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PROPERTIES OF THE DKP EQUATION

by

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Abstract

After recalling the development of relativistic quantum mechanics, I elucidating the properties of the Duffin-Kemmer-Petiau first-order wave equation for spin-0 and -1 mesons. The DKP equation is formally compared to the Dirac equation, and physically compared to the Klein-Gordon second-order equation for mesons. I point out where the DKP and KG equations predict the same results, and where their predictions are different. I conclude with an example of where these differences might interest people studying quark models of nuclei.

This discussion of the DKP equation is based upon work I was involved in , in the 1970's, with Ephraim Fishbach and C. Keith Scott, with some very important help by Henry Primikoff. Later, Richard Krajcik and I looked at the extension to Bhabha equations, which I'll mention briefly. There was also a stirling cast of supporting players[1] who joined us in some of our papers. Our first paper was a 1971 Phys. Rev. Letter.[2] The work culminated in a review in the Springer Verlag Physics Lecture Notes, No. 94.[3] You can work your way through the literature, not only ours but everyone else's, by consulting Ref. 3 and our historical article in the American Journal of Physics.[4] This last article is basically a tree and you can find out what everybody did by starting there and working back.

Now, to understand the basis behind the DKP equation, let's go back and think about the relationship between energy and momentum in quantum mechanics. The first great equation of quantum mechanics that we learned, is the quantum analog to the non-relativistic relationship between energy and momentum:

$$E = \left(\frac{\bar{p}^2}{2m}\right) + V \cdot \tag{1}$$

As you all know, Schrödinger, in his brilliant theory of 1926, decided to turn this into an operator equation on what we now call a wave function:

$$\left(-\frac{\hbar}{i}\frac{\partial}{dt}\right)\psi^s = \left[\frac{1}{2m}\left\{\frac{\hbar}{i}\left(\frac{\partial}{d\,\vec{x}}\right)\right\}^2 + V\right]\psi^s \cdot \tag{2}$$

ψ^s is a probability attitude. Energy is replaced by an operator, $\left(-\frac{\hbar}{i}\frac{d}{dt}\right)$, on the wave function. Momentum is replaced by the operator $\left(\frac{\hbar}{i}\frac{d}{dx}\right)$. If you have such a system, then the Hamiltonian density is the quantity

$$(\psi^s)^* \left(-\frac{\hbar}{i}\frac{\partial}{dt}\right)\psi^s = \text{units [energy/volume]} \cdot \tag{3}$$

It has units of energy per unit volume. Well if $\left(-\frac{\hbar}{i}\frac{d}{dt}\right)$ has units of energy, that means that $\psi^*\psi$ has units of (1/volume) or ψ has units of

$$\psi = \text{units [length]}^{-3/2} = \text{units [mass]}^{3/2} \tag{4}$$

O.K., we understand that $\psi^*\psi$ is a positive-definite probability density. That seems fine, what's the problem? Well, with the success of the Schrödinger equation for the Rydberg states, the hope was that one could get the fine structure by adding relativity. The obvious guess to generalize the Schrödinger equation was to look at the relativistic relationship between energy momentum:

$$E^2 = \vec{p}^{\,2} c^2 + m^2 c^4 \tag{5}$$

and turn it into a quantum equation. This was tried:

$$\left(-\frac{\hbar}{i}\frac{d}{dt}\right)^2 \phi = \left[c^2 \left(\frac{\hbar}{i}\frac{\vec{\partial}}{d\,\vec{x}}\right)^2 + m^2 c^4\right] \phi \cdot \tag{6}$$

Actually the first person who tried it was Schrödinger, and there were a number of people who "discovered" the Klein-Gordon equation.[5] Eventually the names of Klein and Gordon stuck. When you add minimal electromagnetic substitution, the "atom" can be solved analytically and exactly. The energy levels of the Klein-Gordon atom are not equal to the energy levels of the hydrogen atom. We now know what the answer is, its spin. An interesting thing from a historical standpoint is that the energy levels of the pi-mesic atom were not experimentally verified until 1979. It's just too hard to unwrap the strong-interaction effects in a low orbit. To see the electromagnetic structure you've got to capture the pi-meson in a high orbit with large n and l. That is hard.

If in Eq. (6), I now use the standard help of $\hbar = c = 1$, all I have is one unit, which is mass = 1/(length). Therefore, the units are:

$$[\,\text{mass}\,] \sim m \sim E \sim \vec{p} \sim \frac{1}{[\text{length}]} \sim \left(\frac{\vec{\partial}}{\partial\,\vec{x}}\right) \sim \frac{1}{[\,\text{time}\,]} \tag{7}$$

An example is if you take the Klein-Gordon equation in the time independent limit.

$$\left(\Box + \mu^2\right) \phi = 0 \tag{8}$$

The solution is the Yukawa potential

$$V = \left(\frac{e^{-\mu r}}{r}\right). \tag{9}$$

where μ is the mass in units of 1/(length).

A little thing which was not noticed, because it wasn't important then, is that the Klein-Gordon equation is a second-order wave equation. So, since the Lagrangian density

$$\mathcal{L} = \left(\phi^* m^2 \phi\right) \tag{10}$$

is supposed to have units of (energy/volume), this means that ϕ has units of inverse length (L^{-1}), not $(L)^{-3/2}$.

Other than this, the only problem was that there was a funny thing about the probability density; it could go negative. This was "resolved" by Pauli and Weinkopf. They said that it was a charge density which was a fudge. You know and I know, (we were taught) that the wave-function μ is a probability density not a charge density. But, O.K., they were big guys and nobody was going to fight them.

Now, let's look at what eventually turned out to be the revolution. If you look at the relativistic energy equation (5), and take the square-root, you have

$$E = \pm\left[p^2 c^2 + m^2 c^4\right]^{1/2} = \pm\left[mc^2 + \frac{1}{2}\frac{p^2}{m} + \cdots\right]. \tag{11}$$

As a series you get the rest mass, then the Schrödinger non-relativistic kinetic energy.

Dirac wanted to obtain this directly. He wanted a first-order energy equation by taking the square-root of Eq. (5) in an operator sense. He wanted to break the square root. Dirac didn't say it that way, but that's essentially what he did. What Dirac found were four matrices γ_1, γ_2, γ_3, γ_4 such that

$$(\gamma_1 p_1 + \gamma_2 p_2 + \gamma_3 p_3 - \gamma_0 p_0) = (\gamma \cdot p)^2 = -m^2, \tag{12}$$

so that

$$m = \pm\left[-(\gamma \cdot p)^2\right]^{1/2} \tag{13}$$

This gave a first-order wave equation of the type

$$(\partial\,\gamma + m)\,\psi^D = 0 \tag{14}$$

As with the Schrödinger equation, the wave function has units of $(\text{mass})^{3/2}$ or $(L)^{-3/2}$. Of course, it is now well known that we have two large components, which represent particles with spin-up and spin-down, and two small components, which represent anti-particles with spin-up and spin-down. Note that the number of components in the equation is $2 \times (2S+1)$: two for particle and anti-particle times the number of spin states. Dirac then used minimal electromagnetic substitution in his equation. What is curious, however, is that he didn't solve this equation exactly. He admitted that he was afraid to.[6] He was so scared it wasn't going to work that he did not solve the equation exactly. Dirac did a series expansion which led to the Pauli equation. This was like doing a Foldy-Wouthuysen transformation.

It was actually Gordon and Darwin, very shortly after Dirac's fundamental paper appeared, who solved the problem exactly and got the analytical results for the energy levels of the hydrogen atom. Needless to say, it was big stuff.

Well, people understood the Dirac equation and so understood how to handle what were the only massive particles known in those days, Fermions.

The Klein-Gordon equation was accepted for the "mesons" that were predicted from Yukawa's theory of the strong force. But, even so, in the 1930's, Duffin, Kemmer, and Petiau independently, developed, although there was communication between Duffin and Kemmer, what I will call the DKP equation. This is a first-order wave equation for mesons. An interesting thing about Petiau is that he was a follower of the school of DeBroglie. For decades, up to his death, DeBroglie wanted to have a massive photon. The way he wanted to do this was to combine two electrons to make a photon. As I come to below, with this idea you already have a product of two Dirac spaces.

I am not going to describe how the DKP equation was developed historically. I'm going to give you what is a logical progression. This is not the way you do physics; rather it is the way you present it.

Instead of having Eq. (13), let's consider

$$(p \cdot \beta)^3 = (p \cdot \beta)\left(p^2\right) = -m^2\,(p \cdot \beta)\,. \tag{15}$$

This is a generalization of Dirac operators except that you put one more (p.matrix) in the defining conditions. The algebra is

$$\beta_\mu \beta_\lambda \beta_\nu + \beta_\nu \beta_\lambda \beta_\mu = \beta_\mu \delta_{\lambda\nu} + \beta_\nu \delta_{\mu\lambda}, \tag{16}$$

and with this algebra you get a relativistic first-order wave equation that formally looks exactly like the Dirac equation:

$$(\partial\,\beta + m)\,\psi^{\rm DKP} = 0 \tag{17}$$

Therefore, the wave function has units of $(\text{mass})^{3/2}$. What is cute is that the algebra (16) is satisfied by the symmetric outer product of two gamma matrices:

$$\beta_\lambda = \frac{1}{2}\left[I^{(1)}\gamma_\lambda^{(2)} + \gamma_\lambda^{(1)} I^{(2)}\right] \tag{18}$$

Is this telling you that one is making a meson out of two quarks?

In any event, Eq. (18) says that the β's have a 16×16 (reducible) representation. Now, what are all those components for ? There is a 1×1 space, which is trivial! But there is a 5×5 space which is spin-0 and a 10×10 space which is spin-1. If you write out the components of the spin-1 piece, you can show that they basically are the massive-photon Maxwell equations, or the Proca equations. You can show that the components are the electric field, the magnetic field, and the four-vector potential.[7]

The spin-0 components can be shown to be a field and a four-derivative of that field. The field and the time derivative amount to particle and anti-particle, and are related to the Sakata-Taketani and Feshback-Villars equation. The other three components are built-in subsidiary conditions. There are subsidiary conditions because the β matrices can always be transformed into an angular-momentum operator, which has zero eigenvalues. The subsidiary conditions correspond to the zero eigenvalues of that matrix. This is a problem you don't have with the Dirac equation.

It turns out that the Dirac and the DKP equations are the two simplest special cases of the first-order wave equations for arbitrary spin which were developed by Bhabha, Lubanski, and Madhadvarao.[4] Generally I call them the Bhabha equations, but all three of those guys really deserve the credit for them. These equations have as their "matrices" the representation of so(5), whose algebra is

$$[[\alpha_\mu, \alpha_\nu], \alpha_\lambda] = \alpha_\mu \, \delta_{\mu\lambda} - \alpha_\nu \, \delta_{\mu\lambda} . \tag{19}$$

Well, that's nice about DKP, but what's different about it than the Klein-Gordon equation and what's the same? If it's a free particle, it's the same thing. In electrodynamics, you have the same solution. For example, the pi-mesic atom has the same energy levels in the two formalism. Further, it is cute to look at the back of Akhiezer and Berestetskii,[8] because they do meson QED in the DKP formalism. Take Compton scattering. If you use the Klein-Gordon formalism you have three Feynmann diagrams, because of the contact term. Contrariwise, for DKP it's just like Dirac, there are only the direct and crossed terms. But if you sit down and calculate it you get the same analytic answer from either KG or DKP. You can use whatever formalism you like. Actually, I enjoy using DKP. It has trace theorems, so you just go rumbling through it. I find it less complicated. Even though you're handling more components, you're handling them formally, as matrices.

Well, can KG and DKP ever be different? The answer is, "Yes". Why is that so? Consider the current density for the Dirac, KG and DKP equations. As we have noted before, the fields in the currents go as $(\text{mass})^{1/2}$ for Dirac a[illegible]KP, but as (mass) for KG. If I am not conserving my current, in some naive sense, you would expect that when I go from one mass to another, the fields will extrapolate differently off the mass shell. As a matter of fact, you can analytically show this to be true.

The KG current for a non-conserved vector interaction is

$$\begin{aligned} \partial_\lambda^\pm &= \partial_\lambda \pm \imath\epsilon A_\lambda, \\ j_\lambda^{\rm KG} &= -i\left[\phi_B^* \partial_\mu^- \phi_A - \left(\partial_\mu^+ \phi_B^*\right)\phi_A\right]. \end{aligned} \tag{20}$$

Now look at the DKP current,

$$j_\lambda^{\rm DKP} = i\bar{\psi}_B \beta_\lambda \psi_A. \tag{21}$$

Using the wave equations and the β matrices you can write this DKP current in terms of the property normalized KG fields. When you do that you find

$$j_\mu^{\rm DKP} = -i\left[\begin{array}{c} \left(m_B^{1/2}\phi_B^*\right)\left\{\frac{\partial_\mu^-}{m_A}\left(m_A^{1/2}\phi_A\right)\right\} \\ -\left\{\frac{\partial_\mu^+}{m_B}\left(m_B^{1/2}\phi_B^*\right)\right\}\left(m_A^{1/2}\phi_A\right) \end{array}\right]. \tag{22}$$

Notice that if $m_A = m_B$, Eqs. (20) and (22) are the same. But if $m_A \neq_B$, the currents are different.

As a matter of fact, that's how we actually got into the game.[2] We found that there were certain situations when KG and DKP would yield different results for particle physics processes where the masses of the two mesons involved were different: in particular, for the non-conserved vector-current process $K \to \pi\ell\nu$.

Let me point out something; it is an assumption, when you have symmetry breaking, what units the symmetry breaking has. The wave functions are taken with physical mass values. The vertex function is taken as symmetric. Therefore, depending on what units you give to the wave-function and hence the vertex function, you will end up with different answers.

Well, that brings us to this conference, where you will hear a number of discussions on the use of the DKP equation in nuclear physics. Approximately five years ago, the DKP equation became of interest to certain groups, including Bunny's here at Ohio State. This was partially due to the renewed interest in the use of the Dirac equation in nuclear physics. I want to discuss one last topic to demonstrate that there are areas of interest to this audience, where one can ask questions about the use of the Klein-Gordon vs. the DKP equations. This is something that just came up in conversations at Los Alamos a little while ago.

To begin, let me remind you that, as this workshop has demonstrated, many people are interested in discussing nuclear physics in terms of quarks. There are many groups! Of course, I know best about the group at Los Alamos. Terry Goldman is in the office next to me and I am always wandering by to chastise him for wasting his time on quarks in nuclei when he could be doing gravity with me. So Terry and I talk an awful lot about this work [9] with Gerry Stephenson, Kim Maltman, Kevin Schmidt, and Fan Wang.

What they do is take the quark solution of a Dirac equation, where the potential, rather than being bag-like, has more the shape of a saw-tooth bottom with linear-confinement at the side. This models correlated nuclear positions, which actually are obtained variationally. The full state wave function is $\mathcal{A}\,(\Pi_j \psi_j)$, where ψ_j is the j th quark (or antiquark), and $\mathcal{A}$ signifies Pauli antisymmetrization. For baryon calculation they reproduce the N and Δ with normal properties (very similar to bag models), and are O.K. for the rest of the octet. They obtain ^{4}He with 18 MeV bending energy (without any pions) and predict several dibaryons.

Well, a couple of weeks I happened to meander into Terry's office and he said, "Gee, we are starting to get out some numbers on the spectra of mesons." So I said, "Oh? what do you guys do?" Terry replied, "We do our normal thing, except that this time there is an anti-quark." Now intrigued, I asked, "How do you treat your spin?" He said, they consider

$$H_D^{(1)}(\psi_1\bar{\psi}_2)\,H_D^{(2)} = E_1 E_2 (\psi_1\bar{\psi}_2) \tag{23}$$

where $(\psi_1\bar{\psi}_2)$ is an outer product of wave functions. However, they did not keep track of the full Dirac space inherent in $H_D^{(1)} \otimes H_D^{(2)}$.

Now you probably already know what I meant when I said to Terry, "Let me point out something to you. This is similar to, although not exactly the same as, the type of thing you do in the DKP equation." By dimensional arguments that I've given you before, you can realize that one could well get different results for the meson spectra with this sort of method than you would obtain using the Klein-Gordon equation. Also, there might be some corrections if the spinor spaces were handled exactly.

Terry allowed as how he hadn't considered that there might be a difference between what they are doing and normal KG-mesons. I'm not saying that their's is the only quark model that does this, obviously it isn't. I'm saying that for anybody here who wants to use quarks in nuclei and get mesons out of them, this is a first order top-of-the-line question: are you dealing with a DKP-like meson or are you dealing with a Klein-Gordon-like meson? It's not going to make any difference if it's just one meson; but if you're really worried about the spectra of mesons, then the two types of mesons are different. So, everybody with a quark model for nuclei from which they want to make mesons, if they're interested, has some questions they can start thinking about right now.

That's basically all I want to say except that I think that this formalism offers some interesting opportunities in this field that you guys like so much. I wish you all the best and please keep me informed. Thank you.

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