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**TITLE: THE KEMMER-DUFFIN-PETIAU FORMALISM AND INTERMEDIATE-ENERGY
DEUTERON-NUCLEUS SCATTERING**

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THE KEMMER-DUFFIN-PETIAU FORMALISM AND INTERMEDIATE-ENERGY DEUTERON-NUCLEUS SCATTERING

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ABSTRACT

The spin-1 Kemmer-Duffin-Petiau equations are described and applied to deuteron-nucleus scattering. Comparison with $d + {}^{58}\text{Ni}$ elastic scattering data at 400 MeV shows that the KDP model reproduces experimental spin observables very well.

The success of the Dirac equation in describing intermediate-energy proton-nucleus scattering¹⁾ encourages the use of relativistic wave equations in treating other nuclear probes. Recently the Kemmer-Duffin-Petiau (KDP) formalism²⁾, which yields

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first-order equations for both spin-0 and spin-1 particles, has been applied to pion-nucleus and kaon-nucleus scattering³⁾. The KDP approach has so far yielded results that are similar to those obtained from standard treatments of meson-nucleus scattering. It should be remembered, however, that the most dramatic differences between the Dirac and Schrödinger approaches to proton-nucleus scattering occur in the spin observables, which do not exist for the scattering of spin-0 particles from spinless nuclei. For this reason, and for those mentioned in Ref. 3., we now apply the KDP formalism to the scattering of spin-1 probes. Here we will give a brief description of the spin-1 KDP formalism followed by an application to deuteron-nucleus scattering. We find that the KDP-based deuteron-nucleus optical potentials are in close agreement with those obtained by Yahiro *et al.*⁴⁾, using the usual Watanabe approach to d- Δ scattering. The model is also used to calculate d + ^{58}Ni scattering observables at 400 MeV.

The free-particle KDP²⁾ is

$$(i\beta^\mu \partial_\mu - m) \psi = 0 \quad (1)$$

where the β^μ obey

$$\beta^\mu \beta^\nu \beta^\lambda + \beta^\lambda \beta^\nu \beta^\mu = \delta^{\mu\nu} \beta^\lambda + g^{\lambda\nu} \beta^\mu. \quad (2)$$

A 16 x 16 representation for β^μ that satisfies Eq. (2) is

$$\beta^\mu = \frac{1}{2} (I_1 \bullet \gamma_2^\mu + \gamma_1^\mu \bullet I_2) \quad (3)$$

where I is the 4 x 4 identity matrix, the γ^μ are the Dirac matrices, and \bullet indicates an outer product. The numerical subscript indicates the space in which a given matrix operates. The representation given by Eq. (3) is reducible and can be decomposed into three irreducible representations: a one-dimensional representation in which all $\beta^\mu = 0$, a five-dimensional one that results in a set of spin-0 equations, and a ten-dimensional representation that gives a set of spin-1 equations. For the free-particle spin-1 case, ψ is a ten-component vector given by $(A^\mu, F^{\mu\nu})$ where the A^μ satisfy the free-particle Proca equation and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

As with the Dirac equation, an interaction U can be introduced into the KDP formalism yielding

$$(i\beta^\mu \partial_\mu - m - U) \psi = 0 . \quad (4)$$

Like the spin-0 case, the most general form of U contains two scalars and two vectors⁶⁾. The two scalars are given by the 10×10 identity matrix I and the ten-dimensional projection matrix, $P = \text{diag} (1,1,1,1,0,\dots,0)$. The two vectors are β^μ and $(\beta^\mu = P\beta^\mu p - \beta^\mu P)$. A heuristic argument, employing the two-body nature of the deuteron suggests the use of the following form of the deuteron-nucleus interaction⁷⁾:

$$U^D(E_D) = 2S^N(E_D/2 + 2V^N(E_D/2) \beta^0 , \quad (5)$$

where the $N(D)$ superscript represents a nucleon (deuteron)-nucleus potential and E_D is the deuteron energy in the c.m. frame. We note that this choice is of the same form as the usual Watanabe model when zero-range deuteron wave-functions are assumed.

With this choice, Eq. (4) can be used to derive an effective three-component equation of the form

$$\{ p^2 + 2E_D [U_{\text{CENT.}} + U_{S0}(\vec{L} \cdot \vec{S}) + U_{\text{DARWIN}}(\vec{r} \cdot \vec{p}) + U_T^1(\vec{S} \cdot \vec{r})^2 + U_T^2(\vec{S} \cdot \vec{p})(\vec{S} \cdot \vec{r})] \} \vec{A} \approx k_D^2 \vec{A} , \quad (6)$$

with $E_D^2 = k_D^2 + m_D^2$, where m_D is the deuteron mass. Terms of order m_D^{-3} are neglected in Eq. (6). The expressions for the central and spin-orbit potentials are given by (suppressing energy dependence),

$$2E_D U_{\text{CENT.}}^D = 4 \left[m_D S^N + (S^N)^2 + E_D V^N - (V^N)^2 \right] , \quad (7a)$$

$$2E_D U_{S0}^D = -\frac{1}{r} \frac{\partial}{\partial r} \left[2 \ln(m_D + 2S^N) \cdot \ln(E_D - 2V^N) \right] , \quad (7b)$$

where small terms involving derivatives have been neglected in Eq. (7a).

In obtaining Eq. (7b) we have written the mixed tensor term in terms of $(\vec{S} \cdot \vec{p})$ ($\vec{S} \cdot \vec{r}$) rather than $(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{p})$ or as a Cartesian tensor of the type discussed by Satchler⁸⁾.

$$(i\beta^\mu \partial_\mu - m - U) \psi = 0 . \quad (4)$$

Like the spin-0 case, the most general form of U contains two scalars and two vectors⁶⁾. The two scalars are given by the 10×10 identity matrix I and the ten-dimensional projection matrix, $P = \text{diag}(1,1,1,1,0,\dots,0)$. The two vectors are β^μ and $(\beta^\mu = P\beta^\mu p - \beta^\mu P)$. Heuristic argument, employing the two-body nature of the deuteron suggests the use of the following form of the deuteron-nucleus interaction⁷⁾:

$$U_D^D(E_D) = 2S^N(E_D/2 + 2V^N(E_D/2) \beta^0 , \quad (5)$$

where the $N(D)$ superscript represents a nucleon (deuteron)-nucleus potential and E_D is the deuteron energy in the c.m. frame. We note that this choice is of the same form as the usual Watanabe model when zero-range deuteron wave-functions are assumed.

With this choice, Eq. (4) can be used to derive an effective three-component equation of the form

$$\{ p^2 + 2E_D [U_{\text{CENT.}} + U_{SO}(\vec{L} \cdot \vec{S}) + U_{\text{DARWIN}}(\vec{r} \cdot \vec{p}) + U_T^1(\vec{S} \cdot \vec{r})^2 + U_T^2(\vec{S} \cdot \vec{p})(\vec{S} \cdot \vec{r})] \} \vec{A} \approx k_D^2 \vec{A} , \quad (6)$$

with $E_D^2 = k_D^2 + m_D^2$, where m_D is the deuteron mass. Terms of order m_D^{-3} are neglected in Eq. (6). The expressions for the central and spin-orbit potentials are given by (suppressing energy dependence),

$$2E_D U_{\text{CENT.}}^D = 4 \left[m_D S^N + (S^N)^2 + E_D V^N - (V^N)^2 \right] , \quad (7a)$$

$$2E_D U_{SO}^D = -\frac{1}{r} \frac{\partial}{\partial r} \left[2\ln(m_D + 2S^N) - \ln(E_D - 2V^N) \right] , \quad (7b)$$

where small terms involving derivatives have been neglected in Eq. (7a).

In obtaining Eq. (7b) we have written the mixed tensor term in terms of $(\vec{S} \cdot \vec{p})$ rather than $(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{p})$ or as a Cartesian tensor of the type discussed by Satchler⁸⁾.

The various ways of choosing to write the mixed tensor term constitute, in effect, different approximations as we neglect the contributions of terms of the form $(\mathbf{S} \cdot \mathbf{r})^2$, $(\mathbf{S} \cdot \mathbf{p})(\mathbf{S} \cdot \mathbf{r})$, or $(\mathbf{S} \cdot \mathbf{r})(\mathbf{S} \cdot \mathbf{p})$. Each choice for the mixed tensor representation produces a different spin-orbit term. For example, if we choose the representation $(\mathbf{S} \cdot \mathbf{r})(\mathbf{S} \cdot \mathbf{p})$, then the effective spin-orbit potential,

$$2E_D U_{S0}^D = -\frac{1}{r} \frac{\partial}{\partial r} \ln (m_D + 2S^N) , \quad (7c)$$

is independent of the scalar potential. If no contribution from the mixed tensor term is included in the spin-orbit, then we obtain,

$$2E_D U_{S0}^D = -\frac{1}{r} \frac{\partial}{\partial r} \ln (m_D + 2S^N) , \quad (7d)$$

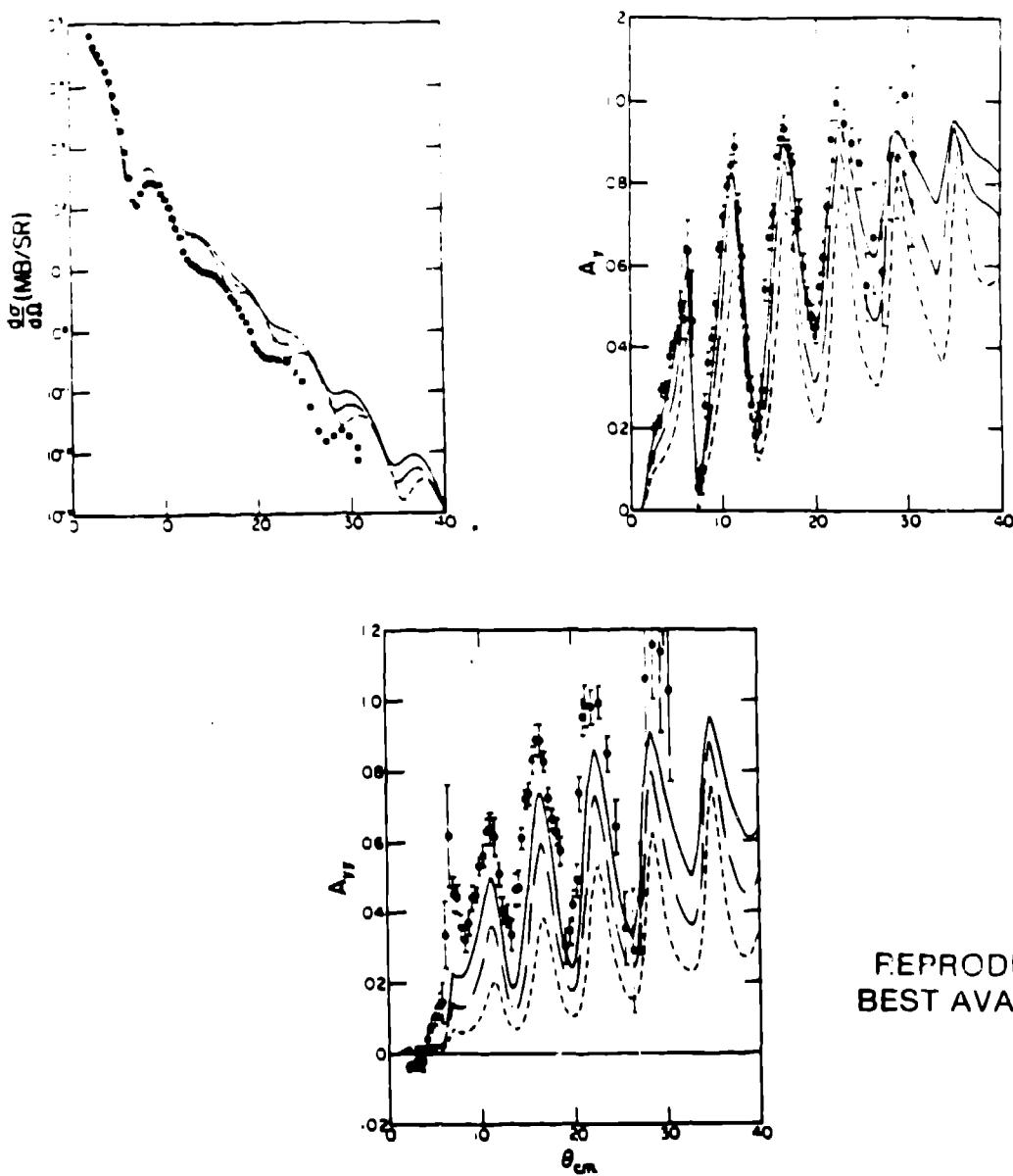
which depends only on the scalar potential. Calculations using these different spin-orbits give similar results for deuteron-nucleus observables, with the form given by Eq. (7b) yielding the best agreement with the spin measurements.

Relativistic deuteron-nucleus scattering has also been considered by Shepard, Rost, and Murdock⁹, employing the Breit equation, and Santos and collaborators, who use the Proca and Weinberg equations¹⁰. In each of these studies, parameters were varied in order to fit the deuteron-nucleus data. Our KDP-based calculation, in contrast, is parameter-free in the sense that once the nucleon-nucleus potentials have been fixed by fitting proton-nucleus data, no other parameters are varied. This provides a clear test of the validity of a relativistic one-body description of deuteron-nucleus scattering data and indicates that the model has predictive power.

The scalar and vector nucleon-nucleus optical potentials, chosen to fit $p + {}^{40}\text{Ca}$ data at 200 MeV, are scaled to represent $p + {}^{58}\text{Ni}$, as was done in Ref. 4. The effective KDP central and spin-orbit potentials are used in the Schrödinger equation with relativistic kinematics and the $d + {}^{58}\text{Ni}$ elastic scattering observables are calculated. The results are shown in Fig. 1, along with the data of Ref. 11. The spin observables obtained using Eq. (7b) are particularly well represented, and are quite similar to the results of Ref. 4. The effect of using the different forms for the spin-orbit can be seen in Fig. 1. The deviation of the calculated cross sections from the data at larger angles is an expected feature of the model, as it contains, for example, no contribution from breakup.

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Fig. 1. Calculated elastic scattering observables for $d + {}^{58}\text{Ni}$ at 400 Mev using the effective KDP central and spin-orbit potentials of Eq. (7). The calculations shown by the smooth lines use the spin-orbit potential of Eq. (7b), the dashed lines use Eq. (7c), and the dashed-dotted lines use Eq. (7d). The data are from Ref. 11.

We have described the spin-1 KDP formalism and used it to calculate deuteron-nucleus optical potentials. We have also found good agreement with deuteron-nucleus data for ${}^{58}\text{Ni}$ at 400 MeV with the spin observables being particularly well represented. It thus appears promising that the KDP formalism can provide a relativistic framework in which to analyze deuteron-nucleus scattering.

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