Residential Demand for Electricity by Time of Day: An Econometric Approach

MASTER

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Final Report
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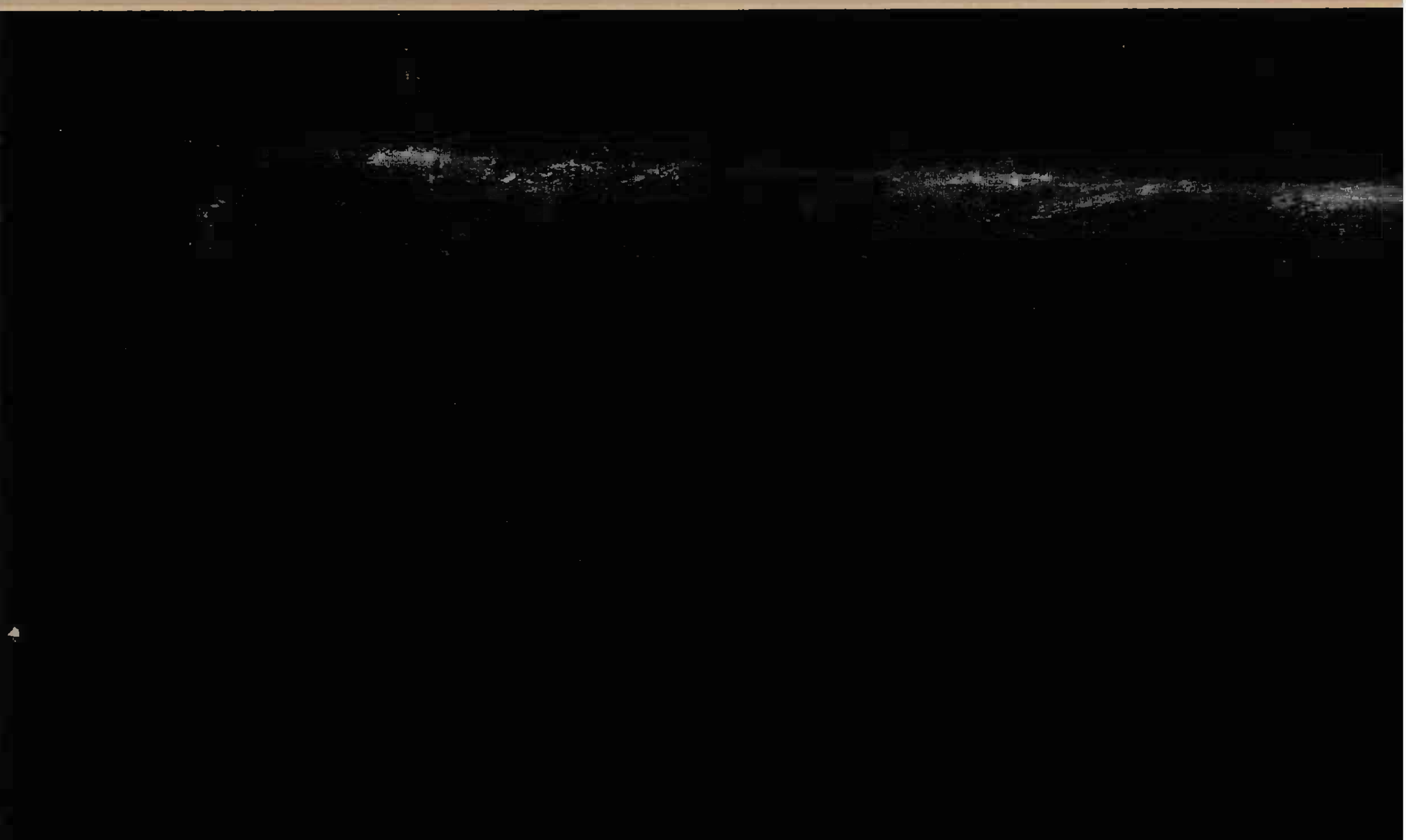
Prepared by
The University of Illinois
Urbana, Illinois



ELECTRIC POWER RESEARCH INSTITUTE

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FOREWORD

The Demand Conservation Program at EPRI is sponsoring an on-going research effort to develop new methodological approaches for load forecasting. The research effort concentrates on developing statistical models that elucidate the long-run behavioral and technological determinants of the diurnal and seasonal load patterns of electricity consumers.

This research report is the second study developing new econometric and statistical methods for modeling household level load patterns. An earlier study "Long Run Residential Load Forecasting" EA 548 develops a two-step scheme for estimating both fifteen-minute and hourly household electricity demands. These micro-load curves can be aggregated into a total residential load curve. The present report deals with the economic theory of optimal peak-load pricing and develops an econometric specification for estimation of the whole time-of-day and seasonal load pattern. The study develops a periodic cubic spline method for estimating the typical winter weekday and summer weekday load patterns at the household level.

Load modeling studies can only be as strong as the data that supports them. For the present study the Connecticut Peak Load Pricing Experiment provided a rich and fertile proving-ground for alternative approaches to household level load modeling. Indeed the only two significant shortcomings of the Connecticut experiment were its relatively short duration and the use of only one experimental peak load rate schedule. The former limits the ability to make statistical inferences about the load pattern's long-run response to time-of-day rates as the residential appliance portfolio is altered to take advantage of the bargain priced off-peak electricity. The latter shortcoming precludes the econometricians from estimating the contemporaneous and non-contemporaneous price elasticities of demand unless highly restrictive assumptions are imposed on the econometric demand model specification. The present study cannot measure the price induced responsiveness of the load pattern to residential consumers. What it can do, and does rather well, is provide a useful and not terribly expensive empirical specification for estimating typical household load curves and measuring how these load curves are affected by the household's appliance portfolios and other socio-economic variables. The model is fully applicable to data with a variety of time-of-day rates and, applied to such data, would provide estimates of the price induced responsiveness of the load pattern to peak load pricing rate structures. As such experimental data becomes available this report should be a valuable foundation upon which further work can be developed.

Further research projects that are in progress or planned will deal with alternative econometric and statistical methods for modeling residential load patterns. Research on commercial and industrial loads by establishment is also under way. This research combined with the residential load studies will provide a firm basis both for long term load forecasting and for conservation analysis. Since the models under development are based on the behavioral and technological structure of the consumers they will be useful for analyzing the effects of load management alternatives including peak load pricing; the load shifts due to new electricity utilizing devices; the effectiveness of conservation regulations and efficiency standards; and the impact of changing economic and social variables.

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ABSTRACT

This report uses data from a time-of-day metering survey of residential electricity use and a FEA-sponsored peak load pricing field test conducted on randomly selected customers of the Connecticut Light and Power Co. to analyze the level and shape of the residential demand cycle for electricity. This analysis is carried out in two steps. The demand cycles for individual customers are first parameterized by a periodic cubic spline. The resulting parameters are then analyzed cross-sectionally, using demographic characteristics of the customers. This analysis provides estimates of the impact of such characteristics as space heating, electric water heating, and stock of electrical appliances, on the level and shape of the demand cycle. The impact of the peak load pricing treatment is also analyzed.

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Section 1

SUMMARY

In the past several years interest in the demand for electricity has shifted from the problem of modeling the integral of demand over some accounting period to the problem of modeling the level and shape of the demand cycle per se. This interest is stimulated by the fundamental technological fact that the cost of generating the power to satisfy a given demand integral is highly dependent on the path of the integrand: the smoother the demand cycle, the lower the cost per kilowatt.

Detailed econometric investigations of the determinants of the shape as well as the level of the residential demand cycle are absolutely essential for long-term load forecasting purposes. Prior to the implementation of the FEA-sponsored peak load pricing experiments, however, the lack of panel data on electricity demand often restricted the investigations to the impact of weather on system peak load.* Econometric work was also inhibited by the lack of an integrated theory of consumer behavior along utility maximization lines for commodities such as electricity that are consumed continuously rather than at discrete points in time.

This report uses data from a time-of-day metering survey of residential electricity use and a FEA-sponsored peak load pricing field test conducted on randomly selected customers of the Connecticut Light and Power Co. to analyze the level and shape of the residential demand cycle. The second section presents a theory of consumer demand formulated in continuous time. The traditional consumers' plus producers' surplus welfare criterion is replaced by a time-additive utility

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*See (1) for examples of this work.

functional which serves to rank alternative consumption paths preferentially. The implications of this theoretical formulation for econometric analyses are discussed. The third section describes the problems that occur when the underlying parameters which are to be estimated from panel data have both systematic and stochastic components. An estimation procedure applicable to these problems is developed. The fourth section gives a brief description of the Connecticut Peak Load Pricing (CPLP) experiment and the data used in the econometric work. The fifth section presents the empirical results on the impact of households' stock of durables, household income, time-of-day electricity prices, and other household demographic variables on the level and shape of the residential demand cycle.

These results are obtained in a two step procedure. In the first step the observed time-series on demand for each household is modeled as a smooth function of time with possible jumps induced by the pricing treatment. A periodic cubic spline is used as a convenient parameterization of this function. It has the advantages of being capable of approximating a wide variety of possible shapes, of being a parsimonious parametric representation, and of having parameters which have direct economic interpretation. Corrections are made for both heteroscedasticity and serial correlation across the demand cycle by estimating the function using generalized least squares. The second step is to explain why the parameters obtained in the first step vary across individual households. Since these parameters have both systematic and stochastic components, the estimation procedure developed in section 3 is used to provide efficient estimates of the impact of household demographics on the level and shape of the demand cycle. The pricing treatment in the CPLP experiment is found to shift the demand cycle significantly away from peak periods. The final section states some conclusions and addresses prospects for future research.

REFERENCES

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Section 2

A MODEL OF CONSUMER DEMAND UNDER PEAK LOAD PRICING

INTRODUCTION

The formulation of socially optimal pricing policies for nonstorable goods with cyclical demand is by now a classical problem of applied welfare economics. Applications may be found in a diverse range of industries, notably transportation, electric power, and telecommunications.

This presentation departs from the received theory of peak load pricing* in three respects. Instead of the usual discrete periodization, the problem is formulated in continuous time. Second, the traditional consumers' plus producers' surplus welfare criterion is replaced by a time-additive utility functional which serves to rank alternative consumption paths preferentially. And third, the familiar fixed proportions production technology is replaced by a more plausible neoclassical technology.**

This formalism is used to derive simple rules for optimal consumption policies for both price-taking and technology-constrained institutional contexts.

First-best pricing policies, which induce consumers to choose the optimal, technology-constrained consumption paths, are then given. When instantaneous

*See for example the seminal contributions of Boiteaux (1), Steiner (2), and Drèze (3).

**Takayama (4) has investigated the problem of maximizing surplus subject to the fixed proportions technology in continuous time. Panzar (5) has recently studied the discrete, surplus maximization problem subject to a neoclassical production technology.

utility varies smoothly in time, the optimal pricing policy is shown to vary smoothly with time also. The problem of approximating the optimal smooth pricing policies by pure-jump pricing policies is treated and a number of implications are drawn for empirical work on time-of-day pricing experiments. Optimal investment rules are also given.

A parametric example is presented to illustrate the general results. An empirically viable specification of the time-additive utility functional is briefly introduced. Results are summarized in a concluding section.

THE PRICE-TAKING CONSUMER

We consider a consumer who faces the problem of selecting a utility-maximizing consumption plan consisting of a vector $x \in \mathbb{R}^K_+$ of conventional commodities and a consumption path $q:[0,T] \to \mathbb{R}_+$ for another commodity whose price may vary within the planning period depending upon when it is consumed.

We suppose that the consumer has a utility function of the form

$$W[x,q] = W\big(V(x),U(q)\big) \ , \tag{2-1}$$

where $V(x)$ is a sub-utility function in the conventional commodities x, and $U[q]$ is a sub-utility functional of the time-additive form*

$$U[q] = \int_0^T u\big(q(t),t\big)dt \ . \tag{2-2}$$

The function u may be interpreted as an instantaneous utility function that gives the utility achieved from consuming q at rate $q(t)$ at time t; it is assumed to be periodic with period T.

The form of Eq. 2-1 implies that there is competition between x and q for shares of the consumer's budget, but intertemporal substitution within the U branch is

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*Additivity of utility in finite dimensional commodity spaces has been discussed by Houthakker (6) and many others. Some remarks on time-additive utility functionals of the type used below may be found in Hadley and Kemp (7) and Bewley (8).

independent of the allocation of spending within the V branch. The form of Eq. 2-2 implies that the utility received from consuming at rate $q(t)$ at time t is independent of past or future consumption rates. However, we will see that in contrast to the traditional peak load pricing literature, demand for q at time t will depend upon the entire time path of the price of q.

The consumer's budget constraint for the horizon of length T is given by

$$m = px + \phi\chi\,[q] + \int_0^T r(t)q(t)dt \ , \qquad (2\text{-}3)$$

where m denotes income allocated to expenditure during the period, p denotes the price vector for the commodities x, $r(t)$ is the price of q at time t, ϕ is a fixed charge imposed on consumers of q, and $\chi[q] = 1$ if $q(t) > 0$ for some $t \in [0,T]$ and is zero otherwise.

The problem of maximizing Eq. 2-1 subject to Eq. 2-3 may be accomplished in two stages.* For $\theta \in [0,1]$, let

$$U^*(\theta) = U[\hat{q}(\theta)] = \max_q \left\{ U[q] \;\middle|\; \theta m \geq \int_0^T r(t)q(t)dt + \phi\chi[q] \right\} \qquad (2\text{-}4)$$

and

$$V^*(\theta) = V\left(\hat{x}(\theta)\right) = \max_x \{V(x) \mid (1-\theta)m \geq px\} \ . \qquad (2\text{-}5)$$

A global optimum is then achieved by selecting a θ to maximize

$$W^*(\theta) = W\left(V^*(\theta), U^*(\theta)\right) \ . \qquad (2\text{-}6)$$

Our attention focuses primarily on the conditionally optimal demand path $\hat{q}(t|\theta)$. If $\hat{q}$ is an interior, optimal consumption path (i.e., the constraint $q(t) \geq 0$ does not bind), then there exists a $\lambda \geq 0$ depending on the expenditure proportion θ, such that

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*This interpretation of separability was introduced by Strotz (9, 10).

$$u'\left(\hat{q}(t|\theta),t\right) = \lambda(\theta)r(t) \qquad t \in [0,T] \tag{2-7}$$

$$\lambda(\theta)\left[\theta m - \phi\chi[q] - \int_0^T r(t)\hat{q}(t|\theta)dt\right] = 0. \tag{2-8}$$

Differentiating Eq. 2-7 with respect to t, the optimal consumption policy is seen to satisfy the first order differential equation

$$\dot{q}(t|\theta) = \frac{u'}{u''}\left[\frac{\dot{r}}{r} - \frac{\dot{u}'}{u'}\right] \tag{2-9}$$

subject to the θ-conditional budget constraint that expressed the initial condition.* Note that when $\dot{r} = 0$, Eq. 2-9 reduces to simply

$$\dot{q}(t|\theta) = -\frac{\dot{u}'}{u''}. \tag{2-10}$$

An attempt to interpret Eq. 2-9 and 2-10 may be facilitated by rewriting 2-9 as

$$\left[q\frac{u''}{u'}\right]\left[\frac{\dot{q}}{q}\right] = \left[\frac{\dot{r}}{r}\right] - \left[\frac{\dot{u}'}{u'}\right]. \tag{2-11}$$

The expression on the left-hand side is seen to be the elasticity of marginal utility with respect to q times the growth rate in q, which is simply the realized rate of change of marginal utility with consumption path q. The utility-maximizing consumer seeks a consumption path that equates this rate to the rate of change in price minus a subjectively evaluated temporal rate of change of marginal utility. Under constant pricing policies, the consumer's subjectively desired and objectively achieved rates of change of marginal utility coincide.

THE PRODUCTION TECHNOLOGY

We assume that the production technology of q may be described by**

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*Throughout this section, prime (′) denotes differentiation with respect to q and dot (•) denotes differentiation with respect to t.

**This so-called neoclassical production technology has been introduced into the peak load pricing literature by Panzar (5), using the discrete-period, consumer-surplus approach.

$$q(t) = F\big(v(t),k\big) \ , \tag{2-12}$$

where k is an N-vector of capital inputs assumed fixed over the horizon T, and v(t) denotes an L-vector of variable inputs. Denoting the vector of variable input prices by ω, we may define the instantaneous variable cost function as

$$c\big(q(t),k\big) = \inf_{v\in\mathbb{R}^L_+} \Big\{\omega\cdot v(t)\,\big|\,q(t) \leq F\big(v(t),k\big)\Big\} \ . \tag{2-13}$$

If the consumption plan q is feasible with capital stock k $\big($i.e., $q(t) \leq \sup_v F(v,k)$ for all $t \in [0,T]\big)$, then total costs are given by

$$C[q,k] = \rho k + \int_0^T c\big(q(t),k\big)dt \ , \tag{2-14}$$

where ρ denotes a vector of rental prices for k for the period $[0,T]$. For convenience, we may set $C[q,k] = \infty$ if $q(t) > \sup_v F(v,k)$ for some $t \in [0,T]$. Finally, we may define the cost functional,

$$C[q] = \inf_k C[q,k] \ . \tag{2-15}$$

OPTIMAL TECHNOLOGY--CONSTRAINED CONSUMPTION POLICIES

We may now consider the problem of maximizing the utility of a single consumer, W [x,q], subject to the revised budget constraint

$$m = p\cdot x + C[q] \ , \tag{2-16}$$

which may be viewed as a requirement either that the firm producing q break even or that the consumer be the sole owner of the firm and absorb any profit or loss it sustains.

As above, the optimization may be separated into two stages. Suppose that $\tilde{q}(t|\theta)$ is an optimal consumption plan and $\tilde{k}(\theta)$ is an optimal capital stock for the problem

$$\max_q \{U[q] \mid C[q] \le \theta m\} \ . \tag{2-17}$$

Then there exists a constant multiplier $\lambda(\theta) \geq 0$ such that the following conditions hold:

$$\frac{u'\left(\tilde{q}(t|\theta),t\right)}{c'\left(\tilde{q}(t|\theta),\tilde{k}\right)} = \lambda(\theta) \quad \text{for all} \quad t \in [0,T] \ , \tag{2-18}$$

$$\lambda(\theta)\left[\theta m - \rho\tilde{k} - \int_0^T c\left(\tilde{q}(t|\theta),\tilde{k}\right)dt\right] = 0 \ , \tag{2-19}$$

$$\rho_i = -\int_0^T c_i\left(\tilde{q}(t|\theta),\tilde{k}\right)dt \qquad i = 1,\ldots,N \ , \tag{2-20}$$

where $c_i = \partial c/\partial k_i$.

Differentiating Eq. 2-18 with respect to t, we see that an optimal consumption plan must satisfy

$$\dot{q} = \left[\frac{c''}{c'} - \frac{u''}{u'}\right]^{-1}\left[\frac{\dot{u}'}{u'}\right] \ . \tag{2-21}$$

This differential equation together with the budget constraint and the optimal capital stock (transversality) conditions 2-20 yield $\tilde{q}(t|\theta)$ and $\tilde{k}(\theta)$. As above, the expenditure proportion θ is chosen to maximize $W^*(\theta) = W\left(U^*(\theta),V^*(\theta)\right)$.

OPTIMAL PRICING POLICIES

We are finally in a position to ask, "Is there a pricing policy $r:[0,T] \to \mathbb{R}_+$ which would induce the price-taking consumer to choose the optimal consumption plan described above?" An affirmative answer is provided by the marginal cost pricing rule

$$\tilde{r}(t) = c'\left(\tilde{q}(t),\tilde{k}\right) \tag{2-22}$$

with the fixed charge

$$\phi = \tilde{\theta}m - \int_0^T c'\left(\tilde{q}(t),\tilde{k}\right)\tilde{q}(t)\,dt \ , \tag{2-23}$$

where $(\tilde{q},\tilde{k},\tilde{\theta})$ denotes the solution to the problem posed by Eq. 2-16.

It would be of obvious interest, reviving a sentiment first expressed by Drèze (3), to have conditions under which decentralized iterative pricing policies based on the scheme introduced above would converge to an optimal policy. For example, suppose we start from the initial policy $\left(r^0(t),\phi^0\right)$ and set

$$r^1(t) = \psi\left[c'\left(q^0(t),k^0\right) - r^0(t)\right] .$$

Here $q^0(t)$ solves the problem of the price-taking consumer above for pricing policy $\left(r^0(t),\phi^0\right)$, k^0 minimizes $C[q^0,k]$, and

$$\phi^1 = \Psi\left(C[q^0] - \int_0^T r^0(t)q^0(t)dt - \phi^0\right) ,$$

with ψ and Ψ denoting sign preserving functions. Continuing this iteration, we have a mapping from the space of pricing policies onto itself for which the solution policy given by $(\tilde{r},\tilde{\phi})$ is readily seen to be a fixed point.

It is frequently claimed that discrete period models of the peak load pricing problem are justified because, ultimately, the price schedule imposed must be a pure-jump function--that is, a series of flat segments with price jumps on a finite set of points within the period. However, for any reasonable model of consumer preferences with smoothly time-varying utility the <u>optimal</u> pricing policy will be smoothly varying in time, and therefore any "jump" pricing policy is at best a decent <u>approximation</u> to the welfare optimal policy.

Any pure-jump pricing policy is completely characterized by the number and position of its jumps and the price levels between the jumps. Therefore such policies constitute a space of finite dimension. Solving the problem above, an indirect utility function may readily be defined on the space of such pricing policies; the problem of selecting an optimal jump-approximation to the optimal smooth pricing policies involves maximizing this indirect utility function subject to a new form of the budget constraint expressed in terms of the price policy parameters. By introducing a cost functional on the space of pricing policies reflecting metering and other costs of price schedule complexity, one could solve other forms of this optimal-approximates-to-welfare-optimal-pricing problem.

A PARAMETRIC EXAMPLE

To illustrate the ideas of the preceding sections, we now present a simple parametric example. The price-taking consumer is treated first, then the technology is introduced and the pricing problem is considered.

Suppose that in addition to q there is a single composite (durable) commodity x and that utility is given by

$$W[q,x] = U[q] + V(x) \ , \tag{2-24}$$

where

$$U[q] = \int_0^T \left[(\alpha_0+\alpha_1 \cos t)q(t) + \tfrac{1}{2} \alpha_2 q^2(t) \right] dt \tag{2-25}$$

and

$$V(x) = \beta_1 x + \tfrac{1}{2}\beta_2 x^2 \ . \tag{2-26}$$

We make the further assumptions that $\alpha_2 < 0 < \alpha_1 < \alpha_0$ and $\beta_2 < 0 < \beta_1$. Under these conditions the marginal utility of q is always positive for some sufficiently small consumption rate but marginal utility is decreasing in both x and q for all $t \in [0,T]$. For obvious reasons we take $T = 2\pi$.

The composite commodity x is taken as numeraire having unit price, so the consumer faces the budget constraint

$$m = x + \phi\chi[q] + \int_0^T r(t)q(t)dt \ , \tag{2-27}$$

where

$$\chi[q] = \begin{cases} 1 & \text{if } \ q(t) > 0 \ \text{ for some } \ t \in [0,T] \\ 0 & \text{otherwise} \end{cases} \ . \tag{2-28}$$

Conditional on the expenditure allocation θm to q and $(1 - \theta)m$ to x, we derive optimal (θ-conditional) consumption policies for q and x and then optimize over $\theta \varepsilon [0,1]$ to determine the unconditionally optimal consumption plan.

The θ-conditional optimal consumption path for q satisfies (by Eq. 2-7)

$$\frac{u'\left(q(t|\theta)\right)}{r(t)} = \lambda(\theta) \qquad t \ \varepsilon \ [0,T] \tag{2-29}$$

for some $\lambda(\theta)$ independent of t. Thus the conditionally optimal policy takes the form

$$q(t|\theta) = \max\left\{0, \frac{1}{\alpha_2}\left[\lambda(\theta)r(t) - \alpha_0 - \alpha_1 \cos t\right]\right\} . \tag{2-30}$$

We may note immediately that for constant pricing policies demand is strictly sinusoidal.

The sub-budget constraint

$$\theta m = \phi\chi[q] + \int_0^T q(t|\theta)r(t)dt \tag{2-31}$$

must be satisfied; and if $\theta \ \varepsilon \ (0,1)$, the marginal condition

$$\frac{d}{d\theta}\left[W^*(\theta)\right] = 0 , \tag{2-32}$$

which in the present context becomes

$$\lambda(\theta) = \beta_1 + \beta_2(1-\theta)m , \tag{2-33}$$

must also be satisfied. Equations 2-31 and 2-33 may be solved for λ and θ and thus yield the demand functionals (mappings from the space of price functions to the space of consumption policies), $q^*(r,\phi)$, $x^*(r,\phi)$. The boundary solutions, for $\theta \ \varepsilon \ \{0,1\}$ when Eq. 2-31 and 2-33 fail to yield an interior solution must of course also be checked.

We now turn to the problem of finding optimal consumption policies for consumers who are directly constrained by the production technology for q.

The production technology for q is assumed to take the form

$$q(v,k) = \left[\frac{e^v - 1}{e^v}\right] k \; , \qquad (2\text{-}34)$$

where v and k denote (scalar) variable and capital inputs, respectively. Isoquants of the production function 2-34 are depicted in Figure 2-1. Note that k is interpreted as units of capacity, since

$$\lim_{v \to \infty} q(v,k) = k \; , \qquad (2\text{-}35)$$

and both inputs are "essential" in the sense that

$$q(0,k) = q(v,0) = 0 \qquad (v,k) \in \mathbb{R}^2_+ \; . \qquad (2\text{-}36)$$

The variable input demand equation arising from Eq. 2-34 is easily seen to be,

$$v(q,k) = \ell n\{k/(k-q)\}, \qquad q < k \; . \qquad (2\text{-}37)$$

For $q \geq k$, we set $v(q,k) = \infty$. Input demand for two capacity levels is illustrated in Figure 2-2. It is assumed that v and k are purchased at fixed prices ω and ρ, so instantaneous variable cost is simply

$$c\{q(t),k\} = \omega \cdot v\{q(t),k\} \; , \qquad (2\text{-}38)$$

while total cost is given by the functional

$$C[q,k] = \rho k + \int_0^T c\{q(t),k\} dt \; . \qquad (2\text{-}39)$$

Given the preferences expressed by Eq. 2-24 and this production technology for q, the consumer faces the problem of maximizing Eq. 2-24 subject to the budget constraint

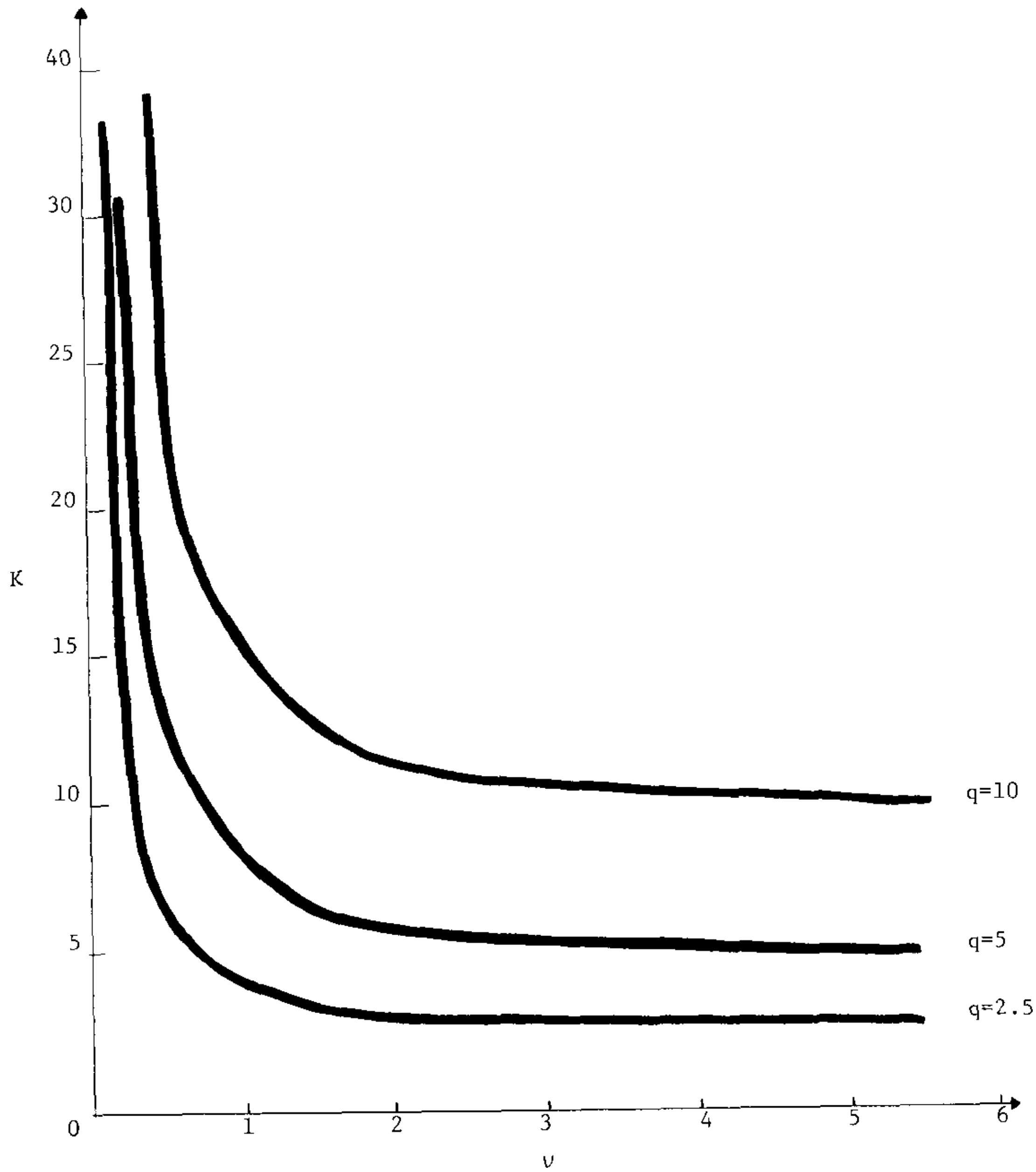

Figure 2-1. The Production Technology

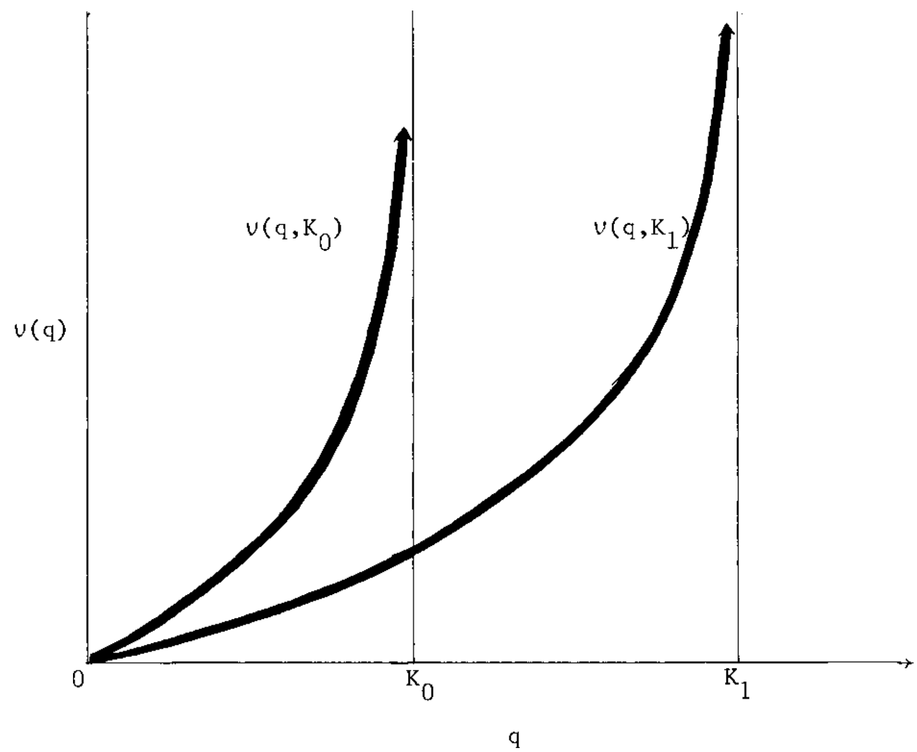

Figure 2-2. The Input Demand Function

$$x + C[q,k] = m \; . \tag{2-40}$$

As above we begin by solving conditional on the expenditure proportion θ. Maximizing Eq. 2-25 subject to the budget constraint,

$$C[q,k] = \theta m \tag{2-41}$$

yields via Eq. 2-21 the differential equation

$$M(q,t) + N(q,t)\dot{q} = 0 \; , \tag{2-42}$$

where

$$M(q,t) = (k-q)\alpha_1 \sin t$$

$$N(q,t) = \alpha_0 + \alpha_1 \cos t + 2\alpha_2 q - \alpha_2 k \; .$$

Equation 2-42 is "exact" and therefore is easily solved to yield

$$\alpha_2 q^2(t) + (\alpha_0 - \alpha_2 k + \alpha_1 \cos t)q(t) - (k\alpha_1 \cos t + Q) = 0 \; , \tag{2-43}$$

where Q is a constant to be determined. The quadratic 2-43 has roots

$$q(t) = \frac{1}{2\alpha_2}\left[\alpha_2 k - \alpha_0 - \alpha_1 \cos t \pm \sqrt{(\alpha_2 k - \alpha_0 - \alpha_1 \cos t)^2 + 4\alpha_2(k\alpha_1 \cos t + Q)}\right] \; . \tag{2-44}$$

The constant Q and the optimal capital stock k are determined conditional on θ by the budget constraint,

$$\frac{\theta m - \rho k}{\omega} = \int_0^T \ell n\big(k/(k - q(t)\big)dt \; , \tag{2-45}$$

and the transversality condition,

$$\frac{\rho}{\omega} = \int_0^T q(t)/k\big(k - q(t)\big)dt \; . \tag{2-46}$$

To determine θ (the optimal expenditure proportion) we require, as in the price-taking context, that

$$\lambda(\theta)m = \beta_1 m + \beta_2(1-\theta)m^2 \ . \tag{2-47}$$

Evaluating $\lambda(\theta) = u'/c'$ at $t = \pi/2$ for convenience, Eq. 2-47 becomes

$$\frac{\alpha_0 + \alpha_2 q(\pi/2)}{\omega/\{k - q(\pi/2)\}} = \beta_1 + \beta_2(1-\theta)m \ . \tag{2-48}$$

A solution to the three non-linear equations 2-45, 2-46, and 2-47 in $\{Q,k,\theta\}$ completes the solution. A compact domain for these nonlinear equations is given by

$$D = \left\{(\theta,k,Q) \in \mathbb{R}^3 \,|\, \theta \in [0,1],\ k \in [0,\theta m/p],\ Q \in [\alpha_1 k, \alpha_0 k]\right\} . \tag{2-49}$$

The bounds on k are obvious from the budget constraint, while the bounds on Q are necessary and sufficient for $q(t) \in [0,k]$ for all $t \in [0,2\pi]$.

Figure 2-3 illustrates optimal consumption, average variable cost, and marginal cost paths for three different pairs of factor prices.* As one would expect, a higher relative price of capital smooths the consumption path which maximizes welfare. In panel (a), where capital is relatively cheap, the amplitude of the demand cycle is large and excess capacity is large--the optimal capital stock is 27.16--so output is always well below the maximal feasible output rate. When capital is dear, as in panel (c), the demand cycle is severely dampened, and the optimal capital stock is only 11.02; this yields extremely high utilization rates throughout the period.

If consumers are price-takers we know from the preceding discussion that a marginal cost pricing policy with an appropriate choice of lump-sum fee will induce optimal consumer behavior. In Table 2-1 we report costs and revenues under each of the

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*The utility function parameters for all of the examples are $(\alpha_0,\alpha_1,\alpha_2) = (2.0,1.0,-0.1)$ and $(\beta_1,\beta_2) = (3.0,-0.1)$, income is taken as 50.00.

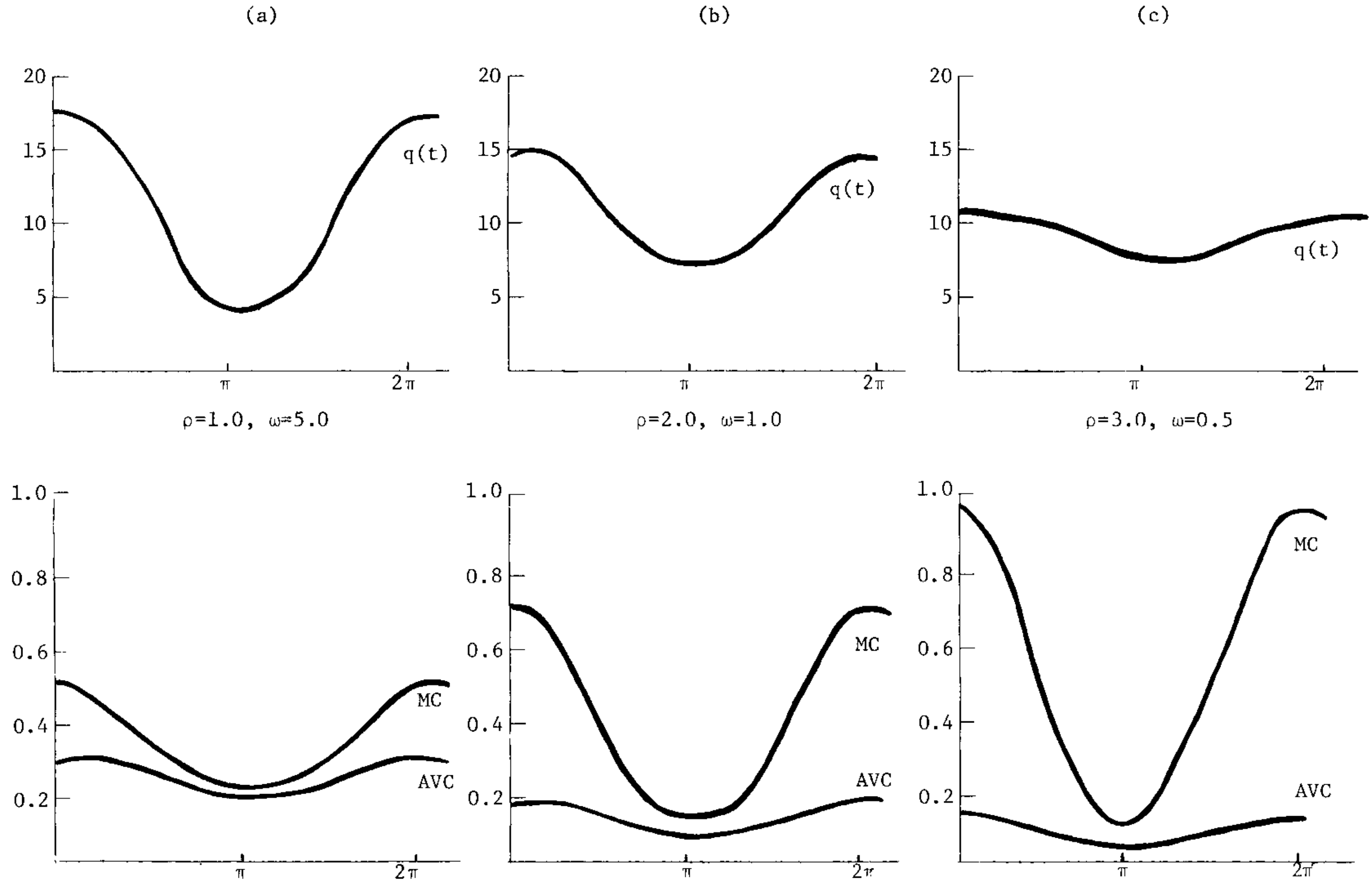

Figure 2-3. Optimal Consumption and Cost Paths for Diverse Factor Prices

factor price regimes illustrated in Figure 2-3. A curious feature of the optimal pricing policy immediately emerges. Usage fees cover capital costs, while lump-sum fees cover variable costs. This curiosity is an immediate consequence of the hypothesized form of the technology. The transversality condition

$$-\int_0^T c_i(q,k)dt = \omega \int_0^T q(t)/k\big(k - q(t)\big)\ dt = \rho \qquad (2\text{-}50)$$

for optimal capital accumulation is given by Eq. 2-46. But the revenue from a marginal cost usage fee is

$$\int_0^T q(t)C_q(q,k)dt = \omega\int_0^T q(t)/\big(k - q(t)\big)dt\ , \qquad (2\text{-}51)$$

which by Eq. 2-50 must equal ρK. It is interesting to note that as capital becomes expensive relative to the variable factor, capital's share of costs rises while the share of revenue financed by the lump-sum fee falls dramatically.

Table 2-1

COSTS AND REVENUES UNDER DIVERSE FACTOR PRICES
WITH OPTIMAL PEAK LOAD PRICING

	Costs			Revenues		
Factor Prices (ρ,ω)	Variable Costs	Capital Costs	Total Costs	Usage Fees	Lump-Sum Fees	Total Revenue
a. (1.0,5.0)	18.22	27.16	45.38	27.16	18.22	45.38
b. (2.0,1.0)	10.03	31.44	41.47	31.44	10.03	41.47
c. (3.0,0.5)	7.00	33.08	40.08	33.08	7.00	40.08

As we have noted above, the smoothly evolving marginal cost pricing policies depicted in Figure 2-3 are frequently viewed as impractical in applications. The technology of metering demand, consumers' aversion to complicated price schedules, and other intrusions of reality constrain the set of feasible pricing policy functions. Interest naturally focuses on the space of jump pricing policies, pricing policies that are constant over the demand cycle except for a finite number of instants at which the price jumps.

An M-jump pricing policy on $[0,\infty]$ has the form

$$r(t) = r_i \qquad t \in [k\tau_i, k\tau_{i+1}]$$

$$i = 0,1,\ldots,M;\ K = 1,2,\ldots ,$$

where we set $\tau_0 = 0$, $\tau_M = T$, and $r_0 = r_M$. The space of M jump pricing policies is 2M-dimensional, consisting of vectors of the form

$$\underset{\sim}{r} = (\tau_1,\ldots,\tau_M;\ r_1,\ldots,r_M) \in J^M.$$

A question naturally arises: If the strict marginal-cost-plus-lump-sum-fee pricing policies above are held infeasible because, for example, continuously varying prices are difficult to meter, what M-jump pricing policy with lump-sum fee is "second best" in the sense of maximizing consumer welfare subject to the condition that the costs of producing q are recovered?

This problem is readily posed in our context. Define the indirect utility functional,

$$\Omega[r,\phi] = w\Big[q^*[r,\phi],\quad x^*[r,\phi]\Big] , \qquad (2\text{-}52)$$

where q* and x* maximize w subject to the usual consumer budget constraint. Letting $\theta^*[r,\phi]$ denote the expenditure proportion on q as a function of the pricing policy, we have the program

$$\max_{(r,\phi)\in J^M\times \mathbb{R}_+} \Omega[r,\phi]$$

$$\text{subject to } \theta^*[r,\phi]m \geq C\Big[q^*[r,\phi]\Big] . \qquad (2\text{-}53)$$

Even in the context of our simple parametric example this nonlinear program poses nontrivial numerical difficulties. In Figure 2-4 we contrast the zero and two-jump pricing policies which solve Eq. 2-53 with the optimal (smooth) marginal cost policy. The zero-jump policy imposes a fixed fee of 23.90, considerably lower than that required by the optimal pricing policy. The consumption path chosen under the zero-jump policy has significantly larger amplitude than the optimal consumption policy, and the consumer spends more on q and less on x than with the optimal pricing policy. Compared to the zero-jump policy, the two-jump policy imposes a slightly higher usage fee during the peak periods and a substantially lower usage fee off-peak. This has the effect of shifting down demand on-peak and shifting up demand off-peak. A discrete jump in consumption occurs at the instant when price jumps. The two-jump pricing policy imposes a lump-sum fee of 24.80. Both jump policies require a substantially larger capital stock than does the optimal policy. The optimal pricing policy achieves a utility level of 128.3, the two-jump policy 127.0, and the zero-jump policy 125.2. Thus when capital is relatively cheap one loses very little with the jump approximations. However, when capital is dear the losses are more substantial.

AN EMPIRICAL SPECIFICATION

In this section we introduce an empirically viable time-additive utility functional. It is a continuous analogue of the well-known Stone-Geary utility function which generates the linear expenditure system.*

Consider the utility functional

$$U[q] = \int_0^T \beta(t)\ell n\big(q(t) - \alpha(t)\big)dt \ , \tag{2-54}$$

which, subject to the budget constraint

$$m = \int_0^T r(t)q(t)dt \ , \tag{2-55}$$

---

*The classic paper is Stone (11); Phlips's recent text (12) offers additional details.

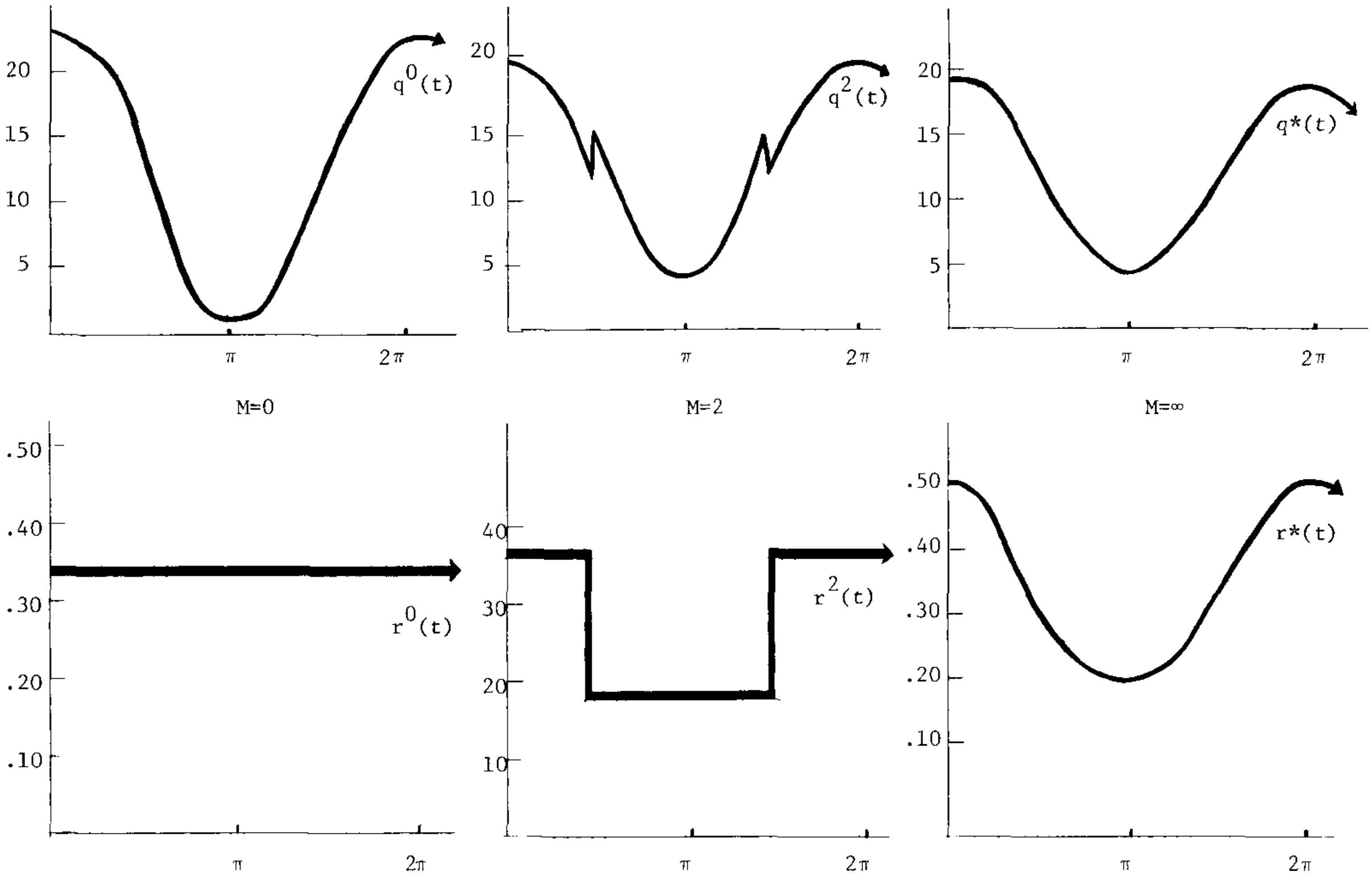

Figure 2-4. M-Jump Approximations to Optimal Pricing and Consumption Policies
(ρ=1.0, ω=5.0)

generates demand paths

$$q(t) = \alpha(t) + \frac{\beta(t)}{r(t)} \left[m - \int_0^T r(\tau)\alpha(\tau)d\tau \right] . \tag{2-56}$$

The "taste" function $\alpha(t)$ has the interpretation of a committed demand path, while $\beta(t)$ is a time path of marginal budget shares. It may frequently be appropriate to make the committed demand path, $\alpha(t)$, depend upon other consumer characteristics, such as past levels of consumption or current stocks of durable goods.

The utility function 2-54 has the obviously appealing feature that it permits demand at any time t to be written explicitly as a functional of the pricing policy r.

We may now briefly address the problem of estimating the functions $\alpha(t)$ and $\beta(t)$ given the model with error $\varepsilon(t)$,

$$q_i(t) = \alpha(t) + \frac{\beta(t)}{r_i(t)} \left[m_i - \int_0^T r_i(\tau)\alpha(\tau)d\tau \right] + \varepsilon(t) , \tag{2-57}$$

and time series data on q and r for a cross-section of individuals $i = 1,\ldots,N$. Data of this type are currently being generated by a number of peak load pricing experiments for electricity. The problem of estimating the continuous time model 2-57 from discrete (unit averaged and/or point samples) data is similar in many respects to the problem of estimating continuous distributed lags recently treated by Sims (13) and others. Although it is obvious that even in principle we cannot hope to identify a continuum of parameters for $\alpha(t)$ and $\beta(t)$ from a necessarily discrete sample, it is perhaps less obvious that $\alpha(t)$ cannot be identified on intervals on which $r(t)$ is constant for all sampled individuals, regardless of how fine the discrete mesh of observations on q and r is made. The latter point is seen immediately by assuming $\beta(t)$ to be known and considering the problem of estimating

$$y_{it} = \beta_t m_i + \sum_{\tau=1}^{T} (\delta_{t\tau} - \beta_\tau) r_{i\tau} \alpha_t + \eta_t , \tag{2-58}$$

where $\delta_{t\tau}$ is the usual kronecker delta, $y_t = r_t q_t$, and $\eta_t = r_t \varepsilon_t$. If $r_{i\tau} = r_{i\tau+1}$ for some τ and all i, then the columns of the design 2-58 differ in only two elements and the design approaches singularity as the observational mesh becomes finer and finer.

Since the function $\alpha(t)$ is crucial to the determination of own and cross price effects, the foregoing identification problem raises serious implications for the design of pricing experiments. If pricing experiments restrict rate design to pure jump functions, then every effort should be made to design rate structures which jump at different time points. These comments hold a fortiori for any attempt to estimate a more complex preference structure than the one represented by the demand function 2-57.

Estimation of the model 2-57 in discrete form involves a number of delicate issues, and we make only one general observation here. The prior information we bring to the problem of estimating the functions $\alpha(t)$ and $\beta(t)$ consists essentially of the claim that both functions are smooth and periodic. A convenient parametrization of such "smoothness priors" is the restriction of $\alpha(t)$ and $\beta(t)$ to the family of periodic cubic splines--periodic piecewise cubic polynomials with continuous first and second derivatives,* or some low-dimensional sinusoid.

The problem of optimal peak load pricing with Stone-Geary utility is, in principle, very straightforward. Substituting Eq. 2-56 in 2-54 we have the indirect utility function

$$\Omega[r,m] = B + \int_0^T \frac{\beta(t)}{r(t)} \ln \left[m - \int_0^T r(\tau)\alpha(\tau)d\tau \right] dt ,$$

where B is a constant independent of r and m.

---

*The cubic spline parametrization has the extremely convenient feature that the ordinate values of the functions at the knots (the abscissa values at which the third derivative is permitted to jump) serve as a linear parametrization of the function space. Thus the discretized version (Eq. 2-57) of the model may be estimated subject to a set of linear restrictions on the parameter vectors $(\alpha_1,\alpha_2,\ldots,\alpha_k)$ and $(\beta_1,\beta_2,\ldots,\beta_K)$, where these vectors denote values taken by $\alpha(t)$ and $\beta(t)$ at knot locations. A valuable reference is Poirier (14).

SOME EXTENSIONS

Although the time-additive form of the utility functional 2-2 has been employed extensively in economic analysis it does impose certain restrictions which may be unreasonable in particular applications. A slightly more general form which is still tractable within the present framework would be to let

$$U[q(t)] = \int_0^T u\left(q(t),\dot{q}(t),t\right)dt \ . \tag{2-59}$$

Such a formulation yields second-order differential equations in lieu of Eq. 2-9 and 2-21.

In many applications, the consumer's utility at time t depends upon some stochastic element--such as the weather in the case of electricity--as well as q(t) and t. Such uncertainty may be incorporated directly into the utility function introduced above.

The formulation of the production technology may also be extended in a number of interesting directions. It may be reasonable to make variable costs a function of $\dot{q}(t)$ as well as q(t) and K to account for adjustment costs. Variable input prices may be made to depend on t. And finally, in some applications, storage of q is not literally impossible, but only expensive. This last situation can be modeled relatively easily within the present framework by introducing a state variable that represents an accumulated inventory of the commodity q. The optimization problem then takes on some of the features of an optimal production-inventory control problem (see, e.g., Nguyen (15)).

Consumers with nonidentical tastes may, in principle, be treated within the framework of a random coefficients model of form (2-57). However, serious technical difficulties may emerge, especially in the attempts to select optimal pricing policies based on weighted sum of individuals' indirect utility functions.

SUMMARY

A formulation of the classical problem of pricing a nonstorable commodity with periodic demand has been suggested based on a time-additive utility functional representing consumer preferences and a conventional (neoclassical) variable cost function defining the production technology. An optimal consumption path is derived. Under mild differentiability conditions these optimal paths are smooth functions of time. The problem of approximating smooth policies by pure jump pricing policies is addressed, and it is noted that discrete jumps in price induce discrete jumps in demand under the assumptions of the analysis.

An empirically viable utility functional based on the well-known Stone-Geary expenditure system is suggested. The results are illustrated with a simple parametric example.

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Section 3

A STOCHASTIC PARAMETER MODEL FOR PANEL DATA

INTRODUCTION

Empirical research in the social sciences, and economics in particular, relies increasingly on panel data. Time series data on several similar cross-section units (household, states, individuals, etc.) are now quite commonplace. These data sources have stimulated the development of a variety of new statistical models designed to "pool" panel data. In this section we propose a natural extension of well-known random coefficient models to incorporate systematic variation in parameters across cross-sectional units.

The existence of systematic and/or stochastic parameter variation is now widely recognized. For evidence, one need only consult the *Annals of Economic and Social Measurement* for October 1973, which is devoted entirely to such issues. The purpose of this section is to explore cross-sectional parameter variation in regression models for which panel data are available.

MODEL NOTATION

Suppose we observe a time series of length T on N distinct individuals or cross-sectional units which can be expressed in the familiar linear regression framework

$$y_n = X_n\beta_n + \varepsilon_n \qquad n = 1,2,\ldots,N\ , \qquad (3\text{-}1)$$

where for the nth individual, y_n is a T-dimensional column vector of observations on the dependent variable, X_n is a T×K matrix of fixed regressors having rank

K,* β_n is a K-dimensional coefficient vector, and ε_n is a T-dimensional disturbance vector such that

$$E(\varepsilon) = 0 \text{ and } E(\varepsilon\varepsilon') = \Sigma \text{ where } \varepsilon = [\varepsilon_1', \ldots, \varepsilon_N']' \; . \tag{3-2}$$

For notational convenience later on, we partition the positive definite matrix Σ into T×T blocks according to

$$\Sigma = \begin{bmatrix} \Sigma_{11} \cdots \Sigma_{1N} \\ \\ \Sigma_{N1} \cdots \Sigma_{NN} \end{bmatrix} \; . \tag{3-3}$$

Stacking the observations in system 3-1 analogous to 3-2, 3-1 can alternatively be written

$$Y = X\beta + \varepsilon \; , \tag{3-4}$$

where $Y = [y_1', \ldots, y_N']'$ is a column vector of length NT, X is a NT×NT block diagonal matrix with diagonal blocks $X_1, \ldots, X_N$, and $\beta = [\beta_1', \ldots, \beta_N']'$ is a column vector of length NK.

Further suppose that the coefficient vectors of each individual can be related according to

$$B = Z\Gamma + U \; , \tag{3-5}$$

where

$$B = \begin{bmatrix} \beta_1' \\ \cdot \\ \cdot \\ \cdot \\ \beta_N' \end{bmatrix} \tag{3-6}$$

---

*The assumption that each cross-sectional unit has observed time series of the same length (T) is not restrictive, and it is made merely for notational convenience. The same is true for the assumption that the number of regressors (K) is identical for each individual.

denotes the $N \times K$ matrix of coefficients whose nth row corresponds to the coefficient vector of individual n, Z is a $N \times J$ matrix of observations on J fixed explanatory variables for each individual,

$$\Gamma = [\gamma_1, \ldots, \gamma_K] \tag{3-7}$$

is a $J \times K$ matrix of unknown coefficients whose kth column γ_k gives the weights attached to the J possible explanatory factors of the kth regression coefficient in β_n for an arbitrary individual n, and

$$U = \begin{bmatrix} u_1' \\ \cdot \\ \cdot \\ \cdot \\ u_N' \end{bmatrix} \tag{3-8}$$

is a $N \times K$ matrix of disturbances whose row vectors are all independent of ε and which satisfy

$$E(u') = 0 \text{ and } E(u'u) = \Omega = (I_N \otimes \Omega_o) \ . \tag{3-9}$$

For later reference, the number of nonzero elements in γ_k will be denoted J_k for $k = 1,2,\ldots,k$. Thus, $J = \max \{J_k\}$.

Ignoring any zero restrictions on the γ_{jk}'s for the moment and stacking the columns of Γ and the rows of U, the coefficient vector in Eq. 3-4 can be written as

$$\beta = \Xi \gamma + u \ , \tag{3-10}$$

where $\gamma = [\gamma_1', \ldots, \gamma_K']'$ is a column vector of length KJ, $u = [u_i', \ldots, u_N']'$ is a column vector of length NK, and

$$\Xi = [Z \otimes e_1 \vdots \ \ldots \ \vdots \ Z \otimes e_K] \ , \tag{3-11}$$

where e_k is the kth standard basis vector of $\mathbb{R}^K$. More explicitly, Eq. 3-11 can be written as

$$
\Xi_{(NK \times JK)} = \left[\begin{array}{cccc|cccc|c|cccc}
z_{11} & z_{12} & \cdots & z_{1J} & 0 & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & z_{11} & z_{12} & \cdots & z_{1J} & \cdots & \vdots & & & \vdots \\
\vdots & & & \vdots & \vdots & & & \vdots & & 0 & 0 & & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & & z_{11} & z_{12} & \cdots & z_{2J} \\
\hline
z_{21} & z_{22} & \cdots & z_{2J} & 0 & 0 & \cdots & 0 & & 0 & 0 & & 0 \\
0 & 0 & & 0 & z_{21} & z_{22} & \cdots & z_{2J} & \cdots & \vdots & & & \vdots \\
 & & & & \vdots & \vdots & & \vdots & & 0 & 0 & & 0 \\
0 & 0 & & 0 & 0 & 0 & & 0 & & z_{21} & z_{22} & \cdots & z_{2J} \\
\hline
\vdots & & & & \vdots & & & & & \vdots & & & \\
\hline
z_{N1} & z_{N2} & & z_{NJ} & 0 & 0 & \cdots & 0 & & 0 & 0 & & 0 \\
0 & 0 & & 0 & z_{N1} & z_{N2} & \cdots & z_{NJ} & \cdots & \vdots & \vdots & & \vdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & 0 & 0 & & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & & 0 & & z_{N1} & z_{N2} & \cdots & z_{NJ}
\end{array}\right].
$$

SUMMARY OF THE LITERATURE

Numerous authors have studied special cases of the general model outlined above. Since as a whole these efforts play a prominent role in the econometric literature, it may be useful to review briefly some of the more important variants.

The most familiar variants are those which deal only with systematic parameter variation. If the disturbance u in Eq. 3-10 and its variance-covariance matrix Ω are assumed null, then 3-10 may be substituted into 3-4 to yield the classical regression model

$$Y = (X\Xi)\,\gamma + \varepsilon \,, \qquad (3\text{-}12)$$

which can be estimated directly using standard techniques. In some cases the following two-step procedure is also attractive. For simplicity assume $\Sigma_{ni} = 0\ (n \neq i)$ and $\Sigma_{nn} = \sigma_{nn} I_T$ for n, i = 1,2,...,N. In the first step, ordinary least squares (OLS) is applied to Eq. 3-4; then in the second step, generalized least squares (GLS) is applied to Eq. 3-10 with the previous OLS estimator of β serving as the dependent variable. One example of such a two-step procedure can be found in (1).

Another particularly interesting case of Eq. 3-12 is the seemingly unrelated (SUR) model of Zellner (2) which corresponds to the case $J_k = N$, for k = 1,2,...,K, $Z = I_N$, and $\Sigma_{ni} = \sigma_{ni} I_T$ for n, i = 1,2,...,N. The same case, but with $\Sigma_{ni} \neq \sigma_{ni} I_t$ due to serial correlation, was studied by Parks (3).

While still less common in applied work, the variants of the general model which do not assume a degenerate distribution for u are by far the most interesting. Rosenberg (4) has provided an extensive survey of these so-called stochastic parameter models.* One notable case is the random coefficient model ($J_k = 1$, for k = 1,2,...,K, $Z = i_N$), which has been extensively studied by Swamy (6).**

When only the constant term is random, then the model reduces to the familiar error components model.+

*See also (5).

**Singh and Ullah (7) have considered a random coefficient variant of the SUR model, and Kelijian (8) and Bandyopadhyak (9) have considered random coefficient variants of simultaneous equation models.

+Of course only the two-component variety is referred to here. For an extension of the error components model to the case of the entire coefficient vector, see (10).

Cases in which the parameter variation is both systematic and stochastic have received relatively little attention. Hanushek (11) considered the comparatively simple case of $K = 1$, $\Omega = \omega^2 I$, $\Sigma_{ni} = 0$ $(n \neq i)$ and $\Sigma_{nn} = \sigma^2 I$ for $n, i = 1,2,\ldots,N$. Saxonhouse (12) has recently considered the case $\Sigma_{ni} = I_T$ for $n, i = 1,2,\ldots,N$, with the added restriction that Ω is diagonal. Lindley and Smith (13) and Smith (14) have considered related models from the Bayesian perspective. None of these authors, however, has discussed in its entirety the general model given above together with the estimation scheme outlined below.*

ESTIMATION

Introduction

Using Eq. 3-4 and 3-10, the model presented above may be expressed compactly as

$$y = X\Xi\gamma + v\ , \qquad (3\text{-}13)$$

where the composite disturbance term

$$v = Xu + \varepsilon \qquad (3\text{-}14)$$

has $Ev = 0$ and $Evv' = \Sigma + X\Omega X'$.

Known Covariance Structure

With known covariance structure $\{\Sigma,\Omega\}$ the problem of estimating Eq. 3-13 is obviously straightforward, giving rise to the Aitken estimator,

$$\hat{\gamma}(\Sigma,\Omega) = [\Xi'X'(\Sigma+X\Omega X')^{-1}X\Xi]^{-1}\Xi'X'(\Sigma+X\Omega X')^{-1}y\ . \qquad (3\text{-}15)$$

An equivalent and potentially more convenient form of Eq. 3-15 is given by**

---

*Many authors (15-25) have considered the similar problem of parameter variation when only a single data series, as opposed to a panel, is available.

**This equivalence is noted in (13) and relies on a matrix identity attributed to C. R. Rao.

$$\hat{\gamma}(\Sigma,\Omega) = [\Xi'((X'\Sigma^{-1}X)^{-1} + \Omega)^{-1}\Xi]^{-1}\Xi'((X'\Sigma^{-1}X)^{-1} + \Omega)^{-1}\hat{\beta} \qquad (3\text{-}16)$$

where $\hat{\beta}$ is the Aitken estimator of β,

$$\hat{\beta}(\Sigma) = (X'\Sigma^{-1}X)^{-1}\, X'\Sigma^{-1}\, y. \qquad (3\text{-}17)$$

The estimating form 3-16 is particularly convenient if Σ can be taken to be block diagonal. Then Eq. 3-17 reduces to N independent computations,

$$\hat{\beta}_i(\Sigma_i) = (X_i'\Sigma_i^{-1}X_i)^{-1}\, X_i'\, \Sigma_i^{-1}y_i \qquad i = 1,\ldots,N, \qquad (3\text{-}17a)$$

and the potentially awesome dimensionality of Eq. 3-15 and 3-17 is avoided.* The purely systematic parameter variation model ($\Omega = 0$ in the notation established above) may now be seen to be rather subtly treacherous. If $\beta = \Xi\gamma$ holds without error, i.e., if all noise in the model is attributed to the first stage, then individuals who "fit well" in the first stage (Eq. 3-17) receive large weight in Eq. 3-16. Not only is such a model implausible a priori since we seldom expect a totally satisfactory model of β, but the weighting scheme it gives rise to is extremely sensitive to variation in the cross section in the goodness of fit of stage 1 estimates.

Unknown Covariance Structure

The covariance structure $\{\Sigma,\Omega\}$ is generally unknown in applications, and this subsection considers problems of estimating Σ and Ω and proposes estimators of γ of the form $\hat{\gamma}(\hat{\Sigma},\hat{\Omega})$. It is generally impractical in models of this type to "integrate out" the nuisance parameters of the covariance structure; the literature is notorious for its ad hoc proposals. An asymptotic justification may be found for a rather wide class of estimators of the form $\hat{\gamma}(\hat{\Sigma},\hat{\Omega})$.

In this section we will assume that Σ is block diagonal and that as the length of the time series on each of the cross section units becomes large ($T \to \infty$), we

---

*Note that the assumption of block diagonality of Σ rules out Zeller's SUR covariance structure in Eq. 3-4, unless of course $X_i = X$ $i = 1,\ldots,N$, in which case Eq. 3-17a is equivalent to 3-17 even under SUR conditions.

have a consistent estimator $\hat{\Sigma}_i$ of Σ_i , $i = 1,\ldots,N$. Thus the estimator $\hat{\beta}_i(\Sigma_i)$ may be shown to be asymptotically efficient (26) under standard normal theory assumptions.

If Σ is block diagonal, then

$$\hat{\beta}_i(\Sigma_i) = (X_i' \hat{\Sigma}^{-1} X_i)^{-1}(X_i'\hat{\Sigma}^{-1} y_i) = \beta_i + \eta_i \ , \qquad (3\text{-}18)$$

where $\eta_i = (X_i' \hat{\Sigma}^{-1} X_i)^{-1} X_i' \hat{\Sigma}^{-1} \varepsilon_i$ for $i = 1,\ldots,N$.
Thus in the notation of Eq. 3-5 we have

$$\hat{B} = B+H = Z\Gamma + U + H \ . \qquad (3\text{-}19)$$

If we now consider the ordinary least squares estimator,

$$\hat{\Gamma} = (Z'Z)^{-1}Z'\hat{B} \ , \qquad (3\text{-}20)$$

we have

$$\begin{aligned} \hat{V} &= \hat{B} - Z\hat{\Gamma} \\ &= (I - Z(Z'Z)^{-1}Z')\,\hat{B} \\ &= M(Z\Gamma + U + H) \\ &= M(U + H). \end{aligned} \qquad (3\text{-}21)$$

Since ε and u are independent,

$$\begin{aligned} E\hat{V}'\hat{V} &= E(U'+H')M(U+H) \\ &= EU'MU + EH'MH \ . \end{aligned} \qquad (3\text{-}22)$$

Consider the ℓkth element of the first term, denoting the kth column of U by u_k,

$$\begin{aligned} Eu_\ell'Mu_k &= E\ \mathrm{tr}\ u_\ell'Mu_k \\ &= \mathrm{tr}\ MEu_k u_\ell' \\ &= \mathrm{tr}\ M\omega_{k\ell} I_N \\ &= (N-J)\omega_{k\ell}. \end{aligned} \qquad (3\text{-}23)$$

So we have

$$E\ U'MU = (N-J)\Omega. \tag{3-24}$$

The second term is a bit more complicated. Again consider the ℓkth element, now denoting columns of H by h_k; we have

$$\begin{aligned} E\ h_\ell' Mh_k &= E\ \mathrm{tr}\ Mh_k h_\ell' \\ &= \mathrm{tr}\ M\ E\ h_k h_\ell' \ . \end{aligned} \tag{3-25}$$

Since $E\ \varepsilon_i\ \varepsilon_j = 0$ for all $i \neq j$, we have

$$E\ h_k h_\ell' = \begin{bmatrix} s_{k\ell}^{(1)} & 0 & \cdots & 0 \\ 0 & s_{k\ell}^{(2)} & & \\ & & \ddots & \\ 0 & & & s_{k\ell}^{(N)} \end{bmatrix} = S_{k\ell}\ , \tag{3-26}$$

where $s_{k\ell}^{(i)}$ is the $k\ell$th element of the ith covariance matrix $(X_i' \Phi_i^{-1} X_i)^{-1}$. Thus,

$$\mathrm{tr}\ MS_{k\ell} = \sum_{i=1}^{N} m_{ii} s_{k\ell}^{(i)} \equiv \hat{\Delta}_{k\ell}\ , \tag{3-27}$$

so finally

$$E\ \hat{V}'\hat{V} = (N-J)\Omega + \hat{\Delta}\ , \tag{3-28}$$

and thus an unbiased estimate of Ω is given by

$$\hat{\Omega} = \frac{\hat{V}'\hat{V} - \hat{\Delta}}{N-J}\ . \tag{3-29}$$

$\hat{\Omega}$ is consistent for Ω as $N \to \infty$ with K and J fixed and therefore $\gamma(\hat{\Phi},\hat{\Omega})$ as developed above is asymptotically efficient. As is typical of unbiased variance estimators in such situations, $\hat{\Omega}$ may fail to exhibit positive definiteness. We will return to this difficulty below.

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Section 4

THE CONNECTICUT PEAK LOAD PRICING DATA

INTRODUCTION

The data used in this paper were provided by Northeast Utilities and were taken from the FEA-sponsored Connecticut Peak Load Pricing (CPLP) experiment. This section briefly describes the data and associated problems. A more detailed description of the experiment is given in reference (1).

CONSUMPTION DATA

Electricity consumption data were gathered by metering electricity usage every 15 minutes for individual households in the test and control groups. Usage data were available for the test individuals for a year prior to the implementation of peak load prices as well as during the pricing experiment. Both groups had approximately 200 customers chosen randomly from five usage strata.*

The pricing experiment began in October 1975 and ended in August 1976. The price schedules for both groups are given in Table 4-1. The control group schedule is a standard declining block rate with a fuel adjustment. The test customers paid a monthly service charge of $2.00 plus a usage charge of 16¢/kWh for peak hour usage, 3¢/kWh for high-use-hour usage, and 1¢/kWh for low-use-hour usage. The analysis in this paper is limited to the winter months of December 1975 and January and February 1976 and the summer months of June, July, and August 1976. No attempt has been made to compare pretest and test period usage within the test

*All nonfarm residential customers with 12-month billing histories were stratified into five groups according to their total usage during the previous year. Fifty prime test customers and 150 alternates were chosen randomly from each stratum. Forty control customers were chosen from the 150 alternates for each stratum.

Table 4-1

PRICES FOR TEST AND CONTROL GROUPS DURING WINTER AND SUMMER MONTHS OF PEAK LOAD PRICING TEST

TEST GROUP

Monthly Customer Charge: $2.00

Energy Charges:

Hours*	Rate	Winter Period	Summer Period
Peak hours (except Saturdays, Sundays, and Holidays)	$0.16/kWh	0900-1100 1700-1900	0900-1100 1200-1400
High use hours	$0.03/kWh	0800-2100 (except peak hours)	0700-2000 (except peak hours)
Low use hours	$0.01/kWh	All other hours	All other hours

CONTROL GROUP

Monthly Rate:**

Amount Consumed	Rate
First 12 kWh	$2.27
13-100 kWh	$0.0635/kWh
101-300 kWh	$0.0505/kWh
301-600 kWh	$0.0354/kWh
601+	$0.0324/kWh

*Eastern Standard Time

**Plus fuel adjustment as follows:

December 1975	-0.00023/kWh
January 1976	-0.00042/kWh
February 1976	-0.00067/kWh

group.* For reasons discussed in Section 5 of this report, our analysis is also restricted to customers who were metered continuously during one or both of the periods. A typical winter week was established for each customer by calculating the mean consumption for each 15-minute period for every weekday (weekends and holidays were excluded). Twelve observations were used for each day. A typical summer week was constructed in an analogous fashion. This reduced the control sample to 147 customers and the test sample to 176 customers for the winter analysis. The corresponding figures for the summer were 96 and 145, respectively.**

DEMOGRAPHIC INFORMATION

The demographic information for individual customers was obtained from several questionnaires. A short questionnaire was administered to both groups prior to the test period. Data were gathered on appliance ownership, type of heating and cooking, type of housing structure, square footage of the dwelling, age of the dwelling, and number of people in three different age groups residing in the household. A second, long questionnaire was also administered to the test group, in which many questions were asked about electricity usage. Data were also obtained on additional appliances, home ownership, income, occupation of the head of the household, race, and marital status. Because our method of analysis involved pooling the test and control groups, this information could not be used since it was available for the test group only.

After the experiment was completed, another questionnaire was administered to both the test and control groups. The major demographic characteristic obtained from this posttest survey was income level. Unfortunately, many customers were not contacted after the experiment, and many more did not respond to the income question. Since a sizable portion of the total sample did not have information on income, our subsequent analysis was done for two samples: the total sample excluding the income variable and the subsample with income data.+ It is

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*See reference (2) for several papers that do utilize this data.

**Considerably fewer customers were monitored during the summer months than during the winter. We do not know the exact reasons for this decline in sample size.

+The income subsample had 87 fewer customers during the winter than the full sample, a decline of more than 27%.

not clear whether the sample with income data is a biased sample of our original test and control groups.

All appliances except electric ranges, water heaters, and air conditioners were grouped into a single appliance index by summing the watts that each appliance would draw in one hour.*

The income variable was coded in broad categories on the survey questionnaires. The categories used also differed from one questionnaire to another. For example, the long questionnaire had six categories of which the highest was $20,000+, while the posttest control questionnaire had fourteen categories of which the highest was $70,000+. Since the categories were of unequal size, income was recoded to the midpoint for the non-open-ended categories. The test group categories of $20,000+ (long questionnaire) and $50,000+ (posttest questionnaire) were recoded to $31,975 and $60,000 respectively.** Income from the long questionnaire was used only when posttest income was not available.

WEATHER DATA

Weather conditions were monitored every 15 minutes at two sites. Weather contours were established for the entire state, and average deviations from site conditions were calculated for both summer and winter months. Each customer was then assigned to both a weather site and a weather contour based on location in the state.

Since our analysis is carried out for a typical week, average temperatures had to be used. The temperature at site 2 was arbitrarily assigned a value of zero. Each customer assigned to this site was given the value for the average (winter or summer) deviation from site conditions for his weather contour. Temperatures at site 1 averaged 4.78°C higher during the winter and 0.4°C higher in the summer than at site 2. Customers assigned to site 1 were given the value for the average (winter or summer) deviation from site conditions for their weather contour plus 4.78° or 0.4°.

*A listing of appliances and assigned wattages is given in Appendix 1.

**There were no respondents in the $70,000+ category in the control group.

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Section 5

EMPIRICAL RESULTS

INTRODUCTION

Application of the econometric theory given in Section 3 above to the problem at hand requires a multistep procedure. For convenience in presentation, we have chosen to outline this procedure under two general stages. State 1 involves the parameterization and estimation of the demand cycle for individual customers. Stage 2 uses the estimated parameters from stage 1, their estimated variance-covariance matrices, and demographic characteristics to estimate the impact of these characteristics on the demand cycle.

STAGE 1: SPECIFYING THE DEMAND CYCLE FOR INDIVIDUAL HOUSEHOLDS

We assume that the observed time-series on demand by the nth household is generated by a model of the form

$$q_n(t) = f_n(t) + e_n(t) , \tag{5-1}$$

where $f_n(t)$ denotes a strictly periodic, purely deterministic function of time and $e_n(t)$ denotes a hopefully relatively simple stationary error process. On the basis of the experience gained in our previous work (1,2), we believe that the periodic cubic spline provides an extremely attractive parametric representation for the functions $f_n(t)$. Leaving a more detailed discussion to Poirier (3,4), a cubic spline over the interval [0,1] can be defined as a continuous piecewise cubic polynomial with continuous first and second derivatives, and with a third derivative which, in general, is a step function. The abscissa values that serve to define the cubic segments and at which possible points of discontinuity in the third derivative occur are known as knots. A periodic cubic spline is simply a cubic spline in which the function and its first two derivatives at 0 equal the corresponding values at 1.

Splines have become a vital tool of modern numerical analysis. They have the visually attractive property that they appear to achieve what one might expect of a talented draftsman working with a French curve (5).

Cubic splines are flexible yet smooth. They arise naturally in variational problems of the form: Find the function f which minimizes the curvature functional,

$$C[f] = \int_0^1 (f''(t))^2 dt \qquad (5\text{-}2)$$

subject to the constraint

$$\sum_{i=1}^{M} (y(t_i) - f(t_i))^2 = Q \ , \qquad (5\text{-}3)$$

where f, f', and f" are continuous, f" is square integrable, and $y(t_i)$: $i=1,\ldots,M$ are observed input.* When the family of functions is further restricted to the class of periodic functions, the solution is a periodic cubic spline. Since on a priori grounds we believe that the daily cycle of a consumer's electricity demand under smooth pricing policies is a smooth periodic function, we have chosen to parametrize it by a periodic cubic spline.

As in the case of polynomials, spline functions can be parameterized in a variety of ways. For the sake of convenience as well as numerical stability, we have chosen to parameterize the periodic cubic spline as a linear function of the ordinate values at the knots.**

Thus, if W(t) is a periodic cubic spline with period P, it can be expressed as

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*For a more detailed discussion of this property and of its relevance in distributed lag models and optimal money growth models, see (3) and (4).

**For a more detailed discussion of the periodic cubic spline, see (4), pp. 43-47.

$$W(t) = \sum_{i=1}^{M} w_{it}\,\alpha_i \quad t=0,\ 1,\ 2,\ \ldots\ , \tag{5-4}$$

where $w_{it} = w_{j(t+pk)}$ constitute elements of a P x M matrix of fixed coefficients predetermined solely as a function of the period (P), the number of knots (M), and the knot locations.

A number of crucial problems of specification must be resolved prior to estimation. We now discuss several of these problems in sequence.

Specification of the Sample Period and Periodicity

The large amount of data from the CPLP experiment obviously allows many different specifications of Eq. 5-4, depending on the sample period and periodicity chosen. Our first step, therefore, was to make a number of specification experiments on a small sample of control customers using data for the month of January 1976.

The primitive data generated by the CPLP experiment is a unit-averaged time series on demand at 15-minute intervals for all control and test customers. At an early stage in our experimentation the decision was made to aggregate these quarter-hour series into hourly unit-averaged data. This temporal aggregation both smooths the series somewhat and reduces the (sizable) dimensionality of the problem. We believed that these benefits more than offset the associated loss of information.

Various additive models with daily periodicities superimposed upon weekly perioditities were tried on the January hourly data. These experiments clearly revealed a "weekend effect," but no significant differences in the level or shape of the weekday demand cycles emerged. The weekly periodicity was removed since it apparently contributed nothing. A limited amount of experimentation with temperature data was also made, but it too exerted an insignificant impact for most households beyond the influence on the daily cycle.

These preliminary tests led us to select as our sample period 12 consecutive weeks of winter data, excluding weekends and holidays, beginning in December and extending into March. A typical winter week was derived by calculating the mean consumption during each hour for the five weekdays. A typical summer week was derived in an analogous fashion using nine weeks of summer data beginning with the last week in June and extending through August.* We are thus left with an extremely simple model of the demand cycle specified as a periodic cubic spline with daily periodicity.

Selection of Knot Locations

In most of the applications discussed in (4) the motivation for using splines lies in the interpretation of the knots as points of structural change. In such cases the knot locations are of critical importance in testing for the equality of adjacent segments, and hence their selection must be grounded in some prior information (4, Chapter 8). However, in the application discussed here and in our previous work (1,2), precise knot locations are less important since the shape of the cubic spline itself is rather insensitive to the precise location of the knots. Although we feel that we have substantive justification for our choice of knot locations, we also feel that their selection was one of our less serious econometric problems.

Based on the discussion in Poirier (4, pp. 151-152) we made parsimonious selection of knots which attempted to isolate at most one inflection point or extremus (maximum or minimum) in each interval--the former appearing near the ends of the intervals and the latter in the center. Since we expected a relatively flat cycle from midnight to 6 A.M., the discussion contained in Poirier (4, Chapter 6) dealing with spline lags suggested the following nine knot locations for the winter sample: midnight, 4 A.M., 5:50 A.M., 7:30 A.M., 10 A.M., 1 P.M., 4 P.M., 8 P.M., and 10 P.M.** Preliminary testing based on a small, randomly selected number of control families confirmed the robustness of this choice of knot locations.

Pricing Period Dummies. If price is constant over the day then there is every reason to believe that the demand behavior of consumers will exhibit a smooth cycle. However, as we noted in section 2 of this report, jumps in price at

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*This period was chosen to avoid mixing time periods when school-age children would be in school and on vacation.

**The locations selected for the summer sample were midnight, 4 A.M., 6 A.M., 8 A.M., 10 A.M., 2 P.M., 5 P.M., 8 P.M., and 10 P.M.

certain instants of the day may induce discrete jumps in demand. To test for these effects it was necessary to introduce a set, $[D_i: i=1,\ldots,N]$, of indicator variables for the pricing periods of the experiment. These variables take values [0,1] at time t depending on whether t is in pricing period i or not. In order to nest the experimental effects hypotheses it was necessary to introduce these indicator variables for both control and treatment households. These pricing period dummies have a daily periodicity just like the spline design, so we may express the demand cycle model for individual households as

$$
\begin{aligned}
q_i &= [W \vdots D] \begin{bmatrix} \alpha_i \\ \cdots \\ \delta_i \end{bmatrix} + \bar{\varepsilon}_i \\
&= X\beta_i + \bar{\varepsilon}_i \quad . \qquad (5\text{-}5)
\end{aligned}
$$

Table 5-1 presents some fits for two control customers and two test customers. Figure 5-1 presents actual versus predicted consumption for weekdays for a single test customer. Figures 5-2 through 5-5 present predicted versus actual consumption for Monday and Tuesday for four randomly selected control customers. It may be noted that while the spline fits the typical day's demand cycle extremely well, in most cases there is considerable day-to-day noise in the original series.

Stochastic Specification

Since the correct generalized least squares (GLS) estimation of the stage 2 regressions utilizes both the estimated ordinates from stage 1 and their estimated variances and covariances, corrections for serial correlation and heteroscedasticity of the error term $\bar{\varepsilon}_j$ could have nontrivial consequences. It is important to remember that whatever cleaning up of $\bar{\varepsilon}_j$ must be performed in stage 1 must be performed for each individual separately, and hence up to 462 times. As a result, nonlinear identification and estimation procedures--specifically ARMA formulations for either $\bar{q}_j$ or $\bar{\varepsilon}_j$ (6)--are rather unattractive.

Serial Correlation. In our previous work (1,2) we found that residuals from an ordinary least squares fit of hourly data to a spline model with daily periodicity appeared to follow a simple second order autoregressive process which was extremely

Table 5-1

COEFFICIENTS FOR TWO CONTROL CUSTOMERS AND TWO TEST CUSTOMERS

Knots	Control *Low*	Control *High*	Test *Low*	Test *High*
0400	0.267 (0.065)	2.23 (0.107)	0.307 (0.038)	4.06 (0.101)
0530	0.340 (0.083)	2.29 (0.136)	0.342 (0.049)	3.96 (0.129)
0730	0.474 (0.134)	2.74 (0.219)	0.310 (0.078)	4.54 (0.208)
1000	0.391 (0.300)	2.70 (0.492)	-0.049 (0.176)	3.78 (0.467)
1300	0.401 (0.339)	2.35 (0.556)	0.466 (0.199)	3.15 (0.527)
1600	0.509 (0.341)	1.73 (0.559)	0.331 (0.200)	2.51 (0.530)
2000	1.47 (0.260)	3.70 (0.425)	0.978 (0.152)	3.30 (0.404)
2200	1.11 (0.090)	3.80 (0.146)	0.893 (0.053)	3.85 (0.139)
2400	0.548 (0.067)	3.00 (0.110)	0.607 (0.040)	3.99 (0.105)
Price Dummies				
0700-0900	-0.063 (0.207)	0.934 (0.339)	0.418 (0.121)	-0.790 (0.321)
09-1100	-0.110 (0.316)	0.594 (0.517)	0.417 (0.185)	-0.671 (0.490)
11-1700	-0.043 (0.335)	0.654 (0.549)	0.219 (0.196)	0.427 (0.520)
17-1900	0.229 (0.310)	1.30 (0.508)	0.236 (0.182)	-0.777 (0.482)
19-2100	-0.469 (0.220)	0.388 (0.361)	-0.192 (0.129)	0.294 (0.342)
R^2(single day)	0.967	0.975	0.965	0.973

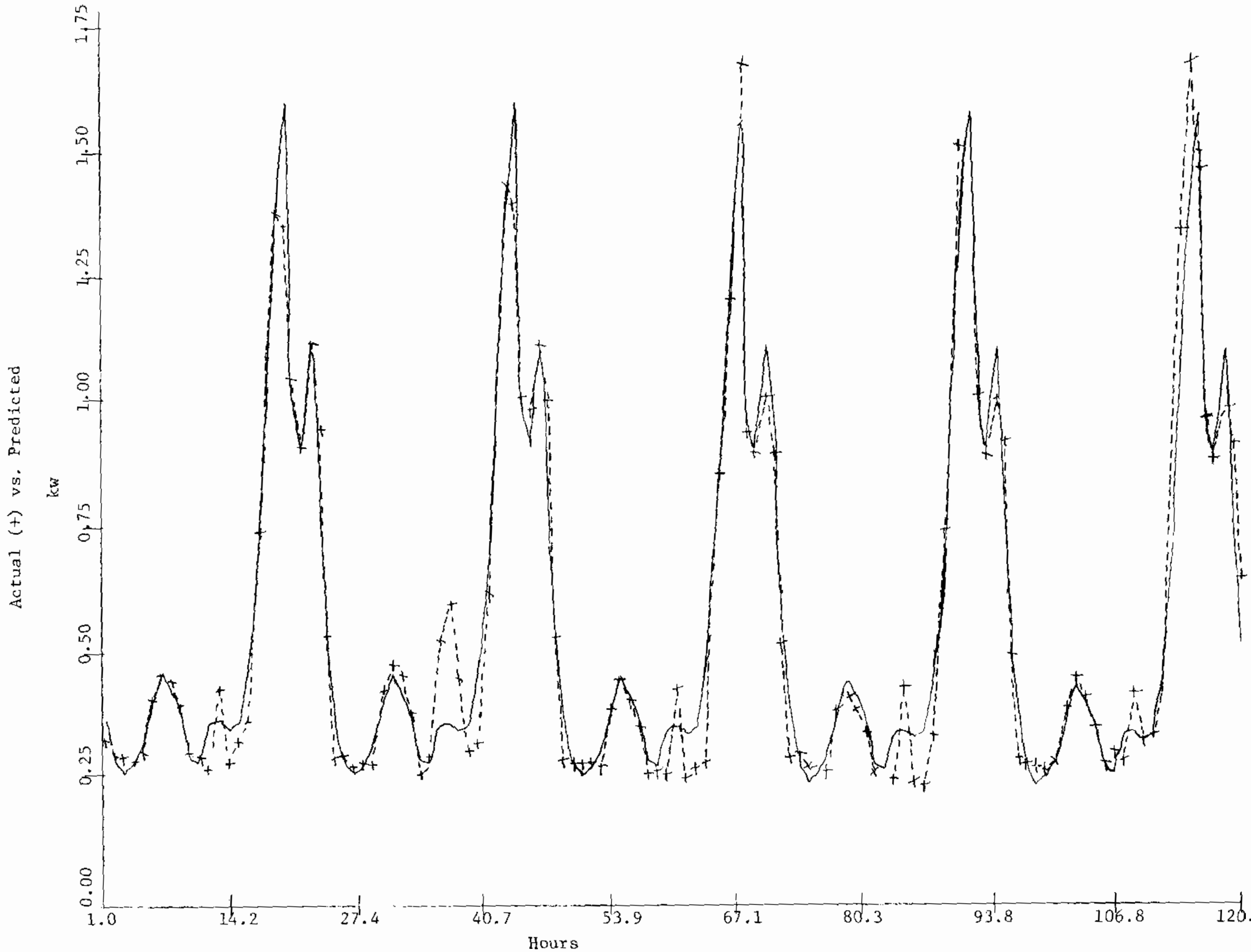


Figure 5-1. Control Individual 1 Winter Aggregate Weekdays

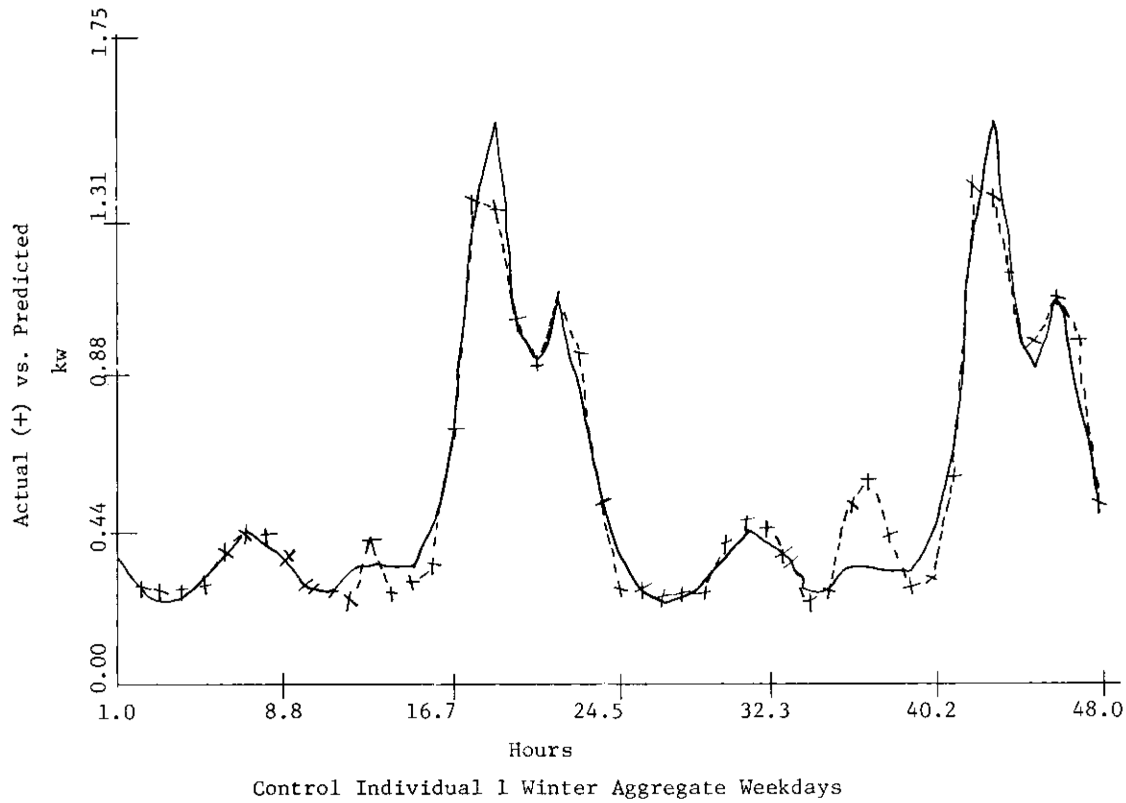


Figure 5-2. Predicted Versus Actual Consumption, Customer 1, Control Group

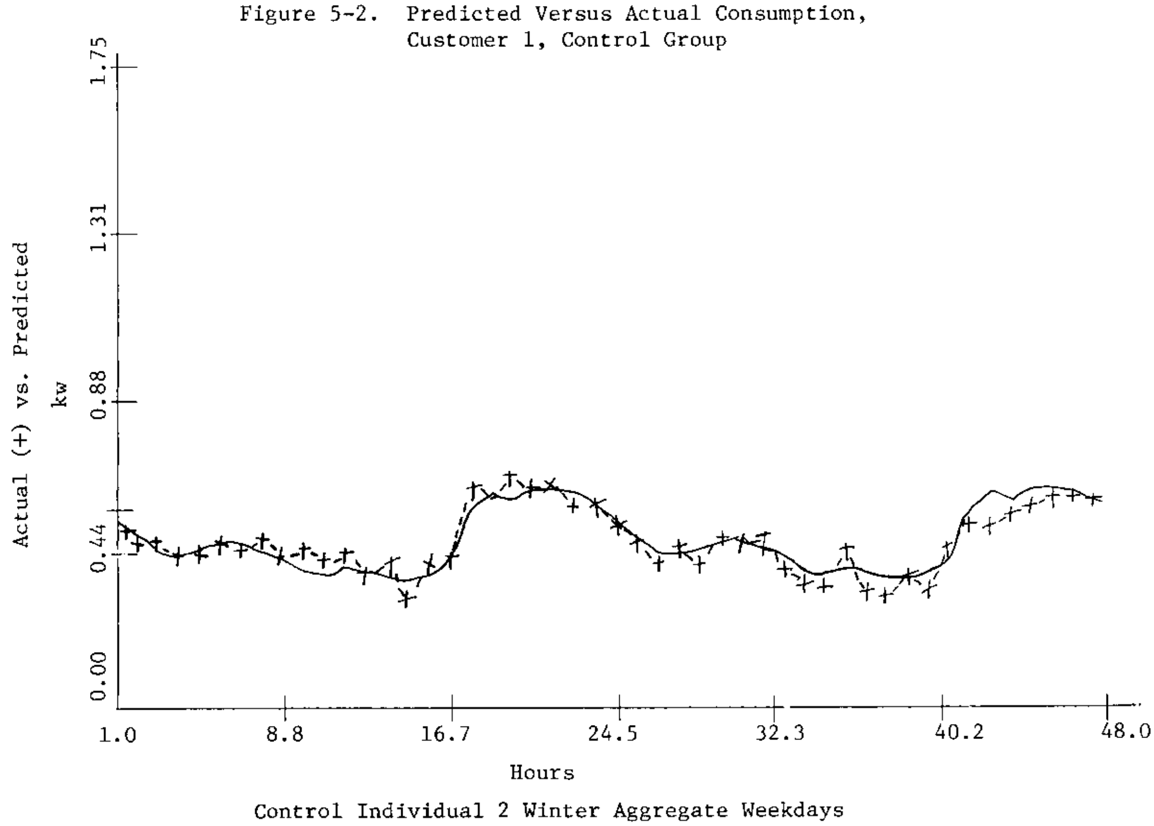
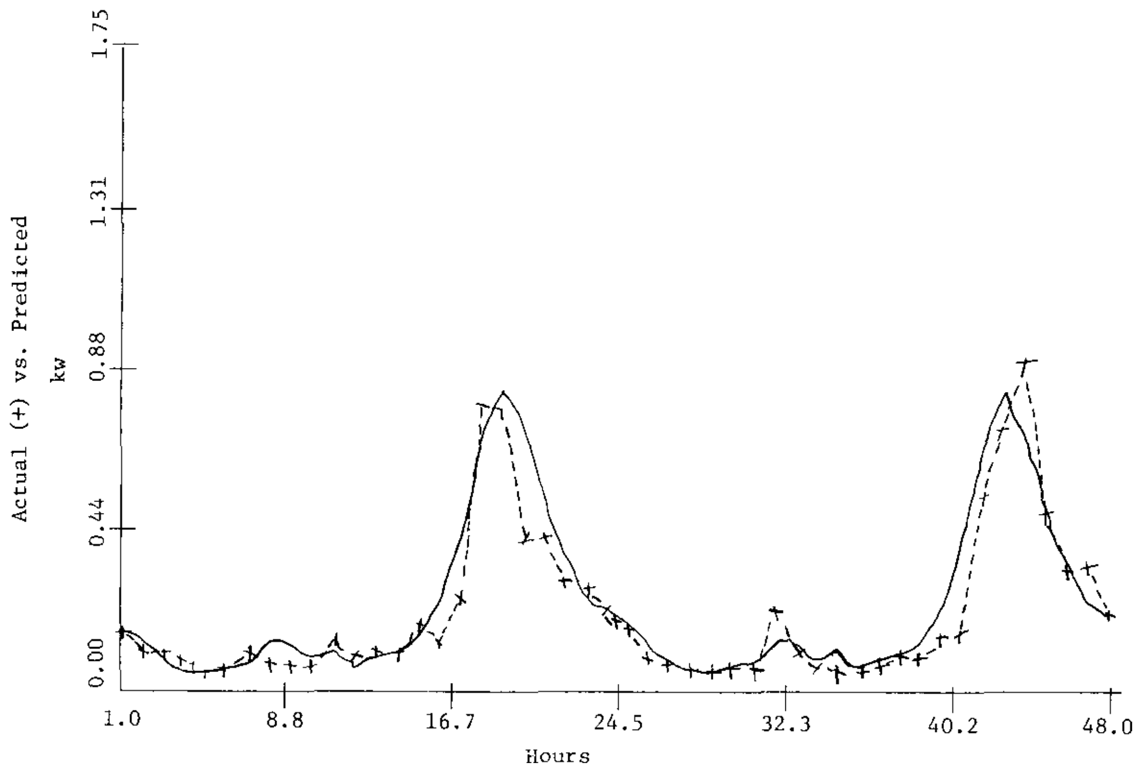
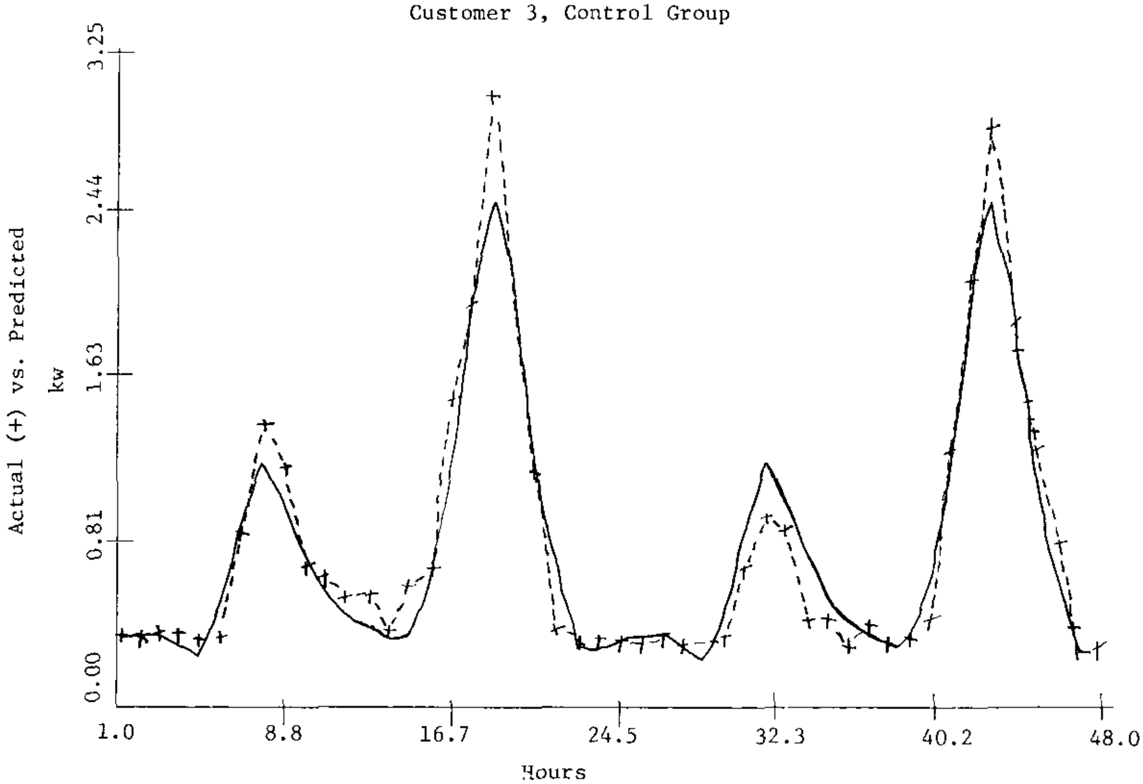


Figure 5-3. Predicted Versus Actual Consumption, Customer 2, Control Group



Control Individual 3 Winter Aggregate Weekdays

Figure 5-4. Predicted Versus Actual Consumption, Customer 3, Control Group



Control Individual 4 Winter Aggregate Weekdays

Figure 5-5. Predicted Versus Actual Consumption, Customer 4, Control Group consistent across observation units. Based on this previous work,* we assumed that the error term in Eq. 5-5 was generated by the mth-order autoregressive process

$$\bar{\varepsilon}_{tj} = \sum_{i=1}^{M} \rho_{ij}\bar{\varepsilon}_{t-i,j} + V_{tj} \qquad (5\text{-}6)$$

where the V_{tj}'s follow a white noise process (i.e., $V_{tj} \sim$ i.i.d. $N(0,\sigma^2,)$) and are independent across individuals. From a time series point of view, autoregression specification 5-6 together with Eq. 5-5 will be satisfactory if Eq. 5-5 can be transformed into

$$G_j\bar{q}_j = (G_jX)\ \beta_j + \tilde{\tilde{\varepsilon}}_j\ , \qquad (5\text{-}7)$$

where G_j is a (120 - m) x 120 transformation matrix and $\tilde{\tilde{\varepsilon}}_j = G_j\bar{\varepsilon}_j$ is a well-behaved disturbance term.

A convenient way to estimate the unknown parameters of an autoregressive process 5-6, and hence G_j, is to regress the residuals $\bar{\varepsilon}_{tj}$ (t = 1,2,...,120) obtained from applying ordinary least squares (OLS) to Eq. 5-5 on their lagged values $\bar{\varepsilon}_{t-1,j}$ (i = 1,2,...,m). This will yield OLS estimates $\hat{\rho}_{ij}$ (i = 1,2,...,m) which are consistent estimates of ρ_{kj} (i = 1,2,...,m).

Our previous work (1,2) and computations made by Poirier (7) using the 15-minute data suggested that the autoregressive process was about two periods. Estimation of Eq. 5-6 with m = 1,2,3,4, for a sample of customers suggested that m = 2 would do a fairly good job in approximating the error structure. While a costly maximum-likelihood approach might aid in identifying the order of the process, it would be prohibitive to apply to the entire sample. Instead, use of standard t-tests and F-tests on the "residual regressions" indicated that different second-order processes for each individual would work adequately. In all cases the estimated autoregressive process was checked and found to be stationary.

*See (7) for the results of some preliminary work with the 15-minute data.

Heteroscedasticity. Even before the data were examined, heteroscedasticity in the error term V_t in Eq. 5-6 was expected. The explanation is that deviations from the daily consumption cycle result, among other things, from the use of discretionary appliances. The use of such appliances is more frequent during some parts of the day than others. For example, during the late night when people are sleeping there is little opportunity for the use of appliances.

Such heteroscedasticity can be modeled by specifying that the variance of V_t varies smoothly with daily periodicity. Again, the periodic spline is the natural parameterization. The ordinates of such a spline can be consistently estimated by modifying the procedure suggested by Glejser (8). Specifically, the absolute value of the residuals from Eq. 5-7 were regressed on the spline regressors. Table 5-2 provides some example results from two control and two test customers for these heteroscedasticity regressions. In each case significant heteroscedasticity across the daily cycle is present. The squares of the predicted values from these heteroscedasticity regressions were then used as weights in a generalized least squares estimation of Eq. 5-7.*

Table 5-3 gives the results for each step of the three-step process in stage 1 for two control and two test customers.

STAGE 2: DEPENDENCE OF THE HOUSEHOLD DEMAND CYCLE ON DEMOGRAPHIC CHARACTERISTICS

The final stage of our empirical work was to estimate the coefficients obtained in our first stage regressions for individual customers as functions of the customer's demographic characteristics.

Of obvious interest is the impact of the indicator variable "on-or-off-the-experiment" on demand behavior by the time of day. Unfortunately, the single peak load pricing "treatment" of the CPLP experiment severely limits the scope of data analysis of price effects. We are placed in the uncomfortable position of estimating a system of demand functions while having only two distinct

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*For a small number of customers the smoothing process yielded very small negative predicted values during the late night. In these cases, the weights were bounded away from zero by substituting the minimum value that existed in the sample for the predicted value.

Table 5-2

HETEROSCEDASTICITY REGRESSIONS FOR TWO CONTROL CUSTOMERS AND TWO TEST CUSTOMERS

	Control Low	Control High	Test Low	Test High
0400	0.005 (0.021)	0.232 (0.166)	0.018 (0.026)	0.335 (0.072)
0530	0.030 (0.025)	0.233 (0.198)	0.037 (0.032)	0.436 (0.087)
0730	0.006 (0.023)	0.070 (0.185)	0.026 (0.030)	0.357 (0.081)
1000	0.029 (0.021)	0.793 (0.173)	0.064 (0.028)	0.319 (0.076)
1300	0.068 (0.021)	1.342 (0.167)	0.055 (0.027)	0.305 (0.073)
1600	0.138 (0.020)	0.574 (0.161)	0.182 (0.026)	0.251 (0.070)
2000	0.091 (0.020)	0.360 (0.159)	0.167 (0.026)	0.239 (0.069)
2200	0.042 (0.023)	0.515 (0.184)	0.011 (0.030)	0.217 (0.080)
2400	0.081 (0.022)	0.528 (0.176)	0.084 (0.028)	0.284 (0.077)

Table 5-3

STAGE 1 REGRESSION COEFFICIENTS FOR TWO CONTROL CUSTOMERS AND TWO TEST CUSTOMERS (WINTER)

	Control Low Use			Control High Use		
	(1)	(2)	(3)	(1)	(2)	(3)
0400	2.267	0.259	0.278	2.23	2.22	2.29
	(0.030)	(0.031)	(0.003)	(0.197)	(0.208)	(0.076)
0530	0.340	0.344	0.332	2.28	2.27	2.21
	(0.038)	(0.039)	(0.014)	(0.251)	(0.232)	(0.066)
0730	0.474	0.463	0.464	2.74	3.09	2.90
	(0.062)	(0.066)	(0.014)	(0.405)	(0.604)	(0.092)
1000	0.391	0.378	0.359	2.69	2.95	2.55
	(0.139)	(0.139)	(0.030)	(0.909)	(1.03)	(0.575)
1300	0.401	0.430	0.471	2.35	2.40	2.84
	(0.157)	(0.161)	(0.097)	(1.03)	(1.16)	(0.960)
1600	0.508	0.534	0.553	1.73	1.78	2.39
	(0.158)	(0.164)	(0.101)	(0.103)	(1.17)	(0.894)
2000	1.47	1.44	1.28	3.69	3.89	4.13
	(0.121)	(0.124)	(0.123)	(0.786)	(0.927)	(0.699)
2200	1.11	1.10	1.07	3.80	3.89	3.98
	(0.042)	(0.045)	(0.028)	(0.271)	(0.437)	(0.322)
2400	0.548	0.567	0.556	3.00	3.04	2.93
	(0.031)	(0.033)	(0.023)	(0.204)	(0.215)	(0.140)
Price Dummies						
07-0900	-0.063	-0.047	-0.042	0.934	0.373	0.860
	(0.096)	(0.102)	(0.020)	(0.627)	(1.09)	(0.215)
09-1100	-0.110	-0.099	-0.088	0.593	0.470	0.796
	(0.146)	(0.150)	(0.039)	(0.955)	(1.26)	(0.907)
11-1700	-0.043	-0.067	-0.092	0.654	0.583	-0.005
	(0.155)	(0.160)	(0.091)	(1.01)	(1.19)	(0.936)
17-1900	0.229	0.264	0.387	1.30	1.25	0.866
	(0.144)	(0.147)	(0.137)	(0.939)	(1.20)	(0.868)
19-2100	-0.469	-0.456	-0.328	0.388	0.033	-0.135
	(0.102)	(0.107)	(0.094)	(0.666)	(1.04)	(0.756)
ρ_1	---	0.0985	0.0985	---	0.797	0.797
ρ_2	---	-0.0921	-0.0921	---	-0.0527	-0.0527

Table 5-3 (continued)

	Test Low Use (1)	Test Low Use (2)	Test Low Use (3)	Test High Use (1)	Test High Use (2)	Test High Use (3)
0400	0.307 (0.037)	0.297 (0.040)	0.323 (0.011)	4.06 (0.093)	4.01 (0.097)	4.02 (0.105)
0530	0.342 (0.047)	0.344 (0.047)	0.336 (0.019)	3.96 (0.118)	3.96 (0.114)	4.01 (0.155)
0730	0.310 (0.076)	0.343 (0.092)	0.298 (0.033)	4.53 (0.192)	4.68 (0.246)	4.72 (0.287)
1000	-0.049 (0.170)	-0.029 (0.198)	-0.235 (0.111)	3.78 (0.431)	3.92 (0.613)	4.03 (0.632)
1300	0.466 (0.193)	0.465 (0.227)	0.183 (0.178)	3.15 (0.487)	3.20 (0.713)	3.15 (0.614)
1600	0.331 (0.194)	0.342 (0.226)	0.022 (0.175)	2.52 (0.489)	2.61 (0.694)	2.59 (0.628)
2000	0.978 (0.148)	1.00 (0.166)	0.694 (0.158)	3.30 (0.370)	3.46 (0.467)	3.38 (0.370)
2200	0.893 (0.051)	0.896 (0.065)	0.910 (0.029)	3.85 (0.129)	3.80 (0.168)	3.78 (0.136)
2400	0.607 (0.038)	0.622 (0.042)	0.633 (0.032)	3.99 (0.968)	4.05 (0.105)	4.08 (0.980)
07-0900	0.418 (0.118)	0.382 (0.153)	0.489 (0.061)	-0.790 (0.297)	-1.09 (0.434)	-1.19 (0.485)
09-1100	0.417 (0.179)	0.406 (0.217)	0.633 (0.131)	-0.672 (0.453)	-0.690 (0.651)	-0.793 (0.664)
11-1700	0.219 (0.190)	0.216 (0.227)	0.509 (0.173)	0.472 (0.480)	0.370 (0.706)	0.404 (0.640)
17-1900	0.236 (0.176)	0.224 (0.214)	0.761 (0.263)	-0.777 (0.445)	-0.992 (0.653)	-0.933 (0.550)
19-2100	0.192 (0.125)	-0.237 (0.156)	-0.184 (0.134)	0.294 (0.316)	0.111 (0.426)	0.202 (0.335)
ρ_1	---	0.329	0.329	---	0.388	0.388
ρ_2	---	0.0881	0.0881	---	0.398	0.398

observations on the vector of relevant prices. Thus, while we unfortunately cannot hope to identify and estimate own- and cross-price effects by time of day, we believe we can nevertheless present some useful quantitative evidence on the net impact of the peak load pricing treatment on the level and shape of residential demand cycles conditional on appliance ownership and a number of other important factors.

Following the notation of section 3 of this report, let Z denote a J-vector of household characteristics variables: stock of appliances of various sorts, physical characteristics of a residence, income, and perhaps other things as well. Let the $N \times (M + L)$ matrix, B, be the matrix of demand cycle parameters for a sample of N households with M knot locations and $(L + 1)$ pricing periods. A row, B, of the matrix defines the demand cycle for the nth household of the sample. Finally, suppose the B's are linear, or approximately linear, in the Z's, i.e.,

$$B = Z\Gamma + U \,, \tag{5-8}$$

where Z is a $N \times J$ matrix of household characteristics, Γ is a matrix of fixed coefficients, and U is a potentially stochastic matrix which may be assumed null under certain circumstances. Unfortunately B is not observed but must be estimated. When Γ is estimated with some $\hat{B}$ substituted for B, the stochastic structure of this second-stage system of equations depends upon both the stochastic structure of the individual household demand cycles and the structure of U.

Since there is little reason to believe that disturbances in the first stage for different individuals are correlated, the variance-covariance matrix, Σ, in the first stage is assumed to be block diagonal. With this assumption and unknown covariance structure, an efficient estimator of the metaparameters Γ was derived in section 3 as

$$\hat{\gamma}(\hat{\Sigma},\hat{\Omega}) = [\Xi'((X'\Sigma^{-1}X)^{-1} + \Omega)^{-1}\Xi]^{-1}\Xi'((X'\Sigma^{-1}X)^{-1} + \Omega)^{-1}\hat{\beta} \tag{5-9}$$

where the coefficients $(\hat{\beta})$ and covariance matrices $([X'\hat{\Sigma}^{-1}X]^{-1})$ were estimated in the first stage.

To find $\hat{\Omega}$ it is first necessary to find the residuals ($\hat{V}$) from the OLS regressions

$$\hat{\beta} = Z\hat{\Gamma} + \hat{V} .$$

From Eq. 3-29 a consistent estimator $\hat{\Omega}$ for Ω is given by

$$\hat{\Omega} = \frac{\hat{V}'\hat{V}-\hat{\Delta}}{N-P}$$

where $\hat{\Delta}$ is defined in Eq. 3-27.

Equation 5-8 constitutes a series of (M + L) equations--one for each variable in our first-stage regressions. In principle, the number of demographic variables which could be used in this step is constrained only by the sample size and the availability of data. It should be noted, however, that the design of Eq. 5-9 requires that the (M + L) equations be estimated as a system once $\hat{\Omega}$ is known. Therefore, the number of explanatory variables in the full design 5-9 is equal to (M + L) x J, where J is the number of demographics. If the number of demographics is too large, inversion of the design matrix in Eq. 5-9 is either impossible or prohibitively expensive.

Our stage 1 treatments of the summer and winter data were identical except for slight changes in knot locations and pricing period dummies. Our stage 2 treatments, however, are considerably different. They are discussed under separate headings below.

<u>Empirical Results for the Winter "Typical" Week</u>

A number of combinations of demographic variables were tried as alternative specifications for Eq. 5-8 for the winter data. As was noted in section 4, these equations were estimated both for the full sample (314 customers) excluding income and for the subsample with income data (259 customers). Our final specification of Eq. 5-8 for the full sample can be written out more fully as

$$\beta_n^i = \alpha^i + \sum_{j=1}^{9} \alpha_j^i Z_{jn}^i + V_n^i \qquad i = 1 \text{ to } 14 , \qquad (5\text{-}10)$$

where β_n^i = the coefficient on the ith variable from the first-stage regression for individual n (i = 1 to 9 are the knot coefficients and i = 10 to 14 are the price period dummy coefficients)

Z_{1n}^i = dummy variable for electric range

Z_{2n}^i = dummy variable for electric water heater

Z_{3n}^i = stock of appliances in watts*

Z_{4n}^i = dummy variable for electric heat

Z_{5n}^i = square footage of the dwelling

Z_{6n}^i = average temperature difference**

Z_{7n}^i = dummy variable for test customer

Z_{8n}^i = number of people in the household

Z_{9n}^i = square footage times electric heat ($Z_{4j}^i \times Z_{5j}^i$).

The average temperature difference variable (Z_{6n}^i) was not significant at any time of the day. This may reflect the substantial errors in measurement of this variable+ or the fact that small temperature differences have only a marginal

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*A description of the method used to calculate the stock of appliance variables and a listing of the wattages for each appliance are given in Appendix 1.

**The temperature at site 2 was arbitrarily assigned a value of zero. Each customer assigned to this site was given the value for the average winter deviation from site conditions for his weather contour. Temperatures at site 1 averaged 4.78°C higher than at site 2. Each customer assigned to this site was given the value for the average winter deviation from site conditions for his weather contour plus 4.78°.

+The temperature difference variable is an average over the entire day and is only an approximation for each customer.

effect once the type of heating and quantity of heating required (partially captured by the square footage variable) are controlled. In any case, for the subsample with income data the temperature difference variable was dropped and an income variable was added.

The OLS regression results for each of the 14 stage 1 coefficients are presented in Table 5-4 for the full sample and in Table 5-5 for the income subsample. The inclusion of income has little effect on the results for other variables in the regressions. We are therefore reasonably confident in our results for the full sample.

The $\hat{\Omega}$'s calculated from the residuals for these regressions and the variance-covariance matrices from stage 1 were not positive definite. In each case, one of the eigenvalues was negative, although small. To insure that the sum $(\hat{\Omega} + X'\hat{\Sigma}X)$ was positive definite, $\hat{\Omega}$ was transformed into a positive semi-definite matrix by substituting zero for the negative eigenvalue. Letting E be the matrix of eigenvectors and D be the diagonal matrix of eigenvalues such that

$$\hat{\Omega}E = ED \qquad (5\text{-}11)$$

and letting $\tilde{D}$ be the diagonal matrix of eigenvalues such that $\tilde{D}_i = \max\{0, D_i\}$, we computed a positive semi-definite matrix $\tilde{\hat{\Omega}}$ as

$$\tilde{\hat{\Omega}} = E\tilde{D}E' \ . \qquad (5\text{-}12)$$

The final GLS regression results for the system of stage 1 coefficients are presented in Table 5-6 for the full sample and in Table 5-7 for the income subsample.

For most customers the pricing period dummies were insignificant in the first stage regressions. Joint F-tests on all the dummy coefficients in the second stage except the 07-0900 dummy indicated that the null hypothesis that all the coefficients were zero could not be rejected at any reasonable significance level.* Thus, there

*The F-test for the 07-0900 dummies was 2.69. None of the other sets of dummy variables had F-tests greater than 1.41.

Table 5-4

STAGE 2 OLS RESULTS FOR FULL SAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.082 (0.102)	0.070 (0.103)	0.074 (0.152)	0.104 (0.168)	0.171 (0.164)	0.127 (0.156)	0.140 (0.167)
Water heater	0.345 (0.094)	0.404 (0.095)	0.791 (0.140)	0.624 (0.155)	0.662 (0.151)	0.530 (0.144)	0.948 (0.154)
Appliances[a]	4.218 (1.780)	4.559 (1.797)	8.204 (2.644)	10.80 (2.918)	7.366 (2.857)	11.27 (2.718)	11.51 (2.901)
Electric heat	1.761 (0.260)	1.847 (0.263)	1.864 (0.387)	2.198 (0.427)	1.934 (0.418)	1.903 (0.398)	1.137 (0.424)
Square footage[a]	4.192 (3.753)	3.329 (3.790)	4.286 (5.575)	-0.651 (6.152)	-0.100 (6.025)	-0.089 (5.730)	4.811 (6.116)
Temperature	-0.033 (0.046)	-0.048 (0.046)	-0.102 (0.068)	-0.031 (0.075)	-0.110 (0.074)	-0.067 (0.070)	-0.029 (0.074)
Test	0.006 (0.082)	0.010 (0.082)	-0.053 (0.121)	-0.369 (0.134)	-0.459 (0.131)	-0.215 (0.125)	-0.587 (0.133)
People	0.035 (0.026)	0.034 (0.026)	0.077 (0.035)	0.083 (0.042)	0.140 (0.042)	0.146 (0.040)	0.217 (0.042)
Square footage x heat[b]	1.203 (0.155)	1.214 (0.156)	1.373 (0.230)	0.953 (0.253)	0.249 (0.232)	0.583 (0.236)	1.435 (0.252)
R^2 =	0.845	0.852	0.777	0.705	0.623	0.635	0.722
N	314	314	314	314	314	314	314

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-4 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.104 (0.152)	0.030 (0.148)	0.160 (0.127)	-0.156 (0.145)	0.082 (0.133)	0.128 (0.132)	-0.062 (0.096)
Water heater	1.063 (0.140)	0.713 (0.136)	0.257 (0.117)	0.212 (0.134)	0.132 (0.121)	0.069 (0.122)	0.005 (0.089)
Appliances[a]	13.05 (2.642)	9.486 (2.566)	-1.330 (2.211)	-3.342 (2.523)	-3.751 (2.302)	-1.510 (2.307)	-1.497 (1.673)
Electric heat	0.635 (0.387)	1.567 (0.376)	-0.998 (0.324)	-1.022 (0.369)	-1.138 (0.337)	-1.124 (0.338)	-0.456 (0.245)
Square footage[a]	7.317 (5.570)	4.907 (5.411)	4.179 (4.662)	5.901 (5.321)	4.332 (4.854)	4.539 (4.864)	2.862 (3.527)
Temperature	0.055 (0.068)	0.052 (0.066)	0.152 (0.057)	0.085 (0.065)	0.107 (0.059)	0.105 (0.060)	0.039 (0.043)
Test	0.262 (0.121)	0.404 (0.118)	0.328 (0.101)	-0.039 (0.116)	0.161 (0.106)	0.043 (0.106)	0.199 (0.077)
People	0.204 (0.038)	0.169 (0.037)	0.035 (0.032)	0.009 (0.037)	-0.009 (0.036)	-0.018 (0.034)	-0.027 (0.024)
Square footage x heat[b]	1.853 (0.230)	1.240 (0.223)	0.632 (0.192)	0.365 (0.220)	0.756 (0.200)	0.577 (0.201)	0.227 (0.146)
R^2	0.778	0.752	0.112	0.053	0.077	0.054	0.049
N	314	314	314	314	314	314	314

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-5

STAGE 2 OLS RESULTS FOR INCOME SUBSAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.169 (0.111)	0.164 (0.113)	0.127 (0.173)	0.199 (0.195)	0.291 (0.188)	0.259 (0.179)	0.201 (0.182)
Water heater	0.289 (0.101)	0.367 (0.102)	0.729 (0.157)	0.598 (0.176)	0.598 (0.171)	0.492 (0.162)	0.803 (0.165)
Appliances[a]	2.899 (1.845)	3.565 (1.875)	6.737 (2.871)	9.437 (3.226)	5.030 (3.119)	9.827 (2.963)	8.161 (3.021)
Electric heat	1.030 (0.315)	1.014 (0.320)	0.894 (0.490)	1.914 (0.551)	1.816 (0.532)	2.064 (0.506)	1.591 (0.516)
Square footage[a]	2.642 (3.840)	2.261 (3.901)	1.782 (5.975)	-2.671 (6.713)	-3.950 (6.491)	-3.388 (6.166)	0.038 (6.286)
Income[a]	0.379 (0.369)	0.182 (0.375)	1.065 (0.574)	0.477 (0.645)	0.462 (0.623)	0.255 (0.592)	2.037 (0.604)
Test	0.050 (0.089)	0.033 (0.090)	-0.033 (0.139)	-0.399 (0.156)	-0.476 (0.151)	-0.228 (0.143)	-0.488 (0.145)
People	0.024 (0.027)	0.023 (0.027)	0.050 (0.041)	0.064 (0.046)	0.151 (0.045)	0.149 (0.043)	0.197 (0.044)
Square footage x heat[b]	1.774 (0.208)	1.859 (0.212)	2.169 (0.324)	1.158 (0.364)	0.668 (0.352)	0.433 (0.334)	1.099 (0.341)
R^2	0.851	0.857	0.776	0.682	0.607	0.610	0.705
N	259	259	259	259	259	259	259

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[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-5 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.160	0.085	0.141	-0.069	-0.010	0.118	-0.048
	(0.162)	(0.161)	(0.151)	(0.172)	(0.153)	(0.159)	(0.114)
Water heater	0.951	0.609	0.211	0.185	0.110	0.083	-0.019
	(0.147)	(0.146)	(0.136)	(0.156)	(0.139)	(0.144)	(0.104)
Appliances[a]	10.73	7.801	-2.086	-4.184	-3.751	-1.438	-1.845
	(2.688)	(2.662)	(2.495)	(2.859)	(2.544)	(2.640)	(1.893)
Electric heat	0.255	0.918	-1.075	-0.992	-1.461	-1.459	-0.865
	(0.459)	(0.454)	(0.426)	(0.488)	(0.434)	(0.451)	(0.323)
Square footage[a]	3.030	1.248	2.959	5.934	5.907	5.241	3.546
	(5.593)	(5.540)	(5.193)	(5.950)	(5.293)	(5.494)	(3.940)
Income[a]	2.099	1.569	0.762	0.672	0.523	0.247	-0.230
	(0.537)	(0.532)	(0.499)	(0.571)	(0.508)	(0.528)	(0.378)
Test	0.344	0.450	0.304	-0.009	0.187	0.016	0.196
	(0.130)	(0.128)	(0.120)	(0.138)	(0.123)	(0.127)	(0.091)
People	0.182	0.161	0.026	0.010	-0.024	-0.017	-0.022
	(0.039)	(0.038)	(0.036)	(0.041)	(0.037)	(0.038)	(0.027)
Square footage x heat[b]	2.244	1.860	0.749	0.364	1.085	0.883	0.604
	(0.303)	(0.301)	(0.282)	(0.323)	(0.287)	(0.298)	(0.214)
R^2	0.792	0.772	0.098	0.057	0.097	0.056	0.067
N	259	259	259	259	259	259	259

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-6

STAGE 2 GLS RESULTS FOR FULL SAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.031 (0.034)	0.050 (0.112)	0.095 (0.119)	0.020 (0.111)	0.060 (0.096)	-0.024 (0.116)	0.011 (0.225)
Water heater	0.287 (0.033)	0.313 (0.106)	0.582 (0.119)	0.551 (0.109)	0.566 (0.106)	0.335 (0.126)	0.895 (0.217)
Appliances[a]	3.808 (0.595)	3.712 (1.929)	5.202 (2.060)	7.318 (1.900)	3.700 (1.595)	6.549 (1.953)	5.351 (3.863)
Electric heat	1.681 (0.130)	1.626 (0.326)	1.385 (0.401)	2.079 (0.355)	1.245 (0.369)	1.353 (0.456)	0.437 (0.631)
Square footage[a]	3.865 (1.199)	4.150 (4.191)	9.594 (4.908)	5.513 (4.136)	8.084 (3.767)	8.122 (4.814)	17.44 (8.475)
Temperature	0.010 (0.161)	0.018 (0.050)	0.024 (0.054)	-0.009 (0.050)	-0.067 (0.043)	0.002 (0.052)	-0.033 (0.100)
Test	-0.002 (0.028)	-0.019 (0.090)	-0.050 (0.100)	-0.406 (0.090)	-0.440 (0.082)	-0.135 (0.099)	-0.706 (0.181)
People	0.035 (0.009)	0.034 (0.028)	0.048 (0.031)	0.035 (0.027)	0.041 (0.023)	0.053 (0.029)	0.111 (0.055)
Square footage x heat[b]	1.118 (0.083)	1.271 (0.199)	1.716 (0.252)	1.130 (0.222)	1.055 (0.229)	1.037 (0.282)	1.931 (0.380)

N = 4396

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-6 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.101	-0.056	0.083	0.039	0.033	0.152	-0.067
	(0.159)	(0.425)	(0.168)	(0.186)	(0.226)	(0.278)	(0.299)
Water heater	0.803	0.521	0.145	0.124	0.256	0.142	0.010
	(0.154)	(0.404)	(0.156)	(0.176)	(0.213)	(0.268)	(0.286)
Appliances	8.393	1.066	-1.257	-2.066	-0.145	3.568	1.370
	(2.790)	(7.282)	(2.962)	(3.280)	(3.940)	(4.902)	(5.226)
Electric heat	-0.191	0.520	-1.211	-1.016	0.213	-0.290	0.408
	(0.499)	(1.150)	(0.522)	(0.655)	(0.796)	(0.945)	(1.006)
Square footage	11.04	23.23	1.203	2.959	3.958	-8.600	-7.849
	(5.949)	(16.70)	(6.045)	(6.564)	(8.047)	(10.14)	(10.65)
Temperature	0.159	0.155	0.098	0.098	0.022	0.093	0.081
	(0.073)	(0.186)	(0.078)	(0.086)	(0.104)	(0.130)	(0.140)
Test	0.283	0.243	0.442	0.198	0.095	0.147	0.299
	(0.131)	(0.339)	(0.136)	(0.153)	(0.186)	(0.230)	(0.246)
People	0.154	0.067	0.057	0.062	0.078	0.112	0.047
	(0.041)	(0.105)	(0.043)	(0.048)	(0.058)	(0.073)	(0.078)
Square footage x heat	2.383	1.926	0.725	0.171	-0.081	-0.159	-0.400
	(0.313)	(0.691)	(0.331)	(0.445)	(0.528)	(0.616)	(0.636)

Table 5-7

STAGE 2 GLS RESULTS FOR INCOME SUBSAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.077 (0.037)	0.136 (0.110)	0.133 (0.130)	0.125 (0.129)	0.191 (0.131)	0.052 (0.172)	0.054 (0.272)
Water heater	0.257 (0.036)	0.287 (0.105)	0.541 (0.129)	0.515 (0.119)	0.538 (0.124)	0.305 (0.172)	0.739 (0.251)
Appliances[a]	2.576 (0.639)	2.381 (1.782)	4.091 (2.119)	7.199 (2.133)	3.697 (2.150)	6.144 (2.750)	4.027 (4.410)
Electric heat	0.840 (0.173)	0.798 (0.402)	0.630 (0.509)	1.896 (0.423)	1.350 (0.433)	1.578 (0.634)	1.007 (0.806)
Square footage[a]	2.982 (1.288)	6.390 (3.976)	8.502 (5.211)	-0.672 (4.635)	-1.997 (4.956)	2.627 (7.218)	4.304 (10.13)
Income[a]	0.397 (0.134)	-0.063 (0.384)	0.551 (0.471)	0.367 (0.435)	0.297 (0.446)	-0.098 (0.620)	1.690 (0.898)
Test	0.018 (0.032)	-0.037 (0.092)	-0.087 (0.111)	-0.395 (0.104)	-0.444 (0.107)	-0.164 (0.143)	-0.556 (0.216)
People	0.032 (0.009)	0.041 (0.025)	0.056 (0.030)	0.063 (0.031)	0.110 (0.031)	0.106 (0.041)	0.156 (0.063)
Square footage x heat[b]	1.797 (0.125)	1.950 (0.279)	2.404 (0.357)	1.269 (0.295)	0.992 (0.298)	0.873 (0.435)	1.560 (0.539)

N = 3626

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-7 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.032	-0.047	0.048	0.014	0.093	0.184	0.015
	(0.173)	(0.439)	(0.192)	(0.227)	(0.251)	(0.339)	(0.362)
Water heater	0.744	0.552	0.108	0.114	0.193	0.114	-0.033
	(0.166)	(0.405)	(0.176)	(0.211)	(0.235)	(0.319)	(0.340)
Appliances	6.789	1.928	-2.066	-3.833	-1.664	1.634	-0.102
	(2.891)	(7.079)	(3.273)	(3.882)	(4.215)	(5.748)	(6.150)
Electric heat	-0.415	0.352	-1.366	-1.312	-0.631	-1.074	-0.298
	(0.627)	(1.291)	(0.688)	(0.903)	(1.025)	(1.276)	(1.361)
Square footage	6.427	0.147	2.272	10.08	10.46	0.591	-1.853
	(6.314)	(0.163)	(6.542)	(7.752)	(8.627)	(12.18)	(12.38)
Income	1.809	1.061	0.609	0.128	-0.130	0.184	-0.188
	(0.610)	(1.460)	(0.651)	(0.784)	(0.861)	(1.159)	(1.244)
Test	0.350	0.219	0.500	0.207	0.159	0.156	0.304
	(0.144)	(0.348)	(0.157)	(0.192)	(0.212)	(0.280)	(0.299)
People	0.151	0.151	0.042	0.047	0.057	0.066	0.034
	(0.042)	(0.100)	(0.047)	(0.055)	(0.060)	(0.083)	(0.090)
Square footage x heat	2.754	2.286	0.876	0.406	0.587	0.475	0.142
	(0.438)	(0.862)	(0.494)	(0.676)	(0.750)	(0.919)	(0.963)

was no evidence except during the early morning period of any discrete jumps in the demand cycle for the typical customer. The 0700-0900 pricing period includes the last hour (0700-0800) of the low price period (1¢/kWh) and one hour (0800-0900) of high prices (3¢/kWh). The results for this period indicate that test customers used significantly larger amounts of electricity than control customers--probably to avoid the peak prices (16¢/kWh) beginning at 0900.

Given these results for the pricing period dummies, our discussion of the impact of demographics on the demand cycle focuses on the estimated effects on the stage 1 knot coefficients. Table 5-8 presents the joint F-tests for each set of demographic coefficients. Electric water heating has a significantly* positive effect on consumption throughout the daily cycle, with a peak in the late evening. The stock of electrical appliances is also significantly positive throughout the day with peak usage occurring in the late morning, early evening, and late evening. The strongest single impact on consumption comes from electric heating. This is reflected in the combination of the electric heating and square-footage-times-electric-heating variables. This impact is significantly positive throughout the day, although it appears to be slightly lower during the afternoon hours.

Table 5-8

JOINT F-TESTS FOR DEMOGRAPHIC VARIABLES IN STAGE 2

Variable	F value
Electric range	0.99
Water heater	23.16
Appliances	15.22
Electric heat	16.01
Square footage	2.23
Temperature	0.92
Test	5.20
People	7.36
Square footage x heat	26.88

*The term significant is used throughout this section to describe coefficients which were at least twice as great as estimated standard errors.

The results for the remainder of the demographic characteristics are much more uneven. Neither the temperature variable nor the electric range variable is significant at any time of the day. The problems with the temperature variable were discussed above. We have no explanation for the insignificance of the electric range variable other than the fact that 85.6% of the sample owned electric ranges. The performance of the square footage variable suggests that the primary impact of house size on consumption is to determine the amount of electric heat that is necessary. It is significant for four of the nine knots in the full sample, but only for one of the nine knots in the income subsample. In accord with our a priori expectations, the estimated impact of the number of people in the household on consumption is lowest during the late night hours and highest during the evening hours, with a peak occurring around 2100 (9:00 P.M.).

The results for the test customer dummy variable are quite interesting. The dummy is insignificant during the hours after midnight, significantly negative throughout the day and early evening (except at the 4:00 P.M. knot), and significantly positive at 10:00 P.M. The insignificant results for the very late night hours were expected, since test customers did not have the means to store heat to use during high- or peak-use pricing periods. The increased usage after midnight under peak load pricing in Europe can be attributed primarily to heat storage. Our results suggest that the impact of peak load pricing was to shift consumption from the day and evening to the late evening and early morning hours.

As we've indicated in our previous work (1, p. 136), the partial elasticity of consumption with respect to income is not unambiguously positive in the long run. At higher income levels, consumers may choose to own more electrical appliances and consume more electricity. However, they might also choose a lower utilization rate for a given stock of appliances. If the elasticity of demand for electricity-consuming goods is positive, the simple elasticity of demand with respect to income would be positive, but the partial elasticity would remain ambiguous. Unfortunately, the results for the income subsample do not help resolve this ambiguity. The income variable in the full model is significant only at the 4:00 A.M. and 10:00 P.M. knots.

To estimate the simple elasticity of demand with respect to income, Eq. 5-10 was reestimated for the income subsample excluding all demographics except income, number of people, and the test dummy. The OLS results for this new specification are given in Table 5-9. In contrast to our previous results (1), the null hypothesis that the impact of income on consumption is zero can only be rejected during the evening and early morning hours. The resulting simple income elasticities (evaluated at the sample means) are also very low (see Table 5-10). While it is quite possible that consumption is inelastic with respect to income at the relatively high income levels in the CPLP experiment samples, we are hesitant to put much faith in these results. There are three reasons for this hesitancy. First, the large number of customers with no income information suggests that the sample was truncated to exclude customers in the lower income groups.* Second, the questionnaires administered to the control and test groups had different income ranges. Third, the results for the test group indicated higher income elasticities than those for the control group.**

Figures 5-6 through 5-10 give plots of the estimated impact of appliances, electric heat,+ number of people, water heat, and test status on electricity consumption over the day for a typical customer. In each case, the results for the full sample were used along with the assumption that all the price dummy coefficients were zero except during the 0700-0900 time period. These plots were obtained by using hypothetical values for demographic characteristics in the stage 2 regressions to estimate the knot coefficients and price dummy coefficient.++ These coefficients were then substituted into the first-stage regression equation to yield predicted consumption. Ninety-five percent confidence intervals were calculated for each point in time by noting that the forecast

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*Customers with low incomes were probably more reluctant to answer the income question. Although the mean values for most demographics were the same for the control and test groups, the control mean income was significantly higher than the test mean income.

**These results were obtained by estimating the elasticities separately for the two groups.

+The electric heat effect is the sum of the electric heat and electric heat-square footage coefficients. The mean value of square footage was used for this calculation.

++The mean value for the sample (3.8) was used for the number of people.

Table 5-9

OLS RESULTS FOR "PURE INCOME" MODEL

	0400	0600	0800	1000	1400	1700	2000
Income[a]	1.238 (0.869)	1.108 (0.903)	2.308 (1.098)	1.358 (1.039)	1.060 (0.894)	0.903 (0.854)	3.002 (0.975)
Test	0.137 (0.224)	0.124 (0.233)	0.073 (0.284)	-0.345 (0.268)	-0.436 (0.230)	-.215 (0.221)	-0.435 (0.252)
People	-0.008 (0.064)	-0.011 (0.067)	0.008 (0.081)	0.032 (0.077)	0.109 (0.066)	0.129 (0.063)	0.159 (0.072)

	2200	2400	0700-0900	0900-1100	1100-1700	1700-1900	1900-2100
Income[a]	3.671 (1.030)	2.678 (0.986)	1.190 (0.467)	0.833 (0.535)	0.952 (0.482)	0.751 (0.494)	-0.028 (0.354)
Test	0.438 (0.266)	0.537 (0.255)	0.335 (0.121)	0.013 (0.138)	0.238 (0.125)	0.044 (0.127)	0.223 (0.091)
People	0.156 (0.076)	0.130 (0.073)	0.021 (0.034)	0.002 (0.039)	-0.029 (0.035)	-0.008 (0.036)	-0.020 (0.026)

[a]Coefficients and standard errors are multiplied by 10^5.

Table 5-10

ESTIMATED INCOME ELASTICITIES BY
TIME OF DAY, INCOME SUBSAMPLE

Time	Simple Elasticity
0400	0.177
0600	0.151
0800	0.318
1000	0.243
1400	0.192
1700	0.101
2000	0.254
2200	0.282
2400	0.254

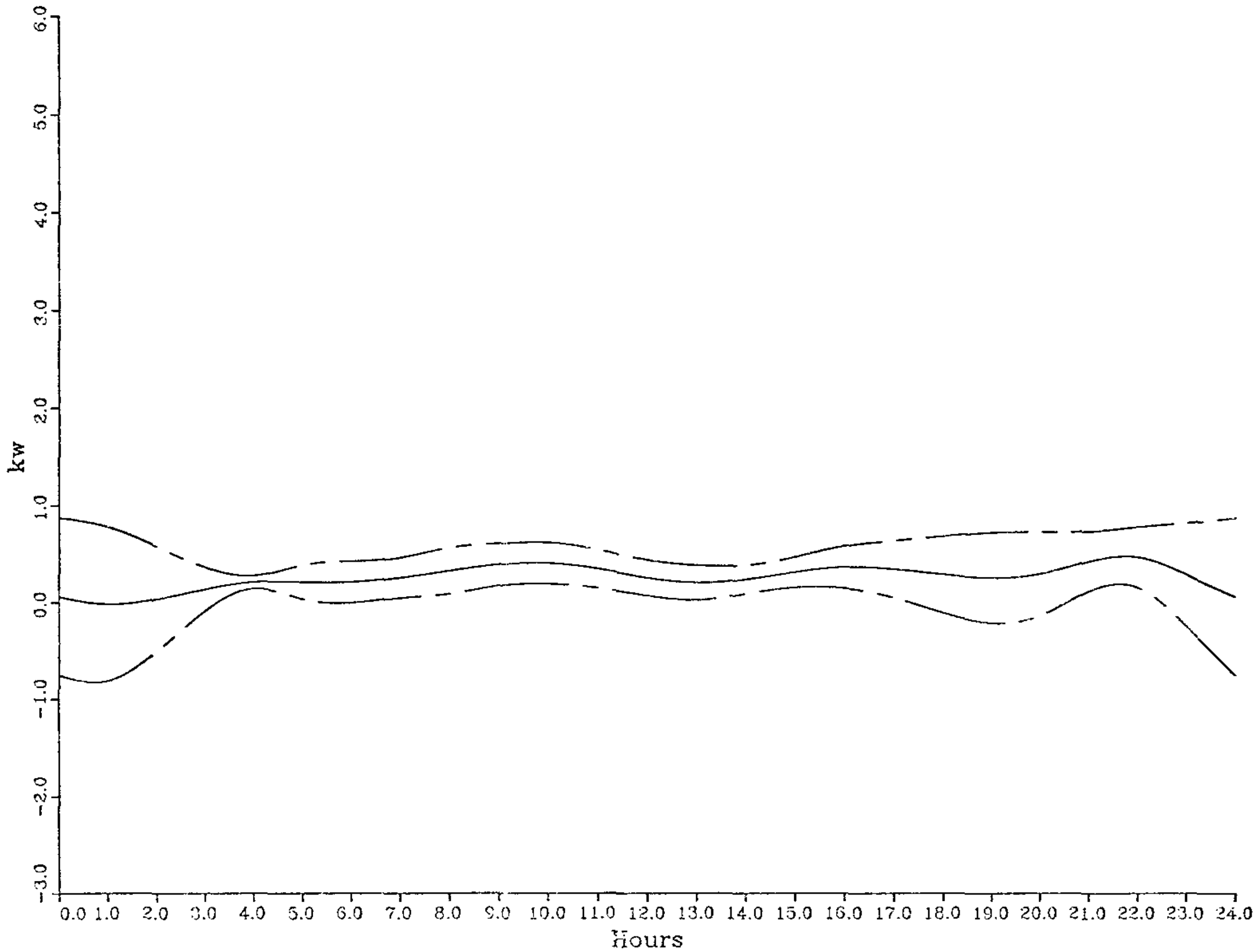

Figure 5-6. Decomposition of the Daily Demand Cycle: Appliance Wattage Effect

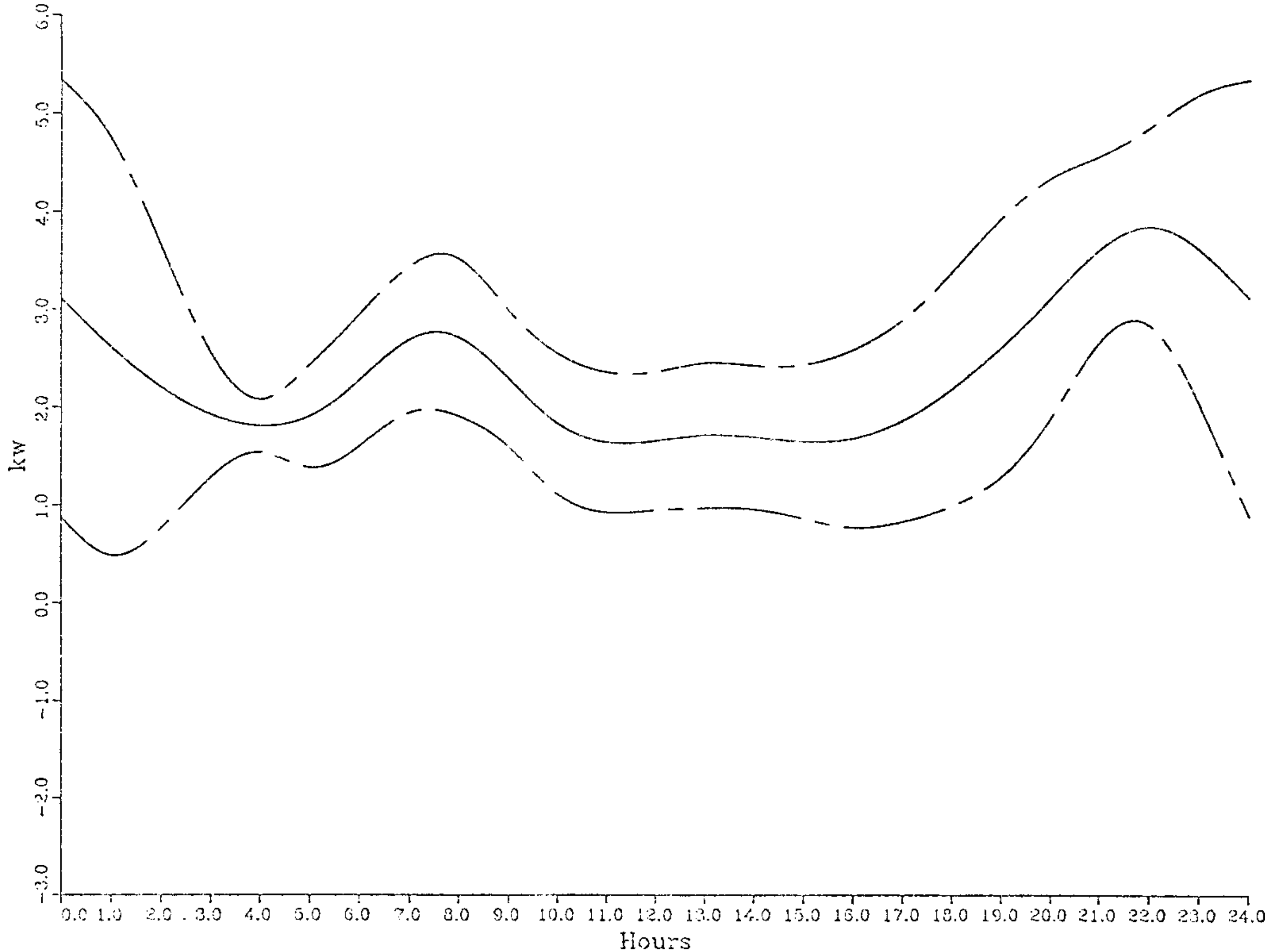

Figure 5-7. Decomposition of the Daily Demand Cycle: Electric Heat-Floorspace Interaction Effect

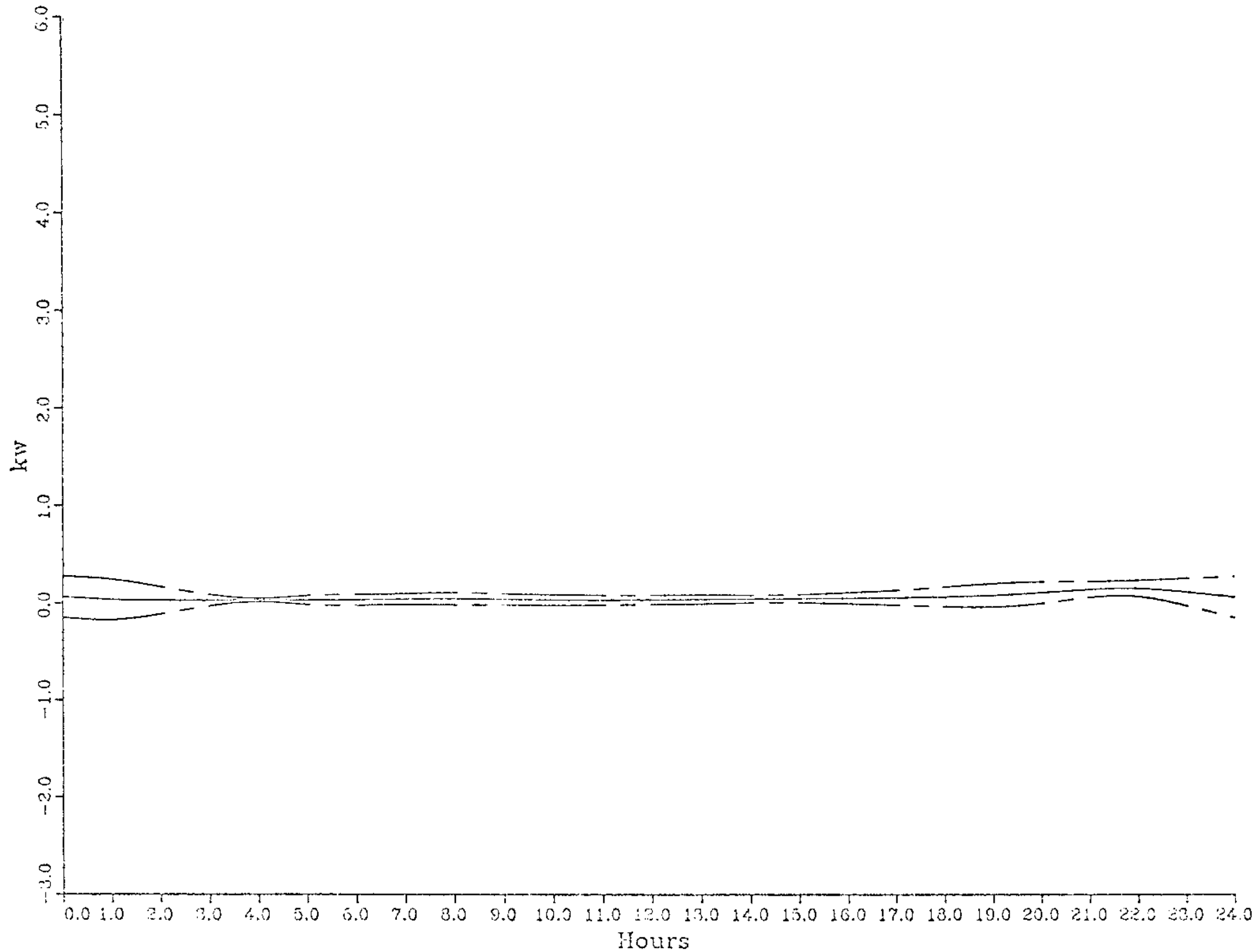

Figure 5-8. Decomposition of the Daily Demand Cycle: Family Size Effect

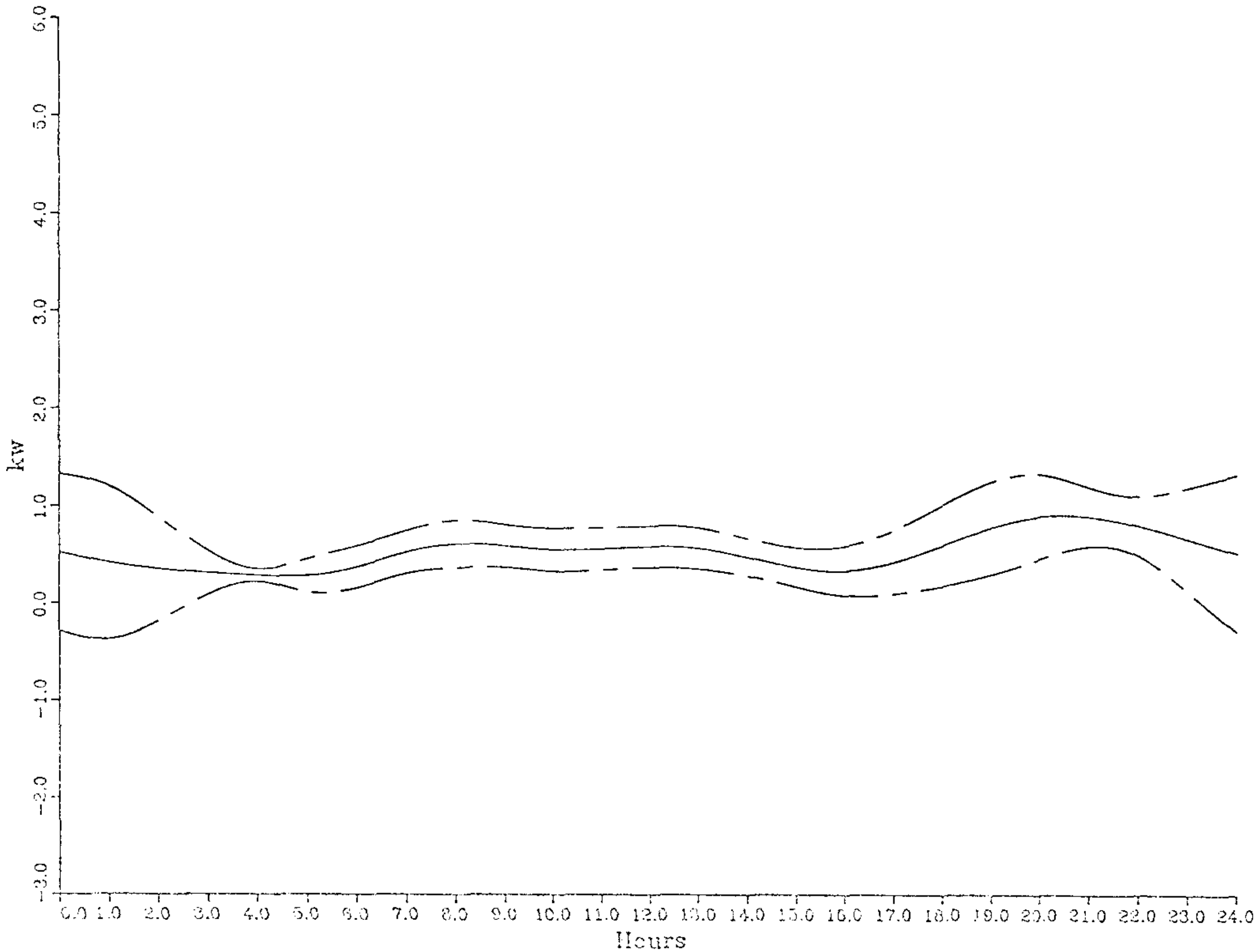

Figure 5-9. Decomposition of the Daily Demand Cycle: Electric Water Heater Effect

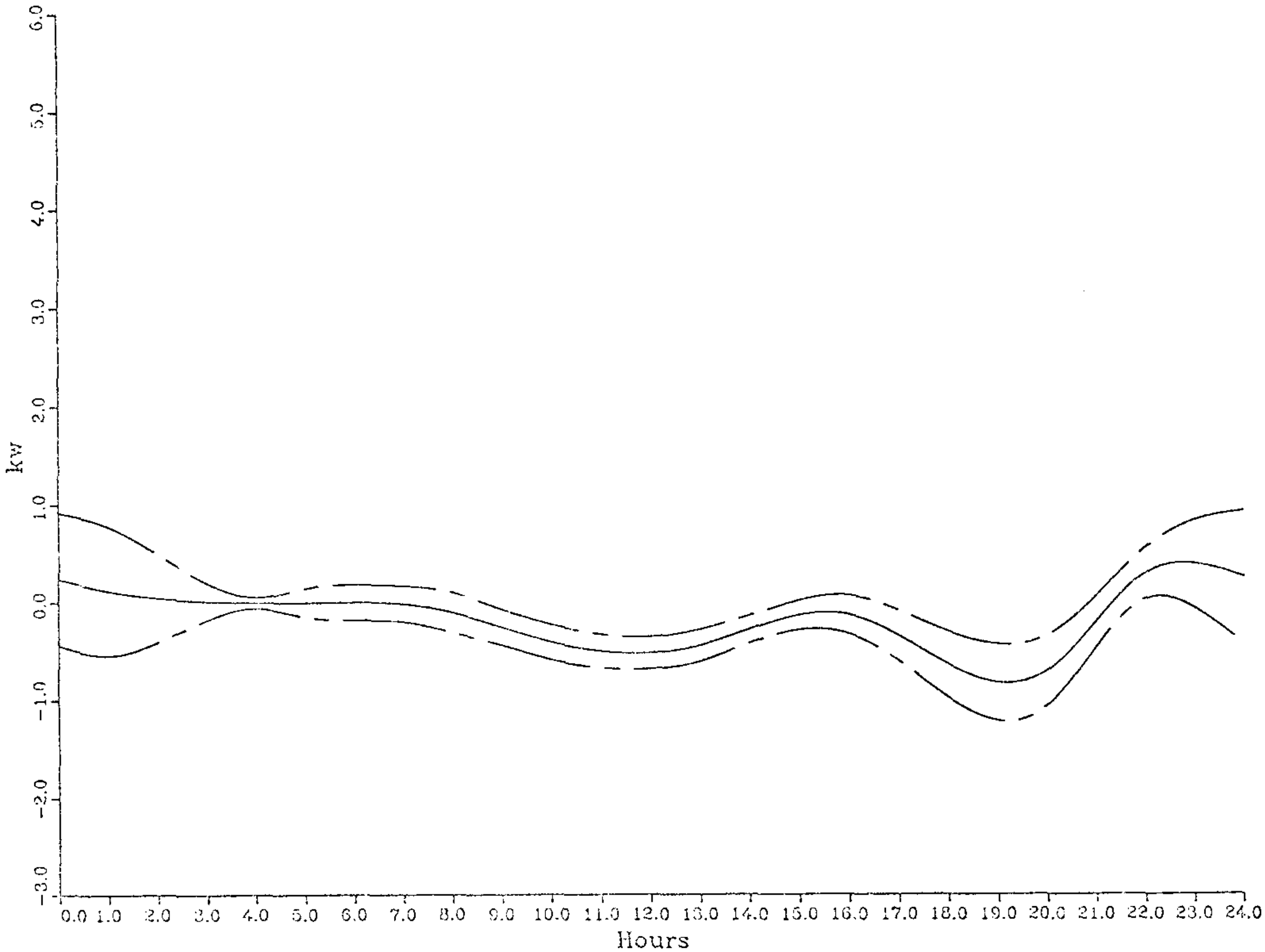

Figure 5-10. Decomposition of the Daily Demand Cycle: Test Effect

$$\hat{y}_i(t) = \alpha_i(t)\hat{\gamma}$$

has forecast variance

$$\hat{\sigma}_i^2(t) = \alpha_i'(t)\Phi\alpha_i(t)$$

where $\alpha_i(t)$ is a vector of the form

$$\alpha_i(t) = w(t) \otimes Z_i$$

and

$$\phi = [\Xi'((X'\Sigma^{-1}X) + \Omega)^{-1}\Xi]^{-1}$$

as derived in section 3.

Figures 5-11 through 5-16 plot estimated consumption for three typical users under the same assumptions. Sample averages were used for square footage (1615 square feet), temperature differences (3.03°), appliance stock (5601 watts), and people (3.765). The typical customer was assumed to own an electric range. "High use" refers to customers with electric heat and electric water heating. "Moderate use" refers to customers with electric water heating. "Low use" refers to customers with neither electric water heating or electric heat.

During the night, the typical test customer's consumption is about the same as the typical control customer's consumption. There is a sizable jump in test consumption in the early morning and a drop during the peak hours from 9 A.M. to 11 A.M. Control usage, however, undergoes a relatively smooth climb to a peak at about noon. Test consumption remains lower than control consumption until the late afternoon. At that time both consumption paths begin a climb to the evening peak. However, control consumption peaks several hours earlier than test consumption, is higher than test consumption throughout the early evening, and is lower than test consumption in the late evening.

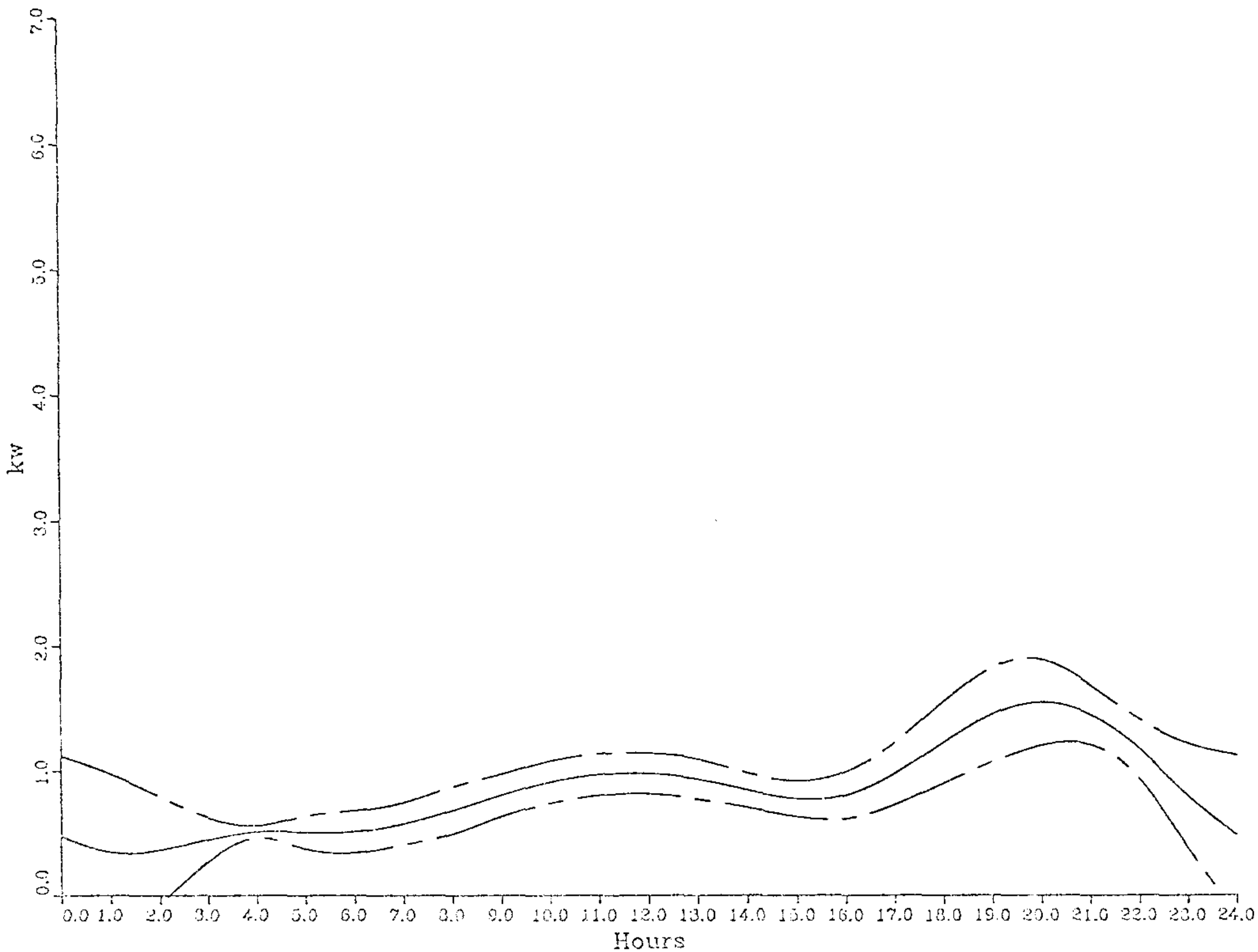

Figure 5-11. Control Customer Daily Demand Cycle: Without Electric Water Heater or Electric Heat

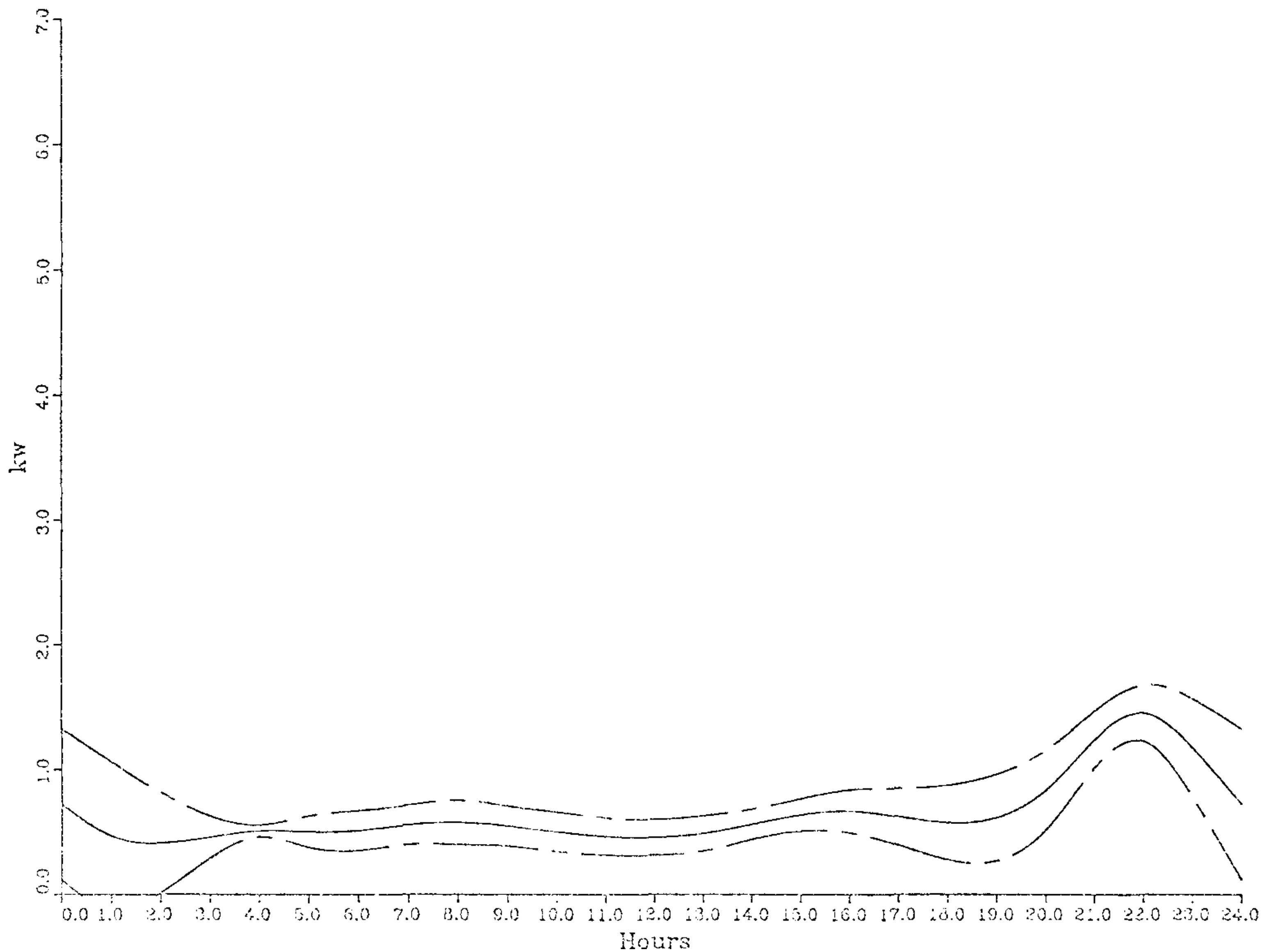

Figure 5-12. Test Customer Daily Demand Cycle: Without Electric Water Heater or Electric Heat

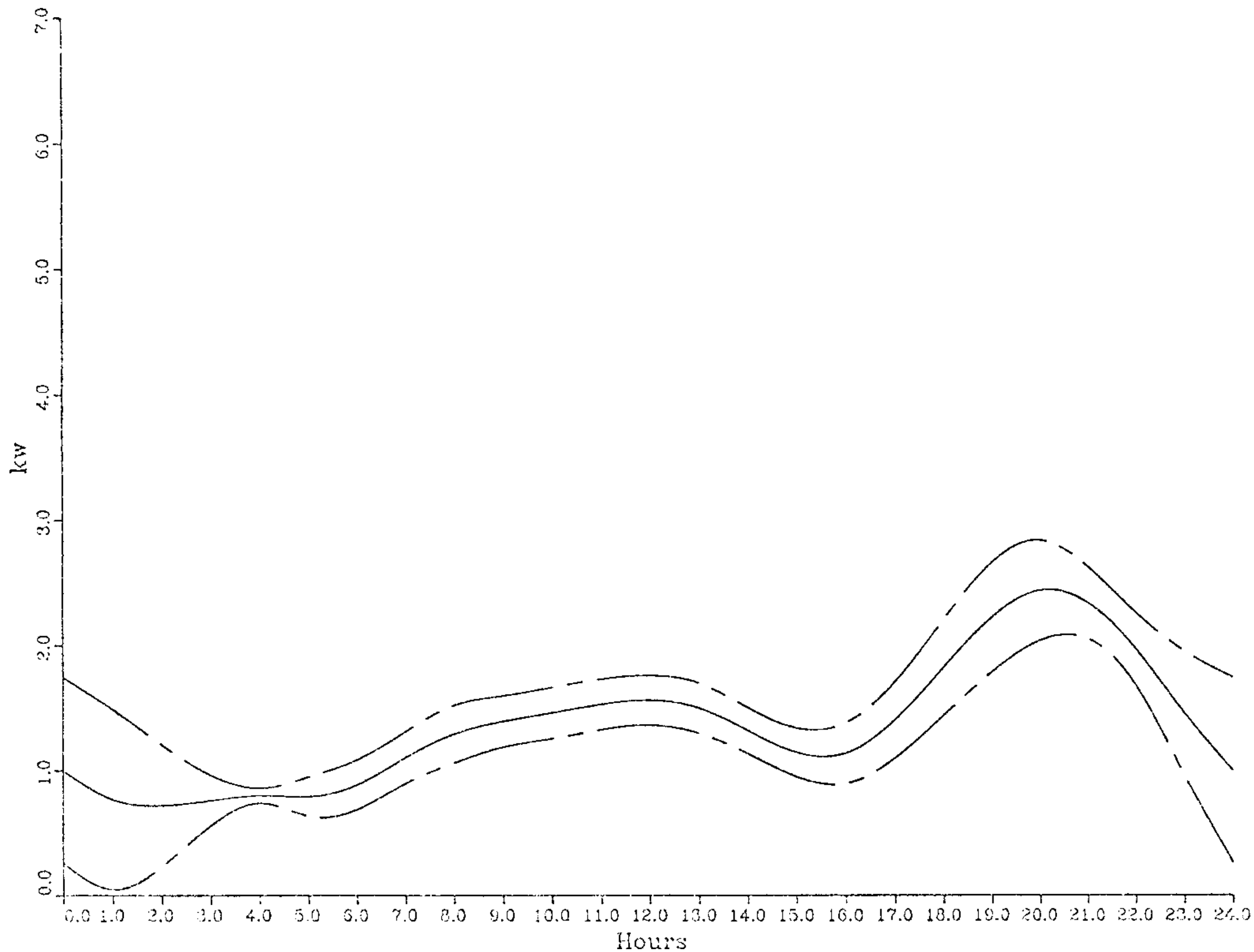

Figure 5-13. Control Customer Daily Demand Cycle: Without Electric Water Heater without Electric Heat

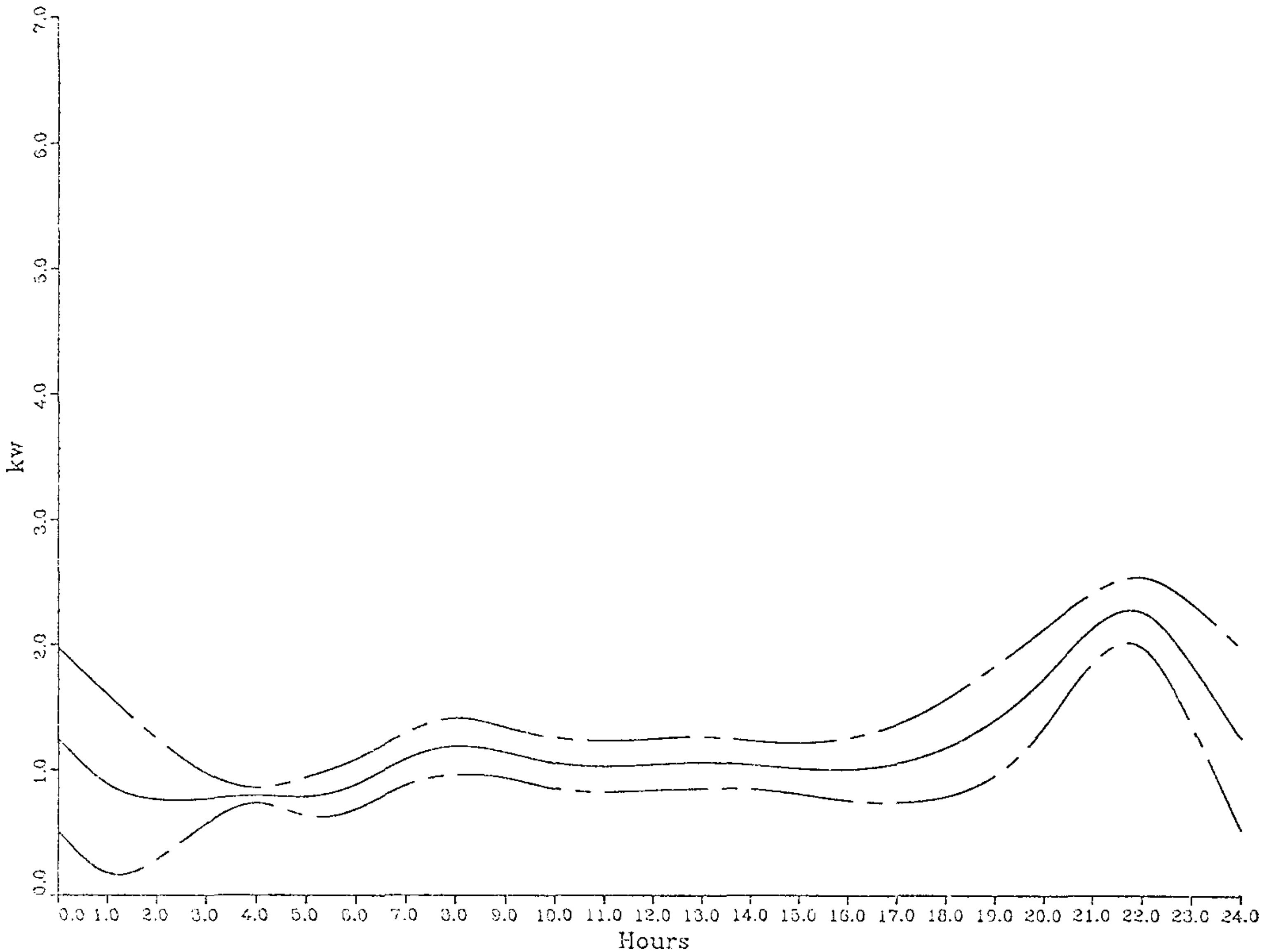

Figure 5-14. Test Customer Daily Demand Cycle: With Electric Water Heater without Electric Heat

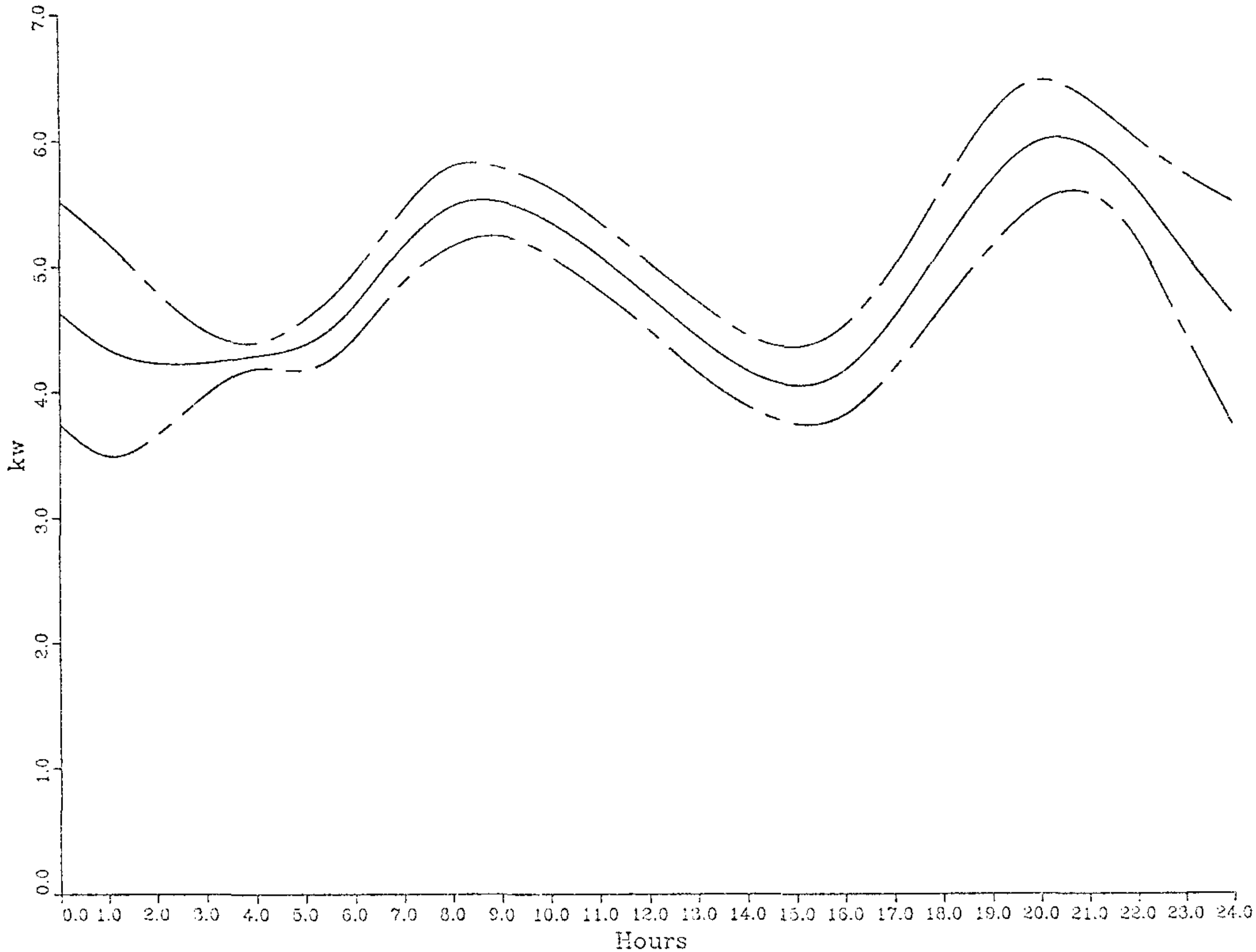

Figure 5-15. Control Customer Daily Demand Cycle: With Electric Water Heater and Electric Heat

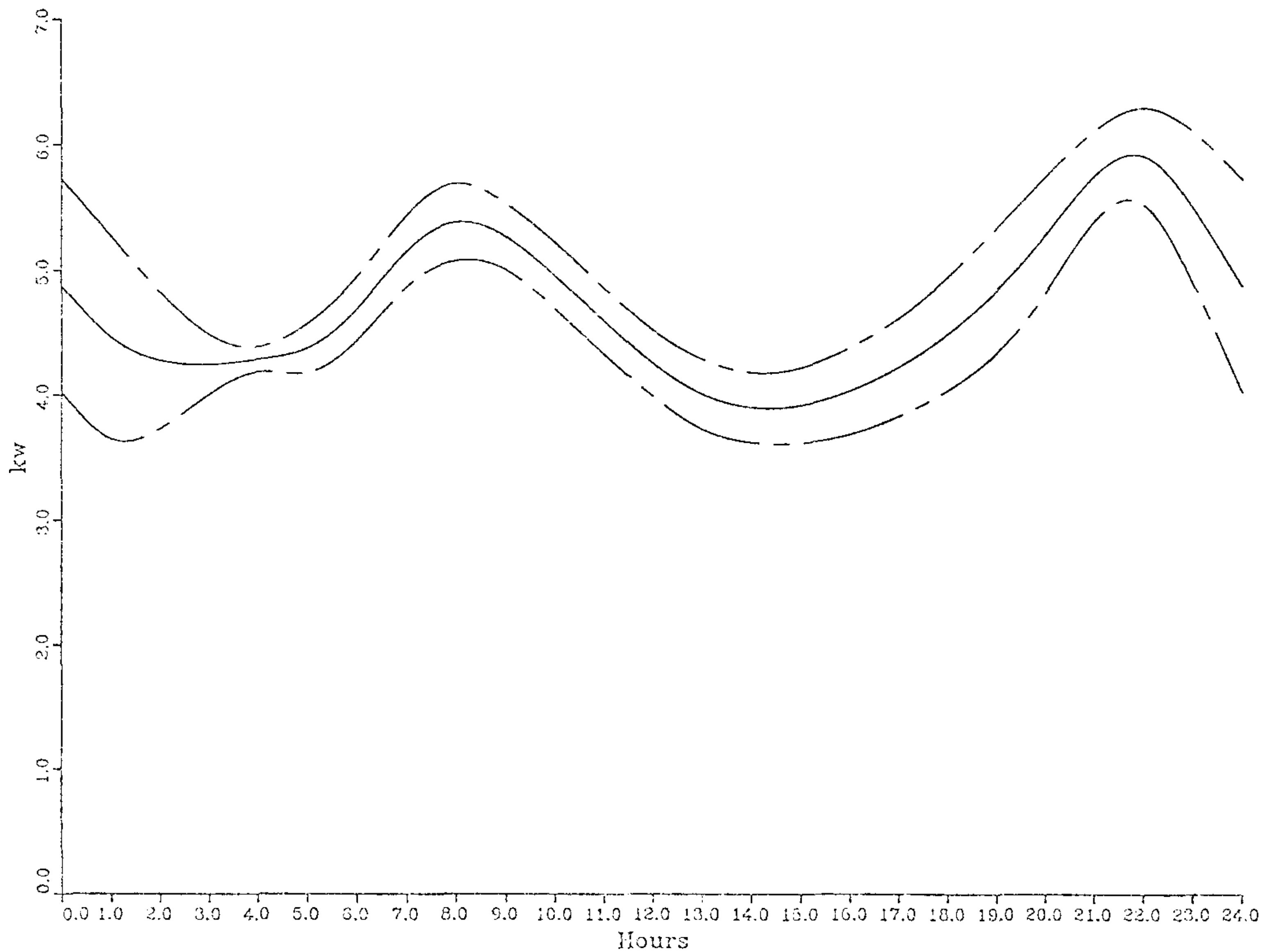

Figure 5-16. Test Customer Daily Demand Cycle: With Electric Water Heater and Electric Heat

The total effect of the pricing treatment as represented in Figures 5-11 through 5-16 is a combination of two effects--a price effect and a budget effect. While it is not possible to separate these effects without restrictive assumptions on the customer's demand functions, it is possible to get an idea of their relative magnitude by calculating total bills under alternative consumption paths. Table 5-11 presents these calculations for the three hypothetical customers defined above. Predicted bills are calculated for control and test pricing schedules with both the control and test predicted consumption patterns.

As per our a priori expectations, the test group saved money by shifting its consumption pattern. This saving ranged from 3.8% for the high use customer to 23.3% for the moderate use customer. As compared to the control group predicted bill, the predicted bill for the test group is higher in the high use category, lower in the moderate use category, and about the same in the low use category. This occurs even though the predicted bill for test group usage in all three categories is essentially the same under both test and control pricing treatments. Test customers would have paid the same predicted bill under control group pricing ($118.06 versus $118.51 for high use; $42.01 versus $41.59 for moderate use; $28.24 versus $29.34 for low use). These calculations suggest that customers with electric heat were induced by the test pricing treatment to increase their use of electricity and shift their consumption pattern. On the other hand, customers with electric water heating were induced by the test pricing treatment both to shift their consumption pattern and to use less electricity.

These casual observations suggest that the size of the test effect at different times of the day might be explained by demographic characteristics. One way to study this question is to reestimate the specification in Eq. 5-10, adding interaction terms between the test dummy and demographic characteristics. Interaction terms between the test dummy and electric heat, electric water heat, stock of appliances and number of people were added for the full sample. In the income subsample, an interaction term with income replaced the interaction term with people.

Unfortunately, the addition of four interaction terms to specification 5-10 increased the number of independent variables in the full design from 140 to 196. The resulting cross-products matrix was too close to singularity to be inverted.

Table 5-11

PREDICTED MONTHLY BILLS FOR THREE USAGE LEVELS
(WINTER)

Group and Usage Level	Predicted Monthly Bill (1)	Bill if Other Group's Predicted Consumption Path Used (2)	Percent Difference (1 - 2)
Test high use	$118.06	$123.25	- 3.8%
Control high use	$112.55	$118.51	- 5.0%
Test moderate use	$42.01	$54.80	-23.3%
Control moderate use	$46.26	$41.59	+10.1%
Test low use	$28.24	$31.63	-10.7%
Control low use	$28.91	$29.34	- 1.5%

An alternative approach which reduces the dimensionality of the problem considerably is to assume that the off-diagonal elements of $(X'\Sigma^{-1}X + \Omega)$ are zero. This allows the 14 equations for the stage 1 coefficients to be estimated independently. The procedure produces less efficient results than those achieved in the more general model, but more efficient results than an OLS procedure which would assume that $((X'\Sigma^{-1}X)^{-1} + \Omega)$ is null.*

The results for this "weighted least squares" estimation of the interaction model are given in Table 5-12 for the full sample and Table 5-13 for the income subsample. They are consistent with the implications from bills predicted above for the noninteraction models. The test-water heat interaction coefficients are negative throughout the day, while the test-electric heat interaction coefficients are positive. The largest shifts occur during the day hours for water heat and the late evening-early night hours for electric heating. There is also some evidence of lower appliance usage in the afternoon in the test group. However, there is no evidence that either size of the household or income level influences the reaction to the test pricing treatment.

Empirical Results For The Summer "Typical" Week

The stage 1 fits for summer customers were worse on average than for winter customers. There are three probable explanations. First, the time period for calculation of the typical week was nine weeks--three weeks shorter than the winter period. Second, customers typically take vacations during this period. Third, there may simply be more day-to-day variation in consumption during the summer than in the winter.

The quality of the stage 2 results depends on both the stage 1 estimations and the degrees of freedom available to estimate the impact of different customer characteristics. As noted in section 4, the number of customers in the summer sample was considerably smaller than in the winter sample (228 versus 315). In fact, the number of summer customers with income information was so small that it was not possible to include this variable in the analysis.

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* The assumption that the off-diagonal elements are zero makes the model very similar to the model presented in (11).

Table 5-12

STAGE 2 WEIGHTED LEAST SQUARES RESULTS FOR INTERACTION MODEL, FULL SAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.059 (0.102)	0.049 (0.103)	0.035 (0.147)	0.064 (0.141)	0.173 (0.132)	0.097 (0.097)	0.138 (0.095)
Water heater	0.434 (0.137)	0.467 (0.139)	0.886 (0.199)	1.097 (0.195)	1.100 (0.190)	0.513 (0.130)	0.712 (0.128)
Appliances[a]	5.839 (2.774)	5.576 (2.800)	10.93 (4.007)	12.22 (3.820)	11.80 (3.574)	9.155 (2.638)	9.513 (2.576)
Electric heat	1.544 (0.291)	1.692 (0.295)	1.512 (0.433)	1.699 (0.483)	1.549 (0.492)	1.124 (0.276)	0.721 (0.271)
Square footage[a]	3.498 (3.715)	2.803 (3.746)	2.937 (5.345)	-1.667 (5.016)	-0.932 (4.673)	0.094 (3.531)	3.658 (3.447)
Temperature	-0.024 (0.045)	-0.038 (0.046)	-0.068 (0.066)	0.053 (0.066)	-0.003 (0.063)	-0.012 (0.043)	-0.017 (0.042)
Test	0.330 (0.272)	0.186 (0.275)	0.355 (0.394)	0.897 (0.380)	0.575 (0.358)	0.273 (0.258)	0.177 (0.253)
People	0.074 (0.046)	0.058 (0.046)	0.120 (0.067)	0.206 (0.067)	0.135 (0.065)	0.080 (0.044)	0.113 (0.043)
Square footage x heat[b]	1.190 (0.155)	1.186 (0.158)	1.341 (0.233)	1.027 (0.272)	0.720 (0.281)	-0.018 (0.148)	0.705 (0.145)
Test x water heater	-0.166 (0.186)	-0.124 (0.188)	-0.201 (0.269)	-0.872 (0.262)	-0.897 (0.251)	-0.473 (0.177)	-0.347 (0.173)
Test x electric heat	0.367 (0.228)	0.310 (0.230)	0.542 (0.336)	0.356 (0.362)	0.159 (0.365)	0.348 (0.216)	-0.119 (0.212)
Test x appliances[a]	-2.355 (3.483)	-1.414 (3.517)	-4.517 (5.052)	-6.532 (4.865)	-8.704 (4.557)	-2.832 (3.311)	-6.217 (3.237)
Test x people	-0.050 (0.054)	-0.028 (0.055)	-0.049 (0.080)	-0.135 (0.079)	-0.009 (0.076)	-0.002 (0.052)	0.013 (0.051)
R^2	0.844	0.849	0.759	0.638	0.512	0.514	0.695
N	314	314	314	314	314	314	314

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-12 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.078 (0.152)	-0.025 (0.171)	0.139 (0.125)	0.150 (0.091)	0.062 (0.097)	0.137 (0.138)	-0.147 (0.142)
Water heater	1.096 (0.206)	0.845 (0.236)	0.179 (0.181)	-0.019 (0.123)	-0.103 (0.131)	-0.134 (0.186)	-0.083 (0.197)
Appliances[a]	13.46 (4.152)	8.884 (4.622)	-3.405 (3.403)	-3.263 (2.483)	-3.961 (2.647)	-1.676 (3.751)	-4.007 (3.851)
Electric heat	1.411 (0.448)	0.706 (0.584)	-1.385 (0.469)	-0.463 (0.260)	-0.704 (0.279)	-1.284 (0.405)	-1.160 (0.487)
Square footage[a]	6.328 (5.538)	3.911 (6.070)	5.120 (4.449)	5.374 (3.325)	3.782 (3.542)	4.836 (5.004)	3.481 (5.058)
Temperature	0.062 (0.069)	0.065 (0.080)	0.116 (0.060)	0.033 (0.041)	0.050 (0.043)	0.060 (0.062)	0.015 (0.066)
Test	0.152 (0.408)	0.110 (0.459)	-0.103 (0.341)	-0.292 (0.243)	-0.158 (0.260)	-0.186 (0.369)	-0.193 (0.383)
People	0.201 (0.069)	0.191 (0.080)	0.040 (0.061)	-0.024 (0.041)	-0.010 (0.044)	0.008 (0.063)	-0.041 (0.067)
Square footage x heat[b]	1.980 (0.241)	2.422 (0.329)	0.799 (0.268)	0.222 (0.139)	0.409 (0.149)	0.518 (0.218)	0.348 (0.274)
Test x water heater	-0.119 (0.279)	-0.112 (0.317)	0.198 (0.239)	0.299 (0.166)	0.409 (0.178)	0.304 (0.252)	0.144 (0.263)
Test x electric heat	0.756 (0.349)	0.776 (0.438)	0.333 (0.347)	-0.276 (0.203)	-0.012 (0.218)	0.284 (0.315)	0.893 (0.365)
Test x appliances[a]	0.159 (5.234)	3.767 (5.887)	4.839 (4.339)	1.731 (3.117)	1.554 (3.326)	1.512 (4.729)	5.004 (4.905)
Test x people	0.001 (0.083)	-0.004 (0.096)	0.015 (0.072)	0.022 (0.049)	-0.014 (0.052)	-0.036 (0.075)	-0.005 (0.080)
R^2 =	0.758	0.688	0.100	0.066	0.078	0.068	0.073
N =	314	314	314	314	314	314	314

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-13

STAGE 2 WEIGHTED LEAST SQUARES RESULTS FOR INTERACTION MODEL
INCOME SUBSAMPLE

	0400	0530	0730	1000	1300	1600	2000
Electric range	0.158 (0.112)	0.156 (0.114)	0.111 (0.174)	0.212 (0.192)	0.304 (0.183)	0.273 (0.176)	0.214 (0.180)
Water heater	0.385 (0.169)	0.442 (0.172)	0.900 (0.262)	1.377 (0.290)	1.375 (0.277)	1.214 (0.266)	1.315 (0.273)
Appliances[a]	5.358 (3.256)	5.023 (3.319)	11.66 (5.061)	15.09 (5.593)	13.81 (5.342)	14.33 (5.138)	13.35 (5.273)
Electric heat	0.799 (0.346)	0.837 (0.353)	0.441 (0.538	1.334 (0.595)	1.445 (0.569)	1.581 (0.547)	1.413 (0.561)
Square footage[a]	1.870 (3.928)	1.754 (4.003)	-0.292 (6.104)	-3.633 (6.746)	-4.579 (6.444)	-3.890 (6.198)	-0.081 (6.360)
Income	0.246 (0.561)	0.079 (0.571)	1.435 (0.871)	0.919 (0.963)	0.529 (0.920)	0.551 (0.885)	2.101 (0.908)
Test	0.124 (0.262)	0.053 (0.267)	0.373 (0.406)	0.589 (0.449)	0.784 (0.429)	0.618 (0.413)	0.350 (0.424)
People	0.027 (0.027)	0.026 (0.027)	0.054 (0.041)	0.063 (0.046)	0.147 (0.044)	0.147 (0.042)	0.193 (0.043)
Square footage x heat[b]	1.701 (0.216)	1.798 (0.220)	2.126 (0.336)	1.203 (0.371)	0.777 (0.355)	0.471 (0.340)	1.196 (0.350)
Test x water heater	-0.150 (0.209)	-0.118 (0.213)	-0.250 (0.325)	-1.192 (0.359)	-1.205 (0.343)	-1.110 (0.330)	-0.794 (0.338)
Test x electric heat	0.491 (0.261)	0.391 (0.265)	0.736 (0.405)	0.740 (0.448)	0.272 (0.427)	0.618 (0.412)	0.027 (0.422)
Test x appliances[a]	-2.977 (3.811)	-1.654 (3.885)	-6.044 (5.923)	-8.366 (6.547)	-13.78 (6.253)	-6.869 (6.015)	-8.458 (6.172)
Test x income[a]	0.308 (0.722)	-0.235 (0.735)	-0.488 (1.121)	-0.432 (1.239)	0.206 (1.184)	-0.217 (1.139)	0.086 (1.169)
R^2 =	0.853	0.858	0.800	0.699	0.636	0.629	0.716
N	259	259	259	259	259	259	259

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

Table 5-13 (continued)

	2200	2400	07-0900	09-1100	11-1700	17-1900	19-2100
Electric range	0.146	0.079	0.142	-0.068	-0.024	0.108	-0.054
	(0.162)	(0.160)	(0.151)	(0.172)	(0.152)	(0.160)	(0.114)
Water heater	1.043	0.616	-0.020	-0.188	-0.319	-0.104	-0.147
	(0.244)	(0.243)	(0.288)	(0.261)	(0.231)	(0.242)	(0.172)
Appliances[a]	10.65	5.611	-7.089	-6.849	-6.277	-1.610	-5.036
	(4.717)	(4.677)	(4.401)	(5.037)	(4.458)	(4.671)	(3.311)
Electric heat	-0.074	0.730	-0.853	-0.529	-1.291	-1.537	-0.953
	(0.502)	(0.498)	(0.468)	(0.536)	(0.474)	(0.497)	(0.352)
Square footage[a]	2.567	1.448	4.193	7.101	5.795	4.213	3.5122
	(5.690)	(5.642)	(5.308)	(6.076)	(5.377)	(5.634)	(3.994)
Income	1.788	1.137	0.392	0.250	0.389	0.641	-0.223
	(0.812)	(0.805)	(0.758)	(0.867)	(.768)	(0.804)	(0.570)
Test	0.041	-0.063	-0.344	-0.426	-0.374	-0.072	-0.272
	(0.379)	(0.376)	(0.353)	(0.405)	(0.358)	(0.375)	(0.266)
People	0.187	0.165	0.027	0.008	-0.021	-0.015	-0.019
	(0.039)	(0.038)	(0.036)	(0.041)	(0.036)	(0.038)	(0.027)
Square footage x heat[b]	2.071	1.686	0.680	0.388	1.020	0.873	0.505
	(0.313)	(0.038)	(0.292)	(0.334)	(0.296)	(0.310)	(0.220)
Test x water heater	-0.146	-0.018	0.346	0.560	0.663	0.305	0.203
	(0.303)	(0.300)	(0.282)	(0.323)	(0.286)	(0.300)	(0.212)
Test x electric heat	0.876	0.682	-0.146	-0.727	-0.104	0.129	0.367
	(0.378)	(0.375)	(0.353)	(0.404)	(0.357)	(0.374)	(0.265)
Test x appliances[a]	1.222	4.053	7.256	3.232	4.316	1.043	5.457
	(5.522)	(5.476)	(5.151)	(5.896)	(5.219)	(5.468)	(3.876)
Test x income[a]	0.608	0.757	0.507	0.541	0.077	-0.716	-0.055
	(1.045)	(1.037)	(0.975)	(1.116)	(0.988)	(1.035)	(0.734)
R^2	0.797	0.777	0.113	0.074	0.123	0.066	0.098
N	259	259	259	259	259	259	259

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^3.

We were left in the uncomfortable position of estimating the impact of demographics on the summer demand cycle with relatively poor stage 1 fits, and relatively few degrees of freedom in the stage 2 data. For example, we expected the impact of central air conditioning to be very important in determining the summer demand cycle. Unfortunately, there were only 13 customers in the entire sample who had central air conditioning.

Obviously the demographics that should affect the summer demand cycle are different than those for the winter. The electric heat, electric heat-square footage interaction, electric range, and temperature difference* variables were dropped from specification 5-10, and a central air conditioning dummy variable and a variable measuring the number of window air conditioners were added. The OLS results for this summer specification are given in Table 5-14 for each of the nine knots and the five pricing period dummies. These results are given for Eastern Standard Time. To find the Eastern Daylight Time equivalents, one hour should be added to each knot location and each pricing period.

The estimated impact of central air conditioning is positive and greater than twice its standard error all during the day and evening except in the late morning. The impact of window air conditioners, however, is estimated to be very small over the entire cycle. It is possible that some of the impact of air conditioning is being picked up by the square footage variable, which is positive and greater than twice its standard error throughout the cycle. We have no other explanation for the small window air coefficients.** Electric water heating and number of people have almost the same effect on the summer demand cycle as they did during the winter. The impact of the stock of appliances, however, is consistently lower than its estimated impact during the winter. The estimated effect is greater than twice its standard error only during the early evening.

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*The temperature difference between sites was only 0.4°C. The small average difference for customers should not affect consumption.

**The largest estimated value for the window air coefficient is approximately 0.1. This indicates that approximately 100 watts would be used by an air conditioner for that hour. This is fairly small, since a typical air conditioner would draw approximately 900 watts per hour.

Table 5-14

STAGE 2 OLS RESULTS - SUMMER

	0400	0600	0800	1000	1400	1700	2000
Central air conditioner	0.133	0.124	0.615	0.158	0.876	1.072	1.058
	(0.112)	(0.174)	(0.299)	(0.287)	(0.293)	(0.315)	(0.310)
Window air conditioner	0.113	0.031	0.066	0.103	0.000	0.050	0.041
	(0.033)	(0.051)	(0.087)	(0.084)	(0.085)	(0.092)	(0.090)
Water heater	0.209	0.460	0.918	0.821	0.544	0.743	0.794
	(0.050)	(0.077)	(0.132)	(0.127)	(0.129)	(0.139)	(0.137)
Appliances[a]	1.751	2.396	2.461	2.457	4.864	7.364	9.557
	(1.132)	(1.756)	(3.014)	(2.895)	(2.953)	(3.177)	(3.123)
Square footage[b]	1.748	2.264	3.858	2.908	3.281	3.509	4.409
	(0.401)	(0.622)	(1.067)	(1.025)	(1.045)	(1.125)	(1.106)
Test	-0.008	-0.034	-0.133	-0.339	-0.140	-0.161	-0.082
	(0.050)	(0.078)	(0.133)	(0.128)	(0.130)	(0.140)	(0.138)
People	0.040	0.031	0.059	0.097	0.139	0.182	0.166
	(0.016)	(0.025)	(0.044)	(0.042)	(0.043)	(0.046)	(0.045)
R^2 =	0.281	0.225	0.265	0.251	0.235	0.306	0.350
N =	228	228	228	228	228	228	228

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^4.

Table 5-14 (continued)

	2200	2400	07-0900	09-1100	11-1200	12-1400	14-2000
Central air conditioner	0.820	0.522	0.290	0.231	-0.229	-0.021	0.071
	(0.270)	(0.173)	(0.221)	(0.214)	(0.218)	(0.251)	(0.209)
Window air conditioner	0.096	0.105	-0.043	0.079	-0.013	0.078	0.062
	(0.079)	(0.050)	(0.064)	(0.062)	(0.064)	(0.073)	(0.061)
Water heater	0.617	0.293	-0.334	-0.117	-0.268	-0.228	-0.230
	(0.119)	(0.076)	(0.098)	(0.094)	(0.096)	(0.110)	(0.094)
Appliances[a]	7.905	2.996	-1.702	-3.305	-0.688	-4.204	-3.212
	(2.725)	(1.739)	(2.227)	(2.153)	(2.197)	(2.531)	(2.110)
Square footage[b]	2.979	2.255	0.223	-0.094	-0.322	-0.566	-0.639
	(0.965)	(0.615)	(0.788)	(0.762)	(0.778)	(0.896)	(0.747)
Test	0.196	0.093	-0.199	-0.077	0.045	-0.340	-0.102
	(0.120)	(0.077)	(0.098)	(0.095)	(0.097)	(0.111)	(0.093)
People	0.197	0.105	0.022	0.015	0.028	-0.025	0.003
	(0.040)	(0.025)	(0.032)	(0.031)	(0.032)	(0.037)	(0.031)
R^2 =	0.343	0.285	0.081	0.024	0.054	0.081	0.051
N =	228	228	228	228	228	228	228

[a]Coefficients and standard errors are multiplied by 10^5.

[b]Coefficients and standard errors are multiplied by 10^4.

The estimated influence of the test treatment is negative throughout the cycle except in the late evening. The strongest influence is in the morning and afternoon hours. The conservation of electricity that occurred during these hours was about the same as during the winter. However, there is no evidence that a shift of consumption to off-peak or high-use pricing periods took place, as had been the case in the winter. Thus the typical test customer used considerably less electricity than the typical control customer.

We have not attempted to improve the efficiency of these OLS estimates because of the problems with the summer data mentioned above. Based on our experience with the winter data, we are confident that the coefficients would remain about the same, although the estimated errors would probably be higher.

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Section 6

CONCLUSIONS AND FURTHER RESEARCH

The principal purpose of this report was to analyze the major determinants of the level and shape of the residential demand cycle for electricity. The empirical results presented in Section 5 have direct implications for medium-term forecasts of time-of-day demand and for estimation of the impact of peak load pricing on the demand cycle. We will not attempt to reiterate these results in this section. Instead, we would like to provide some observations about the use of individual data from time-of-day experiments to estimate and forecast residential load curves.

A large amount of day-to-day fluctuation occurs in the residential demand cycle for individual customers. Our preliminary results and those obtained in (1) suggest that only a small amount of this fluctuation can be attributed to weather conditions.* This does not mean that weather has little influence. Rather, it suggests that if daily fluctuations are to be forecast, this influence should be estimated using aggregate, as opposed to individual, data. The strength of individual data is in estimating medium-term forecasts in which typical weather for that period is known. We believe therefore that the use of a typical week in analysis of individual data is appropriate. This use reduces the dimensionality of the estimation process considerably and eliminates some of the day-to-day noise while providing an estimate of the cycle for typical weather during the period in question.

An issue that always arises with individual data is the appropriate time period for analysis. Some authors** have suggested that relatively long periods

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*At least to the weather conditions we were able to measure.

**See (2), for example.

(e.g., peak versus off-peak hours) should be used, since usage of appliances within the periods is basically random. We rejected this approach since we had strong prior beliefs that typical usage across the day was a smooth function of time. When shorter time periods are used (e.g., one hour) the main problem is how to parameterize the time series. The cubic spline method which we have adopted has several advantages as the appropriate specification of this time series. It provides a smooth representation of the daily cycle; it can be estimated using OLS techniques; the estimated parameters have ready economic interpretation and can be associated with demographic characteristics; and it is a relatively parsimonious representation of the original series. The advantages of this parsimony in the first stage can be easily overlooked. Not only are computation costs considerably reduced, but problems such as heteroscedasticity and serial correlation can be handled without unsurmountable costs.

Our results and those obtained in (1) indicate that the errors in the estimation of time series parameters using demographic data are considerably larger than errors in the estimation of the parameters themselves. This suggests that future experiments should concentrate on obtaining a relatively large number of sample customers. This is especially true for data from the summer months.

We have two other observations about the conduct of experiments. First, every effort should be made to gather good data on income levels. A great deal more must be known about the impact of income on the daily cycle and on reaction to peak load pricing if regulatory commissions are to evaluate the impact of different price structures on different income groups. Second, experiments must be designed with at least several pricing treatments. Only then can we hope to estimate the cross-price elasticities which are crucial in any cost-benefit evaluation of peak load pricing.

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Appendix 1

APPLIANCE WATTAGES*

Appliance indices were constructed with information from the short form questionnaire for the pooled sample of test and control groups.

Appliances	Wattage
Electric range	12,200
Electric washer	512
Electric dryer	4,856
Dishwasher	1,201
Sink disposal	445
Trash compactor	400
Microwave oven	1,450
Black and white television	45
Color television	145
Self-defrosting refrigerator	205
Manual-defrosting refrigerator	80
Self-defrosting freezer	227
Manual-defrosting freezer	151
Electric water heater	2,475
Electric blanket	177
Humidifier	177
Swimming pool pump	2
Dehumidifier	257

*Wattages were obtained from the Illinois Power Company and from the Electric Energy Association handout, "Annual Energy Requirements of Electrical Appliances" (New York, yearly).

Appendix 2

DESCRIPTION OF SAMPLE DEMOGRAPHIC CHARACTERISTICS

Description	Frequency (%)	Mean	Summed means
Electric range	82		
Electric washer	91		
Electric dryer	77		
Self-defrosting refrigerator	79		
Manual-defrosting refrigerator	34		
Self-defrosting freezer	15		
Manual-defrosting freezer	30		
Dishwasher	58		
Sink disposal	21		
Trash compactor	2		
Microwave oven	2		
Electric blanket	24		
Black and white television		0.789	1.653
Color television		0.864	
Humidifier	20		
People in household (age under 18)		1.446	
People in household (age 18 to 64)		2.146	3.744
People in household (age 65 and over)		0.152	
Heating system (main) electric	22		
Water heating electric	51		
One-family structure	90		