PPPL--2517

DE88 015867

Nonlinear Tearing Instabilities in Tokamaks with Locally Flattened Current Profiles

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Nonlinear tearing stability is evaluated for current profiles which are linearly stabilized by flattening the current in the neighborhood of the rational surface. When marginally stable to the linear instability, these profiles remain unstable in the presence of a small but finite island. The growth of the island saturates only when the island reaches the width it would have attained in the absence of flattening. Implications are discused for proposed methods of tearing mode stabilization and for theories of the tokamak sawtooth oscillation.

Investigations of linear tearing mode stability have often been led to consider current density profiles with reduced gradients, or entirely flattened currents, in some neighborhood of the low order rational surfaces. This sort of profile emerges naturally when one attempts to construct tearing stable current profiles[1,2] because of the sensitivity of linear tearing modes to the current gradient near the rational surface. Analysis of experimental data also leads to a consideration of current density profiles of this sort.[3] In this paper we determine the nonlinear stability of tearing modes for current profiles which have been linearly stabilized by flattening in the vicinity of the rational surface. We find that profiles which are marginally stable to linear tearing modes are nonlinearly unstable in the presence of small but finite islands. The growth of the islands saturates only when they reach the width they would have attained in the absence of flattening.

One conclusion that emerges from this work is that the use of a linear stability analysis as a guide to construct tearing stable current profiles is inadequate and can be misleading. Current profile modifications suggested by a linearized stability analysis often lead to a linearly stable but nonlinearly fragile equilibrium, formed by the bifurcation of the initially unstable axisymmetric equilibrium into a stable axisymmetric equilibrium and a nearby unstable nonaxisymmetric equilibrium.

For experimentally determined current profiles which are flat near the rational surface, we conclude that a linear stability analysis tells only part of the story. A linearly stable profile can be unstable to small but finite perturbations. The growth rate can increase rapidly as the island grows, giving a growth time much shorter than would otherwise be expected from a tearing mode going through marginal stability. As we will discuss, this suggests a possible resolution of one mystery that has arisen in connection with tokamak sawtooth oscillations.[4]

For our nonlinear analysis of tearing modes, we use Rutherford's theory of nonlinear tearing mode growth[5] and the quasilinear theory of tearing mode saturation of White *et al.*[6] Valid for small (but finite) islands, the quasilinear theory of tearing mode saturation was shown to agree well with numerical calculations. As we will discuss, our conclusions rely on the quasilinear saturation mechanism, but are not sensitive to the detailed assumptions made in Ref. 6 in calculating saturated island widths. Our analysis does not include finite β. We neglect toroidal effects in the island, but do not preclude toroidal effects on Δ'.

Because our analysis does not preclude toroidal effects on Δ', we will be able to discuss the $n = 1$, $m = 1$ mode. The growth rate of small $n = 1$, $m = 1$ islands near marginal stability is small, so inertia can be neglected, and Rutherford's analysis can be applied.

To establish the basic ideas, we first consider a slab model. The analysis is then redone in cylindrical geometry, where there are some additional wrinkles that enhance the effect. Finally, we discuss the implications of our results.

We first consider slab geometry. Equilibrium quantities are functions only of x, and are symmetric about $x = 0$. The poloidal flux is expressed as the sum of an equilibrium part and a perturbation,

$$\psi(x) = \psi_0(x) - \psi_1(x)\cos(ky). \tag{1}$$

There is no z dependence.

Stability of the tearing mode is expressed in terms of the solution of the equation for the exterior region,

$$\psi_1'' - k^2\psi_1 = -(j_0'/\psi_0')\psi_1, \tag{2}$$

where $j_0(x)$ is the equilibrium current density and the prime denotes d/dx. Following Ref. 6 we define

$$\Delta'(w) \equiv [\psi_1'(w) - \psi_1'(-w)]/\psi_1(x = 0). \tag{3}$$

For $w = 0$, this corresponds to the conventional Δ' used in linear stability analysis.[7,8] The tearing mode is linearly unstable when $\Delta'(0) > 0$, and linearly stable when $\Delta'(0) < 0$. Stability of the linearized equations does not, however, preclude the possibility that a small but finite island will grow.

The analysis of Ref. 6 shows that the growth of thin islands is governed by the equation

$$dw/dt = 1.66\eta[\Delta'(w) - \alpha w], \tag{4}$$

where w is the width of the island (assumed to be greater than the width of the resistive layer), η is the local resistivity, and α is a complicated expression depending on ψ_0 and j_0. It follows that the island growth saturates when

$$\Delta'(w) - \alpha w = 0. \tag{5}$$

For current profiles of practical interest the contribution of α is small,[9,10] and the saturation condition is well approximated by

$$\Delta'(w) = 0. \tag{6}$$

The nonlinear stability of small but finite islands is determined by the quantity $\Delta'(w)$. We will see that it is possible to have $\Delta'(0) < 0$ while $\Delta'(w) > 0$ for some small value of w. When this is the case, any small islands which are present will grow, regardless of the linear stability.

The analysis of this paper relies on the saturation condition of Eq. (6), and on the observation that an island of width w grows when $\Delta'(w) > 0$ (more precisely, when $\Delta'(w) - \alpha w > 0$). We neglect α. We make no use of the value of the coefficient in front of the right-hand side of Eq. (4). Nor do we use the value of α calculated in Ref. 6, except insofar as it demonstrates that the α term may be neglected. Our conclusions are, therefore, insensitive to the assumptions concerning the current profile in the island, etc. made in Ref. 6 to calculate these quantities.

Now suppose that our equilibrium current profile, $j_0(x)$, has been flattened in the region $-x_1 < x < x_1$. For $|x| < x_1$, the solution to Eq. (2) can be expressed as,

$$\psi_1 = A\cosh(kx) \pm B\sinh(kx),$$

where the plus (minus) signs correspond to $x > 0$ $(x < 0)$. Substituting this into Eq. (3), we get

$$\Delta'(w) = 2k[C\cosh(kw) + \sinh(kw)],$$

for $0 \leq w \leq x_1$, where $C = A/B$. Solving for C in terms of $\Delta'(0)$ gives

$$\Delta'(w) = [\Delta'(0) + 2k\tanh(kw)]\cosh(kw) \tag{7}$$

for $0 \leq w \leq x_1$. In this equation, the quantity in square brackets is a monotonically increasing function of w.

Figure 1 is a sketch of $\Delta'(w)$. (The case shown corresponds to $\Delta'(0) < 0$.) For $w \geq x_1$, $\Delta'(w)$ has the same value as it does in the absence of the flattening (assuming that the equilibrium current profile is unchanged for $|x| > x_1$). For $w < x_1$, $\Delta'(w)$ is governed by Eq. (7). The value of $\Delta'(0)$ is determined by demanding continuity at $w = x_1$. As x_1 is increased (i.e.

the region over which the current is flattened is increased in size), $\Delta'(0)$ is decreased. The discontinuity in the derivative of $\Delta'(w)$ at $w = x_1$ is caused by the discontinuity of j' at $x = \pm x_1$ in our analytically soluble model. We could clearly smooth the current profile in the neighborhood of $x = x_1$, and thereby remove the discontinuity in the derivative of Δ', without changing the value of Δ' very much.

When the linearized tearing mode is marginally stable, $\Delta'(w)$ is positive for $0 < w \leq x_1$. An island whose width is greater than that of the resistive layer is nonlinearly unstable, and it continues to grow until its width is greater than the width over which the current profile has been flattened. When the island width is greater than x_1, $\Delta'(w)$ has the same value as it does in the absence of the flattening. The island growth saturates when the island reaches the width it would have attained in the absence of flattening. Although flattening the current profile in the neighborhood of the rational surface has stabilized the linear tearing mode, it has had no effect at all on the saturated island width.

A slight further increase in x_1 (the width of the region over which the current profile is flattened) makes $\Delta'(0)$ negative but small. (This corresponds to the case shown in Fig. 1.) There is now a small but finite critical island width, w_c. An island whose width is less than w_c shrinks. An island whose width is just above w_c grows, and saturates only when it reaches the width it would have attained in the absence of flattening, w_s. An island whose width is exactly equal to w_c represents an unstable nonaxisymmetric equilibrium.

Equation (7) gives

$$w_c = \tanh^{-1}[-\Delta'(0)/2k]/k \approx -\Delta'(0)/(2k^2).$$

When $w_c = x_1$, $\Delta'(w) \leq 0$ throughout the region $w \leq x_1$, and the tearing mode is completely stable. This corresponds to flattening the current profile over a region whose width is equal to that of the saturated island in the absence of flattening. In practice we may be satisfied with a smaller value of w_c, depending on the magnitude of the resonant perturbation.

To suppress the tearing mode, it is desirable to modify the current profile in such a way that the saturated island width is reduced. When the tearing mode is completely stabilized, the stable equilibrium corresponding to the saturated island merges with the unstable axisymmetric equilibrium to produce a robustly stable axisymmetric equilibrium. If we flatten the current profile in a small region about the rational surface, in contrast, we decrease $\Delta'(0)$ without affecting the saturated island width. As we increase the width over which the current profile is flattened, we further decrease $\Delta'(0)$ until we reach marginal stability for the tearing mode, still without affecting the saturated island width. At marginal stability, the unstable axisymmetric equilibrium bifurcates to a stable axisymmetric equilibrium and a nearby unstable nonaxisymmetric equilibrium. The unstable equilibrium has an island of width w_c. Although the axisymmetric equilibrium is now stable, it remains sensitive to small but finite perturbations until the profile is flattened further. The linear stability analysis provides a poor guide to the suppression of tearing. We will see that the linear stability analysis leads us even further astray in the cylinder.

For our analysis, we have adopted an analytically soluble model with $j_0'(x) = 0$ for $|x| < x_1$. It is clear from Eq. (2) that nothing singular happens in the limit as $j_0'(x)$ approaches zero. If we had instead imposed a current profile with a small but nonvanishing gradient in the region $|x| < x_1$, the results would not be very different.

The slab model has been useful for establishing some of the basic ideas we wish to present, but it cannot be used to get a feeling for the magnitude of the effect in cases of practical interest. We will see next that the effect is greatly enhanced when the initial (unflattened) current profile has a nonvanishing j_0' at the rational surface.

We now extend our analysis to cylindrical geometry, with the ordering $B_z \gg B_\theta$ assumed. Equilibrium quantities are functions only of r. Perturbed quantities are functions of r and $m\theta + kz$. The helical symmetry allows the introduction of a helical flux function,

$$\psi(r) = \psi_0(r) - \psi_1(r)\cos(m\theta + kz). \tag{8}$$

Stability of the tearing mode is determined by the solution to

$$\psi_1'' + \frac{1}{r}\psi_1' - \frac{m^2}{r^2}\psi_1 = -\frac{j_0'}{\psi_0'}\psi_1, \tag{9}$$

where the prime now denotes d/dr. For the cylinder we define

$$\Delta'(w) \equiv [\psi_1'(r_0 + w) - \psi_1'(r_0 - w)]/\psi_1(r_0), \tag{10}$$

where r_0 denotes the location of the rational surface.

In the cylinder there is no symmetry about the rational surface, so that $\psi_1'(r_0 + w) \neq -\psi_1'(r_0 - w)$ in general. We define the quantity

$$s(w) \equiv (1/2)[\psi_1'(r_0 + w) + \psi_1'(r_0 - w)]/\psi_1(r_0). \tag{11}$$

(Our definitions of $s(w)$ and $\Delta'(w)$ differ slightly from those of Ref. 6. The definitions are equivalent to the order of w we retain, and our form of the definitions will be more convenient for our purposes.) We can express ψ_1'/ψ_1 in terms of $\Delta'(w)$ and $s(w)$,

$$\psi_1'(r_0 \pm w)/\psi_1(r_0) = s(w) \pm \frac{1}{2}\Delta'(w). \tag{12}$$

When j_0' is nonvanishing at the rational surface, Eq. (9) has a singularity there. The singularity dominates in the evaluation of $s(w)$, and allows us to obtain an analytical expression for this quantity,[6]

$$s(w) \approx -(j_0'/\psi_0'')\ln(w), \tag{13}$$

where the radial derivatives in this expression are evaluated at the rational surface, and we have used the approximation $\psi_0(r) \approx (1/2)(r - r_0)^2\psi_0''$ for $r - r_0$ small. If $j_0(r)$ is a decreasing function of r, j_0'/ψ_0'' is a negative quantity.

Suppose that the current profile is flattened in the region $r_0 - \delta_- < r < r_0 + \delta_+$, with $\delta_\pm > 0$. The solution to Eq. (9) gives

$$\psi_1'(r)/\psi_1(r_0) = m(C_\pm r^{m-1} - r^{-m-1})/(C_\pm r_0^m + r_0^{-m}), \tag{14}$$

where the plus and minus signs correspond, respectively, to the regions $r_0 < r < r_0 + \delta_+$ and $r_0 - \delta_- < r < r_0$. The coefficients C_- and C_+ can be determined in terms of the Δ' and s functions of the original (unflattened) current profile by matching at $r_0 \pm \delta_\pm$ using Eq. (12),

$$C_\pm = \frac{m(r_0 \pm \delta_\pm)^{-m-1} + [s_0(\delta_\pm) \pm \Delta_0'(\delta_\pm)/2]r_0^{-m}}{m(r_0 \pm \delta_\pm)^{m-1} - [s_0(\delta_\pm) \pm \Delta_0'(\delta_\pm)/2]r_0^m}, \tag{15}$$

where the "0" subscripts on s and Δ' refer to the original current profile.

We can use Eqs. (14) and (15) to determine the effect of the flattening on $\Delta'(0)$. For $\delta_\pm$ small we find that

$$\begin{aligned}\Delta'(0) \approx (1/2)[\Delta_0'(\delta_+) + \Delta_0'(\delta_-)] - (j_0'/\psi_0'')\ln(\delta_+/\delta_-) \\ - (j_0'/\psi_0'')[\delta_+\ln(\delta_+) + \delta_-\ln(\delta_-)]/r_0.\end{aligned} \tag{16}$$

We have discarded here terms of order $\delta_{\pm}$ and $\delta_{\pm}\ln(\delta_{-}/\delta_{-})$. The easiest way to make Eq. (16) negative is to take δ_{-}/δ_{-} small (subject to the constraint that δ_{-} is greater than the width of the resistive layer). The $\ln(\delta_{-}/\delta_{-})$ term then dominates. Flattening the current profile also reduces $\Delta'(0)$ due to the $\delta\ln(\delta)$ terms, and due to the fact that $\Delta'(w)$ is in general a decreasing function of w for the unflattened current profile. The linear tearing mode can be stabilized by flattening the current profile over a narrow region if δ_{+}/δ_{-} is sufficiently small.

Now we again consider what happens near marginal stability. Using Eqs. (13–15), assuming $\delta_{\pm}$ small, and expanding to first order in w, we find

$$\Delta'(w) \approx \Delta'(0) + [2m^2/r_0 + (j_0'/\psi_0'')\ln(\delta_{+}\delta_{-})]w/r_0, \qquad (17)$$

for $w < \min(\delta_{-}, \delta_{+})$. We have retained the $2m^2w/r_0^2$ term in this expression to make contact with the corresponding expression for the slab, Eq. (7), when $j_0' \to 0$. For nonvanishing j_0', the term proportional to j_0' dominates at small δ. As in the slab, $\Delta'(w)$ is an increasing function of w for small w. At marginal stability, the tearing mode is again nonlinearly unstable for an island of small but finite width. For $\Delta'(0)$ negative, but small, there is again a critical island width above which islands grow and below which they shrink. The rate of increase of $\Delta'(w)$ with w is larger than in the slab, leading to smaller values of the critical island width w_c. Equation (17) breaks down when $\Delta'(0)$ becomes sufficiently negative that $w_c > \min(\delta_{-}, \delta_{+})$. If δ_{+}/δ_{-} is small and δ_{-} is smaller than the width of the saturated island, there is still a critical island width $w_c < \delta_{-}$ above which the mode is nonlinearly unstable. $\Delta'(w)$ is then unchanged from the initial unflattened current profile for $w \geq \delta_{-}$. The island continues to grow until it attains the width it would have reached in the absence of flattening.

As a guide to construct tearing stable current profiles, the results of the linear stability analysis are now even more troubling than in the case of the slab. For the slab the linear stability analysis was at least leading us in the right direction. It suggested that the tearing mode would be stabilized if we flattened the current profile over a sufficiently broad region in the neighborhood of the rational surface, although it did not correctly determine how broad that region needs to be. For the cylinder, Eq. (16) suggests a strategy of nonsymmetric flattening over a very narrow region. By decreasing δ_{+}, we make $\Delta'(0)$ increasingly negative for a fixed value of δ_{-} (subject to the constraint that δ_+ remain larger than the width of the resistive layer). This leads to no improvement in the nonlinear stability.

We now consider the implications of our results. A linearized stability analysis assumes that modes are initially excited by infinitesimal perturbations. In practice we expect islands of finite width to exist in tokamaks, arising from field errors or originating in the tokamak start-up scenario. We must ask whether these finite perturbations grow or shrink. In addition, if we propose to use external means, such as rf, to modify the current profile and thereby control tearing modes, we must also consider the islands that begin growing before the profile is modified.

In general, in applying any linearized stability analysis we implicitly assume that the stability properties are not sensitive to the amplitude of the initial perturbation. However, linear tearing stability is sensitive to small changes in the current in the neighborhood of the rational surface. This current does not enter into the stability for a finite width island. There is cause for concern that a linearized analysis does not adequately represent the true stability of tearing modes. Our results show that such concern is justified for current profiles which are flat, or nearly flat, in the neighborhood of the rational surface. Linear stability can sometimes be a poor guide to the true stability properties of tearing modes.

Our conclusions bear particularly on attempts to use a linearized analysis to determine the nonlinear evolution of the plasma. One approach to studying the effects of plasma instabilities is to assume that the presence of instability causes the plasma to evolve to a marginally stable state. If the assumption is valid, it allows us to say a good deal about the nonlinear evolution of the instability in terms of its linearized stability properties. Our results indicate that this assumption is not valid for tearing modes. We might try to represent roughly a magnetic island of width $2w$ in an axisymmetric equilibrium by flattening the current profile from $r_0 - w$ to $r_0 + w$, where r_0 is the radius of the rational surface.[11] As we have seen, the further growth of the island is not well represented by the linear stability properties of the resulting axisymmetric equilibrium. Moreover, in flattening symmetrically from $r_0 - w$ to $r_0 + w$, we already make use of the results of a nonlinear analysis. The linearized analysis alone suggests a nonsymmetric flattening about the rational surface. When the flattening of the current profile is caused by stochastic regions due to island overlap, there are similar objections to applying a linearized stability analysis.

Nor is a linear analysis a useful guide if we want to modify the equilibrium current profile of a tokamak by external means. We now have some capability to control the current profile using auxiliary heating and nonohmic current drive. There have been a number of theoretical studies of tearing mode stabilization via current profile control, both in the context of particular schemes for modifying the current profile,[12-16] and in the context of a more general characterization of stable profiles.[1,2] Some of these studies have calculated the nonlinearly saturated island width, while other studies have relied on a linear analysis. Generally speaking, the nonlinear saturation has been calculated in studies where a computationally expensive model was used to determine the current profile modification, so that the nonlinear calculation represented a small additional effort. (Some of the studies, however, focused on effects due to current profile modification in the island,[13,14] and were therefore intrinsically nonlinear.) Studies which have attempted to map the stability boundaries for a range of parameters, or have looked at stability for a large range of different mode numbers, have generally relied on a linear stability analysis. It has been recognized that a nonlinear stability analysis is preferable to a linear analysis when the incremental computational effort required is small. There has not been a recognition of the dangers inherent in relying on a linear analysis as a guide to current profile modification. The linear analysis suggests a nonsymmetric flattening of the current profile in a small region about the rational surface as an efficient method of controlling the tearing mode. A nonlinear analysis shows that, as a result of this strategy, the unstable axisymmetric equilibrium bifurcates into a stable axisymmetric equilibrium and a nearby unstable nonaxisymmetric equilibrium. The axisymmetric equilibrium is stable but fragile. Perturbations large enough to push the system over the small potential hill to the nearby nonaxisymmetric equilibrium are unstable. A small but finite island grows, and saturates only when it attains the width it would have reached in the absence of flattening. We remark that to determine properly the nonlinear evolution of an island in the presence of nonohmic current drive, it is necessary to modify the quasilinear analysis to take into account the nonohmic current.[13]

For experimentally determined current profiles which are flat near the rational surface, our results suggest that a linear stability analysis tells only part of the story. In Ref. 3, current profiles are inferred from experimental measurements and adjusted slightly, within the experimental error bars, to give linear tearing stability. Near the $q = 1$ surface the current is nearly flat. (Because of toroidicity, j is not a constant on the flux surfaces, nor is the gradient of j.) The flattening is primarily on the inner side of the rational surface. In the notation of this paper, $\delta_- \ll \delta_+$. Although an island at the $q = 1$ surface could, in principle, cause a flat spot to appear in the measured current profile there, the experimental measurements indicate that there is no sizable island at the $q = 1$ surface, and that the flattening of the current is (at least approximately) axisymmetric.[17]

For the $n = 1$, $m = 1$ tearing mode, toroidal effects must be retained in Δ', which is otherwise infinite. The toroidal effects may be neglected in a narrow island if the aspect ratio at the $q = 1$ surface is sufficiently large. Similarly, toroidal effects may be neglected in a narrow region of flattened current about the $q = 1$ surface. The approximations we have made in this paper do not preclude treatment of the $n = 1$, $m = 1$ mode.

The profile of Ref. 3 is marginally stable to an $n = 1$, $m = 1$ tearing mode. Our results suggest that the stability in this case is a fragile one. A small island will grow. (This is consistent with a nonlinear numerical calculation, in which a small island was observed to grow, accelerate, and reconnect through the magnetic axis.[18]) As long as the island is sufficiently narrow, the growth rate remains small and inertia may be neglected. The nonlinear Rutherford analysis employed in this paper remains valid. When the growth rate becomes sufficiently large that inertial effects become important, the mode is no longer in the domain of validity of our analysis.

These conclusions suggest a possible resolution of one mystery that has arisen in connection with the sawtooth oscillation in tokamaks.[4] A new generation of experiments on large tokamaks has found sawthooth crash times too fast to be accounted for by the Kadomtsev[19] model. Models to account for this behavior are fundamentally limited by the Alfvén time scale. This is not in itself inconsistent with the experiments. The problem is that any such model must account for how the system goes through marginal stability. At any given time, the growth rate in such a model is $\gamma = \alpha(t)\tau_A$, where $\alpha(t) \ll 1$ measures how far the system has gone through marginal stability, and τ_A is the Alfvén time scale. For example, if we think of q as being the trigger for the instability, then $\alpha(t) \sim \delta q/q$ measures how far the q profile has evolved past its critical value. A quantity such as q which is tied to the global current profile evolves on a resistive time scale, $\alpha \sim t/\tau_R$. We therefore obtain an e-folding time intermediate between the resistive and Alfvén time scales, even if the instability itself can grow on an Alfvén time scale. An instability model of this sort does not in itself resolve the difficulties in interpreting the experimental data.

The scenario we have described in this paper suggests one way out of this dilemma. It shows that the plasma can maintain a metastable state while a potential hill is being built up. Starting from a profile of the sort described in Ref. 3, the passage through marginal stability is now associated with a rapid increase in $\Delta'(w)$ as the island width increases.

The subsequent evolution of the fast crash is beyond the scope of this paper. A flat current profile is not itself sufficient to give a fast crash. It is still necessary to explain why the subsequent evolution occurs on a fast time scale. Various models have been invoked to explain this, involving a large localized anomalous resistivity, the onset of large scale stochasticity, or a transition to an ideal instability. The point is that none of these mechanisms is in itself capable of giving a fast crash. It is still necessary to explain the rapid passage through marginal stability. Our scenario does provide a possible explanation of this.

In conclusion, the results of this paper suggest that studies of tearing mode stabilization via profile control, and studies attempting to characterize tearing stable profiles, should not rely on a linear stability analysis alone. We have also found that a flattening of the current profile at the rational surface allows a metastable state to be maintained while a large potential hill builds up for the tearing mode. This can allow the tearing mode to pass quickly through marginal stability, giving rapid growth.

This paper has benefited from discussions with Don Monticello and Wolfgang Stodiek. The work was supported by the United States Department of Energy under Contract DE-AC02-76-CHO-3073.

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Figures

FIG. 1. Sketch of $\Delta'(w)$ for the slab model. The case shown corresponds to $\Delta'(0) > 0$. There is a critical island width, w_c, above which the island grows. The growth of the island does not saturate until it reaches the width it would have attained in the absence of flattening, w_s.

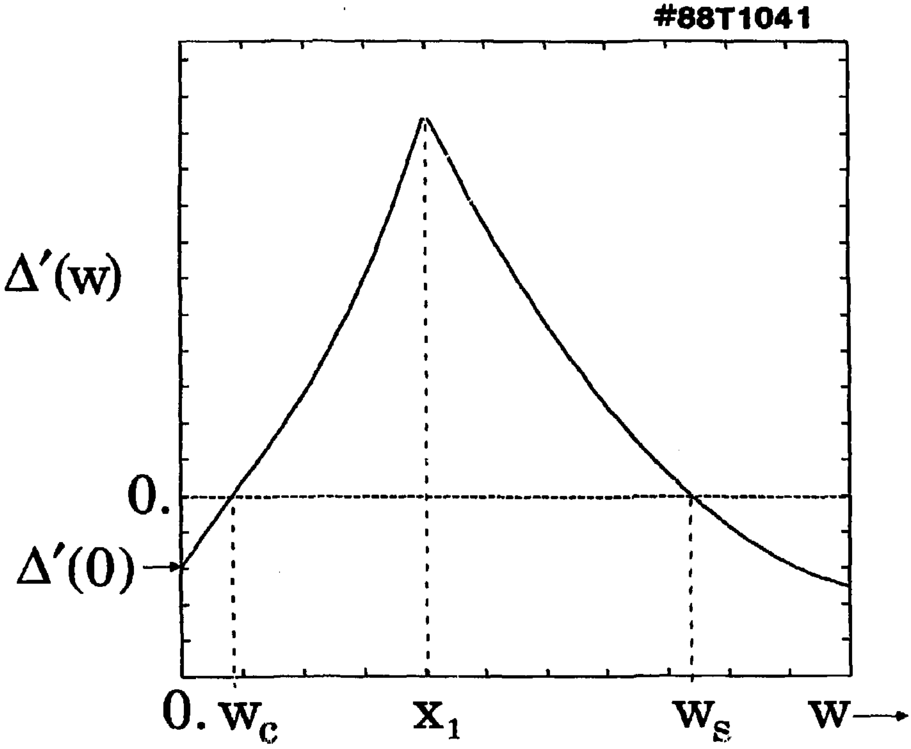

Fig. 1

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