CONF-9008103--1

STIFFNESS CONTROL OF TELEOPERATORS WITH REDUNDANT, DISSIMILAR KINEMATICS

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CONF-9008103--1

DE90 011642

Paper to be presented to the
IEEE International Conference on System Engineering
August 9-11, 1990
Pittsburgh, Pennsylvania

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*Operated by Martin Marietta Energy Systems, Inc. for the U.S. Department of Energy under Contract No. DE-AC05-84OR21400.

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ABSTRACT

This paper addresses the problem of dissimilar kinematic teleoperator systems. Since the next generation of teleoperator systems will likely include dissimilar kinematics, a workable control scheme compatible with modern microprocessor computing capability is needed. The control scheme presented in this paper incorporates the work and ideas of numerous researchers over the past 40 years. Due to the ongoing nature of this research and limited space allowed, only a brief summary will be given concerning the overall control strategy; instead, the master controller and orientation representation for both the master and slave will be the main focus of this paper.

I. INTRODUCTION AND OBJECTIVES

In the late 1940's, Goertz [**Goertz,54**] and colleagues at Argonne National Laboratory developed one the earliest recognizable mechanical master/slave manipulators without force reflection and, later, one with force-reflecting capabilities. In the early 1950's, Goertz and his colleagues developed an electric master/slave teleoperator with each slave joint servo tied directly to the master joint servo because the master and slave were kinematically similar. The control structure for these manipulators was the classical position-position controller. A positional difference between the desired and actual slave position is reflected back as a drive signal to the master to provide the human operator with a feel of the environment. The positional-positional control scheme has been the basic controller for almost all master/slave manipulators used by industry until now. When the master and slave are not kinematically similar, the design of the controller is particularly difficult. Three major problems are associated with this controller design: (1) orientation representation, (2) accurate force-refection, and (3) redundancy resolution.

Representing the orientation between the master and slave and using this information for force reflection are difficult problems in the control of any teleoperated system with kinematically dissimilar master and slave. The objective of this paper is to show how to incorporate Euler parameters (which are related to quaternions) into the controller design. To achieve accurate force-reflection, a type of stiffness controller will be designed for both master and slave. Using both the master and slave Jacobians [**Miyazaki,86**] allows accurate force reflection for kinematically dissimilar manipulators. Differences in the application of a stiffness controller for robotic mode and teleoperation are pointed out. When the slave has more than six DOF (degrees-of-freedom), redundancy resolution must be addressed. With dissimilar kinematic designs, simple joint positional differences are no longer adequate for a force-reflecting manipulator. Because of space limitations and the ongoing nature of this research, the master manipulator is the primary focus of this paper. A brief discussion is given concerning the slave controller, but details are deferred until a later paper. The results are applied to the 6-DOF Kraft master manipulator and the 7-DOF Center for Engineering Systems Advanced Research Manipulator (CESARm) slave manipulator at Oak Ridge National Laboratory (ORNL).

II. DEFINITION OF EULER PARAMETERS

<u>Euler Parameters</u>

The notation followed in this paper is similar to Craig's notation [**Craig,89**], and differences will be clear from the context. Many of the matrix and vector relationships that are stated but not proven are given in [**Yuan,88**] (or at least their references).

Let frame A, {A}, and frame B, {B}, be two arbitrary frames that are initially coincident. If {A} is fixed and {B} is rotated about a normalized vector ${}^{A}\hat{K}$ by an angle θ according to the right-hand rule then the rotational matrix ${}^{A}_{B}R$ relating a vector in {B} to {A} can be written in terms of ${}^{A}\hat{K}$ and θ. Defining the following Euler parameters:

$$\varepsilon_1 = k_1 \sin(\theta/2) \tag{1}$$

$$\varepsilon_2 = k_2 \sin(\theta/2) \tag{2}$$

$$\varepsilon_3 = k_3 \sin(\theta/2) \tag{3}$$

$$\varepsilon_4 = \cos(\theta/2) \tag{4}$$

where ${}^{A}\hat{K} = [\,k_1\,, k_2\,, k_3\,]^T$. From [**Craig,89,p.55**], it can be shown that ${}^{A}_{B}R$ can be written as

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*Research performed at Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the U.S. Department of Energy under Contract No. DE-AC05-840R21400.

$$
{}^{A}_{B}R = \begin{bmatrix} 1-2\varepsilon_2^2-2\varepsilon_3^2 & 2(\varepsilon_1\varepsilon_2-\varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_4) & 1-2\varepsilon_1^2-2\varepsilon_3^2 & 2(\varepsilon_2\varepsilon_3-\varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4) & 1-2\varepsilon_1^2-2\varepsilon_2^2 \end{bmatrix} . \quad (5)
$$

Since only three pieces of information are needed to adequately represent a rotational matrix, the Euler parameters must satisfy the following constraint **[Yuan,88]**:

$$
\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2+\varepsilon_4^2 = 1 \ . \quad (6)
$$

Let the first three Euler parameter terms be combined into a vector

$$
\hat{\varepsilon} = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T \quad , \quad (7)
$$

which is given with respect to {A} because ${}^{A}\hat{K}$ is given with respect to {A}. Equation (6) can be rewritten in vector notation as

$$
\varepsilon_4^2 + \hat{\varepsilon}^T\hat{\varepsilon} = 1 \ . \quad (8)
$$

In this paper, the Euler parameters are represented by the set $\{\varepsilon_4, \hat{\varepsilon}\}$.

A further property of Euler parameters is that if the rotational angle θ is restricted between $-180 \le \theta \le 180$, then ε_4 is nonnegative and the Euler parameter representation is unique **[Yuan,88]**. Outside this range, the representation is nonunique. By substitution into Eq. (5), both $\{\varepsilon_4, \hat{\varepsilon}\}$ and $\{-\varepsilon_4, -\hat{\varepsilon}\}$ can be shown to represent the same orientation. For teleoperation, restricting the range between ± 180 is more than adequate.

IV. EULER PARAMETER RATES

Time derivatives of the Euler parameters will be used in the design of the stiffness controller. The Euler parameter rates can be written as:

$$
\dot{\varepsilon}_4 = -\frac{1}{2}\hat{\varepsilon}^T\hat{\omega} \quad (9)
$$

$$
\dot{\hat{\varepsilon}} = \frac{1}{2}\left(\varepsilon_4 I_3 - \hat{\varepsilon}^{\times}\right)\hat{\omega} \quad , \quad (10)
$$

where $\hat{\omega}$ is the angular velocity vector with respect to {A}. A slightly different formulation is given in **[Yuan,88]**.

V. RELATIVE ORIENTATION

The relative orientation between two rotational matrices can be easily defined in terms of the Euler parameters. Let ${}^{0}_{M}R$ and ${}^{0}_{S}R$ be two arbitrary matrices relating frames {M} and {S} to frame {0} respectively. The rotational matrix ${}^{M}_{S}R$ describing the orientational differences between these two frames is ${}^{M}_{S}R = ({}^{0}_{M}R)^T\,{}^{0}_{S}R$. The Euler parameters of ${}^{M}_{S}R$, $\{\delta\varepsilon_4, \delta\hat{\varepsilon}\}$, can be written in terms of the Euler parameters of ${}^{0}_{M}R$, $\{\varepsilon_M, \hat{\varepsilon}_M\}$ and ${}^{0}_{S}R$, $\{\varepsilon_S, \hat{\varepsilon}_S\}$ **[Yuan,88]** as:

$$
\delta\hat{\varepsilon} = \varepsilon_M\hat{\varepsilon}_S - \varepsilon_S\hat{\varepsilon}_M - \hat{\varepsilon}_M^{\times}\hat{\varepsilon}_S \quad (11)
$$

and

$$
\delta\varepsilon_4 = \varepsilon_M\varepsilon_S + \hat{\varepsilon}_M^T\hat{\varepsilon}_S \quad , \quad (12)
$$

where $\delta\hat{\varepsilon}$ is with respect to {M}.

VI. STIFFNESS CONTROLLER USING EULER PARAMETERS

Master

The master manipulator will incorporate a stiffness controller **[Salisbury,80]**. The torque signal is

$$
\tau_m = J_m^T \{[K_{pm}(x_s - x_m) + K_{vm}(\dot{x}_s - \dot{x}_m)]\} + \tau_{m\,grav} \quad ,(13)
$$

where the m subscript indicates master terms and
- J_m = master Jacobian,
- K_{pm} and K_{vm} = positional and velocity gain matrices respectively,
- $\tau_{m\,grav}$ = torque signal to compensate for gravity effects,
- x_s and $\dot{x}_s$ = slave position and velocity respectively,
- x_m and $\dot{x}_m$ = master position and velocity respectively.

For the Kraft manipulator, counterbalance weights were incorporated in the design, making $\tau_{m\,grav} \cong 0$. Typically, K_{pm} and K_{vm} are diagonal matrices.

Slave

The CESARm slave manipulator has 7 DOF. The slave manipulator will incorporate a stiffness controller **[Salisbury,80 and Miyazaki,86]**. The torque signal is

$$
\tau_s = J^T[K_{ps}(x_m - x_s) + K_{vs}(\dot{x}_m - \dot{x}_s)] + \tau_{s\,grav} + \tau_{red} \ . \quad (14)
$$

where the s subscript indicates slave terms and
- J^T = transpose of the slave Jacobian,
- K_{ps} and K_{vs} = positional and velocity gain matrices respectively,
- $\tau_{s\,grav}$ = torque signal to compensate for gravity,
- x_s and $\dot{x}_s$ = slave position and velocity respectively,
- x_m and $\dot{x}_m$ = master position and velocity respectively,
- τ_{red} = redundancy torque.

Assume that feedforward compensation has been incorporated to make $\tau_{s\,grav} \cong 0$; consequently, for the rest of the discussion it will be set to zero.

The redundancy torque will be defined based on extended task-space techniques **[Oh,84 and Colbaugh,89]**. The basic idea is to simply add further constraints to the system such that the end-effector Jacobian can be extended to have full rank. For the CESARm manipulator, adding a constraint to the elbow positions it in a desired (e.g. upright) location; that is

$$
\tau_{red} = J_{red}^T\left[K_{pselb}\left(\hat{x}_{elb}^{des} - \hat{x}_{elb}\right)\right] - k_{damp}\,\dot{q}_s \quad , \quad (15)
$$

where
- $\dot{q}_s$ = slave joint velocity vector,
- $\hat{x}_{elb}$ = elbow position in Cartesian position,

$\hat{x}_{elb}^{des}$= desired elbow position in Cartesian position,
k_{damp}= positive damping constant,
K_{pselb} = positive semidefinite matrix,
J_{red} = redundancy Jacobian.

J_{red} has the property that $J_{red}\dot{q}_s = \tilde{I}\,\dot{\hat{x}}_{elb}$ where $\tilde{I} = [I_3\ 0_3]^T$, I_3 = (3×3) identity matrix, and 0_3 = (3×3) zero matrix. The redundancy torque τ_{red} is the signal used to exploit the redundancy of the extra degree-of-freedom without resorting to pseudoinverse techniques.

The main difficulty with Eq. (13) is representing the slave position x_s and the master position x_m. Both x_s and x_m are vectors and must be at least of dimension 6 by 1, because six pieces of information are required to specify the spatial location and orientation in three-dimensional space. The first three terms of these vectors should be the linear Cartesian position (i.e., the x, y, z coordinates). In Eq. (13), replace the first three terms in $x_s - x_m$ with Δx. The next three variables in $x_s - x_m$, as proposed in this paper, should be the $\delta\hat{\varepsilon}$ vector discussed in Section V. Modifying the stiffness controller of the master and slave to include Euler parameters produces:

$$\tau_m = J_m^T \left\{ K_{pm} \begin{bmatrix} \Delta\hat{x} \\ \delta\hat{\varepsilon} \end{bmatrix} + K_{vm} \begin{bmatrix} \Delta\dot{\hat{x}} \\ \delta\dot{\hat{\varepsilon}} \end{bmatrix} \right\} + \tau_{m\,grav} \quad , \quad (16)$$

$$\tau_s = -J_s^T \left\{ K_{ps} \begin{bmatrix} \Delta\hat{x} \\ \delta\hat{\varepsilon} \end{bmatrix} + K_{vs} \begin{bmatrix} \Delta\dot{\hat{x}} \\ \delta\dot{\hat{\varepsilon}} \end{bmatrix} \right\} + \tau_{s\,grav} + \tau_{red} \quad . \quad (17)$$

In the control algorithm, there are two Jacobians: the *force-reflecting* Jacobian and the *master* Jacobian. Two Jacobians are necessary because the stiffness controller, which uses the master Jacobian, is based on Euler parameter coordinates; whereas, the force-reflecting Jacobian is not. The force-reflecting Jacobian for a 6-DOF manipulator is a (6×6) matrix whose i^{th} column vector j_i is given by [Asada,86].

$$j_i = \begin{cases} \begin{bmatrix} z_{i-1} \times {}^{i-1}p_6 \\ z_{i-1} \end{bmatrix} & \text{if joint i is rotational} \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{if joint i is translational} \end{cases} \quad , \quad (18)$$

where z_{i-1} is the unit vector along the axis of motion of joint i expressed in the base coordinate frame and ${}^{i-1}p_6$ is the position of the origin of the hand coordinate frame from the origin of the $(i-1)^{th}$ coordinate frame, expressed in the base coordinate frame. The force-reflecting Jacobian J_{mfr} of the master can be written as

$$J_{mfr}\,\dot{q} = \begin{bmatrix} \dot{x} \\ \hat{\omega} \end{bmatrix} \quad , \quad (19)$$

where $\dot{q}$ (6×1 vector) is the joint actuator rates and $\hat{\omega}$ is the angular velocity vector (3×1 vector) with respect to the base frame. The master Jacobian is different from the force-reflecting Jacobian because the angular rates will be based on Euler parameters. Define the (3×3) matrix W, which relates the angular velocities $\hat{\omega}$ to $\delta\dot{\hat{\varepsilon}}$ (i.e. $\delta\dot{\hat{\varepsilon}} = W\hat{\omega}$). The master Jacobian J_m is simply

$$J_m = \begin{bmatrix} I_3 & 0 \\ 0 & W \end{bmatrix} J_{mfr} \quad . \quad (20)$$

Likewise, the slave Jacobian J_s and slave force-reflecting Jacobian J_{sfr} can be defined similarly, where

$$W = 0.5\left[\hat{\varepsilon}_s\hat{\varepsilon}_M^T + \left(\varepsilon_s I_3 - \hat{\varepsilon}_s^{\times}\right)\left(\varepsilon_M I_3 - \hat{\varepsilon}_M^{\times}\right)\right] \quad . \quad (21)$$

Equation (21) shows that W has no artificial singularities. Next, the stability properties will be examined.

VIII. LIAPUNOV STABILITY

The human operator will be modeled simply as a spring/dashpot model. To show positional stability of stiffness control, a Liapunov function candidate can be written as

$$\begin{aligned} L = {} & 0.5\,\dot{q}_m^T M_m \dot{q}_m + 0.5\,\alpha\,\dot{q}_s^T M_s \dot{q}_s \\ & + 0.5\,(x_m - x_s)^T K_{p1}(x_m - x_s) + 0.5\,(x_{des} - x_m)^T K_{p2}(x_{des} - x_m) \\ & + 0.5\left(\hat{x}_{elb}^{des} - \hat{x}_{elb}\right)^T K_{elb}\left(\hat{x}_{elb}^{des} - \hat{x}_{elb}\right) \quad , \end{aligned} \quad (22)$$

where K_{p1}, K_{p2}, and K_{elb} are >0 and α is >0. The dynamic model for the master is

$$M_m\ddot{q}_m + C_m\dot{q}_m + \tau_{mgrav} + J_{mf}^T F_{hand} = \tau_m \quad , \quad (23)$$

where F_{hand} is the force exerted by the operator's hand. The F_{hand} term will be modeled as a simple spring model with damping, that is,

$$F_{hand} = K_{phand}(x_m - x_{des}) + K_{vhand}\dot{x}_m \quad , \quad (24)$$

where K_{phand} and K_{vhand} are > 0. The slave dynamic model is

$$M_s\ddot{q}_s + C_s\dot{q}_s + \tau_{sgrav} + J_{sf}^T F_{sext} = \tau_s \quad . \quad (25)$$

The Liapunov function, L of Eq. (22), is a continuously differentiable, positive-definite function in terms of Δx and $\dot{q}_s$. According to Liapunov's second method, one needs to show for global stability that

$$\frac{dL}{dt} = \dot{L} < 0 \quad (26)$$

for all nontrivial trajectories.

Case 1: Set F_{sext} = 0. This condition implies that the slave is moving in free space. Set $K_{p2} = K_{phand}$. Using the matrix relationships [Asada,86 p. 137]:

$$\dot{M}_m - 2\,C_m = 0 \text{ and } \dot{M}_s - 2\,C_s = 0 \quad . \quad (27)$$

Let $K_{pm} = \alpha K_{ps}$ and $K_{vm} = \alpha K_{vs}$ which means that the slave and master gains are related by an arbitrary positive constant and $K_{p1} = K_{pm}$. Set $K_{elb} = \alpha K_{pselb}$; it can be shown that $\dot{L}$ is negative semidefinite. By LaSalle's Theorem **[Miyazaki,86]**, asymptotic stability can be proven.

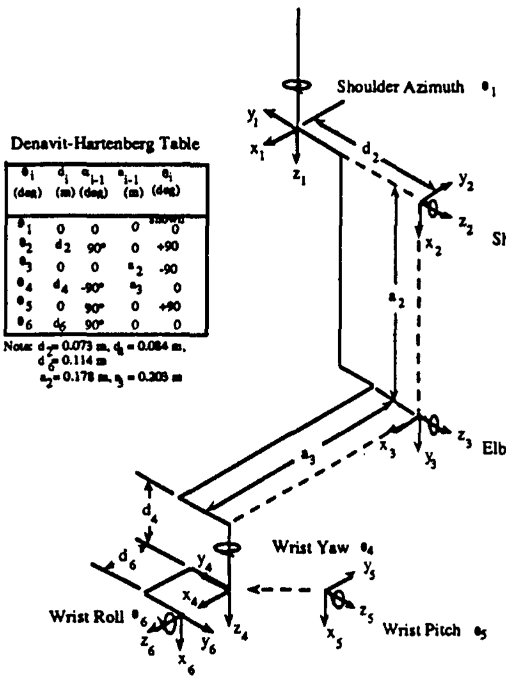
Case 2: Set $F_{sext} = K_{psext}(x_s - x_E) + K_{vsext}\dot{x}_s$, which is the case when the slave touches the environment and the force is reflected by means of a force/torque sensor back to the master. Augment the Liapunov function in Eq. (22) with two additional terms

$$L \rightarrow L + 0.5(x_s - x_E)^T K_{p3}(x_s - x_E) \quad . \tag{28}$$

Without going through the details, $\dot{L}$ can be shown to be negative semidefinite. Again by LaSalle's Theorem, asymptotic stability can be shown.

X. APPLICATION TO THE KRAFT MASTER

The controller was implemented on the Kraft master controller shown schematically in Fig. 1. The Kraft KMC 9100-MC is a lightweight 6-DOF master arm designed, manufactured, and sold by Kraft Telerobotics, Inc., of Overland Park, Kansas. Position is measured at each joint by potentiometers. The first five joints are actuated by ac servomotors for force feedback (wrist roll is not actuated). The Kraft arm comes with the KMC 9100S electronics interface unit.



θ_i (deg)	d_i (m)	α_{i-1} (deg)	a_{i-1} (m)	θ_i (deg) shown
θ_1	0	0	0	0
θ_2	d_2	90°	0	+90
θ_3	0	0	a_2	-90
θ_4	d_4	-90°	a_3	0
θ_5	0	90°	0	+90
θ_6	d_6	90°	0	0

Note: d_2 = 0.073 m, d_4 = 0.084 m, d_6 = 0.114 m
a_2 = 0.178 m, a_3 = 0.203 m

Fig. 1 - Kinematic diagram and Denavit-Hartenberg Table for the Kraft master controller.

The control algorithm was programmed in C on a Motorola 68020 with a 68881 floating point coprocessor. The control algorithm was optimized in two ways for real-time implementation. First, the Jacobian was factored so that common terms were not recalculated. Second, a custom assembly language routine was written to determine simultaneously the sine and cosine of each joint angle.

XI. GAIN SELECTION FOR THE STIFFNESS CONTROLLERS

While both the master and the slave incorporate a type of stiffness controller, the purpose is different from that of robotic operation. For teleoperation, the purpose is to reflect the environment forces and stiffness accurately to the human operator. The human operator will vary his/her impedance according to changes in the slave impedance **[Hogan,85]**. To achieve this, the positional gain matrices for both the master and slave K_{pm} and K_{ps} will be set to large values while avoiding a limit cycle phenomenon.

XII. CONCLUSION AND SUMMARY

The stiffness controller worked well on the Kraft master manipulator. With the Euler parameter formulation, no artificial singularities were present as with the Euler angle formulation. Orientation representation and accurate force-refection for dissimilar kinematic teleoperator systems can be implemented based on Euler parameters.

Further, the control algorithm could easily be modified to create artificial walls and surfaces in Cartesian space. This makes the control algorithm useful for obstacle avoidance in a cluttered environment.

ACKNOWLEDGMENTS

This work was partially funded by the Biological Acoustics Branch, Biodynamics and Bioengineering Division, Armstrong Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base.

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(10 refs, 1 fig.)

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where ${}^A\hat{K} = [k_1, k_2, k_3]^T$. From [Cr be shown that ${}^A_BR$ can be written as