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TITLE: STATISTICAL THEORIES OF RAYLEIGH-TAYLOR INSTABILITY FOR COMPRESSIBLE FLUIDS

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STATISTICAL THEORIES OF RAYLEIGH-TAYLOR INSTABILITY FOR COMPRESSIBLE FLUIDS

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ABSTRACT

Statistical theories for the outer envelope of the Rayleigh-Taylor mixing layer refer to a simplified dynamics of fundamental modes and their interactions. These modes are bubbles of light fluid entrained in the mixing layer between the undisturbed light and heavy fluids. The dynamics can be understood in terms of the motion of a single mode and the interactions between modes. The single mode dynamics has to be solved self-consistently in a background field of random bubbles. The dominant interaction is bubble merger, i. e. the spreading of larger bubbles at the bubble envelope. Merger leads to dynamically increasing length scales, and thus to a dynamic renormalization of scaling dimensions. The mechanism for bubble merger is the differential motion of physically adjacent single bubble modes.

This paper is focused on the above topics: single bubble motion, bubble interactions and statistical models.

1. Introduction

Density gradients at an accelerated interface result in Rayleigh-Taylor instability. At late times, the interface evolves into a chaotic regime characterized by a mixing layer and the entrainment of one fluid in the other. Sensitivity to the initial conditions and to random heterogeneities as well as the complexity of the mixing process call for statistical descriptions. Here we focus on statistical theories for the outer envelope

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for the penetration of the light fluid into the heavy, i.e. the boundary of the mixing layer.

As with mixing theories in general, the issues discussed here fall within the area of nonequilibrium statistical mechanics. The essential issue is to determine transport behavior in the nonlaminar, chaotic regime. In this chaotic regime, macroscopic continuum events (i.e. interactions between coherent structures) rather than molecular collisions are the driving mechanisms. Pursuing this analogy, our current investigation could be viewed as an effort to characterize the two body potential of this process.

The elementary modes for the description of the outer envelope of the Rayleigh-Taylor mixing layer are bubbles of light fluid penetrating into undisturbed heavy fluid. §2 describes a recent refinement [8] of the single mode (bubble) theory developed previously [1]. An extension of this theory, based on superposition, describes the interaction of a single bubble with a random background field in the limit of small compressibility. §3 reviews current statistical theories and develops ingredients which may be needed in a new generation of statistical theories. Extending earlier work [1] using the Sharp-Wheeler model, we find a greater variability in α than was observed in the Read experiments [4]. This variability may be due to the smaller sample size in our computation as well as the ability to vary initial condition systematically.

2. Single Bubble Theory

2.1. The Periodic Array

The periodic array of bubbles, or equivalently the single bubble with periodic boundary conditions allows a detailed study of the long time behavior of a single mode. In this regime, the bubble goes through three successive time periods of exponential growth, bounded acceleration and approach to a constant (terminal) velocity. There are four parameters which effectively describe the entire motion, and in [8], the four parameter ODE for the velocity v

$$\frac{dv}{dt} = \frac{\sigma\, v\,(1 - \frac{v}{v_\infty})}{\frac{\sigma}{b}\frac{v}{v_\infty} + (1 - \frac{v}{v_\infty}) + [\frac{\sigma v_\infty}{g_R} - (1 + \sqrt{\frac{\sigma}{b}})^2]\,\frac{v}{v_\infty}\,(1 - \frac{v}{v_\infty})} \tag{2.1}$$

is proposed as a model for the single bubble dynamics. Here σ, g_R, v_∞ and b are the linear growth rate, the renormalized gravity, the terminal velocity and the decay constant to terminal velocity respectively. The reason for generalizing our previous three parameter ODE for single bubble motion was the realization that it contained an ansatz or prediction concerning the decay rate to terminal velocity which seemed to be lacking a physical basis. The solution for Eq. (2.1) is

$$t - t_0 = \frac{1}{\sigma}\ln(\frac{v_t}{v_0}) + [\frac{1}{g_R} - (\frac{1}{\sqrt{\sigma}} + \frac{1}{\sqrt{b}})^2\frac{1}{v_\infty}]\,(v_t - v_0) - \frac{1}{b}\ln(\frac{v_\infty - v_t}{v_\infty - v_0}). \tag{2.2}$$

Extensive computations with the compressible two fluid Euler equations show a good fit for a range of Atwood numbers A and compressibilities $M^2 = \lambda g/c^2$ [8] for the equation (2.2). Here we give an intuitive, or physical interpretation for these parameters. We also note that two of the four parameters have been effectively determined and the remaining two must be obtained as a function of A and M^2 through explicit numerical solution of the single bubble problem to complete this theory. This determination has been made for a limited range of these variables only.

The two parameters which are known govern the initial period of bubble growth. The exponential growth rate σ is a solution of a transcendental equation, and its dependence on A and M^2 has been partially explored [1,8]. The constant which gives the maximum acceleration is $g_R = \frac{1}{2}Ag$ on the basis of the examination of a large number of numerical solutions to the two fluid Euler equations [1]. The final two parameters specify the terminal velocity v_∞ and the rate b of approach to v_∞.

2.2. The Superposition Hypothesis

The bubble velocity for the periodic case does not agree with the experimentally observed values for chaotic flow [2]. The essential idea we propose is to consider the bubble as a short wave length mode. Then an envelope is constructed through the tips of adjacent bubbles. This envelope defines a long wave length mode in the interface motion. We consider a bubble which is further advanced than its neighbors. Then its location can be regarded as a bubble on the long wave length envelope. In other words the long and short wavelengths are in phase in this case. Similarly, a bubble which is less advanced than its neighbors is a spike on the long wave length envelope, or in other words, the long and short wave lengths are phase reversed.

The superposition hypothesis states that the bubble velocity is the sum of the single bubble velocity plus the single bubble (or spike) velocity of the envelope. These two velocities are determined from the single bubble model of §2.1, using only the bubble radius and amplitude as geometrical parameters and so finally the bubble in a chaotic flow also has a velocity depending only on long and short wavelength radii and amplitudes (and dimensionless physical parameters A and M^2).

Experiment	34 $A = 1$			36 $A = 0.5$	104 $A = 0.946$		
t (ms)	23.6	29.2	34.8	60.8	46.6	57.6	68.6
V_{exp}	117	145	170	161	76.1	95.5	110
V_{th}	106	142	168	152	84.1	106	127
$\Delta V/V$	9%	2%	1.2%	5.9%	9.5%	9.8%	13.5%

Table 1. Verification of the superposition hypothesis from experiments of Read [4]. The hypothesis is satisfactory for those experiments in which envelopes can be clearly identified. In all these cases, $A \geq 0.5$.

The superposition hypothesis has been confirmed for the incompressible case by analysis of Read's experiments. Only cases with clearly formed bubbles and envelopes were analyzed. In the remaining cases (presumably with small surface tension) the interface was too irregular to define a long wave length envelope. The results are shown in Table 1.

Numerical solution of the two fluid Euler equation by the front tracking method shows agreement with the superposition hypothesis for $M^2 \approx 0$ in cases with clearly formed bubbles (no bubble splitting secondary instability). We find disagreement as M^2 is increased, see Table 2. We observe three cases of disagreement, all outside the range of experiments. Our proposed explanation in this case concerns the density stratification of the fluids in the gravitational field. In hydrostatic isothermal equilibrium, the density of the heavy fluid decreases exponentially with height. The density profile is more strongly stratified as M^2 increases. As a result, when the light fluid penetrates the stratified heavy fluid, the effective density ratio will be less than it was initially, thereby decreasing the velocities. We also observe disagreement with superposition in cases where bubble splitting occured presumably due omission of high frequency bubble splitting modes in the envelope description. Finally we observe disagreement with superposition for small Atwood number, for reasons not presently understood.

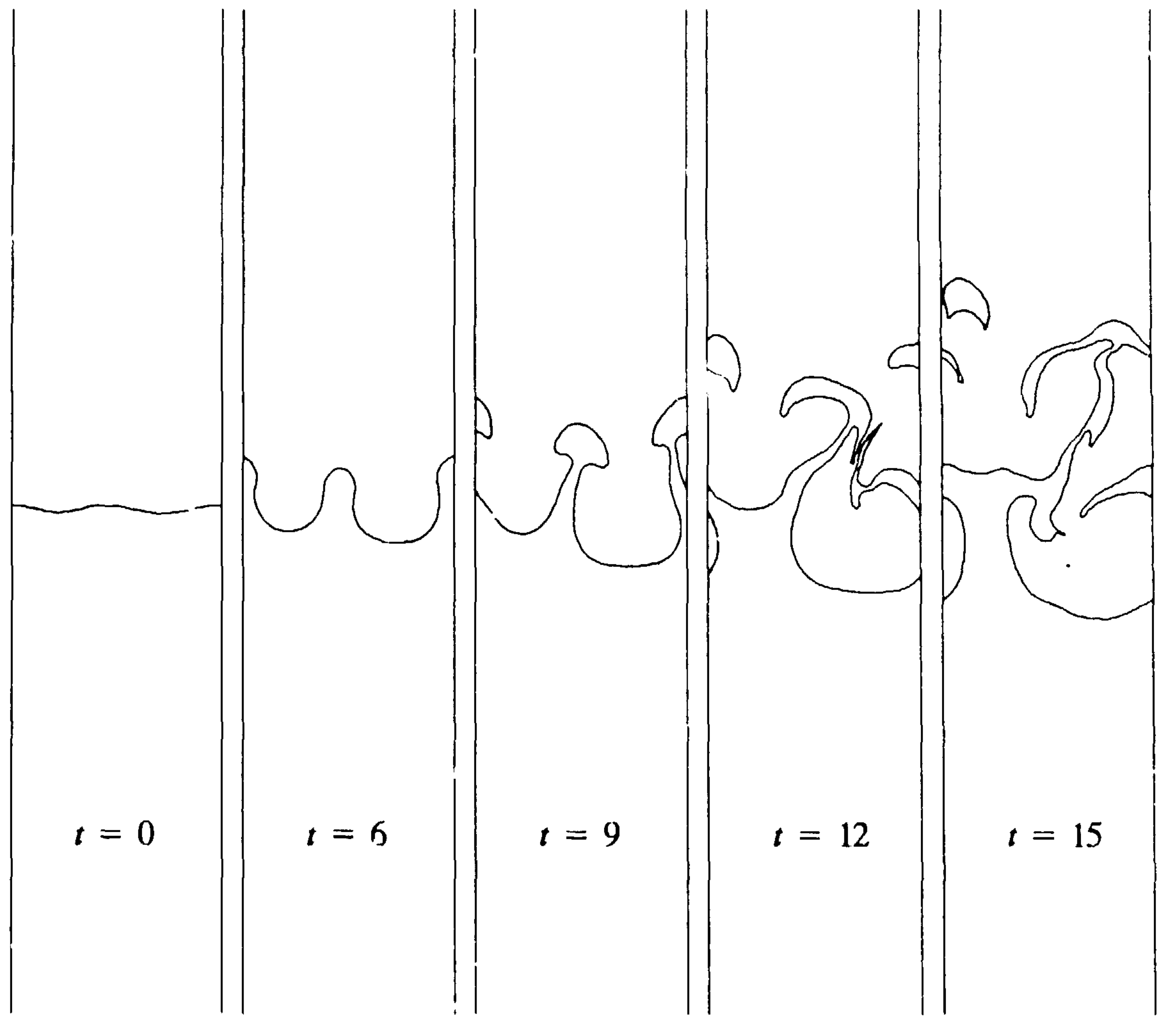

Figure 1. Successive times in a two bubble merger process. The compressibility and density ratio for this case are $M^2 = 0.1$ and $D = 5$ respectively. It can be seen that the large bubble overtakes the smaller one at $t = 12$. The velocity of the large bubble is accelerated during the merger while the velocity of the small bubble is reversed, see Figure 2.

Case	D	M^2	Error
Experiment	3 – 600	0.001-0.005	1.2-13.5%
Simulation	5 – 10	.1	1-19%
Simulation	2 – 10	0.5	72-105%

Table 2. The deviation of experimental and numerical results from the superposition hypothesis.

2.3. Mode Mode Interaction

The bubble merger process appears to have two stages. As illustrated in §2.2 with the superposition hypothesis, smaller bubbles develop a negative envelope velocity (contributions out of phase with their single bubble velocity), and at sufficient envelope amplitude, their total velocity becomes negative. At this point, they move rapidly away from the bubble envelope, and the position they previously occupied can be regarded as an oversized spike between the remaining larger bubbles. The second stage of the merger process involves an equilibration of radii, whereby the remaining (large) bubbles increase in size, while the spike region between them reduces to its equilibrium value. Fig. 1 shows the interface evolution of the two bubble interface during merger and Fig. 2 is the plot of the velocities of the two bubbles.

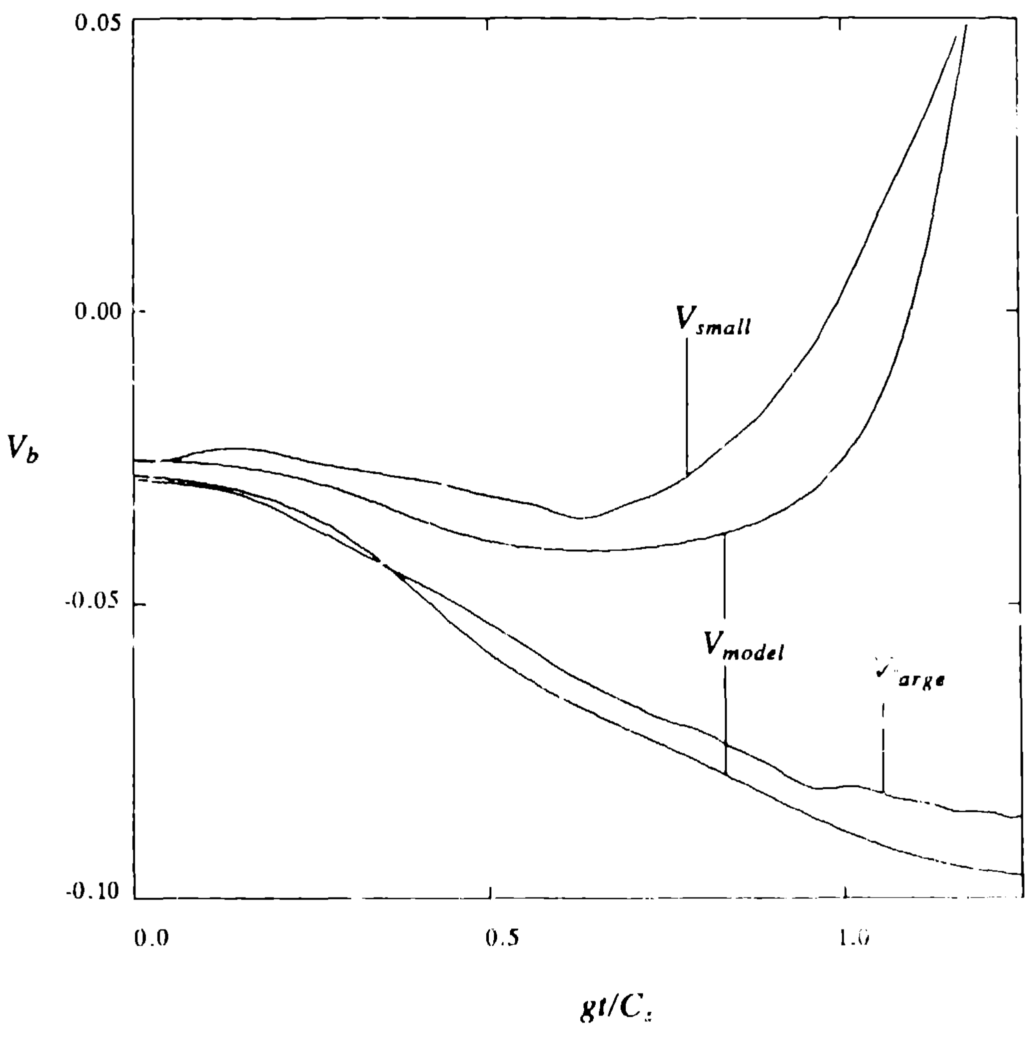

Figure 2. The plots of bubble velocities vs. time for the two bubble merger simulation. The result shows that the small bubble is accelerated at the beginning and is then decelerated after about t .5. The small bubble is washed out downstream after its velocity is reversed. The large bubble is under constant acceleration. The smooth curves represent the bubble motion as predicted by the superposition hypothesis.

3. Statistical Theories

Let $h(t)$ be the distance from the initial bubble interface to the outer bubble envelope. Then

$$h(t) = \alpha A g t^2 \tag{3.1}$$

and in two dimensions, $\alpha = .06$ (experiment) [4].

Computations of the acceleration constant α have been given by several authors, based on the full two fluid Euler equations with a random interface. Youngs [7] used an incompressible MAC code with van Leer advection. Special interface enhancements (e.g. the method of LeBlanc) which minimize diffusive mixing were not used, and the computation presents considerable diffusive mixing of the two fluids. His computations used small amounts of viscosity. He considered initial configurations of 12 bubbles with 200 horizontal mesh blocks, or about 16 blocks per bubble. He used a variety of initial conditions and Atwood numbers, and obtained values for α in the range .04 to .05.

Zufiria [9] used a vortex-in-cell code for the incompressible case. He considered only $A = 1$, with small surface tension. His initial conditions were various 4 bubble configurations, and he used a range of mesh sizes, the coarest of which was 16 grid cells per bubble. His result was $\alpha = .05$ to .06.

We report here on recent compressible front tracking computations. A wide range of physical parameters have been varied in our simulation. Those include the Atwood number A, the compressibility M and a variation in the number and size of bubbles on the initial front. For these simulations, we have traced the height of the largest bubble during the run. We have two methods for analyzing the acceleration coefficient α, namely from plots of h vs. t^2 and from plots of v vs. t. The first type of analysis is similar to Read's analysis and is close to the experimental data. This first method gives integrated time averaged acceleration, α_h, relative to the instanteous acceleration, α_v, in the second method and is consequently more regular. We find that α_h in most cases is nearly time independent, and varies in the range 0.05 to 0.065 in agreement with Read [4]. However, some initial conditions give rise to significantly smaller values of α; namely extreme values $\alpha_h = 0.038$ were recorded. α_v shows even larger fluctuations, both between different runs and also at different times within a single run. In Figs. 3 and 4 we examine a case for which $\alpha_h = .066$. The bubble motion can be observed to have three stages, as recorded in Fig. 3b. The sharp increase in α_v in the time period $7.5 \le t \le 10$ is associated with the collision of two spikes which lie above the bubble interface and are falling into the larger bubble. Upon collision, they create a jet, which accelerates the bubble. The sign reversal for $10 \le t \le 14$ appears to be due to the formation of a secondary bubble splitting instability. This detail of structure is missing in the plots of Fig. 3a, which are once integrated from Fig. 3b. The more regular quantities plotted in Fig. 3a are the same as measured and plotted by Read, which provides a partial explanation of the regularity of his results in comparison to ours. A further explanation is that Read has about 10 times the number of initial bubbles in his experiments; if 10 of our runs were combined into an ensemble of 50 bubbles, we would obtain the same leading bubble behavior for α_h as Read.

None of the above computations or experiments have examined many generations of bubble merger. Computations have been limited to one or two generations of bubble merger. The experiments contain one observable generation of merger. According to the theory of the most unstable wavelength, the experiments have an additional one to two generations of bubble merger which is not directly observable from the experimental pictures. The fact that initial conditions can play an important role after one or two generations is not surprising.

The number of generations of bubble merger in the laser fusion application can be bounded as follows. The theory of the most dangerous wave length [3,6] gives an estimate of the final bubble size in terms of the aspect ratio of the spherical shell of the container; the initial bubble size could be fixed by (a) photon wave length, (b) surface tension, (c) surface finish, and (d) asymmetries of the driving source. An order of magnitude estimate of the number of bubble merger generations from the initial bubbles set by these sources would be (a) 5 generations, (b) unknown but presumed very large, (c) at most 5 generations, (d) 0 to 1 generation. From this analysis we conclude that, aside from the driving source asymmetries, there is a potential for more generations of bubble merger in the laser fusion application than in present computations or experiments. We turn next to statistical models, and the possibility of universal behavior independent of initial conditions.

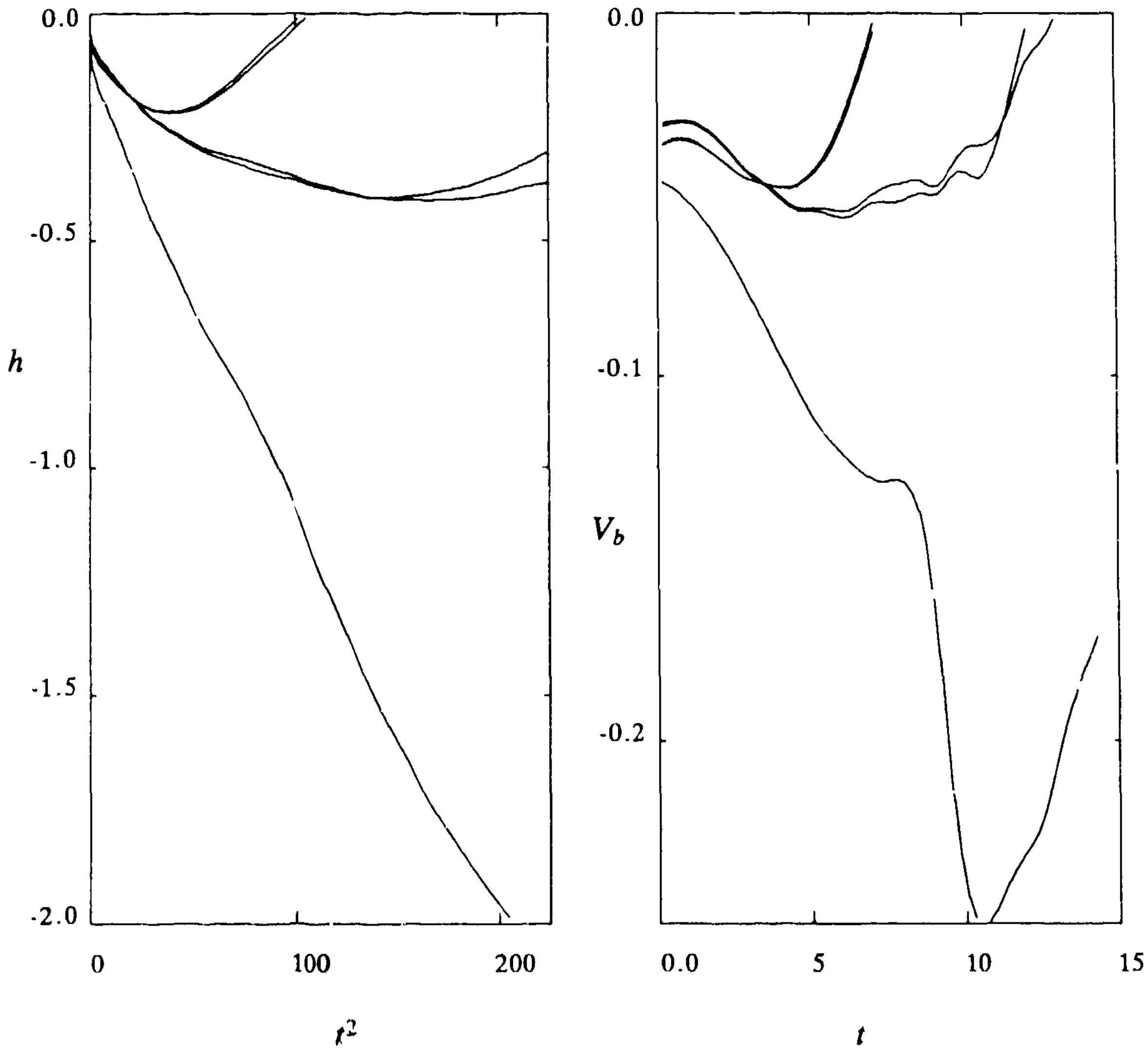

Figure 3. The left plot displays bubble heights vs. t^2 in a simulation with 5 initial bubbles. The Atwood number in this case is $A = 0.818$, and the compressibility $M^2 = 0.1$. The right picture shows the velocity vs. t in the same case.

Two statistical models for the bubble envelope have been proposed [5, 10]. These models are coupled systems of differential or difference equations for the bubble growth and merger. The essential differences between these models are: The Zufiria model has no free parameters and is limited to the case $A = 1$, $M^2 = 0$. It allows continuous relaxation of bubble width, as an aspect of bubble merger. The Sharp-Wheeler model has two emipirical parameters and appears to be applicable to a range of values of A and M^2. Merger in this model is discrete in all its aspects. They both result in a constant acceleration, with an acceleration constant α in reasonable agreement with experiment.

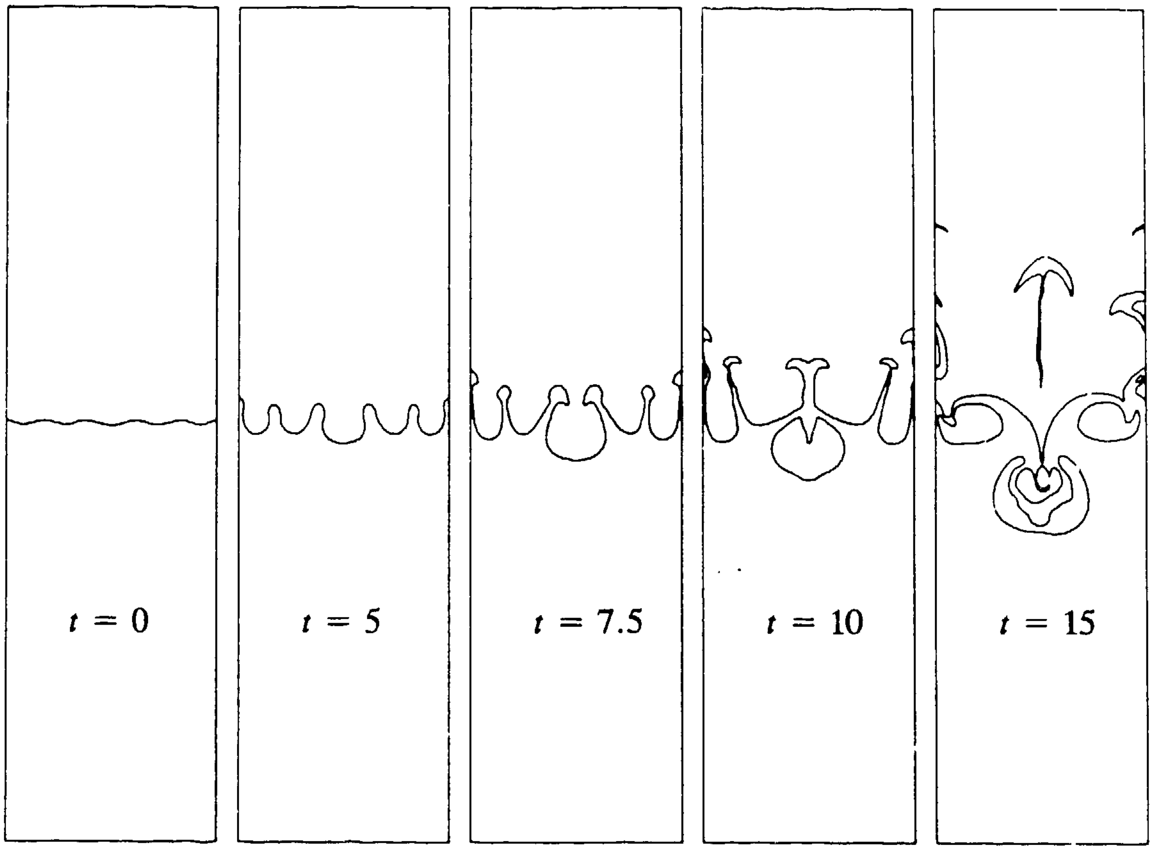

Figure 4. The interface positions at successive times in a computation with five initial bubbles. The physical parameters of this run are the same as in Figure 3.

Two phenomena have been observed in our random interface computations which are not contained in the above statistical models. One is the role of stratified initial conditions, which implies that for times large in proportion to the compressibility, the light fluid bubbles rise into a rarefied portion of the heavy fluid, to an extent that the effective Atwood number is diminished or even becomes zero. This observation raises the question of initial conditions which are not density stratified. It appears to be related to the breakdown of superposition for small Atwood numbers and moderate or large compressibilities. Also note that the increase in wave number due to bubble merger leads to an increase in the effective compressibility. A second phenomena is a change of flow regime to a bubbly, frothy or slug flow regime in the mixing layer, in which the light fluid spatially disconnected. This multiphase regime also reduces the effective Atwood number at the interface. The occurence of a slug flow regime is dependent on initial conditions, in particular on the relative size of adjacent bubbles. It could also depend on the distinction between exactly two-dimensional computations as opposed to approximately two-dimensional experiments.

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