CONF-8805175--2

June 1988 BNL-41495

CHIRAL GAUGE THEORY ON A LATTICE*

Are anomalous gauge theories renormalizable ?

Sinya Aoki

Physics Department
Brookhaven National Laboratory
Upton, NY 11973, USA

BNL--41495

DE88 014288

ABSTRACT

We discuss the quantization of chiral gauge theories by lattice regularization, carefully treating the effects of the chiral anomaly. We derive a chiral gauge invariant lattice fermion action from a chiral gauge variant Wilson fermion action without changing its partition function. By lattice power counting for this formula we show that anomalous gauge theories as well as anomaly-free gauge theories are renormalizable even in 4-dimensions. Some applications and implications of this result and problems therein are discussed.

* Talk at the Lattice Higgs Workshop, SCRI, Florida State University, May 16-18, 1988.

1.PROBLEM OF CHIRAL GAUGE THEORY

1.1 Motivation

Lattice regularization[1] provides us with a powerful tool to investigate Quantum Fields Theories, such as QCD or Gauge-Higgs system. However successes of lattice regularization have been limited to purely bosonic systems or to systems with fermion-gauge field vector-like interactions. The reason for this is that we can not define the lattice fermion coupled to gauge field in a chirally invariant way[2]. This fact is closely related to the local gauge anomaly[3]. Because the lattice regularization is well-defined, an anomalous symmetry cannot be maintained on a lattice. In the continuum case, the anomalous symmetry is broken by the regularization, for example, Pauli-Villars regularization or dimensional regularization.

From this observation, it seems possible to construct a chirally gauge invariant action for the anomaly free case. However it is not so easy. There are two ways of breaking chiral gauge symetry: one is by the anomaly, the other is the breaking which can be written as gauge variant local terms. Anomaly free means that a pure anomaly is canceled out among all fermions contributions; however it does not mean the cancellation of the gauge variant local terms. We should add by hand the gauge variant local counterterms order by order to recover the gauge invariance of the final results. Furtheremore the cancellation of a pure anomaly is only true for the infinite cut-off limit. Because of the above two reasons, we cannot generally have chiral gauge invariant action with finite cut-off even for the anomaly free case.

In this talk we will discuss the quantization of anomalous chiral gauge theories as well as anomaly free theories by using lattice regularization. One important physical motivation for this work is to put the full Weinberg-Salam model(with fermions) on a lattice in order to investigate it by non-perturbative methods such as Strong Coupling Expansion or Monte-Carlo simulation. The other theoretical motivation is to investigate the consistencey; for example, renormalizability or unitarity, of anomalous gauge theories.

In this talk we concentrate on this point and show that anomalous gauge theories are renormalizable in the sense of lattice power counting[4].

1.2 Quantization of anomalous gauge theories in the continuum approach

First we briefly sketch how difficut is the quantization of anomalous gauge theories in the continuum approach by following a paper by Gross and Jackiw[5].

We denote action by $S(q)$ where q represents any field involved in the theory. In this short-hand notation a gauge transformation is written as

$$q'(x) = q(x)^{\omega(x)}, \qquad \omega(x) = \exp[i\theta(x)]$$

where $\omega(x) \in U(N), SU(N)$ or other Lie groups, is a gauge function. For example, $q'(x)$ is explicitly given by

$$\begin{aligned}
\psi'(x) &= [\omega(x)P_L + P_R]\psi(x) \\
\bar{\psi}'(x) &= \bar{\psi}(x)[\omega^{-1}(x)P_R + P_L] \\
A'_\mu(x) &= \omega(x)A_\mu(x)\omega^{-1}(x) + \frac{1}{g}\omega^{-1}(x)\partial_\mu\omega(x).
\end{aligned}$$

In order to make a perturbative expansion (more precisely, to get a propagator for a gauge field) we have to fix the gauge. That is achieved by inserting ΓΓΓΓ

$$\prod_x \int d\omega(x) \exp[S_{GF}(q^{\omega^{-1}})] = 1$$

into the partition function. Here the Faddeev-Popov determinant[6] is included in the gauge fixing function S_{GF}. Then the partition function becomes

$$\begin{aligned}
Z &= \int \mathcal{D}q\ \exp[S(q)] \int \mathcal{D}\omega\ \exp[S_{GF}(q^{\omega^{-1}})] \\
&= \int \mathcal{D}q^\omega\, \mathcal{D}\omega\ \exp[S(q^\omega) + S_{GF}(q)] \\
&= \int \mathcal{D}q\, \mathcal{D}\omega\ \exp[S_{eff}(q,\omega) + S_{GF}(q)]
\end{aligned}$$

where we define

$$\mathcal{D}q\ \exp[S_{eff}(q,\omega)] \equiv \mathcal{D}q^\omega\ \exp[S(q^\omega)], \qquad S_{eff}(q,1) = S(q).$$

Since the theory is anomalous,

$$S_{eff}(q,\omega) = S_{eff}(q,1) + \int dx\ \theta(x)\mathcal{A}(x) + O(\theta^2)$$

for infinitesimally small θ , where $\mathcal{A}(x)$ is the so-called non-Abelian anomaly[3]. For finite θ,

$$S^{WZ}(q,\omega) \equiv S_{eff}(q,\omega) - S_{eff}(q,1)$$

is nothing but the Wess-Zumino term[7]. The field ω is called the Wess-Zumino scalar. Finally we get

$$Z = \int \mathcal{D}q\,\mathcal{D}\omega\ \exp[S(q) + S^{WZ}(q,\omega) + S_{GF}(q)].$$

This form was first obtained by Harada-Tsutsui[8] and it is equivalent to the proposal by Fadeev-Shatashivili[9] who add the Wess-Zumino term to the original action by hand to quantize the anomalous gauge theory. We have to integrate both q and ω interacting through the Wess-Zumino term to quantize anomalous gauge theory correctly.

It is easy to see that $S(q) + S^{WZ}(q,\omega)$ has gauge invariance such that

$$q'(x) = q^h(x), \qquad \omega'(x) = h^{-1}\omega(x).$$

From this property there arises a hope that the theory defined by $S(q) + S^{WZ}(q,\omega)$ may be renormalizable. However, since $S^{WZ}(q,\omega)$ is highly non-linear in 4-dimensions, it is difficult to work with it.

2. LATTICE QUANTIZATION

Next let us apply the above procedure to the lattice action. From the observation in the continuum case, we should take a gauge ΓΓΓΓΓBEtion at the begining in order to quantize an anomalous gauge theory. Therefore we take the Wilson fermion action[10] for the fermionic part of the action, which is given by

$$
\begin{aligned}
S_F &= \int \bar{\psi}\gamma_\mu(D_\mu P_L + \partial_\mu P_R)\psi + ar\bar{\psi}\partial^2\psi \\
&= a^d \sum_{n,\mu} \frac{1}{2a}\bar{\psi}_n[(U_{n,\mu}P_L + P_R)\psi_{n+\mu} - (U^\dagger_{n-\mu,\mu}P_L + P_R)\psi_{n-\mu}] \\
&\quad - a^d \sum_{n,\mu} \frac{r}{2a}\bar{\psi}_n(\psi_{n+\mu} + \psi_{n-\mu} - 2\psi_n) \\
&\equiv S_{\Gamma\Gamma I}(\varphi) + S_W(\varphi).
\end{aligned}
$$

Here φ represents $U_{n,\mu}, \bar{\psi}$ and ψ , $S_{\Gamma\Gamma I}$ is a gauge invariant part of S_F and S_W is a gauge variant part of S_F, that is, the Wilson term. The total action is $S_F + S_G$ where S_G is the pure gauge action. We call a field φ of this original action the physical field and denote it by φ^p, if necessary. An gauge transformation is given by $\varphi'_n = \varphi^g_n$ and explicitly written as

$$
\begin{aligned}
\psi'_n &= (g_n P_L + P_R)\psi_n \\
\bar{\psi}'_n &= \bar{\psi}_n(g^\dagger_n P_R + P_L) \\
U'_{n,\mu} &= g_n U_{n,\mu} g^\dagger_{n+\mu}.
\end{aligned}
$$

As in the continuum case, we can insert unity :

$$
\prod_n \int dg_n \exp S_{GF}(\varphi^{g^{-1}}) = 1
$$

into the partition function to fix the gauge in the perturbative expansion. However , for non-pereturbative calculations it is not necessary to fix the gauge on the lattice. In that case we can take $S_{GF} = 0$ since $\int dg_n = 1$. This is possible only on a lattice. By inserting the gauge fixing condition into the partition function and making a change of integration variables such as $\varphi^r_n = \varphi^{g_n^{-1}}_n$ where we call this new field a renormalizable field and attach the suffix r, we get

$$
\begin{aligned}
Z &= \int \mathcal{D}g\, \mathcal{D}\varphi\ \exp[S_{\Gamma\Gamma I}(\varphi^g) + S_W(\varphi^g) + S_{GF}(\varphi)] \\
&= \int \mathcal{D}g\, \mathcal{D}\varphi\ \exp[S_{\Gamma\Gamma I}(\varphi) + S_W(\varphi^g) + S_{GF}(\varphi)]
\end{aligned}
$$

where the suffix r is omitted and

$$S_W(\varphi) = a^d \sum_{n,\mu} \frac{r}{2a}[\bar\psi_n g_n^\dagger P_R(\psi_{n+\mu} + \psi_{n-\mu} - 2\psi_n) + (\bar\psi_{n+\mu} + \bar\psi_{n-\mu} - 2\bar\psi_n) g_n P_L \psi_n].$$

This term represents the violation of gauge invariance and includes all information concerning anomaly. For example the non-Abelian anomaly can be obtained by this term in lattice perturbation theory[11].

It is noted that $S_{\Gamma\Gamma}(\varphi) + S_W(\varphi^g)$ also is gauge invariant under

$$\varphi_n' = \varphi_n^{h_n}, \qquad g_n' = h_n^{-1} g_n.$$

Because the above action is bilinear for fermions, it is possible to do Monte-Carlo simulation with this action. This form[4,12] is also suitable for a perturbative calculation of the 2-dimensional chiral Schwinger model[13].

Since there are no coupling among g_n in the action, we can integrate the g_n field[4] explicitly. We define

$$\exp[K(\varphi)] = \prod_n \int dg_n \ \exp[S_W(\varphi^g)] \qquad ;$$

then we get , for example, for $g_n \in U(1)$,

$$\exp[K(\varphi)] = \prod_n I_0(2\sqrt{\bar A_n A_n})$$

$$\bar A_n = -a^d \frac{ar}{2} \sum_\mu \bar\psi_n P_R \partial_\mu^2 \psi_n$$

$$A_n = -a^d \frac{ar}{2} \sum_\mu (\partial_\mu^2 \bar\psi_n) P_L \psi_n.$$

Furthermore, because of the Grassmann property of fermions , $K(\varphi)$ is finite polynomial of $\bar\psi$ and ψ . For example, $g_n \in U(1)$ in 2 dimensions, it is given by

$$K(\varphi) = a^{2d} \frac{a^2 r^2}{4} \sum_n \sum_{\mu,\nu} \bar\psi_n P_R \partial_\mu^2 \psi_n \times (\partial_\nu^2 \bar\psi_n) P_L \psi_n$$

and $g_n \in U(1)$ in 4 dimensions, it is given by

$$K(\varphi) = a^{2d}\frac{a^2r^2}{4}\sum_n\sum_{\mu,\nu}\bar{\psi}_n P_R \partial_\mu^2 \psi_n \times (\partial_\nu^2 \bar{\psi}_n) P_L \psi_n$$
$$-\frac{1}{4}a^{4d}\sum_n [\frac{a^2r^2}{4}\sum_{\mu,\nu}\bar{\psi}_n P_R \partial_\mu^2 \psi_n \times (\partial_\nu^2 \bar{\psi}_n) P_L \psi_n]^2$$

It is noted that $K(\varphi)$ for general g_n is also local and gauge invariant but non-linear.

3. RENORMALIZABILITY AND PHYSICAL INTERPRETATION

From lattice power counting[14], which means that we count the order of lattice spacing a of the given diagram, for the non-linear action $S_{\Gamma\Gamma}(\varphi) + K(\varphi)$, we have shown[4] that the superficial degree of divergence D of a given Fenyman diagram in four dimensions is

$$D = 4 - (E_G + E_g) - \frac{3}{2}E_f$$

where E_G, E_g and E_f are the numbers of external lines for gauge field, ghost field, and fermion, respectively. This shows that anomalous gauge theories as well as anomaly free theories are renormalizable even in four dimensions. The effect of the anomaly induced by g integral is renormalizable. If we add the kinetic term for the g field to the action before the g integration, we can identify g as a polar part of the Higgs field[15]. However this makes the g integration impossible and the theory un-renormalizable.

There are some remarks.

1) Though the physical field φ^p does not have species doubling modes, the gauge invariant, non-linear action for the renormalizable field φ^r may have doubling problems. However it is stressed that we do not attempt to propose the chiral gauge invariant lattice action for the physical fields though Nielsen-Ninomiya's theorem[2] does not apply to the non-linear action. We introduce φ^r to show the renormalizability of the theory and the physical interpretation should be made for φ^p. This relation between φ^r and φ^p reminds us of

the relation between the renormalizable gauge and the unitary gauge in the gauge-Higgs system.

2) Since the action for φ^p is gauge variant, there exist a non-trivial relation between φ^p and φ^r. For example, to see the relation between fermion fields with $g_n \in U(1)$, we introduce a generating function for fermion field such as

$$Z(\eta,\bar{\eta}) = \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}U \;\; \exp[S_{\mathrm{Trr}} + S_W + \bar{\eta}\psi^p + \bar{\psi}^p\eta]$$
$$= \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}U \;\; \exp[S_{\mathrm{Trr}} + \bar{\eta}P_R\psi^r + \bar{\psi}^r P_L\eta] \prod_n I_0(2\sqrt{\bar{D}_n D_n})$$

where

$$\bar{D}_n = \bar{A}_n + \bar{\psi}^r_n P_R \eta_n, \quad D_n = A_n + \bar{\eta}_n P_L \psi^r_n.$$

From the above identity it is easy to get equations such that

$$< (P_R\psi^p_n)^\alpha(\bar{\psi}^p_m P_L)^\beta > =< (P_R\psi^r_n)^\alpha(\bar{\psi}^r_m P_L)^\beta >$$

$$< (P_L\psi^p_n)^\alpha(\bar{\psi}^p_m P_L)^\beta > =< (P_L\psi^r_n)^\alpha(\bar{\psi}^r_m P_L)^\beta A_n >$$

$$< (P_L\psi^p_n)^\alpha(\bar{\psi}^p_m P_R)^\beta > =< (P_L\psi^r_n)^\alpha(\bar{\psi}^r_m P_R)^\beta A_n \bar{A}_m > +\delta_{nm} < (P_L\psi^r_n)^\alpha(\bar{\psi}^r_n P_R)^\beta >$$

in 2-dimensions. Similar but more complicated equations can be also obtained in 4-dimensions. By using these equation we can calculate all vacuum expectation values of physical field φ^p if we know all vacuum expectation values of renormalized field φ^r.

3) If we interpret $\bar{\psi}^p$, ψ^p and A^p_μ as real physical fields, we can observe interesting properties as follows:

The physical fermion field is not confined because gauge invariance does not hold for the physical fields.

Mass generation for gauge field by quantum effect is allowed because of the same reason.

The above two properties may correspond to the fact that the weak gauge bosons have mass and the leptons are not confined.

4) It is noted that from the renormalizable theory , which has finite number of counterterms, by some non-linear transformation we may construct the theory which seems to be un-renormalizable because the number of counter terms is infinite. However in this case the

number of independent counter terms is finite. The non-linear sigma model in 4-dimensions may be one example of this type of theory[16]. It is interesting to see whether the anomalous theory is of this type or not.

Now we are at the starting point to investigate an (anomalous) chiral gauge theory on a lattice; we should carefully study both perturbative and non-perturbative properties of this theory. As the first the perturbative expansion for a non-linear action is now under investigation. After getting some information from that calculation we would like to do Monte-Carlo simulation in the near future.

I would like to thank Prof. T. L. Trueman for careful reading of the manuscript and useful comment. I also thank Prof. M. Creutz, Prof. R. Shrock and Dr. I-Hsiu Lee for useful discussion. This work was supported by the U.S. Department of Energy under contract DE-AC02-76CH00016.

REFERENCES

1 . K. G .Wilson, Phys. Rev. D10, 2445 (1974).

2 . H. B. Nielsen and M. Ninomiya, Nucl. Phys. B185, 20 (1981); B193, 173 (1981).

3 . W. A. Bardeen, Phys. Rev. 184, 1828 (1969).

4 . S. Aoki, Phys. Rev. Lett. 60, 2109, (1988); Phys.Rev. D37, ? (1988).

5 . D. J. Gross and R. Jackiw, Phys. Rev. D6, 477 (1972).

6 . L. D. Faddeev and V. Popov, Phys. Lett. 25B, 29 (1967).

7 . J. Wess and B. Zumino, Phys. Lett. 37B, 95 (1971).

8 . K. Harada and I. Tsutsui, Phys. Lett. 183B, 311 (1987).

9 . L. D. Faddeev and S. L. Shatashvili, Phys. Lett 167B, 225 (1986).

10. K. G. Wilson, 'Quark and Strings on a Lattice.' in: New Phenomina in Subnuclear

Physics, ed. A. Zichichi (Plenum, New York, 1977).

11. A. Coste, C. Korthals-Altes and O. Napoly, Phys. Lett. 179b, 125 (1986); S. Aoki, Phys. Rev. D35, 1435 (1987).
12. K. Funakubo and T. Kashiwa, Phys. Rev. Lett. 60, 2113 (1988).
13. R. Jackiw and R. Rajaraman, Phys. Rev. Lett. 54, 1219 (1985).
14. H. Kawai, R. Nakayama and K. Seo, Nucl. Phys. B189, 40 (1981).
15. L. H. Karsten, 'The lattice Fermion Problem and Weak Coupling Perturbation Theory.' in: Field Theoretical Method in Particle Physics, ed. W. Rühl (Plenum, New York, 1980); J. Smit, Acta Physinica Polonica B17, 531 (1986); P. V. D. Swift, Phys. Lett. 145B, 256 (1984).
16. P. Hasenfratz, in ; this workshop proceeding.