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LIGHT NUCLEI: AN EXPERIMENTAL PROVING GROUND FOR THE MICROSCOPIC CLUSTER MODEL*

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ABSTRACT

A selected review is given of comparisons of experimental data for low-mass nuclear systems with results of calculations using microscopic cluster models. [illegible] tems. Topics include influence [illegible] force, some consequences of the Pauli principle, effects of the [illegible]-exchange interaction, [illegible] distortion, absorption [illegible] el[illegible] scattering, and future [illegible]. Some as yet unpublished results are presented.

INTRODUCTION

Microscopic cluster models [illegible] posite [illegible] ons; the [illegible] accou[illegible] tran[illegible] excitat[illegible] ment [illegible] poi[illegible] equ[illegible]

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both methods--it has been called[7,8,22] a resonating-group method with a complex-generator-coordinate technique. It is basically a RGM in which the matrix elements are evaluated in the simpler manner of the GCM rather than using the former cluster-coordinate technique, thus making the labor arising from the complete antisymmetrization of ψ much less tedious.

Many of these recent developments in technique have now enabled calculations to be accomplished for systems having many more nucleons than was thought possible a short time ago. Some examples of such calculations are in Refs. 23-27. Many groups are presently working in this field and are making exciting progress, and it is unfortunately impossible in this very restricted survey to give adequate recognition to all these efforts.

Light nuclei have played a special role in the realm of the microscopic cluster model. From a historical viewpoint, they were important when the available techniques restricted most calculations to the interaction of two [illegible]-shell nuclei, thus limiting these studies to systems of [illegible] nucleons or fewer. Even with this restriction it was deemed [illegible] calculations, not only to test the micro[illegible] and to study few-nucleon systems, but also, as was [illegible] and others,[illegible] to gain a general understanding [illegible] interactions which could be applied to more massive [illegible] such detailed investigations were not practical. At present we are at a stage where we have a reasonably good, though not [illegible], understanding of interactions in few nucleon [illegible]. Thus, for example, calculat[illegible], now often are used to test [illegible] or numerical methods.

In this [illegible] I present comparisons of theory with experiment [illegible] the understanding gained by such comparisons. [illegible] to the systems containing 4, 7, and 8 nucleons, [illegible] selected for presentation [illegible] that with which I am most [illegible]. Suggestions are made for future work in this area.

FOUR-NUCLEON SYSTEM

The 4-nucleon system is the lightest system which has been studied seriously with the microscopic cluster model, the 2- and 3-nucleon systems enjoying their own special methods of study. A number of significant calculations on light systems in the low-energy region have been done by the Köln group[28-30] using the computer code developed by the late H. H. Hackenbroich. These calculations use a rather complete nucleon-nucleon force containing a central component with a soft repulsive core[31] and tensor and spin-orbit components.[32,33] In addition, several channels are often included, and a Jastrow-type correlation is handled in an approximate manner. This group uses the Kohn method[34] to obtain their solutions. An example[35] of their work is given in Fig. 1, which shows a calculation of differential cross sections for n+t scattering compared with experimental data.[36] It is seen that this calculation yields quite reasonable results. An interesting feature which emerged from the calculation was the existence of a significant amount of $^3P_1$-$^1P_1$ coupling, which

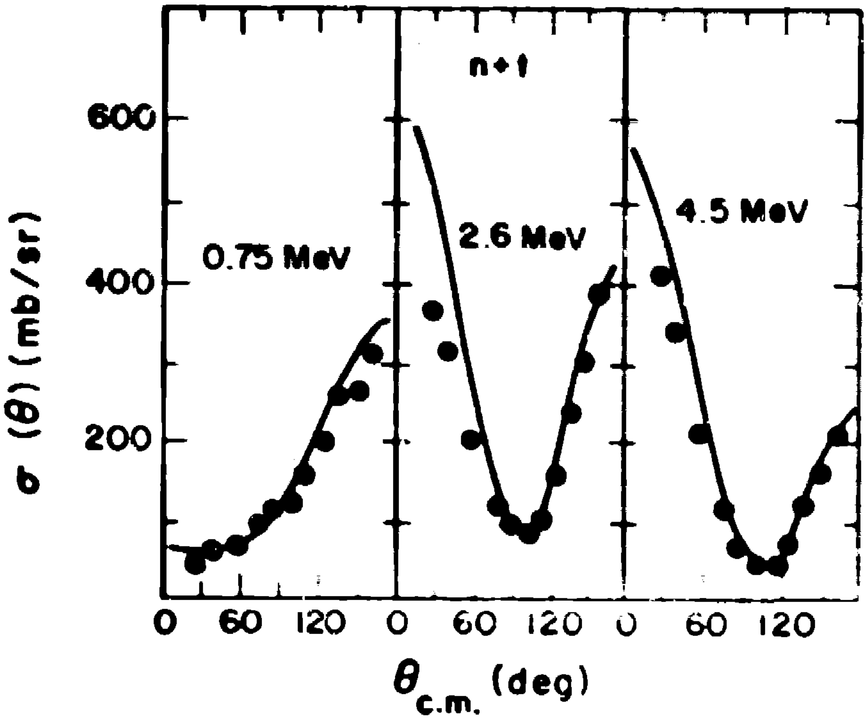

Fig. 1. Differential [illegible] indicated [illegible] curves [illegible]

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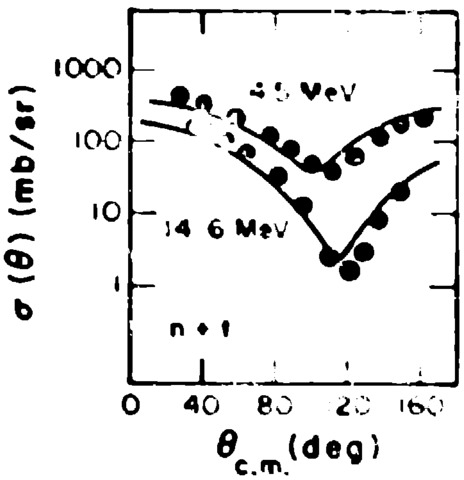


Fig. 2. Differential [illegible] sections [illegible] tering at the [illegible] energies. The [illegible] data from [illegible] and the curves show [illegible] tions from Ref. [illegible]

[illegible]

in the calculations to account approximately for the effects of reactions on the elastic channel. Figure 2 illustrates an important characteristic of the calculations that is verified by the data. This is that the angular position of the minimum in the cross section moves toward larger angles as the energy increases. Such behavior is opposite to that expected for a diffraction-type minimum. In the present case, the minimum is generated by interference between a forward-peaked direct amplitude and a backward-peaked exchange amplitude, where I should emphasize that the exchange amplitude arises from the Pauli principle and occurs naturally in the calculations through the use of a fully antisymmetrized wave function. Thus, the calculations [illegible], and the data verify, that, as the energy increases, the exchange amplitude becomes weaker with respect to the direct amplitude.

Next, p+ [illegible] scattering is considered. Figure 3 shows that a calculation[illegible] from [illegible] reproduces well the experimental differential cross sections[illegible] and vector analyzing powers[illegible] in the low-energy region. At higher energies of about 20-50 MeV, a new phase-shift study has been made of some of the most recent data. The data base consisted of differential cross sections from Manitoba[49] at 11 energies, vector analyzing powers from Lawrence Berkeley Laboratory[illegible] at [illegible] energies, and total reaction cross sections from Manitoba[illegible] at 10 energies--a total of [illegible] data points. These data were subjected to an energy dependent, phase-shift analysis using the energy parameterization of the R-matrix formalism[illegible] and a code developed at the Los Alamos Scientific Laboratory by D. C. Dodder and G. M. Hale.[53,54] Partial waves through $\ell=7$ were allowed in the elastic channel, and absorption through $\ell=6$ was incorporated in the unitary formalism by including d+pp and d*+pp as two-body channels. Singlet-triplet and tensor couplings through $\ell=2$ in the elastic channel were allowed after the initial phase of the search. No s or

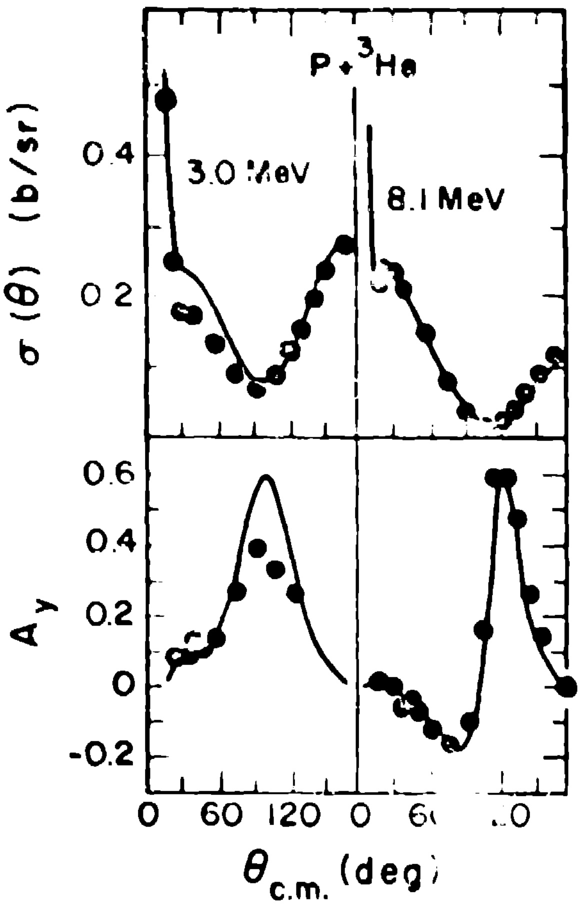


Fig. 3. Differential cross sections (c.m.) [illegible] and vector analyzing powers Ay for p+^{3}He scattering at the indicated c.m. energies. The points show data from Refs. [illegible] and 48, and the curves show calculations from Ref. 46.

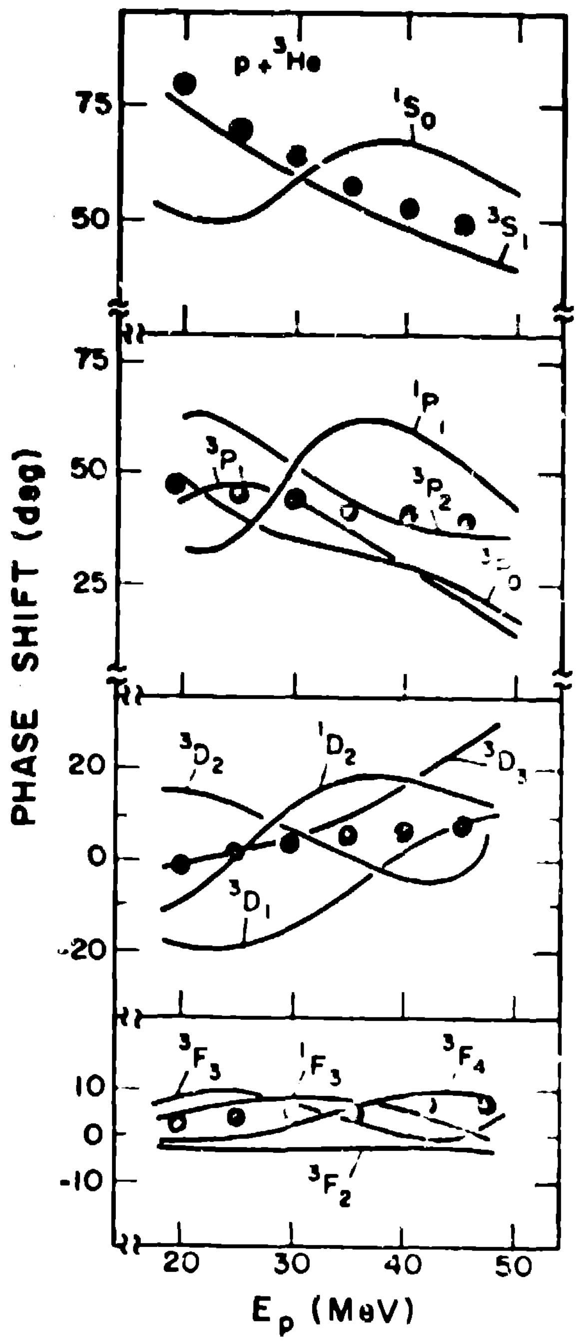


Fig. 4. Phase shifts through ℓ=3 vs lab energy for p+^{3}He scattering. The curves show the results of an energy-dependent phase-shift analysis, and the points show the results of a resonating-group calculation using a purely central nucleon-nucleon force.

j splittings were allowed for ℓ=6 or 7. The initial parameters were chosen to yield a smooth, structureless dependence on energy for the phase-shifts and to yield phase-shift values at 20 MeV approximately equal to those obtained in [illegible] R-matrix analysis[illegible] of the low-energy data. At first about 60 parameters were varied, with this [illegible] increased to about [illegible] during the middle phase of the search, and being decreased to about 80 during the final phase. A [illegible] minimum was found with a [illegible] tum of [illegible]. The [illegible] shift [illegible] the first four [illegible] waves are shown as [illegible] in Fig. 4. The [illegible] obtained should [illegible] tive, [illegible] be definit[illegible] that [illegible] data are [illegible] ergy range [illegible] nalysis should [illegible] low-energy data [illegible] as to [illegible] range [illegible] in Fig. 4 are points which represent the results of a resonating-group calculation using a purely central nucleon-nucleon potential. This calculation is much like the Minnesota work of Ref. 5e; however, an imaginary potential and the Coulomb exchange terms have now been included. The energy dependent strength of the imaginary potential was chosen to reproduce the measured total reaction cross sections.[3] There is an apparent kinship between

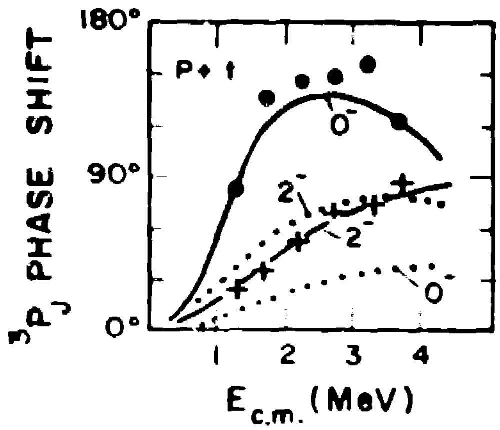


Fig. 5. $^3P_0$ and $^3P_2$ phase shifts at low energies for p+t scattering. The dot[illegible] are the J = 0⁻ and 2⁻ pha[illegible], respectively, from the analysis of Refs. 58 and [illegible]. [illegible] are the results of a calculation both with the odd-[illegible] tensor force (solid curve) and without it (dotted curve).

the calculated and empirical phases, but calculations with noncentral forces clearly are needed in this energy range before anything more can be said.

Next, I mention some consequences of a recent experiment by D. Fick and collaborators,[illegible] who used the polarized triton beam[60] at the Los Alamos Scientific Laboratory to measure vector analyzing powers and differential cross sections for t+p scattering in the c.m. energy range 1.26–3.71 MeV. This work was stimulated by Hackenbroich's comments[illegible] on the sensitivity of the low-energy p+t phase shifts to "weak" components of the nucleon-nucleon force (especially, it appears, to the odd-ℓ tensor force). In Fig. 5 we show the $^3P_0$ and $^3P_2$ phases extracted from the data. Two cluster model calculations[illegible] are also shown. The solid curves illustrate the results when the odd-ℓ tensor force is included and the dotted curves show the results when it is not. The sensitivity just mentioned is clearly seen in the figure, and the calculation does well in reproducing the empirical phase [illegible]. It [illegible] be that these studies will be able to provide information on the weak components of the nucleon force. I will comment no further on this problem, as I believe it will be discussed in more detail later in the conference.

In concluding this section on the 4-nucleon system, I wish to point out a very recent calculation from KBIn by Heiss, Bauer, Aab[illegible]kamp, and St[illegible] in which they take into account all possible two-body fragmentations of the ^{4}He system, namely p+t, n+^{3}He, and d+d. They include partial waves through ℓ=[illegible], which results in a (36 × 36) S matrix. They compare with experiment observables for elastic scattering in all three channels and for the reactions [illegible](p,n)^{3}He and ^{2}H(d,p)^{3}H. As the authors discuss,[illegible] there are some shortcomings in the calculation, but the agreement with experiment is generally good, and I consider this calculation a tour-de-force in the area of microscopic cluster models.

^{3}H AND ^{3}He INTERACTIONS WITH ^{4}He

The study of the interaction of ^{4}He with mass-3 nuclei (^{3}H or ^{3}He) has been very fertile ground for the microscopic cluster model. The fact that the two interacting nuclei differ by only one nucleon has interesting consequences--for example, exchange effects are particularly strong in this mass-7 system.[64,65] This system has also been used extensively to study those features of the nucleus-nucleus interaction which might have some general relevance to heavier systems. I will first give a short historical review of cluster calculations for this system, will then select some specific examples for more detailed discussion, and will compare some new data with a current calculation.

To my knowledge, the first cluster-model treatment of ^{3}He+^{4}He as a scattering system was the RGM calculation in 1963 of Tang, Schmid, and Wildermuth.[67] They performed a one-channel calculation (as are all the mass-7 calculations I will mention) and used a purely central nucleon-nucleon force, with the further simplification that the ranges (but not the strengths) of the singlet and triplet potentials were equal. The parameters of the potential [illegible] were chosen to fit the nucleon-nucleon, low-energy scattering data as well as possible. Some adjustment in the exchange mixture of the potential was made to fit the $2J+1$ averaged energy of the $^2P_{3/2}$ and $^2P_{1/2}$ bound states of ^{7}Be--this resulted in a nearly Serber exchange mixture. The exchange terms involving the Coulomb potential were neglected in this calculation. The agreement that was obtained with the available differential cross sections and phase shifts was very encouraging.

Five years later, a modest update and extension of this first calculation was published by Brown and Tang.[68] (If I may digress momentarily on a personal note, that paper served as my initiation into the ideas of cluster models and the RGM. I was converted immediately, because even an experimentalist as I am could see the possibilities in a theory that allowed one to proceed from the nucleon-nucleon force to the interactions between complex nuclei in an antisymmetrized and translationally invariant way without the need for some form of shell-model potential along the way.) In that paper some minor improvements were made in the intrinsic wave functions of the mass-3 and mass-4 clusters, bringing the rms radii into line with the most recent electron-scattering experiments. The systems ^{3}H+^{4}He and ^{3}He+^{4}He were treated together with the same nuclear force, the only difference being the Coulomb interaction. By adjusting the nucleon-nucleon exchange mixture, as mentioned above, the ^{7}Li and ^{7}Be bound state energies ($2J+1$ averaged) were fit to within 60 keV. A rough estimate was made of the effect of the specific distortion of the asymptotic shape of the mass-3 clusters while in the region of strong interaction with the ^{4}He. (The term "specific distortion" is used to distinguish this distortion from that already contained in the calculation by the action of the Pauli exclusion principle.) It was found that specific distortion probably would not cause any major changes in the results. I should mention that the adjustment of the exchange mixture compensates to some extent for the lack of allowance for specific distortion in the calculation.[illegible] Probably the most significant finding reported in that paper resulted from our study of effective potentials between the ^{4}He and the mass-3 nuclei. Such studies had been done previously for the $\alpha+\alpha$ interaction[illegible] and had resulted in an understanding of the features of the phenomenological potentials,[72,73] especially their angular-momentum dependence, energy independence, and short-range repulsion for low angular momenta. In our mass-7 study we found the even-ℓ potentials to be quite similar to those for the α+α system. However, the odd-ℓ potentials were found to be quite different from and significantly more attractive than the even-ℓ potentials. We termed this an "odd-even effect." It is produced by the exchange kernels, and hence arises from the Pauli principle. The odd-even effect is strikingly illustrated when the phase shifts are plotted vs orbital angular momentum ℓ, as was done in Ref. 64. The phase shifts then exhibit a distinct zigzag pattern. It was also demonstrated in Ref. 64 that if the odd-even structure in the phase shifts is removed and replaced with a smooth dependence on ℓ, then the large backward rise in the differential cross section disappears. By now the odd-even effect has been noticed in many systems;[illegible] for some the even-ℓ interaction is stronger than the odd-ℓ, and often simple oscillator-model arguments[74] can be used to decide which will occur. The effect is known to be strong when the number of nucleons in the two interacting clusters is nearly the same,[illegible] and it has its origin in the so-called core-exchange term.[illegible]

Further improvements were made in the mass-7 calculation in a paper by Koepke et al.[illegible] A new nucleon-nucleon potential (purely central), potential B of Ref. 78, was used. This potential allows the ranges in the triplet and singlet states to be different, and hence it fits the low-energy nucleon-nucleon data better than did the previous potential. The nuclear part V_{ij} of this improved potential is given by

$$V_{ij} = \left[\tfrac{1}{2}(1+P^{\sigma}_{ij})V_t + \tfrac{1}{2}(1-P^{\sigma}_{ij})V_s\right]\left[\tfrac{1}{2}u+\tfrac{1}{2}(2-u)P^{r}_{ij}\right], \tag{2}$$

where P^{σ}_{ij} and P^{r}_{ij} are spin- and space-exchange operators, respectively, and V_t and V_s denote the triplet and singlet nucleon-nucleon potentials, respectively. These latter potentials are given by

$$V_t = -V_{0t}\exp(-\kappa_t r^2_{ij}),$$

$$V_s = -V_{0s}\exp(-\kappa_s r^2_{ij}). \tag{3}$$

The constants in Eq. (3) are chosen[78] to fit the nucleon-nucleon effective range parameters; they are found to be

$$V_{0t} = 66.92\ \text{MeV},\quad \kappa_t = 0.415\ \text{fm}^{-2},$$

$$V_{0s} = 29.05\ \text{MeV},\quad \kappa_s = 0.292\ \text{fm}^{-2}. \tag{4}$$

The quantity u in Eq. (2) governs the exchange mixture, with u=1 giving a pure Serber force.

Another improvement in this calculation was the inclusion of the exchange terms associated with the nucleon-nucleon Coulomb interaction. This Coulomb-exchange force was found to be quite significant here. In addition, account was taken of the fact that the rms radii of ^{3}He and ^{3}H are different. Finally, an imaginary potential containing a dependence on the relative-motion parity was included to provide absorption in the elastic scattering. Further discussion of absorption effects will be given below.

When the exchange constant u in Eq. (2) was adjusted to fit the ^{7}Li and ^{7}Be bound-state energies, a value of 0.984 was found to fit them to within 15 keV. This improvement over the previous value of 60 keV is due principally to the inclusion of the Coulomb-exchange interaction. Calculated differential cross sections for ^{3}He+^{4}He scattering were compared with experiments[79-81] over a broad energy range. At each energy two parameters were adjusted to obtain the best visual fit--these were the strength and parity dependence of the imaginary potential. A sample of some of the results is shown in Fig. 6 where comparisons are made at c.m. energies (from top to bottom) of 10.11, 24.56, and 44.5 MeV. To indicate the amount of absorption needed in Fig. 6, I mention that the calculated total reaction cross sections are 291, [illegible], and 447 mb, in order of increasing energy. It is seen that the calculation does rather well in reproducing the data, and in particular the agreement with the rise in the cross section in the backward hemisphere indicates that exchange processes [illegible] are accounted for in a reasonable manner.

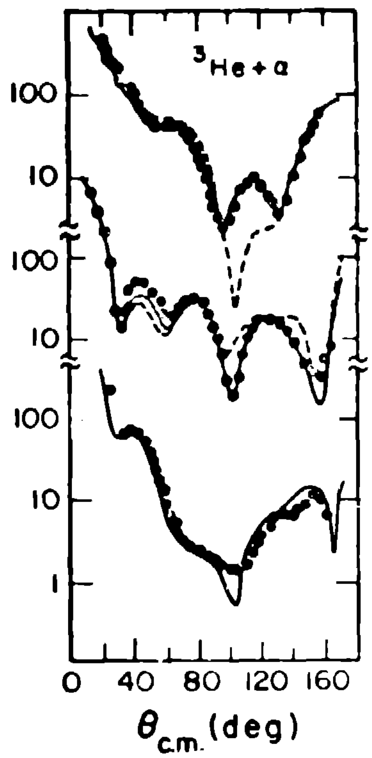


Fig. 6. Differential cross sections (c.m.) for ^{3}He+α elastic scattering at c.m. energies (from top to bottom) of 10.11, 24.56, and 44.5 MeV. The points show data from Refs. 79-81, and the curves are from the RGM calculation of Ref. 77. The dashed curves illustrate the best fits obtainable when Coulomb-exchange was omitted.

An intriguing prediction[illegible] of the various mass-7 RGM calculations was that in a measurement of the back-angle differential cross section vs energy there should occur a resonance effect, which one might term an exchange resonance because it vanishes in the calculation when the exchange terms are omitted. This resonance structure was predicted to be rather broad in energy but to be observable only in the angular range of about 165°-180°. (Earlier, Temmer[illegible] had suggested the possibility of resonance transfer processes in nuclei based on the known occurrence of such

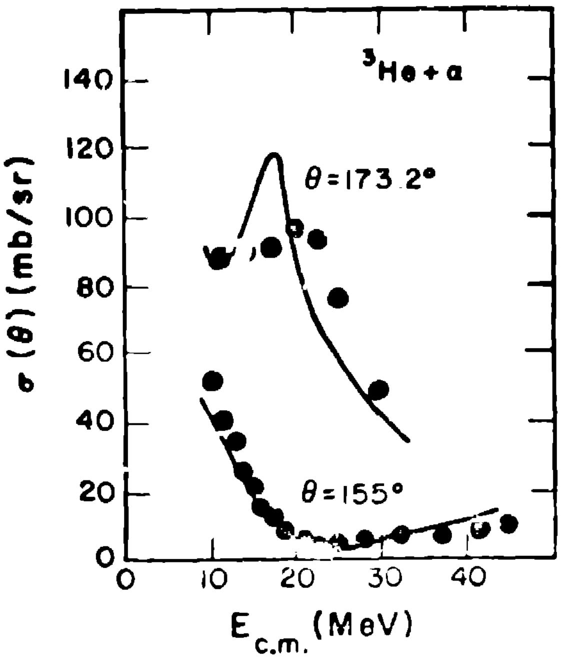


Fig. 7. Differential cross sections (c.m.) vs c.m. energy for ^{3}He+α scattering at c.m. angles θ. The points show data from Refs. 79-81, 83, and the curves are from the RGM calculation of Ref. 77.

transfer of electrons in atomic scattering. However, that was expected to be a forward-angle effect, and its connection to the present process is not clear.) An experiment was performed[83] on the Oak Ridge cyclotron to test this prediction. A ^{4}He gas target was bombarded with a variable energy ^{3}He beam, and the recoil ^{4}He particles were detected in a magnetic spectrograph at a lab angle of 3.4°. The result of the measurement is shown in Fig. 7 as points labelled by the ^{3}He c.m. angle of 173.2°. A clear structure is evident. Also shown are data[79-81] at 155° to illustrate that the structure is quite backward peaked. The curves are from the calculation just described of Koepke et al.[77] Because the amount of absorption to use in the calculation is not precisely determined by the data, there is some uncertainty, of roughly ± 20%, in the calculated cross sections of Fig. 7; however, the general trend should be as shown. If no absorption had been included in the calculation, then the curve for 173.2° would rise to a peak value of about 160 mb/sr at about 25 MeV.[83] The experimental confirmation of this exchange effect predicted by the RGM calculations is very pleasing to me, and for that reason I have described it here, even though the experiment is now 8 years old!

The latest addition to this series of mass-7 calculations was made by Furber,[84] who has added a nucleon-nucleon, spin-orbit force while retaining the other features of the calculation of Koepke et al.[77] (Wurster-Kanellopoulos[85] has also reported a calculation for this system employing a nucleon-nucleon, spin-orbit force.) The spin-orbit potential $V_{ij}(so)$ used by Furber is given by

$$V_{ij}(so) = -(V_\lambda + V_{\lambda\tau}\vec{\tau}_i \cdot \vec{\tau}_j)(\vec{r}_i - \vec{r}_j) \times (\vec{p}_i - \vec{p}_j) \cdot (\vec{\sigma}_i + \vec{\sigma}_j)(2\hbar)^{-1} \exp(-\lambda r_{ij}^2) . \quad (5)$$

Equation (5) is to be regarded as an effective noncentral potential for 1p-shell nuclei and not as the true nucleon-nucleon, spin-orbit potential. The reason for this is that the important, more complicated, nucleon-nucleon tensor interaction is not taken into account; and it is desired that the spin-orbit potential of Eq. (5) help to compensate as much as possible for this omission. Therefore, values for the constants V_λ, $V_{\lambda\tau}$, and λ in Eq. (5) were obtained by fitting (1) the $^2P_{3/2}$-$^2P_{1/2}$ bound-state splittings in ^{7}Li and ^{7}Be and (2) the p+^{4}He scattering data below the reaction threshold. Item (2) was accomplished through a RGM calculation like the one described in Ref. 44. The following values were chosen as a good compromise to requirements (1) and (2):

$$V_\lambda = -50 \text{ MeV}, \; V_{\lambda\tau} = 270 \text{ MeV}, \; \lambda = 2 \text{ fm}^{-2}. \qquad (6)$$

Some calculations using this spin-orbit potential will be presented below; however, I first would like to discuss briefly the experimental situation. For ^{3}He+^{4}He there is a copious quantity of differential-cross-section data.[86] Until just recently, the only significant amount of polarization data for this system was that of the Rice group[87] and the Rice-Caltech collaboration,[88] who measured analyzing-power excitation functions using a ^{4}He beam incident on a polarized ^{3}He target. Earlier work[86,89] used double-scattering techniques. The recently operational polarized ^{3}He beam at the University of Birmingham now has been used by Lui et al.[90] to measure analyzing-power angular distributions for ^{3}He lab energies from 18 to 32 MeV in 2 MeV steps, thus greatly increasing the data base for this quantity.

For ^{3}H+^{4}He the available differential-cross-section data are not nearly as plentiful as for ^{3}He+^{4}He scattering.[36] The principal published work in this area is due to Spiger and Tombrello,[91] Ivanovich, Young, and Ohlsen,[92] and Chuang.[93] The published polarization data are sparse. There is the early Los Alamos work[94-96] using double-scattering techniques and more recent work[50,97] using the Los Alamos polarized triton beam. At present there is a project underway at Los Alamos[98-100] to measure differential cross sections and analyzing-power angular distributions in the triton energy range of about 5 to 17 MeV and to phase-shift analyze the data.

In Fig. 8 I show some of the recent data compared with calculations using Furber's RGM computer code.[84] The ^{3}H+^{4}He data are from the recent Los Alamos studies[99,100] and are at 9.69 MeV (c.m.) [E(^{3}H) = 17 MeV, lab]. The ^{3}He+^{4}He data are at 17.09 MeV (c.m.) [E(^{3}He) = 30 MeV, lab], the differential cross section is from Ref. 79, and the analyzing power is from the recent Birmingham study.[90] A phenomenological imaginary potential iW was included in the calculations to account approximately for absorption. It has the form[77]

$$W = (1+C_I P^r)U(r), \qquad (7)$$

where P^r is a Majorana operator which exchanges the position of the c.m. of the ^{4}He with that of the mass-3 particle. When a partial wave expansion is carried out, Eq. (7) results in an absorptive term of the form $[1+C_I(-1)^\ell]U(r)$, which depends in a simple way on the

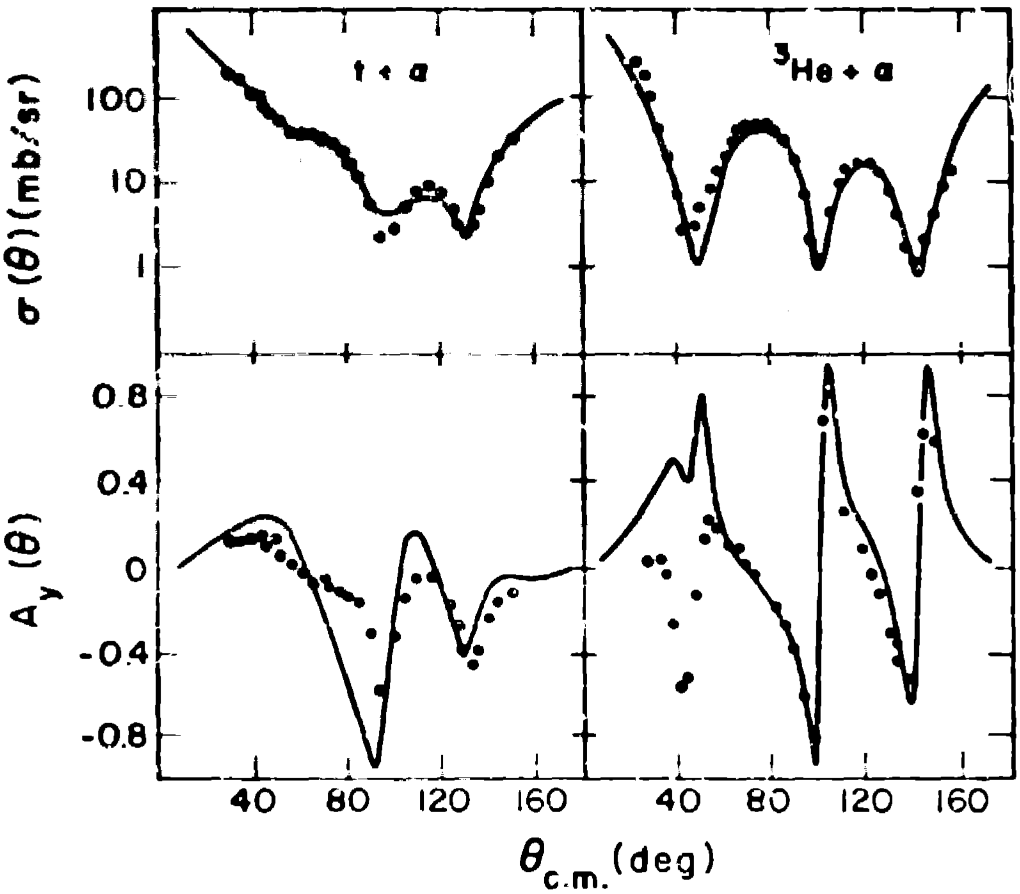


Fig. 8. Differential cross sections [illegible] and analyzing power A_y([illegible]) for t+[illegible] and [illegible] scattering. The t+[illegible] data (points) are from Refs. [illegible] and are at a c.m. energy of [illegible] MeV. The [illegible] data (points) are at a c.m. energy of 17.09 MeV. The [illegible] data are from Ref. 79, and the [illegible] data are from Ref. [illegible]. The curves show results of a RGM calculation [illegible] the code of Ref. [illegible].

relative motion parity. Further discussion is given below on this form of absorption. The function [illegible] of Woods-Saxon shape [illegible] volume and surface components of equal strength U_o and [illegible] radius of 3.2 fm and a diffuseness of 0.5 fm (see Ref. [illegible]). The [illegible] adjustable parameters are U_o and C_I, and in the calculations shown in Fig. 8 they had the values U_o = 1.00 MeV, C_I = -0.55 for $^3H+^4He$, and U_o = 1.45 MeV, C_I = -0.65 for $^3He+^4He$.

It is seen in Fig. 8 that the calculation does reasonably well in reproducing the data. The discrepancy in A_y at forward angles for $^3He+^4He$ tends to lessen at somewhat higher energies, as there the calculation also develops a minimum near [illegible]. The calculation of A_y for $^3H+^4He$ is more structured than the data and gives the impression of fitting less well than the other cases shown. This may be somewhat misleading, however, because, even though the Los Alamos phase-shift study is still in a preliminary state, we do know[100] that it does not take very much of a change in the phase shifts away from the RGM values to produce an excellent fit.

The Birmingham phase-shift analysis of their data has been completed, and they quote[24] three solutions. Their solution C gives a slightly lower overall χ^2 value than the other two, and it was obtained by starting at 18 MeV (lab) with the RGM values and using the solution found at each energy as starting values for the search at the next higher energy. Their result for the $j=\ell+\frac{1}{2}$ phase at 30 MeV (lab) is plotted vs ℓ in Fig. 9. Also shown are the $j=\ell+\frac{1}{2}$ RGM phases (crosses) from the calculation illustrated in Fig. 8. Two features are apparent: (1) the RGM and empirical phases are reasonably close to each other, and (2) the odd-even effect described above is clearly present in the empirical solution. A plot of the $j=\ell-\frac{1}{2}$ phases vs ℓ shows similar features. Feature (2) was also observed[1] in an analysis of the differential cross section at 28 MeV (lab) using unsplit phases, but the new result is more definitive as it includes the effect of the analyzing powers. The other two phase-shift solutions of the Birmingham group show very little or no odd-even effect. In such a situation I would suggest that one adopt theoretical guidance in deciding which phase-shift set is the most physically realistic. There is little doubt in my mind that in this case solution C deserves that status.

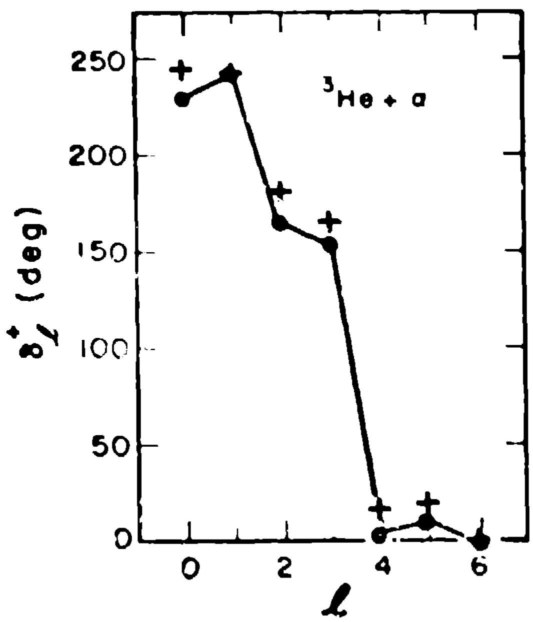


Fig. 9. ^{3}He+α phase shift for $j=\ell+\frac{1}{2}$ at 17.09 MeV (c.m.) plotted vs orbital-angular-momentum quantum number ℓ. The points connected by straight lines are from the phase-shift analysis of Ref. 24, and the crosses show the results of a RGM calculation using the code of Ref. 84.

^{4}He SCATTERING BY ^{4}He

There has been a strong interest for many years in understanding the nature of the interaction between two α particles, and consequently a great deal of theoretical and experimental study has been carried out on the α+α system. Discussions of and references to the early work on this system can be found in the review by Afzal, Ahmad, and Ali.[?]

The strong binding and the spin-isospin saturation of the α particle make the α+α interaction especially amenable to cluster-model treatments, particularly below the first reaction threshold of 17.35 MeV (c.m.). RGM studies of this system were already being done in the 1950's, and in the 1960's these studies yielded a remarkable explanation[3-7,10] for the characteristics of effective α+α potential [illegible] which had been [illegible] the lowest [illegible] principle [illegible] nearly [illegible] proposed [illegible]

Calculat[illegible] time that [illegible] It was [illegible] simplified [illegible] were equal [illegible] are scatter[illegible] piri[illegible] mixture [illegible] the [illegible] calculat[illegible] (4), and [illegible] However, [illegible] late [illegible] ter, [illegible] In [illegible] The late[illegible] soft [illegible]

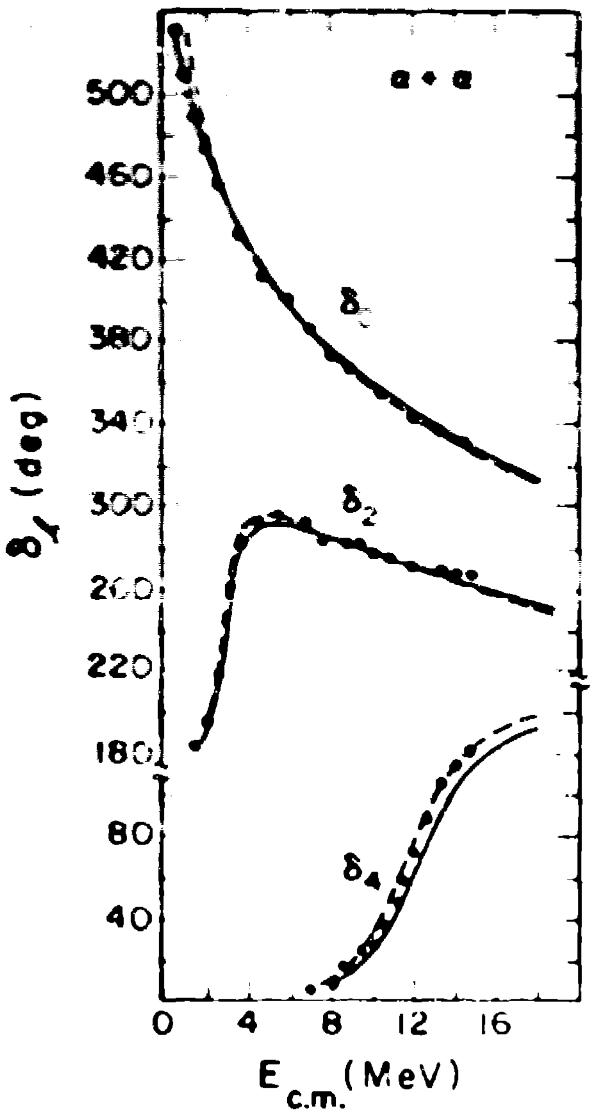
for [illegible]

$$V_{[illegible]} [illegible]$$

The [illegible]

[illegible]

The [illegible] account [illegible] fr[illegible] tortion [illegible] symmetric[illegible] distortion [illegible] that specific distortion [illegible] is trying to [illegible] empirical [illegible] exchange par[illegible] are chosen [illegible] calculation [illegible] correct [illegible] resonance [illegible] shown as solid curve in Fig. [illegible] for the [illegible] and [illegible] states, but the problem [illegible] state has per-sisted. Also shown in Fig. [illegible], as dashed [illegible], are results of a calculation[illegible] from K9in, which used the central potential of Ref. 33.



[illegible]

and which took distortion into account by including the [illegible] channel, where [illegible] denotes the first excited T 0, [illegible] state of the [illegible] particle. That calculation does very well for the [illegible] and 4 states, but shows some deviation from the [illegible] empirical [illegible] about 2 MeV. The [illegible] are two rather careful [illegible] [illegible] model—still, they show some disagreement with experiment and with each other. The two [illegible] of [illegible] and [illegible] in an [illegible] However, it is not clear [illegible] the [illegible] difference [illegible] the two results [illegible] the [illegible] state. It may be that the [illegible] treatment of low energy [illegible] scattering is yet to be done, although we are certainly very close.

There is some nice experimental work on [illegible] scattering in the c.m. energy range 15–35 MeV by Bacher et al.,[illegible] but to my knowledge, no calculation has yet been compared with these data. A calculation[illegible] does exist in the c.m. energy range 25–60 MeV which yielded modes [illegible] agreement with [illegible] results of Darriulat et al.[illegible] Finally, [illegible] calculations [illegible] have been performed to test or illustrate new theoretical [illegible] and methods. A few of these are the development of the [illegible] model by Saito,[illegible] and treatments using generator-coordinate techniques by several workers.[illegible]

ABSORPTION IN ELASTIC SCATTERING

Ideally, absorption in elastic scattering should be taken into account in calculations by including the reaction channels responsible for it. This has often been done by the Köln group in their low-energy calculations. However at higher energies this becomes impractical, and other methods are needed if one wishes to obtain reasonable agreement with experimental data. To date, these methods have been to incorporate phenomenological imaginary components into the microscopic calculations. For example, in a GCM calculation of $^{16}O+^{16}O$ scattering, Canto[112] has included an absorptive kernel proportional to the GCM nuclear-interaction kernel and has found appreciably improved agreement with experiment. Instead of using imaginary kernels, the Minnesota group has added imaginary potentials of Woods-Saxon form to the direct nuclear potentials. It was originally hoped[65] that, because the real part of the cluster-cluster interaction is reasonably well understood from the microscopic calculations, some detailed information on the imaginary potential could be found by fitting experimental data. This hope has not yet been borne out; however, one interesting proposal has arisen from these studies. It was found, first in a study of p+α scattering[113] and later for other systems (see Ref. 77, for example), that the inclusion of a Majorana component in the imaginary potential, as given in Eq. (7), was very helpful in obtaining better fits to experimental data. This Majorana component results in an odd-even, orbital-angular momentum dependence of the imaginary potential, and hence it introduces a nonlocal effect somewhat like that produced by the real-interaction kernels. It has to be admitted, however, that no very convincing independent evidence exists for this form of imaginary potential.

Some measured total reaction cross sections for light systems have been used to check the amount of absorption used in cluster model calculations. For p+α it was found[illegible] that the total reaction cross section from the calculation of Ref. 113 was too small [for example 88 mb compared to 120 mb at about 14 MeV (c.m.)]. However, the absorption here is small enough that it was possible to obtain nearly as good a fit as before to the p+α scattering data while reproducing the measured total reaction cross sections. For d+d the total reaction cross section has been measured[115] at 17.5 MeV (lab) by summing the partial reaction cross sections. In Fig. 11 is shown a comparison of d+d differential-cross-section data[115] with a RGM calculation[114] similar to that of Ref. 115, but which includes Coulomb exchange. The total reaction cross sections from experiment and calculation are shown, and are in reasonable agreement. For $^3He+^4He$ the total reaction cross section has been measured[17] at 15.95 MeV (c.m.), again by summing the partial reaction cross sections. The result is shown as a solid circle in Fig. 12, along with cross sections (triangles) given by two phase-shift analyses[71,117] using complex phases. The result from the RGM calculation of Ref. 77 is displayed in Fig. 12 as a shaded region, which gives some idea of the accuracy to which it was felt the fits to the differential cross sections could determine the total reaction cross sections. The measured value (455±10 mb) does not agree with the "RGM value" (285±40 mb) at 15.95 MeV, but it does agree with the higher energy values at which the calculations level off. The points indicate that the general shape of the cross section from the RGM work is reasonable, but that the energy region

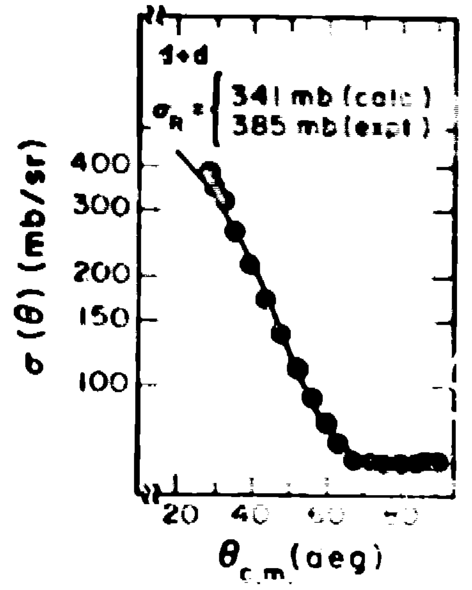


Fig. 11. [illegible]

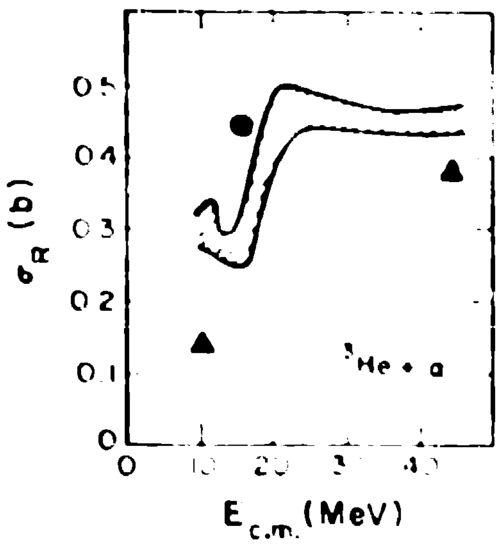


Fig. 12. [illegible] sections for [illegible] show [illegible] He; the [illegible] from the [illegible] of Refs. [illegible] shaded [illegible] culation of [illegible]

where the cross section rises rapidly is not given correctly by the calculation. [illegible] the amount of absorption included in cluster-model [illegible] agreement with [illegible] has been reasonable.

p+^{4}He SCATTERING [illegible]
HIGHER [illegible]

The [illegible]

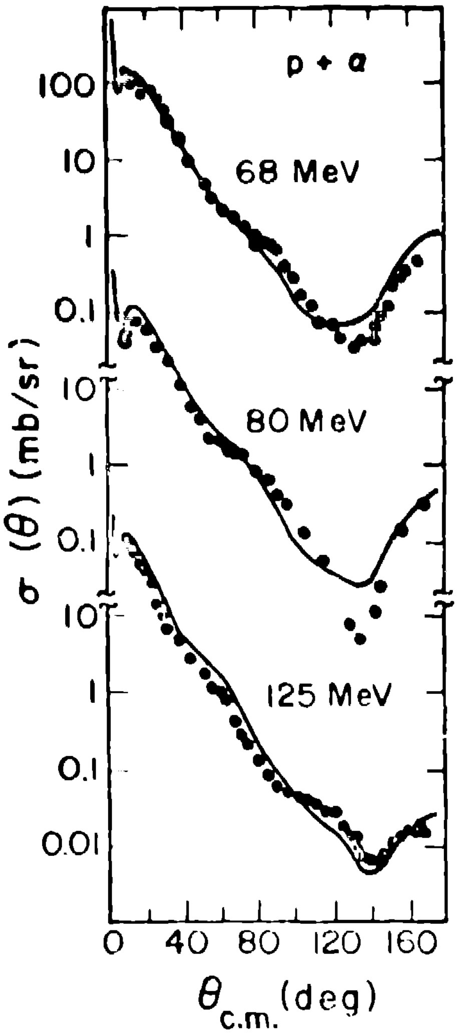


Fig. 13. Differential cross sections (c.m.) for p+α scattering at indicated c.m. energies. The points show data from Ref. 119; the curves are from the RGM calculation of Ref. 118.

CONCLUSION

In this discussion I have reviewed selected topics concerned with the application of the microscopic cluster model to few-nucleon systems. We have seen how the interplay of theory and experiment has led not only to a general understanding of interactions among light nuclei, but also to improvements in the calculations. Information gained from studying light systems has helped in understanding effects in heavier systems, such as the role played by the Pauli principle.

As for future work in light nuclei, there is certainly a need to improve the calculations further if we are to understand these systems in the detail we would like. For instance: more realistic nucleon-nucleon potentials with tensor and repulsive-core components should be used in all energy regions, theoretical guidance on how better to include absorption would be most welcome, the treatment of multiparticle breakup should be investigated, and practical techniques for applying these microscopic calculations in the intermediate energy region would be extremely useful. On the experimental side, high quality data are always valuable; however, it appears to me that there are much useful data available below about 10 MeV, and it is in the higher energy region where future effort might prove most profitable.

I have benefited immeasurably these past 11 years from the friendship of Y. C. Tang. His patience in initiating me into the ideas of the microscopic cluster model and his continued help and advice have been more valuable than words can express.

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