-1 (and illustrated in Fig. III-1) was not performed, and thus the results shown must be seen as only intermediate results, susceptible of further refinement via a series of man-machine iterations.

1. Example of Application of the "Continuous and Static" Procedure

a. Input Data

The system considered has 31 nodes and 58 branches. It corresponds to an existing subsystem of the BPA network, in the central Washington area.

*In this case, the number of constraints in the linear programming problem of Eq. (III-5) is also increased, following the number of new critical points that must be tested.

The system is partitioned into one 24-branch active subsystem and one 34-branch passive subsystem, as shown in Fig. III-3.

The types of lines considered are listed in Table III-2. The initial state (at the beginning of 1969) of the entire 58-branch system is shown in Tables III-3 and III-4. However, a set of projected additions* is also given for the passive system only in Table III-4; no such a set is given for the active system, since the set is the subject of the optimization.

In this example, the outage tests will consist only of line outages (see Sec. C-2-3(i)) in the active network. Since a static optimization is sought, only one set of node injection forecasts is used. These injections have been taken for August 1969, and are shown on Table III-5 for the nodes belonging to the active subsystem. A linear cost model is assumed.

b. First Optimization

Table III-6 explains how the linear programming method proceeds (see flowchart, Fig. III-2). The initial overloads are shown on Tableau 0, with the outages causing them**. The overload ratio is the ratio $\psi_k^{(m)*}/\bar{\bar{\psi}}_k$.

Table III-2. Characteristics of the Transmission Lines Used in the Example of Fig. III-3.

Voltage	R/Mile (Ω)	X/Mile (Ω)	Max. Current (Amps.)	Max. Flow (MW)	Cost/Mile ($)
69 kv	.510	.78	300	36	
115 kv	.225	.75	900	125	45,000
230 kv	.129	.812	1200	480	52,000
525 kv	.0302	.571	3000	2700	108,000

*Two of these projected additions consist of transformers, which in the computations have been represented by short high-capacity lines.

**These are line outages in the corresponding branches; see Sec. C-2-e(i).

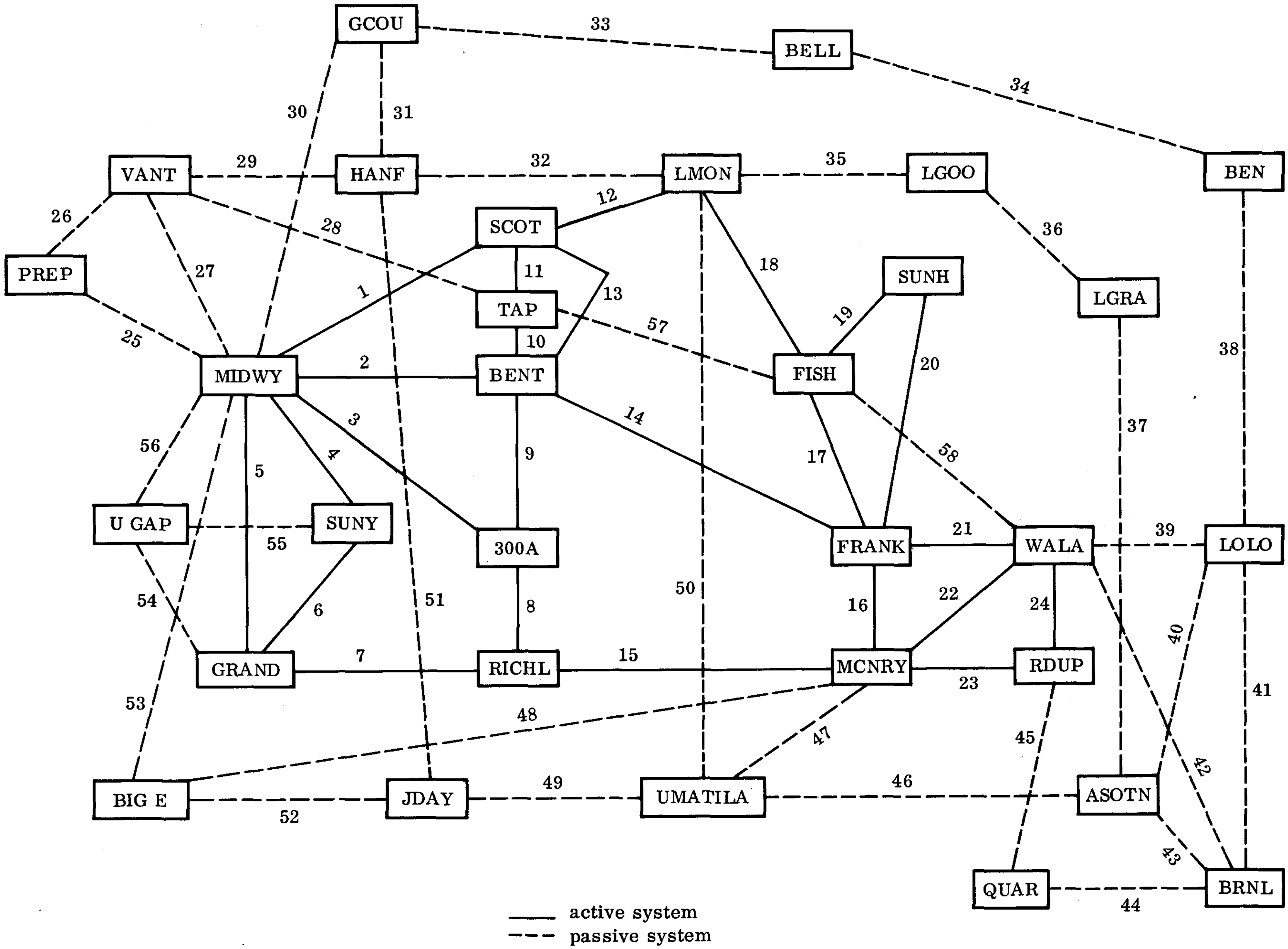


Fig. III-3. Topology of the Example System

Table III-3. Active System in the Example of Fig. III-3.

Branch	Mileage	Existing Lines (1969)	Branch	Mileage	Existing Lines (1969)
1	47	-----	13	20	115 kv
2	16	115 kv	14	20	2 x 115 kv
3	35	-----	15	30	115 kv
4	25	-----	16	27	69 kv + 230 kv
5	25	2 x 115 kv	17	4	-----
6	12	-----	18	30	-----
7	29	115 kv	19	9	-----
8	11	115 kv	20	12	115 kv
9	6	115 kv	21	36	69 kv + 115 kv
10	5	-----	22	40	69 kv
11	15	-----	23	38	69 kv + 230 kv
12	25	-----	24	55	-----

Table III-4. Passive System in the Example of Fig. III-3

Branch	Mile-age	Existing Lines (1969)	Projected Additions	Branch	Mile-age	Existing Lines (1969)	Projected Additions
25	6	3 x 230		42	153	230	
26	15	230		43	10	---	525 (1979)
27	20	230		44	43	230	
28	20	230		45	81	230	
29	28	525		46	123	---	525 (1973)
30	105	3 x 230		47	0	---	Transformer (1974)
31	90	---	525 (1974)	48	97	230	
32	54	525		49	71	525	
33	83	5 x 230 2 x 115		50	67	525	
34	37	230		51	97	525	
35	27	---	525 (1972)	52	20	525	
36	28	---	525 (1972)	53	103	230	
37	31	---	525 (1972)	54	25	---	
38	70	230		55	36	115	
39	80	230		56	39	230	
40	0	---	Transformer (1976)	57	12	230	
41	119	230		58	40	230	

Table III-5. Node Injection Forecasts in August 1969
(Active subsystem only)

Node	Injection (MW)
MIDWY	611
SUNY	-30
GRAND	-49
SCOT	-30
TAP	33
BENT	-91
300A	0
RICHL	-45
LMON	-184
FISH	0
FRANK	235
MCNRY	356
SUNH	-45
WALA	-475
RDUP	-285

Table III-6. Example of Continuous and Static Optimization, Via Repetitive Application of Linear Programming

Initial State (Tableau 0)

Branch Overloaded	Outage Causing Overload	Overload Ratio
21	21	1.43
21	23	1.48
22	23	1.16
23	23	2.79

First Approximation of $\Delta\gamma$

Branch	Capacity Addition
17	463.3
23	238.8

Cost = $\$2.71 \times 10^5$

Remaining Overloads (Tableau 1)

Branch Overloaded	Outage Causing Overload	Overload Ratio
17	21	5.26
	(and eleven other outages overloading branch 17)	
21	21	1.15
21	23	1.07
23	23	1.71

Second Approximation of $\Delta\gamma$

Branch	Capacity Addition
17	1496.
23	476.

Cost = $\$5.87 \times 10^5$

Table III-6. (Cont.)

Remaining Overloads (Tableau 2)

Branch Overloaded	Outage Causing Overload	Overload Ratio
17	21	2.89
	(and eleven other outages overloading branch 17)	
23	23	1.22

Third Approximation of $\Delta\gamma$

Branch	Capacity Addition
2	165.6
17	3048.
23	616.

Cost = $9.06 x 10^5

Remaining Overloads (Tableau 3)

Branch Overloaded	Outage Causing Overload	Overload Ratio
17	21	1.76
	(and ten other outages overloading branch 17)	
23	23	1.04

Fourth Approximation of $\Delta\gamma$

Branch	Capacity Addition
17	4444.
21	64.
23	640.0

Cost = $10.41 x 10^5

Table III-6. (Cont.)

Remaining Overloads (Tableau 4)

Branch Overloaded	Outage Causing Overload	Overload Ratio
17	21	1.20
17	23	1.04
23	23	1.00

Fifth Approximation of ΔY

Branch	Capacity Addition
17	5618.
23	646.

Cost = $10.82 x 10^5

Remaining Overloads (Tableau 5)

Branch Overloaded	Outage Causing Overload	Overload Ratio
17	21	1.04
23	23	1.00

A first solving of the linear program yields the vector $\Delta\gamma_{k,1}$ where $k = 1, \ldots, 24$. Capacity additions are suggested in Branch 17 and in Branch 23. Once these additions are performed, the overloads are computed again, and the results shown on Tableau 1.

The overload in Branch 22 has vanished, and those in Branches 21 and 23 have been reduced. But the addition of some capacity in Branch 17, where there was nothing initially, now compels us to watch for overloads also in Branch 17, which turn out to be huge. This is an illustration of the "fundamental difficulty" discussed in Sec. B-4.

A second application of the linear programming optimization now takes into account the overloads in Branch 17, and yield a second approximation of $\Delta\gamma$. The addition suggested in Branch 17 is considerably strengthened, and the corresponding cost increases also. The remaining overloads are shown on Tableau 2. They have all been reduced, but a 2.89 overload still subsists in Branch 17.

Three further approximations of $\Delta\gamma$ are then computed, using the same method. Capacity additions in Branch 2 and in Branch 21 are temporarily suggested, then abandoned in the next approximation. Finally, all the overloads are reduced to below 1.05 after the fifth approximation of $\Delta\gamma$, which consists of additions in Branches 17 and 23 only.

c. Second Optimization, Using Lock Feature

In the example just discussed, the first approximation* suggested additions in Branch 17 which had later on to be considerably strengthened in order to eliminate overloads in Branch 17 itself. This is a case when a slight modification decided by a human operator can greatly improve the optimization. We now decide to "lock" Branch 17; i.e., to forbid any capacity addition in that empty branch. This is done by substituting a huge positive number for C_{17} in the vector of the cost coefficients. The results obtained are shown on Table III-7.

The initial overloads are obviously the same, but the first approximation of $\Delta\gamma$ consists of capacity additions in Branches 19, 21, 23, and none in Branch 17. The corresponding cost is slightly higher than that without the lock on Branch 17 (compare with Table III-6), since there is less freedom

*In which Branch 17 was not watched, since it was initially empty.

Table III-7. Same Example as in Table III-6 but With Lock on Branch 17

Initial State (Tableau 0)

Branch Overloaded	Outage Causing Overload	Overload Ratio
21	21	1.43
21	23	1.48
22	23	1.16
23	23	2.79

First Approximation of $\Delta\gamma$ (with lock on branch 17)

Branch	Capacity Addition
19	359.6
21	75.
23	238.

Cost = $3.70 x 10^5

Remaining Overloads (Tableau 1)

Branch Overloaded	Outage Causing Overload	Overload Ratio
19	21	1.69
19	23	2.48
21	21	1.16
21	23	1.07
23	23	1.71

Second Approximation of $\Delta\gamma$ (with lock on branch 17)

Branch	Capacity Addition
19	601.
21	159.
23	497.

Cost = $7.47 x 10^5

Table III-7. (Cont.)

Remaining Overloads (Tableau 2)

Branch Overloaded	Outage Causing Overload	Overload Ratio
19	21	1.17
21	21	1.03
23	23	1.21

Third Approximation of $\Delta\gamma$ (with lock on branch 17)

Branch	Capacity Addition
19	744.
21	166.
23	652.

Cost = $9.32 x 10^5

Remaining Overloads (Tableau 3)

Branch Overloaded	Outage Causing Overload	Overload Ratio
19	21	1.02
21	21	1.00
23	23	1.03

of choice in the linear programming optimization. After three iterations, we finally obtain a set of additions in Branches 19, 21, and 23 that reduces all overloads to below 1.05 with a cost of 9.32×10^5; i.e., the cost is lower than the cost obtained when additions are permitted in Branch 17 (10.82×10^5). Thus the decision of locking Branch 17 is beneficial. This is not always the case, but the example has illustrated how human intervention can profitably guide the optimization.

2. Example of Complete Solution

We now show an example of discrete and dynamic optimization, although without making use of the man-machine idea for successive approximations.

The system considered is the same as in the continuous example above, although the active/passive partition has been slightly modified*. Thirteen sets of injection forecasts were provided, from January 1969 to August 1980, for both the active and the passive networks. A set of projected additions was also given in the table for the passive network only.

Sixteen possible branch states were defined (Table III-8). For each branch, only a subset of these branch states is available (for some branches all sixteen states are permitted). Tearing down lines is ruled out in this example, although provisions for this possibility are included in the dynamic-programming procedure (see Ref. 5, pp. 72-73).

A separate application of the continuous and static procedure for each time period yields the results of Table III-9. Then, the branch-by-branch dynamic programming procedure explained in Ref. 5, pp. 66-73 is applied to the results, and yields the five discrete additions of Table III-10.

This set of additions, although of interest in itself, should be the starting point of a series of man-machine successive approximations, as explained above, rather than a final answer.

3. Remark on Computation Times

In the examples so far discussed, the linear-programming problems that were solved had 24 rows and about 1200 columns. They usually needed three to five simplex iterations (pivotings), and were solved in less than one second, which

*See Table III-10 on page 41

Table III-8. Definition of the Branch States

(The entries in this table indicate the line additions corresponding to each state; they do not include the initial line compositions.)

States	69 kV	115 kV	230 kV	525 kV
1	0	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	0	0	1	1
6	0	1	0	1
7	0	1	1	0
8	0	1	1	1
9	0	2	0	0
10	0	0	2	0
11	0	2	1	0
12	0	1	2	0
13	0	0	0	2
14	0	1	0	2
15	0	0	1	2
16	0	1	1	2

Table III-9. Continuous and Static Capacity Additions ($\Delta\gamma$ in MW)

Branches	1969	1970	1971	1972	1974	1976	1979	1980
2	114.7	490.9	0	0	1095.	873.3	0	1849.
3	0	0	0	0	0	0	0	494.8
10	0	0	0	0	0	1407.	0	2533.
11	0	0	0	0	0	47.1	0	64.5
23	543.3	106.3	0	0	0	0	0	0

Notes:

- No addition in any other branch of the active network.
- The active/passive partition has been modified, as explained in Table III-10.

Table III-10. Recommended Line Additions
(First Dynamic-Programming Iteration)

Year	Type	Branch #	From	To
1969	230 kV	2	MIDWY	BENT
	230 kV	23	MCNRY	RDUP
1976	115 kV	10	TAP	BENTON
	115 kV	11	TAP	SCOT
1980	230 kV	3	MIDWY	300A

Note:

For this example, the active/passive network partition of Fig. III-3 has been modified as follows:

- Branches 28, 57, 58 belong to the active network;
- Branches 13, 20, 24 belong to the passive network.

is a remarkably short time for linear programs of that size. This very fast computation time is due to the particular nature of the problem, and it is one of the main advantages of the method proposed.

The dynamic programming computations also required less than one second.

4. Parametric Studies

Variations in the initial assumptions, such as the given generator-addition plan, bus-load forecasts, equipment costs, and possibly some other input data, will have an effect on the best transmission-system expansion policy. The prototype transmission planning program can be used efficiently to perform parametric studies, evaluating the effects of changes in these assumptions.

Effects of changes in the assumed loads have been investigated earlier in the context of a specific planning example (see Ref. 7). It was found for the specific cases studied that as long as the deviations from the forecasted loads were not unreasonably large, the branches whose capacities needed expansion remained the same; however, the amounts of capacity additions $\Delta\gamma$ and their timing was affected by changes in the load assumptions.[7]

Additional parametric studies were performed to investigate the effects of alternate generation plans and modified line-loading limits upon the optimum transmission expansion plan. The results of these computer runs are summarized in Tables III-11 and III-12. The base case in these tables refers to the data used in the example above.

Table III-11 shows the effects of relaxing the transmission-line power-flow constraints $\bar{\bar{\psi}}_k$ in each branch by 10 percent. Part (a) of this Table indicates the changes in the planned capacity additions $\Delta\gamma$ (branch capacity γ is treated as a continuous variable). Part (b) of the Table indicates the changes in the corresponding discrete line additions and their discounted costs. It is seen that overly conservative assumptions on line-loading limits can result in additional cost; this is one important reason for acquiring a better understanding of the stability-related power-flow constraints and developing methods for fast evaluation of the transient-stability properties of an interconnected system (see Chapter V-B).

Table III-12 illustrates the use of the program for studying the effects of alternative generation plans on the optimal transmission-system expansion. The

Table III-11. Comparison of Transmission System Expansion Plans Obtained with the Given Line-Loading Limits and the Same Limits Increased by 10 Percent

(a) Recommended Branch-Capacity Additions -- Continuous Case

Given Angular Constraints: $\bar{\bar{\psi}}$			Modified Angular Constraints: $1.1\bar{\bar{\psi}}$		
Branch	Year	Addition $\Delta\gamma$	Branch	Year	Addition $\Delta\gamma$
2	1969	114.7	2	1969	None
	1970	490.9		1970	249.2
	1974	1095.0		1974	867.5
	1976	873.3		1976	582.0
	1980	1849.0		1980	1993.0
3	1980	494.8	3	1980	None
9	1980	None	9	1980	1581
10	1976	1407	10	1976	1675
	1980	2533		1980	2367
11	1976	47.07	11	1976	None
	1980	64.51		1980	8.892

(b) Recommended Transmission Line Additions -- Discrete Case

Given Angular Constraints: $\bar{\bar{\psi}}$				Modified Angular Constraints: $1.1\bar{\bar{\psi}}$			
Branch	Year	Line	Discounted Cost*	Branch	Year	Line	Discounted Cost*
2	1969	230 kV	230.0	2	1970	115 kV	190
3	1980	230 kV	51.6	3	----	None	0
9	----	None	0	9	1980	115 kV	7.7
10	1976	115 kV	20.2	10	1976	115 kV	20.2
11	1976	115 kV	60.6	11	1980	115 kV	19.2
Total discounted cost*			362.4	Total discounted cost*			337.1

*All costs given in $1,000

Table III-12. Comparison of Transmission System Expansion Plans Obtained with the Given Generation Plan and with a Modified Generation Plan where 500 MW Capacity is Removed in Grizzly and Added in John Day in the Year 1974.

(a) Recommended Branch-Capacity Additions -- Continuous Case

Given Generation Plan			Modified Generation Plan*		
Branch	Year	Capacity Addition $\Delta\gamma$	Branch	Year	Capacity Addition $\Delta\gamma$
2	1969	99.2	2	1969	99.2
	1970	489.6		1970	489.6
	1974	1095.		1974	1021.
	1976	873.3		1976	512.5
	1980	1849.		1980	1526.
3	1980	494.8	3	1980	407.7
10	1976	1047.	10	1976	1880.
	1980	2533.		1980	2285.
11	1976	47.07	11	1976	None
	1980	64.51		1980	60.61
17	1980	None	17	1980	1750.

(b) Recommended Transmission Line Additions -- Discrete Case

Given Generation Plan				Modified Generation Plan			
Branch	Year	Line	Discounted Cost*	Branch	Year	Line	Discounted Cost*
2	1969	230 kV	230.0	2	1969	230 kV	230.0
3	1980	230 kV	51.6	3	1980	115 kV	44.7
10	1976	230 kV	20.2	10	1976	230 kV	20.2
11	1976	115 kV	60.6	11	1980	115 kV	19.2
17	----	None	0	17	1980	115 kV	5.1
Total discounted cost*			362.4	Total discounted cost*			319.2

*All costs given in $1,000

expansion schedule obtained with the base case is compared with the schedule based on a fictitious modified generation plan in which 500 MW of generation is transferred from Grand Coulee to John Day in 1974. This assumed change in generation capacities (introduced for illustration purposes only) results in savings in transmission-line construction costs due to the availability of an additional 500 MW production at John Day in 1974. Part (a) of Table III-12 shows again the continuous capacity additions, while Part (b) gives the corresponding discrete line additions and their costs.

E. Possible Improvements of the Prototype Program

The numerical results presented in this chapter, can be no better than the input data that were used, and the data were subjected to numerous simplifications. The systematic use of a man-machine approach, as discussed earlier, should be the way to make these simplifications acceptable.

Several important improvements can be suggested:

(1) Better understanding and modeling of line-flow constraints (This point is discussed in Chapter V-B.)

(2) Use of sparsity - programming techniques. (The computation times and storage requirements could be reduced by a systematic use of sparsity programming, especially in the computations associated with linear program.)*

(3) Use of More Accurate Power-Flow Programs. The inaccuracies introduced by the use of dc power-flow equations could be considerably reduced if the dc model were replaced by a nonlinear model. It would be possible, in particular, to use a so-called "ac real power only" model, which differs from the ac power-flow in that it still disregards reactive flows. This is a reasonable assumption in a planning context, since the investment expenses for reactive transmission are usually much smaller than the investment expenses for active transmission.

(4) Data Preparation Program. It would be of great importance to be able to make direct use of the information already available on BPA data files, and have a computer program for data preparation. In particular, the network decomposition

*At the time of publication of this report, Dr. E. C. Ogbuobiri of the Branch of System Engineering of BPA had already introduced sparsity-programming techniques into the prototype program delivered in September 1969, resulting in considerable reduction in computation time and storage requirements.

could be performed automatically, possibly with a method using the dc power flow, instead of the manual decomposition that led to the examples discussed in this chapter.

It is believed that most of the effort required before these planning techniques can be routinely used by the staff of the Branch of System Engineering is in the domain of automatic data preparation.

Chapter IV

PLANNING UNDER UNCERTAINTY

A. Introduction

All planning problems are subject to uncertainty. The use of deterministic planning methods is only a first-order approximation. These methods are accurate only if future predictions are themselves accurate.

Within the context of power system planning, the effects of future uncertainties may be quite substantial. Such uncertainties as future demand, future costs, and varying discount rates are of major importance. Unforeseeable changes such as major technological improvements, better equipment, better system controls, etc., are also factors the planner should account for.

This chapter discusses some powerful mathematical and computational tools that can cope with most of the problems encountered in planning under uncertainty. This chapter deals with two essential topics:

(1) Methods of stochastic planning -- Realistic and computational approaches to the stochastic planning problem involves, first, substantial reduction of the dimensionality of the system into a smaller number of configuration states; and second, the introduction of various stochastic optimization techniques.* These techniques have their respective advantages and disadvantages from the performance vs. computational aspects of the problem. Considerations such as implementation lead times, equipment age, and varying interest rates are some of the practical considerations that are dealt with. The various approaches are illustrated with an example.

(2) Methods of input data generation -- The prediction of future operating costs of the system takes into account the demand forecasts with their allocated statistical properties and the detailed simulation of future system operation to determine system costs. This procedure is often referred to as "production costing." The results of the simulation must provide the system planner with data that is

*For details on stochastic optimization theory and methods see Ref.'s 14-18 adequate for his stochastic optimization planning. In addition, some relevant material on load forecasting as it applies to planning is presented and discussed.

B. Methods of Stochastic Planning

The work described in this section is a continuation of the subjects treated in Ref. 2, Chapter IX, under the title, "Subsystem Optimization in the Presence of Uncertainty." A detailed description and analysis of the entire effort along these lines is found in Ref. 6. (It is recommended that the reader consult this Ref. 6 in addition to Refs. 2 or 4 for further detail).

Several contributions have been added under the present contract in the area of planning under uncertainty. These include:

(1) Development of several alternative stochastic optimization schemes with various advantages and disadvantages from the planning and computational points of view.

(2) Solution of problems related to lead times, equipment aging, and sensitivity of the investment plan to changes in certain assumptions and variables.

(3) Extension of a procedure originally designed for small systems -- the Olympia to Port Angeles expansion[2,4] -- to one that accommodates almost all power system planning problems, including the simultaneous expansion of the generation and transmission facilities of the whole system.

1. Survey of Stochastic Optimization Methods

As in Refs. 2 and 4, the important concept of generating system configurations x_t which correspond to the state of the system in year t is retained. This concept leads to a remarkable reduction in the dimensionality of the problem. Without such reduction, planning under uncertainty is virtually impossible from the computational point of view.

Let X_t be the set of allowable states (configurations) at Year t. This set depends on state x_{t-1}, since only a limited number of transitions are allowed. Let u_t be the decision made at time t as to the configuration at time $t+1$. Let d_t correspond to the demand at Year t. The equations of the system are

$$x_{t+1} = u_t \qquad t=1,\ldots,T-1 \tag{IV-1}$$

$$d_{t+1} = f(d_t, w_t, t) \qquad t=1,\ldots,T-2 \tag{IV-2}$$

where T is the planning period, w_t is a random variable corresponding to uncertainty in future demand (as obtained in a load-forecasting program) and $f(\,.\,)$ is a generally nonlinear function of its argument. The initial conditions are given by

$$\left.\begin{aligned} x_1 &= c \\ d_1 &= \delta \end{aligned}\right\} \tag{IV-3}$$

In addition,

$$\left.\begin{aligned} &x_t \in X_t \\ &u_t \in U_t(x_t) \end{aligned}\right\} \tag{IV-4}$$

where X_t and $U_t(x_t)$ are the respective sets of allowable states and controls at Year t.*

Define the cost function J as follows:

$$J \triangleq \sum_{t=1}^{t=T-1} \frac{\left[\ell_t'(x_t, x_{t+1}) + \ell_t''(x_t, d_t)\right]}{(t+\xi)^t} + \frac{L_{TI}(x_T)}{(1+\xi)^T} \tag{IV-5}$$

where $\ell_t'(x_t, x_{t+1})$ is the transition cost from state x_t to state x_{t+1}; $\ell_t''(x_t, d_t)$ is the cost of operating the system in state x_t when demand is d_t; $L_{TI}(x_T)$ is the salvage cost of the system at the end of the planning period and ξ is the interest discount rate. Included in cost ℓ_t'' is a reliability cost that corresponds to a very high cost if demand exceeds supply. (This is known as a soft constraint in system theory).

Since demand d_t is uncertain, the cost J is randomly distributed. Hence, it makes no sense to minimize J itself. The following methods correspond to various ways of formulating an optimization problem out of the above considerations:

a. Pure Feedback Approach

This is the approach already used in Refs. 2 and 4. The sequence of optimal decisions u_t^*, $t=1,..,T-1$ will be selected in order to minimize

$$J_{PF} = \underset{w_1,\dots w_{T-1}}{E} [J] \qquad \text{(IV-6)}$$

where $\underset{w_1,\dots,w_{T-1}}{E} [\,.\,]$ is the expectation of $[\,.\,]$ given the probability distributions of $w_1,\dots,w_{T-1}$. In this case the solution u_t^* is called a "policy" since u_t^* will depend strictly on x_t.

This approach corresponds to a stochastic optimization problem as described in the literature. Using the method of Dynamic Programming,[14] it is possible to obtain a solution, which leads to the following recursive algorithm:[6]

Define

$$I_s(x_s, d_s) \triangleq$$

$$\min_{u_s, u_{s+1}, \dots, u_{T-1}} E \left\{ \sum_{t=s}^{T-1} \frac{\ell_t'(x_t, u_t) + \ell_t''(x_t, d_t)}{(1+\xi)^t} + \frac{L_{TI}(x_T)}{(1+\xi)^T} \right\} . \qquad \text{(IV-7)}$$

*For simplicity, the present discussion does not take into account the problems of lead times and equipment aging. A discussion of these problems and their solutions will appear later.

Then, $I_t(x_t, d_t)$ is determined recursively according to

$$I_t(x_t d_t) = \min_{u_t(x_t, d_t)} \left\{ \frac{\ell_t'(x_t, u_t) + \ell_t''(x_t, d_t)}{(1 + \xi)^t} + \underset{w_t}{E} [I_{t+1}(u_t, f(d_t, w_t, t))] \right\} \quad \text{(IV-8)}$$

with the end condition

$$I_T(x_T, d_T) = I_T(x_T) = \frac{L_{TI}(x_T)}{(1 + \xi)^T} \quad \text{(IV-9)}$$

Since the number of states x_t is small, this backward recursive probram is easy to solve and is computationally feasible.

b. Pure Certainty Equivalent Approach

This is useful and accurate whenever the uncertainty introduced by the random variables w_t is very small. In this case, d_t is replaced by its expected value $\bar{d}_t$. The decisions u_t^* are selected as a schedule $u_1^*, \ldots, u_{T-1}^*$ which will minimize the deterministic cost:

$$J_{PCE} = \sum_{t=1}^{T-1} \frac{\ell_t'(x_t, u_t) + \ell_t''(x_t, \bar{d}_t)}{(1 + \xi)^t} + \frac{L_{TI}(x_T)}{(1 + \xi)^T} \; . \quad \text{(IV-10)}$$

The optimization problem here is purely deterministic and may be solved by forward or backward dynamic programming algorithms.

c. Certainty Equivalent Feedback Approach

In the pure certainty equivalent approach, expectations $\bar{d}_t$ are computed under the condition that d_1 is known. Since the present demand is always known, one can update the decisions made each year on the basis of this information.

The sequence of decisions, $u_t^*, \ldots, u_{T-1}^*$, is thus obtained by minimizing

$$J_{CEF} = \sum_{s=t}^{T-1} \frac{\ell_s'(x_s, u_s) + \ell_s''(x_s, \bar{d}_s)}{(1 + \xi)^s} + \frac{L_{TI}(x_T)}{(1 + \xi)^T} \tag{IV-11}$$

where

$$\bar{d}_s = E[d_s | d_t] \; . \tag{IV-12}$$

By solving the above problem, u_t^* is selected as the decision at Year t. At the next year, Year $t + 1$, a similar computation is made on the basis of the new state x_{t+1} and the new demand d_{t+1}, and so on. The resulting decisions are no longer schedules; they have become policies.

d. Pure Open-Loop Approach

Define

$$\ell_t'''(x_t) = \underset{d_t | d_1}{E} \; \ell_t''(x_t, d_t) \; ; \tag{IV-13}$$

i.e., $\ell_t'''(x_t)$ is the conditional expectation of $\ell_t''(x_t, d_t)$ given d_1. With this information, a schedule $u_1^*, \ldots, u_{T-1}^*$, of decisions can be computed that will minimize

$$J_{OL} = \sum_{t=1}^{T-1} \frac{\ell_t'(x_t, u_t) + \ell_t'''(x_t)}{(1 + \xi)^t} + \frac{L_{T1}(x_T)}{(1 + \xi)^T} \; . \tag{IV-14}$$

This is a deterministic open-loop optimization problem that can be solved via forward or backward dynamic programming algorithms.

e. Open-Loop Feedback Approach

This is similar conceptually to the certainty equivalent feedback approach. At every Year t, a decision schedule $u_t^*, \ldots, u_{T-1}^*$, is obtained by solving an open-loop optimization problem given d_t. However, only the decision u_t^* is employed for planning. In the following year, a new open-loop problem is solved, which utilizes the new information on d_{t+1}.

Mathematically, one writes

$$\ell'''_{s|t}(s_s) = \mathop{E}_{d_s|d_t} \ell''_s(x_s, d_s) . \qquad \text{(IV-15)}$$

The schedule $\{u^*_{s|t}, \; s = t, \ldots, T-1\}$, will minimize

$$J_{OLF} = \sum_{s=t}^{T-1} \frac{\ell'_s(x_s, u_{s|t}) + \ell'''_s(x_s)}{(1 + \xi)^s} + \frac{L_{TI}(x_T)}{(1 + \xi)^T} . \qquad \text{(IV-16)}$$

At Year t, only decision $u^*_{t|t}$ is used. Hence, at every year an open-loop approach is used depending on the new information gathered about d_t.

Table IV-1 gives a brief summary of the above approaches and their main characteristics.

Table IV-1. Criteria

Method	Form of Strategy	Goal
Pure Feedback	Feedback	Minimize Expected Present Cost
Pure Certainty Equivalent	Schedule	Minimize Present Cost of Mean Demand
Certainty Equivalent	Feedback	Use First Decision of Successive Pure Certainty Equivalent Optimizations
Pure-Open-Loop	Schedule	Minimize Expected Present Cost
Open-Loop Feedback	Feedback	Use First Decision of Successive Pure Open-Loop Optimizations

2. Example of Subsystem Expansion

a. Assumptions

The subsystem expansion plan considered in Refs. 2 and 4 is presented here in order to illustrate the various optimization approaches just discussed. The set of possible configurations is given in Fig. IV-1. The demand model is of the form

$$d_{t+1} = d_t (1 + a + b\, w_t), \quad t = 1, \ldots, 14$$
$$d_1 = 105 \text{ MW} \qquad \text{(IV-17)}$$

where

d_t = the peak demand observed in Year t at Port Angeles. The variable d_t is chosen here as "demand equivalent" in Year t .

a = the predictable mean increase per year.

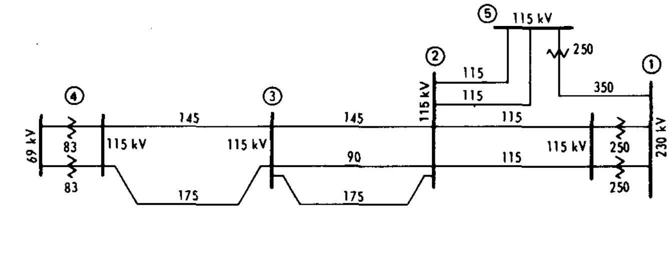
w_t = a white random scalar deviation, uniformly distributed with 0 mean and unit variance.

b = a constant used to normalize the white noise w_t to a desired variance.

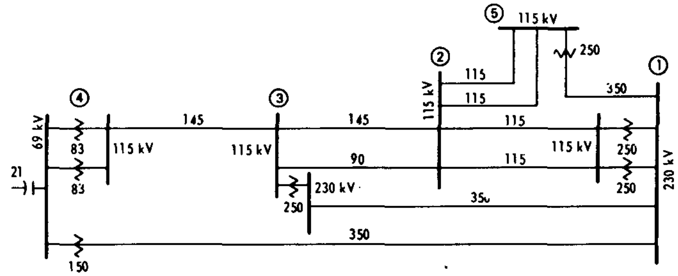
The parameters a and b used here are, respectively, 3% and 2.5%, which corresponds to a maximum demand growth of 7.5% per year and a minimum growth of -1.5% per year. Figure IV-2 gives the possible future peak demands for this model together with their probabilities of occurrence. Fig. IV-2 indicates the minimum, the average, and the maximum peak demands that can be expected in the future at Port Angeles, together with the known future peak loads at the other nodes.

The transisions costs $\ell_t'(x_t, x_{t+1})$ are given in Table IV-2.

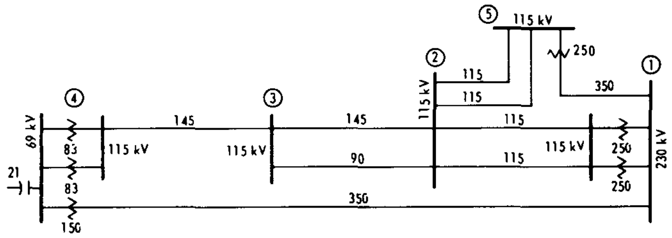
The cost $\ell_t''(x_t, d_t)$ of operating the system in Year t , if its configuration is then x_t and the actual peak demand is d_t , comprises two components: the cost of losses, and the penalties incurred for insufficient capacity. Other operating costs, such as maintenance costs, are neglected.



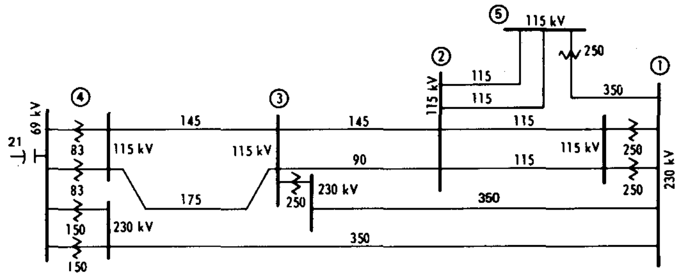
CONFIGURATION 0 ($\bar{x}^{(0)}$ = 145)



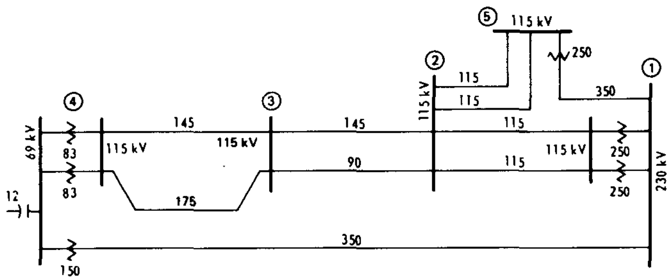
CONFIGURATION 4 ($\bar{x}^{(4)}$ = 145)



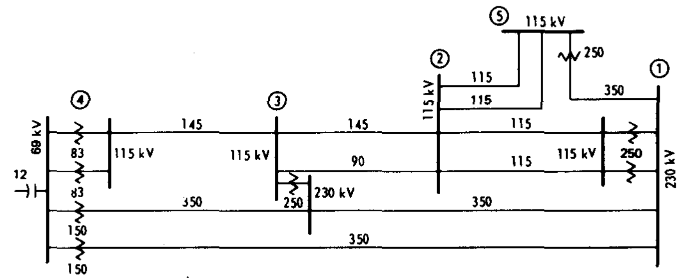
CONFIGURATION 1 ($\bar{x}^{(1)}$ = 145)



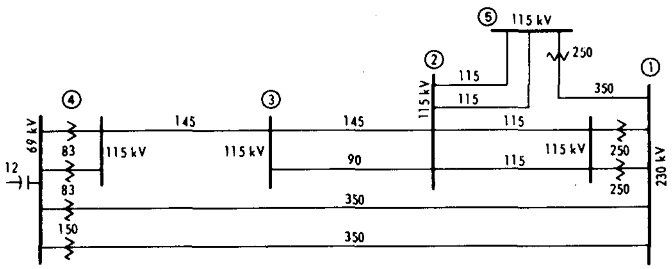
CONFIGURATION 5 ($\bar{x}^{(5)}$ = 166)



CONFIGURATION 2 ($\bar{x}^{(2)}$ = 166)



CONFIGURATION 6 ($\bar{x}^{(6)}$ = 295)



CONFIGURATION 3 ($\bar{x}^{(3)}$ = 295)

Transmission-line thermal limits and transformer ratings are shown in MVA units. The value $\bar{x}^{(i)}$ denotes the maximum power that can be delivered to Node 4 by Configuration i , with the largest-capacity line out of service.

Fig. IV-1. System Configurations i = 0, 1, . . . , 6 for the Olympia Port Angeles Expansion Plan

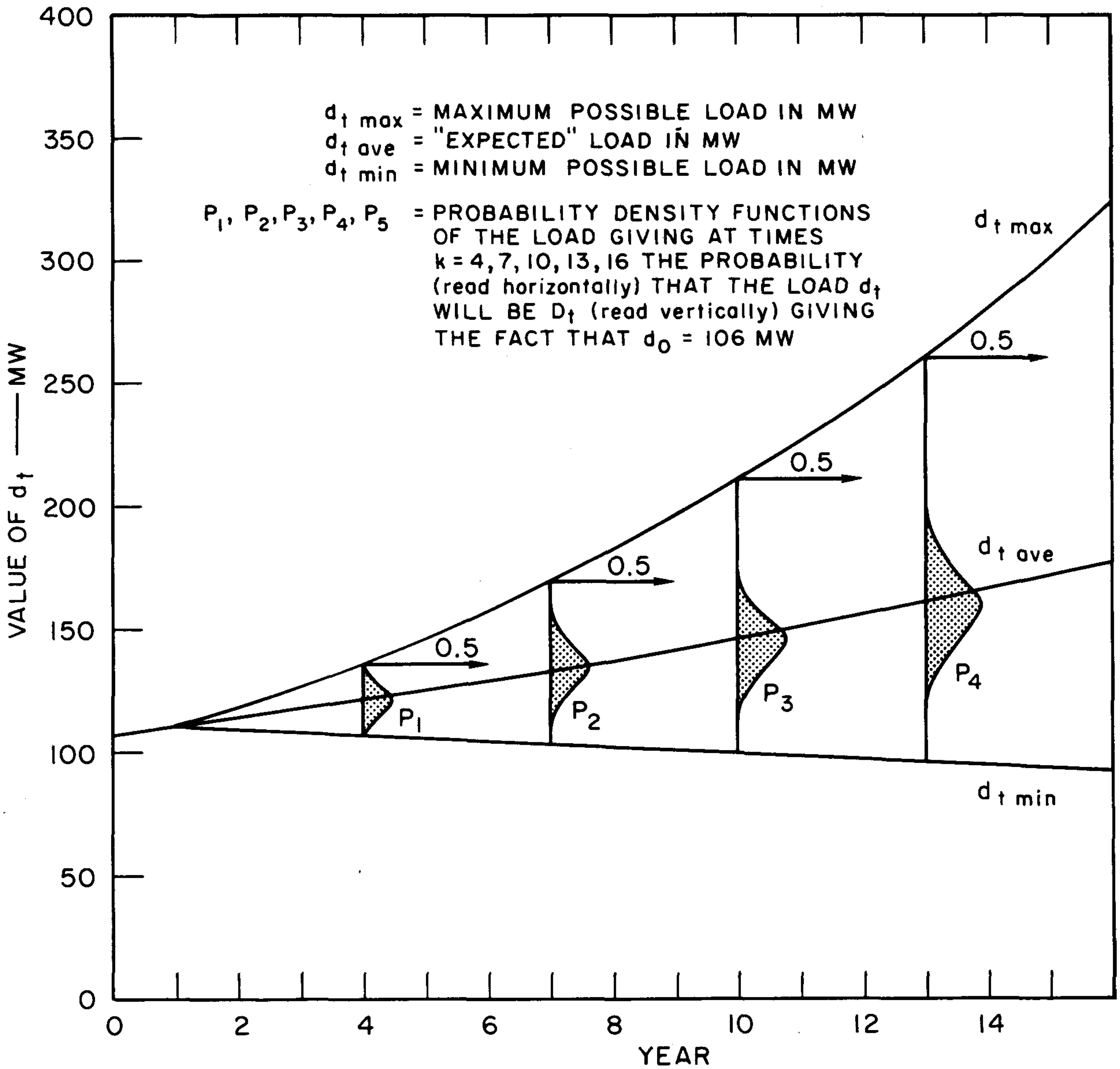


Fig. IV-2. Possible Peak Demands at Port Angeles in the Olympia/ Port Angeles Expansion

Table IV-2. Transition Costs $\ell_t(x_t, x_{t+1})$ Associated with an Investment Decision u_t To Go From Configuration x_t To Configuration x_{t+1} For The Olympia/Port Angeles Expansion.†

x_t \ x_{t+1}	Transition Costs (Millions of Dollars)						
	0	1	2	3	4	5	6
0	0	2.440	2.868	5.463	5.103	6.686	6.886
1		0	.618	2.998	2.663	4.471	4.491
2			0	2.555	2.708	3.818	4.018
3				0	.408	1.443	1.463
4					0	1.808	1.828
5						0	.200
6							0

†Configurations 0, 1, . . . , 6 are shown in Fig. 1. Transitions from higher to lower investment states are not considered here.

The power losses are computed by replacing first the actual configurations by the simplified, electrically equivalent, network models. A linearlized version of the standard load flow program is then used to generate the peak losses as a function of the system configuration and the actual peak demands.[2,4] Finally, the average cost of losses is obtained by assuming that average losses are half of peak losses (i.e., a 50% load factor); that the system is operated 8,760 hours per year; and that the unit cost of lost energy is $2/MWh. The resulting figure is taken as the cost of losses.

The penalty cost assumed is $1 million whenever the system capacity $\bar{x}_t$ is less than the actual peak demand d_t at Port Angeles.

In transmission system planning, an overriding reason for investing is to ensure the reliability of the transmission. This reliability constraint is defined in terms of either the tolerable loss of equipment without load curtailment, or the tolerable degree of curtailment, for given contingencies. The reliability criterion used here is as follows:

The system configuration x_t in Year t is said to be reliable if it is able to supply the peak demand at Port Angeles, with any one piece of equipment out of service and without any curtailment (simultaneous outages are not considered).

Therefore, the capacity $\bar{x}_t$ of the system configuration x_t in Year t is defined as the maximum peak demand at Port Angeles that can be satisfied that year, with the prescribed reliability. These capacities, as computed for the seven configurations considered here, are listed in Figure IV-1.

b. Results

Figures IV-3, IV-4 and IV-5 summarize the results of the expansion strategy using pure feedback, certainty equivalent, and pure open-loop approaches, respectively. Only in the case of pure feedback is a schedule of policy decisions obtained that is dependent on the observed peak demand in Year t . In the cases of certainty equivalent feedback and open-loop feedback, only the decisions of Year 1 (Figs IV-4 and IV-5, respectively) are applicable.

3. Some Practical Considerations

One of the main reasons for introducing the various stochastic optimization methods just described is to permit a large degree of computational flexibility, especially when additional constraints and/or problems are encountered. As observed in Ref. 6, these fall into the following three categories:

(a) Problems of lead times

(b) Problems of equipment aging

(c) Sensitivity analysis (see Sec. B-4).

In this section, a brief discussion of these topics and their relation to the various stochastic optimization schemes is given.

a. Implementation of Lead Times

In the above formulation it was assumed that it takes zero time to implement a decision u_t . In reality, however, the implementation of u_t will require some time, λ ; the value of λ will depend on u_t , the state x_t , and t itself (technological improvements).

The lead time is assumed to be an integral number of years; it is also assumed that during one transition no other transition can be decided, and that the system configuration remains as it was before the transition; finally, it is assumed that no configuration is in the process of

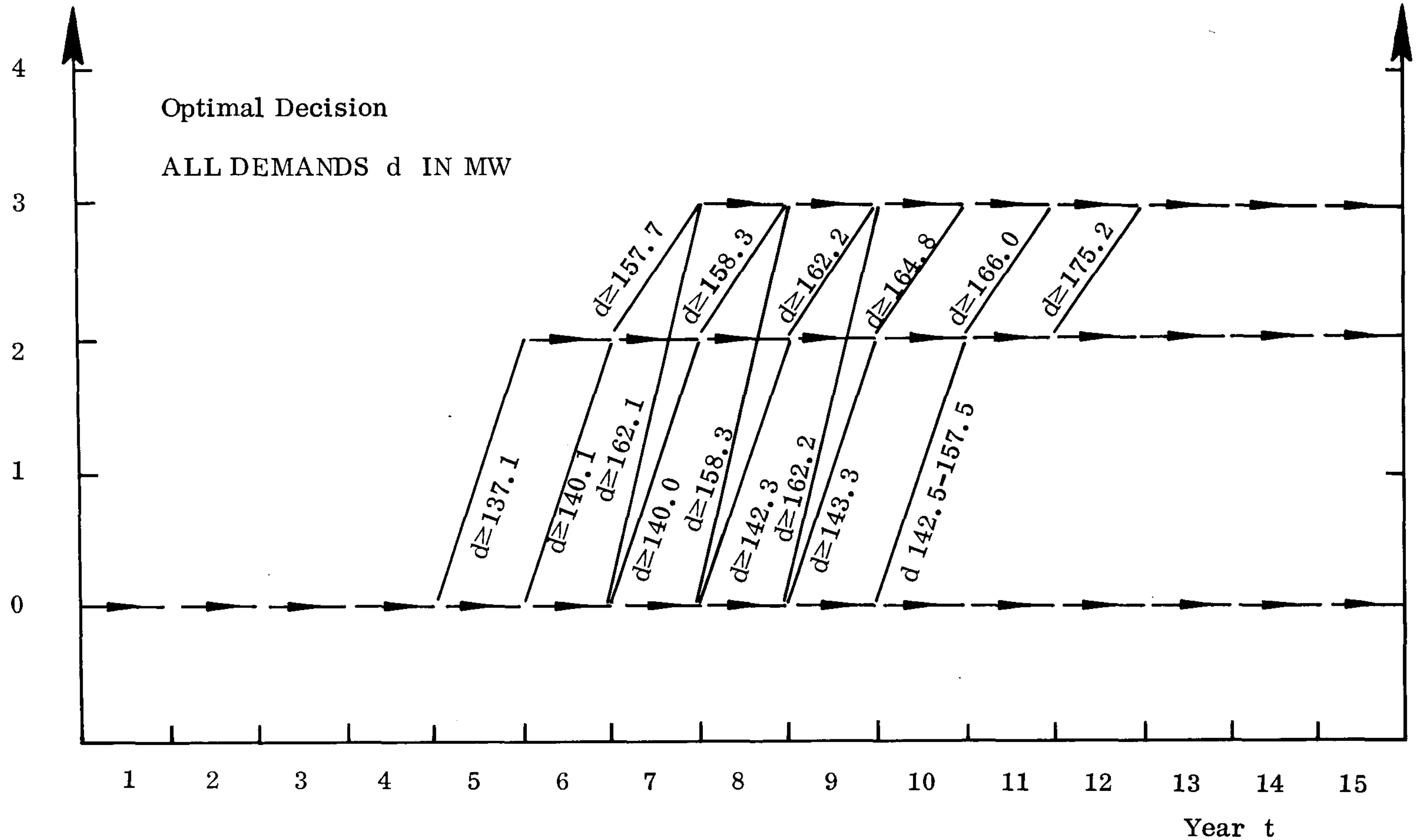


Fig. IV-3. Olympia/Port Angeles Expansion Strategy, as Obtained by the Pure Feedback Approach. An arrow indicates the transition decided at the end of Year t. This transition is made if the demand equivalent d_t exceeds (or is within) the values indicated. These values are only approximate because of quantization. New equipment is added instantaneously.

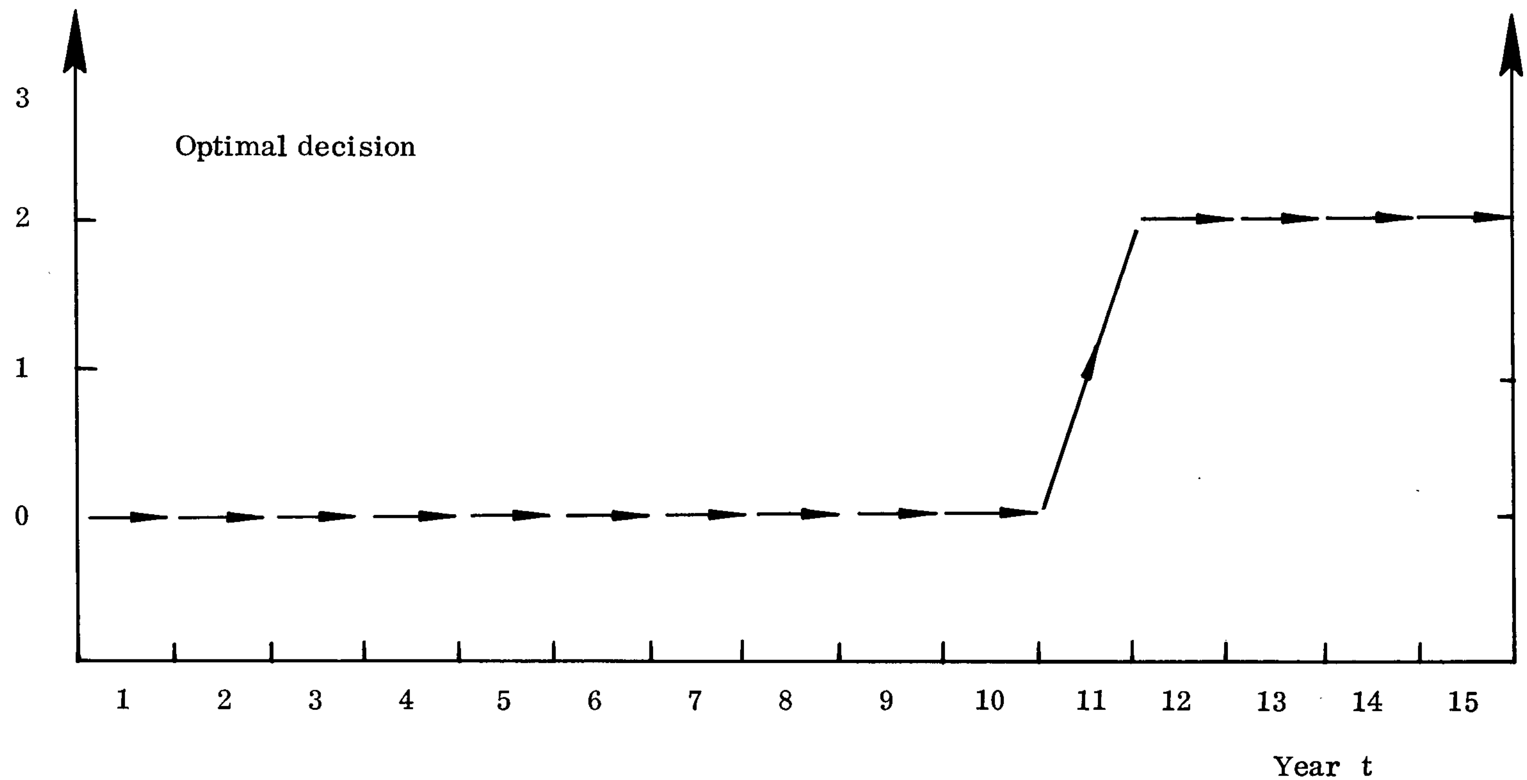


Fig. IV-4. Olympia/Port Angeles Expansion Schedule, as Obtained by the Pure Certainty Equivalent Approach. An arrow indicates the transition decided at the end of Year t. New equipment is added instantaneously.

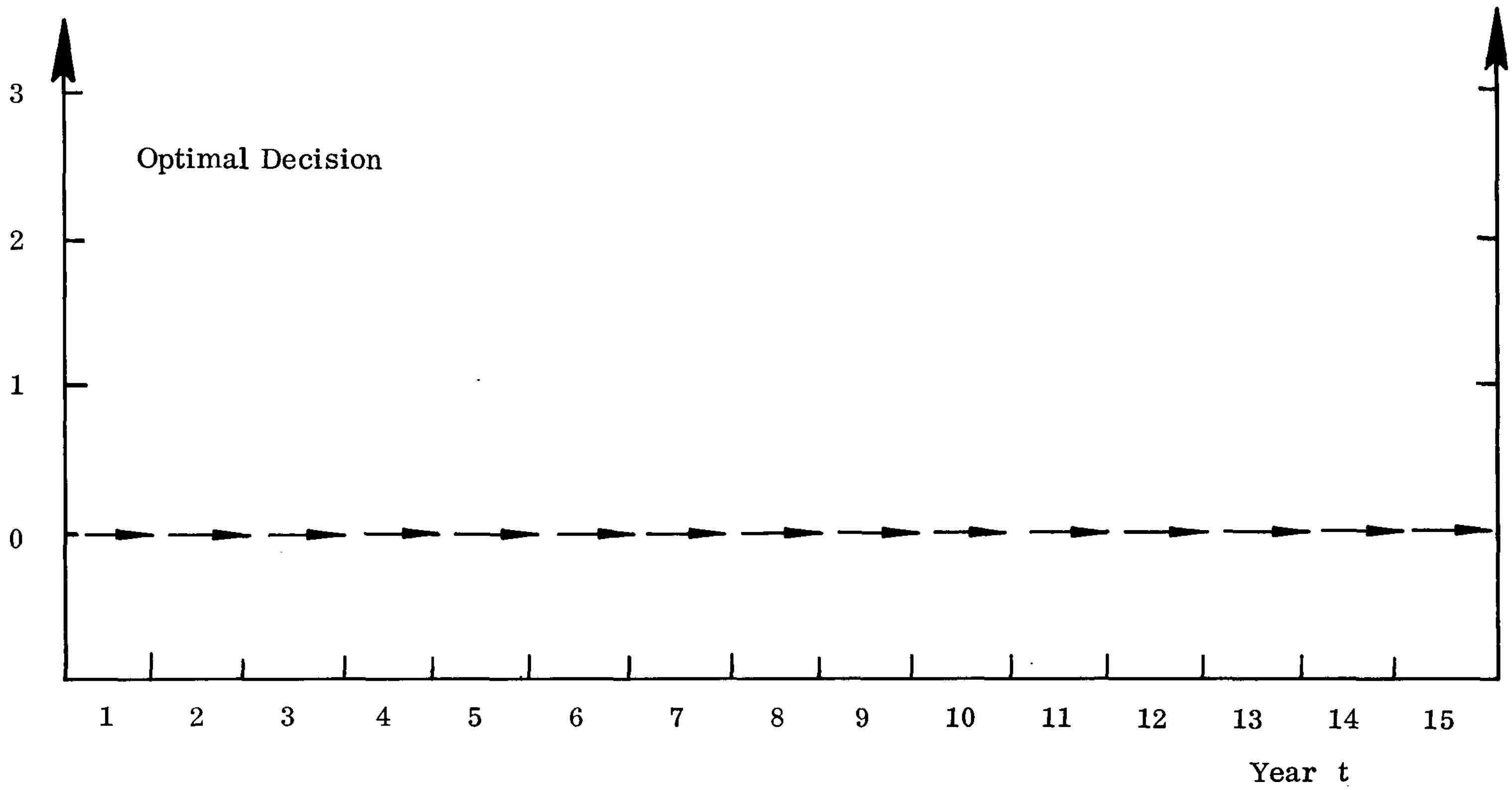


Fig. IV-5. Olympia/Port Angeles Expansion Schedule, as Obtained by the Pure Open-Loop Approach. An arrow indicates the transition decided at the end of Year t . New equipment is added instantaneously.

being implemented at horizon time. Mathematically one writes the following equality constraints:[6]

$$x_{t+\lambda_t+1} = u_t,\ t = 1,\ \lambda_1 + 1,\ \lambda_1 + \lambda_2 + 2,\ T - \lambda' - 1$$

$$x_{t+i} = x_t\ ,\ i = 1, 2, \ldots, \lambda_t,\ \text{if}\ \lambda_t \neq 0 \qquad \text{(IV-18)}$$

$$d_{t+1} = f(d_t, w_t, t)\ ,$$

where λ_t is the lead time in years given x_t , and u_t is the decision to be made; i.e., $\lambda_t = \lambda_t(x_t, u_t)$. Also, λ' is the lead time associated with the last decision before the end of the planning period. The cost criterion J is defined as follows:

$$J = \sum_{\substack{t=1,\lambda_1+1\\ \lambda_1+\lambda_2+2,\ldots,T-\lambda'-1}} \frac{\ell_t'(x_t, u_t)}{(t+\xi)^t} + \sum_{t=1}^{T-1} \frac{\ell_t''(x_t, d_t)}{(1+\xi)^T} + \frac{L_{T1}(x_T)}{(1+\xi)^T}\ . \qquad \text{(IV-19)}$$

Given these equality constraints and the cost J , the various stochastic optimization approaches are applied. Thus, conceptually the problem of lead times can be accounted for in the formulation.

On the computational level, it is noted that the pure feedback approach will be more difficult to apply than the other approaches, since in the other approaches, forward dynamic programming can be used. No knowledge of future decisions is needed in order to make the present decision. Since the pure feedback approach requires backward dynamic programming, some problems of excessive storage will result.

b. Equipment Aging

It is important to keep track of equipment age for at least two reasons:

(a) When equipment is removed, it can generally be sold or used elsewhere in the system. In both cases, it has a salvage value, depending on its age, which should be accounted for by the planner.

(b) At the end of the planning period, the system has a certain value representing the services it may render thereafter. This terminal value also depends on the age of equipment.

Mathematically, these requirements are dealt with by introducing a new variable, a_{xt}, which corresponds to equipment age. The time evolution of a_{xt} may be expressed as

$$a_{xt+1} = g(x_{t+1}, x_t, a_{xt}) \ . \tag{IV-20}$$

Furthermore, all of the terms in the cost criterion are modified to take into account the age-state a_{xt}, . This is illustrated in Ref. 6.

It is easy to see that the use of backward dynamic programming will lead to substantial computation difficulties. This occurs because the latest investments are determined before the early investments. The dates at which equipment is installed, and thus the age of the equipment, are not known when equipment is removed. The solution is to compute one investment plan for each possible age of the equipment at each time, but this approach significantly increases the dimensionality of the problem. In the deterministic case, or in other stochastic optimization procedures, the computations may proceed forward so that equipment aging can be taken into account without substantially increasing problem size.

4. Sensitivity Analysis

The investment plan obtained by any procedure cannot be better than the assumptions made in applying that procedure. The purpose of sensitivity analysis is to determine (qualitatively or, better, quantitatively) how the investment plan or the costs it yields are modified when the conditions of the problem are changed. It is preferable to achieve this goal without rerunning the program.

The use of dynamic programming in the optimal as well as the suboptimal procedures proposed readily permits a number of such sensitivity analyses. These depend to a large extent on the direction of the computations in the solution algorithms used, and on the presence or the absence of uncertainty. Four types of sensitivities are considered:

a. <u>Sensitivity to Changes in the System State</u>

In the deterministic case, the state of the system is defined as the state of its equipment at any time. In the case of uncertainty, the system state includes the actual values of the demand(s) which this equipment must satisfy. Changes in the system state occur when equipment is modified and/or when demands are varied. The results obtained provide sensitivity information in the following situations:

- When, because of circumstances unknown at the time of planning, it has not been possible to follow the computed investment plan until time t. The state of the equipment then is not the planned state. However, if backward dynamic programming has been used to determine that plan, then it is possible to determine readily what the next course of action should be between time t and the end of the planning period. The corresponding costs are also known.

- When the age of equipment may be not clear. The use of backward dynamic programming permits the planner to determine how variations in equipment age affect the investment plan and/or its cost.

- When, uncertainty exists, it is possible to evaluate the sensitivity of the investment plan and of costs to variations in the demands which are imposed on the system.

- Also, when uncertainty exists, the planner can determine which equipments are economically attractive and which ones are uneconomical, regardless of future demands.

b. <u>Sensitivity to Changes in Control</u>

The decision maker controls the expansion of the system by his investment decisions. Since many constraints--e.g., capital limitations--which are unforeseen at the time of planning may arise to limit the number of feasible decisions, it is of interest to determine the additional costs due to these constraints. This is easily done when backward dynamic programming has been used to compute the investment plans.

To provide for the situations where several investment alternatives are desirable, forward or backward dynamic programming can be used to yield second-best, or even n^{th}-best, investment plans. Finally, in all cases it is possible to determine how sensitive investment plans and costs are to variations in implementation lead times.

c. Sensitivity to Changes in Stage

The use of forward dynamic programming permits the planner to determine most readily the sensitivity of the investment plan and its cost to changes in the terminal value put on the system at the end of the planning period. This is a very important result, for terminal values are usually not well known. Forward dynamic programming also allows the planner to analyze the effects of reductions in the length of the planning period. These studies are usually not possible if backward dynamic programming is used in computing investment plans, since quantitative measures cannot be obtained easily, although qualitative conclusions can often be drawn. On the other hand, if backward dynamic programming is used, it is quite simple to determine the optimum time to make an investment, a result that forward dynamic programming will not provide.

d. Sensitivity to Changes in a Parameter

The uncertainties inherent in the determination of most parameters entering any planning problem make this type of sensitivity analysis especially desirable. Good cost estimates, for instance, are hard to obtain; discount rates are not well defined; etc. Unfortunately, it is difficult to measure the sensitivity of an investment plan to parameter variations when that plan has been obtained by dynamic programming. The amount of computation required for the measurement is usually equivalent to rerunning the program for all possible values of the parameters under study. However, an approximate method of evaluating cost variations due to parameter changes is described in Ref. 6. Two attractive features of this method are its simplicity and the fact that it provides pessimistic estimates of these variations, allowing the planner to make safe decisions

5. Comparison of the Various Methods

Certain advantages and disadvantages are associated with the above stochastic optimization methods. There are two ways of comparing the methods one with another.

First, from the practical and computational points of view, all those methods which employ forward dynamic programming have a definite advantage. These methods include: pure certainty equivalent (CE), certainty equivalent feedback (CEF), pure open-loop (OP) and open-loop feedback (OPF). Implementation of lead times, equipment aging and sensitivity analysis are all possible with little extra effort. The drawback of these methods is that, mathematically speaking, they are suboptimal.

Second, from the analytic point of view, it is quite difficult to assess the ordering of these methods. Denoting by I_{CE}, I_{CEF}, I_{OL}, I_{CL}, the minimum costs obtained by certainty equivalent, certainty equivalent feedback, open-loop, open-loop feedback and pure feedback (closed-loop), respectively, the following analytic results are affirmed (proofs are given in Ref. 6):

(a) If future demands are known perfectly, all of the above approaches are equivalent; i.e., for a deterministic problem,

$$I_{CE} = I_{CEF} = I_{OL} = I_{OLF} = I_{CL} .$$

(b) If future demands are not known perfectly, then the pure certainty equivalent approach is always inferior to or equivalent to the pure open-loop approach. The latter in turn is inferior or equivalent to the open-loop feedback scheme, which is itself inferior or equivalent to the pure feedback approach; i.e.,

$$I_{CE} \geq I_{OL} \geq I_{OLF} \geq I_{CL} .$$

The pure certainty equivalent approach does not take account of the high penalties that could be incurred for lack of reliability if mean values of demands do not actually materialize. The pure open-loop approach does weight these penalties, but does not incorporate the use of subsequent information about demands. The open-loop feedback approach uses actual demands to yield a better decision but the computations do not take account of this fact. The pure feedback approach uses all of the information available at any time t to yield the optimal decision at that time, and the fact that feedback is knowingly used results in a better decision scheme.

(c) The certainty equivalent feedback scheme is never better than (although it may be as good as) the open-loop feedback scheme. The certainty equivalent feedback scheme is often better than the pure certainty equivalent scheme, but not always; i.e.,

$$I_{CEF} \geq I_{OLF}$$

and

$$I_{CEF} \gtrless I_{CE} .$$

It is possible that the certainty equivalent feedback approach that incorporates feedback in the decision process may be inferior to a pure certainty equivalent approach that does not use feedback.

(d) A sufficient condition for the pure certainty equivalent approach to be as good as the pure feedback approach is to have the expected operating cost ℓ'' equal the cost obtained by replacing in ℓ'' the demands by their expected values for every period; i.e.,

$$\underset{d_t/d_1}{E} \ \ell''(x_t, d_t) = \ell''(x_t, \bar{d}_t) . \qquad \text{(IV-21)}$$

Apart from the very particular case where cost ℓ'' would be a linear function of demand, there does not seem to be any practical situation where the condition would hold.

(e) A necessary and sufficient condition for the pure open-loop approach to be equivalent to the pure feedback approach is to have the policy $u_t(x_t, d_t)$ obtained by the pure feedback approach be such that all decisions are the same for all demands d_t at any stage t.

Again this condition will generally not hold in real world situations, especially when reliability is a critical consideration.

(f) It has not yet been possible to derive theoretically the conditions under which the open-loop feedback approach or the certainty equivalent feedback approach would be equivalent to the pure feedback approach. These conditions obviously could be far less restrictive than Conditions (d) and (e), since they would require only that the initial decisions obtained by the open-loop scheme or the pure certainty equivalent scheme be the same as their pure feedback counterparts, without any consideration of cost.

It is clear that the certainty equivalent feedback method will be inferior in most circumstances because penalty costs are not correctly weighted. This is especially true in transmission system planning, where the penalties for lack of reliability can be very high. *

The open-loop feedback approach on the other hand does weight these penalties and will often be equivalent to the pure feedback approach for the following reasons:

- The planning problem is essentially discrete so that, although the pure open-loop and the pure feedback approaches may have different costs, they may yield the same first decisions.
- The set of possible investments is bounded at any stage. It is possible then that the optimal decision is determined by one of these bounds, whatever the decision scheme is.
- Demands are quite well known in the near future, say for the next five or six years, but not thereafter. The effects of uncertainty are therefore quite remote and are smoothed by discounting, so that their effects on present decisions are negligible.

It is believed that such conditions can be found in many practical planning situations encountered by BPA.

*These penalties may or may not have a practical meaning. For instance, they may represent the actual cost of unsatisfied demand, or they may be evaluated subjectively for no other purpose than to avoid additions.

C. Methods of Input Data Generation

1. Computation of Production Cost

The planning problem under conditions of uncertainty has been formulated as an optimization with respect to all possible system configurations x_t^i and demand d_{t+1}^i. The objective is to minimize the functional J in Eq. (IV-5). As stated in Sec. A, the production cost ℓ'' (x_t^i, d_t^i), which is required in the dynamic programming solution method, may be predicted by simulating the detailed operation of the system under consideration over the time interval, here one year, between subsequent stages. This simulation allows the stochastic planning techniques discussed in Sec. B to accommodate almost all of the power-system planning problems,* including the simultaneous expansion of the generation and transmission facilities of the whole system.

Several such simulation schemes, often referred to as production-costing schemes, have been reported in the literature[19-21]. Most of these approaches simulate the system operation by assuming that

$$w_t = 0 \qquad \text{(IV-22)}$$

or, equivalently, assuming a deterministic formulation of the problem. This basic production-costing scheme is briefly described in the following, followed by a brief discussion of a possible extension of the basic scheme to take into account various uncertainties in the future operation of a power system (e.g., when $w_t \neq 0$).

a. Deterministic Production Costing

The objective is to simulate the hour-by-hour operation of the system in accordance with some stated operational policy, in order to estimate the total cost of power generation during the simulation period (here one year).

An important input to the simulation is the forecast of daily load shapes or the load-duration curves for the major load nodes of the system (the generation of these forecasts would take into account the annual peak load d_t). The other inputs required include the cost and operating characteristics of generating units, river-water flow sequence, contractual obligations for

*Assuming that the planning engineer knows how to define good alternative system configurations $x_t^{(i)}$.

energy sales and purchases, maintenance schedules, and the operational policy.

The hourly costs are computed by estimating the total cost of operating the appropriate generation units in that time period in order to satisfy the forecast load. Most of the existing simulation techniques allocate the hydro and thermal generation on the basis of incremental costs[24]. However, such a procedure would not be economically optimum for a predominantly hydroelectric system because the operation of such a system influences not only the present operating costs but also the subsequent costs (the stored energy in the limited amount of water should be used optimally to maximize the cost of the replaced thermal generation). Since the correct utilization of the hydro capacity can lower the production cost considerably, the optimum hydro-scheduling is one of the most important aspects of a production-costing scheme. A detailed exposition of the problem can be found in Ref. 23; therefore, the discussion here will be limited to only a brief summary.

The optimization of hourly operation on the basis of yearly load and water-flow forecasts is computationally impractical. Hence, the problem is divided into two subproblems--long-term optimization and short-term optimization. While the long-term optimization yields an estimate of hydro energy to be used each day that would result in the minimum cost of thermal generation over the whole year, the short-term problem optimally allocates the daily hydro energy to different periods of the day. This daily allocation minimizes the cost of overall thermal generation during that day, taking into account the current stage of the storage reservoirs, the current water flow conditions, the availability of the generation units and the daily load-duration curve. In Ref. 23, the long-term optimization is solved by applying forward dynamic programming, while the technique of dynamic programming with successive approximations is used to solve the short-term optimization.

For computational convenience, the input/output curve of a steam unit may be quasi-linearized, leading to a finite number of load bands with constant incremental fuel rates. Then the loading of these steam units may be accomplished by forming two priority lists:

(1) Order of placing units on line

(2) Blocks of loads in the order of ascending incremental costs, for loading the units on line.

These lists should be updated frequently to account for the unit-outages for maintenance. Given the load curve for the day and the two priority lists, unit commitment can be accomplished. The system must also be checked for satisfactory spinning reserves and, if necessary, the loading procedure can be revised. Figure IV-6 shows a simplified Flow Chart for the simulation procedure.

b. Stochastic Production-Costing

A stochastic simulation scheme is required when it is necessary to estimate the production cost with several uncertainties entering in the future operation of a power system, such as the following:

(1) Errors in load forecasts (i.e., $w_t \neq 0$).

(2) Deviations of the river water flows from the projected sequence.

(3) Uncertainty in equipment availability due to forced outages.

(4) Uncertainty in fuel, facility and labor costs

The stochastic production-costing scheme is very similar to the deterministic one described above. The only difference is in the description of the input quantities, which are random in the stochastic case. A stochastic simulation scheme requires description of the inputs in the form of probability-density functions. These probability-density functions may be generated from existing historical data of the input quantities such as hourly load data, annual water flow data, and the unscheduled-equipment-outage data.[25,26]

The stochastic simulation consists of generating a value for each input from its probability-density function and using these values as inputs to the deterministic production-costing scheme. This procedure, repeated a number of times ($\sim$100) for each hour of system operation, would yield a probability-density function of the annual production cost. The simulation would also provide a means of checking the constraints on loss-of-load probability -- i.e., the inability to satisfy demand 1 day out of 10 years.

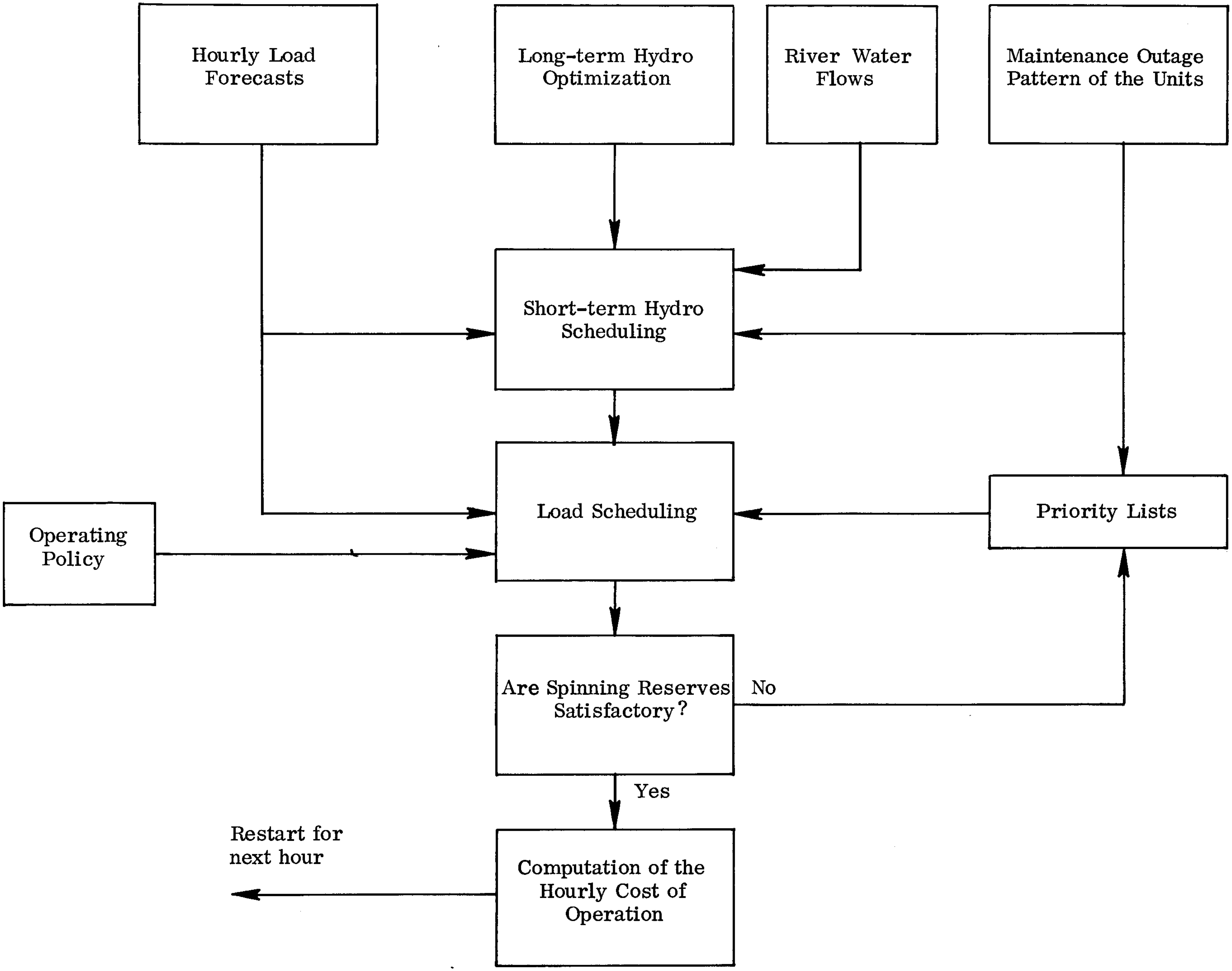


Fig. IV-6. Production Costive Simulation

c. Computational Considerations

While it seems conputationally quite feasible to use deterministic simulations to generate the production cost $\ell''(x_t, d_{t+1})$ for optimum planning of power systems, the stochastic case presents some problems. For illustration, let there be 100 possible system configurations for a planning period of 10 years. This means that 1000 simulations might be needed in the deterministic formulation and 100,000 in the stochastic one. The computational requirements of each simulation would depend on the complexity and the size of the system under consideration and may be considerable for a power system of the size of the BPA. Therefore, it seems that a direct application of the stochastic simulation procedure is computationally impractical. However, there are methods available to modify the procedure in order to make it computationally feasible, but the discussion of those methods is outside the scope of this report.

2. Long-Term Demand Forecasting

The planning techniques under uncertainty recognize that future demands cannot be predicted exactly and, hence, they entail the use of a probabilistic forecasting method with the following characteristics:

1. The load forecast at a specified future time is represented as a random variable defined by its probability distribution function.

2. Since annual, seasonal, or monthly demands can be computed from weekly demands in a systematic manner, it is desirable to forecast weekly peak demands since this type of forecast could be used for several other purposes in addition to providing annual or seasonal peaks.

The forecasting procedure[27-30] described here uses probabilistic techniques to determine the weekly peak demands directly.

Growth of the base load is extrapolated by a time polynomial which is determined by an exponentially weighted regression, with the more recent data being assigned progressively higher weights in an exponential manner. It is important to remove seasonal effects from the data before attempting to fit the time polynomial. This is accomplished by separating the weekly peak demand into a weather-sensitive component and a nonweather-sensitive component by using weather-load models determined from past demand and weather data. When the base load component of the nonweather-sensitive demand is "trended" by an exponentially weighted regression program, no difficulty should be encountered in fitting a second-order polynomial to the curve.

Since the dominant factor in the seasonal component of peak demand is the weather-sensitive component (heating and/or cooling load) useful weather load models are obtained by correlating demand with the appropriate weather variables. In making a forecast, extrapolation of weather-load models derived from historical data is necessary to project the model forward to the desired time of forecast. Since detailed weather forecasts are not available on a long-range basis, weather statistics rather than weather forecasts are used with the model to calculate corresponding statistics for the weather-induced demand. A similar procedure is used to extrapolate the economy-dependent component of demand, which is isolated by correlating the nonweather-sensitive demand with an appropriate economic indicator.

The forecast of each of the three components is obtained in form of probability distribution functions. These probability distribution functions are <u>not</u> assumed to be Gaussian, which makes it necessary to use nonparametric techniques to compute and combine the distribution functions. The forecasts are obtained in terms of discrete-quantized probability distributions. A quantitative measure of the forecast errors in readily available from the shape and spread of the computed probability distribution functions.

Chapter V

RELATED TOPICS

This chapter includes three important topics closely related to the subject matter of the previous two chapters. First, an approach to the integrated planning of interconnected systems and subsystems is outlined in Sec. A. Thereafter, methods for establishing the line-loading constraints used in the planning procedures are discussed in Sec. B. Finally, related work by other utilities is summarized in Sec. C.

A. Integrated System-Subsystem Planning

1. Introduction

So far, it has been assumed that planning for the main grid system can be independent of that for parts of the lower-voltage transmission subsystems. Part of the reason for assuming this independence is that whenever a detailed network including all subsystems is to be planned for the problem assumes very large dimensions.

If an efficient computational tool could be developed to tackle this problem of high dimensionality, then trade-offs among the various system and subsystem plans would be possible. The result could be a large saving.

In this section we propose two planning techniques for systems of large dimensions. One of those techniques makes use of the developing theory on hierarchical control and decomposition techniques. So far, several authors have suggested and/or used this theory for various aspects of the operation of power systems.[33-38] Methods of network reduction[32] are employed in the other technique.

The overall objective of these techniques is to generate a "continuous nominal" plan for all systems and subsystems. In the case of subsystems, this will become the basis for generating network configurations to be used for obtaining the overall planning policy.

2. Problem Statement

Denote by S_o the main-grid network and by $S_1, \ldots, S_N$ the subsystem networks, which are either connected to S_o or to one another. Let $J_o, J_1, \ldots, J_N$ denote the expansion costs of $S_o, S_1, \ldots, S_N$, respectively, over the planning period. The objective of integrated planning is to minimize

$$J = J_o + J_1 + \ldots + J_N \qquad \text{(V-1)}$$

subject to the following constraints:

(a) Load and generation forecasts,

(b) Line-flow limits and,

(c) Reliability.

Let $\gamma^o(t)$, $\gamma^1(t), \ldots, \gamma^N(t)$ denote the branch-capacity vectors of all lines in S_o, $S_1, \ldots, S_N$, respectively, at Year t. These capacity vectors will be the state vectors of $S_o, \ldots, S_N$. At every Year t the above constraints may be written in abbreviated form as follows:

$$f(\gamma^o(t), \gamma^1(t), \ldots, \gamma^N(t), t) \leq \beta . \qquad \text{(V-2)}$$

The problem of primary concern at this stage is the following: Given $\gamma^o(0), \ldots, \gamma^N(0)$, at Year 0; and the forecast of load and generation at Year t, it is required to determine $\Delta\gamma^o, \ldots, \Delta\gamma^N$, such that the cost

$$J = \langle C^o, \Delta\gamma^o \rangle + \langle C^1, \Delta\gamma^1 \rangle + \ldots + \langle C^N, \Delta\gamma^N \rangle \qquad \text{(V-3)}$$

is minimized subject to the constraints

$$f(\gamma^o(0) + \Delta\gamma^o, \ldots, \gamma^N(o) + \Delta\gamma^N, t) \leq \beta; \Delta\gamma^i \geq 0, \; i = 0, \ldots, N . \qquad \text{(V-4)}$$

In this formulation, the following assumptions have been used

(1) $\Delta\gamma^i$, $i = 0, \ldots, N$, is allowed to vary continuously (continuous nominal)

(2) $J_i = \langle C^i, \Delta\gamma^i \rangle$, $i = 0, \ldots, N$ (i.e., the expansion cost is a linear function of the expansion capacity)

(3) Only expansions of branch capacities are allowed (i.e., no lines are removed).

Thus the objective of the above problem is to generate a continuous nominal expansion which takes into consideration system-subsystem interactions.

3. Decomposition and Reduction

The objective of decomposition and/or reduction techniques is to generate a set of independent optimization problems of low dimensions out of the given high-dimensional problem. The proposed methods in this section correspond to certain simplifications pertinent to electrical networks. In addition, an iterative technique is used to arrive at the optimal solution.

Let $\gamma_k^i(t)$ be the k^{th} iteration step in computing $\gamma^i(t) = \gamma^i(0) + \Delta\gamma^i$, $i = 0, \ldots, N$, $K = 0, 1, 2, \ldots$ Initially,

$$\gamma_o^i(t) = \gamma^i(0), \quad i = 0, \ldots, N. \qquad \text{(V-5)}$$

Due to decomposition and/or reduction there will be two forms of iteration:

(a) Over the index $k = 0, 1, 2, ..$

(b) Over the index $i = 0, 1, \ldots, N$.

a. Integrated Planning Using Network Reduction

In this technique the expansion of each S_i is considered at an iteration step. The rest of the systems and subsystems are represented by reduced equivalent.

1) Initialization of Algorithm:

$$\gamma_o^i(t) = \gamma^i(0) \; ; \; i = 0, \ldots N. \qquad \text{(V-6)}$$

2) Reduced Inequality Constraints:

$$\text{Given } \gamma_k^o(t), \ldots, \gamma_k^{i-1}(t), \gamma_{k-1}^i(t), \ldots, \gamma_{k-1}^N(t),$$

it is required to compute

$$\gamma_k^i(t), \; i = 1, \ldots, N. \qquad \text{(V-7)}$$

Let θ denote the vector of voltage node angles at all the buses of the overall system. The electrical equations of the overall system can be written as

$$g(\gamma^o, \ldots, \gamma^i, \ldots, \gamma^N, \theta) = 0 . \tag{V-8}$$

Let θ be the solution to this set of equations. Suppose that a change of γ^i is required, so that

$$\gamma^i \rightarrow \gamma^i + \delta\gamma^i ; \tag{V-9}$$

then, one can write

$$\delta\theta \rightarrow \theta + \delta\theta . \tag{V-10}$$

Using first-order sensitivity relations, one can show

$$\delta\theta = - \left(\frac{\partial g}{\partial \theta}\right)^{-1} \left(\frac{\partial g}{\partial \theta^i}\right) \delta\gamma^i . \tag{V-11}$$

Let B_i be the bus-branch incidence matrix restricted to subsystem S_i ; i.e.,

$B_i\theta = \psi^i \stackrel{\Delta}{\equiv}$ vector of angular differences belonging to S_i .

One may write

$$\delta_\psi{}^i = B_i \delta\theta = -B_i \left(\frac{\partial g}{\partial \theta}\right)^{-1} \left(\frac{\partial g}{\partial \delta^i}\right) \delta_\psi^i . \tag{v-12}$$

Define

$$G_k^i = \left. \frac{\partial g\left(\gamma_k^o, \ldots, \gamma_k^{i-1}, \gamma^i, \gamma_{k-1}^{i+1}, \ldots, \gamma_{k-1}^N, \theta\right)}{\partial \gamma^i} \right|_{\gamma_{k-1}^i}$$

$$H_k^i = \left. \frac{\partial g\left(\gamma_k^o, \ldots, \gamma_k^{i-1}, \gamma_{k-1}^i, \ldots, \gamma_{k-1}^N, \theta\right)}{\partial \theta} \right|_{\theta_k^i} \tag{V-13}$$

where θ^i_k is the solution of

$$g\left(\gamma^o_k, \ldots, \gamma^{i-1}_k, \gamma^i_{k-1}, \ldots, \gamma^N_{k-1}, \theta\right) = 0 . \tag{V-14}$$

The vector γ^i_k is given by

$$\gamma^i_k = \gamma^i_{k-1} + \delta^i_k \tag{V-15}$$

where δ^i_{k-1} is the optimal solution to the linear program:

$$\underline{\underline{\psi}}^i \leq \psi^i_k - B_i (H^i_k)^{-1} G^i_k \delta^i_k \leq \bar{\bar{\psi}}^i$$

$$\delta^i_k \geq \alpha^i(0) - \alpha^i_{k-1} \tag{V-16}$$

$$J_i = \langle C^i , \delta^i_k \rangle, \; i=1, \ldots, N .$$

3) Iterative Algorithm:

In the above algorithm one computes sequentially γ^1_k, $\gamma^2_k, \ldots, \gamma^N_k$. The next step is to compute γ^o_{k+1} given $\gamma^o_k, \ldots, \gamma^N_k$ by applying the above linear programming procedure. In summary, the iterative algorithm using network reduction is the following:

(i) $\gamma^i_o = \gamma^i(0)$, $i = 0, \ldots, N$

(ii) Given γ^i_k, $i = 0, \ldots, N$, compute γ^o_{k+1} using linear programming.

(iii) Given $\gamma^o_k, \ldots, \gamma^{i-1}_k, \gamma^i_{k-1}, \ldots, \gamma^N_{k-1}$, compute γ^i_k using linear programming.

We have no proof that this algorithm will converge. However, experience with the "continuous nominal" algorithm has shown very fast convergence properties.

b. Integrated Planning Using Decomposition

The major difficulty with network reduction is that in iterating on subsystems S_i, full account is taken of the other subsystems. In reality, however, the effects of disturbances in S_i are mostly felt within S_i and its nearest neighbors.

Let θ^i be the vector of nodal angles of a network that contains S_i; i.e., θ^i will correspond to all buses within S_i plus some outside buses. Let θ^{-1} correspond to the rest of the angles of the overall network; i.e.,

$$\theta = \begin{bmatrix} \theta^i \\ \cdots \\ \theta^{-i} \end{bmatrix} . \tag{V-17}$$

The electrical equations of the overall system can be written as

$$\begin{aligned} & g^i (\gamma^o, \ldots, \gamma^N, \theta^i, y^i) = 0 \\ & g^{-i} (\gamma^o, \ldots, \gamma^{i-1}, \gamma^{i+1}, \ldots, \gamma^N, \theta^{-i}, y^i) = 0 \end{aligned} \tag{V-18}$$

where

$$y^i = y^i (\theta^i, \theta^{-i}) .$$

The vector variable y^i is called a "coordination" variable. In general, it will correspond to line flows between θ^i and θ^{-i} nodes. For computational purposes, we postulate that the network corresponding to θ^i variables is sufficiently large that a disturbance in S_i will cause little change in y^i. Hence, we can write

$$\frac{\partial g^i}{\partial \gamma^i} \delta \gamma^i + \frac{\partial g^i}{\partial \theta^i} \delta \theta^i = 0 . \tag{V-20}$$

This implies

$$\delta\theta^i = -\left(\frac{\partial g^i}{\partial\theta^i}\right)\frac{\partial g^i}{\partial\gamma^i}\,\delta\gamma^i \tag{V-21}$$

By including ψ^i, which is the vector of angular differences among θ^i variables, the above equation will yield the inequality constraints required for the iteration step involving changes in S_i.

By definition,

$$\psi^i \triangleq B_i\,\theta^i . \tag{V-22}$$

The inequality constraints can be written as

$$\underline{\underline{\psi}}^i \le \psi^i - B_i\left(\frac{\partial g^i}{\partial\theta^i}\right)^{-1}\frac{\partial g^i}{\partial\gamma^i}\,\delta\gamma^i \le \bar{\bar{\psi}}^i . \tag{V-23}$$

The rest of the iteration algorithm is the same as discussed in two uses of network reduction.

4. Discussion

The differences between the decomposition and network reduction algorithms are the following:

(1) Network reduction involves the inversion of a large matrix $\partial g/\partial\theta$ whose dimension is the same as the overall system. Decomposition will involve the inversion of a matrix $\partial g^i/\partial\theta^i$ whose dimension is comparable to that of subsystem S_i.

(2) The linear program required by network reduction is considerably larger than that required by decomposition.

(3) Decomposition allows for overlapping among neighboring systems. This is not the case with network reduction. Thus the decomposition algorithm may result in considerably faster convergence than the reduction algorithm.

(4) Then main difficulty in the decomposition algorithm is in the selection of networks containing each subsystem S_i and guaranteeing that disturbances within S_i are only felt within those networks.

The techniques of network reduction and network decomposition are applicable to integrated system-subsystem planning. These techniques will generate a "continuous nominal" plan which takes into account system-subsystem interactions due to disturbances and outages.

It is conjectured that, from the computational point of view, the method of decomposition is faster and simpler than that of reduction. The validity of these approaches and the rates of convergence should be ascertained by experiments.

B. Transmission Line Loading Limits

1. General

The maximum capacity of a transmission line is required as an input to any program developed to optimize the transmission system expansion. In Chapter III, Sec. D-4, it was shown experimentally (via parametric studies) that the transmission facility expansion requirements (and hence investment costs) are very sensitive to the line-loading limits. Hence, there is a significant economic advantage to be gained by accurate modeling of these constraints.

In addition to being one of the main inputs for facility expansion optimizations, which are the main concern of this report, accurate knowledge of the line loading constraints is also required for the following:

(a) Optimum operation of the power system from the point of view of minimizing the operating cost subject to line loading constraints[39-41]

(b) Statistical methods for reliability analysis of a power system in which the probability of transmission outages is considered.[42]

(c) Determination of the capacity cost to be charged to neighboring utilities that use the line for power wheeling.

The physical phenomena to be considered when setting the line loading limits are the following:

- Line heating
- Ability to maintain near-nominal voltage throughout the system
- Ability of the system to survive certain specified faults that set up a transient which may lead to loss of synchronism.

The purpose of the following discussions is to determine how the capacity constraints resulting from these phenomena enter into the optimum transmission-system expansion program that has been developed in the course of the project. It will be seen that the thermal and voltage phenomena can be handled relatively easily. Transient-stability, however, is very difficult to deal with, although some encouraging progress has been made as part of the project; for details see Chapter V of Ref. 12.

2. Thermal Case

The assumption usually made is that the maximum line flow, as determined by heating considerations, must not be exceeded in the course of normal operation nor after the occurrence of any one of a list of postulated contingencies.[12]

While this assumption can be accomodated by the optimization procedures developed, it may well be overly conservative for the following reason: The increase in temperature to be avoided does not occur instantaneously; hence, some time (of the order of minutes) remains to shift generation, to modify the interchanges with neighboring utilities or to drop interruptible customers after the actual occurrence of an outage. Automatic control systems capable of protecting the lines against excessive heating have been described in the literature.[43]

Although the thermal constraint presently used may be too conservative, this constraint is a property of the line itself and is independent of the remainder of the system. Specifically, it is possible to associate with each line k a number ψ_k, which is the maximum angular difference that can exist for a prolonged period between its terminals.

For long radial lines, the steady-state stability limit that corresponds to an angular difference of 90 degrees could be treated as a property of the line, somewhat like the thermal limit for shorter lines. However, steady-state stability is a valid consideration only if system transients can be eliminated with the help of suitable stability augmentation controls yet to be developed. (For more details of this possible long-term development, see Ref. 44)

It can be shown that under certain very restricted assumptions, notably a system consisting of short radial lines, the line should be operated at a loading limit for which the incremental cost of yearly losses equals the incremental cost of yearly investment. The argument is purely economic, and altogether excludes thermal, voltage, and stability phenomena. For complex networks, particularly those containing long lines, it can be easily shown that this economic loading limit is not optimal and often not even feasible, as it is independent of line length.

Some system planning departments use the surge impedance concept to develop line-loading limits. When operating at the surge impedance loading, the voltage at the receiving terminal approximately equals the voltage at the sending terminal. Again it can be easily shown that for a transmission system, as opposed to a single line, a loading limit derived from surge impedance considerations will be non-optimal; further, the relative costs of VAR sources to improve the system's voltage performance need to be weighed against the costs of additional transmission capacity.

3. Planning With Thermal Constraints Only

In the previous sections of this report two distinct approaches toward finding the optimum facility-expansion plan have been described. These approaches may be categorized as semi-discrete (LP with contingency check on the strongest known line or on an entire branch), and discrete (dynamic programming with exact contingency test); each approach satisfies a different planning objective. In particular, semi-continuous approaches are relatively simple but are not optimum in a strict mathematical sense, whereas discrete approaches are complex from the computational standpoint but are optimum. Each of these approaches involves a somewhat different way of taking the thermal constraint into account:

(a) In the semi-discrete approach, the composition of a branch is not considered accurately, but the optimization is required to produce a network capable of keeping the branch angles within specified limits after a branch (or its strongest known line) has been lost.

(b) In the discrete approach, the detailed composition of a branch is retained in the computer memory and the facility expansion is optimized subject to the constraint that no line overload may occur after any line in the network has been lost (or subject to a similar logical constraint statement).

4. The Transient Stability Case

The already considerable computational difficulties encountered for the thermal case are made more complex, by orders of magnitude, when transient stability considerations are included as constraints in the optimization.

The difference in the constraining relations that correspond to the thermal and the transient stability case are illustrated in Fig. V-1 (a) and (b) below for two branches k and l.

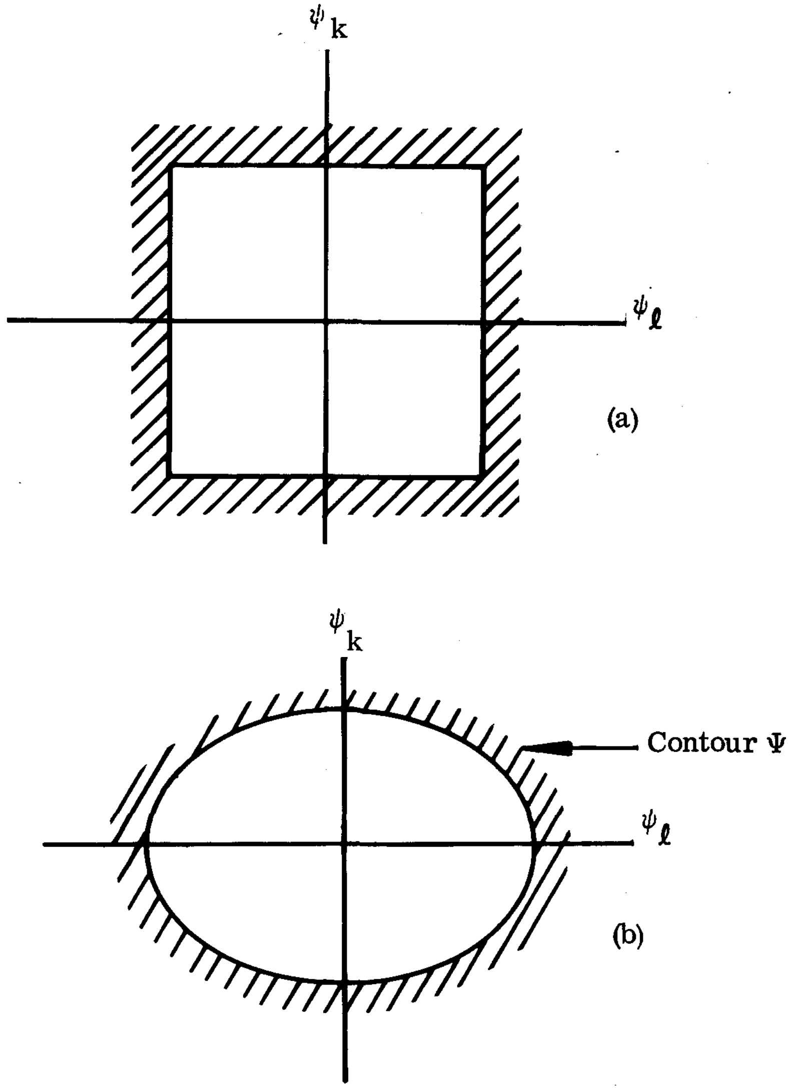


Fig. V-1. Line Loading Limits for a Thermal Case (a) and a Hypothetical Transient Stability Case (b) in the Space of Angular Differences ψ

The boundary Ψ shown defines a region such that any of a number of transient conditions can be survived. Note that Ψ depends on the exact composition of the network and the generation/load pattern.

The contingency hypotheses usually made when testing a system for its transient stability properties are either a three-phase short circuit followed by an unsuccessful reclosure or the loss of a major block of generation. In either case, it is not possible to associate line-loading limits with certain branches because the problem is global in nature. By this we mean that the loading that separates a stable case from an unstable case on a given line is not a characteristic of that line, but depends on the injection pattern throughout the system, the precise nature of the contingency chosen, the configuration of the remainder of the system, and the protection and transient-stability augmentation systems in use. The global nature of this constraint is illustrated in Fig. V-1(b), where a safe region of operation is defined.

Whereas testing a transmission system for thermal overload in the event of a contingency merely involves the solution of a power-flow equation (usually of the dc variety), testing for transient stability and possible loss of synchronism requires consideration of the dynamic effects. In Section B-6 the analytical and simulation methods available and developed in the course of this and other BPA contracts for stability analysis are reviewed. At this point, it is sufficient to categorize these methods as follows:

- Transient stability programs for simulating the effect of faults, mainly short circuits.
- Energy-balance procedures[45,46] analyzing the long-term transients caused by sudden changes of generation or load.
- Simplified approaches, such as the single-machine/infinite-bus reductions[2], the concepts in Chapter V of Ref. 12, and others.

Whereas it is very time consuming to run a full-fledged transient-stability program, the energy-balance simulation and the simplified transient-stability analysis procedures discussed in Ref. 12 can be made very fast and probably sufficiently accurate, considering the numerous uncertainties that affect the specific details of future prime movers, control practices, etc.

<u>Note</u>:

In some system planning departments, a prefault angular limit of the order of 35 to 40 degrees is used with the hope of accommodating transient stability phenomena. The justification of this approach can be traced back to Ref. 53, in which a very specific situation (a single generator connected to an infinite bus through two parallel lines of which one is lost) is considered with the equal-area criterion. The results of that study[53] cannot easily be extrapolated to the transient-stability properties of more complex systems, and the angular limit developed there applies only to the very specific case considered. In particular, interarea stability problems caused by upsets in generation or load are in no way related to this angular limit.

5. Planning With a Transient-Stability Constraint

It is useful to review in what way the transient stability constraints can be associated with the semi-discrete and discrete models developed for optimum power-system expansion.

For purposes of discussion, it is easiest to start with the discrete case; the outcome of a number ℓ of specified contingencies that cause transients (one for every postulated fault condition) would be analyzed by the computer for each facility addition that it considers at every stage and every state of dynamic programming optimization or of some other discrete search procedure. If any one of these ℓ contingency tests, performed at time t for state x, led to unacceptable transient performance, then this state would be judged infeasible and omitted from further consideration. This is illustrated in Fig. V-2.

Although this approach could lead, in theory, to a very large number of transient stability checks, it may still be feasible for the following reasons:

(i) It is not necessary to test for a very large number of fault conditions since most of the weak spots of a system can be identified, often by inspection or by means of a limited number of transient stability simulations performed separately for systems of the type being expanded.

(ii) Transient stability runs can be parameterized and suitable interpolations can be made; such interpolations are discussed by Dy-Liacco[37] and in Chapter V of Ref. 12 for operating purposes, but the same principles could be adapted to planning. If the states x in Fig. V-2 were suitably ordered, a feasible region X corresponding to each state t of the planning process could thus be found.

(iii) If, as often stated, the stability problems with the outside world are of greatest concern, greatly simplified models can be used for transient-stability analysis. These models would most likely be of the energy-balance type discussed in Ref. 12.

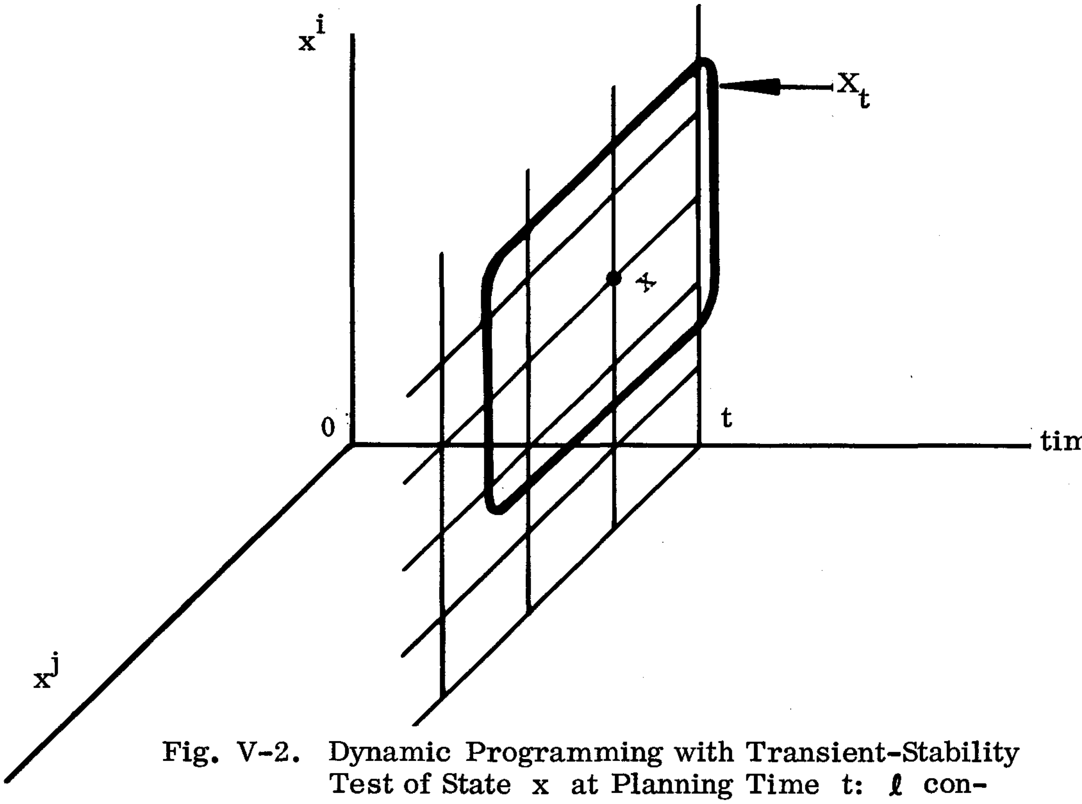


Fig. V-2. Dynamic Programming with Transient-Stability Test of State x at Planning Time t: ℓ contingencies are analyzed and x is retained only if all ℓ contingencies are survived. The contour X_t contains all the states that satisfy the ℓ contingency tests at stage t .

For the semi-discrete (linear-programming) approach, a difficulty already pointed out in Chapter III in conjunction with the angular-difference constraint arises: The detailed composition identified in Fig. V-2 by the state x of the planned system is not known at time t. Rather, the linear-programming procedure finds an optimum branch capacity vector γ_t in the presence of the stability-related constraints on ψ, which may be represented by a contour Ψ of the type illustrated in Fig. V-1(b) (or a piecewise linear approximation thereof to avoid nonlinear constraint expressions). Since the composition of the network is not known at t, the contour Ψ cannot correspond to t, but at best to a comparable network, -- most likely the one retained in the previous planning stage t-1. Thus, a semi-discrete optimization based upon this constraint contour is mathematically speaking not optimum, though it appears to be the only practical way to introduce stability-related constraints.

6. Approximate Procedures for Establishing Stability-Related Line-Loading Limits

An approximate method of estimating the transient-stability power limits at various nodes of a power system was presented in Ref. 2, pp 73-90. This method is an extension of the well-known equal-area criterion; it is aimed at obtaining simplified stability criteria for multi-machine systems. The method has been successfully tested on a very small (8-node, 13 branch) example[2] but its accuracy has not yet been verified on a system of realistic size.

In the course of this study, the above approach has been extended and further developed, and a stability index has been derived that is applicable to power system operation and planning. This index would provide a measure of stability (instead of merely a "stable/unstable" type of answer), and could be used to evaluate the outputs of a transient-stability simulation run. While further development is necessary before the actual use of such a stability index, it appears to be a very promising and useful aid for fast checking of transient-stability limits on line power flows or, equivalently, on node power injections.

Research effort leading to the formulation (and preliminary testing) of approximate and fast transient-stability analysis techniques was expanded under this contract as well as under another contract [12] with BPA. Since the results of this research are reported in a Final Report published earlier (see Ref. 12, Chapter V), they are not repeated here.

7. Concluding Comments

In this section, the line loading limits have been related to the two optimum planning procedures discussed in Chapters III and IV of the report. The limits determined by thermal overloads can be handled easily. However, the limits determined by transient effects are much more difficult to incorporate into the planning procedures. If only discrete expansion alternatives are considered (as in dynamic programming), the infeasible states could conceivably be eliminated by new methods capable of performing very fast transient stability analyses. For the semi-discrete expansion program discussed in Chapter III, the transient stability effects must be included as angular difference limits or functions of the angular difference limits; these can be determined approximately from separate transient-stability studies performed on networks that are similar to the one being expanded. The principle of several approximate, but computationally efficient, methods to perform these transient stability studies has recently been developed and could be applied to determine the required stability-related constraints.

C. Related Work by Other Utilities

1. General

In the course of 1969, numerous professional contacts with the planning and research departments of other utilities were established; detailed knowledge of their attempts to improve the power-system planning process was acquired, and interpreted in the context of the present project.

One of the authors of this report (J. Peschon) was invited by R. B. Shipley of TVA to provide a comprehensive reply to the following question asked by a Cooperating Group of eastern and midwestern utilities concerned with the improved use of computers for power system planning:

> "Develop...long range objectives for computer work in the system planning area. Specifically, what should our future computer programs be doing in the fields of system optimization and creation of system expansion plans?"

In this formal talk, present and future planning methods were classified as shown in Table V-1. Note that the work performed to-date in conjunction with BPA is almost exclusively concerned with the Direct Optimization Techniques, Items D1, D3, and D4. A slightly modified version of this talk was given to BPA by J. Peschon and R. E. Larson on November 25, 1969, in Portland, the major addition being an exposition of certain complicating factors introduced by nuclear generation and the associated nuclear fuel cycle.

2. Alternative Direct Optimization Techniques

In the course of this and prior projects performed by the authors for BPA, several optimization techniques were selected for detailed development, because they appeared to be particularly well adapted to resolve the planning problems of BPA. The following alternative techniques (developed elsewhere) were studied in particular, and these could well be used in the future to supplement the existing programs or to optimize planning situations that are beyond the scope of the studies undertaken to date.

a. Best Investment Policy (BIP)

This optimization program was developed in the course of a joint effort between Societes Reunies D'Energie Du Bassin De L'Escaut (E. Jamoulle) and the Center of Operations Research and Econometrics (P. Douillez) of the University of Louvain, Belgium.[47,48] It contains the following key ideas:

Table V-1. Classification of Power System Planning Methods: Present and Future

A. Present: Engineering judgment plus detailed analysis of case situations by the following standard computer programs:

1. AC power flow
2. Detailed transient stability
3. Short-circuit programs
4. Plant availability (maintenance, hydro, etc.)

B. Present and under development: Deterministic simulations (production costing)

Given the capacity expansion schedule of the system x_t, $t=0,\ldots,T$ Find the discounted investment and operating cost and check for constraint violations.

C. Present and under development: Stochastic simulations (Monte Carlo) to check probability of occurrence of constraint violations. The principle of stochastic simulations is to perturb the deterministic simulation of (b) with random inputs representing unscheduled outages, hydrological conditions etc.

D. Under development: Direct Optimization techniques for the following typical situations

1. Static optimization with planning horizon of 1 year
2. Short-term optimization with planning horizon of 4 years
3. Long-term optimization with planning horizon of 25 years
4. Long-term optimization in the presence of uncertainty

E. Suggested: Man-machine approaches based primarily on (D); for suggestions, see Chapter III.

F. Under development: Corporate modeling, which is an expansion of the deterministic simulation of (B) to encompass all the operations (technical, financial, etc.) of a utility.

(1) The network is (usually) represented by the first law of Kirchoff only, but the d.c. power flow may also be used; the remaining time span over which this model satisfies all the specified contingencies for given load growth is determined and a list of discrete remedies (e.g., line additions) is found, either by the computer or as a result of a man-machine interaction.

(2) A decision tree in a one-dimensional state space is formed with the alternatives found previously and starting from the present time.

(3) The number of paths of this decision tree are sharply reduced by a branch and bound argument.

(4) The optimum path through what remains of the original tree is found by a single-state forward dynamic programming.

The computation times of the BIP algorithm are quite acceptable: running times of several minutes on a modern high-speed machine for networks of 29 branches indicate the order of magnitude.*

Roughly speaking, BIP constitutes a very astute compromise between the predominantly continuous technique of Chapter III and the entirely discrete technique of Chapter IV. It does not supersede either of these techniques, but could be developed into a major optional addition for both.

b. Mixed-Integer Programming (MIP)

Two mixed-integer programs were applied by M. Arnaud of Electricite de France[42] (EDF) to optimize the planning of the high-to-medium voltage (HV-MV) transformers. The first program has a one-year planning horizon; discrete choices are available for the transformers, but a continuous model (the transmission capacity γ can be increased continuously) is used for the HV and MV portions of the transmission system. With these two assumptions, the result given by the program is optimum. The second program "connects" the yearly optimizations obtained for several consecutive years to provide an optimum expansion path. Dynamic programming is used to compute this path; optimality of the solution is not guaranteed.

*These estimates cannot be made more precise because of the heuristics and the "branch and bound" arguments used to reduce the decision tree.

The work of Arnaud is important to BPA for the following reason:

- The capability of mixed-integer programs (MIP) has very rapidly increased in the last several years[49]; MIP problems arise in a variety of power system studies other than those related to planning, for example the treatment of transformer tap settings in optimum power flows.

- The MIP application of Arnaud could easily be adapted to certain situations of combined generation and transmission planning, the generators being treated like HV/MV transformers and the transmission system being approximated by a continuous model.

D. The Generalized Reduced Gradient (GRG)

Abadie, Carpentier and Guigou of EDF developed the method of generalized reduced gradient[50,51] to solve nonlinear programming problems with constraints on dependent variables and without using a penalty approach (as was done successfully for the optimum power flow of Ref. 40). After reaching a dependent-variable constraint, linear programming is used to transform (or relabel) the constrained dependent variables x into independent variables u ; the gradient of u can be projected upon its constraints in the subsequent iteration.

While the computational efficiency of the optimum power flow appears to be much better than that of GRG, the latter technique remains of great importance for the following reasons:

- It may converge in cases where the optimum power flow does not.

- It may be very well suited (after certain modifications have been made) to handle the frequently encountered case of finding an optimum (minimum loss or minimum investment) in the presence of logical inequality constraints which ensure that the next contingency does not create an overload; these complex constraints were dealt with by linear programming (see Chapter III of this report and also Chapter IV of Ref. 12), which is an important element of GRG. Therefore, a very efficient specialized version of GRG could be developed for this important class of situations.

1. Stochastic Simulation

A detailed exposition of the various stochastic simulations pioneered by EDF and used routinely (SOPHY, MEXICO, PERU, ORGIE, LOG, GRETA) was given to WMS by M. Arnaud[42] in the course of his six-week visit to WMS, Palo Alto. These techniques have not yet been included in any WMS project in support of BPA, but a detailed understanding and documentation is available.

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