

Summary of Working Group
on
Collective Instabilities*

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Abstract

In this paper we summarize the efforts of the Working Group on Collective Instabilities at the Workshop on the RHIC Performance. Impedance estimates have been made for some of the main hardware in RHIC, including bellows, pickup electrodes, abort kicker, and transverse damper. In general, these impedances are not expected to limit the beam intensity for Au ions, but might limit the proton intensity. We have also calculated the higher-order modes of the standard 26.7-MHz RF cavity for use in estimating coupled-bunch instability growth rates. Predictions of intrabeam scattering confirm the results in the RHIC Conceptual Design Report. For the standard assumptions, there is a threefold growth in transverse emittance. Varying the initial transverse emittance by a factor of two changes the final emittance value (after 10 hours) by less than 20%. If a 214-MHz RF system is considered, the growth is more severe—about a factor of five—and a beam lifetime of 10 hours requires an RF voltage in excess of 32 MV. Coupled-bunch calculations show that the transverse instabilities are dominated by the resistive-wall impedance for either RF choice. A modest damping system should be adequate to deal with this. Longitudinal growth times of about 20 ms are expected for the low-frequency RF case; growth times for the high-frequency RF system are a factor of 10 longer and the instability is predicted to be Landau damped. Copper plating of the dipole vacuum chambers has been found to have no deleterious effects, provided the coating is uniform and not overly thick.

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MASTER

INTRODUCTION

The purpose of this Workshop was to investigate the influence of various collective effects on the predicted performance of the Relativistic Heavy Ion Collider (RHIC) being designed at Brookhaven National Laboratory. In this paper we summarize the results of our efforts in three main areas:

- impedances and thresholds
- intrabeam scattering (IBS)
- coupled-bunch instabilities.

In the first area, our task was to investigate the expected longitudinal and transverse impedances that might be present in RHIC and to estimate the corresponding thresholds. An attempt was made to estimate the impedance contributions arising from bellows, beam position monitors, the abort kicker, and a transverse damper system.^{1,2} In addition, we have considered³ the effects of copper plating the inside of the dipole chambers, to see if any problems were likely to arise.

Because the information was not already available, it was necessary to calculate the higher-order modes of the RHIC 26.7 MHz RF cavity during the Workshop.⁴ These results were subsequently used to explore the effects of coupled-bunch instabilities. As the Workshop progressed, it was concluded that there might be significant benefits to the design of the machine if a higher frequency RF system (214 MHz rather than the presently conceived 26.7 MHz) were utilized. Some implications of such a change were also explored by our Working Group.

In the second area, intrabeam scattering, a great deal of work has already been performed by G. Parzen.⁵ Our task here was mainly to confirm, via independent calculations,⁶ the present estimates of this effect. In addition, we looked briefly at the sensitivity of the IBS estimates to the starting assumptions and evaluated the changes in growth rates associated with the higher frequency RF system alternative mentioned above.

Because RHIC will be operated with many bunches in the ring, the issue of coupled-bunch instabilities is an important one. This aspect of machine performance had been neglected up to now. Calculations were performed⁷ with ZAP⁸ to estimate the growth rates that might be expected, based upon the calculated RF modes. In addition, simulations have been made of coupled-bunch instability growth in the RHIC booster.⁹

Our results in each of these areas will be summarized in the sections that follow.

IMPEDANCES AND THRESHOLDS

Impedances for some of the major ring components were estimated by Lambertson and Ng.^{1,2} Their results, summarized below, show that beam position monitors (BPM's), bellows, and kickers contribute to the impedance about equally. These impedance contributions can be held to a level that will not limit the heavy ion performance of RHIC, but may well limit the use of the machine as a proton storage ring. The estimates made in Refs. 1 and 2 are based upon relatively crude "designs" that are intended only for purposes of illustration. Thus, the "moral" here is that *the accumulation of impedances from these components must be carefully controlled to avoid excessive contributions to the overall transverse impedance.*

Beam Position Monitors

The BPM's for RHIC are striplines having a split cylinder design. Each electrode, of length ℓ and subtending an angle ϕ_0 transverse to the beam axis, is terminated at each end with a characteristic impedance of $Z_s = 50 \Omega$. The longitudinal and transverse impedances are given by:¹

$$\left(\frac{Z}{n}\right)_{\text{BPM}} = -i Z_s \frac{2\ell}{R} \left(\frac{\phi_0}{2\pi}\right)^2$$

and

$$Z_{\perp} = \frac{R}{\beta b} \left(\frac{4}{\phi_0} \right)^2 \sin^2 \left(\frac{\phi_0}{2} \right) \left(\frac{Z}{n} \right)_{\text{BPM}}.$$

In each ring there are 250 BPM units of length 0.2 m and subtending an angle of 90°, so the total longitudinal and transverse impedances are, respectively, $Z/n = -0.5i \, \Omega$ and $Z_{\perp} = -0.4i \, \text{M}\Omega/\text{m}$.

Bellows

Longitudinal and transverse impedances for the bellows can be obtained by running a code such as TBCI¹⁰ and taking Fourier transforms. In general,¹ the results can be characterized in terms of broadband resonators having a shunt impedance R_s , a resonant (angular) frequency ω_r , and a quality factor Q (typically $Q \approx 3-5$).

Three possible bellows designs (having different corrugation depths and lengths) are presently being contemplated for RHIC. Their impedance contributions, summarized in Table 1 (taken from Ref. 1), are not excessive, but the 500 units nonetheless constitute a significant portion of the overall impedance. It will be important to keep this impedance contribution under control, either by keeping the corrugation depth to a minimum or by utilizing a shielded design.

Because of the relatively long bunches in RHIC, parasitic heating of the bellows is expected to be unimportant. Lambertson and Ng¹ have estimated typical losses to be a fraction of a watt for protons, with Au ions giving even less heating. It is worth noting, however, that a high-frequency RF system would increase the parasitic heating to tens of watts.

Transverse Damper

As will be discussed below, transverse oscillations at injection energy (driven by the resistive-wall impedance) could give rise to a 23-ms growth time for the Au beam. (We note, however, that this estimate is based upon the pessimistic assumption of a stainless steel wall everywhere; the planned use of cryogenic copper in the dipole chambers will be of significant benefit—about a factor of two—in this regard.) To counteract this growth, it was considered beneficial to investigate the parameters for a transverse damper.

The model design considered in Ref. 2 would use capacitive plates of length 0.5 m, tuned to a central frequency of 50 MHz. A bandwidth of 4.5 MHz, i.e., twice the minimum requirement of $\Delta f = 57f_0/2$, would be provided. If we take an initial amplitude of 1 mm at a location having $\beta = 100$ m, then the required power to damp a gold beam at injection energy is estimated² to be:

$$P_{\text{[watts]}} \approx \frac{1.6 \times 10^{-4}}{\tau}$$

where τ is the growth time of the instability. Thus, for a 23 ms growth time, $P = 0.3$ W. Clearly this is not a problem.

The transverse impedance of such a device is expected to be $Z_{\perp} = 45 \text{ k}\Omega/\text{m}$.² This value is not a concern.

Abort Kicker

Based upon a conceptual model of an abort kicker with a shielded liner, Lambertson and Ng¹ have calculated the expected longitudinal and transverse impedance values. For the longitudinal impedance, a low-frequency value of $|Z_{\parallel}| = 2.2 \text{ }\Omega$ was obtained. The transverse impedance was about $0.5 \text{ M}\Omega/\text{m}$ at a frequency of 0.5 MHz. Values this low are acceptable, but we note that the results are sensitive to the assumptions about the tolerable liner conductance and yoke geometry.

Single-bunch Thresholds

To put the impedance values in context, we must estimate the thresholds to which they would correspond. For RHIC, the threshold that appears to be the most severe limitation is that arising from transverse mode coupling. From Ref. 8 we have

$$I_b = \frac{4 \left(\frac{E_v}{q_e} \right) v_s}{\text{Im}(Z_{\perp}) \langle \beta_{\perp} \rangle R} \frac{4\sqrt{\pi}}{3} \sigma_z$$

where E_t is the total energy of the beam particles (of charge state q), v_s is the synchrotron tune, $\langle\beta_\perp\rangle$ ($= 55$ m) is the average beta function, R ($= 610$ m) is the machine radius, and σ_z is the rms bunch length.

ZAP calculations (summarized in Table 2) indicate that, for Au ions at injection energy, the transverse impedance corresponding to the required intensity of 1.1×10^9 particles per bunch is about 10 M Ω /m for the low frequency (26.7 MHz) scenario or about 45 M Ω /m for the high frequency (213.9 MHz) case. If these values are converted to longitudinal impedances via

$$\left| \frac{Z}{n} \right| = \frac{\beta b^2}{2R} Z_\perp,$$

the equivalent values are $|Z/n| = 10 \Omega$ or 45Ω , respectively.

Similar estimates for protons at injection energy, however, do not lead to such comfortable impedance values. A transverse impedance below 1.2 M Ω /m is required to fill the bunches to 1×10^{11} particles. Given the estimates—albeit crude—above, *it does not seem safe at present to assume that the desired proton intensity can be reached.*

Because the beta-weighted transverse impedance depends on machine details that are presently uncertain, it is premature to draw definite conclusions about the achievable proton intensities. However, the estimates here should serve as a warning.

Copper Plating of Dipole Chambers

One topic we were asked to investigate as part of the workshop was that of copper plating of the dipole vacuum chambers. This can have ramifications in several areas:³

- resistive-wall instability growth rate
- parasitic heating

- mechanical stresses during a quench
- effects arising from gaps in the plating.

For the resistive-wall instability, the growth times for Au ions at injection energy are expected^{3,7} to be about 20 ms if the entire machine circumference were constructed from stainless steel. As mentioned above, a damper to cope with this growth requires only modest power, and is not expected to be difficult to construct. Because the cold copper coating serves to reduce the instability growth rates even further, it can only be of benefit in this context.

Parasitic heating must also benefit directly from the increase in conductivity associated with copper plating of the vacuum chamber. If the entire RHIC vacuum chamber were copper plated, the reduction compared with a stainless steel wall would be about a factor of 30. In any case, Lambertson and Ng¹ have shown that the parasitic heating is not a major concern for RHIC due to the relatively long bunches.

Of more concern are mechanical effects associated with eddy currents induced in the copper layer during a magnet quench. These currents generate an outward Lorentz pressure that has its maximum value at the midplane of the pipe. In combination with the increase in external pressure from the helium vaporized during a quench, the beam pipe could collapse. For the present RHIC pipe dimensions, Ng has estimated³ that the maximum allowable helium pressure is about 30 atmospheres, which presents no problem. This effect *could* pose a severe problem, however, if the wall thickness were decreased significantly from its design value of 1.65 mm.

Another mechanical effect of potential concern arises from the possibility of a nonuniform copper coating. If the copper thickness varies with the polar angle, there is a torque induced that can lead to stress failure of the keys holding the beam pipe in place. Based on the estimates in Ref. 3, the nonuniformity must exceed about 0.5 mil (i.e., a 50% variation in the nominal 1 mil coating) before failure will occur. This tolerance should be easily achievable.

It is presently envisioned that only the dipole chambers—not the entire RHIC vacuum

chamber—will be copper coated. (The quadrupole beam pipes will remain as unplated stainless steel to ensure that the capability for rapid field penetration—required, for example, to perform transition jumps—is maintained.) Given that only about half of the circumference is occupied by dipoles, the copper plating will reduce the parasitic heating by about a factor of two compared with an entirely stainless steel chamber.³ As mentioned, for the presently planned long bunches, the parasitic heating is not expected to be a concern.

In the context of the resistive-wall instability, it is expected³ that most of the wall current will cross the copper-to-stainless-steel gaps. Although some additional impedance results from the gaps, the resistive-wall growth rate still decreases by about a factor of two compared with the case of a bare stainless steel vacuum chamber covering the entire ring. As discussed earlier, even the growth rate associated with the stainless steel chamber is easily manageable, so the gap is of no concern in this regard either.

Curvature Effects

In a curved vacuum chamber, it is possible for the electromagnetic wave generated by the beam to propagate and act back on the beam. This results in a resonant situation in which the beam feels an impedance. Because the resonant frequencies are generally expected to be rather high, the mode-coupling instability is not affected, but microwave growth is possible.

Our estimate for this effect¹¹ is that the impedance seen by the beam is $|Z/n| \approx 2 \Omega$. This value is low enough to make it of no concern in driving the microwave instability. We find, however, that in the long-wavelength limit (i.e., below the lowest resonance), an asymptotic expansion arising from the toroidal geometry yields terms of order $(b/R)^2$ and $(nb/R)^2$. The n -dependent terms typically give a significant contribution to the impedance—tens of ohms at a few gigahertz. Although such a value is not inconsistent with impedance measurements for some proton machines, it does appear to be inconsistent with the few-ohm impedances measured at various electron rings.

We note here that the asymptotic expansion of the relevant Bessel functions is in an awkward regime when the order n is not zero, and it can be expected that additional terms will be required. Whether the predicted impedance is a symptom of the divergence of an asymptotic expansion, or is indeed physical, remains an open question for further investigation.

Caveats

In the present impedance study, we have not considered several items that can contribute significantly to the ring impedance. In particular, we have not looked at possible designs for the injection kicker nor have we considered the "transitions" between differing vacuum chamber cross sections. An obvious suggestion to the RHIC design team is to properly taper the transition sections where possible.

INTRABEAM SCATTERING

For heavy ion beams (we use 100 GeV/amu Au ions as an example), the effects of intrabeam scattering (IBS) are quite significant. The beam lifetime of 10 hours is defined, for example, as the time required for the longitudinal beam emittance to blow up sufficiently to fill the entire RF bucket area. As mentioned earlier, Parzen⁵ has already studied this problem in detail. To be safe, however, the code ZAP⁸ was modified⁷ to permit equivalent calculations to be performed.

In general, the calculations in Ref. 7 are in good agreement with those of Parzen. For the nominal RF parameters, the transverse normalized emittance grows by about a factor of three. If the high-frequency RF system is considered, the bunches are about a factor of three shorter. In this circumstance, the growth is more severe—about a factor of five in transverse emittance. (In performing these calculations, the number of particles per bunch was kept fixed, rather than readjusting this parameter to reoptimize the luminosity.) In this case, a voltage of more than 32

MV would be required to ensure that the beam momentum spread does not exceed the RF bucket height after 10 hours of growth.

COUPLED-BUNCH INSTABILITIES

Because the operating scenario for RHIC utilizes 57 equally spaced bunches, the topic of coupled-bunch instabilities must be explored. For the standard 26.7 MHz cavity, the required information on higher-order cavity modes was obtained by Schoessow⁴, who utilized the URMEL code¹² to model the RHIC cavity. This information was then used in ZAP⁸ to calculate the predicted growth rates and estimate whether or not the instabilities were Landau damped. For the high-frequency case, also explored here, cavity modes were taken from published values¹³ for the similar SPS 200-MHz cavities.

The present calculations have been performed for a gold beam at injection energy and full energy; longitudinal bunch parameters were calculated as described in Ref. 14. Both longitudinal and transverse instabilities were considered. In addition to the higher-order modes of the RF cavities, impedance contributions from the resistive wall, space charge, and a broadband ($Q = 1$) resonator were considered. To be pessimistic, we have taken the case where all cavities are identical, so that the modes add up exactly. For the low-frequency case, we use six cavities to produce the requisite 1.2 MV; for the high-frequency case we use 15 cavities to produce 15 MV.

Transverse Instabilities

At injection energy, the dominant growth comes from the resistive-wall impedance. Even without a broadband impedance contribution, this growth is not predicted⁷ to be Landau damped. It is assumed that a transverse damper will be utilized; we have already shown that the requirements for such a device are easily met. The fastest growth times for the various cases are summarized in Table 3.

For the 100 GeV/amu Au beam, we again find that the transverse instability is dominated by the resistive-wall term. At the higher energy, of course, the growth rates are lower than those at injection energy by nearly a factor of ten. With a betatron tune spread of 0.0005, the rigid dipole ($a=0$) mode instability would be Landau damped. In any case, the transverse damper is available to control the growth.

In the high-frequency case, the calculations give growth rates comparable to those from the 26.7-MHz cavities. However, the frequency shifts are about three times larger. A betatron tune spread of 0.0013 would provide sufficient Landau damping of the $a=0$ mode.

Longitudinal Instabilities

ZAP calculations have also been performed to estimate longitudinal coupled-bunch instability growth rates. The results for the fastest growing modes are summarized in Table 4.

For the low-frequency 26.7-MHz system, the fastest growing dipole ($a=1$) mode has a growth time of about 20 ms at injection energy. At 100 GeV/amu, the fastest growth time was about 15 ms. Artificially locating an RF mode exactly on a rotation line increased the growth rate by a factor of 10. This represents the worst-case scenario. Without a broadband resonator contribution, the synchrotron tune spread in the bunch is insufficient for Landau damping. We conclude, therefore, that some combination of feedback and higher-order mode damping will be required.

Calculations at full energy were also carried out for the high-frequency RF scenario. In this case, the predicted growth rates were found to be nearly 20 times lower than for the low-frequency system. This is probably because the bunches are still relatively long ($\sigma_z = 0.15$ m) and thus do not sample the high-frequency parasitic modes very effectively. For this case, ZAP predicts that the growth will be Landau damped by the synchrotron tune spread. To see how much broadband impedance could be tolerated without losing the Landau damping, the calculations were repeated for progressively larger values of $|Z/n|$. A 10 Ω broadband impedance

was sufficient to eliminate the damping.

RHIC Booster

In a separate investigation,⁹ the simulation code ESME was used to study coherent coupled-bunch dipole oscillations in the RHIC booster. This code tracks particles in longitudinal phase space and simulates the developing instability that is driven by the higher-order modes of the booster RF cavity.

The RF modes of the ferrite-loaded cavity were calculated and are tabulated in Ref. 9. The mode that is responsible for the calculated growth in effective longitudinal emittance is at $f_r = 20.68$ MHz. During the acceleration cycle, the beam harmonics sweep across the resonant frequency as the revolution frequency changes.

The simulations in Ref. 9 give evidence for large-amplitude coherent dipole oscillations. The presence of a beam gap (i.e., filling only two of three booster RF buckets) appears to permit the growth to start more easily, due to the presence of additional Fourier components in the beam spectrum. In this circumstance, large-amplitude oscillations appear more quickly in the simulation. We note, of course, that the present version of ESME generates only approximate excitation fields and that these are evaluated only at revolution harmonics, rather than at the actual synchrotron sideband frequencies $f = n\omega_0 \pm \omega_s$. Therefore, the growth-time estimates in Ref. 9 must be viewed with some caution.

SUMMARY

In this paper we have explored many of the collective effects that could affect the performance of the Relativistic Heavy Ion Collider.

Impedance estimates do not point to serious limitations in beam intensity, with the possible exception of the proton case. Transverse coupled-bunch instabilities are dominated by the resistive-wall impedance, and will require a damping system. Longitudinal instabilities may also

be a problem (about 20 ms growth time for Au ions) for the low-frequency RF scenario. With the higher frequency system, the growth rates are lower and Landau damping should be sufficient.

Intrabeam scattering growth has been examined. Our results are in good agreement with earlier predictions upon which the RHIC design is based. The high-frequency RF system is less favored in the IBS context. Its resultant shorter bunches lead to more severe transverse emittance growth, and the voltage required to contain the beam for 10 hours is in excess of 32 MV.

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DISCLAIMER

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Table 1
RHIC Bellows Impedances^{a)}

Bellows	Depth (mm)	Length (cm)	f_r (GHz)	$Im(Z/n)^{b)}$ (Ω)	$Im(Z_{\perp})^{c)}$ (M Ω /m)
Option 1	4	10.1	14.6	0.52	0.5
Option 2	6	9.0	9.9	0.67	0.6
Formed	9	12.9	6.8	1.40	1.3

a) Total for 500 units.

b) $Im(Z/n)$ at zero frequency, which is equivalent to $(R_{||}/n_r Q)$ at the resonant frequency.

c) $Im(Z_{\perp})$ at zero frequency, which is equivalent to $(R_{\perp}/Q)$ at the resonant frequency.

Table 2
RHIC Transverse Impedance Requirements^{a)}

		Protons	Gold Ions
Intensity	[10^{10} /bunch]	10.0	0.11
Energy	[GeV/amu]	28.5	10.7
$Z_{\perp}$ ^{b)}	[M Ω /m]	1.2	10
$Z_{\perp}$ ^{c)}	[M Ω /m]	—	45

^{a)} Assumed bunch area is 0.3 eV-s/amu.

^{b)} For 26.7-MHz RF system.

^{c)} For 214-MHz RF system.

Table 3
Transverse Coupled-Bunch Growth Times^{a)}

E (GeV/amu)	f_{RF} (MHz)	τ (ms)	Damping ^{b)}
10.7	26.7	23	U (0.05)
100.0	26.7	204	U (0.005)
100.0	213.9	204	U (0.0013)

- a) Au ions; $N_b = 10^9$ /bunch. Only the rigid dipole ($a = 0$) mode is shown. Taken from Ref. 7.
- b) Landau damping for $a = 0$ is absent without a betatron tune spread. The parenthetical number is the tune spread required for stability.

Table 4
Longitudinal Coupled-Bunch Growth Times^{a)}

E (GeV/amu)	f_{RF} (MHz)	τ (ms)	Damping ^{b)}
10.7	26.7	21	U
100.0	26.7	14	U
100.0	213.9	233	D

a) Au ions; $N_b = 10^9$ /bunch. Only the dipole ($a = 1$) mode is shown. Taken from Ref. 7.

b) Landau damping indicator: U is unstable; D is Landau damped.