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ABSTRACT

In the event that the SSC does not observe any resonances such as a Higgs boson or a techni-rho meson, we would like to know if the SSC can still discover something about the nature of the electroweak symmetry breaking. In particular, we consider the question of whether there is a "no-lose" corollary at the SSC. We will use chiral Lagrangian techniques to address this question and analyze their utility for studying events containing W and Z gauge bosons at the SSC.

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ABSTRACT

In the event that the SSC does not observe any resonances such as a Higgs boson or a techni-rho meson, we would like to know if the SSC can still discover something about the nature of the electroweak symmetry breaking. In particular, we consider the question of whether there is a "no-lose" corollary at the SSC. We will use chiral Lagrangian techniques to address this question and analyze their utility for studying events containing W and Z gauge bosons at the SSC.

1. Introduction

One of the central questions facing the SSC is the origin of the electroweak symmetry breaking. The non-zero W and Z masses are experimental evidence for the existence of the symmetry breaking. The only question is where and how it occurs. In the simplest possible case, the symmetry breaking occurs through the coupling of a complex $SU(2)$ scalar doublet to $SU(2) \times U(1)$ gauge fields. The Higgs mechanism then gives the W and Z gauge bosons their masses, leaving a single physical scalar, the Higgs boson H. The mass of the Higgs boson is, however, a free parameter in the theory and it is necessary to search for it in all mass regimes. In building future high energy colliders, it is crucial to know what experimental limits they will place on the Higgs mass. It is generally believed that the SSC will be able to observe a Higgs boson with a mass less than about 800 GeV.[1]

Another possibility for the breaking of the electroweak symmetry is that of dynamical symmetry breaking. Although no completely satisfactory model exists, these models share some general features, among them the existence of a heavy particle with the quantum numbers of the ρ, usually called the techni-rho, ρ_{TC}. Depending on the details of the model, the SSC will be able to discover a techni-rho up to a mass near 2 TeV.[2]

Clearly, the most attractive possibility for electroweak symmetry breaking is the existence of a resonance which can be seen at the SSC. In this report we consider the following depressing scenario: It is the year 2001 and the two major detector collaborations at the SSC have just announced their results: $M_H > 800\ GeV$ and $M_{\rho_{TC}} > 1\ TeV$. Furthurmore, they have studied vector boson scattering *ad infinitum* and have found no hint of any resonant structure. What then? Must we plead for yet higher energies or are we guaranteed to learn something about the electroweak symmetry breaking by the very absence of resonances? This idea that the SSC guarantees information about the nature of the electroweak symmetry breaking has been dubbed the "no-lose corollary".[3]

Why might one reasonably expect there to be a "no-lose" corollary? The basic logic is as follows: We know the electroweak symmetry is broken and so if we see no resonances associated with the symmetry breaking, we must be below the scale of the symmetry breaking, $\sqrt{s} << \Lambda_{EWSB}$. However, the standard model with the Higgs boson removed to a very heavy energy scale contains three and four gauge boson self couplings which grow with energy. Hence such a model describes strongly interacting gauge bosons. Similarly, models with dynamical symmetry breaking are strongly interacting by construction. Some vestige of these strong interactions should hopefully be observable, presumably in an enhancement of the vector boson scattering amplitudes. Hence the "no-lose" corollary in its simplest form says that either there is an observable resonance or the W and Z gauge bosons are strongly interacting. It is the purpose of this work to analyze this conjecture within the framework of chiral perturbation theory. We have no definitive answers, but will rather point the way to where more work remains to be done.

To analyze theories with strongly interacting gauge bosons, we recall the example of strongly interacting particles observed experimentally, the pion system. The experimental results from pion scattering are well described through the use of the chiral Lagrangian, which is an expansion in powers of energy.[4] Because of the symmetries of the theory, the low energy behaviour obeys $\pi - \pi$ scattering theorems.

The low energy theorems can easily be generalized from pions to the longitudinal components of the W and Z gauge bosons. These scattering theorems are valid at low energy, that is $\sqrt{s} \ll \{M_{Res}, \Lambda_{EWSB}\}$, where M_{Res} is the mass of the lowest lying resonance interacting with the W_L's and Z_L's. (Of course if there is a resonance at low energy, it will be observed directly and the use of the chiral Lagrangian is not justified.) The lowest order term in an energy expansion of a scattering amplitude, that of $\mathcal{O}(s)$, is completely fixed by the global symmetry of the theory. The next terms in the expansion, however, are quite sensitive to the nature of the symmetry breaking mechanism. It has been suggested that by extracting the energy dependence of longitudinal gauge boson scattering cross sections, it may be possible to differentiate between different mechanisms of electroweak symmetry breaking, even if a resonance is not observed.[5] In Section 2 of this paper, we will pursue the analogy between pions and the W and Z gauge bosons and analyze the utility of the chiral Lagrangian for studying the interactions of longitudinal gauge bosons.

In Section 3, we extend our analysis by gauging the chiral Lagrangian with respect to $SU(2)_L \times U(1)_Y$. This allows us to study the interactions of all polarizations of gauge bosons and to analyze the production of gauge bosons from both vector boson scattering and from quark- antiquark annihilation. As an example, we discuss the reaction $q\bar{q} \rightarrow W^+Z$ which depends on the three gauge boson vertex and so is affected by higher order terms in the chiral Lagrangian. Section 4 uses chiral Lagrangian techniques for theories which have larger gauge groups than the standard model. These models are meant to simulate technicolor models which typically have an $SU(N) \times SU(N)$ global symmetry. We pay particular attention to reactions involving gluons in the initial state. Finally, Section 5 contains some conclusions.

2. Longitudinal Gauge Bosons and the Chiral Lagrangian

We begin this section by reviewing the use of chiral Lagrangians for the study of longitudinal gauge boson interactions. The formalism of effective Lagrangians provides a well defined computational framework for investigating the physics of electroweak symmetry breaking. The infinite set of terms in the effective Lagrangian can be organized in an energy expansion. At low energies, only a finite number of terms contribute to any given process. At higher energies, more and more terms become important, until the whole process breaks down at the scale of the symmetry breaking. This procedure gives an acceptable description of $\pi\pi$ scattering amplitudes up to energies of about $500\ MeV$.[6] For QCD, the scale of chiral symmetry breaking is about $1\ GeV$, so we naively expect the effective Lagrangian for vector boson scattering to be reasonable up to roughly half the scale of electroweak symmetry breaking, in most cases about $1.5\ TeV$.

If we assume that there are only three Goldstone bosons $(w^{\pm}, z)$, and that $\rho = 1$, the standard model has a global $SU(2)_L \times SU(2)_R$ symmetry which is broken to a custodial $SU(2)_V$. When the model is gauged with respect to $SU(2)_L \times U(1)_Y$ the Goldstone bosons become the longitudinal components of the W and Z gauge bosons. We use the electroweak equivalence theorem in which the longitudinal gauge bosons are replaced by their corresponding Goldstone bosons, producing amplitudes which are correct to $\mathcal{O}(M_W^2/s)$.[7]

The interactions of the Goldstone bosons can be described in terms of the $SU(2)$ field,

$$\Sigma = exp\left(2i\Pi \cdot T/v\right), \tag{1}$$

where $\Pi = (w^{\pm}, z)$, T are $SU(2)$ generators normalized such that $Tr(T_iT_j) = \frac{1}{2}\delta_{ij}$ and $v = 246\ GeV$. Under $SU(2)_L \times SU(2)_R$ transformations, Σ transforms as,

$$\Sigma \rightarrow L\Sigma R^{\dagger} \quad . \tag{2}$$

The $w^{\pm}$ and z interactions can now be found by an expansion in powers of energy, s/Λ^2_{EWSB}. The first term in the expansion, which is of $\mathcal{O}(s)$, has two derivatives and is,

$$\mathcal{L}_2 = \frac{v^2}{4} Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^{\dagger}\right). \tag{3}$$

The absolute normalization of $\mathcal{L}_2$ is completely determined by the requirement that the Goldstone bosons have canonical kinetic energy; there are no free parameters in Eq. (3).

The Lagrangian of Eq. (3) gives cross sections which are rising with energy and will eventually violate unitarity. For example,

$$\mathcal{A}(w^+w^- \rightarrow zz) = \frac{s}{v^2}, \tag{4}$$

violates partial wave unitarity at $\sqrt{s_c} \sim 1.7\ TeV$. The Lagrangian of Eq. (3) is obviously only valid for $s << s_c$.

To attempt to extend the range of validity of the chiral Lagrangian, we can include the $\mathcal{O}(s^2)$ terms in the energy expansion; that is, terms with four derivatives,

$$\mathcal{L}_4 = \frac{L_1}{16\pi^2} Tr\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) Tr\left(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right) + \frac{L_2}{16\pi^2} Tr\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right) Tr\left(\partial^\mu \Sigma \partial^\nu \Sigma^\dagger\right) \quad . \tag{5}$$

The sum $\mathcal{L}_2 + \mathcal{L}_4$ is the most general Lagrangian consistent with the chiral symmetry and involving only Goldstone bosons. Possible mass terms break the chiral symmetry and we neglect them. The coefficients L_1 and L_2 are *a priori* unknown and contain dynamical information about the model. What a given model does is to tell you the values of L_1 and L_2. In the range of validity, $\sqrt{s} << \Lambda_{EWSM}$, and with the minimal global symmetry group, **all models have the form** $\mathcal{L}_2 + \mathcal{L}_4$. The power of the chiral lagrangian formalism is that all models take this form; the weakness is that the validity of the energy expansion is severely restricted by the requirement of perturbative unitarity.

The Lagrangian of Eqs. (3) and (5) can be used to compute the Goldstone-Goldstone scattering amplitudes to order s^2. There are two types of contributions to this order. The first is a direct coupling that follows from the tree-level Lagrangian. The second is a one- loop correction that must be included at order s^2. The one loop contribution renormalizes the parameters L_1 and L_2 and depends on an arbitrary renormalization scale μ. There are also finite logarithmic corrections that cannot be absorbed into a redefinition of the couplings. We find the complete result to $\mathcal{O}(s^2/v^4)$[8],

$$\begin{aligned} \mathcal{A}(w^+w^- \to zz) =& \frac{s}{v^2} + \frac{4}{16\pi^2 v^4}\left(2L_1(\mu)s^2 + L_2(\mu)(t^2+u^2)\right) \\ &+ \frac{1}{16\pi^2 v^4}\left\{-\frac{1}{12}\left(3t^2+u^2-s^2\right)\log\left(-\frac{t}{\mu^2}\right)\right. \\ &\left. -\frac{1}{12}\left(3u^2+t^2-s^2\right)\log\left(-\frac{u}{\mu^2}\right) - \frac{s^2}{2}\log\left(-\frac{s}{\mu^2}\right)\right\} \quad . \end{aligned} \tag{6}$$

Isospin relationships can be used to find all of the other Goldstone boson scattering amplitudes.

An important question is the energy regime in which Eq. (6) is valid. A naive answer is given by setting $L_1 = L_2 = 0$ and computing the energy scale at which partial wave unitarity is first violated as done above. At one loop, L_1 and L_2 can be tuned to push this scale slightly higher, although the highest scale possible seems to be about $2\ TeV$.[9] It is also possible to use various unitarization techniques, such as K-matrix or Padé, to extend the range of validity,[10] but this is contrary to the spirit of the chiral Lagrangian formalism. Also, the results obtained in this manner are extremely sensitive to the exact nature of the unitarization prescription.

In Fig. 1, we show the allowed range of L_1 and L_2 for which partial wave unitarity is respected in the first five partial waves for Goldstone boson scattering below $1\ TeV$ and $1.5\ TeV$. The results of this figure include the one-loop results given above with $\mu = 1.5\ TeV$. From this figure we clearly see that the L_i are naturally of $\mathcal{O}(1)$ when normalized as in Eq. (5).

Figure 1

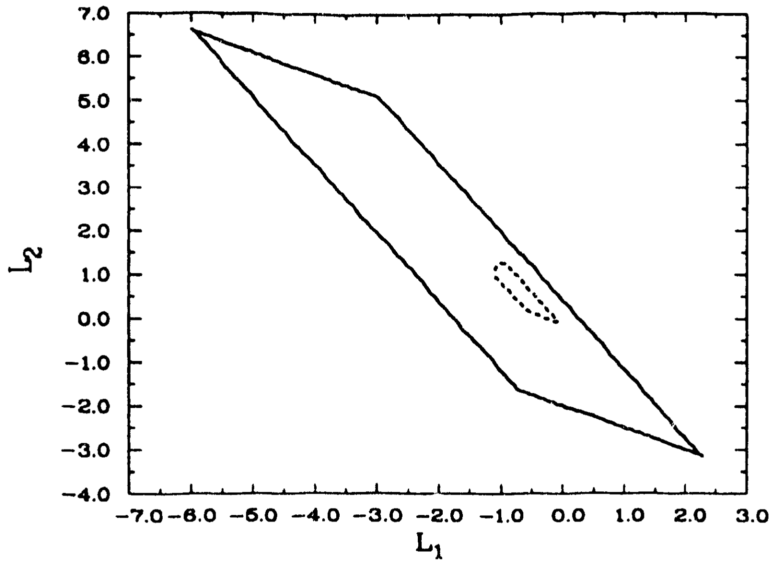

Fig. 1. *Values of $L_1(1.5\ TeV)$ and $L_2(1.5\ TeV)$ allowed by partial wave unitarity. The solid (dashed) line is the boundary of the allowed region when one requires the partial waves with $J < 2$ to satisfy $\mid Re(a^I_J) \mid < \frac{1}{2}$ for $\sqrt{s} < 1\ TeV$ ($\sqrt{s} < 1.5\ TeV$).*

It is instructive to compute some typical values for the L_i. In a model with three Goldstone bosons interacting with a scalar, isoscalar resonance like the Higgs boson, we have[11]

$$\begin{aligned} L_1(M_H) &= \frac{64\pi^3}{3}\frac{\Gamma_H v^4}{M_H^5} > 0 \\ L_2(M_H) &= 0 \quad . \end{aligned} \tag{7}$$

Using the standard model relationship between M_H and Γ_H (probably a poor assumption for a heavy Higgs boson) gives $L_1(M_H) = .55$ for a 1.5 TeV Higgs boson. In this case, the chiral Lagrangian would be valid at energies much below the Higgs boson mass. On the other hand, the best fit to the pion data (which presumably corresponds to a model with a ρ resonance) gives $L_1 = -L_2$. Using large N scaling arguments for the mass and width of a techni-rho meson, we can find $L_1(M_{\rho TC}) \sim -L_2(M_{\rho TC}) \sim -.6$ for a 1.5 TeV techni-rho meson.

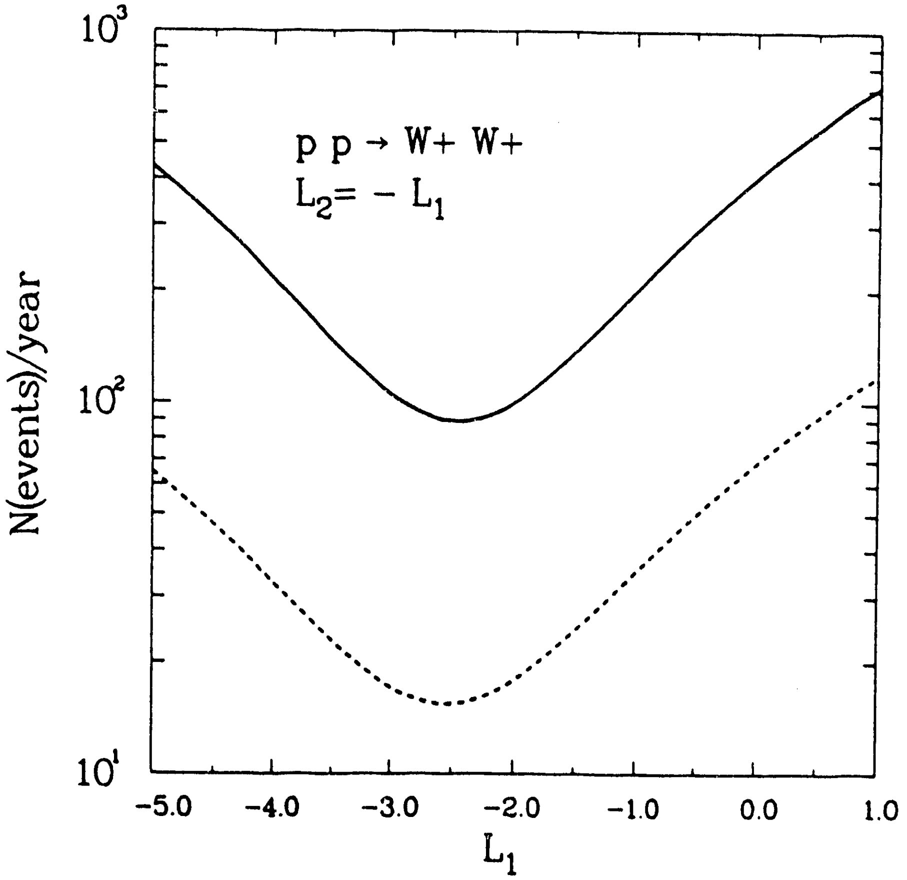

Fig. 2. *The number of $W_L^+ W_L^+$ events per year at the SSC (solid) and LHC (dotted), assuming an integrated luminosity of $10^{40}/cm^2$, for $.5 < M_{WW} < 1.$ TeV as a function of $L_1(1.5\ TeV)$, with $L_1 = -L_2$. The values of $L_1(1.5\ TeV)$ and $L_2(1.5\ TeV)$ shown preserve partial wave unitarity below 1 TeV.*

The coefficients L_1 and L_2 will be best measured in the process $pp \to W_L^+ W_L^+$ for which vector boson scattering is the dominant contribution.[12] In Fig. 2, we show the event rate in the interval $.5 < M_{WW} < 1\ TeV$ as a function of $L_1(1.5\ TeV)$. We have taken $L_1 = -L_2$ (which corresponds to a heavy techni-rho meson) and plotted only values which preserve unitarity up to $1\ TeV$. We see that the production rate is quite sensitive to L_1 and L_2, although to determine the exact values which can be probed at the SSC requires detailed detector simulations. If we make the naive assumption that the longitudinally polarized gauge bosons can be extracted efficiently from the total W^+W^+ signal[1] and define the signal to be observable if it introduces a 50% change in the integrated cross section, then the $W_L^+ W_L^+$ is sensitive to $| L_1, L_2 | > 1.0$. We note that this value is slightly larger than that of our "typical" values given above.

3. Gauging the Chiral Lagrangian

We can extend our analysis to include transversely polarized gauge bosons and also the three and four gauge boson vertices by gauging the chiral Lagrangian with respect to $SU(2)_L \times U(1)_Y$. This is simply done for the terms involving only Σ fields:

$$\begin{aligned}\mathcal{L} \to & \frac{v^2}{4} Tr\Big(D_\mu \Sigma D^\mu \Sigma^\dagger\Big) + \frac{L_1}{16\pi^2} Tr\Big(D_\mu \Sigma D^\mu \Sigma^\dagger\Big) Tr\Big(D_\nu \Sigma D^\nu \Sigma^\dagger\Big) \\ & + \frac{L_2}{16\pi^2} Tr\Big(D_\mu \Sigma D_\nu \Sigma^\dagger\Big) Tr\Big(D^\mu \Sigma D^\nu \Sigma^\dagger\Big) \quad ,\end{aligned} \tag{8}$$

where

$$D^\mu \Sigma = \partial^\mu \Sigma + ig \vec{W}^\mu \cdot \vec{T} \Sigma - i \Sigma g' B^\mu T_3 \quad , \tag{9}$$

and g and g' are the $SU(2)_L$ and $U(1)_Y$ couplings respectively. In the unitary gauge, $\Sigma = 1$, it is easy to see that the above Lagrangian gives the correct W mass, $M_W = \frac{1}{2} g v$. If we do not assume a custodial $SU(2)_V$ then there is an additional term of $\mathcal{O}(s)$,

$$\mathcal{L}' = g^2 \beta_1 v^2 \Big[Tr\Big(T_3 \Sigma^\dagger D_\mu \Sigma\Big)\Big]^2 \quad . \tag{10}$$

Since this term violates the custodial $SU(2)_V$, the coefficient is related to the ρ parameter, $\rho = 1 - 2g^2\beta_1$.

To $\mathcal{O}(s^2)$ there are 13 CP conserving terms involving gauge fields and Goldstone bosons which were first written down by Longhitano for the $SU(2)_L \times U(1)_Y$ case.[13] Most of these terms violate the custodial $SU(2)_V$ symmetry and are presumably small, although they do not all contribute to the ρ parameter. We will assume that $SU(2)_V$ is broken only by terms proportional to the hypercharge coupling, as is the case in the standard model. In this case there are three additional terms of $\mathcal{O}(s^2)$:

[1]The detailed studies of Ref. [12] suggest that this is indeed possible.

$$\mathcal{L}'_4 = -ig\frac{L_{9L}}{16\pi^2}Tr\Big(W^{\mu\nu}D_\mu\Sigma D_\nu\Sigma^\dagger\Big) - ig'\frac{L_{9R}}{16\pi^2}Tr\Big(B^{\mu\nu}D_\mu^\dagger\Sigma D_\nu\Sigma\Big)$$
$$+ gg'\frac{L_{10}}{16\pi^2}Tr\Big(\Sigma B^{\mu\nu}\Sigma^\dagger W_{\mu\nu}\Big) \quad , \tag{11}$$

where $B_{\mu\nu} = (\partial_\mu B_\nu - \partial_\nu B_\mu)T_3$ and $W_{\mu\nu} = (\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu)\cdot\vec{T} - ig[\vec{W}_\mu\cdot\vec{T}, \vec{W}_\nu\cdot\vec{T}]$. One can think of other terms, such as $Tr\Big(D^2\Sigma^\dagger D^2\Sigma\Big)$, but they can all be absorbed in Eqs. (8) and (11) by using the lowest-order equations of motion, $\Sigma D^2\Sigma^\dagger = (D^2\Sigma)\Sigma^\dagger$.

L_{10} is just the famous S parameter of Peskin and Takeuchi, $L_{10} = -\pi S$.[14] A best fit to current data translates to $L_{10}(1.5\ TeV) = .5 \pm 1.6$.[15] We see that at TeV energies the error is so large as to make the limit on L_{10} relatively meaningless. The terms with L_9 contain anomalous three and four gauge boson couplings which contribute to the processes, $q\bar{q} \to W^\pm Z, W^+W^-$, and $W^\pm\gamma$. Hence the chiral Lagrangian is just another means of parameterizing the deviation of the three gauge boson couplings from those of the Standard Model. The chiral Lagrangian provides a particularly convenient and consistent framework. Note that Eq. (11) is manifestly gauge invariant with respect to $SU(2)_L \times U(1)_Y$.[16]

It is straightforward to use Eqs. (8) and (11) to compute the amplitudes for various longitudinal gauge boson scattering production rates. As an example, we present the rate for $pp \to W_L^+ Z_L$. This process has also been studied extensively by Falk, Luke and Simmons, with similar results.[17] In Fig. 3, we show the rate for the process $pp \to W_L^+ Z_L$ at the SSC, $\sqrt{S} = 40\ TeV$, and at the LHC, $\sqrt{S} = 17\ TeV$. This process is dominated by the $q\bar{q}$ contribution and hence is primarily sensitive to L_{9L} (we have taken all other $L_i = 0$). With no anomalous couplings, the total number of W^+Z events per year is expected to be around $88,500$ at the SSC and $34,000$ at the LHC. Of these, the number of $W_L^+ Z_L$ events is about $10,000$ at the SSC and $4,000$ for the LHC. Hence in order for there to be any hope of measuring the effect of the anomalous coupling parameterized by L_{9L}, it is necessary to be able to extract the longitudinally polarized gauge bosons from the overwhelming number of transverse gauge bosons. There have been many serious phenomenological studies of the prospects for doing this, with encouraging results.[18] If we again define a 50% deviation from the lowest order result as "observable", then the SSC is sensitive to $\mid L_9 \mid > 3$. For comparison, we note that in a model coupled to a scalar, isoscalar particle, $L_{9L}(1.5\ TeV) \sim 0$, while in a model coupled to a techni-rho, $L_{9L}(1.5\ TeV) \sim 2.4$.

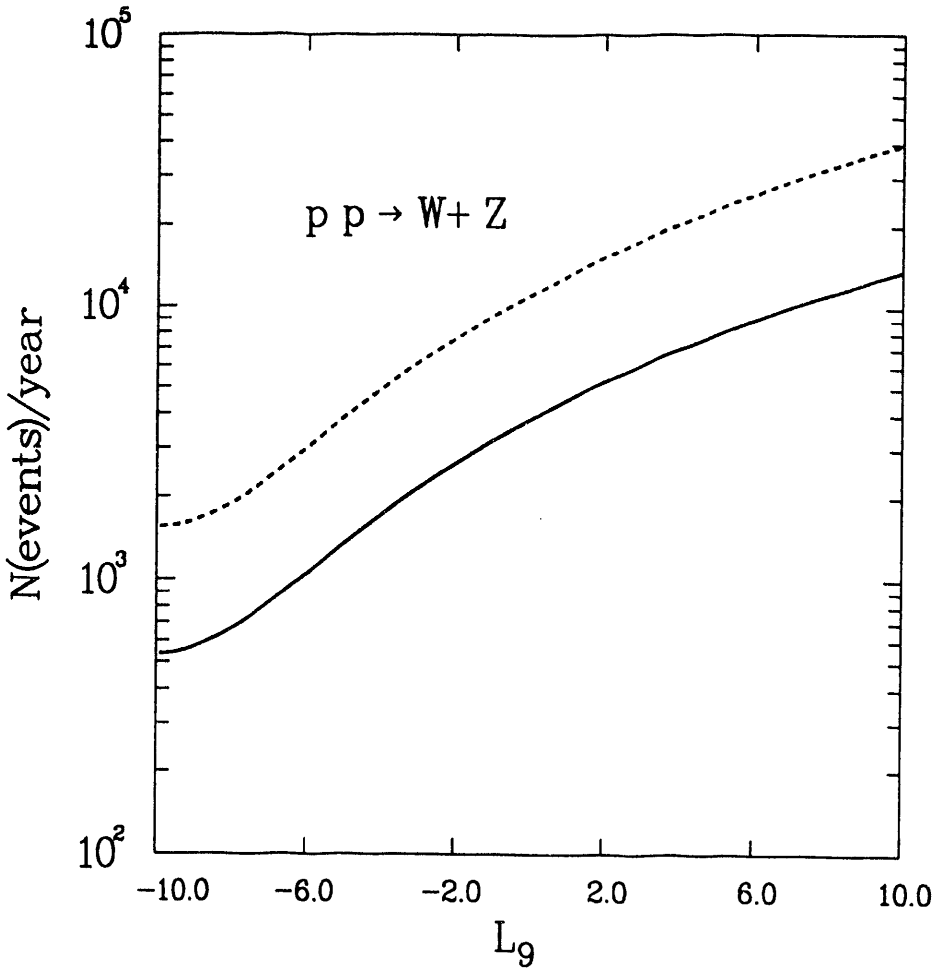

Fig. 3 *The number of W^+Z events per year assuming an integrated luminosity of* 10^{40} cm^{-2} *at the SSC (dotted) and LHC (solid) as a function of* $L_{9L}(1.5\ TeV)$.

4. Beyond the Standard Model

It is in extentions of the standard model that the real power of the chiral Lagrangian approach becomes apparent[19]. Chiral Lagrangians provide a systematic and model-independent approach to electroweak symmetry breaking. If future colliders have insufficient energy to produce any new resonances which may be associated with the electroweak symmetry breaking, then the chiral Lagrangian may yield important information in the low energy region below the scale of the new resonances.

In this section, we will make the simplifying assumption that whatever physics breaks the electroweak symmetry, it has a global chiral symmetry group $SU(N)_L \times SU(N)_R$ which is spontaneously broken to the diagonal subgroup G. We have in mind a generic technicolor model. There are then $dim\ G$ Goldstone bosons Π^A which can be parameterized by the matrix $\Sigma = exp(2i\Pi^A T^A/f)$, where the T^A are now the generators of G (normalized such that $Tr(T^A T^B) = \frac{1}{2}\delta_{AB}$) and $f = v/\sqrt{N/2}$. Of course, three of the Goldstone bosons are exactly massless and become the longitudinal components of the $W^\pm$ and Z gauge bosons. The remaining $dim\ G - 3$ Goldstone bosons must acquire mass; they are known as pseudo-Goldstone bosons.

Once the embedding of the $SU(3) \times SU(2)_L \times U(1)_Y$ standard model gauge group in the chiral symmetry group is specified, all of the various interactions can be computed. To define the embeddings, we first construct the matrices $X^\alpha = X^{\alpha A}T^A$, $X^a = X^{aA}T^A$ and $X = X^A T^A$ which generate $SU(3)$, $SU(2)_L$, and $U(1)_Y$, respectively. The embedding is then defined by the covariant derivative,

$$D_\mu \Sigma = \partial_\mu \Sigma + g_s G_\mu^\alpha [X^\alpha, \Sigma] + igW_\mu^a X^a \Sigma - ig' B_\mu \Sigma X \tag{12}$$

where G_μ^α is the $SU(3)$ color gauge field and W_μ^a and B_μ are the $SU(2)_L \times U(1)_Y$ gauge bosons. The gauge couplings explicitely break the global symmetry.

The self-interactions of the Goldstone bosons and their interactions with the standard model gauge fields are given by the chiral Lagrangian. The Lagrangian is non-renormalizable, but it makes sense as an effective theory at low energies. As before, to lowest order in the energy expansion, the effective Lagrangian is

$$\mathcal{L} = \frac{f^2}{4} Tr\Big(D_\mu \Sigma D^\mu \Sigma^\dagger\Big) \quad , \tag{13}$$

where the covariant derivative is now given in Eq. (12). The mass term for the pseudo-Goldstone bosons is extremely model dependent, so we will neglect possible mass effects. In typical models, $m \sim 100\ GeV$, so we will always present results for $\sqrt{s} > 200\ GeV$.

The lowest order effective Lagrangian describes pseudo-Goldstone boson scattering to $\mathcal{O}(s)$. To this order, the predictions are universal in the sense that they depend only on the symmetry group and the decay constant f. At $\mathcal{O}(s^2)$, the pseudo-Goldstone scattering amplitudes depend on the next to leading order effective Lagrangian, which now contains four operators:

$$
\begin{aligned}
\mathcal{L}_4 = & \frac{L_1}{16\pi^2} Tr\Big(D_\mu \Sigma^\dagger D^\mu \Sigma\Big) Tr\Big(D_\nu \Sigma^\dagger D^\nu \Sigma\Big) \\
& + \frac{L_2}{16\pi^2} Tr\Big(D_\mu \Sigma^\dagger D^\nu \Sigma\Big) Tr\Big(D_\mu \Sigma^\dagger D^\nu \Sigma\Big) \\
& + \frac{N L_3}{16\pi^2} Tr\Big(D_\mu \Sigma^\dagger D^\mu \Sigma D_\nu \Sigma^\dagger D^\nu \Sigma\Big) \\
& + \frac{N L_4}{16\pi^2} Tr\Big(D_\mu \Sigma^\dagger D^\nu \Sigma D_\mu \Sigma^\dagger D^\nu \Sigma\Big) \quad .
\end{aligned}
\tag{14}
$$

The coefficients L_i are expected to be of order one; they are determined by the dynamics that underlie the symmetry breaking.

The problem of unitarity violation is even more severe in models with extended gauge groups than in the standard model since some of the partial waves grow with N.[20,21] In Fig. 4, we plot the singlet, antisymmetric adjoint and the symmetric adjoint amplitudes for pseudo-Goldstone boson scattering. We have chosen arbitrary values for the L_i in order to illustrate the effects of these terms. We see that there can be a considerable difference between the energies at which the $\mathcal{O}(s)$ and the $\mathcal{O}(s^2)$ partial waves first violate unitarity. We associate the unitarity violation with the appearance of some structure in the fundamental theory such as a resonance. Clearly the chiral Lagrangian is only valid below this scale.

It is useful to compare the results of the chiral Lagrangian with a model in which we couple the pseudo-Goldstone bosons to vector and/or scalar resonances. We choose a model where the couplings are given by

$$
\mathcal{L} = i\frac{g_\rho f^2}{m_\rho} Tr\Big(\rho^{\mu\nu}\xi^\dagger D_\mu \Sigma D_\nu \Sigma^\dagger \xi\Big) + \frac{g_\sigma f^2}{m_\sigma} \sigma Tr\Big(D_\mu \Sigma D^\mu \Sigma^\dagger\Big) \tag{15}
$$

As an example, we arbitrary parameters and plot the results with the solid line in Fig. 4 .

In a general model there will be pseudo-Goldstone bosons which are not color singlets. As an example we will compute the contribution of these colored particles to longitudinal vector boson production via gluon fusion. The gluon fusion amplitude is model independent to $\mathcal{O}(s^2)$ in the sense that it does not depend on the L_i. Since the gluon fusion amplitude is independent of the L_i, it probes different physics than Goldstone boson scattering. For concreteness we shall present our results in terms of the Farhi-Susskind[22] model which has $G = SU(8)$, so there are 63 Goldstone bosons. The embeddings are such that there is one color octet and two color triplets of weak -triplet pseudo Goldstone bosons.

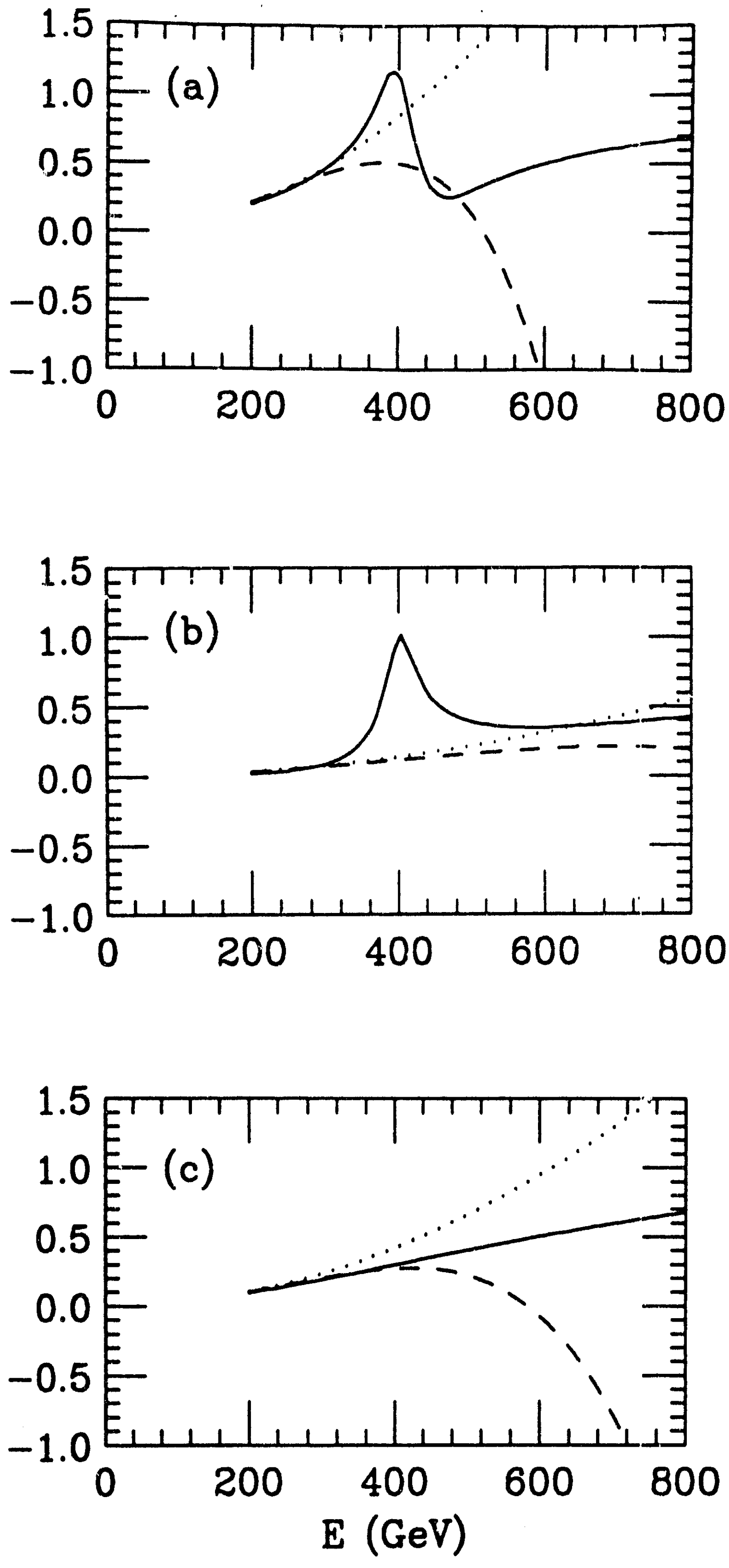

Fig. 4. *Partial wave amplitudes for the singlet channel (a); the anti-symmetric adjoint (b); and the symmetric adjoint (c). The dotted curves represent the real part of the lowest-order results and the dashed curves are the real part of the order s^2 results for $L_1(\mu) = -.1$, $L_2(\mu) = .3$, $L_3(\mu) = -.09$, and $L_4(\mu) = -.13$ at $\mu = 383$ GeV. The solid line is the modulus of the amplitude for a tree-level model with a vector resonance of mass 400 GeV and width 40 GeV and a scalar resonance of mass 400 GeV and width 50 GeV.*

We will compute the amplitude for producing two longitudinally polarized Z_L bosons or a $W_L^+W_L^-$ pair due to loops of colored pseudo-scalar bosons[23]. We work in the high energy limit and use the equivalence theorem to replace the longitudinal vector bosons by the corresponding Goldstone bosons. Since the global symmetry group is always larger than $SU(2) \times SU(2)$, the effective Lagrangian has a custodial $SU(2)_V$ symmetry which guarantees that the two amplitudes are identical, $\mathcal{A}(gg \to Z_L Z_L) = \mathcal{A}(gg \to W_L^+ W_L^-)$. The amplitude for $g_\mu^\alpha(q_1) g_\nu^\beta(q_2) \to z(p_1) z(p_2)$ is required by gauge invariance to have the form

$$\begin{aligned}\mathcal{A} =\Big\{&\tilde{A}(s,t,u)\Big(-\frac{s}{2}g_{\mu\nu} + q_{2\mu}q_{1\nu}\Big)\\ &+ \tilde{B}(s,t,u)\Big(-\frac{ut}{2}g_{\mu\nu} - sp_{1\mu}p_{1\nu} - tq_{2\mu}p_{1\nu} - up_{1\mu}q_{1\nu}\Big)\Big\}\epsilon^\mu(q_1)\epsilon^\nu(q_2)\delta_{\alpha\beta} \quad .\end{aligned} \tag{16}$$

The amplitude vanishes at tree level. The one-loop amplitude is finite, and is given by

$$\begin{aligned}\tilde{A}(s,t,u) =&T(R)\Big(\frac{\alpha_s}{\pi v^2}\Big)\\ \tilde{B}(s,t,u) =&0\end{aligned} \tag{17}$$

where $T(R) = 1/2$ for each color triplet and $T(R) = 3$ for each color octet.

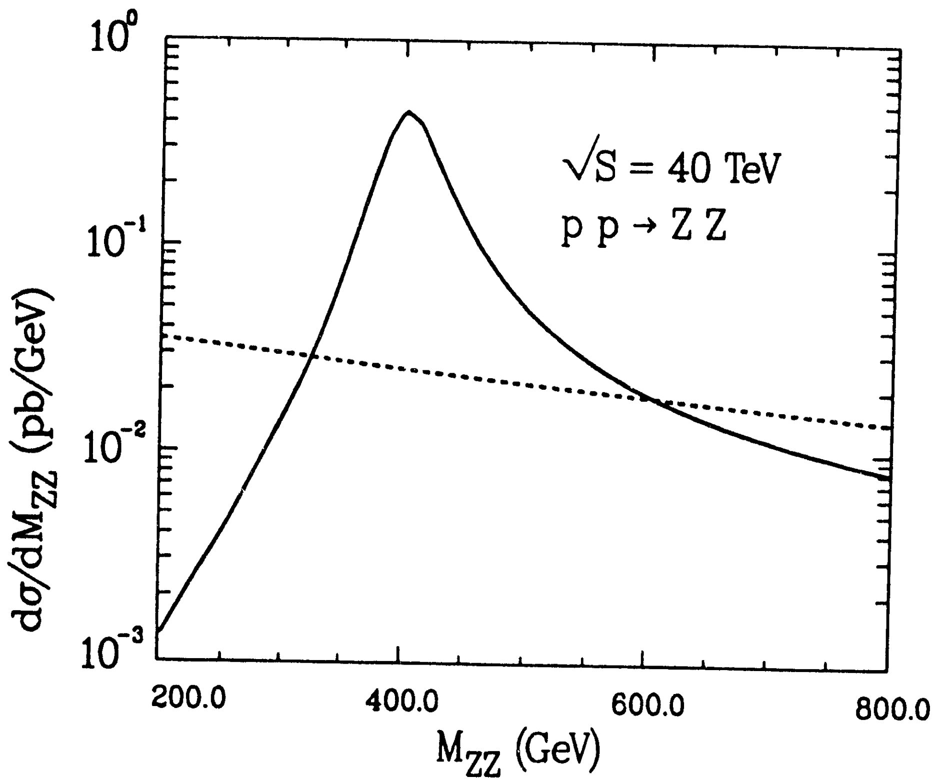

Fig. 5 *Production of $Z_L Z_L$ pairs by gluon fusion at the SSC with a rapidity cut $|\, y \,| < 2.5$. The dotted line corresponds to one color octet of pseudo-Goldstone bosons. The solid line corresponds to a techni-sigma of mass 400 GeV, width 50 GeV, and $h_\sigma = .8$*

In Fig. 5, we see that the contribution of colored pseudo-Goldstone bosons to Z pair production (dotted line) can be very large. In the Farhi- Susskind model, it dominates the standard model contribution from $q\bar{q} \rightarrow ZZ$ for $M_{ZZ} > 300\ GeV$. This results from the large octet color factor and from the fact that the gluon luminosity is much larger than that for quarks. We did not include a mass for the pseudo-Goldstone bosons, so the figure is valid only for $M_{ZZ} \geq 2\mu$, where μ is the pseudo-Goldstone boson mass. Below threshold, the curves are extremely sensitive to the precise form of the mass matrix. From this figure we see that at future hadron colliders such as the SSC or the LHC, the contributions from colored pseudo-scalars are potentially large and dominate the standard model contribution. We have extended this figure to an energy scale of $800\ GeV$, although above about 600 GeV, it is probably unreliable due to the effects of unitarity violation illustrated in Fig. 4.

For comparison, we consider a model with a direct coupling between the techni-σ and the gluons,

$$\mathcal{L} = \frac{\alpha_s h_\sigma}{m_\sigma} \sigma G^a_{\mu\nu} G^{a\mu\nu} \quad . \tag{18}$$

We show in Fig. 5 the contribution from a $400\ GeV$ techni-sigma which is coupled via Eq. (18) to ZZ production from gluon fusion . We choose $h_\sigma = .8$, as suggested by the model of Cahn and Suzuki.[20] Again we see a large rate. The important point is that the order of magnitude found from coupling a techni-sigma to the chiral Lagrangian and that from computing the pseudo-Goldstone boson loops is similar. Clearly, there are too many assumptions to take these numbers as more than an indication that the rate in this channel may be significantly larger than in the standard model if there is an extended gauge group.

5. Conclusions

We have studied high energy scattering of longitudinal gauge bosons using the equivalence theorem and chiral perturbation theory to $\mathcal{O}(s^2)$. A strongly interacting electroweak symmetry breaking sector could produce light resonances which should be seen by the next generation of colliders. It could also happen that there are no relatively light resonances (with mass less than $\sim 1.5\ TeV$). It is in this case that our analysis becomes relevant since the scattering amplitudes should show deviations from the low energy theorems that depend on the nature of electroweak symmetry breaking. Different processes will measure different coefficients: W^+W^+ is sensitive to L_1, L_2, W^+Z measures L_9, and L_{10} is measured by precision tests at LEP. The precise values of the coefficients which will be probed requires detailed studies, which are beyond the scope of this work.[18] However, the power of the approach is clear in that within the range of validity of the energy expansion, the chiral lagrangian approach describes **any** mechanism of spontaneous symmetry breaking. Clearly, much more work needs to be done studying the range of predictions of various models.

I have focussed by attention on the SSC; similar conclusions can be drawn for the LHC.

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