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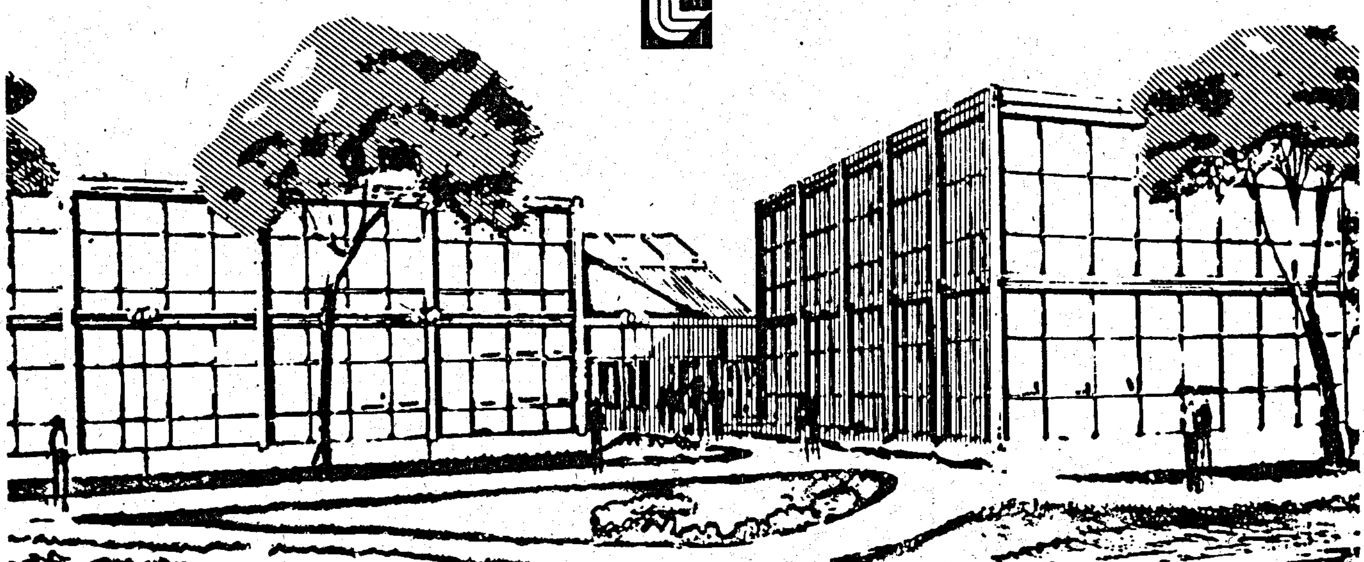

A LIGHT SCATTERING TECHNIQUE FOR DETERMINING DROPLET SIZE DISTRIBUTIONS IN TWO-PHASE LIQUID-DOMINATED NOZZLE JETS

T. W. Alger[1] W. H. Giedt[2]

ABSTRACT

A method for determining the droplet size distribution from measurements of the scattered light intensity in the forward scattering lobe has been developed for two-phase, single-component, liquid-dominated nozzle jet flows containing small droplets (diameters < 10 μm for visible light wavelengths). The technique is based upon matching the measured scattered light intensity profile with a summation of the intensity contributions of a series of appropriately spaced narrow band size distributions. A numerical optimization technique is used to determine the strengths of the individual bands which yields the best agreement with the measured scattered light intensity profile. The narrow band intensity contributions are calculated using Mie scattering theory which is accurate for small multiparticle light scattering. Application of the technique to the determination of the droplet size distribution (Sauter mean diameter near 1.8 μm) in a low quality steam-water nozzle jet is described.

NOMENCLATURE

a = upper limit distribution function (ULDF) shape parameter defined by eq. (13)
C = optimization constant defined in eq. (6)
C' = normalization constant defined by eq. (12)
D = particle diameter
$f(\alpha)$ = particle size distribution function defined by eq. (4)
$J(\theta)$ = scattered light intensity per unit solid angle
I_o = incident parallel light intensity
$i(\theta,\alpha,m)$ = Mie scattering function
K = optical constant defined by eq. (2)
m = particle refractive index
N = number of particles scattering light
n = upper limit index of sum in eq. (6)
Q = ULDF normalized width defined by eq. (15)
S = exponential size distribution shape parameter defined by eq. (17)
α = particle size parameter = $\pi D/\lambda$
$\bar{\alpha}$ = size distribution modal size parameter
α_o = minimum size parameter
α_∞ = maximum size parameter
$\alpha_{\pm 1/2}$ = size distribution half width size parameters
δ = ULDF shape parameter
θ = scattering angle
λ = incident light wavelength

Subscripts
k = index counter
m = measured value

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INTRODUCTION

Geothermal Energy is one of the current alternate energy sources to which considerable developmental attention is being given. Major emphasis is being devoted to hot water deposits rather than dry steam sources because they constitute approximately 85% of the known geothermal deposits and because dry steam sources can be utilized with conventional steam-turbine technology. One of the methods proposed for power generation from geothermal hot water deposits is the total flow process [1], [2], in which the total wellhead product (consisting of a liquid-dominated two-phase mixture of steam and water plus dissolved salts and impurities) is expanded through an energy conversion machine. One such device (which provided the impetus for this study) is an impulse turbine that utilizes a high-velocity two-phase mixture exiting from a series of converging-diverging nozzles for motive power. Because of the high vapor volume fraction, the high velocity, and the negligible pressure gradient, the two-phase mixture in the nozzle jet can be considered to consist of small spherical droplets suspended in a vapor continuum. Note that such a two-phase flow from a nozzle can be produced from energy sources other than geothermal deposits (e.g., by solar heat or from the waste heat of industrial processes).

The efficient conversion of the energy in a high-velocity two-phase fluid by an impulse turbine requires small droplets [3], [4]. The reason for this is that large droplets (> 4 μm) will not follow the vapor streamlines within the blade passages and will collide with the blade walls. This results in reduced momentum exchange and liquid film pumping losses. However, small droplets (< 4 μm), in conjunction with appropriate blade passage design, will allow more efficient turbine operation (the impulse wheel efficiency increases approximately linearly for droplet sizes < 4 μm [4]). In view of this requirement, a nozzle design to produce droplets with diameters less than 4 μm is essential. The primary objective of this work was therefore to develop a method for determining droplet size distributions in nozzle jets in order to be able to evaluate particular nozzle designs.

The measurement of droplet size distribution in single-component, two-phase, liquid-dominated nozzle jet flows is difficult because of the high velocities, the small droplet diameters, the high droplet number densities, and the complex interphase transfer mechanisms. Conventional diagnostics such as probing techniques, become unacceptable because of the serious perturbations introduced into the flow, or due to the harsh environment of the flow itself. For these reasons, a passive multiparticle light scattering technique based upon the intensity variation of the forward scattering lobe was developed for the required droplet size distribution measurements. Since analytical studies [5] , [6] have shown that droplet diameters $< 10\ \mu m$ were to be expected, attention was focused on developing a method for this size range.

LIGHT SCATTERING BY SPHERICAL PARTICLES

The light intensity scattered by a spherical particle depends upon the incident light intensity, wavelength and polarization, the particle diameter and refractive index, and the observation angle relative to the illumination direction. The basic theory for the scattering of collimated monochromatic light by a single spherical particle in a homogeneous medium was developed by Mie [7]. The result for the intensity per unit solid angle of the scattered light $J(\theta)$ as a function of the angle from the incident direction is expressed in terms of the Mie scattering function $i(\theta,\alpha,m)$ by [8]

$$J(\theta) = K\ i(\theta,\alpha,m) \qquad (1)$$

where

$$K = (\lambda/2\pi)^2 I_0 \qquad (2)$$

Calculations based upon equation (1) have shown that for a particle whose diameter is nearly equal to the incident light wavelength, the structure of the scattered light intensity distribution is very complex. For particles much smaller than the incident light wavelength the Mie theory calculations can be approximated by the simpler Rayleigh-Gans theory. For particles much larger than the incident light wavelength the simpler diffraction theory becomes applicable as an approximation to Mie theory.

The results obtained from the scattering of light by a single spherical particle can be directly extended to the scattering of light by a cloud of monodiameter particles using superposition to obtain [8]

$$J(\theta) = K\ N\ i(\theta,\alpha,m) \qquad (3)$$

where N is the number of particles scattering light. This equation is applicable for a monodispersion provided that the light incident upon all particles has not been significantly altered by the scattering effects of the other particles, i.e., single scattering occurs. The topic of monodisperse particle sizing is discussed in detail in [8] - [10].

The light scattered from a dispersion containing particles of many different sizes can be determined by the superposition of the scattering effects of each particle in a manner similar to the monodisperse light scattering represented by equation (3). By defining a particle size distribution function $f(\alpha)$ as

$$\int_0^\infty f(\alpha)d\alpha = N \qquad (4)$$

the scattered light intensity variation caused by a polydispersion is given by [9]

$$J(\theta) = K \int_0^\infty i(\theta,\alpha,m)f(\alpha)d\alpha \qquad (5)$$

The use of this result is again limited to dispersions with sufficiently low particle concentrations so that single scattering effects dominate.

PARTICLE SIZE DISTRIBUTION DETERMINATION FROM SCATTERED LIGHT PATTERNS

In principle, the scattered light intensity variation can be used to determine the particle size distribution which produced this variation. Many different investigators have studied this problem for both large and small particle polydispersions. For large particle polydispersions ($\alpha > 20$), the Mie scattering function $i(\theta,\alpha,m)$ is replaced by the much simpler diffraction theory scattering function in equation (5). The resulting equation can then be analytically inverted to yield an explicit relationship between the particle size distribution function $f(\alpha)$ and the scattered light intensity variation. Descriptions of the basic large particle polydispersion size distribution inversion technique and several variations and applications are given in [11] - [19]. The large particle techniques could not be used for the nozzle jet flow work because the droplets involved are in general smaller than the lower limit for which diffraction theory is applicable.

Several different multiparticle light scattering techniques have been developed for the determination of particle size distributions in polydispersions with diameters less than or nearly equal to the incident light wavelength (e.g. see [20] - [23]). These methods generally require an a priori assumption of the form of the size distribution function, with the experimental data providing the information necessary to determine the distribution function shape parameters. Also, the distribution functions must be monomodal and of narrow width, otherwise the characteristic patterns in the light scattering curves will be masked by the superposition of the patterns characteristic to each of the different sizes. The experimental complexity (large angular and/or many wavelength variations are required) in combination with the aforementioned restrictions limited the usefulness of these size distribution inversion techniques for the complex nozzle flow conditions.

Because of the geometrical constraints imposed by the nozzle exhaust chamber (only small angle measurements could be reasonalby obtained) and the small diameter droplets produced by the nozzles to be used, it was found that no existing method for size distribution determination could be used. Because of this, a modified optical arrangement was devised and a new procedure was developed for evaluating the particle size distribution using the measured scattered light intensity profile. This new method uses the exact Mie theory in order to be applicable to the small spherical particle range of interest.

SMALL PARTICLE SIZE DISTRIBUTION INVERSION METHOD

The numerical inversion of equation (5), a Fredholm integral equation of the first kind, for the size distribution function $f(\alpha)$ has been studied by several authors (e.g., see [24]-[27]). The inversion techniques of Phillips [24] and Hanson [25] require small $\Delta\alpha$ steps (consistent with the numerical integration quadrature formula used) and values of the scattered light intensity at relatively large values of the forward scattering angle in order to obtain an accurate result for the size distribution $f(\alpha)$. These techniques are relatively insensitive to groups of small particles with comparatively broad size distributions and require high accuracy in the scattered light intensity measurements. These requirements involve undesirably long computer run times and greater experimental complexities for the nozzle jet flow measurements. The statistical inversion method of Turchin, et. al. [26], as studied in detail by Shifrin, et. al. [27], was of limited value because of its insensitivity to the small particle range ($\alpha < 6$) for broad size distributions.

Measured Size Distribution Function Representation by Series of Narrow Individual Size Distributions

During the investigation of possible size distribution inversion techniques it was determined that a conceptually simple inversion method based upon scattered light intensity profile matching could be developed. This inversion process is accomplished by representing the measured scattered light intensity profile by a summation of contributions from the intensities produced by a series of suitably spaced narrow individual particle size distributions. That is, the measured scattered light intensity $J_m(\theta)$ is expressed by

$$J_m(\theta) = \sum_{k=1}^{n} C_k J_k(\theta) \qquad (6)$$

in which the $J_k(\theta)$ are the intensities calculated from an appropriately spaced series of narrow band particle size distributions and the C_k are constants which determines the contribution of each individual distribution. The constants C_k are determined by a minimization of the sum of squared error between the measured intensity profile and the calculated intensity profile using the numerical optimization technique of Powell [28]. Each of the individual intensity profiles $J_k(\theta)$ are calculated using equation (5) as

$$J_k(\theta) = K \int_0^\infty i(\theta,\alpha,m) f_k(\alpha) d\alpha \qquad (7)$$

The individual size distribution functions $f_k(\alpha)$ and their spacing must be specified appropriately to approximate the size distribution function to be determined. In the limit as $n \to \infty$ in equation (6) and as the individual size distribution functions of equation (7) become delta functions, equation (6) simply reduces to the integral equation (5).

The measured scattered light intensity represented by equation (6), when combined with equation (7), yields

$$J_m(\theta) = K \int_0^\infty i(\theta,\alpha,m) [\sum_{k=1}^{n} C_k f_k(\alpha)] d\alpha \qquad (8)$$

The measured light intensity can also be expressed in terms of the size distribution function of the polydispersion $f_m(\alpha)$ as

$$J_m(\theta) = K \int_0^\infty i(\theta,\alpha,m) f_m(\alpha) d\alpha \qquad (9)$$

By a direct comparison of equations (8) and (9) the measured size distribution function $f_m(\alpha)$ becomes

$$f_m(\alpha) = \sum_{k=1}^{n} C_k f_k(\alpha) \qquad (10)$$

Thus, by the optimized reconstruction of the measured scattered light intensity profile $J_m(\theta)$ from a series of individual narrow band intensity profiles $J_k(\theta)$ (calculated using the specified size distribution functions $f_k(\alpha)$) the size distribution function of the polydispersion $f_m(\alpha)$ can be determined using equation (10). Example calculations and experimental measurements using this inversion method are presented in the following sections of this report.

VERIFICATION OF PROPOSED INVERSION METHOD

Several numerical experiments were performed to investigate the applicability of the scattered light intensity profile matching procedure for the inversion of small particle size distribution functions. Particular attention was given to both the maximum forward scattering angle required and the number of narrow band size distributions necessary for a given size distribution inversion accuracy. Many calculations were performed for which a monomodal size distribution function was specified, a scattered light intensity profile was calculated with equation (5) using this distribution function and Mie theory (the numerical quadrature formula used was Simpsons 1/3 rule), and then the optimization technique was applied to determine the strengths of the series of assumed narrow band distributions. The process for determining an appropriate group of narrow band distributions is basically an iterative one. The first step is to calculate a scattered light intensity profile for an assumed broad monomodal size distribution. Comparison of this calculated intensity distribution with the true intensity profile provides a guide for a second assumed size distribution, which may be bimodal or greater. Continuation of this procedure leads to a final series of narrow band size distributions which, when their respective intensity distributions are combined using the optimization procedure, yields an acceptable agreement with the known intensity distribution.

It was found through these calculations that for the small particle range of interest the inverted size distribution function was very nearly the same as the specified distribution function, irrespective of the initial solution guess (an initial guess for the C_k's is required to start the calculation). It was also determined that constraints had to be applied to the optimization technique to preclude any large negative C_k's that could result in negative values for $f_m(\alpha)$. Similar results should be possible for large particle polydispersions but due to the increased number of specified input distribution functions $f_k(\alpha)$ (for broad distributions in α) and the increased calculational time required for the Mie scattering functions, the optimization technique becomes inefficient and an inversion method based upon diffraction theory would be more efficient.

To demonstrate the versatility of the constrained optimization size distribution inversion method a theoretical bimodal size distribution was chosen and its related scattered light intensity profile calculated. The theoretical bimodal function was obtained by the addition of two upper limit distribution functions (ULDF). The ULDF is represented by [29]

$$f(\alpha) = C' \exp(-\{\delta \ \ln[a\alpha/(\alpha_\infty - \alpha)]\}^2) \,/\, [\alpha^4(\alpha_\infty - \alpha)] \quad (11)$$

where C' is defined for normalization such that

$$\int_0^{\alpha_\infty} f(\alpha)d\alpha = 1 \quad (12)$$

The constant a is given by

$$a = (\alpha_\infty/\bar{\alpha} - 1) \exp[(1/\delta^2)(2.5\bar{\alpha}/\alpha_\infty - 2)] \quad (13)$$

and δ is related to the constant a and the function width by the transcendental equation in terms of the half width size parameters $\alpha_{\pm\frac{1}{2}}$

$$(\alpha_{\pm\frac{1}{2}}/\bar{\alpha})^4(\alpha_\infty-\alpha_{\pm\frac{1}{2}})/(\alpha_\infty-\bar{\alpha})-2\exp[-\delta^2(\{\ln[a\alpha_{\pm\frac{1}{2}}/(\alpha_\infty-\alpha_{\pm\frac{1}{2}})]\}^2 - \{\ln[a\bar{\alpha}/(\alpha_\infty-\bar{\alpha})]\}^2)] = 0 \quad (14)$$

The bimodal distribution function was formed by the addition of a ULDF with $\bar{\alpha} = 5$, Q = 1, and $\alpha_\infty = 50$ to a second ULDF with $\bar{\alpha} = 25$, Q = 0.5, and $\alpha_\infty = 50$. The normalized function half width Q is

$$Q = (\alpha_{+\frac{1}{2}} - \alpha_{-\frac{1}{2}})/\bar{\alpha} \quad (15)$$

The second ULDF was multiplied by six to amplify the comparison between the two functions.

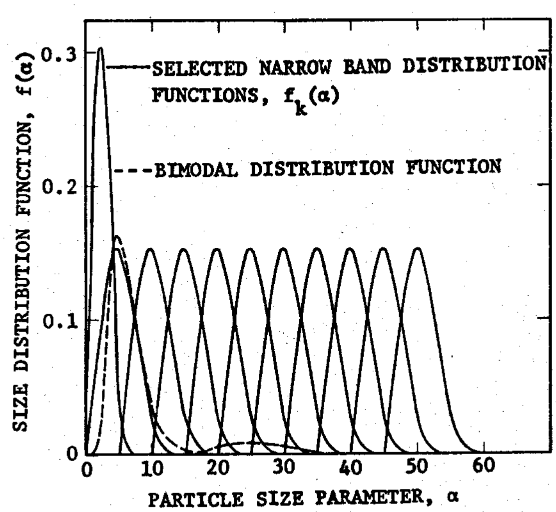

Figure 1. Selected narrow band (exponential) and bimodal (ULDF) size distribution functions.

Figure 1 presents this bimodal distribution function plotted with the specified narrow band distribution functions $f_k(\alpha)$'s assumed for the constrained optimization inversion method. These functions are exponential distribution functions given by the equation [9]

$$f(\alpha) = C'(\alpha - \alpha_o) \exp\{-[(\alpha - \alpha_o)/S]^3\} \quad (16)$$

where the normalization constant C' is calculated using equation (12) and the constant S is defined by

$$S = (\bar{\alpha} - \alpha_o)\ 3^{1/3} \quad (17)$$

The exponential functions were chosen because of the simplicity in specifying both the modal size and the amount of overlap between successive functions. In general, the numerical experiments have given the best results with a function overlap near one half, as shown in Figure 1.

Figure 2 presents a graph of the input scattered light intensity profile (calculated from the bimodal distribution and used as experimental data) and the resulting optimization reconstructed scattered light intensity profile. The intensity values plotted were divided by the optical constant K of equation (2) for the comparison. The agreement between the two curves is excellent and can be used as a direct measure of the size distrubiton inversion accuracy.

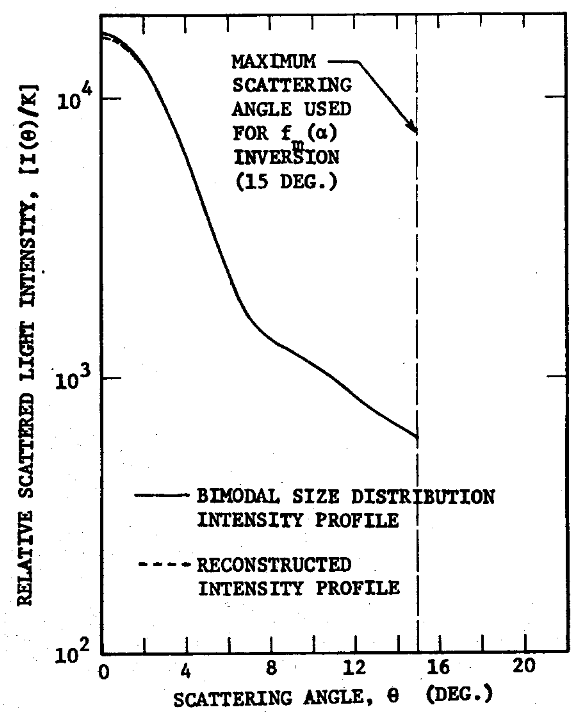

Figure 2. Comparison of the bimodal and the reconstructed light intensity profiles.

Figure 3 presents the input bimodal distribution function, the inverted size distribution function $f_m(\alpha)$ (m = 1.33) and the mass fraction calculated for the input and inverted distribution functions (the determination of which is very important for impulse turbine blade flow passage design). The mass fraction is calculated by the equation

$$\text{mass fraction} = \int_0^\alpha \alpha^3 f(\alpha)d\alpha \,/\, \int_0^\infty \alpha^3 f(\alpha)d\alpha \quad (18)$$

From this figure, the accuracy and uniqueness of the constrained optimization size distribution inversion method can be seen. The capability for bimodal size distribution inversions is very important to the understanding of nozzle flows since flow channeling and boundary layer-liquid film shearing action appears to cause complex particle size distribution functions [18],[30]. Further calculational studies will be undertaken to determine the full range of applicability of this inversion technique.

EXPERIMENTAL LIGHT SCATTERING SYSTEM

The purpose of this work was to develop an experimental procedure for the measurement of droplet size distributions in two-phase liquid-dominated jets. *Figure 4 shows a schematic of the experimental* arrangement. A photograph of the actual nozzle installation as shown in Figure 5 illustrates the geometrical complexity involved.

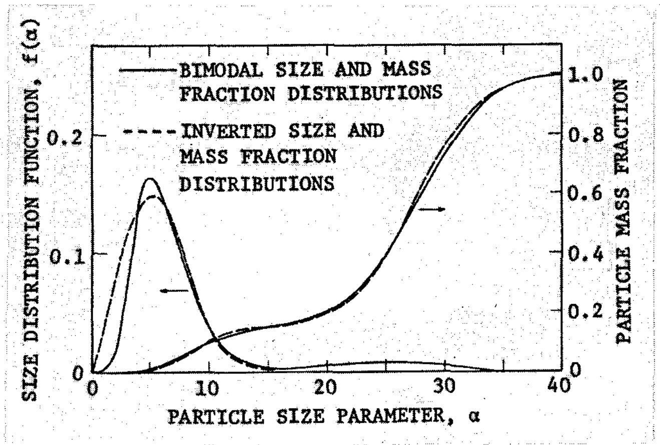

Figure 3. Comparison of the input bimodal distribution function, the inverted distribution function and their respective mass fractions.

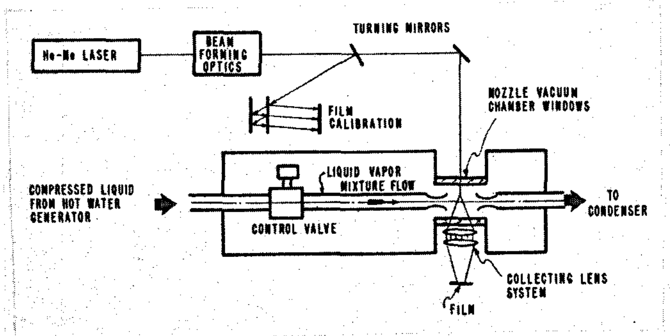

Figure 4. Schematic of the nozzle jet flow experimental arrangement.

The scattered light intensity measurement is based upon the use of a calibrated film. The light beam from a horizontally polarized He-Ne laser is shuttered and attenuated appropriately for the film sensitivity (KODAK[3] linagraph shellburst 2474 film was used) and aperatured to eliminate the scattered light from the optical components before traversing through the nozzle jet flow. The light scattered from the droplets is collected with a lens combination and directed to the film plane at the focal length. The collecting lens system was designed after the arrangement used by Chin, et. al. [11] . However, modification was necessary to eliminate spherical aberrations that can be encountered at the larger scattering angle measurments. This was accomplished by using a combination of two 7 cm diameter 60 cm focal length plano-convex lenses in series to form an effective 30 cm focal length system. This allows a scattering angle near 15 degrees to be monitored.

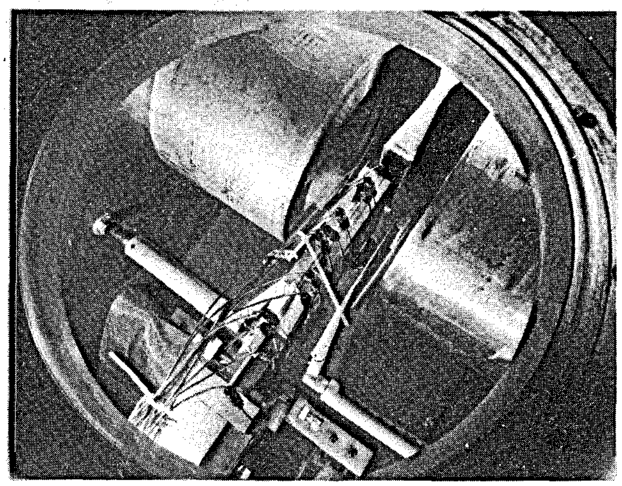

Figure 5. Photograph of the two-phase nozzle arrangement as installed in the exhaust chamber.

Each individual piece of film is calibrated to ensure high accuracy because of possible variations in the emulsions and development processes. The film is calibrated using a partially transmitting mirror combination, scanned for film density using a microdensitometer, and the film density converted to energy density using an adaptation of the analysis developed by Weaver, et. al. [31] . The light intensity measurements were corrected for extraneous scattering (optical components, windows, etc.) by subtracting the no flow scattered light intensity profile from the light scattering profile with flow.

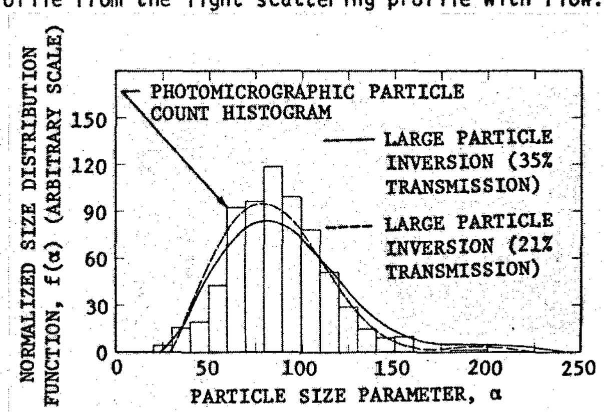

Figure 6. Comparison of the size distribution functions obtained using a large particle inversion technique [13] and a photomicrographic count for a glass beads-in-water polydispersion.

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[3] Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U. S. Department of Energy to the exclusion of others that may be suitable.

An experimental demonstration of the optical system was accomplished by substituting a glass beads-in-water polydispersion in place of the nozzle flow of Figure 4. The scattered light intensity profiles for this polydispersion were determined according to the previous description, and the size distribution function inverted according to the large particle technique of Shifrin, et. al. [13]. The particle size distributions obtained for light transmissions of 35% and 21% are compared to a photomicrographic particle count (≈ 700 particles counted) in Figure 6. The transmission of the dispersion was maintained $> 20\%$ to ensure dominant single scattering effects, as recommended in [17], [29], and [32]. The two particle size distributions and the particle count were normalized to equivalent areas under each curve for the comparison. As can be seen from the figure, the agreement is excellent.

MEASURED NOZZLE JET FLOW DROPLET SIZE DISTRIBUTION

For nozzle inlet conditions approximating geothermal wellhead conditions, rectangular nozzle cross-sectional geometry with a very short optical path length was necessary for the light scattering measurements. The thickness of the nozzle jet parallel to the incident laser beam direction (0.325 cm) was chosen short enough to satisfy the single scattering requirement, but large enough to avoid severe boundary layer effects. The nozzle throat height was 0.635 cm with a radius of curvature of 0.318 cm. To avoid external (to the jet) scattering and exhaust chamber window wetting, a diffuser was located immediately following the nozzle jet measurement area (see Figures 4 and 5).

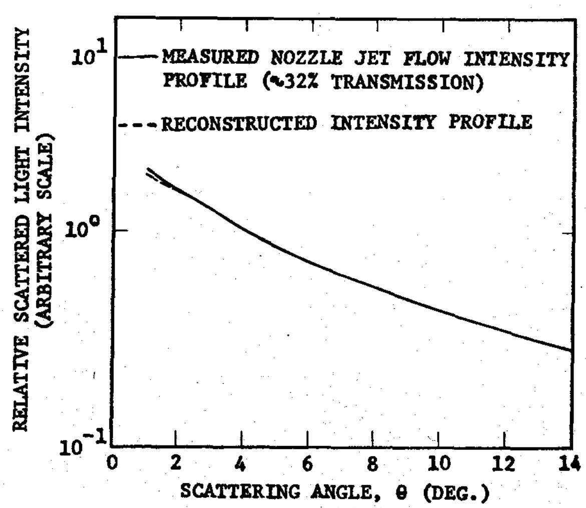

Figure 7. Comparison of the measured and reconstructed relative scattered light intensity profiles for the nozzle jet flow experiment.

The measured scattered light intensity profile is shown in Figure 7. The measurement location was the nozzle centerline approximately 1 cm from the exit. The nozzle was operating with a two-phase steam-water mixture at an inlet pressure of 690 kPa (6.80 atm), an inlet quality of 24.5% and an exit pressure of 14 kPa (0.14 atm). The ordinate of Figure 7 is plotted in arbitrary units because the measured light energy density need not be converted to absolute intensity (using the optical system constants) unless the total number of droplets traversed by the light beam need be known. In this way, the inverted size distribution function will be identical to the absolute size distribution except for the difference of a constant multiplier.

Figure 7 also presents the reconstructed scattered light intensity profile. The eleven exponential size disribution functions $f_k(\alpha)$ shown in Figure 1 were used to calculate the individual intensity profiles ($m = 1.33$) for the sum given by equation (6), and the constants C_k were determined using the optimization method of Powell for the size distribution inversion represented by equation (10). The agreement between the measured and the reconstructed intensity profiles indicates high accuracy for the size distribution inversion.

The inverted size distribution function $f_m(\alpha)$ and the resulting droplet mass fraction distribution are shown in Figure 8. The waviness of the mass fraction curve is caused by a combination of the multimodal nature of the droplet size distribution and the degree of approximation afforded by the narrow band distributions (the mass fraction is proportional to α^3, which amplifies any differences due to the degree of approximation). Interestingly, the droplet size distribution has a modal diameter less than 1 μm, but the Sauter mean diameter is near 1.8 μm and the mass median diameter is near 2.5 μm. This indicates that the nozzle jet flow is dominated by a large number of very small droplets, but they constitute only a small fraction of the total droplet mass. In view of this information, it can be concluded that there are several possible droplet size distributions occurring in the nozzle jet. The small droplets (< 2.5 μm) can be associated with the core flow region of the jet, while the larger droplets are generated by the boundary layer-liquid film shearing action occurring along the nozzle walls and particularly near the nozzle throat (see [18] and [30] for similar conclusions concerning two-phase nozzle flows).

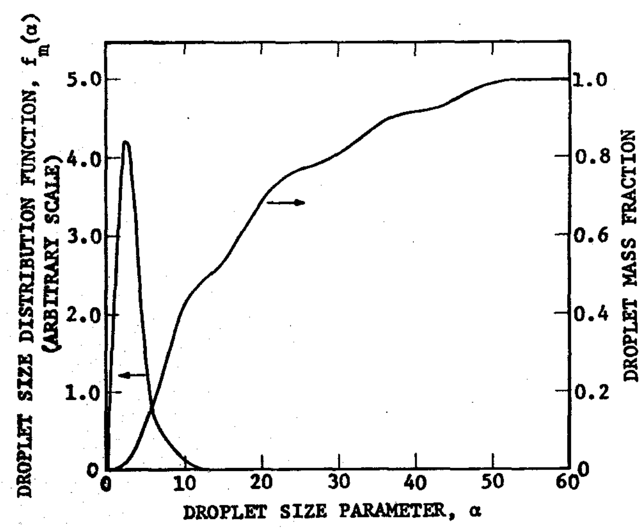

Figure 8. Measured size distribution function and mass fraction distribution for the nozzle jet flow experiment.

Measurements of the droplet size distribution in nozzles with small jet widths as used for this work will produce information that is biased towards the large droplet sizes because the nozzle wall boundary effects are much more significant than would be found in the full size impulse turbine nozzles. Thus, the measurement presented herein is conservative, in that a greater fraction of the liquid mass would be expected to be present as small droplets for the impulse turbine blade flow using similar but larger cross-section nozzles.

SUMMARY AND CONCLUSIONS

1) A new method for the determination of small particle size distributions from multiparticle light scattering measurements of the scattered light intensity profile has been developed. Numerical experiments indicate that the technique has the capability to invert bimodal size distribution functions, which is important to the understanding of two-phase liquid-dominated nozzle flows.

2) An optical arrangement has been modified to eliminate spherical aberrations for the larger scattering angles required for the small particle measurements. The use of the optical system for light scattering measurements was demonstrated using a glass beads-in-water polydispersion.

3) An experimental measurement of the droplet size distribution in a two-phase, single-component, liquid-dominated, steam-water nozzle jet was accomplished. Results indicate the size distribution Sauter mean diameter is near 1.8 μm but the mass median diameter is near 2.5 μm.

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