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Intrabeam Scattering

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Intrabeam scattering is the scattering of the particles in the beam from each other through the coulomb forces that act between each pair of particles. It depends on the ion charge and mass like Z^4/A^2 and is usually larger for the heavier ions.

In RHIC, extra aperture is provided to allow the beam to grow transversely because of intrabeam scattering. For Au ions, enough aperture has to be provided to allow the transverse emittances to grow by a factor of 3 over 10 hours. The beam will also grow longitudinally, and enough RF bucket area has to be provided for the longitudinal growth. For Au ions at γ=100, the beam energy spread will grow by about a factor of 3 over 10 hours.

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IBS Theory

The treatment of IBS that is used to obtain the results given below, was done by A. Piwinski (1974) for the case where the beta functions, β_x and β_y, and the horizontal dispersion, D_p, were assumed to be constant around the accelerator. These results were generalized in β_x, β_y and D_p by Bjorken and Mtingwa (1984) and by Möhl, Piwinski, Sacherer and Martini (1984). The latter version, as written by Martini (1984), is used here.

The particle distribution function is assumed to be gaussian with the 3 parameters $\bar{\epsilon}_x$, $\bar{\epsilon}_y$ and σ_p, where $\bar{\epsilon}_x$ and $\bar{\epsilon}_y$ are the rms emittances, and σ_p is the rms relative momentum $\Delta p/p$. Also

$$\bar{\epsilon}_x = 2\ \sigma_x^2/\beta_x$$

$$\bar{\epsilon}_y = 2\ \sigma_y^2/\beta_y$$

where σ_x, σ_y are the rms betatron amplitudes.

In the absence of intrabeam scattering this distribution would be constant with time. It is assumed that in the presence of intrabeam scattering the distribution function keepts the same form but $\bar{\epsilon}_x$, $\bar{\epsilon}_y$ and σ_p now vary slowly with time. This time variation is given by a set of equations of the form

$$\frac{1}{\bar{\epsilon}_x} \frac{d}{dt} \bar{\epsilon}_x = f_1 (\bar{\epsilon}_x, \bar{\epsilon}_y, \sigma_p)$$

$$\frac{1}{\bar{\epsilon}_y} \frac{d}{dt} \bar{\epsilon}_y = f_2 (\bar{\epsilon}_x, \bar{\epsilon}_y, \sigma_p) \qquad (1)$$

$$\frac{1}{\sigma_p} \frac{d}{dt} \sigma_p = f_3 (\bar{\epsilon}_x, \bar{\epsilon}_y, \sigma_p)$$

The expressions for f_1, f_2, f_3 are complicated involving integrals that have to be evaluated numerically. Equation (1) can be regarded as a set of differential equations for $\bar{\epsilon}_x$, $\bar{\epsilon}_y$, and σ_p, and they can be integrated numerically to find $\bar{\epsilon}_x$, $\bar{\epsilon}_y$, σ_p as a function of time for a given set of initial values for $\bar{\epsilon}_x$, $\bar{\epsilon}_y$, σ_p.

IBS at High γ

High γ means $\gamma > \gamma_t$, where γ_t is the transition energy.

For a lattice consisting only of regular cells (no special insertions), σ_x and σ_p are related by the time invariant

$$\sigma_E^2 - \sigma_x^2 = \text{constant}$$

$$\sigma_E = D_p \sigma_p$$

for bunched beams and no x,y coupling.

The σ_p, σ_x growth rates are related by

$$\frac{1}{\sigma_p} \frac{d\sigma_p}{dt} = \left(\frac{\sigma_x}{\sigma_E}\right)^2 \frac{1}{\sigma_x} \frac{d\sigma_x}{dt} ,$$

$$\sigma_E = D_p \sigma_p ,$$

for bunched beams.

The vertical growth rate is smaller than the σ_x or σ_p growth rates at high γ, $\gamma > \gamma_t$. The contribution due to each element in the lattice goes like

$$\frac{1}{\sigma_y} \frac{d\sigma_y}{dt} = -\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \frac{(\beta_x/D_p)^2}{\gamma^2} r^2 \frac{2-r}{1+r}$$

$$r = (\sigma_x/\beta_x) / (\sigma_y/\beta_y)$$

$\beta_x/D_p \simeq \gamma_t$ and the vertical growth rate is smaller by $(\gamma_t/\gamma)^2$. The vertical motion is slightly damped if σ_y is not too small ($r < 2$).

At high γ, for a lattice consisting only of regular cells, one can find the growth rates from

$$\frac{1}{\sigma_x}\frac{d\sigma_x}{dt} = \frac{20\ N_b c r_o^2}{\epsilon_x \epsilon_y \sigma_p \sigma_s (\beta\gamma)^3}\ \frac{D_p \sigma_p}{(\sigma_x^2 + D_p^2\ \sigma_p^2)^{1/2}}\ \frac{D_p}{(\beta_x \beta_{x,av})^{1/2}} ,$$

$$\frac{1}{\sigma_p}\frac{d\sigma_p}{dt} = \left(\frac{\sigma_x}{\sigma_E}\right)^2 \frac{1}{\sigma_x}\frac{d\sigma_x}{dt} .$$

This result holds for bunched beams, N_b particles/bunch, with $\epsilon_x = \epsilon_y$ and a lattice for which $\beta_{x,av} = \beta_{y,av}$. ϵ_x, ϵ_y are the 90% emittances

$$\epsilon_x = \frac{4\sigma_x^2}{\beta_x} , \quad \epsilon_y = \frac{4\sigma_y^2}{\beta_y}$$

$$r_o = Q^2/MC^2$$

For a 90° cell one may take $\beta_{x,av} = L_{cell}$, the cell length.

<u>At High γ</u> (Some observations from the results on the previous slide)

$$\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \simeq \frac{N_b}{\text{6-dimensional Phase Space}}$$

$$\simeq \frac{N_b}{\epsilon_x \epsilon_y \sigma_p \sigma_s} \quad .$$

$$\frac{1}{\sigma_x} \frac{d\sigma_x}{dt} = \frac{1}{\gamma_t}$$

$$\gamma_t \sim \frac{(\beta_x \beta_{x,av})^{1/2}}{D_p}$$

Results for RHIC

Assumptions for Au and other ions

Initial $\epsilon_x = \epsilon_y = 10 \times 10^{-6}$, 95% emittance
RF, $V = 1.2 \times 10^6$ volts, $h = 6 \times 57 = 342$
RHIC lattice, $\gamma_t = 25$

Initial bunch area = .3 eV-sec/amu, $\gamma < \gamma_t$
= 1 ev-sec/amu, $\gamma > \gamma_t$

$N_b = 1.2 \times 10^9$/bunch
ϵ_x, ϵ_y are normalized emittances throughout

Assumptions for Protons

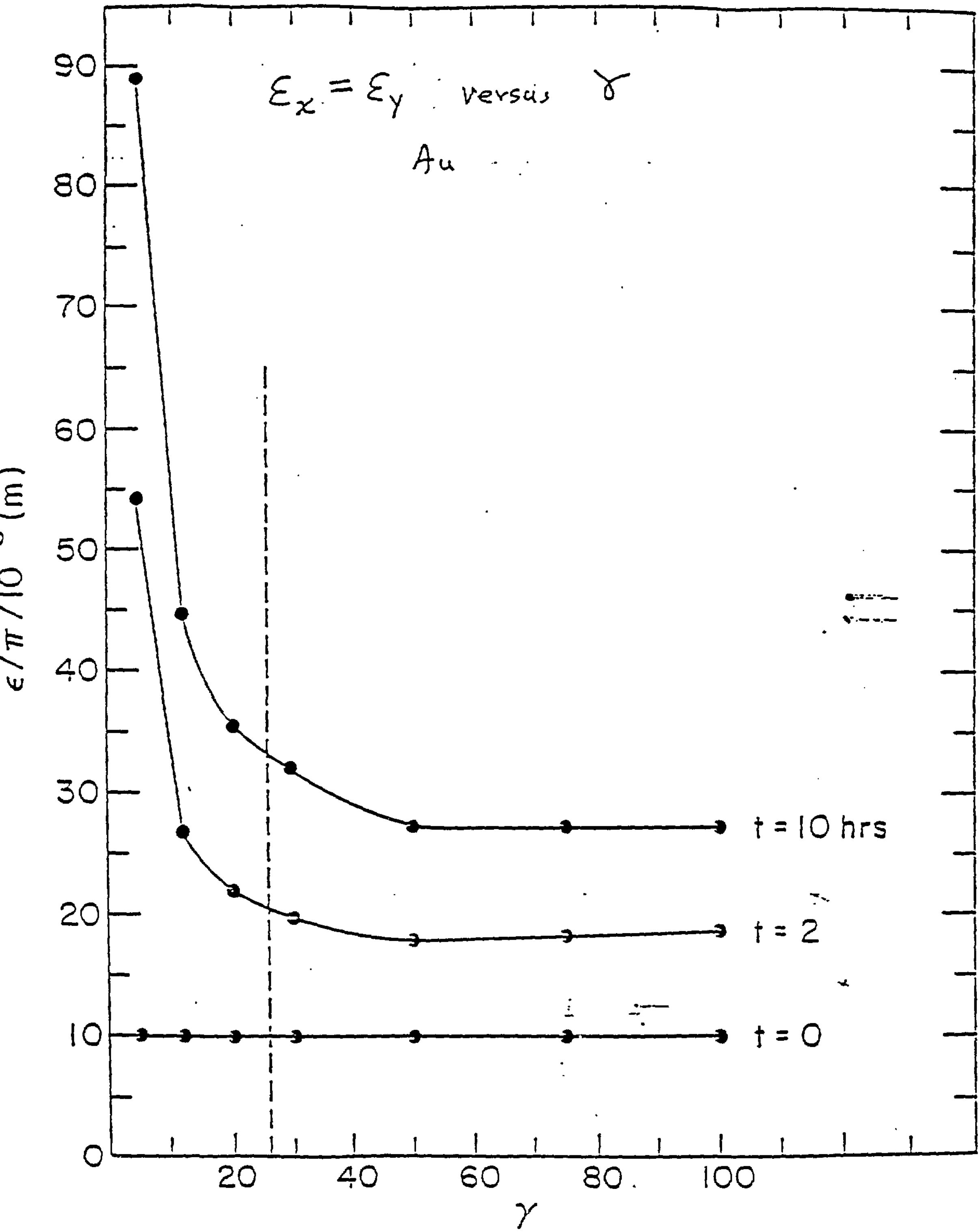
Initial $\epsilon_x = \epsilon_y = 20 \times 10^{-6}$

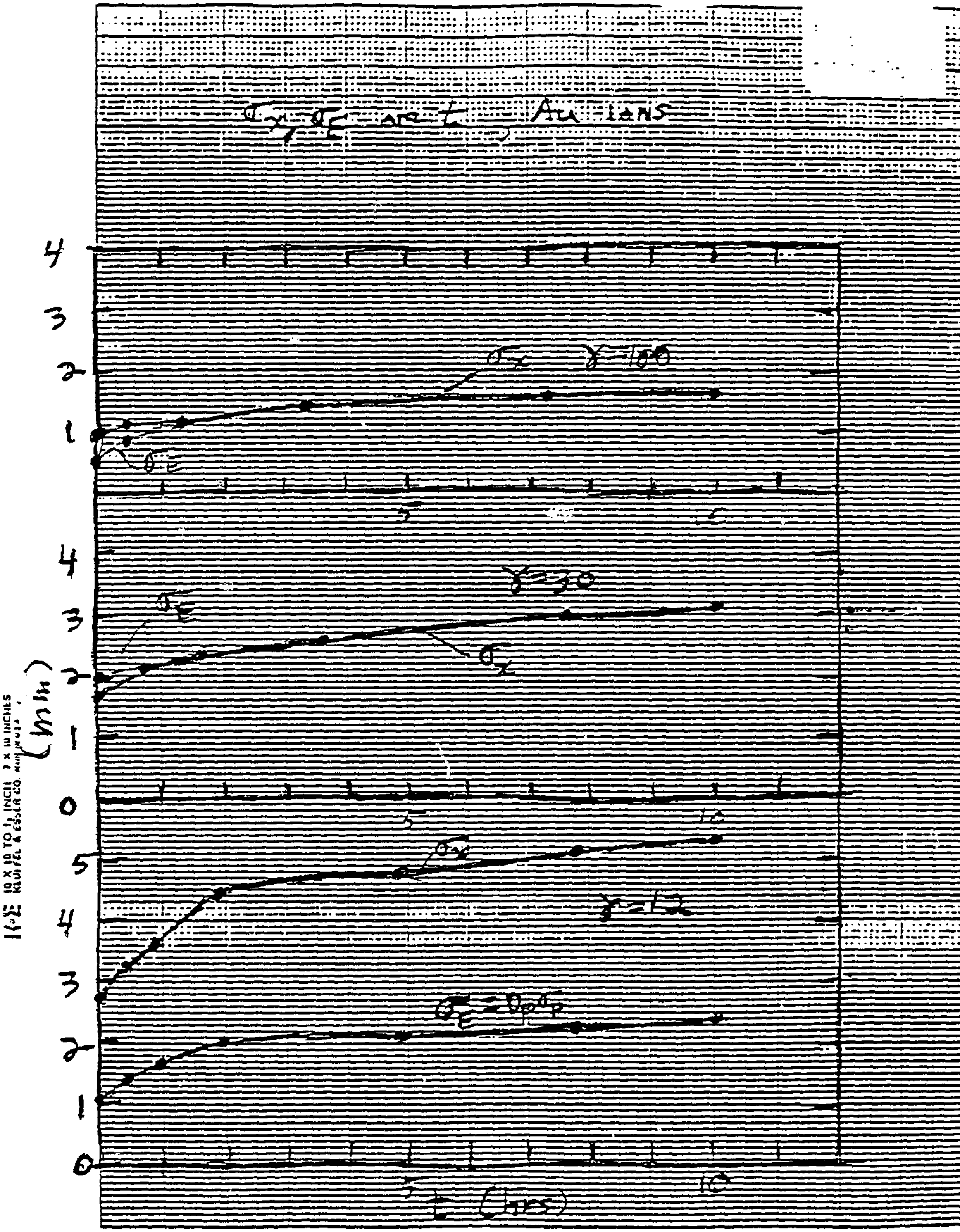
Initial bunch area = .3 eV-sec
$N_b = 1\ 10^{11}$/bunch

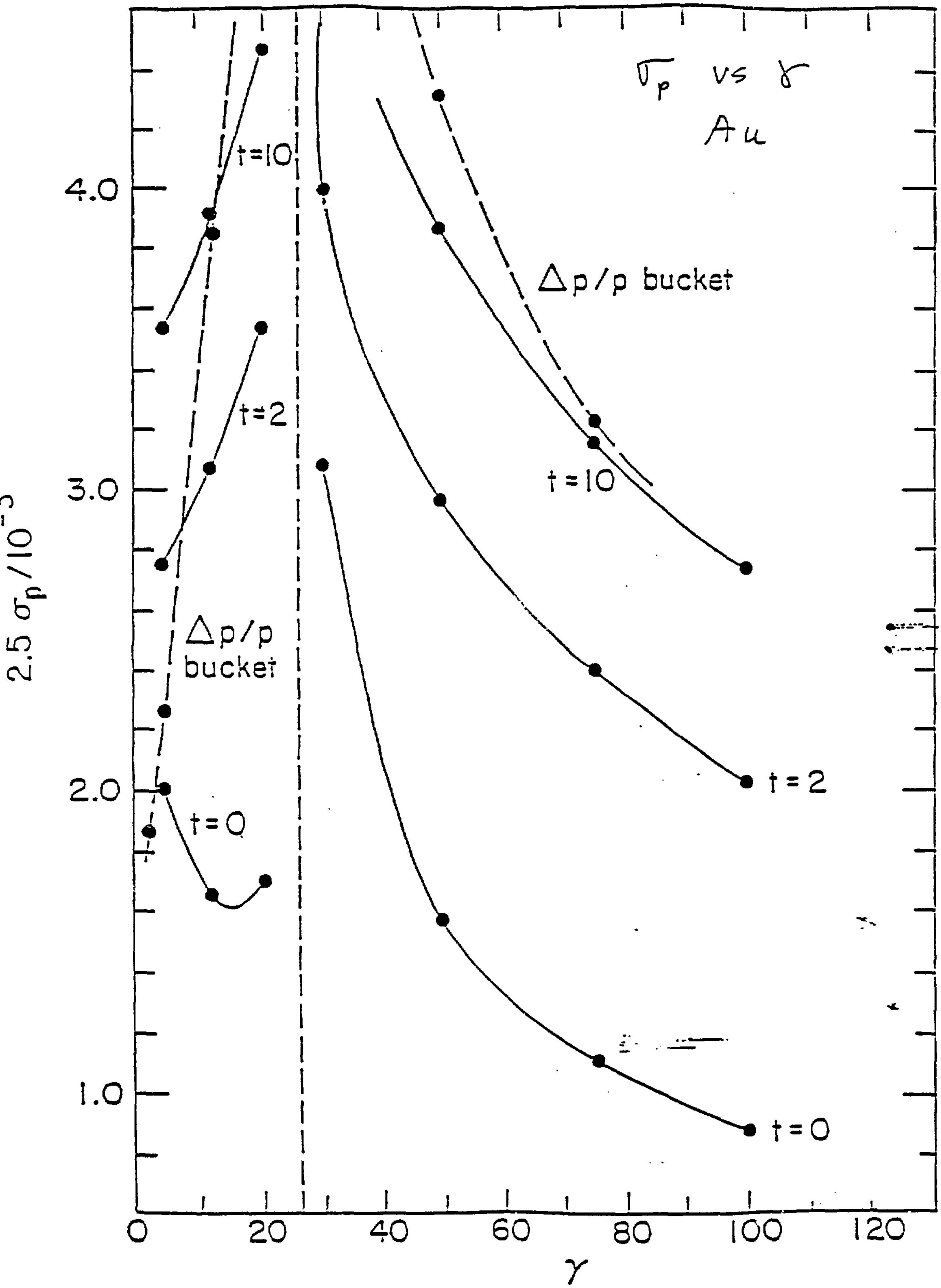
Coupling

x and y motions are assumed to be coupled, and ϵ_y grows with ϵ_x. The larger growth rate for σ_x and σ_y is used for both x and y.

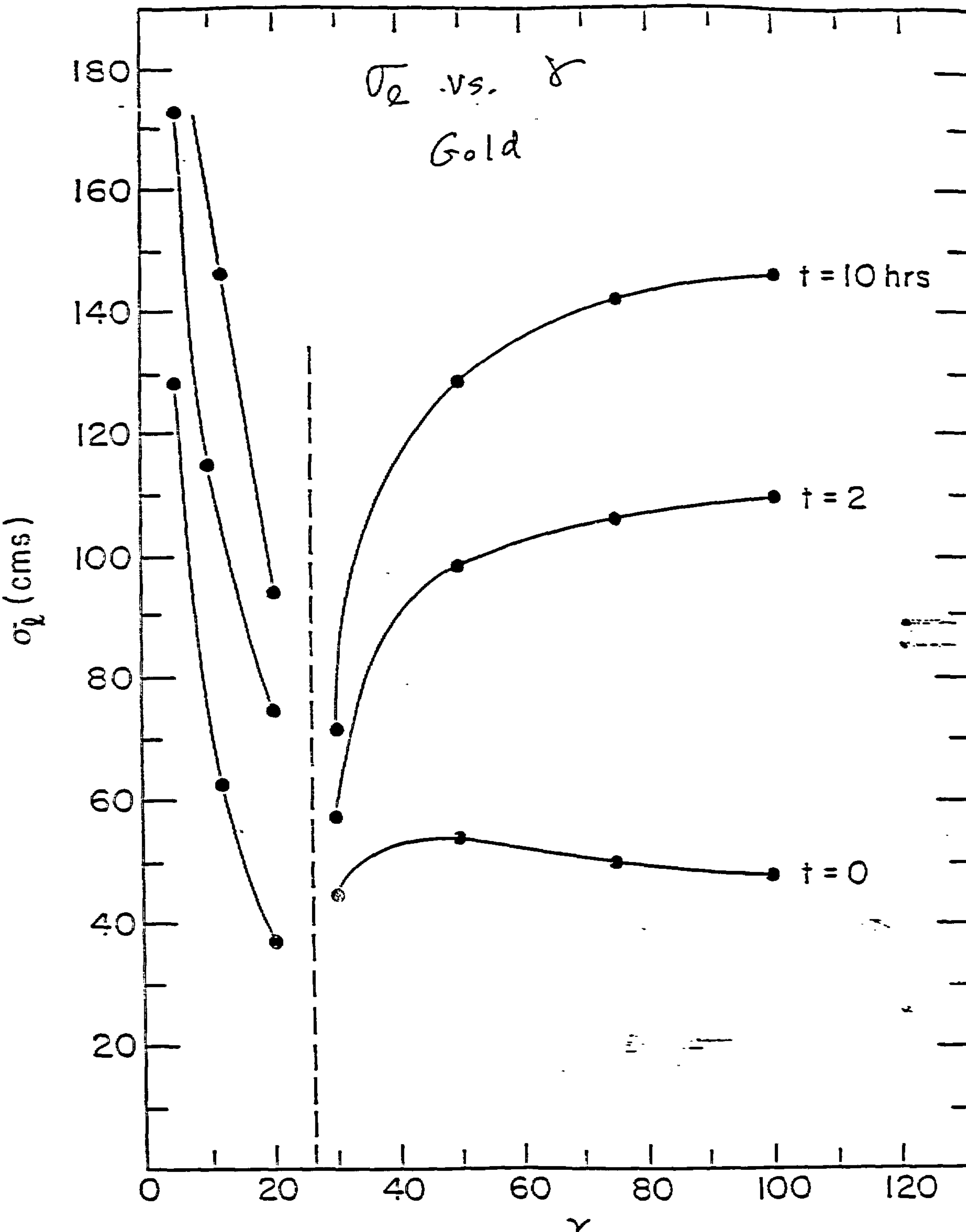
Somewhat better treatment of the coupling will be presented later.


$\mathcal{E}_x = \mathcal{E}_y$ versus γ
Au
$\epsilon/\pi/10^{-6}$ (m)
90
80
70
60
50
40
30
20
10
0
20
40
60
80
100
γ
t = 10 hrs
t = 2
t = 0

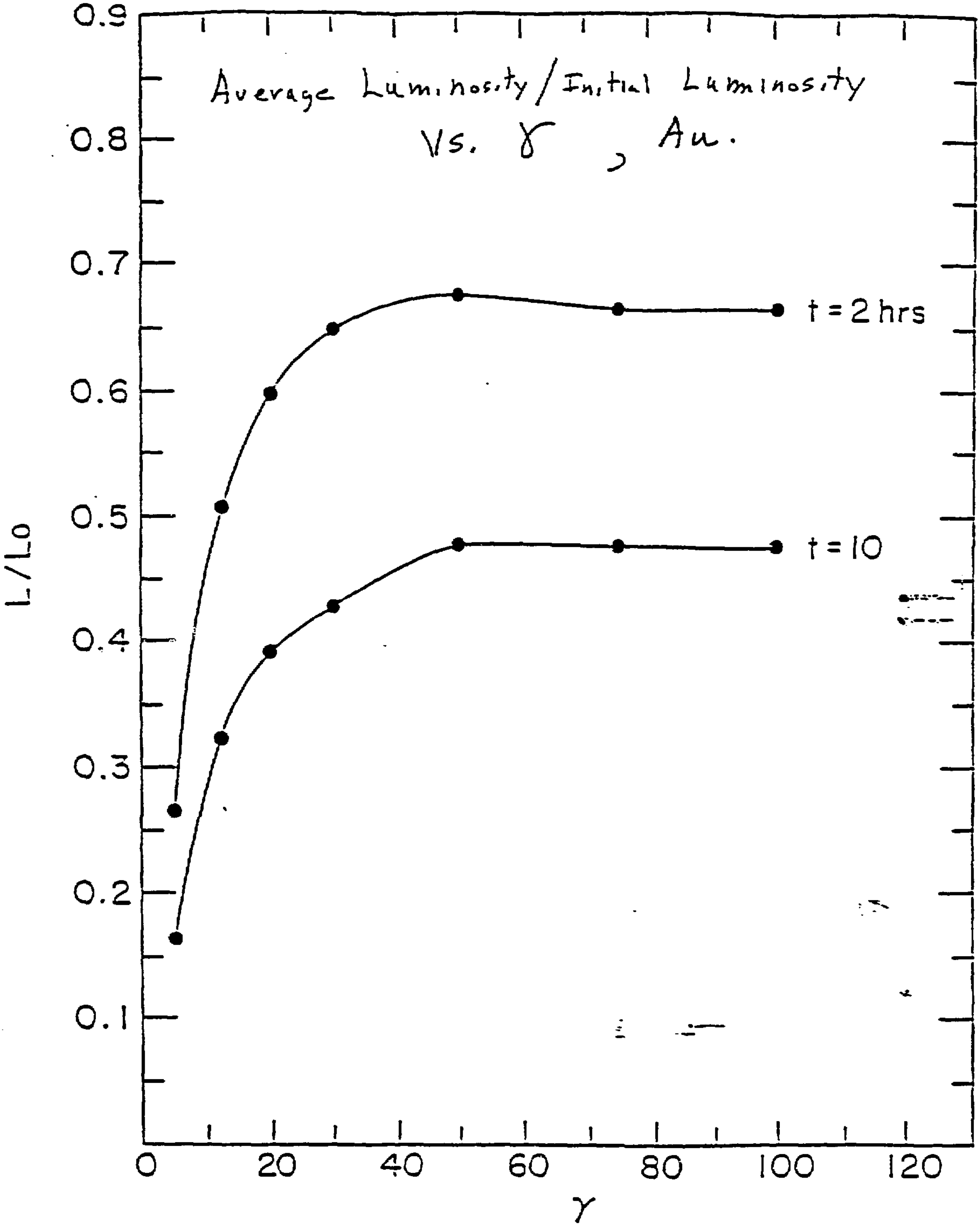

γ=100
γ=30
γ=12
(mm)
t (hrs)

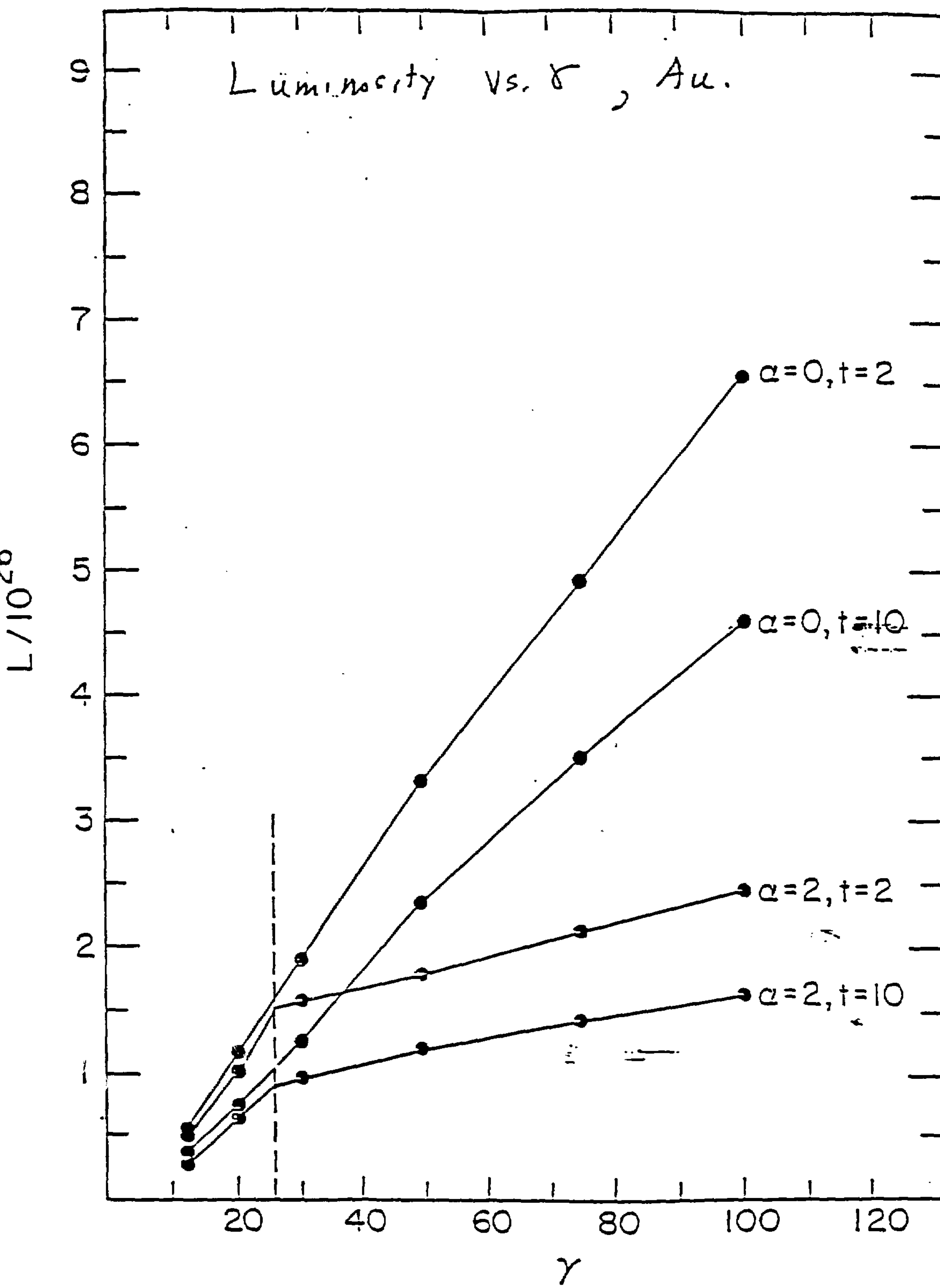


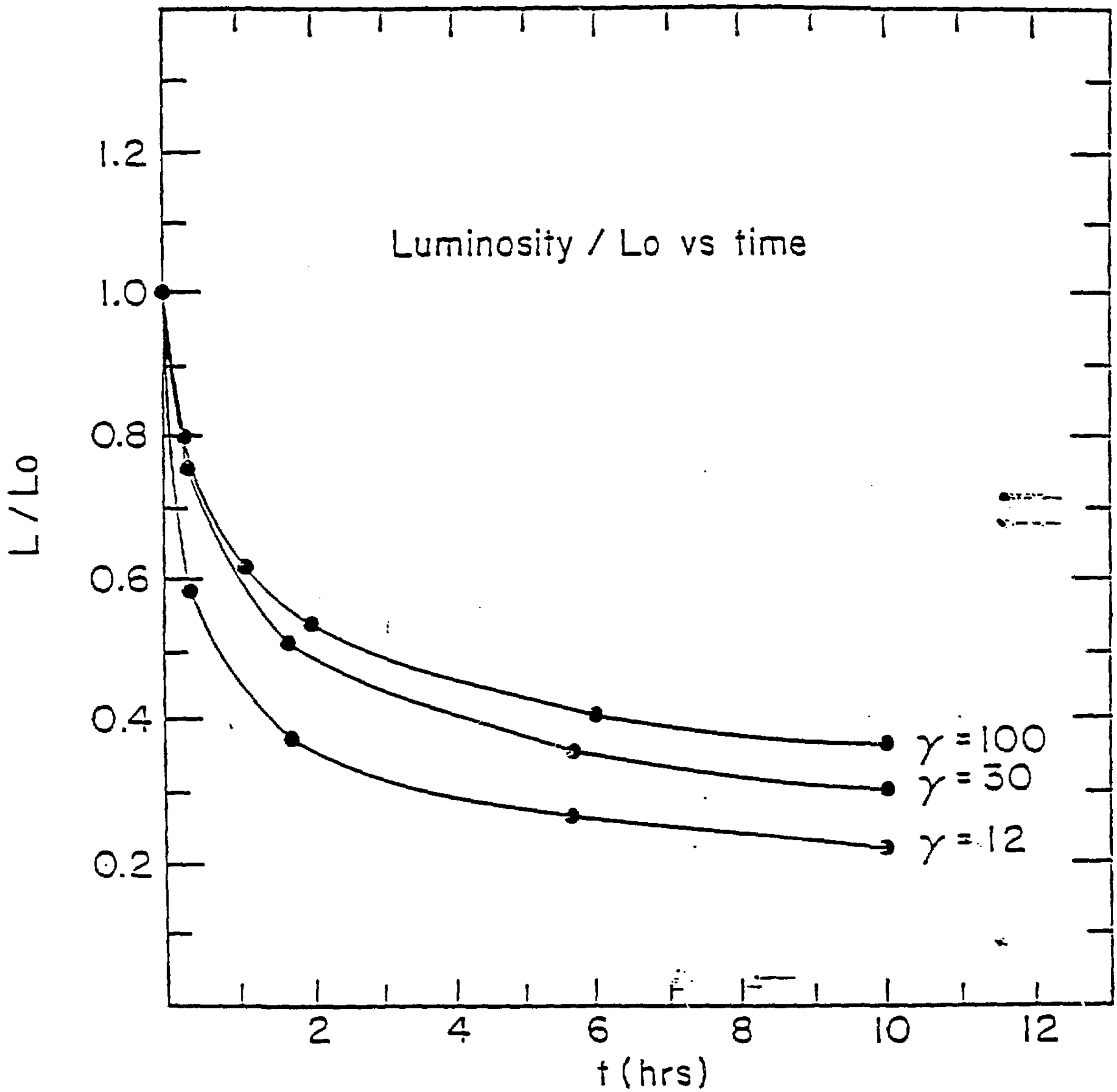
σ_p vs γ
Au
t=10
t=2
t=0
Δp/p bucket
t=10
t=2
t=0
Δp/p
bucket
4.0
3.0
2.0
1.0
0
20
40
60
80
100
120
$2.5\ \sigma_p/10^{-3}$
γ

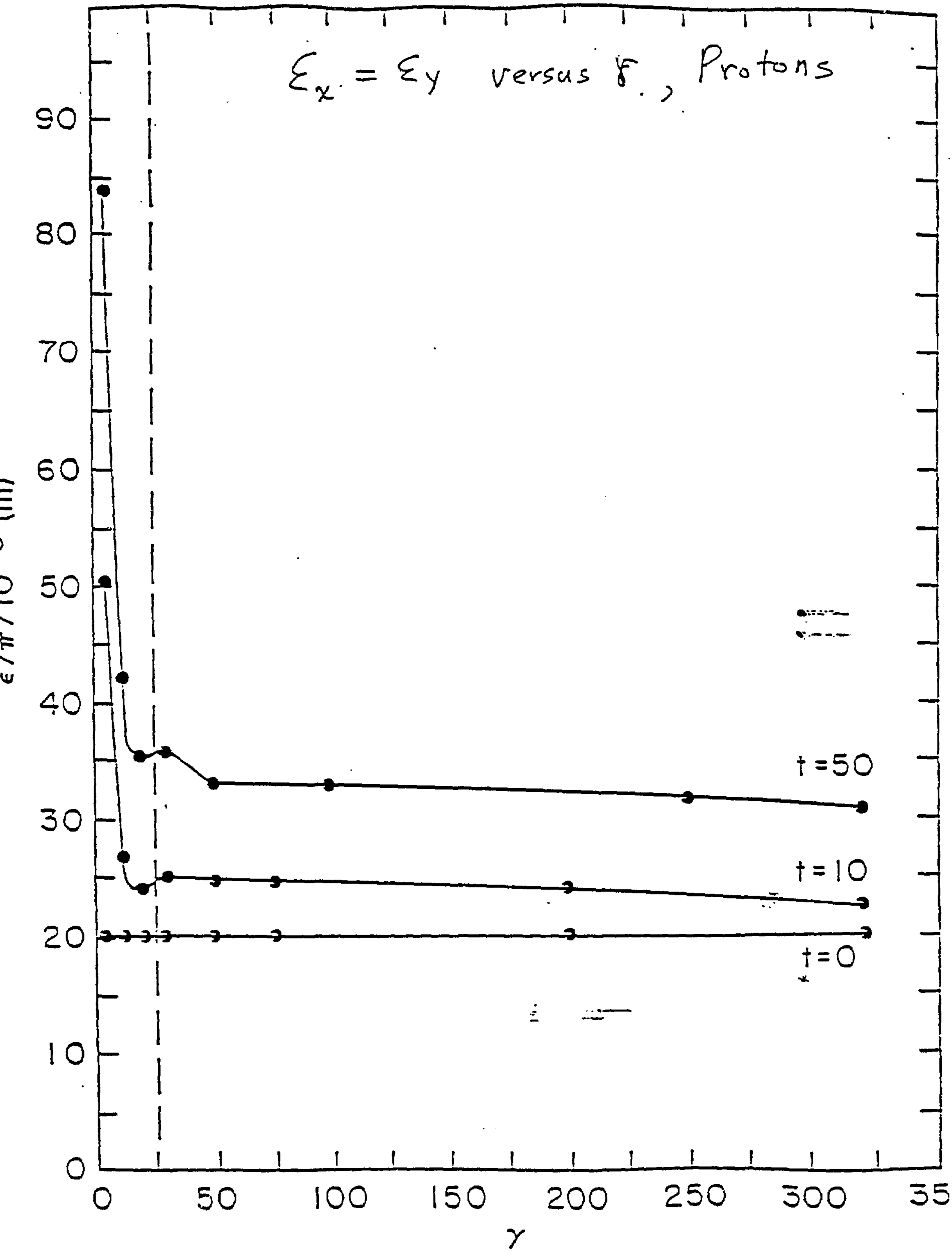


σ_ℓ vs. γ
Gold
t = 10 hrs
t = 2
t = 0
σ_ℓ (cms)
180
160
140
120
100
80
60
40
20
0
20
40
60
80
100
120
γ

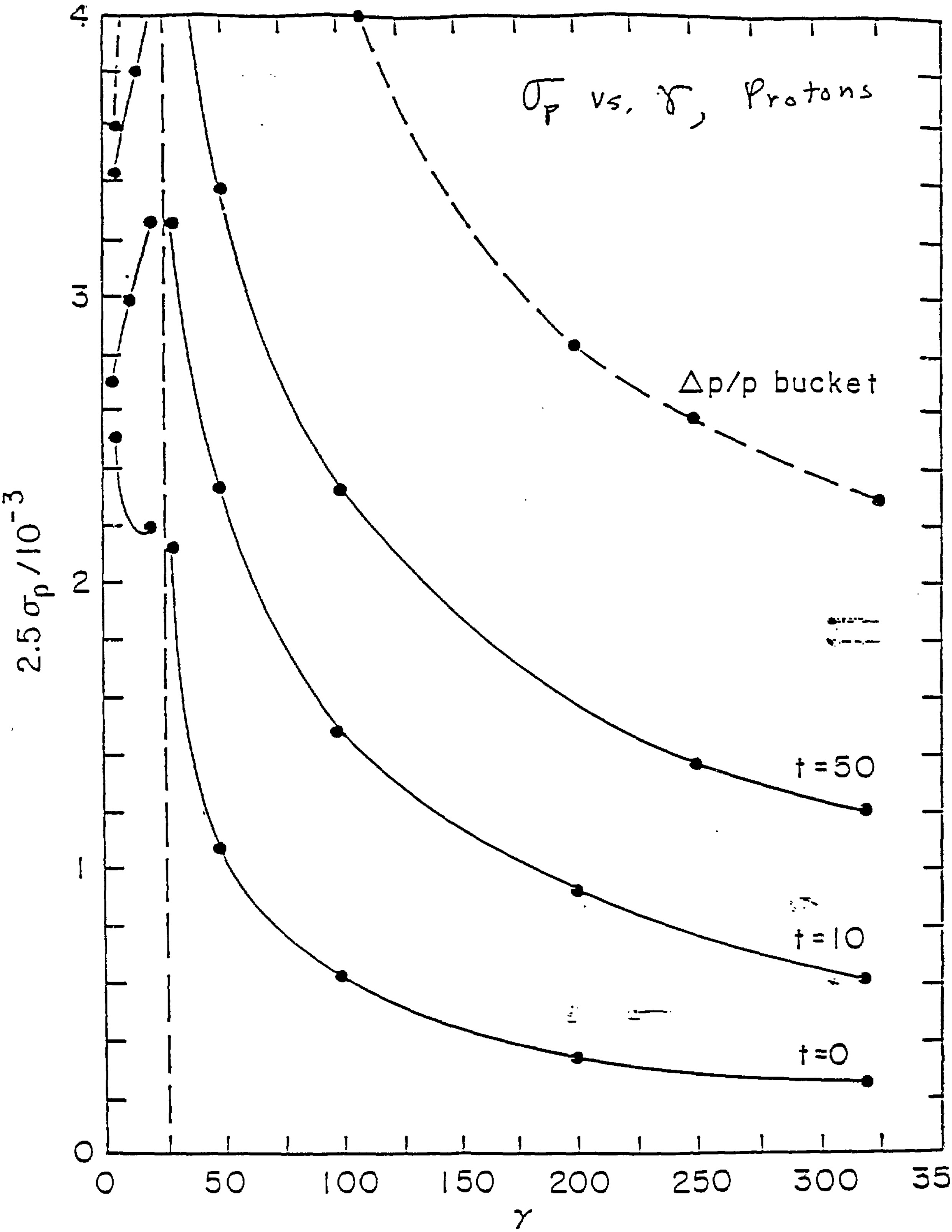

Average Luminosity / Initial Luminosity
Vs. γ , Au.
t = 2 hrs
t = 10
L/Lo
γ
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
120


Luminosity vs. γ, Au.
L/10^26
9
8
7
6
5
4
3
2
1
20
40
60
80
100
120
γ
α=0, t=2
α=0, t=10
α=2, t=2
α=2, t=10

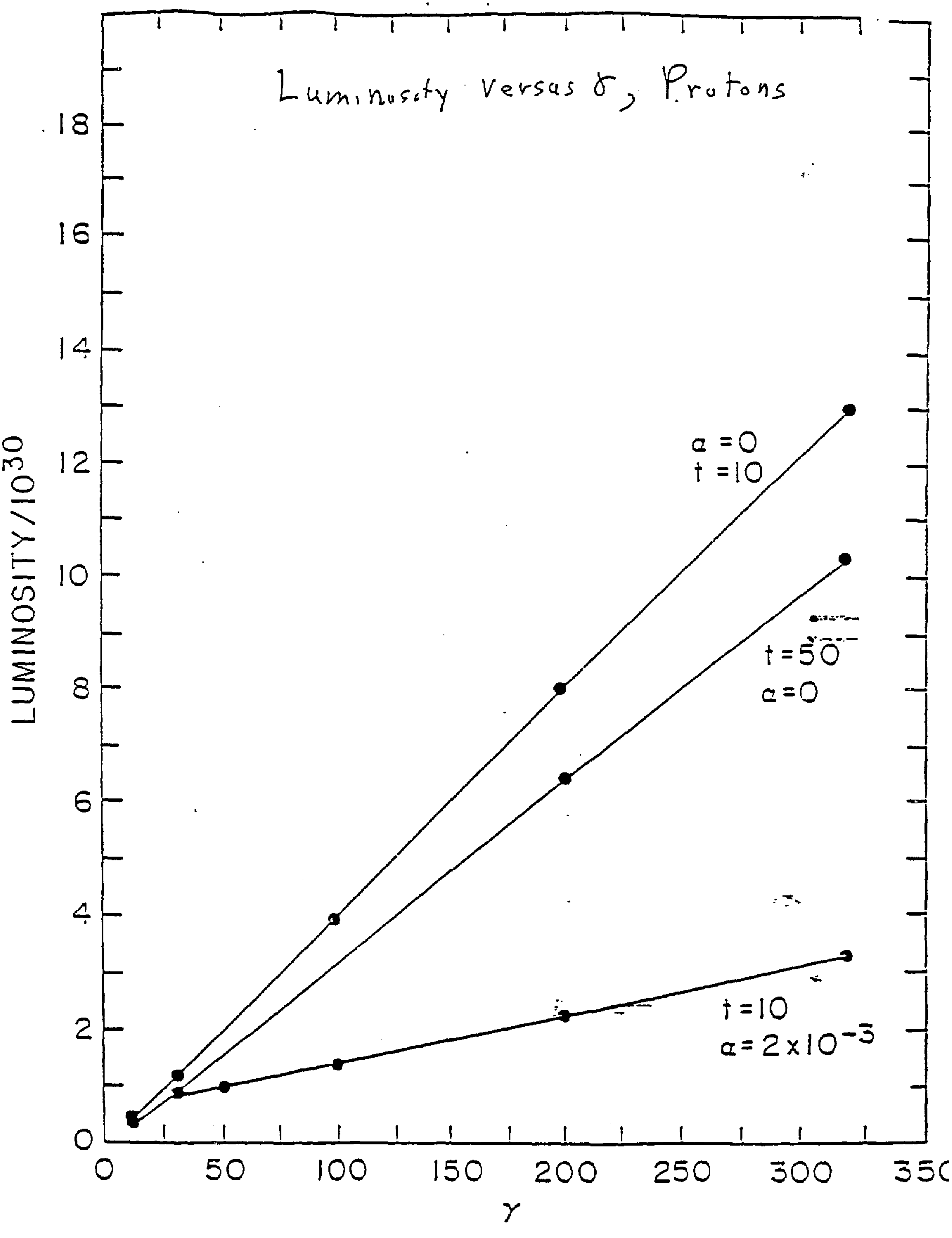

Luminosity / Lo vs time
L / Lo
1.2
1.0
0.8
0.6
0.4
0.2
t (hrs)
2
4
6
8
10
12
γ = 100
γ = 30
γ = 12



$\mathcal{E}_x = \mathcal{E}_y$ versus γ, Protons
$\epsilon/\pi/10^{-6}$ (m)
t=50
t=10
t=0
γ
0
50
100
150
200
250
300
350
10
20
30
40
50
60
70
80
90



σ_p vs. γ, Protons
Δp/p bucket
t=50
t=10
t=0
$2.5\,\sigma_p/10^{-3}$
γ
0
1
2
3
4
0
50
100
150
200
250
300
35



Luminosity versus γ, Protons
LUMINOSITY / 10^{30}
γ
α = 0
t = 10
t = 50
α = 0
t = 10
α = 2 x 10^{-3}
0
2
4
6
8
10
12
14
16
18
0
50
100
150
200
250
300
350

Dependence of Beam Size on Intensity

Dependence is weak

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{N_b}{\text{6-dimensional Phase Space}} = \frac{N_b}{\sigma^6} .$$

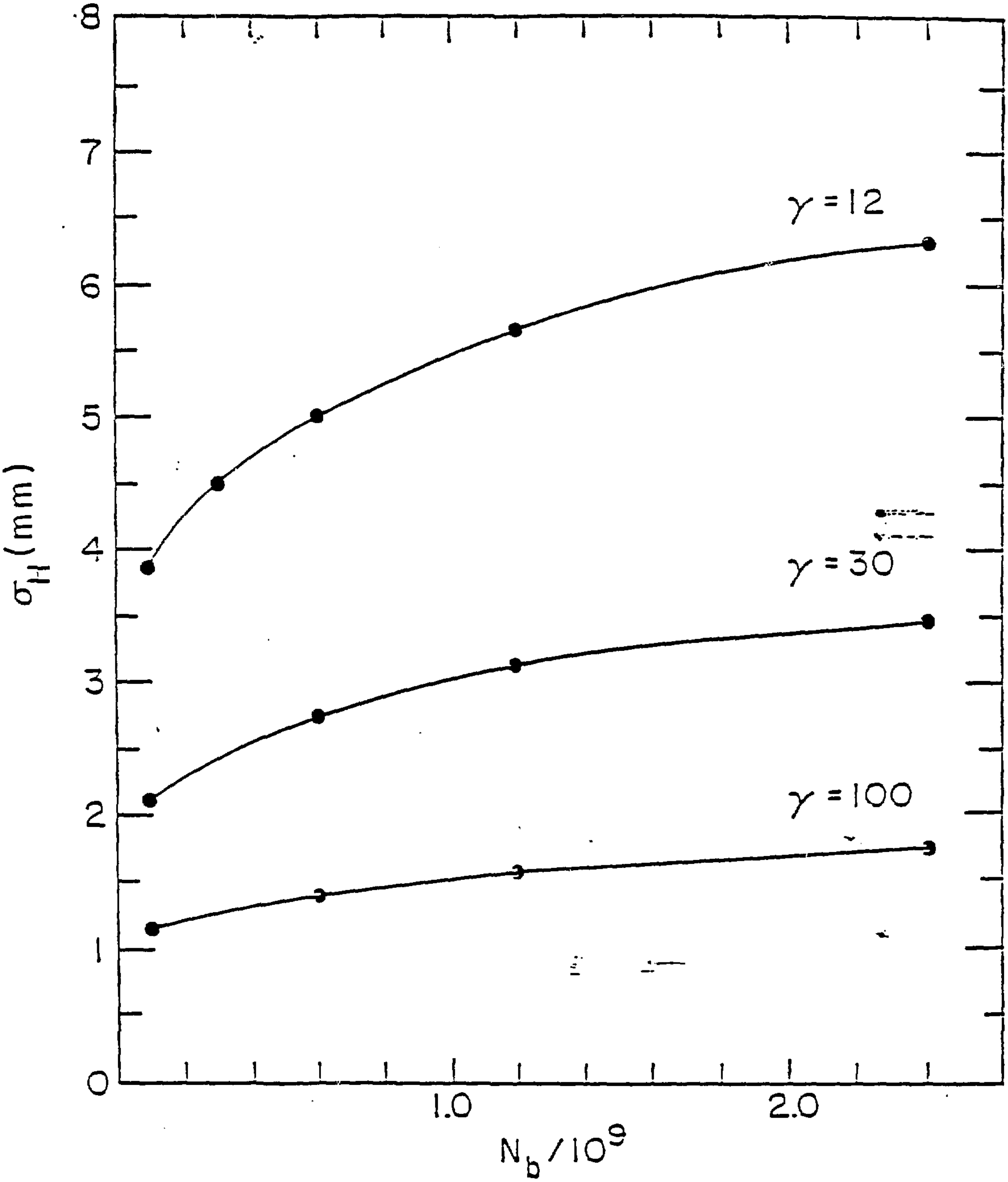
A small change in σ can compensate for a large change in N_b.

For the final state (full coupling)

$$\sigma_x \sim N_b^{1/6} ,$$

$$\sigma_p \sim N_b^{1/6} .$$

σ_x versus N_b Au



Results for a beam of gold ions showing the effects of coupling on the beam growth due to intrabeam scattering.

γ	12	12	30	30	100	100
Coupling	No	Yes	No	Yes	No	Yes
<u>Initial Beam</u>						
$\sigma_p/10^{-3}$	.678	.678	1.261	1.26	.359	.359
σ_ℓ (cms)	63.3	63.3	45.2	45.2	47.7	47.7
ε_x/π (mm-mr)	10	10	10	10	10	10
ε_y/π (mm-mr)	10	10	10	10	10	10
<u>Final Beam</u>						
t = 10 hrs						
ε_x/π (mm-mr)	44.3	44.5	32.4	33.2	29.3	27.8
ε_y/π (mm-mr)	43.5	44.5	23.3	33.2	9.82	27.8
$\sigma_p/10^{-3}$	1.560	1.563	1.990	1.985	1.216	1.099
σ_ℓ cm	146	146	71.4	71.2	162	146
$\sigma_E = X_p\sigma_p$ (mm)	2.17	2.17	2.77	1.76	1.69	1.53
σ_x (mm)	5.62	5.64	3.04	3.08	1.66	1.54
σ_y (mm)	5.57	5.64	2.58	3.08	.961	1.54
<u>Beam Half-Width</u>						
$2.5\ (\sigma_x + \sigma_E)$ (mm)	19.1	10.1	14.2	14.3	8.20	7.52
$2.5\ \sigma_y$ (mm)	13.6	14.1	6.32	7.70	2.35	3.85

Intrabeam Scattering with Coupling

At high γ, $\gamma > \gamma_t$, vertical growth rate is almost zero. Coupling might reduce growth rates by as much as a factor of 2 (A. Ruggiero).

However, smaller growth rates need not cause large changes in the final state of the beam after 10 hours.

$$\text{Growth rate} \simeq \frac{1}{\sigma}\frac{d\sigma}{dt} \simeq \frac{N_b}{\text{6-dimensional Phase Space}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dt} \sim \frac{N_b}{\sigma^6}$$

Factor 2 in growth rate can be compensated for by changing σ by $2^{1/6}$ which is a 12% change σ.

Intrabeam Scattering with Complete Coupling

It is assumed that

$$\epsilon_t = \epsilon_x (x,x') + \epsilon_y (y,y') \text{ is}$$

a constant of the motion. One assumes for $\rho(x,x',y,y')$

$$\rho(x,x',y,y') = \exp [-\epsilon_t(x,x',y,y')/\bar{\epsilon}]$$

where $\bar{\epsilon} = \bar{\epsilon}(t)$ grows slowly with time

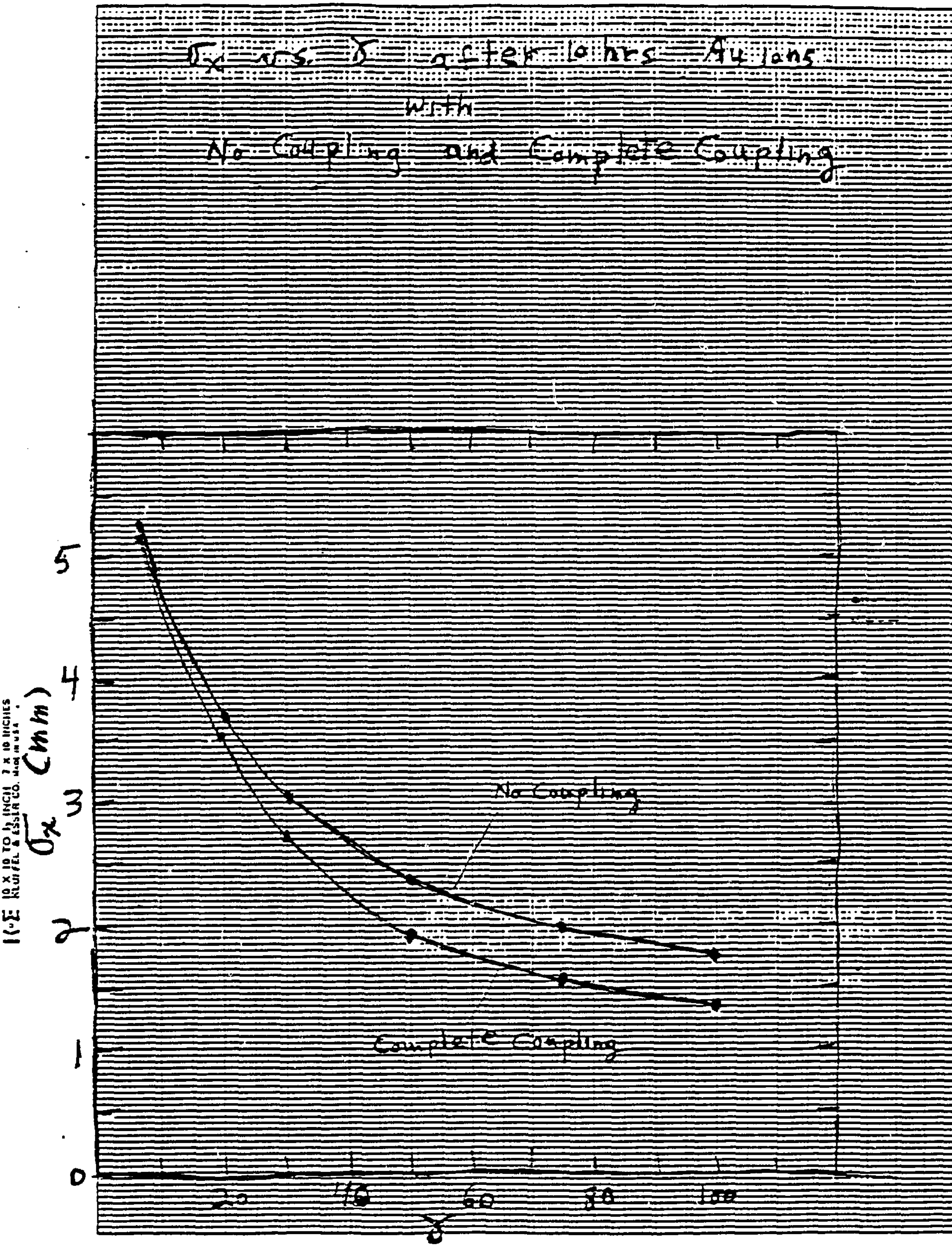
$$\bar{\epsilon} = \frac{1}{2} \bar{\epsilon}_t = \frac{1}{2} \int dxdx'dydy' \; \rho(x,x',y,y') \; \epsilon_t(x,x',y,y')$$

Eqs. (1), slide 3, are replaced by

$$\frac{1}{\epsilon_t} \frac{d}{dt} \bar{\epsilon}_t = \frac{1}{2} [f_1(\bar{\epsilon}_t/2, \bar{\epsilon}_t/2, \sigma_p) + f_2 (\bar{\epsilon}_t/2, \bar{\epsilon}_t/2, \sigma_p]$$

$$\frac{1}{\sigma_p} \frac{d}{dt} \sigma_p = f_3 (\bar{\epsilon}_t/2, \bar{\epsilon}_t/2, \sigma_p) \quad .$$

These emittances can be integrated to find $\bar{\epsilon}_t$, σ_p as functions of t.


σx vs. γ after 10 hrs Au ions
with
No Coupling and Complete Coupling
No Coupling
Complete Coupling
σx (mm)
γ
K+E 10 X 10 TO 1/2 INCH 7 X 10 INCHES KEUFFEL & ESSER CO. MADE IN U.S.A.
0
1
2
3
4
5
20
40
60
80
100