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Pion-Nucleon Scattering
and Pion Production in Nucleon-Nucleon
and Nucleus-Nucleus Collisions*

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Outline:

1) Basic aspects of πN interactions

Properties of pions and nucleons

SU(3) & SU(6) classification

phenomenology of πN scattering
[(3,3) resonance ; phase shift analysis

Bag model approach to πN

2) Pion production and absorption in the two nucleon system

$NN \to NN\pi$ (isobar model)

$\pi d \leftrightarrow NN$ (existence of dibaryon res.?)

3) Pion absorption in complex nuclei
multiparticle aspects
cascade calculations

4) Pion production with nuclear targets
a) nucleon - nucleus
b) nucleus - nucleus (Fermi-averaged 2-body vs. thermodynamic models)
c) $\pi\pi$ interoferometry

A selection of review articles on pion-nucleus interactions:

1) J. Hüfner, "Pions Interact with Nuclei" Phys. Rep. 21, 1 (1975)

2) G. E. Brown + W. Weise, "Pion Scattering + Isobars in Nuclei", Phys. Rep. 22, 279 (1975

3) G. E. Brown + W. Weise, "Pion Condensates" Phys. Rep. 27, 1 (1976)

4) G. E. Brown, B. K. Jennings + V. I. Rostokin, "The Pion-Nucleus Many-Body Problem" Phys. Rep. 50, 227 (1979)

5) J. Alster + J. Warszawski, "Pion-Nucleus Charge Exchange Reactions", Phy. Rep. 52, 87 (1979)

6) A. W. Thomas + R. H. Landau, "Pion-Deuteron and Pion-Nucleus Scattering" Phys. Rep. 58, 121 (1980)

7) J. M. Laget, "Pion Photoproduction in Few Body Systems", Phys. Rep. 69, 1 (1981).

Properties of Pions and Nucleons:

Pions: come in 3 charge states

$\pi^{\pm}$ mass 139.6 MeV

π^{0} mass 135 MeV

form a triplet in isospin space ($I = 1$)

range associated with pion exchange $\sim 1.4f$
$\sim m_\pi^{-1}$

spin-parity : 0^- (pseudoscalar)

parity relative to nucleon (+) from fact that $\pi^- + d \to n + n$ observed from $^3S_1$ state

charge conjugation C (transforms particle into antiparticle)

$$C|\pi^{\pm}\rangle = -|\pi^{\mp}\rangle \; ; \; C|\pi^0\rangle = +|\pi^0\rangle$$

strong + EM interactions conserve P, C

G-parity : $G = C e^{i\pi I_y}$

$$G|\pi^{\pm},\pi^0\rangle = -|\pi^{\pm},\pi^0\rangle \; ; \; G = (-)^n \text{ for } n \text{ pions}$$

Decay Modes of Pions:

1) $\pi^\pm$ decay weakly: $\tau_{\pi^\pm} \approx 2.6 \times 10^{-8}$ sec

$\pi^+_{\mu 2} \to \mu^+ + \nu_\mu$ dominates (0.999)

$$R = \frac{\Gamma(\pi \to e \nu_e)}{\Gamma(\pi \to \mu \nu_\mu)} = \frac{f_\pi^2(e)}{f_\pi^2(\mu)} \left(\frac{m_e}{m_\mu}\right)^2 \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2}$$

$$\approx 1.2 \times 10^{-4}$$

$\rightarrow$ $f_\pi^2(e) = f_\pi^2(\mu)$ μ-e universality

2) π^0 decays electromagnetically

$\pi^0 \to 2\gamma$ (98.85%)

$\quad \to \gamma e^+ e^-$ (1.15%)

$\tau_{\pi^0} \approx 0.83 \times 10^{-16}$ sec

with respect to time scale $10^{-22} - 10^{-23}$ of strong interactions, pions are effectively stable

Nucleon Properties:

two charge states : proton: 938.28 MeV
neutron: 939.57

$\rightarrow$ p, n form isodoublet ($I = 1/2$)

spin-parity $1/2^+$ (+ assumed by convention)

no good C or G for baryons: $C|p\rangle = |\bar{p}\rangle$
$C|n\rangle = |\bar{n}\rangle$

Decay Modes:

1) neutron β-decay: $n \rightarrow p + e^- + \bar{\nu}_e$

$$\tau_n \approx 10^3 \text{ sec}$$

2) proton is essentially stable ($\tau_p \geq 6 \times 10^{30}$ yr;

However, note that baryon-number violating processes are predicted in grand unified theories : $p \rightarrow e^+ \pi^0$ ($\Delta(B-L) = 0$ in SU(5))

$$n \rightarrow \bar{n} \quad (\Delta B = 2, \Delta L = 0)$$

PCAC and Goldberger - Treiman relatio[n]

weak hadronic current $J_\mu(x) = J_\mu^V + J_\mu^A$

$$\frac{\partial}{\partial x_\mu} J_\mu^V = 0$$ conserved vector current

$$\frac{\partial}{\partial x_\mu} J_\mu^A = \frac{f_\pi}{\sqrt{2}} \phi(x)$$ PCAC

pion field

f_π = pion decay constant ($\Gamma_{\pi \to \mu \nu_\mu} \sim f_\pi^2$)

hadronic current for neutron β-decay:

$$\bar{\psi}(x) \gamma_\mu (1 + g_A \gamma_5) \frac{\tau_1 \pm i\tau_2}{2} \psi(x)$$

$g_A = G_A/G_V$ = ratio of axial vector to vector couplings

$$f_\pi = \sqrt{2}\, m_N m_\pi^2 g_A / g$$ Goldberger-Treiman

πNN strong coupling co[nstant]

Isospin Structure of πNN and NN Couplings

Define Pauli matrices: $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Commutation relations: $[\tau_i, \tau_j]_- = 2i\,\epsilon_{ijk}\,\tau_k$

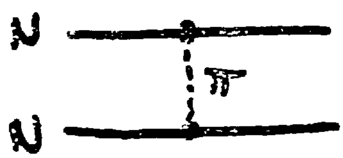
$\uparrow$ Lie algebra of SU(2)

In isospin space: $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\pi^+ = -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$ $\pi^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$ $\pi^0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

<u>NN coupling</u>: $\vec{I} = \frac{1}{2}\vec{\tau}^{(1)} + \frac{1}{2}\vec{\tau}^{(2)}$

$H = H_0 + H_1\, \vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)}$



<u>πNN coupling</u>:

$N_1^\dagger\, \vec{\tau}\, N_2 \cdot \vec{\phi}$

$\uparrow$ isovector pion field

CLASSIFICATION OF PIONS AND NUCLEONS IN SU(3) → "eightfold way"

SU(3) group has representations

$[1], [3], [\bar{3}], [6], [8], [10] \ldots$

adjoint ... octet ... decuplet

<u>SU(3) nonets</u> = $[8] + [1]$ mixed

0^- : $\pi^\pm \pi^0 \; \eta \; \eta' \; K^\pm \; K^0 \bar{K}^0$

1^- : $\rho^\pm \rho^0 \; \omega \; \varphi \; K^{*\pm} \; K^{*0} \; \bar{K}^{*0}$

mesons (B=0)

$\frac{1}{2}^+$: $n \; p \; \Lambda \; \Sigma^\pm \Sigma^0 \; \Xi^- \Xi^0 = [8]$

$\frac{3}{2}^+$: $\Delta(1236), Y^*(1385), \Xi^*(1530), \Omega^- = [10]$

B

SU(n) has $n^2 - 1$ generators which satisfy a Lie algebra

$$[F_i, F_j] = i f_{ijk} F_k \; ; \quad i = 1 \ldots 8$$

~~for~~ SU(3)

SU(3) has <u>strangeness degree of freedom</u> in addition to SU(2) of isospin → can write SU(3) invariant MBB′ couplings : $\pi NN \to K^+ \Lambda N$ ....

CLASSIFICATION IN QUARK MODEL + SU(6)

<u>Quark Model:</u> species of quark $\{u, d, s, c, b, t\}$

for nuclear and hypernuclear physics, need low mass u, d, s only

	u	d	s
Q	$+2/3$	$-1/3$	$-1/3$
I	$1/2$	$1/2$	0
S	0	0	-1
B	$1/3$	$1/3$	$1/3$

hypercharge $Y = B + S$

mesons are $\bar{q}q$ $(B=0)$

baryons are qqq $(B=1)$

<u>Quark composition:</u> $\pi^+ \sim u\bar{d}$, $K^- \sim s\bar{u}$

$p \sim uud$, $\Lambda \sim sud$

$\Delta^{++} \sim uuu$

we get SU(6) by combining SU(3) of isospin and strangeness with SU(2) of ordinary spin $\vec{\sigma}/2$: $SU(6) \supset SU(3) \otimes SU($

Features of SU(6) classification

representations: [6], [20], [35], [56], [70]

$\uparrow$ fundamental

SU(6) unifies different SU(3) representations for mesons and baryons:

baryons: $[56] = (10,4) + (8,2) \to \frac{1}{2}^+, \frac{3}{2}^+$

$\uparrow$ SU(3) $\quad \uparrow$ $2J+1$

mesons: $[1] + [35] = \underbrace{(8,1) + (1,1)}_{0^- \text{ nonet}} + \underbrace{(8,3) + (1,3)}_{1^- \text{ nonet}}$

Δ and N in same [56]: $NN\pi$, $N\Delta\pi$, $\Delta\Delta\pi$ vertices are related (not in SU(3))

In the quark model, Δ and N are <u>distinguishable</u> particles → in "bootstrap" picture, on the other hand, Δ is a πN composite (and ρ is $\pi\pi$ composite)

πN scattering: isospin amplitudes

πN system has isospin $I = \vec{1/2} + \vec{1} \to 1/2, 3/2$

construct eigenstates of good (I, I_3):

$I = 3/2$: $\quad I_3 = 3/2 \quad |p\pi^+\rangle$

$I_3 = 1/2 \quad \sqrt{2/3}\,|p\pi^0\rangle + \sqrt{1/3}\,|n\pi^+\rangle$

$I_3 = -1/2 \quad \sqrt{1/3}\,|p\pi^-\rangle + \sqrt{2/3}\,|n\pi^0\rangle$

$I_3 = -3/2 \quad |n\pi^-\rangle$

$I = 1/2$: $\quad I_3 = 1/2 \quad \sqrt{1/3}\,|p\pi^0\rangle - \sqrt{2/3}\,|n\pi^+\rangle$

$I_3 = -1/2 \quad \sqrt{2/3}\,|p\pi^-\rangle - \sqrt{1/3}\,|n\pi^0\rangle$

amplitudes:

$$\langle \pi^+ p | T | \pi^+ p\rangle = T^{3/2}$$

$$\langle \pi^- p | T | \pi^- p\rangle = \frac{1}{3} T^{3/2} + \frac{2}{3} T^{1/2}$$

$$\langle \pi^- p | T | \pi^0 n\rangle = \frac{\sqrt{2}}{3}\left(T^{3/2} - T^{1/2}\right)$$

cross sections:

$$\sigma(3/2) = \sigma(\pi^+ p)$$

$$\sigma(1/2) = \frac{1}{2}\left(3\,\underbrace{\sigma_{TOT}}_{\text{incl. charge exch}}(\pi^- p) - \sigma(\pi^+ p)\right)$$

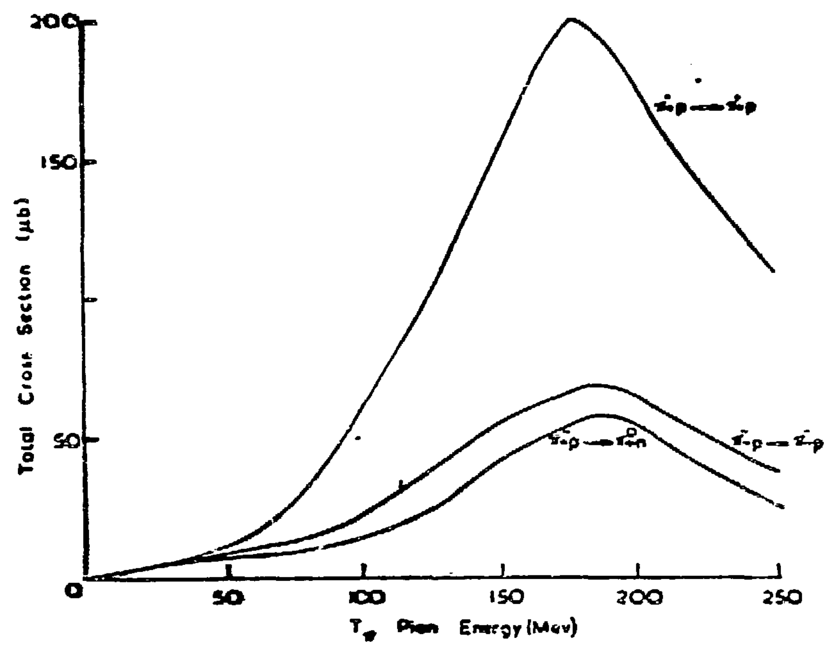


Fig. 2-2. Total cross sections for π^+ and π^- scattering by protons for laboratory pion energies T_π in the range 0 to 250 MeV.

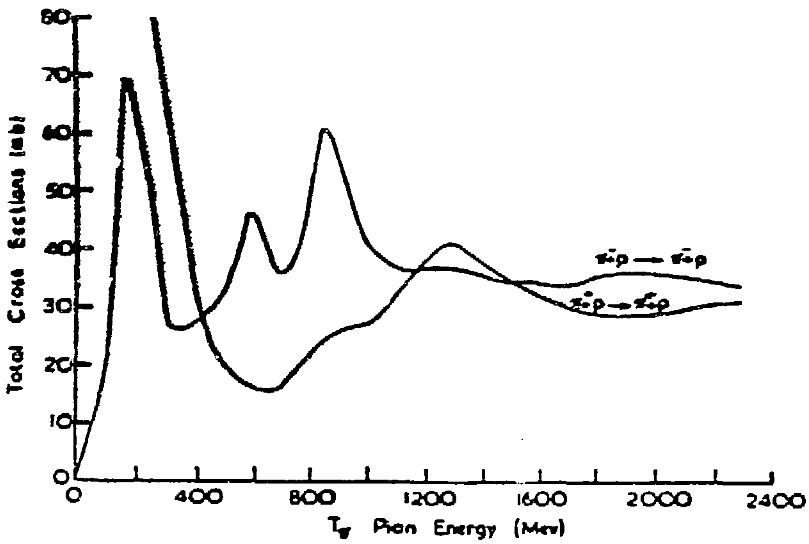


Fig. 2-3. Total cross sections for π^+ and π^- scattering by protons for laboratory pion energies T_π in the range 250 MeV to 2.4 GeV.

Properties of peak at 1236 MeV in $\sigma_{\pi N}$

1) $\sigma(\pi^+ p) : \sigma(\pi^- p \to \pi^- p) : \sigma(\pi^- p \to \pi^0 n)$

$= 9 : 1 : 2$

$\to \Delta(1236)$ has $I = 3/2$

2) resonance has form

$$\sigma_{\ell j} = \frac{\pi(2j+1)}{q^2} \frac{\Gamma_\ell^2}{(W_R - W)^2 + (\Gamma_\ell/2)^2}$$ Breit-Wigner

unitarity limit $\sigma_{\ell j} \leq 4\pi(2j+1)/q^2$

threshold properties: $\Gamma_\ell \sim q^{2\ell+1}$

$\sigma_{\ell j} \sim q^{4\ell}$

data consistent with $\ell = 1,\ j = 3/2$

$\therefore$ $\boxed{\Delta(1236) \text{ is } 3/2^+,\ I = 3/2 \text{ resonance in } P_{33} \text{ partial wave of } \pi N \text{ system}}$

MORE DETAILS ON $\Delta(1236)$:

$$\sigma_{\pi^+ p} = \sigma_{res} + B(q)$$

$\uparrow$ background

$$\Gamma(q) = 2(qR)^3\gamma/(1+(qR)^2) \qquad \ell = 1$$

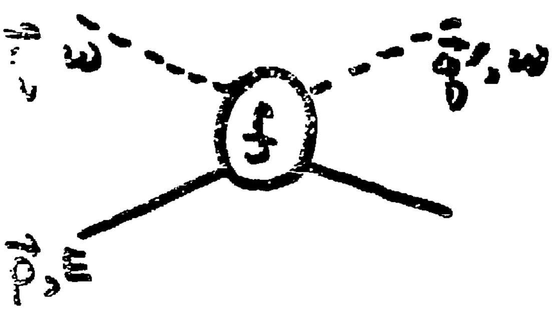
$\gamma = 71$ MeV
$R = 0.81\ (\hbar/m_\pi c)$
$B = 4$ mb (small)
$\Gamma(q_{res}) = 120$ MeV

$\Delta(1236)$ also seen in <u>photoproduction</u>

$$\gamma + p \to \pi^0 + p,\ \pi^+ + n$$

same resonance form, with $\Gamma^2 \to \Gamma\Gamma_\gamma$

AMPLITUDES FOR πN SCATTERING (GENERAL)



Invariants:

$$s = (p_\mu + q_\mu)^2 = (E+\omega)^2$$
$$t = (q_\mu - q'_\mu)^2 = -2q^2(1-\cos\theta)$$
$$u = (p_\mu - q'_\mu)^2 = (E-\omega)^2 - 2q^2(1+\cos\theta)$$

$$\cos\theta = \hat{q}\cdot\hat{q}'$$

$$s+t+u = 2(m_N^2 + m_\pi^2)$$

f is an <u>operator in spin-isospin space</u>:

$$f(\vec{q},\vec{q}') = f_0 + f_1\,\vec{T}_\pi\cdot\vec{\tau}_N + f_2\,\vec{\sigma}_N\cdot\hat{n} + f_3\,\vec{\sigma}_N\cdot\hat{n}\,\vec{T}_\pi\cdot\vec{\tau}_N$$

where $\hat{n} \equiv \hat{q}\times\hat{q}'$ is perpendicular to scattering plane

<u>Partial wave expansion</u> for $0^- + \frac{1}{2}^+$:

$$f_I(\vec{q}',\vec{q}) = \sum_\ell \left[(\ell+1) f_{I,\ell+}(s) + \ell f_{I,\ell-}(s)\right] * P_\ell(\cos\theta)$$
$$+ i\vec{\sigma}_N\cdot\hat{n}\sum_\ell \left[f_{I,\underset{\uparrow\, j=\ell+1/2}{\ell+}} - f_{I,\ell-}\right] P'_\ell(\cos\theta)$$

S matrix, K matrix and scattering lengths

$$f_\alpha(s) = \frac{1}{2iq}\left(S_\alpha(s) - 1\right)$$

$$S_\alpha(s) = e^{i\,2\delta_\alpha(s)}$$

δ_α real below
$\pi N \to \pi\pi N$ threshold
$f_\alpha = (\eta_\alpha e^{2i\delta_\alpha} - 1)/2iq$
above threshold
$(\eta_\alpha \leq 1)$

define K-matrix: $S_\alpha = (1 + iq K_\alpha)/(1 - iq K_\alpha)$

$$K_\alpha = 1/q \tan\delta_\alpha$$

<u>scattering lengths</u> $a_\alpha = \lim_{q\to 0} K_\alpha/q^{2l} = \lim_{q\to 0} \frac{\tan\delta_\alpha}{q^{2l+1}}$

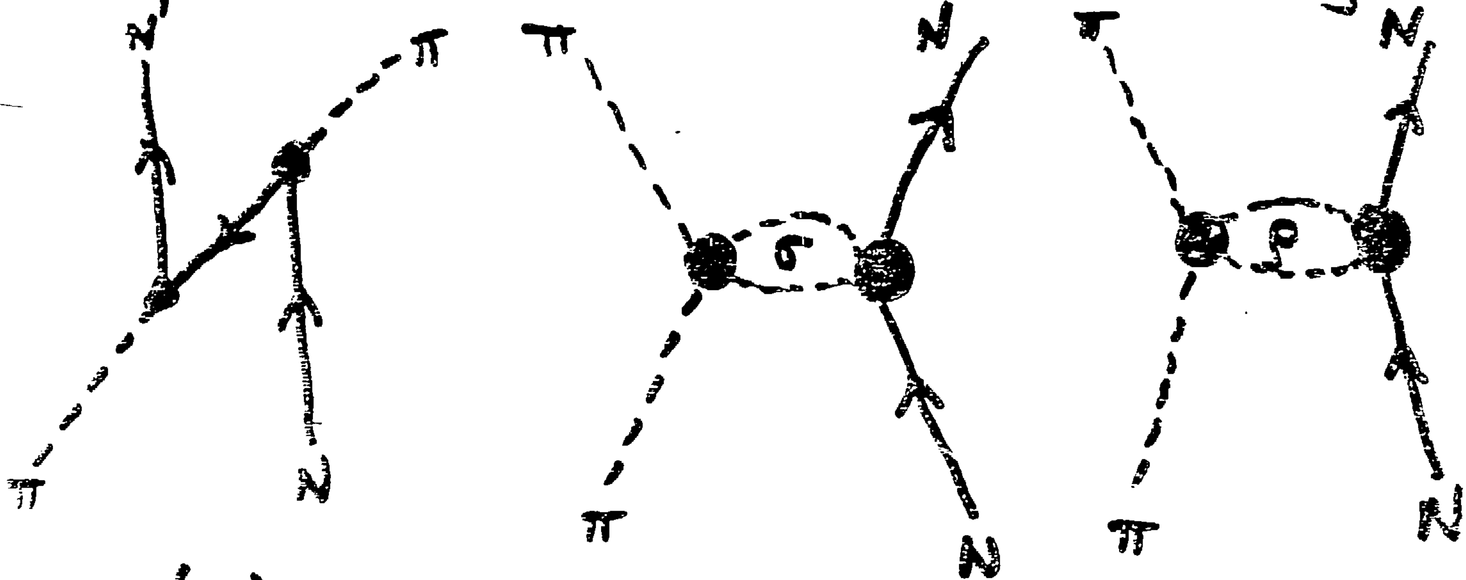
Expeiment: $a_1 \approx 0.17 \pm .004\ m_\pi^{-1}$
$a_3 \approx -0.092 \pm .002\ m_\pi^{-1}$ (Bugg)

<u>isoscalar</u> $a_1 + 2a_3 \approx -0.01$ small

<u>isovector</u> $a_1 - a_3 \approx 0.26$ large

$a_1 + 2a_3 \approx 0$ follows from soft-pion theorem of Weinberg (PCAC)

old picture in terms of meson exchanges



(a) (b) (c)

(a) + (b) tend to cancel → "pair suppression"
→ isoscalar small

(c) gives isovector

Phase shift analysis for πN scattering:

see B.H. Bransden + R.G. Moorhouse, "The Pion-Nucleon System", Princeton (1973)

Define
$$f(\theta) = \sum_{\ell} \left[(\ell+1) f_{\ell+} + \ell f_{\ell-} \right] P_\ell(\cos\theta)$$
$$g(\theta) = \sum_{\ell} (f_{\ell+} - f_{\ell-}) \sin\theta \, P_\ell'(\cos\theta)$$
(spin-flip)

need both single and double scattering experiments with polarized target to get complete set of observables

Density matrix formalism:

initial target polarization $\langle \vec{\sigma}_i \rangle = \frac{1}{2} \vec{P}_i$

initial density matrix $\vec{\rho}_i = \frac{1}{2}(1 + \vec{P}_i \cdot \vec{\sigma})$

final density matrix

$$\rho_f = \frac{f \rho_i f^\dagger}{\mathrm{Tr}(f \rho_i f^\dagger)}$$

$\vec{P}_f$ = final nucleon polarization = $\mathrm{Tr}(\vec{\sigma}\rho_f)$

observables are $\frac{d\sigma}{d\Omega}$, P_f, $\underbrace{A, R}_{\text{spin rotation parameters}}$

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2 + 2\hat{n}\cdot\vec{P}_i \, \mathrm{Im}(fg^*)$$

$$\frac{d\sigma}{d\Omega}\vec{P}_f = 2\hat{n}\,\mathrm{Im}(fg^*) - 2\,\hat{n}\times\vec{P}_i \, \mathrm{Re}(fg^*)$$
$$+ (|f|^2+|g|^2)(\hat{n}\cdot\vec{P}_i)\,\hat{n} - (|f|^2-|g|^2)(\hat{n}\times(\hat{n}\times\vec{P}_i))$$

$$\frac{d\sigma}{d\Omega}A = (|f|^2-|g|^2)\sin\theta + 2\,\mathrm{Re}(fg^*)\cos\theta$$

$$\frac{d\sigma}{d\Omega}R = (|f|^2-|g|^2)\cos\theta - 2\,\mathrm{Re}(fg^*)\sin\theta$$

spin rotation parameters A and R
require measuring polarization of recoil nucleon in a second scattering

Define $\hat{m} = \hat{q} \times \hat{n}$ ($\perp$ to scattering plane)

target pol. $\parallel$ to $\vec{q}$

$\epsilon_A = P^{(3)} A P^{(1)}$

$\epsilon_R = P^{(3)} R P^{(1)}$

target pol. $\parallel$ to $\hat{m}$

analyzing power of second scattering

ϵ_A = up-down asymmetry w.r.t. scattering plane defined by first collision

$\boxed{\text{meas. of } d\sigma/d\Omega,\ P,\ R,\ A \text{ for } \pi^+ \text{ and } \pi^- \rightarrow f \text{ and } g \text{ (4 complex nos. for each } I)}$

also have optical theorem + Coulomb-nuclear interference

object of πN phase shift analysis:
extract non-strange baryon resonance spectrum ($I = 1/2$ and $3/2$)

Results for lowest-lying states:

mass	isospin	J^π	Partial wave ($2I, 2J$)
1236	3/2	$3/2^+$	P_{33} ← $\Delta(1236)$
1470	1/2	$1/2^+$	P_{11} ← Roper
1500	1/2	$1/2^-$	S_{11}
1520	1/2	$3/2^-$	D_{13}
1620	3/2	$1/2^-$	S_{31}
1700	3/2	$3/2^-$	D_{33}
1850	1/2	$3/2^+$	P_{13}
1900	3/2	$1/2^+$	P_{31}

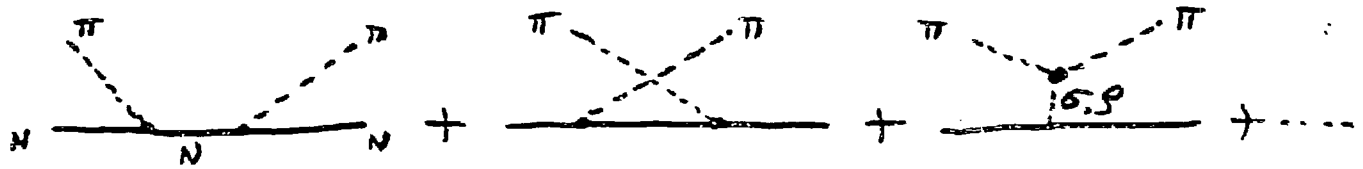
↑ in πN

DYNAMICS OF THE πN SYSTEM:

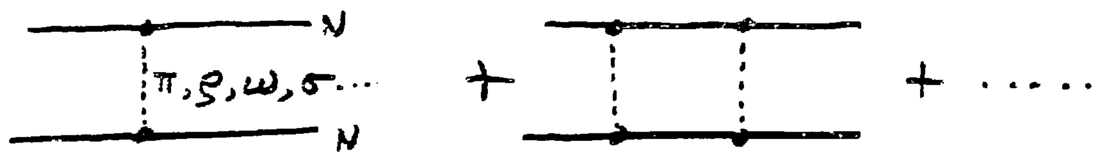
old fashioned approach:

dispersion relations
meson exchange models

$\underline{\pi N}$:



$\underline{NN}$:



one boson exchange models (OBE)

<u>Idea</u>: one meson or one baryon exchange diagrams generate a <u>potential V</u> ("driving term") for insertion into some <u>dynamical equation</u> which iterates V to all orders and generates a unitary S-matrix

(Schrödinger, Lippmann-Schwinger, Blankenbecler-Sugar, Bethe-Salpeter .... wave equations)

<u>Qualitative Idea</u>: obtain long and medium range parts of V from meson exchange theory; parametrize the short range interaction

ranges: $1/\mu_\pi \approx 1.4\,\text{fm}$ (one pion exchange)

$1/\mu_\sigma \approx 0.5\,\text{fm}$

$1/\mu_{\rho,\omega} \approx 0.25\,\text{fm}$

danger signal: $1/\mu_{\rho,\omega} \ll \langle r^2 \rangle^{1/2}_{\text{proton}} \approx 0.8\,\text{fm}$

<u>disadvantages</u> of one boson exchange models

a) <u>too many fields</u> → adjustable coupling constants

b) <u>arbitrary cutoffs</u> (form factors) to describe short distance behavior of V

c) <u>no relation to underlying theory</u> of strong interactions → quantum chromodynamics (QCD), involving quark and gluon degrees of freedom old Chew-Low model is limit $h(k) \to 0$

then $t_{eL}(k, k', \omega) = \dfrac{g(k')\, g(k)}{1 - \frac{2\omega}{\pi} \int_0^\infty dq \frac{q^2 g^2(q)}{\omega + i\varepsilon - \omega_q}}$

$\sim \dfrac{1}{\omega - \omega_0 + i\Gamma/2} \to$ Breit-Wigner form

<u>Numerical fit in cloudy bag model</u>:

$\omega_\Delta \approx m_\Delta - m_N = 294$ MeV

$f_{\Delta N\pi} \approx 0.42$

<u>$R \approx 0.72$ fm</u>

<u>Conclusions</u>: elementary Δ carries 80% of the strength at the (3,3) resonance (far from Chew-Low!

R rather well determined

Structure of the Low equation:

$$t(E) = (V_{cL} + V_\Delta) + (V_{cL} + V_\Delta) G_0(E)\, t(E)$$

Chew-Low driving term

If driving terms V_{cL} and V_Δ are treated as separable, i.e. $\langle k | V | k' \rangle \sim f(k) f(k')$, then

$$\boxed{t(k,k',\omega) = N(k,k',\omega) / D(\omega)}$$

$$N(k,k',\omega) = g(k')g(k)\, D_2(\omega) + h(k)h(k')\, D_1(\omega) + \omega\left[g(k')h(k) + h(k')g(k)\right] D_3(\omega)$$

$$D(\omega) = D_1(\omega)\, D_2(\omega) - \omega D_3^2(\omega)$$

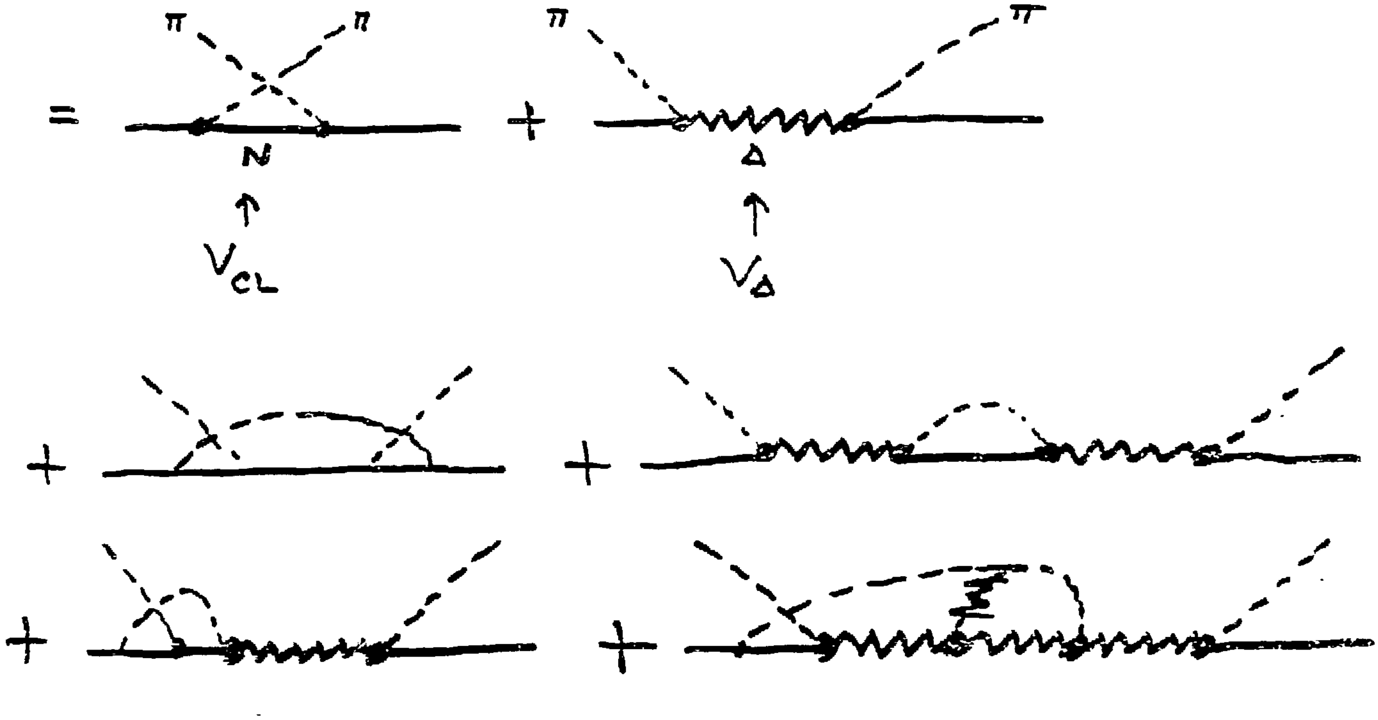
$$D_1(\omega) = 1 - \frac{2\omega}{\pi} \int_0^\infty dq \frac{q^2\, g^2(q)}{\omega + i\varepsilon - \omega_q} \qquad \text{Chew-Low}$$

$$D_2(\omega) = \omega - \omega_\Delta - \frac{2}{\pi} \int_0^\infty dq \frac{q^2\, h^2(q)}{\omega + i\varepsilon - \omega_q} \qquad \Delta \text{ propagator}$$

$$D_3(\omega) = \frac{2}{\pi} \int_0^\infty dq \frac{q^2\, g(q)\, h(q)}{\omega + i\varepsilon - \omega_q} \qquad \text{interference}$$

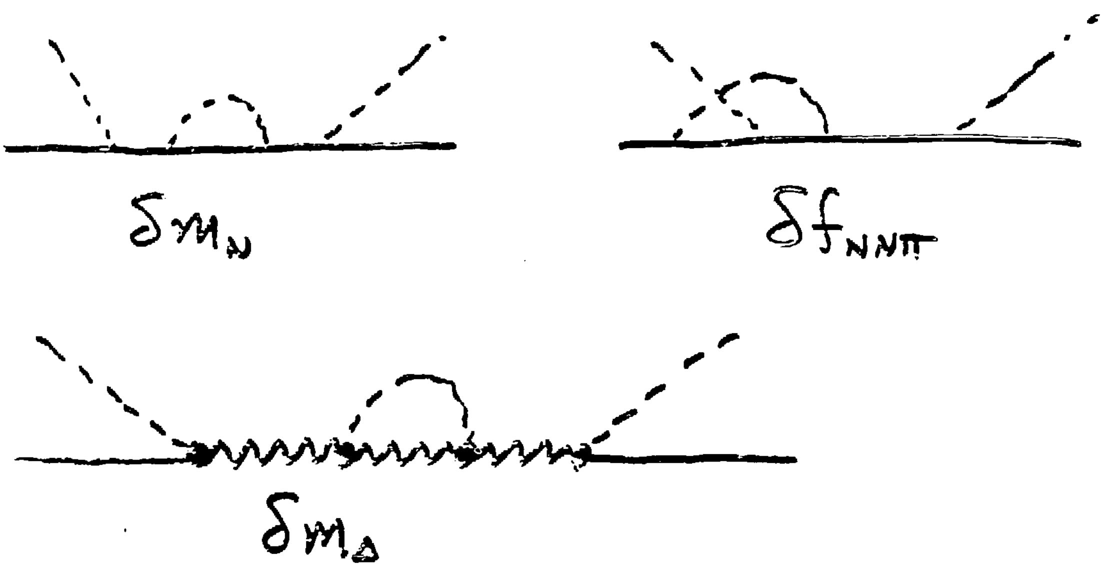
Graphical interpretation of the Low equation:

$t =$



$+ \cdots\cdots$

other graphs are included as renormalizations of masses and coupling constants:



application to πN scattering:

$$H\,|\tilde{N},k\rangle_+ = (m_N + \omega_k)\,|\tilde{N},k\rangle_+$$

$$|\tilde{N},k\rangle_+ = a_k^\dagger\,|\tilde{N}\rangle + |\chi\rangle_+$$

outgoing wave boundary conds

recall $H_{int} = \sum_{\alpha,\beta=\{N,\Delta\}} \sum_{\underset{\sim}{k}} \left(V_k^{\alpha\beta} a_k + \text{herm. conj.} \right)$

πN t-matrix is

$$t(\tilde{N}'k', \tilde{N}k) = \langle \tilde{N}'k' | \sum_{\alpha\beta} V_k^{\alpha\beta} | \tilde{N} \rangle$$

t satisfies the <u>Low equation</u>

$$t(\tilde{N}'k', \tilde{N}k) = \sum_{|\tilde{n}\rangle} \left[\frac{t^\dagger(\tilde{N}'k, \tilde{n})\, t(\tilde{n}, \tilde{N}k')}{m_N - \omega_{k'} - E_{\tilde{n}}} \right.$$

$$\left. + \frac{t^\dagger(\tilde{N}'k', \tilde{n})\, t(\tilde{n}, \tilde{N}k)}{m_N + \omega_{k'} - E_{\tilde{n}} + i\varepsilon} \right]$$

$|\tilde{n}\rangle$ is a complete set of states

approximate $|\tilde{n}\rangle$ by $\tilde{N}$, $\pi\tilde{N}$ only

Approximations in cloudy bag model

1) no self-consistency
2) $\sum_\alpha |\alpha\rangle\langle\alpha|$ truncated with N, Δ
3) pion field does not influence quark wave functions → no deformation of the bag

Simple picture of the physical nucleon:

$$|\tilde{N}\rangle = Z^{1/2}|N\rangle - Z^{1/2}\sum_{\underset{\sim}{k}}\left(\frac{v_k^{NN}|N,\underset{\sim}{k}\rangle}{\omega_k} + \frac{v_k^{\Delta N}|\Delta,\underset{\sim}{k}\rangle}{m_\Delta - m_N + \omega_k}\right)$$

bare nucleon $\qquad$ $N+\pi$ $\qquad$ $\Delta+\pi$

$$Z^{-1} = 1 + \sum_{\underset{\sim}{k}}\left[\frac{|v_k^{NN}|^2}{\omega_k^2} + \frac{|v_k^{\Delta N}|^2}{(m_\Delta + \omega_k - m_N)^2}\right]$$

like the quasiparticle residue in nuclear physics

RESULTS:

61% bare nucleon $(Z^2 = 0.61)$
25% $N\pi$
14% $\Delta\pi$

NNπ vertex:

$$H_{int}^{NN\pi} = |N\rangle\langle N| \frac{1}{(2\pi)^{3/2}} \sum_j \int \frac{d^3\vec{k}}{(2\omega_k)^{1/2}} \left(v_{jk}^{NN} a_k + (v_{jk}^{NN})^\dagger a_k^\dagger \right)$$

$$v_{jk}^{NN} = \frac{i}{2f} \frac{\omega}{\omega - 1} \frac{j_1(kR)}{kR} \langle N | \sum_a \vec{\sigma}_a \cdot \vec{k}\, \tau_{aj} | N \rangle$$

compare to usual form

$$v_{jk}^{NN} = i\,(4\pi)^{1/2} \left(\frac{f_{NN\pi}^{(0)}}{m_\pi} \right) u_N(k)\; v_N^\dagger \vec{\sigma}\cdot\vec{k}\, \tau_j\, v_N$$

identify

$$\boxed{\begin{array}{l} \text{form factor} \quad u_N(k) = \dfrac{3 j_1(kR)}{kR} \rightarrow \begin{cases} 1 & k \to 0 \\ 1/k^2 & k \to \infty \end{cases} \\[2ex] \text{coupling constant} \quad (4\pi)^{1/2} \dfrac{f_{NN\pi}^{(0)}}{m_\pi} = \dfrac{5}{18} \dfrac{\omega}{\omega - 1} \dfrac{1}{f} \end{array}}$$

For $\underline{\Delta N\pi \text{ vertex}}$, we have

$$v_{jk}^{\Delta N} = i\,(4\pi)^{1/2} \left(\frac{f_{\Delta N\pi}^{(0)}}{m_\pi} \right) u_\Delta(k)\; v_\Delta^\dagger\, \vec{S}\cdot\vec{k}\, \vec{T}_j\, v_\Delta$$

transition spin + isospin

form factor $\underline{u_\Delta(k) = u_N(k)}$

coupling constant $\underline{f_{\Delta N\pi}^{(0)} = \sqrt{\frac{72}{25}}\, f_{NN\pi}^{(0)}}$ from SU(6)

COUPLING CONSTANTS AND FORM FACTORS:

Expand the quantized pion field:

$$\phi_j(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\vec{k}}{(2\omega_k)^{1/2}} \left(a_{jk}\, e^{i\vec{k}\cdot\vec{x}} + a^{\dagger}_{jk}\, e^{-i\vec{k}\cdot\vec{x}} \right)$$

creation operator for π

$$H_\pi = \sum_j \int d^3\vec{k}\, \omega_k\, a^{\dagger}_{jk}\, a_{jk}$$

$$H_{int} = \sum_{\alpha,\beta} |\alpha\rangle\langle\beta| \left[\frac{i}{2f} \langle\alpha| \int d^3x \sum_a \bar{q}_a(x)\, \gamma_5\, \vec{\tau}\, q_a(x)\, \vec{\phi}(x)\, \delta(r-R)\, |\beta\rangle \right]$$

baryonic bag states N, Δ ....

<u>$NN\pi$, $\Delta N\pi$, $\Delta\Delta\pi$ vertices</u> can be evaluated using bag model wave functions $q(\vec{r})$:

$$q(\vec{r}) = \frac{N}{(4\pi)^{1/2}} \begin{pmatrix} j_0(\omega r/R) \\ i\vec{\sigma}\cdot\hat{r}\, j_1(\omega r/R) \end{pmatrix} \nu$$

spin-isospin wave fcn.

where $\omega = \omega_{1,-1} = 2.04$

$$N^2 = \frac{1}{R^3} \left\{ \omega^2 + \omega/2(\omega-1) \right\}$$

Applications of chiral bag models:

a) $NN\pi$ and $BB'\pi$ vertices

→ $f_{\pi NN}$ and form factors as fcns. of ω and R

b) one pion exchange potential

c) sizes of baryons $\langle r^2 \rangle^{1/2}_{p,n}$

form factor G_{E_n} of neutron

d) axial vector coupling constant g_A

e) theory of πN scattering

→ soft pion limit

dynamics of (3,3) resonance

Hamiltonian for cloudy bag model:

$$H = H_{MIT} + H_\pi + H_{int}$$

$$H_{MIT} = \int d^3x \left(\frac{i}{2} \sum_\alpha q_\alpha^\dagger \overleftrightarrow{\partial_0} q_\alpha + B \right) \theta(R-r)$$

$$H_\pi = \frac{1}{2} \int d^3x \left(\vec{\pi} \cdot \vec{\pi} + \vec{\nabla}\phi \cdot \vec{\nabla}\phi + m_\pi^2 \phi^2 \right)$$

$$H_{int} = \frac{i}{2f} \int d^3x \sum_\alpha \bar{q}_\alpha \gamma_5 \vec{\tau} \cdot \vec{\phi}\, q_\alpha\, \delta(r-R)$$

differences between CBM and Stony Brook versio

1) CBM is linearized in $\vec{\phi}$ → pion fields are not large corrections to MIT bag

2) CBM allows pion field inside the bag → SB has factor $\theta(r-R)$ in H_π

3) CBM has quantized π field
SB emphasizes non-linear effects in classical limit

How to build in PCAC:

for Gell-Mann and Levy $\mathcal{L}$, minimum for $\sigma^2 + \vec{\pi}^2 = v^2$

→ a continuous symmetry

→ there exists a massless excitation (the pion)
[Goldstone theorem]

now break chiral symmetry by $\mathcal{L} \rightarrow \mathcal{L} + c\sigma$

→ pick a preferred direction in $(\sigma, \vec{\pi})$ space

then $\partial_\mu A^\mu = -c\pi^\mu$

If $c = -f m_\pi^2$, recover PCAC

The breaking of chiral symmetry implies that the pion acquires a finite mass m_π

Elimination of σ field:

$$\vec{\pi} = f\,\hat{\phi}\,\sin(\phi/f)$$
$$\sigma = f\cos(\phi/f)$$

such that $\sigma^2 + \vec{\pi}^2 = f^2$

$$\mathcal{L}(x) = (i\bar{q}\not{\partial}q - B)\,\theta(R-r) - \frac{1}{2}\bar{q}\,e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/f}\,q\,\delta(r-R) + \frac{1}{2}(D_\mu\phi)^2$$

where $D_\mu = (\partial_\mu\phi)\,\hat{\phi} + f\sin(\phi/f)\,\partial_\mu\hat{\phi}$

Exploration of non-linear effects in $\vec{\phi}$: Vento et al, Nucl. Phys. (1980)

If we linearize $\mathcal{L}(x)$, we obtain the cloudy bag model

$$\mathcal{L}(x) = (i\bar{q}\not{\partial}q - B)\,\theta(R-r) - \frac{1}{2}\bar{q}q\,\delta(r-R) + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\pi^2\phi^2 - \frac{i}{2f}\bar{q}\gamma_5\vec{\tau}\cdot q\vec{\phi}\,\delta(r-R)$$

not D_μ

linear in $\vec{\phi}$

ow to make $\mathcal{L}$ chiral invariant ?

σ model of Gell-Mann and Lévy (1960)

introduce new fields $(\sigma, \vec{\pi})$ → span 4 degrees of freedom of $SU(2) \otimes SU(2)$

σ: $I=0$, $\vec{\pi}$: $I=1$

$$\mathcal{L}(x) = i\bar{\psi}\not{\partial}\psi + g\bar{\psi}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{\lambda^2}{4}(\vec{\sigma}^2 + \vec{\pi}^2 - v^2)^2$$

invariant under chiral transformation

$$\psi \to \psi - i\frac{\vec{\tau}\cdot\vec{\alpha}}{2}\gamma_5\psi$$

$$\bar{\psi} \to \bar{\psi} - i\bar{\psi}\gamma_5\frac{\vec{\tau}\cdot\vec{\alpha}}{2}$$

$$\sigma \to \sigma - \vec{\alpha}\cdot\vec{\pi}$$

$$\vec{\pi} \to \vec{\pi} + \sigma\vec{\alpha}$$

Note $e^{-i\vec{\tau}\cdot\vec{\alpha}\gamma_5/2} \approx 1 - \frac{i}{2}\vec{\tau}\cdot\vec{\alpha}\gamma_5 + \cdots$

$\sigma^2 + \vec{\pi}^2$ constant

Technical diversion on chiral invariance

$$\vec{V}_\mu = \bar{q}(x)\,\vec{\tau}\,\gamma_\mu\, q(x) \qquad \text{vector}$$
$$\vec{A}_\mu = \bar{q}(x)\,\vec{\tau}\,\gamma_\mu\gamma_5\, q(x) \qquad \text{axial vector}$$

$$L_\mu = \bar{q}(x)\,\vec{\tau}\,\gamma_\mu(1-\gamma_5)\, q(x) \qquad \text{left-handed}$$
$$R_\mu = \bar{q}(x)\,\vec{\tau}\,\gamma_\mu(1+\gamma_5)\, q(x) \qquad \text{right-handed}$$

$1 \pm \gamma_5$ is chirality

L_μ, R_μ form independent $SU(2)$ algebras

chiral invariance $\rightarrow$ $\mathcal{L}$ does not mix left and right-handed particles

$\rightarrow$ $\underline{SU(2)_L \otimes SU(2)_R \text{ is good symmetry}}$

see H. Pagels, Phys. Rep. 16, 219 (1975)

<u>relation to helicity</u>: define $\psi_\pm = (1 \pm \gamma_5)/_2\, \psi$

then $\underline{\vec{\sigma}\cdot\hat{p}\,\psi_\pm = \mp\,\psi_\pm}$

$\therefore$ $\frac{1+\gamma_5}{2}$ projects out negative helicity

Consequences: axial current $A^\mu = \bar{q}\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2} q\,\theta(R-r)$

$$\partial_\mu A^\mu(x) = -\frac{i}{2}\,\bar{q}(x)\gamma_5\vec{\tau}\,q(x)\,\delta(r-R) \neq 0$$

MIT bag model does not conserve axial current

or chiral symmetry is broken

attempts to restore chiral symmetry

A. Chodos + C.B. Thorn, Phys. Rev. D12, 2733 (1975)

G.E. Brown + M. Rho, Phys. Lett. 82B, 177 (1979)

G.E. Brown, M. Rho and V. Vento, Phys. Lett. 84B, 383 (1979)

V. Vento et al, Nucl. Phys. A345, 413 (1980)

Théberge, Thomas + Miller: Phys. Rev. D22, 2838 (1980)
D24, 216 (1981)
Can. J. Phys. 60, 59 (1982)

"cloudy bag model"

Idea: introduce a pion cloud around the bag to restore chiral symmetry ($m_\pi \to 0$)

- recover Yukawa π exchange potential
- let $m_\pi \neq 0$, get PCAC
- soft pion theorems + (3,3) resonance

Examples of currents:

<u>charge current</u>: $q(x) \to q(x) + i\varepsilon q(x)$ $\qquad q^\dagger \gamma^0 = \bar{q}$

$\bar{q}(x) \to \bar{q}(x) - i\varepsilon \bar{q}(x)$

$\mathcal{L} = \mathcal{L}(\bar{q}q)$ is invariant $\to j^\mu = \bar{q}(x)\gamma^\mu q(x)\,\theta(R-r)$ is conserved $(\partial_\mu j^\mu(x) = 0)$

<u>isospin current</u>: $q(x) \to q(x) + i\dfrac{\vec{\tau}\cdot\vec{\varepsilon}}{2} q(x)$

$\bar{q}(x) \to \bar{q}(x) - i\bar{q}(x)\dfrac{\vec{\tau}\cdot\vec{\varepsilon}}{2}$

$I^\mu = \bar{q}(x)\gamma^\mu \dfrac{\vec{\tau}}{2} q(x)$ conserved $\to \vec{t} = \int d^3x\, I^0(x)$ is isospin

<u>axial current</u>: $q(x) \to q(x) - i\dfrac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\gamma_5 q(x)$

$\bar{q}(x) \to \bar{q}(x) - i\bar{q}(x)\gamma_5\, \underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}/2$

↑note sign! $\qquad \gamma_5\gamma_0 = -\gamma_5$

surface term $-\frac{1}{2}\bar{q}(x)q(x)\delta(r-R)$ not invariant

technical: surface term has odd chirality

physical: confinement forces quarks to be reflected at bag surface, but no spin flip in the reflection, so "chirality" or "handedness" or "helicity" of quark is changed

Introduce a <u>Lagrangian for the Bag Model</u>

$$\mathcal{L}_{MIT}(x) = \left[\frac{i}{2}\bar{q}(x)\overleftrightarrow{\not\partial} q(x) - B\right]\theta(R-r) - \frac{1}{2}\bar{q}(x)q(x)\delta(r-R)$$

$$\overleftrightarrow{\not\partial} = \gamma^\mu(\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\mu})$$

no current flow through surface

<u>Conserved currents</u>: Noether's theorem

→ invariance of $\mathcal{L}(x)$ associated with a conserved quantity

Consider a variation $\underline{\delta\varphi_i(x) = f_i(\{\varphi_i(x)\})\,\varepsilon(x)}$

infinitesimal

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_i} f_i\,\varepsilon(x) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)}\varepsilon(x)\,\partial_\mu f_i + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} f_i\,\partial_\mu\varepsilon(x)$$

If $\delta\mathcal{L} = 0$, we can integrate by parts and use $\delta\int d^4x\,\mathcal{L} = 0$ (equations of motion) to find

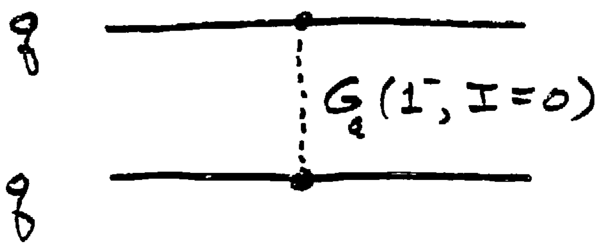
$$\int d^4x\,\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} f_i\right]\varepsilon(x) = 0$$

$\therefore$ $\underline{j^\mu(x) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} f_i}$ is a conserved current

i.e. $\underline{\partial_\mu j^\mu(x) = 0}$

Aside on <u>residual interactions in bag model</u>:

N, Δ degeneracy lifted by quark-quark interactions (<u>one gluon exchange</u>)



a = color index = 1....
$G = SU(3)_c \; [8]$

work to first order in α_c = QCD strong coupling constant

one gluon energy shift $= \Delta E_G = \alpha_c \sum_{a=1}^{8} \int d^3\vec{x} \, (\underset{\sim}{E}^a \cdot \underset{\sim}{E}^a - \underset{\sim}{B}^a \cdot \underset{\sim}{B}^a)$

color electric   color magneti

for quarks of same mass in same orbits, $\underset{\sim}{E}^a \cdot \underset{\sim}{E}^a \to c$

$$\boxed{\Delta E_G = \frac{\lambda \alpha_c}{R} \sum_{i<j} \overline{M}(m_i, m_j, R) \, \underset{\sim}{\sigma_i} \cdot \underset{\sim}{\sigma_j}}$$

$\lambda = 1$ for baryons.

$\longrightarrow \quad m_\Delta - m_N > 0 \quad$ determines α_c

resolution of problem of energy – momentum conservat

$$T^{\nu\mu}_{MIT} = (T_D^{\nu\mu} + B g^{\nu\mu})\, \theta(R-r)$$

Then $\partial_\nu T^{\nu\mu}_{MIT} = (-P_D + B)\, n^\mu\, \delta(r-R) = 0$

if $B = P_D = -\frac{1}{2}\, n\cdot\partial\left[\bar{q}(x)\, q(x)\right]_{r=R}$

This is the non-linear boundary condition of the MIT Bag Model

Interpretation of B:

total energy of bag $= \int d^3x\, T^{00}(x) = E(R)$

$$E(R) = 3\omega_{1,-1}/R + \frac{4\pi R^3}{3} B$$

boundary cond. $\rightarrow \partial E(R)/\partial R = 0$

$$\rightarrow R^4 = 3\omega_{1,-1}/4\pi B \quad \text{or} \quad B^{1/4} \sim R^{-1}$$

$$E(R) = \frac{4}{3}(3\omega_{1,-1})/R$$

if $E(R) \approx \frac{m_N + m_\Delta}{2}$, then $R \approx 1.4\,\text{fm}$, $B^{1/4} \approx 100$ MeV

Now $i n_\mu \bar{q} \gamma^\mu q = (\bar{q}\, i\gamma\cdot n) q = \bar{q}\,(i\gamma\cdot n q)$

$$= -\bar{q}q \quad = \bar{q}q$$

$$= 0$$

requires $\bar{q}q\big|_{r=R} = \bar{\Psi}\Psi\big|_{r=R} = 0$

This is just eigenvalue condition in Bogoliubov model

so no flow of quark current through surface!

<u>Problem with Bogoliubov model</u> → <u>violates energy-momentum conservation</u>

$$T^{\mu\nu} = T_D^{\mu\nu}\,\theta(R-r) = \frac{i}{2}\bar{q}(x)\,\gamma^\mu \overleftrightarrow{\partial}^\nu q(x)\,\theta(R-r)$$

↑ energy-momentum tensor $\qquad \overleftrightarrow{\partial}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu$.

$$\partial_\mu T^{\mu\nu} = \underbrace{(\partial_\mu T_D^{\mu\nu})}_{=\,0 \text{ from Dirac equation}}\theta(R-r) + T_D^{\mu\nu}\, n_\mu\,\delta(r-R)$$

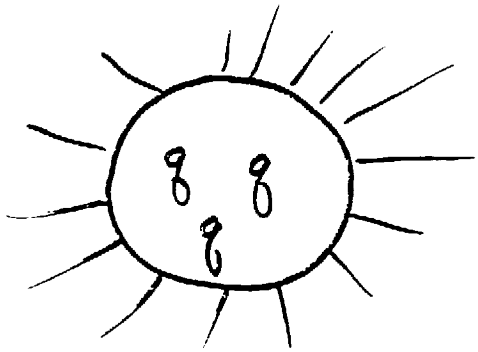
$$= \frac{i}{2}\bar{q}\,\gamma\cdot n\,\overleftrightarrow{\partial}^\nu q\;\delta(r-R) = -P_D\, n^\nu\,\delta(r-R)$$

where $P_D = -\frac{1}{2}\, n\cdot\partial(\bar{q}q)\big|_{r=R}$ = pressure exerted on bag surface by quarks

so $\partial_\mu T^{\mu\nu} \neq 0$ since $P_D \neq 0$ → violate energy-momentum conservation

MIT Bag Model: Phys. Rev. D9, 3471 (1974)
D12, 2060 (1975)
D12, 2733 (1975)

Physical picture:



nucleon = three quarks confined to sphere of radius R

obtain relation to Bogoliubov model: $\psi(x) \to q(x)$

MIT model requires no flow of quark current through surface at $r = R$

→ boundary condition $n_\mu \bar{q} \gamma^\mu q \big|_{r=R} = 0$

$$n^\mu = (0, \hat{r}) = \text{normal to surface}$$

implemented through linear boundary condition

$$i \gamma \cdot n \, q = q$$

pf: recall $\bar{q} = q^\dagger \gamma^0$, $\gamma^\mu = \gamma^0 \gamma^{\mu\dagger} \gamma^0$, so l.b.c. implies
$-i q^\dagger \gamma^\dagger \cdot n = q^\dagger$ or $-i \bar{q} \gamma \cdot n = \bar{q}$

Consider now the <u>"confinement limit" $m \to \infty$</u>

<u>eigenvalue eq.</u> for $j^\pi = \frac{1}{2}^+$ fermion:

$$j_0(ER) = j_1(ER)$$

<u>energy levels</u>: $E_{n\varkappa} = \omega_{n\varkappa}/R$

principal quantum no.

$\omega_{1,-1} = 2.04$, $\omega_{2,-1} = 5.4$ ....

<u>wave function</u>: $$\psi^{\mu}_{n,\varkappa=-1} = N_{n,-1}\begin{pmatrix} j_0\left(\frac{\omega r}{R}\right)\chi^{\mu}_{-1} \\ -i j_1\left(\frac{\omega r}{R}\right)\chi^{\mu}_{1}\end{pmatrix}$$

Note $\vec{\sigma}\cdot\hat{r}\,\chi^{\mu}_{\varkappa} = -\chi^{\mu}_{-\varkappa}$

Properties: 1) <u>Quark density ρ</u> $= \bar{\psi}\gamma^0\psi$; $\bar{\psi} = \psi^{\dagger}\gamma^0$

$$\sim \left(j_0^2\left(\frac{\omega r}{R}\right) + j_1^2\left(\frac{\omega r}{R}\right)\right)\theta(R-$$

$\rho \neq 0$ for $r = R - \varepsilon$, $\varepsilon \to 0$

2) $$\underline{\bar{\psi}\psi\Big|_{r=R}} \cong (j_0, i\vec{\sigma}\cdot\hat{r}j_1)\begin{pmatrix} j_0 \\ i\vec{\sigma}\cdot\hat{r}j_1\end{pmatrix}$$

$$\sim j_0^2(\omega) - j_1^2(\omega) = \underline{0}$$

eigenvalue

Consider a scalar field $W(r)$ in the Dirac equation

$$H\psi(r) = [\vec{\alpha}\cdot\vec{p} + \beta(m+W(r))]\psi(r) = E\psi(r)$$

two component form: $\psi_{\varkappa}^{\mu}(r) = \begin{pmatrix} g(r)\,\chi_{\varkappa}^{\mu} \\ if(r)\,\chi_{-\varkappa}^{\mu} \end{pmatrix}$

$\varkappa = -1$ is positive parity

coupled equations:
$$df/dr = (\varkappa-1)f/r - (E-u)g$$
$$dg/dr = (E+u)f - \frac{(\varkappa+1)}{r}g$$

where $u = m + W(r)$

Now consider square well: $W(r) = \begin{cases} -m & r \le R \\ 0 & r > R \end{cases}$

then $f = (E+u)^{-1}\, dg/dr$; $g = u/r$ for $\varkappa = -1$

$$\frac{d^2u}{dr^2} + (E^2-u^2)\,u = 0$$

Solution: $u(r) = \begin{cases} A\sin Er & r \le R \\ A\sin ER\, e^{-(m^2-E^2)^{1/2}(r-R)} & r \geq R \end{cases}$

also demand $f \sim dg/dr$ continuous at $r = R \longrightarrow$

$$\cos(ER) + \frac{(1-(E/m)^2)^{1/2}}{1+E/m}\sin(ER) = \frac{\sin(ER)}{ER}\left(1 - E/E+m\right)$$

eigenvalue equation for $E = E(R)$; $\varkappa = -1$

Simple Derivation of the MIT Bag Model:

Consider a fermion moving freely in a spherical volume of radius R, with a scalar potential operating for $r > R$ ("confinement")
[Bogoliubov (1967)]

Notation: Dirac eq. for spin ½ $\qquad H\psi = i\partial\psi/\partial t$

$$H = \underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta m$$

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} = \gamma_0 \vec{\alpha} \qquad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad g_{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$\not{p} = \gamma_\mu p^\mu = i\not{\partial}$$

Conserved quantities: $\quad \vec{j} = \vec{l} + \vec{\sigma}/2 \qquad (\text{not } \vec{l})$

$$K = \beta(\underset{\sim}{\sigma}\cdot\underset{\sim}{l} + 1)$$

$$[H, \vec{j}] = [H, K] = 0$$

Write $\psi(r,t) = \psi(r)\, e^{-iEt}$

Eigenfunctions: $\quad \vec{j}^2 \psi^\mu_x = j(j+1)\psi^\mu_x \quad , \quad j_z \psi^\mu_x = \mu \psi^\mu_x$

$$K \psi^\mu_x = -x\, \psi^\mu_x \quad ; \quad x = \pm(j + 1/2)$$

QCD has had many successes in describing high energy, high momentum transfer processes (deep inelastic electron scattering → scaling, quark counting rules, "jet" phenomena ....)

However, for low energy, low q processes, QCD calculations are difficult → must make some concessions to phenomenology

discuss <u>MIT bag model with chiral symmetry</u> as a phenomenological representation of QCD

<u>Outline</u>:

a) simple derivation (quarks in a square well)

b) incorporating energy-momentum conservation
→ boundary conditions

c) incorporate chiral symmetry
→ recover one pion exchange potential, PCAC, soft pion theorems

d) application to πN scattering in (3,3) resonance region

Fundamental treatment of πN scattering:

Concepts in QCD : quark and gluon fields fundamental
asymptotic freedom
confinement

quark flavors : $u, d, \underbrace{s, c, b, t}_{\text{heavy}}$

for light quarks (u,d), flavor is $SU(2)$ of isospin

quarks have three colors : group $SU(3)_c$, $\{3\}$ repr.

confinement $\rightarrow$ physical hadrons are "colorless"
$\rightarrow$ $\{1\}$ of $SU(3)_c$ group
$\rightarrow$ isolated Q or QQ systems not observed $(\{3\}\times\{3\} = \{\bar{3}\}+\{6\})$
$\rightarrow$ only $Q\bar{Q}$, QQQ are observable (in general $\alpha(Q\bar{Q})+\beta(Q^3)$)

asymptotic freedom : QCD coupling constant α_s has form

$$\alpha_s(q^2) = \alpha_s(\Lambda^2) / (1+\xi \ln(q^2/\Lambda^2))$$

so $\alpha_s(q^2) \rightarrow 0$ for $q^2 \rightarrow \infty$ $\Rightarrow$ weak interaction at short distances

CLOUDY BAG MODEL FITS TO πN (3,3) RESONANCE

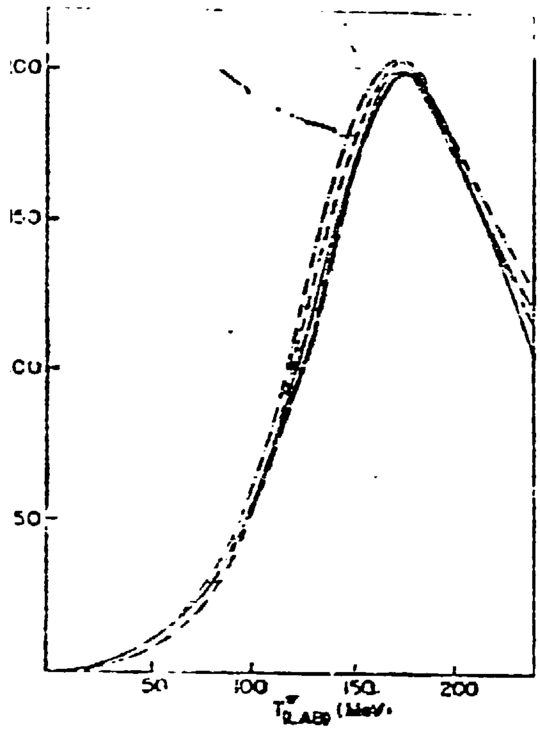
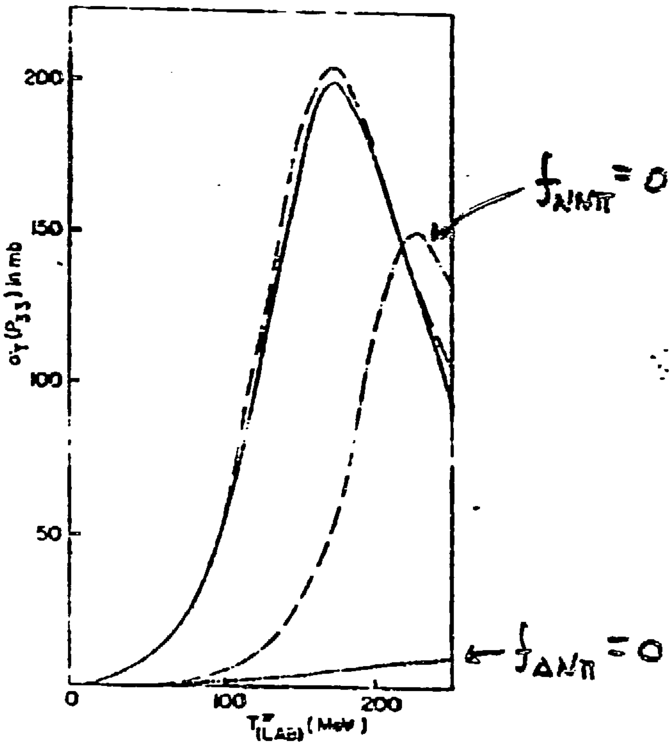


FIG. 13. Best-fit calculations using the CBM form ... but retaining only the delta graphs ...



... dashed ... ection (solid). ... arily setting ... in all other ... unchanged.

PION PRODUCTION IN NN COLLISIONS

$$NN \to NN\pi$$
$$\to d\pi$$

Pion Production in Nucleon-Nucleon Collisions

channels: $pp \to pn\pi^+$, $pp\pi^0$, $d\pi^+$ $\qquad pn \to nn\pi^+$, $pp\pi^-$, $pn\pi^0$, $d\pi^0$

thresholds: single pion production: $s = (2M_N + \mu_\pi)^2$

lab kinetic energy $= \underline{2\mu_\pi} + \mu_\pi^2/2m_N$

≈ 280 MeV

double pion production:

lab k.e. $= \underline{4\mu_\pi} + 2\mu_\pi^2/m_N \approx 600$ MeV

π production on nuclear target: $N + A \to N + A + \pi$

$$S_{thr} = (m_N + Am_N + \mu_\pi)^2 = \left(\sqrt{m_N^2 + p_L^2} + Am_N\right)^2 - p_L^2$$

$$T_L = (p_L^2 + m_N^2)^{1/2} - m_N = \frac{A+1}{A}\mu_\pi + \mu_\pi^2/2m_N A \quad \text{threshol}$$

for large A, $\underline{T_L \to \mu_\pi}$

Statistical Theory of Pion Production: Fermi (1950)

$p+p$ initial state fuses into compound system in statistical equilibrium in volume Ω

observer in c.m. system sees meson cloud of nucleon as Lorentz contracted sphere

→ looks like circle of radius $\hbar/\mu_\pi c$ perpendicular to direction of motion; along direction of motion, get Lorentz contraction factor $\gamma = E_{cm}/2M$

total energy concentrated in volume $\Omega = \Omega_0/\gamma$

where $\Omega_0 = \frac{4\pi}{3}\left(\hbar/\mu_\pi\right)^3$, so $\Omega = 8\pi M/3\mu_\pi^3 E_{cm}$

cross section for n particles in final state

$\propto$ probability that n particles contained in $\Omega \propto \Omega^{n-1}$

$$\boxed{\sigma(E, m_1 \ldots m_n) \sim \Omega^{n-1} \rho(E, m_1 \ldots m_n)}$$

$\rho(E, m_1 \ldots m_n)$ = density of states

$$\sim \int d\vec{p}_1 \cdots d\vec{p}_n \, \delta\Big(E - \sum_{i=1}^{n} (m_i^2 + p_i^2)^{1/2}\Big) \, \delta^{(3)}\Big(\sum_{i=1}^{n} \vec{p}_i\Big)$$

Can make into Lorentz-invariant phase space.

STATISTICAL THEORY OF PION PRODUCTION VS. EXPERIMENT

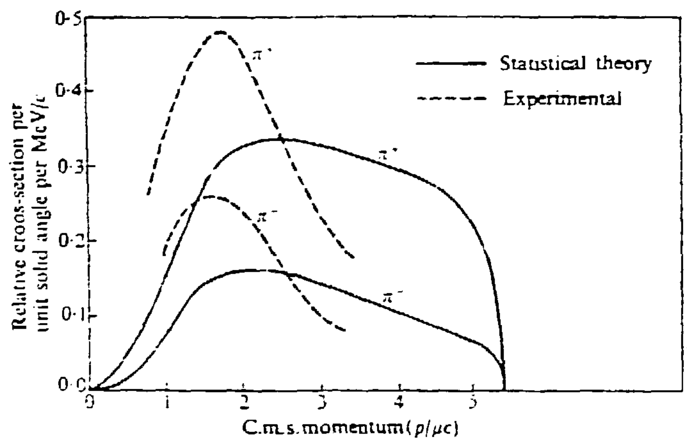


FIG. 6 ?. Comparison of the experimental results for the production of pions in Be and H (positive pion spectrum only) with the statistical theory predictions. ([11]. (1956). *Phys. Rev.* 103, 404)

from S. Lindenbaum, Oxford (1973)

THE ISOBAR MODEL FOR PION PRODUCTION

S.J. Lindenbaum and R.M. Sternheimer, Phys. Rev. 105, 1874 (1957); Phys. Rev. 106, 1107 (1958), Phys. Rev. 109, 1723 (1961)

Idea: NN energy high enough to excite one or both nucleons to isobaric levels

isobars characterized by variable mass m = total energy of decay products in isobar c.m.s.

General form: $d\sigma_{NN\to NN\pi}/dm_1 = \alpha_s F_s \sigma(m_1) a_s(\theta) d\Omega$

$d\sigma_{NN\to NN\pi\pi}/dm_1 dm_2 = \alpha_d F_d \sigma(m_1)\sigma(m_2) a_d(\theta) d$

F_s, F_d are phase space factors

$a_s(\theta), a_d(\theta)$ contain angular information

$\sigma(m)$ contains information on isobar mass and widt

no assumptions about detailed mechanism for isobar formation

later, peripheral model with one pion exchange was introduced → pins down the mechanism

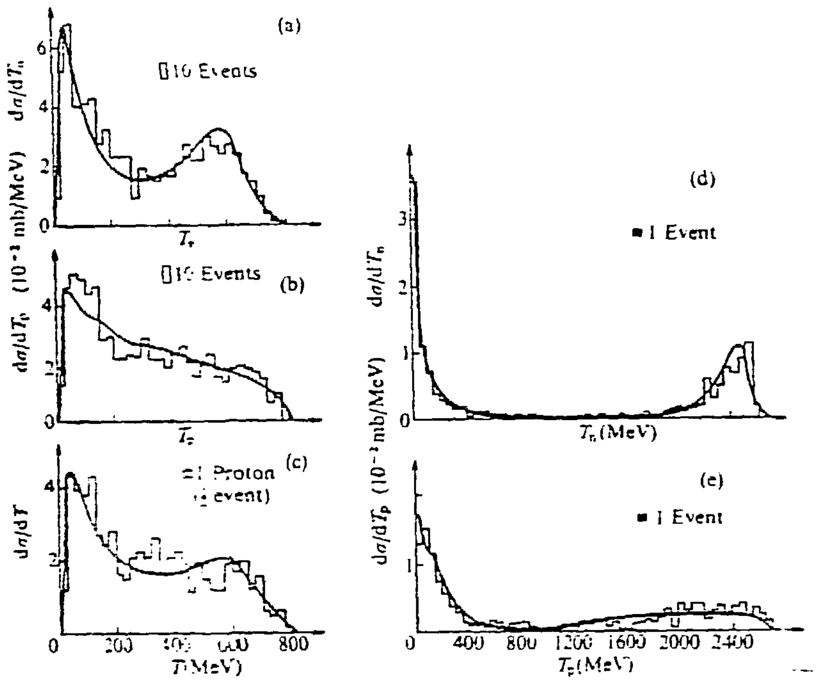


FIG. 6.11. Example of laboratory kinetic energy spectra fitted by the one-pion exchange model with form factors. (Taken from Ferrari and Selleri (1963), which also contains the experimental bibliography.) (a) Neutrons from $pp \to pn\pi^+$, (b) protons from $pp \to pn\pi^+$, (c) protons from $pp \to pp\pi^0$ (at an incident energy of 970 MeV), (d) neutrons, and (e) protons from $pp \to pn\pi^+$ (at 2.85 GeV), where only events in which the system $p\pi^+$ is in the region of the resonance N^{*++} have been selected. (From Bertocchi and Ferrari, [20].)

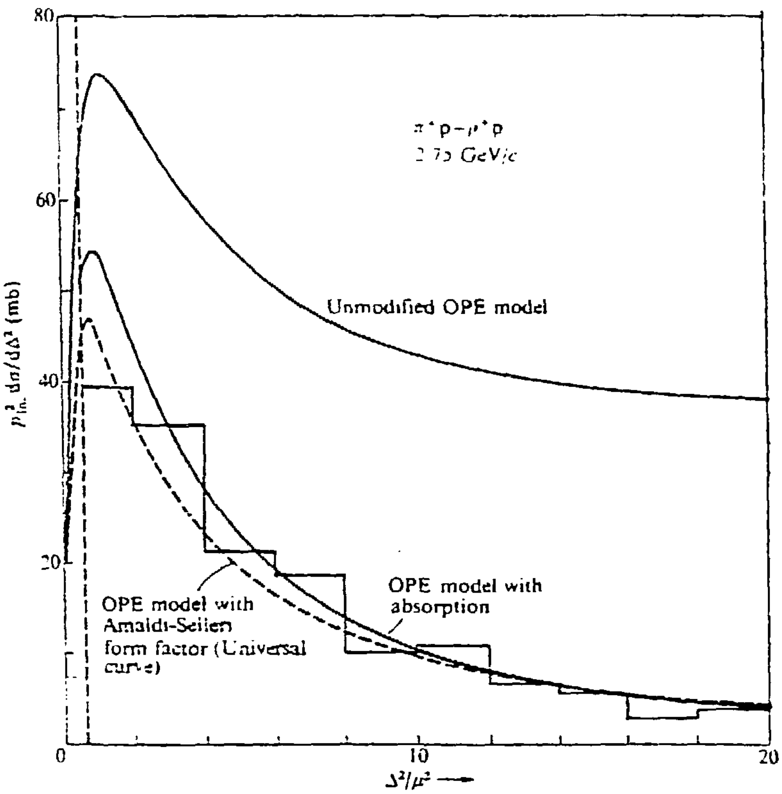
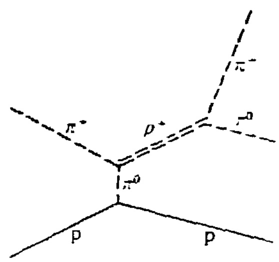


FIG. 6.12. Comparison of theory and experiment for the differential cross-section of the reaction, $\pi^+ p \to \rho^+ p$ at 2·75 GeV/c. The histogram represents the data. The upper solid curve is the cross-section predicted by the unmodified OPE. model. The dashed curve is given by the cross-section multiplied by the square of the form factor. The lower solid curve is that predicted by the OPE. model modified to include absorptive effects as described in §§ 5 and 6. The abscissa is Δ^2 in units of $\mu^2 = m_\pi^2 = 0{\cdot}0195\ (\mathrm{GeV}/c)^2$. (After review by Lindenbaum [38].)



PERIPHERAL MODEL FOR $\pi^+ p \to$ meson + N^*

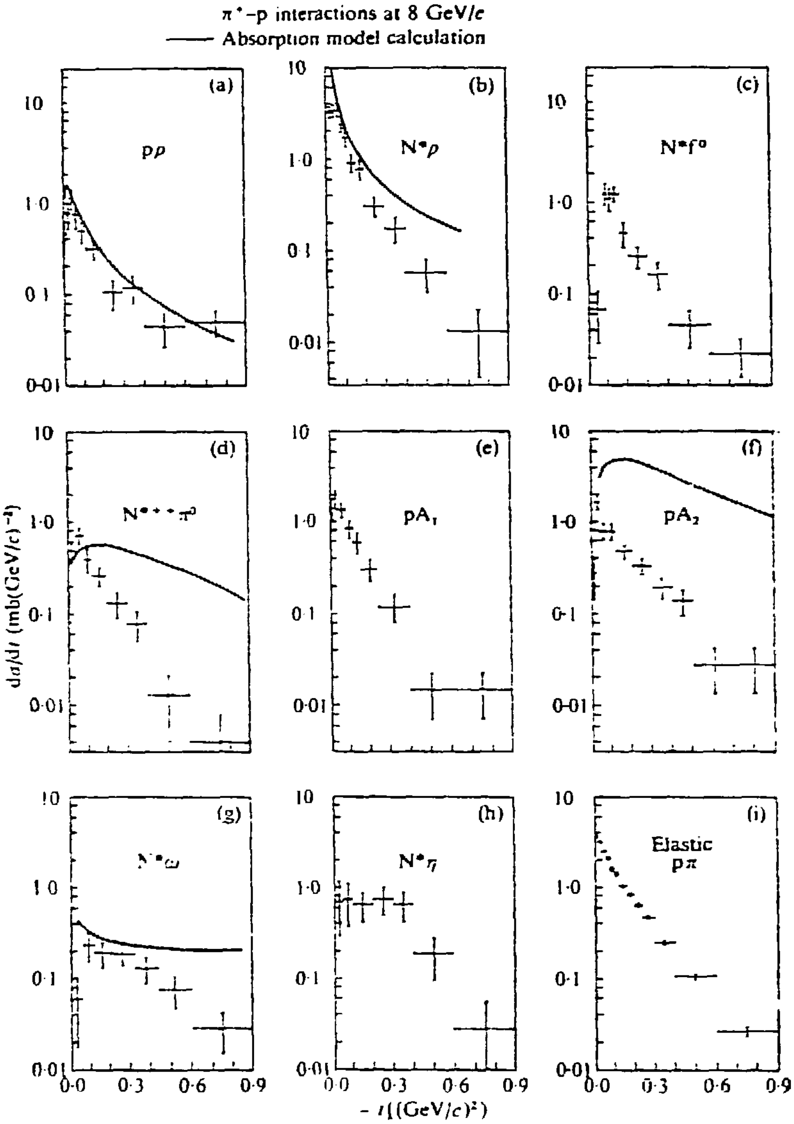


FIG. 6.13. 8 GeV/c π^+-p inelastic channels and their comparison with predictions of the OME (ρ or π) model, with absorption effects included. The graph was presented by the Aachen-Berlin-CERN collaboration at the Oxford Conference (1965). The figure is from Lindenbaum (Oxford, 1965)

ISOSPIN DECOMPOSITION OF REACTION CROSS SECTIONS

4 independent isospin cross sections for $\underbrace{NN}_{I} \rightarrow \underbrace{NN}_{I'}\pi$

Process	Cross Section
$pp \rightarrow d\pi^+$	σ_{10}^d
$pp \rightarrow pp\pi^0$	σ_{11}
$pp \rightarrow pn\pi^+$	$\sigma_{10} + \sigma_{11}$
$np \rightarrow d\pi^0$	$\frac{1}{2}\sigma_{10}^d$
$np \rightarrow np\pi^0$	$\frac{1}{2}(\sigma_{10} + \sigma_{01})$
$np \rightarrow nn\pi^+$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$
$np \rightarrow pp\pi^-$	$\frac{1}{2}(\sigma_{11} + \sigma_{01})$
$pp \rightarrow$ inelastic	$\sigma_{10}^d + \sigma_{10} + 2\sigma_{11} = \sigma_{I=1}$
$np \rightarrow$ inelastic	$1/2(\sigma_{I=1} + \sigma_{I=0})$; $\sigma_{I=0} = 3\sigma_{01}$

B. J. VerWest and R. A. Arndt, Phys. Rev. C25, 1979 (1982)

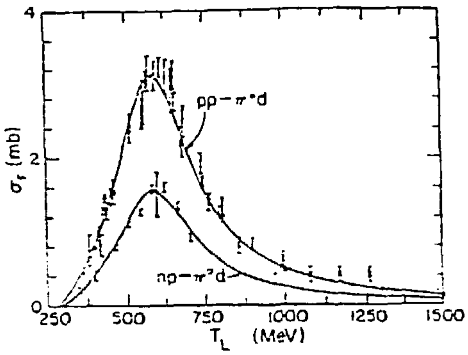


FIG. 3. Cross sections for reactions $pp \to d\pi^+$ and $np \to d\pi^0$. Data indicated as in Fig. 2.

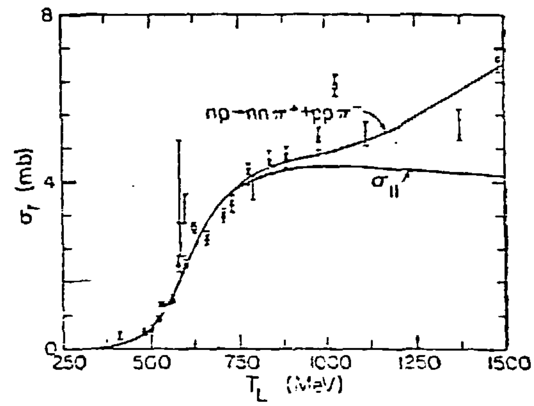


FIG. 4. Reaction cross section for $np \to nn\pi^+ + pp\pi^-$ and isospin cross section σ_{11}. Data are as indicated in Table II with data from Ref. 5 indicated by Δ, from Ref. 6 by $\square$, and Ref. 7 by $\times$.

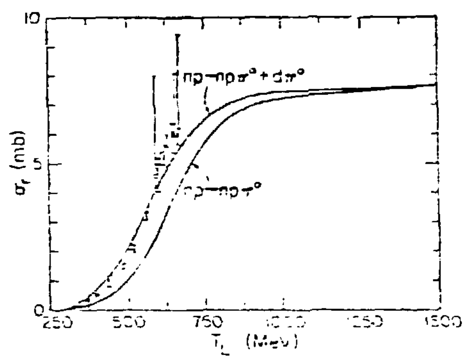


FIG. 5. Reaction cross sections for $np \to np\pi^0 + d\pi^0$ and $np \to np\pi^0$. The data are all for the first reaction as indicated in Table II.

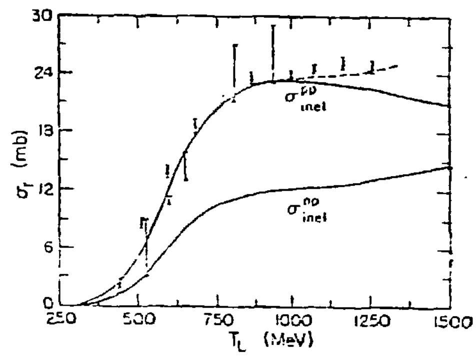


FIG. 6. Total reaction cross sections for single pion production for pp and np scattering. The data are as indicated in Table II except for those indicated by $\times$ which were obtained from $\sigma_{tot} - \sigma_{el}$ from Ref. 4. Also the dashed curve indicates the result of adding the cross section for 2π production from Ref. 4 to the single pion production curve.

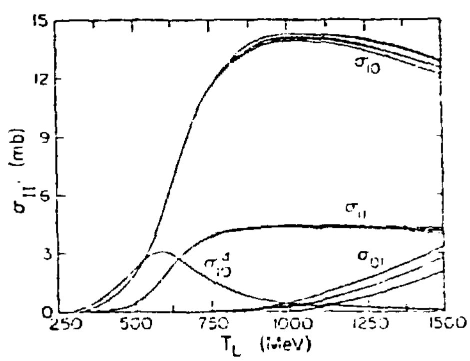


FIG. 7. Isospin reaction cross sections. The shaded areas indicate the relative uncertainties of their determination.

TWO PION PRODUCTION

$pp \to \pi^+ \pi^- pp$

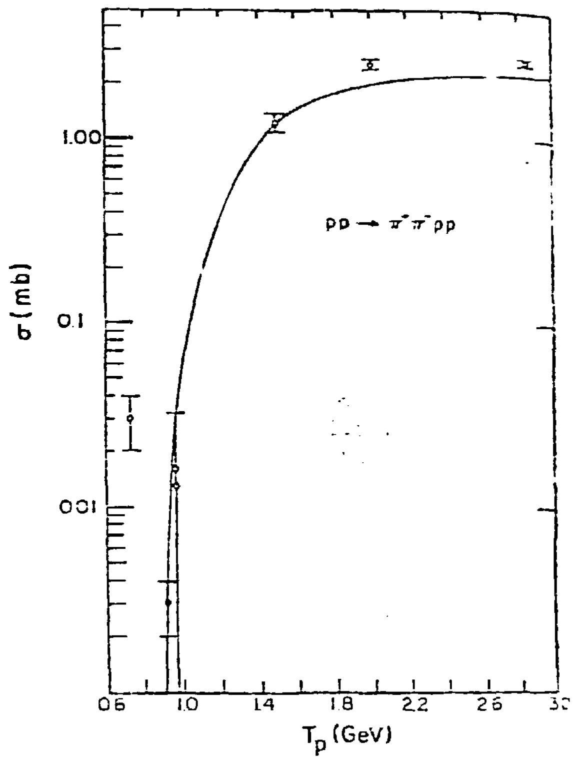


FIG. 13. Total cross section for double-pion production in proton–proton collisions ~~vs~~ incident proton energy (laboratory reference frame). Solid line is an extrapolation from the calculation of Lindenbaum and Sternheimer (Ref. 23).

Cverna et al, Phys Rev. C23, 1698 (1981)

PARAMETRIZATION OF $NN \to NN\pi$ REACTION CROSS SECTIONS

single most important feature: dominance of P_{33} πN resonance as substate of πNN system

$$\boxed{\sigma_{10}^{d}(s) = \frac{\pi(\hbar c)^2}{2p^2}\,\alpha\left(\frac{p_r}{p_0}\right)^{\beta}\frac{m_0^2\Gamma^2}{(s_{\pi N}-m_0^2)^2+m_0^2\Gamma^2}}$$

where $s = 4m_N^2 + 2m_N T_L$, $s_{\pi N} = (\sqrt{s} - m_N)^2$, $p^2 = s/4 - m_N^2$

$p_r^2(s) = \frac{1}{4s}\left[s-(m_d-m_\pi)^2\right]\left[s-(m_d+m_\pi)^2\right]$; $p_0^2 = \frac{s_0}{4} - m_N^2$; $s_0 = (m_N + m_0)$

For $NN \to NN\pi$, use effective Δ mass $\langle M(s)\rangle$:

$$\langle M(s)\rangle = (s^*)^{1/2} = \int_{m_N+m_\pi}^{\sqrt{s}-m_N} dM\,\sigma(M)\,M \Big/ \int_{m_N+m_\pi}^{\sqrt{s}-m_N} dM\,\sigma(M)$$

with $\sigma(M) = \frac{1}{\pi}\frac{\Gamma_0/2}{(\Gamma_0/2)^2+(M-M_0)^2}$

use same form as above for $\sigma_{II'}(s)$, with $s_{\pi N} \to s^*$,

$$p_r^2(s) = \frac{1}{4s}\left[s-(m_N-\langle M(s)\rangle)^2\right]\left[s-(m_N+\langle M(s)\rangle)^2\right]$$

$$\alpha \to \alpha\,(q/q_0)^3$$

$$q^2 = \frac{1}{4s^*}\left[s^*-(m_N-m_\pi)^2\right]\left[s^*-(m_N+m_\pi)^2\right]$$

$$q_0 = q(m_0^2)$$

PARAMETERS FOR $NN \to NN\pi$ CROSS SECTIONS

(VerWest + Arndt, Phys. Rev. C25, 1979 (1982))

	σ_{10}^{d}	σ_{11}	σ_{10}	σ_{01}
α	6.03	3.77	15.28	146.3
β	1.7	1.26	0	0
m_0 (MeV)	1203	1188	1245	1472
Γ (MeV)	134.3	99.	137.4	26.5

For $I=1$ initial state, $M_0 = 1220$ MeV, $\Gamma_0 = 120$ MeV (Δ

$I=0$ initial state, $M_0 = 1430$ MeV, $\Gamma_0 = 200$ MeV (P_{11} Roper

Conclusions: a) $\sigma_{01} \approx 0$ below $T_L = 1$ GeV

b) π production mostly peripheral with strong πN resonance quasiparticles

c) little π production from NN impulse approximation terms (non-resonant intermediate states)

d) no large violations of isospin symmetry

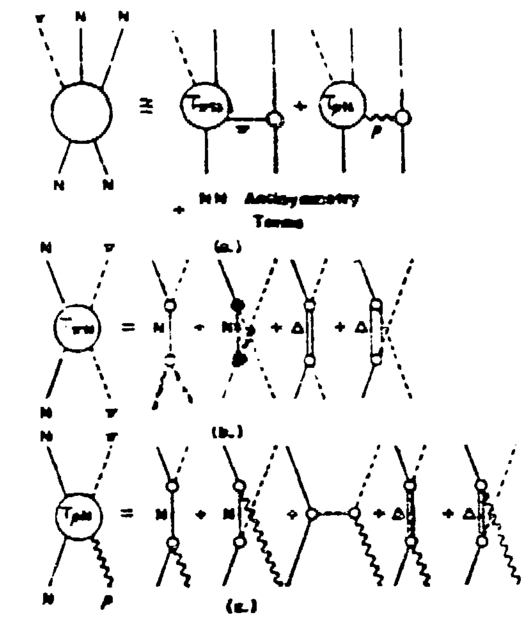
LOWEST ORDER PERIPHERAL MODELS FOR $NN\pi$ PRODUCTION

some examples: J. Hudomalj-Gabitzsch et al, Phys. Rev. C18, 2666 (1978)

B.J. VerWest, Phys. Lett. 83B, 161 (1979)

F.H. Cverna et al, Phys. Rev. C23, 1698 (1981)

B.J. VerWest, Phys. Lett. 83B, 161 (1979)



π and ρ exchange

πN → πN ampl.

πN → ρN ampl.

Fig. 1. (a) Feynman diagrams included in the model for NN → NNπ, (b) diagrams included for the reaction πN → πN, (c) diagrams included for the reaction ρN → πN.

Table 1
Lagrangians and coupling constants. $m_\pi = 139.0$ MeV, $m_\rho = 765.0$ MeV, $m_N = 938.9$ MeV.

Vertex	Lagrangian	Coupling constants
πNN	$L = -(f_\pi/m_\pi)\bar{\psi}\gamma_5\gamma_\mu\tau\cdot\partial^\mu\phi\psi$	$f_\pi/m_\pi = g_0/2m_N, g_0 = 13.61$
πNΔ	$L = (f_\pi^*/m_\pi)\bar{\psi}_\mu T\cdot\partial^\mu\phi\psi + \mathrm{h.c.}$	$f_\pi^* = 1.882$
ρNN	$L = -g_\rho\bar{\psi}\gamma_\mu\tau\cdot\rho^\mu\psi - (f_\rho/2m_N)\bar{\psi}\sigma_{\mu\nu}\tau\cdot\partial^\mu\rho^\nu\psi$	$g_\rho = 2.556, f_\rho = 12.781$
ρNΔ	$L = \mathrm{i}[g_\rho^*/(m_\Delta + m_N)]\bar{\psi}_\mu T\cdot[\partial^\nu\rho^\mu - \partial^\mu\rho^\nu]\gamma_\nu\gamma_5\psi + \mathrm{h.c.}$	$g_\rho^* = 29.910$
ρππ	$L = g_{\rho\pi\pi}(\phi\times\partial^\mu\phi)\cdot\rho_\mu$	$g_{\rho\pi\pi} = 2g_\rho$

$$\Gamma_{\pi NN} = \frac{\Lambda_1^2 - m_\pi^2}{\Lambda_1^2 - q_\pi^2}, \quad \Gamma_{\pi N\Delta} = \left[\frac{2\Lambda_1^2 - m_\pi^2}{2\Lambda_1^2 - q_\pi^2}\right]^2,$$

$$\Gamma_{\rho NN} = \Gamma_{\rho N\Delta} = \frac{\Lambda_2^2 - m_\rho^2}{\Lambda_2^2 - q_\rho^2}, \quad \Lambda_2 = 1500\ \mathrm{MeV}. \qquad (1)$$

form factors

B. J. VerWest, Phys. Lett. 83B, 161 (1979)

lowest order Feynman diagram approach for $NN \to NN\pi$

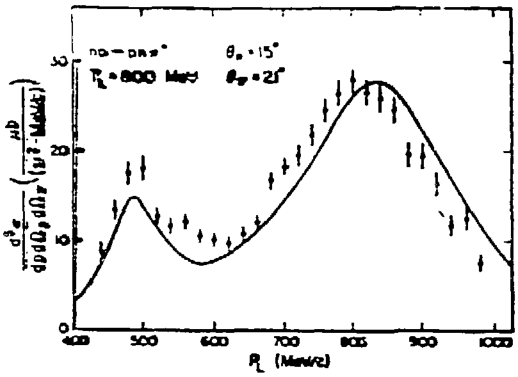
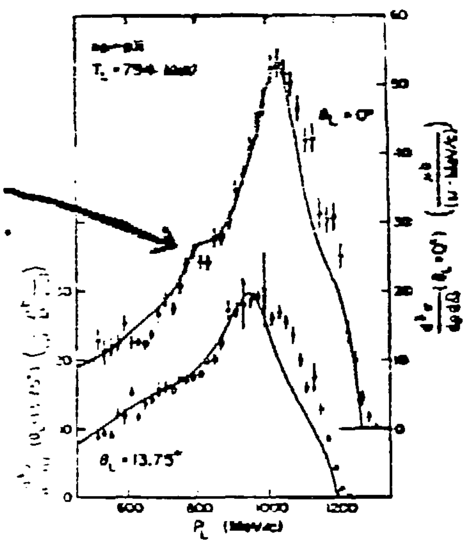


Fig. 4. Differential cross section $d^5\sigma/dp_p d\Omega_p d\Omega_\pi$ in the lab for $p + p \to p + n + \pi^+$ as a function of proton momentum for $\theta_p = 15°$ and $\theta_\pi = 21°$. The data are from ref. [illegible].

$d^5\sigma/dp_p\, d\Omega_p\, d\Omega_\pi$

$pp \to pn\pi^+$

lower peak from low energy NN final state interactions

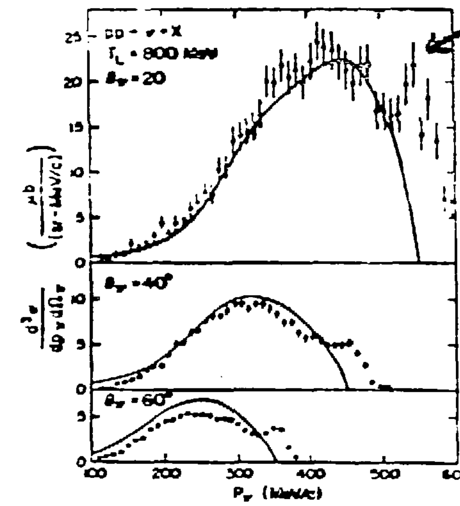


NN final state int.

Fig. 2. Differential cross section $d^3\sigma/dp_p d\Omega_p$ in the lab for $n + p \to p + X$ as a function of proton momentum [illegible] and $\theta_p = 13.75°$. The data are from ref. [illegible] and the curve is from the present calculation.

$d^3\sigma/dp_p\, d\Omega_p$

$np \to p + X$



from π final state

Fig. 3. Differential cross section $d^3\sigma/dp_\pi d\Omega_\pi$ in the lab for $p + p \to \pi^+ + X$ as a function of pion momentum for $\theta_\pi = 20°$, [illegible]. The data are from ref. [9].

$d^3\sigma/dp_\pi\, d\Omega_\pi$

$pp \to \pi^+ + X$

, Phys. Rev. C18, 2666 (1978)

$pp \to pn\pi^+$

$d^5\sigma/dp\,d\Omega_1\,d\Omega_2$

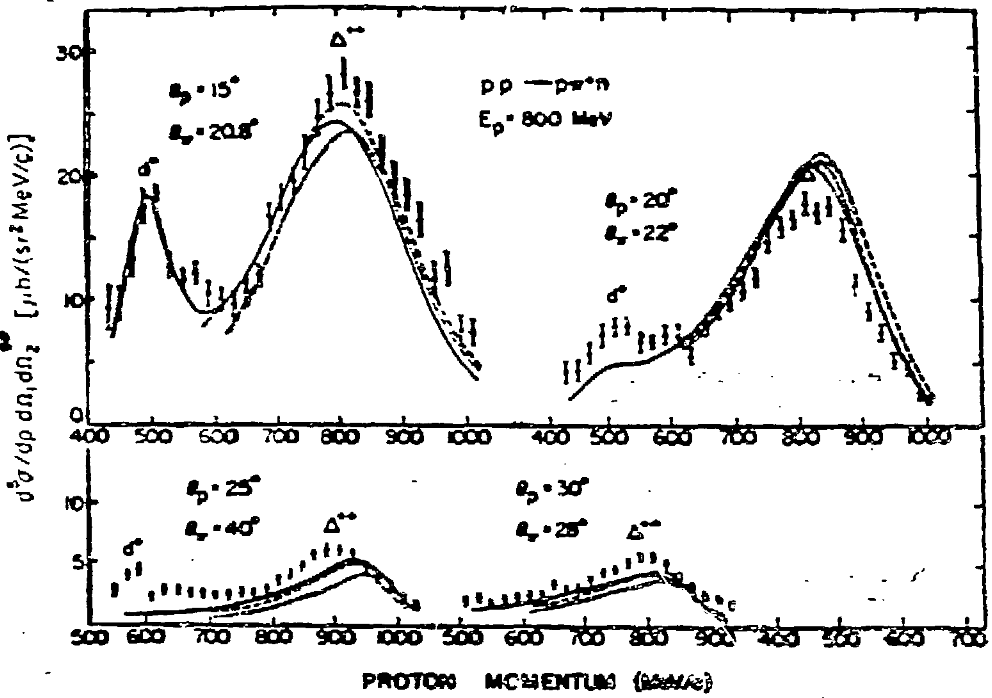


FIG. 8. Differential cross section $d^3\sigma/dp\,d\Omega_1\,d\Omega_2$ as a function of proton momentum and angle. The parameters for the curves are in units of GeV/c: (a) solid curve $\Lambda_\pi = 0.78$, $\Lambda_\rho = 1.3$, (b) dashed curve $\Lambda_\pi = 0.62$, $\Lambda_\rho = 1.5$, and (c) dot-dashed curve [illegible] curves; Galilean invariance was used.

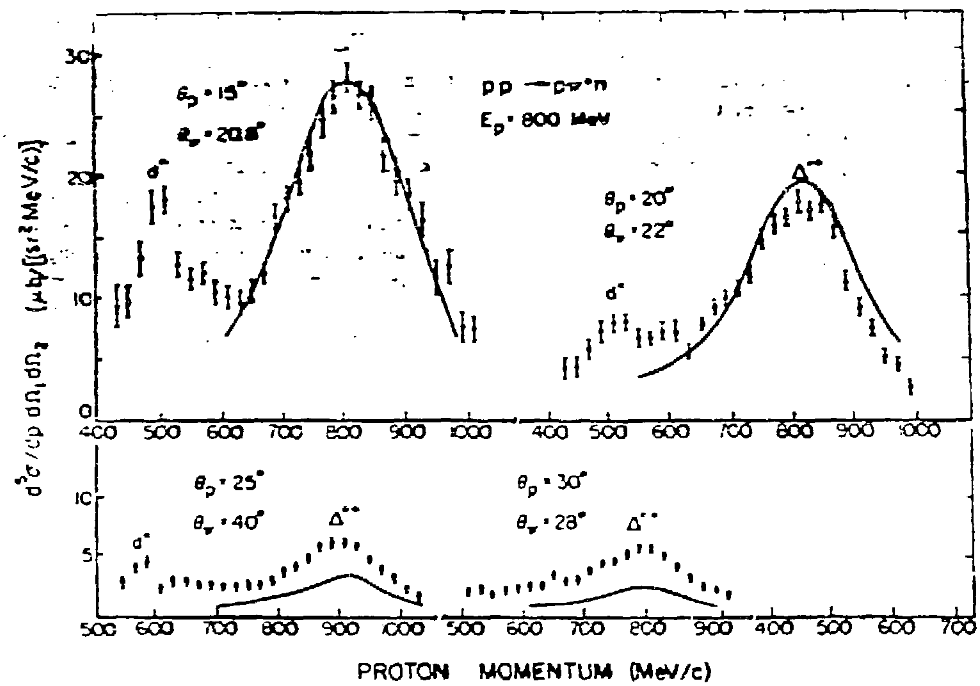


FIG. [illegible] Same as in Fig. 6. The calculation used the static approximation. Parameters are $\Lambda_\pi = 0.72$ GeV/c and Λ_ρ [illegible] GeV/c.

solid curves include first order Feynman diagrams with π and ρ exchange

$pp \to \pi^+ p n$ at $T_p = 800$ MeV

F. H. CVERNA *et al.*, Phys. Rev. C23, 1698 (1981)

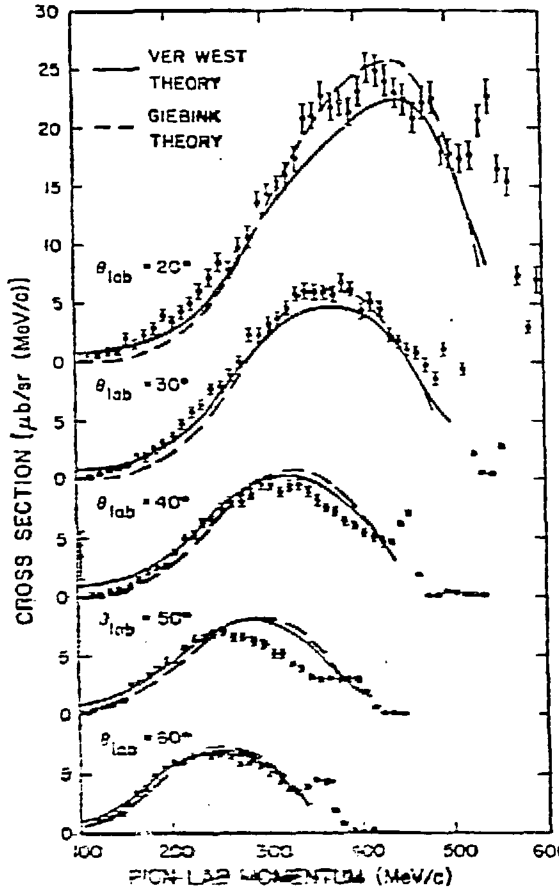


FIG. 3. Momentum spectra of π^+ (laboratory reference frame).

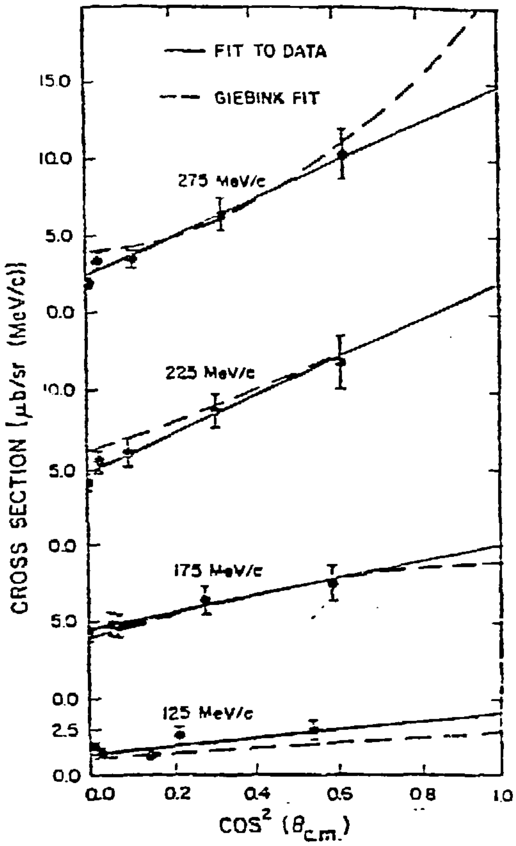


FIG. 4. Angular distributions of π^+ at selected over-all-center-of-mass (OACM) momenta of 125, 175, 225, and 275 MeV/c. Solid lines are least-squares fits to the data. Dashed lines are from the calculations of Giebink (α = 730 MeV/c).

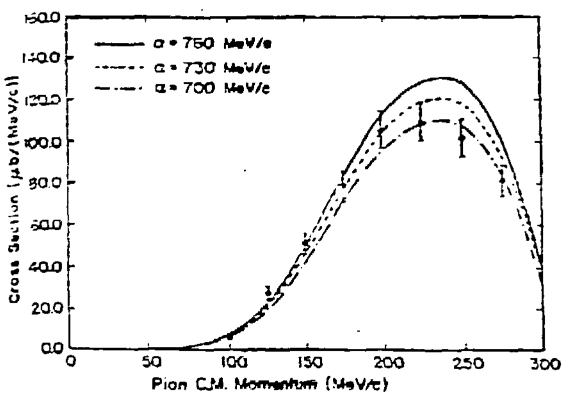


FIG. 5. Cross section for π^+ production integrated over angle ($d\sigma/dq = 8\pi^2 c_0$). Lines are from the calculation of Giebink using different form factors. Solid line, α = 760 MeV/c. Dashed line, α = 730 MeV/c. Dot-dash line, α = 700 MeV/c.

use <u>isospin invariance</u> to write

$$\mathcal{M}(pp \to np\pi^+) = -\sqrt{2}\,(M_C^+ - M_B^+) - M_A^{CEX} + M_D^{CEX}$$

u.s.

$\pm$ refers to $\pi^\pm p$ elastic, CEX to $\pi^- p \to \pi^0 n$

<u>Approximations</u>:

a) <u>no off-shell corrections for</u> $\pi N \to \pi N$ partial wave amplitudes $\underline{T_{\ell\pm}}$

b) take into account <u>full off-shell kinematics</u> of angular momentum projection operators

c) use <u>form factor</u> $e^{\lambda t_i}$ at πNN vertex
$\lambda \approx 0.4\ \mathrm{GeV}^{-2}$ is only parameter

<u>Example</u>: $\mathcal{M}^{off}(\pi N \to \pi N) = \bar{u}(p_i)\left[A^{off} + \not{p}_\pi B^{off}\right] u(p_j$

$$A^{off} = \frac{4\pi}{q_i^3}\Big\{ (E_i - m)(\sqrt{s_i} + m)\, T_{0+}^{on} - (E_i + m)(\sqrt{s_i} - m)\, T_{1-}^{on} + \dots \Big\}$$

E_i = energy of outgoing nucleon in πN_i c.m. syste

For $q_\pi^2 = m_\pi^2 \to$ recover usual partial wave expans

Deck model includes isobar and nucleon pole models as special cases:

$T_{1^-} \to$ nucleon pole (P_{11}) , $T_{1^+} \to \Delta$ pole (P_{33})

Other off-shell extensions of the πN amplitude have been tried:

Example: $A^{off} = A^{on}$, $B^{off} = B^{on}$

This works for integrated $NN \to NN\pi$ cross sections ($I=$
but fails for angular distribution of pions

Conclusions:

1) for $p_L = 1.5$ GeV/c, cross sections systematically below data in magnitude → this could indicate normalization errors in data or presence of dibaryon resonances
2) good agreement with angular distributions
3) strong spin dependence for $NN \to NN\pi$
4) can neglect ρ exchange at low p_L

Inclusion of dibaryon resonances in deck model

no information available on relative phases of amplitude

so add dibaryon resonance contribution incoherently

$$\sigma^{res} = 2\pi \frac{4s}{\lambda(s, m_a^2, m_b^2)} \frac{x(1-\Gamma_{el}/\Gamma)\, m_r^2 \Gamma^2}{(s-m_r^2)^2 + m_r^2 \Gamma^2} \left(\frac{s - s_{thr}}{m_r^2 - s_{thr}}\right)^{J/2}$$

$$x = (2J+1)\, \Gamma_{el}/\Gamma$$

np channel best testing ground for dibaryon resonance since Δ does not contribute to $I=0$ amplitudes

→ smaller background from peripheral mechanism

Dibaryon resonance parameters obtained from nucleon-nucleon scattering data:

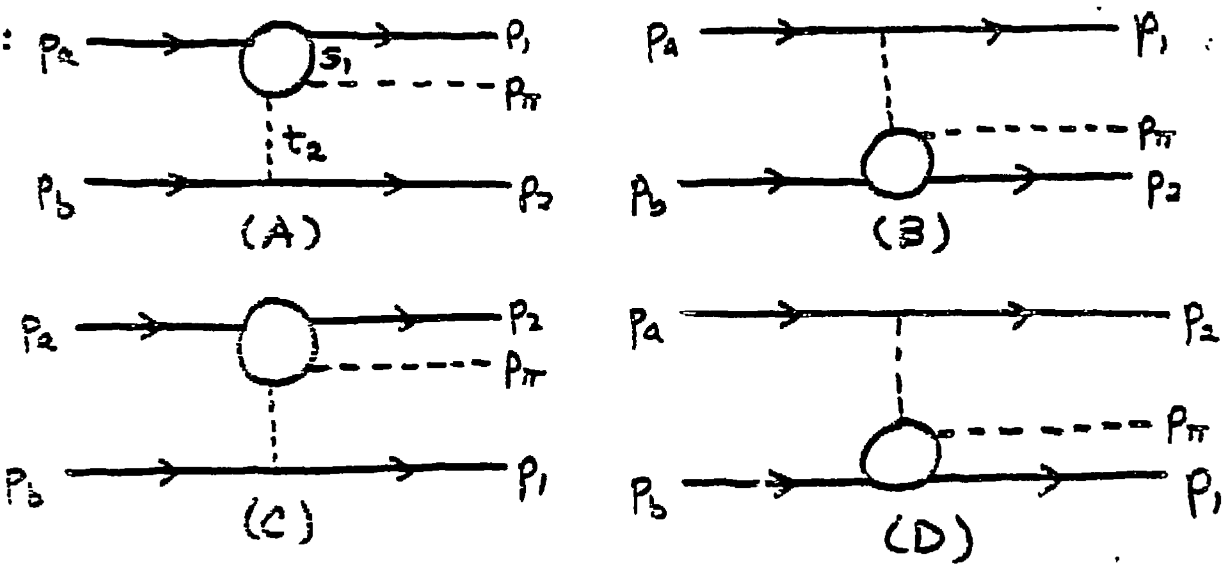
Mass (MeV)	J^π	I	Γ (MeV)	x	
{2240 {2340	3^-	1	{120 {140	{1.4 {0.98	PL 70B, 47 PRL 41, 35 NP B137, 17 (1978
2390	$0^+, 2^+$, or 4^+	1	90	0.33	
2250	1^+ or 3^+	0	100	0.91	PL 96B, 17 (1980)

The "*Deck Model*" for $NN \to NN\pi$:

König + Kroll, Nucl. Phys. A356, 345 (1981)
R.T. Deck, Phys. Rev. Lett. 13, 169 (1964)

a refined form of the one pion exchange peripheral model $\to$ describe $NN \to NN\pi$ (both $I = 0, 1$) from threshold to 6 GeV/c.

Graphs:



Work in the helicity representation (λ)

$$M_A(\lambda_1 \lambda_2 ; \lambda_a \lambda_b) = \frac{g}{\sqrt{4\pi}} \, \bar{u}(p_1\lambda_1)\left[A^{off}(t_2, s_1, t_1) + \not{p}_\pi B^{off}\right]$$

$$\times \, u(p_a\lambda_a) \frac{i}{t_2 - m_\pi^2} \, \bar{u}(p_2\lambda_2)\gamma_5 \, u(p_b\lambda_b)$$

$g^2/4\pi = 14.28$ is πNN coupling, $t_1 = (p_1 - p_a)^2$, $s_1 = (p_1 + p_\pi)^2$

A^{off} and B^{off} are off-shell $\pi N \to \pi N$ invariant amplitud

Isospin 1 CROSS SECTION FOR $NN \to NN\pi$

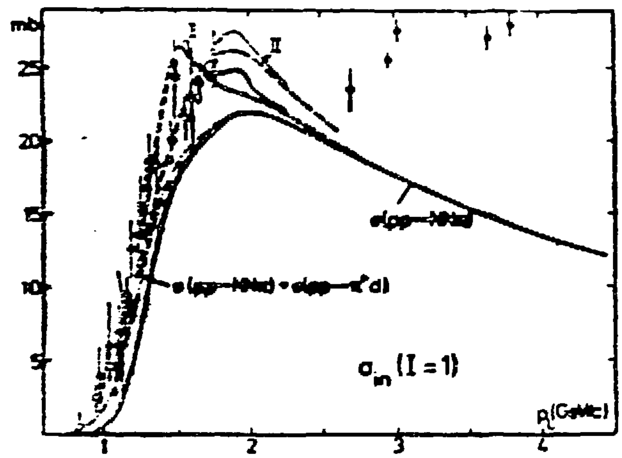


Fig. 11. The isospin one inelastic total cross section versus lab momentum. Data are taken from refs. [illegible] (●) and [33] (○), where the latter points are obtained from integrated elastic differential cross sections which have been extrapolated to $t = 0$ with the aid of the forward amplitudes from dispersion theory [illegible]. The solid line is the prediction of the Deck model; the dashed-dotted line is the Deck cross section plus the experimental values for the $pp \to \pi^+ d$ cross sections [illegible] (smoothed by hand). The dashed lines are the additional contributions from the 3^- dibaryon resonance (I: m_{res} = 22[illegible]0 MeV, II: m_{res} = 2320 MeV). Finally, adding the 4^+ resonance one arrives at the dotted lines.

solid line = prediction of König + Kroll,
Nucl. Phys. A356, 345 (1981)
(Deck model)

dashed lines include additional contributions of 3^-, 4^+ dibaryon resonances

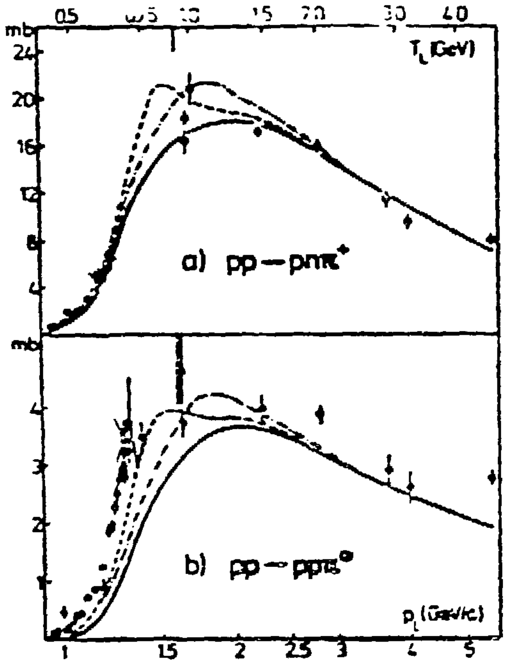


Fig. 2. The reaction cross sections for $pp \to pn\pi^+$ (a) and $pp \to pp\pi^0$ (b) versus lab momentum p_L and lab kinetic energy T_L. Data are taken from ref. [18]. The solid lines are the Dock model predictions. The dashed (dashed-dotted) lines are the [illegible] $J = 1$ dinucleon resonance with mass 2240 (2320) [illegible] (see text).

REACTION CROSS SECTIONS FOR

$pp \to pn\pi^+$

$pp\pi^0$

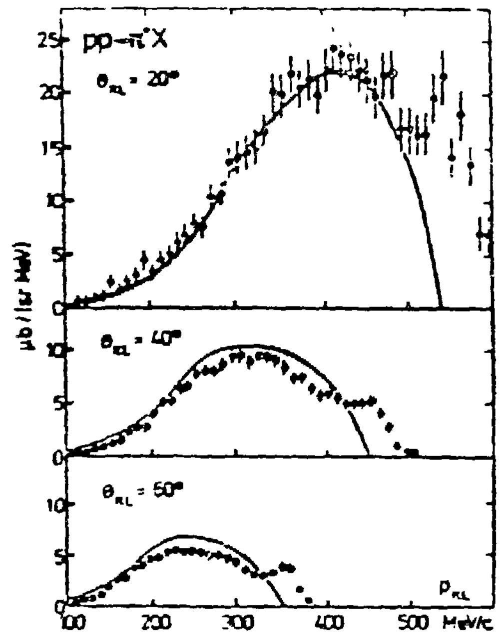
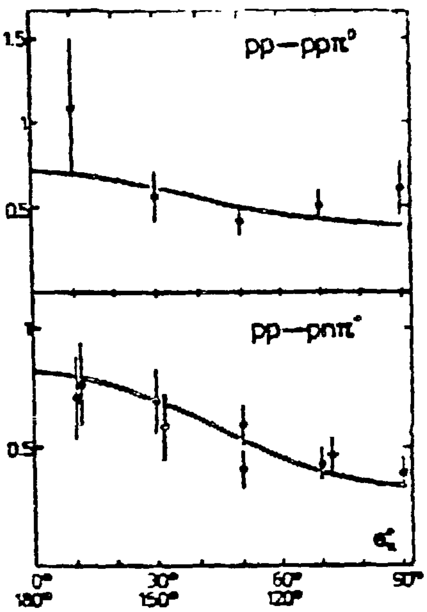


Fig. 3. The differential cross section [illegible] system for the inclusive reaction $pp \to \pi^+X$ versus pion momentum [illegible] kinetic energy T_L is 800 MeV. Data [illegible] are taken from ref. [illegible] the Dock model predictions.

DIFFERENTIAL CROSS SECTION FOR $pp \to \pi^+ X$ (inclusive)

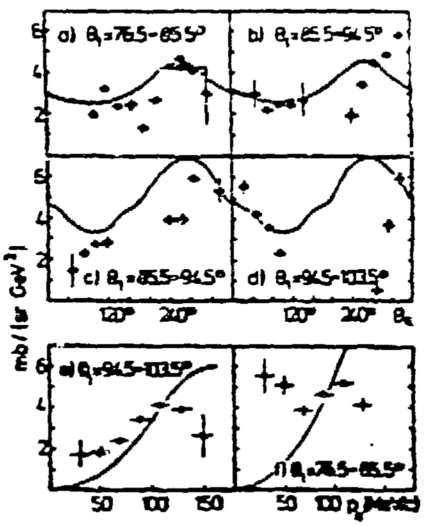
$d^2\sigma / 2\pi\, dp_{\pi L}\, d\cos\theta_{\pi L}$

$T_p = 800$ MeV



$\frac{1}{\sigma}\, d\sigma/d\cos\theta^*_\pi$

Fig. 4. Angular distributions (in c.m. system) $(1/\sigma)\,d\sigma/d\cos\theta^*_\pi$ for π^0 and π^+ production at T_L = 650 MeV compared to the Deck model predictions (solid lines). Data [18]) are not normalized



pp → ppπ°
differential
cross section

$\frac{1}{p^{*2}}\, d^4\sigma/2\pi\, dp^*_\pi\, d\Omega^*_\pi\, d\cos\theta^*$

Fig. 5. The $pp \to pp\pi^0$ differential cross section $(1/p^{*2})\,d^4\sigma/2\pi\,dp^*_\pi\,d\Omega^*_\pi\,d\cos\theta^*_\pi$ in the c.m. system for coplanar events at T_L = 620 MeV. The solid lines are the normalized Deck model predictions (compare text). Data are taken from [illegible] and (d) p^*_π = 100–120 MeV/c; (e) [illegible] = 253–276°

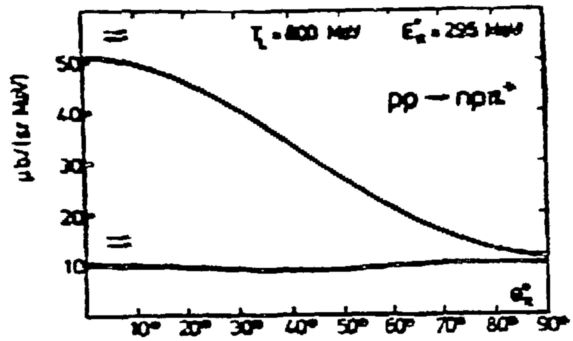


Fig. 6. The differential cross section $d^2\sigma/d\Omega\, dE_\pi^*\, d\cos\theta_\pi^*$ for longitudinally polarized incident protons at $T_L = 800$ MeV as predicted by our model

ANGULAR DISTR. WITH POLARIZED PROTONS

strong spin dependence predicted !

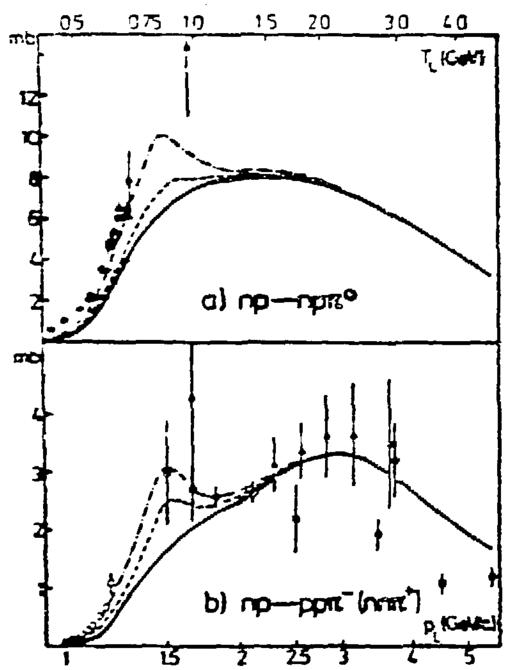


Fig. 7. The reaction cross sections for $np \to np\pi^0$(a) and $np \to pp\pi^-$, $nn\pi^+$(b) versus lab momentum and kinetic energy. The data are taken from ref. [16]) (●), DESY [20]) (▲), DUBNA [21]) (■), FREIBURG [22]) (○), BIRMINGHAM [23]) [illegible] the latter two are [illegible] data. The solid lines are the Deck model predictions. The dashed [illegible] resonance. The dashed-dotted lines are these fr[illegible] MeV and η = 1 (see text).

solid lines = Deck model prediction

dashed lines include I=0 dibaryon resonan

MORE COMPLETE APPROACHES TO $NN \to NN\pi$, $d\pi$

1) Helsinki group (Green, Niskanen, Sainio)

NN, NΔ, ΔΔ as coupled channels (CCM)

A.M. Green and J.A. Niskanen, Nucl. Phys. A271, 503 (1976)

J.A. Niskanen, Nucl. Phys. A298, 417 (1978)

2) Rochester group (Koltun, Myhrer, Mizutani)

non-relativistic Fadeev theory (emphasis on πd elasti

T. Mizutani & D. Koltun, Ann. Phys. 109, 1 (1977)

F. Myhrer & D. Koltun, Nucl. Phys. B86, 441 (1975)

3) Australian group (Afnan, Blankleider, Thomas)

NN, πd, NΔ incorporated in one set of unitary, coupled three body integral equations

J.R. Afnan + A.W. Thomas, Proc. Int. Conf. on Few Particle Problems, UCLA (1972); Phys. Rev. C10, 109 (1974)

I.R. Afnan & A.T. Stelbovics, Phys. Rev. C23, 1384 (1981)

B. Blankleider and I.R. Afnan, Phys. Rev. C22, 1638 (1980

APPROACHES TO $NN \to NN\pi$, $d\pi$ (continued)

4) Argonne group (Betz, Coester, + Lee)

coupled channels method with bare N and Δ, including $\Delta N\pi$ but not $NN\pi$ vertex

M. Betz and F. Coester, Phys. Rev. C21, 2505 (1980)

M. Betz and T.-S. H. Lee, Phys. Rev. C23, 375 (1981)

5) Los Alamos group (Dubach, Kloet, Silbar)

coupled channel 3-body model → emphasis on πNN continuum states

W. M. Kloet and R. R. Silbar, Nucl. Phys. A338, 281 (198[illegible]) + 317

Phys. Rev. Lett. 45, 970 (198[illegible])

J. Dubach, W. M. Kloet, and R. R. Silbar, Phys. Rev. D22, 27[illegible] (1981) + preprints

6) Review articles

A. W. Thomas + R. H. Landau, Phys. Rep. 58C, 121 (1980)

D. Bugg and A. W. Thomas, review talks at Versailles (1981)

J. A. Niskanen, 5th Int. Symposium on Polarization Phenomena, Sante Fe (1980)

M. Betz, B. Blankleider, J. A. Niskanen and A. W. Thomas, Workshop on Pion Production and Absorption in Nuclei

SPIN OBSERVABLES IN $NN \to \pi d, \pi NN$

pion asymmetry in $pp \to \pi^+ pn$ has been measured

$$\left(\frac{d^2\sigma}{d\Omega dq}\right)_{pol} = \left(\frac{d^2\sigma}{d\Omega dq}\right)_{unpol} + \vec{P}_B \cdot \hat{n} \sin\theta_{cm} \left[d_0 + d_1 P_1(\cos\theta_{cm}) + \cdots \right]$$

pion angle

beam polarization

beam direction

asymmetry for $pp \to \pi^+ d$ has also been measured

introduce partial wave helicity amplitudes

$$\underbrace{F_{\mu_1 \mu_2}}_{\text{nucleon helicities}};\lambda = \frac{1}{4\pi} \sum_J (2J+1) F^J_{\mu_1 \mu_2;\lambda}\, d^J_{\mu,-\lambda}(\theta)$$

deuteron helicity

$$\mu = \mu_1 - \mu_2$$

pp state labeled by ${}^{2S+1}L_J$; $\pi^+ d$ state L'_J

apply Pauli principle and parity conservation

example: $J =$ even $\quad {}^1J_J \to (J \mp 1)_J \quad$ parity $+$

$\qquad\qquad\qquad\qquad {}^3(J \mp 1)_J \to J_J \quad$ parity $-$

$J =$ odd $\quad {}^3J_J \to (J \mp 1)_J \quad$ parity $-$

examples of transitions : ${}^1S_0 \to P_0$, ${}^3P_1 \to S_1, D_1$

${}^1D_2 \to P_2, F_2$

introduce <u>partial wave amplitudes</u> $T^J_{\underset{pp}{LS};\underset{\pi d}{L'}}$

J = even
$$F^J_{\frac12\frac12;\pm1} = \left(\frac{J+1}{2J+1}\right)^{1/2} T^J_{J0;J-1} + \left(J/_{2J+1}\right)^{1/2} T^J_{J0;J+1}$$
$$\pm\left(\left(J/_{2J+1}\right)^{1/2} T^J_{J-1,1;J} - \left(J+1/_{2J+1}\right)^{1/2} T^J_{J+1,1;J}\right)$$

similar for other helicities and odd J

Now
$$\sigma^{pp\to\pi^+d}_{TOT} = \frac{\pi}{4p^2}\sum(2J+1)\left|F^J_{\mu_1\mu_2;\lambda}\right|^2$$

$$\frac{d\sigma}{d\Omega} = \frac14\left(\frac{2\pi}{p}\right)^2\sum\left|F_{\mu_1\mu_2;\lambda}\right|^2$$

$$P(\theta)\frac{d\sigma}{d\Omega} = \left(\frac{2\pi}{p}\right)^2\frac12\sum \mathrm{Im}\left(F_{\frac12\mu_2;\lambda}F^*_{-\frac12\mu_2;\lambda}\right)$$

<u>asymmetry</u> (or polarization) of scattered pion

In general, <u>both beam and target can be polarized</u>

general form defining asymmetries A_{ij} for $pp \to \pi^+ d$

$$\boxed{\sigma_{ij}(\text{polarized}) = \sigma_{00}(\text{unpol.})\,(1 + P_i A_{i0} + P_j A_{0j} + P_i P_j A_{ij})}$$

example: if beam and target pol. along $+z$,

$$\sigma_{zz}(\theta) = \sigma_{00} A_{zz}(\theta)$$

need spin-spin correlation and spin transfer measurements to form a complete set of observables
→ non co-planar geometries

<u>observation</u>: simple peripheral model cannot account for fine details of spin observables

Development of the theory: <u>non-perturbative few-body calculations of π production</u>

<u>Afnan + Thomas</u>, Phys. Rev. C10, 109 (1974)

3-body description in bound state model
NN channel as $N + (\pi N)_{P_{11}}$ bound state

$\pi d \to \pi d$, $\pi d \leftrightarrow NN$, $NN \to NN$ ok for low energies

<u>Blankleider + Afnan</u>, Phys. Rev. C22, 1638 (1980)
Flinders preprint FIAS-R-76 (1981)
formalism, three-body equations for πNN

<u>Afnan + Blankleider</u>, Phys. Lett. 93B, 367 (1980)

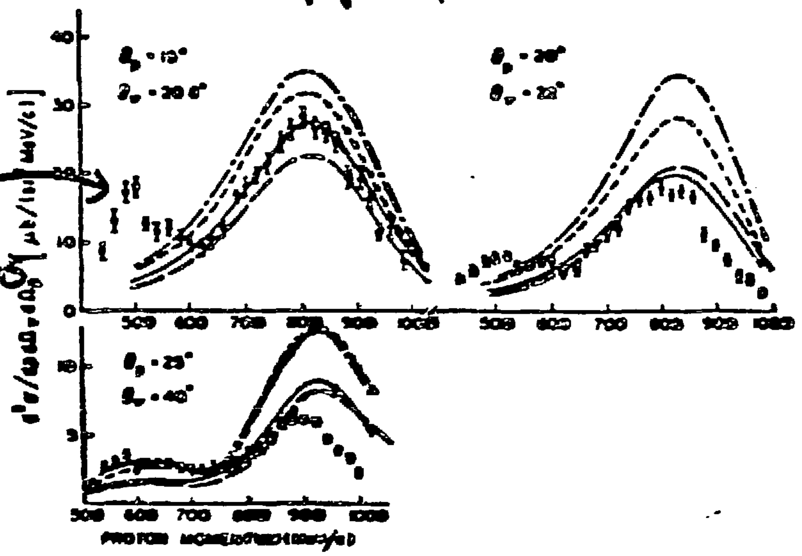
first attempt to solve full set of πNN eqs.

<u>Fayard et al</u>, Versailles Conf. (1981)
more ambitious 3-body calculation

DIFFERENTIAL CROSS SECTIONS
$pp \to pn\pi^+$ at 800 MeV

NN final state interactions

$$\frac{d^5\sigma}{dp\, d\Omega_\pi\, d\Omega_p}$$



solid curves include NN initial state interaction: → lower σ

Fig. 3. Differential cross sections for kinematically complete measurements[58] of $pp \to pn\pi^+$ at T_L = 800 MeV. The geometries are coplanar; θ_p and θ_π are the laboratory angles of the proton and the pion, respectively. The data is plotted versus the lab proton momentum. The parameters of the NN → NΔ transition operator corresponding to each curve are (in MeV/c): a) --- $\Lambda_\pi = \Lambda_\rho$ = 1200; b) —— Λ_π = 1200, Λ_ρ = 1600; c) —·— Λ_π = 700, no ρ-exchange; d) -·-·- Λ_π = 700, no ρ-exchange, no initial state interactions. The momentum transfers for the $pp \to n\Delta^{++}$ process corresponding to the three geometries are roughly 300, 360 and 460 MeV/c.

ANALYZING POWER A_y

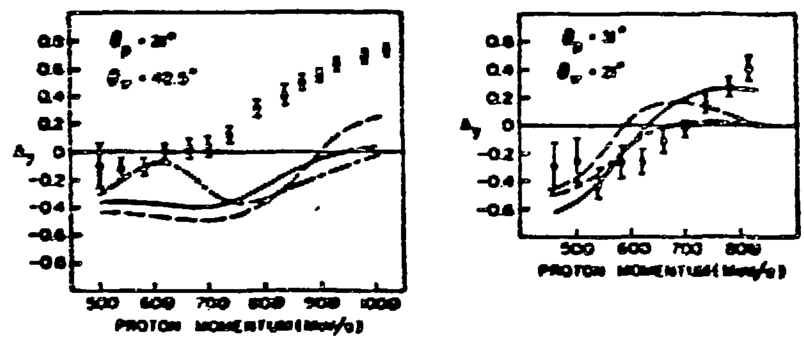
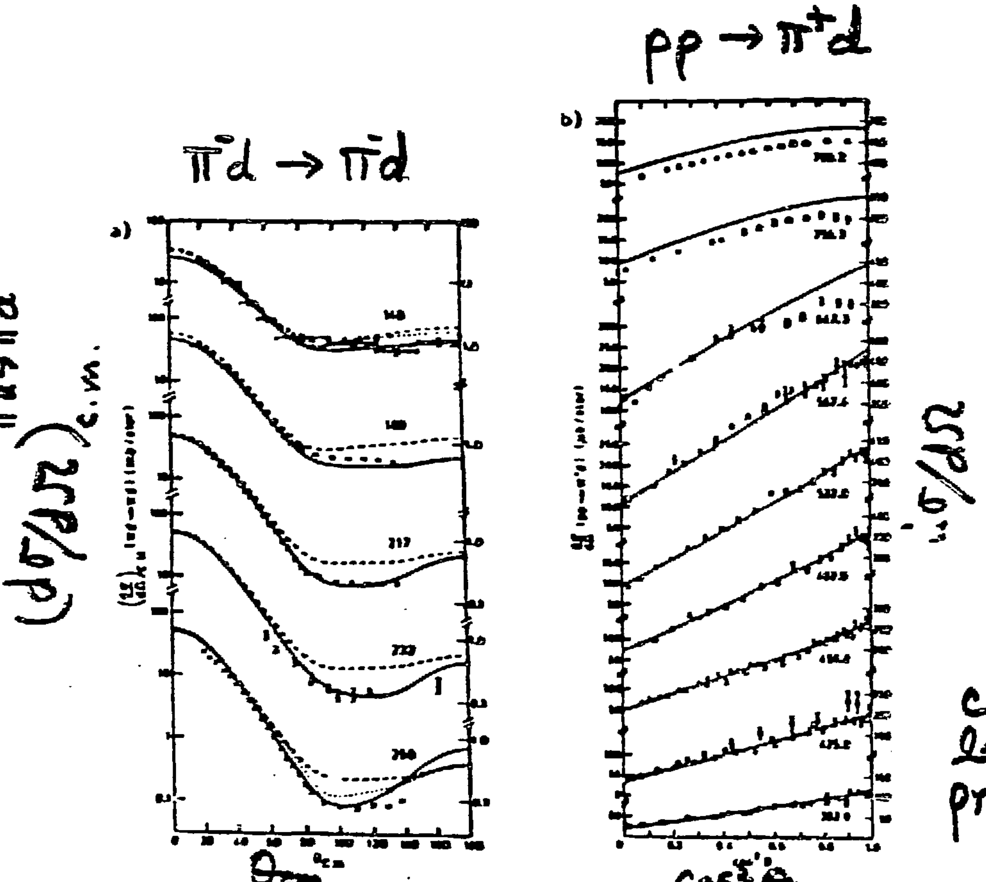


Fig. 4. Analyzing powers for kinematically complete measurements of $pp \to pn\pi^+$ at T_L = 800 MeV.[56] The meaning of each curve is the same as in Fig. 3. The momentum transfers for the $pp \to n\Delta^{++}$ process corresponding to the two geometries are roughly 400 and 550 MeV/c.

M. Betz, B. Blankleider, J.A. Niskanen + A.W. Thomas, Bloomington Workshop Proc. (1981)



curves labeled by proton energy

Fig. 9. Simultaneous calculations of the differential cross sections for the coupled processes: a) $\pi d \to \pi d$: for various pion lab energies we show the full calculation (—), no absorption (--) and no absorption or P_{11} rescattering (··). The experimental data are those of Cole et al,[85] Gabathuler et al.,[86] Holt et al.[87] and Pewitt et al.[88] b) $pp \to \pi^+ d$; curves are labelled by the proton lab energy. The experimental results are those of Axen et al.,[105] Dolnick,[89] Aebischer et al.,[90] Richard-Serre,[61] Mann et al.[91] and Hürster et al.[92] The data of Hürster was scaled since it was normalized to 1 at zero degrees.

Blankleider & Afnan, Phys. Rev. C22, 1638 (1980)
Flinders preprint FIAS-R-76 (1981)

Some conclusions

1) πNN dominates πd above 500 MeV

2) non-resonant NN and πN interactions play a rôle for large q → simple Δ model not valid

3) importance of NN initial state interactions
   a) lower the cross section

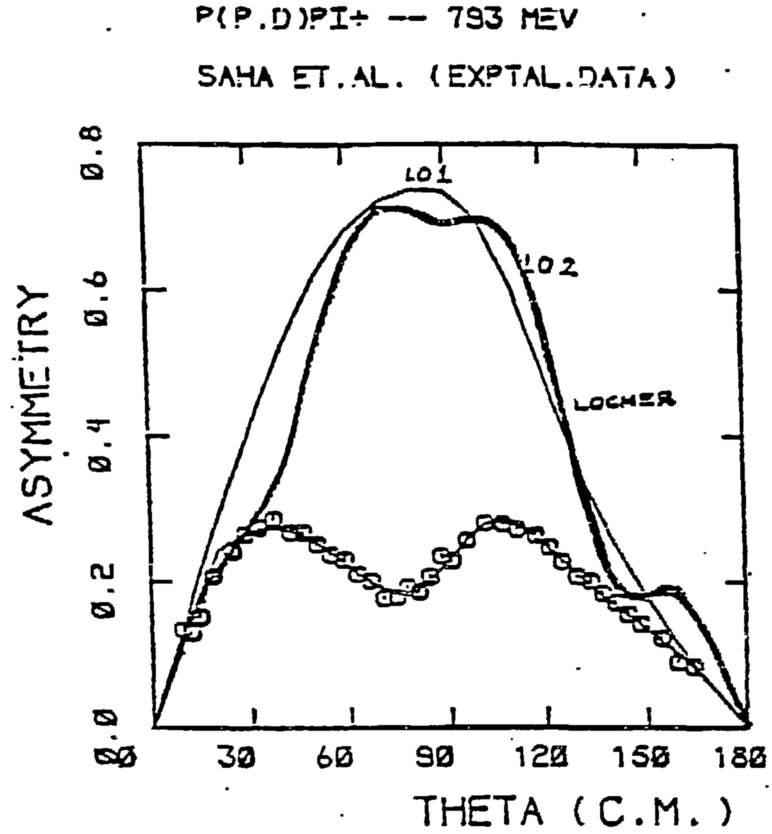
   b) perturbative treatment inadequate

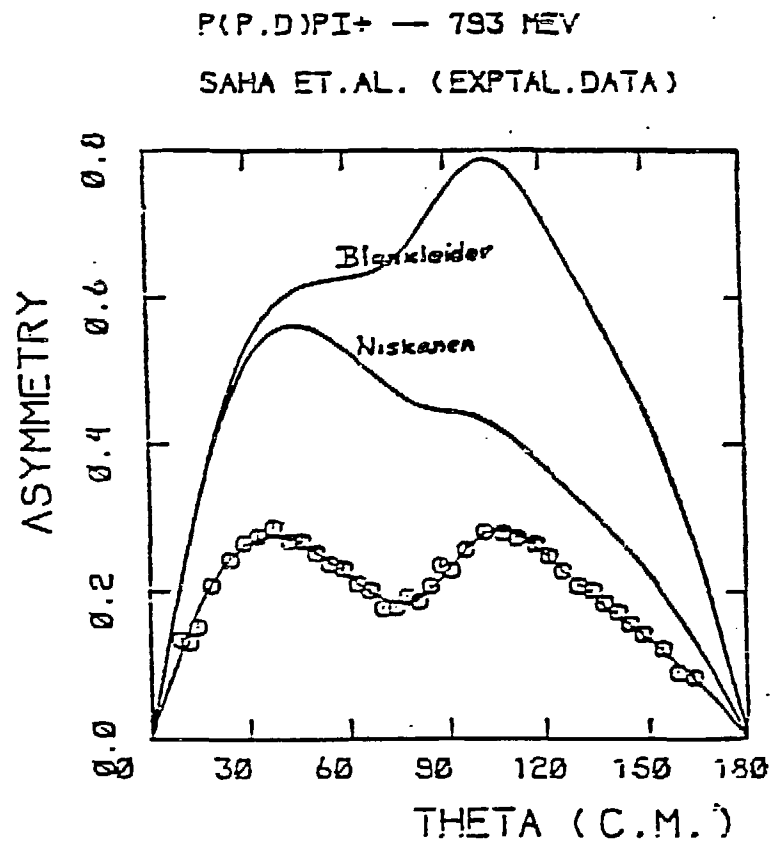
4) need detailed study of three-body channels to separate s-channel dibaryon resonance from pseudo-resonance phenomenon due to resonant behavior of πN subsystem

5) convergence of multiple scattering series is poor ⟶ large changes in spin observables

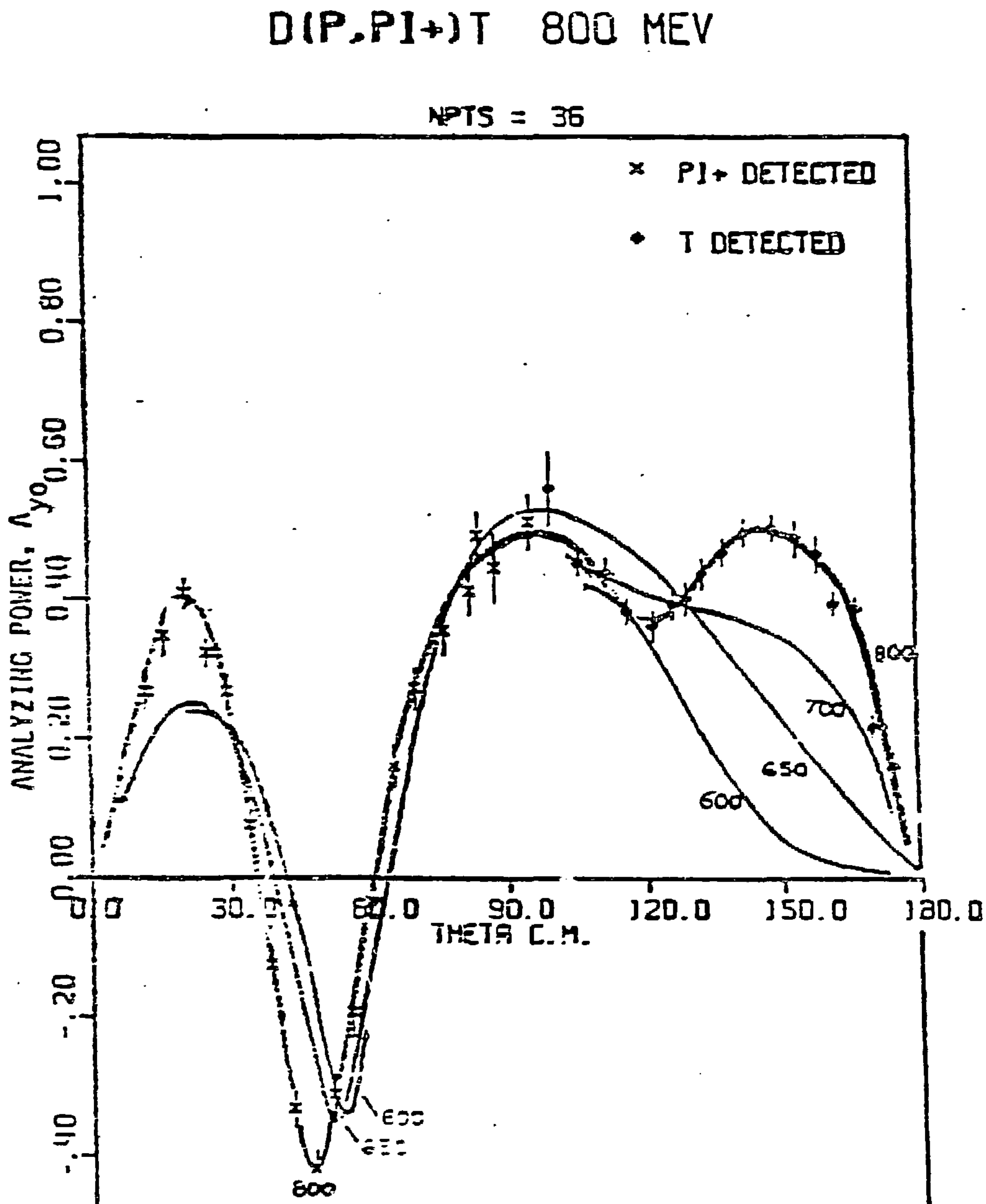
6) substantial disagreements between theory and experiment for $\vec{p}\,\vec{p} \to \pi^+ d$ (A_{y0} too large)

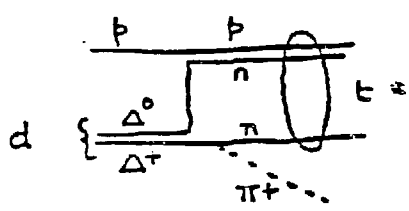
PP → dπ⁺ WITH POLARIZED PROTONS → ASYMMETRY





ANALYZING POWER FOR $pd \to \pi^+ t$





π Production in Proton-Nucleus Collisions

Data: D. Cochran et al, Phys. Rev. D6, 3085 (1972)

measured $d^2\sigma/d\Omega dE$ for 15°–150° lab and $E \approx 25$–550 MeV

Theory: M. Sternheim and R. Silbar, Phys. Rev. D6, 3117 (1972)
(transport model) C8, 492 (1973)

J. Ginocchio and M.B. Johnson, Phys. Rev. C21, 1056 (1980)
(cascade model)

D. Long and M. Sternheim, Phys. Rev. C26, 1558 (1982)
(one particle cascade model)

transport models are an approximation to cascade models which include absorption and charge exchange but neglect multiple elastic scattering

cascade models build in multiple scattering, nuclear medium corrections, π absorption, Fermi motion, Pauli blocking ....

complete cascade code VEGAS follows all recoiling particles, whereas Long + Sternheim follow only one particle (pion or faster nucleon)

Philosophy: processes in nucleus built up from series of independent collisions, using $\pi N \to \pi N$, $NN \to NN$, $NN \to NN\pi$, $\pi NN \to \ldots$ data as input

a semi-classical approach: probabilities rather than amplitudes for series of independent collisions for semi-classical approach to apply, want

$\lambda_M \gg r_0$ $\qquad$ r_0 = interparticle spacing ~ 1.5 f

$\bar{\lambda} = 1/k \ll r_0$ $\qquad$ λ_M = mean free path

These <u>conditions not satisfied</u>, since

$\lambda_M \sim 1/\rho\sigma \sim 0.5$ fm for π

$\bar{\lambda} \sim 2$ fm for 50 MeV pion

Even so, it works!

<u>Ingredients of cascade calculation</u> (Long + Sternheim)

a) projectiles enter nucleus at random points on circle of radius R_M such that $\rho(R_M)/\rho(0) = 0.01$

b) if N projectiles → n outgoing pions in E, Ω bin

→ cross section $\pi R_M^2 (n/N)$

c) propagating nucleon can elastically scatter, charge exchange, or produce a pion

d) pion can elastically scatter, charge exchange, or be absorbed with mean free path

$\lambda_M = 1/\sigma_{TOT}\bar{\rho}$; $\bar{\rho}$ = average density along path

-ngredients (cont.)

e) distribution of path lengths given by $x = -\lambda_M \ln r$

$0 < r < 1$ is random number

f) cross sections (input) are Fermi averaged

g) momentum of struck nucleon selected randomly according to Fermi gas momentum distribution

h) follow π after πN collision or $NN \to \pi NN$

follow faster nucleon after $NN \to NN$ (E < 400 MeV terminates cascade

i) $NN \to NN$, $\pi N \to \pi N$, $NN \to \pi NN$ taken from free space data (no off-shell effects)

j) pion absorption on two nucleons treated according to several different models

$$\lambda_{abs} = \frac{1}{\epsilon_{abs} \rho^2} \text{ or } \frac{1}{\sigma_{abs} \rho}$$

→ neglect real part of pion optical potential

π⁺ Production in Proton-Nucleus Collisions

Cascade Model

DAVID G. LONG AND MORTON M. STERNHEIM

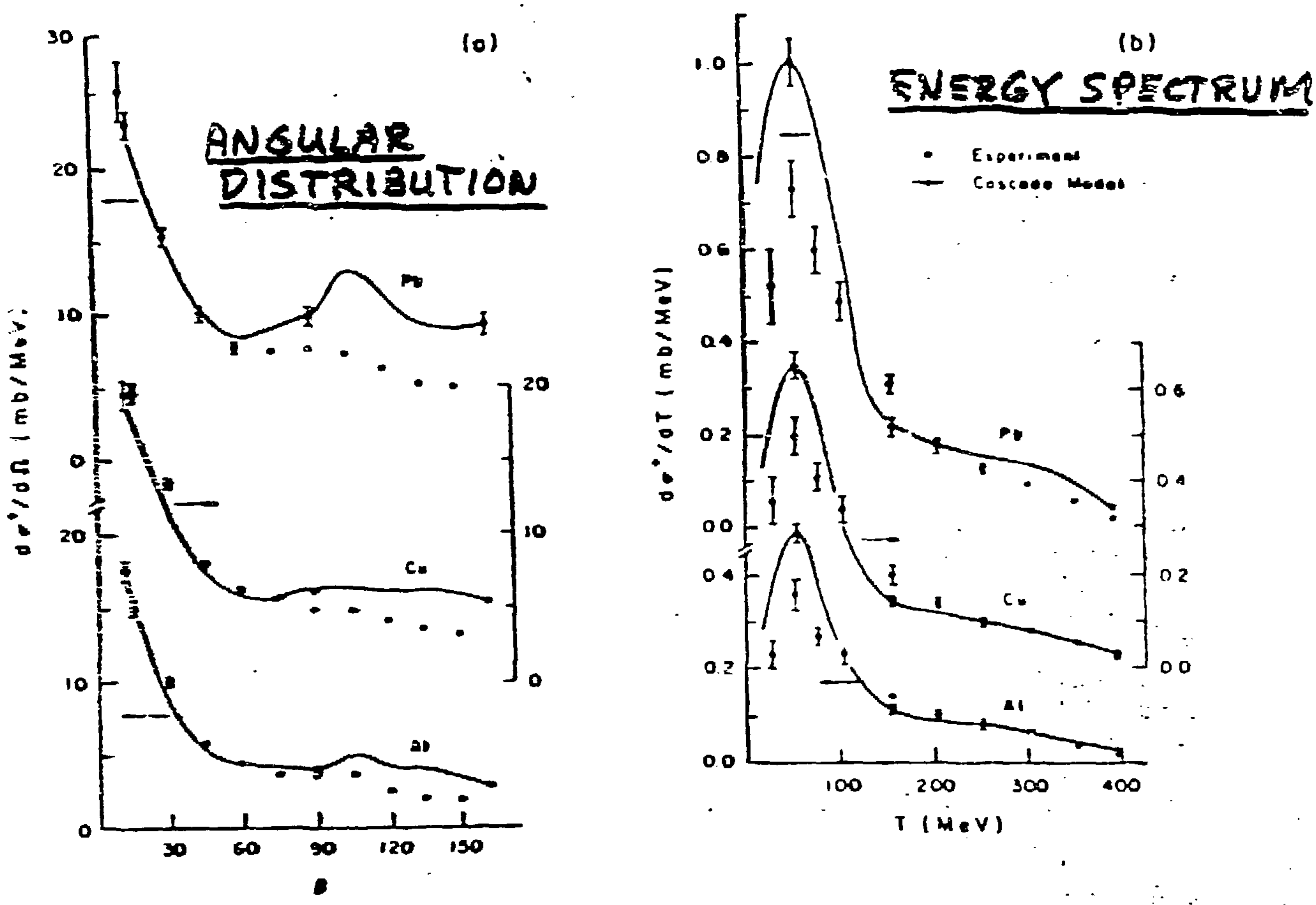


FIG. 2. π^+ production by 740 MeV protons as calculated with the single particle cascade model. Data (filled circles) from Ref. 9. (a) Angular distributions; (b) energy spectra. Representative error bars (open circles) are shown for the cascade calculations.

Phys. Rev. C26, 1558 (1982)

Results: (Long + Sternheim)

a) π^+/π^- ratios a bit too large, but much better than impulse approximation values obtained using Δ dominance and neglecting nuclear medium effect
$\to$ improvement from $\pi^0 n \to \pi^- p$

b) <u>multiple scattering series converges rapidly</u>
50% π^+, 25% π^- from impulse approximation
0, 1 rescatterings most common

c) after only one scattering, pion loses its original bias toward small angles
$\to$ even get backward peak from π's made in back half of nucleus

d) large effect on angular distr. and energy spectra from different absorption models
(only 30% of pions escape from nucleus)

e) use of realistic $\rho(r)$ rather than square well is important
diffuse edge gives larger range of impact parameter b such that N or π can travel through low density region

f) small effects from Pauli blocking and mom. distr. of struck nucleon

PION PRODUCTION IN NUCLEUS-NUCLEUS COLLISIONS

Subthreshold Pion Production

Experimental:

W. Benenson et al, Phys. Rev. Lett. 44, 54 (1980)
43, 683 (1979)

S. Nagamiya et al, LBL-14033 (1982) [90°

T. Johansson et al, Phys. Rev. Lett. 48, 732 (1982) $\frac{183\ \text{KeV}}{A}$ $85\ \frac{\text{MeV}}{A}$

Theoretical:

J. I. Kapusta, Phys. Rev. C16, 1493 (1977)

J. Gosset, J. I. Kapusta, G. D. Westfall, Phys. Rev. C18, 844 (1978)

} thermal models

G. Bertsch, Phys. Rev. C15, 713 (1977)

P. Hecking, LBL-12671 (1981)

} hard scattering models

A. H. Blin, S. Bohrmann + J. Knoll, Zeitsch. f. Physi

S. Nagamiya et al (above) ← phase space models

B. Jakobsson et al, Phys. Lett. 82B, 35 (1979)

J. P. Bondorf, C. Guet, B. Jakobsson and M. Dinwarl

Mechanisms for subthreshold pion production

a) NN → NNπ, dπ "hard scattering" models
includes diffuse Fermi momentum distribution

b) collective phenomena

1) collisions between clusters

V. Burov et al, Phys. Lett. 67B, 46 (1977)

A.M. Baldin, Dubna Report E1-80-174 (1980)

2) coherent production of Δ isobar

P. Deutschmann + L.W. Townsend, Phys. Rev. Lett. 45, 1622 (1980)

G.E. Brown, Saclay Workshop on High Resolution Heavy Ion Physics at 100 MeV/A, 1978

3) Pion Condensation

V. Ruck, M. Gyulassy and W. Greiner, Zeit. Physik A277, 391 (1976)

H.J. Pirner, Phys. Rev. C22, 1962 (1980)

Manifestations of pion condensation:

M. Gyulassy, Nucl. Phys. A354, 395 (1981)

a spin-isospin instability shows up as a non-vanishing expectation value for the nuclear axial current $\vec{J}_{\mu 5}$

$$(\Box + \mu_\pi^2)\,\phi(x,t) = g\,\partial^\mu J_{\mu 5}(x,t)$$

number of coherent pions radiated per baryon $\sim 10^{-4}$

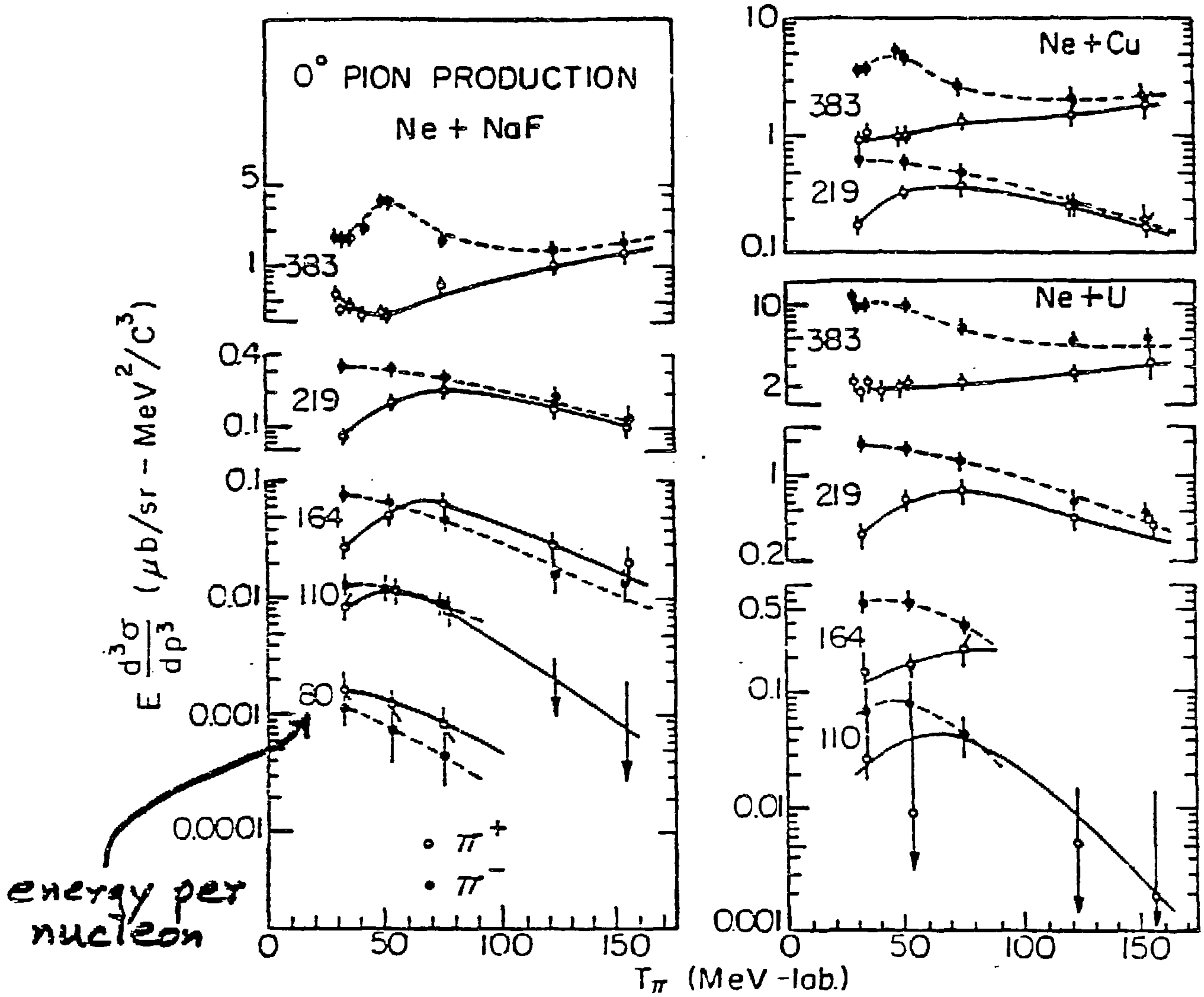
number of incoherent pions/baryon from $NN \to NN\pi$ $\sim 10^{-1}$

$\therefore$ pion condensation does ~~not~~ lead to copious production of pions

coherent pions should appear as a bump in energy spectrum at ~ 200 MeV $\Rightarrow$ at higher energies this effect is swamped by background

$\rightarrow$ look at subthreshold production with heaviest possible target and projectile (U+U)

Subthreshold Pion Production



W. Benenson et al, Phys. Rev. Lett. 44, 54 (1980)

r. Johansson et al. Phys. Rev. Lett. 48, 732 (1982)

π^+ production in C+C and C+Au at 85 MeV/A

for $20 \leq E_\pi \leq 80$ MeV and $\theta_{lab} > 55°$

scale of cross section:

$$\int (d^2\sigma/d\Omega dE)\,dE \approx \begin{cases} 0.8\ \mu b/sr & \text{for } 55° \\ 0.5 & \phantom{\text{for }} 90° \quad \text{for C+C} \\ 0.4 & \phantom{\text{for }} 130° \end{cases}$$

C+Au is ~9 times larger $\longrightarrow$ $A^{0.8}$ dependence

↑ target A

C+Au π production strongly backward peaked in NN c.m., slightly forward peaked in nucleus-nucle c.m

refer to J. P. Bondorf, C. Guet, B. Jakobsson, M. Prakash!

$NN \to NN\pi$, $d\pi$ model

can reproduce absolute cross sections if diffuse internal momentum distribution is used

contr. of $pp \to d\pi^+$ largest, since Pauli blocking weaker than for $NN \to NN\pi$

B. Jakobsson, J. P. Bondorf and G. Fai, Phys. Lett. 82B, 35 (197

π production from 200-500 MeV/A

Basis: independent NN interactions

Glauber multiple scattering

isobar model for pion absorption → Monte Carl

$$P(b) = 1 - e^{-\bar{\sigma}\langle T(b)\rangle}$$ = prob. that incident beam nucleon scatters at least on

$$\langle T(b)\rangle = \iint T_A(\underset{\sim}{b}')\, T_B(\underset{\sim}{b}-\underset{\sim}{b}')\, d^2\underset{\sim}{b}' \Big/ \iint T_A(\underset{\sim}{b}')\, d^2\underset{\sim}{b}'$$

$$T_{A,B}(\underset{\sim}{b}) = \int_{-\infty}^{+\infty} \rho_{A,B}(\underset{\sim}{b}, z)\, dz$$

$\underset{\sim}{b}$ = impact parame

$\bar{\sigma}$ = free NN cross section × Pauli blocking factor

$$P(N_i, b) = \binom{A}{N_i}\,[P(b)]^{N_i}\,[1 - P(b)]^{A-N_i}$$

= prob. that N_i beam nucleons scatter at least once

small b → larger N_i favored

calculate $S(E)$ = distribution of energy E of two nucleon system in their c.m.

from Fermi momentum distr. with $p_F = 260$ MeV/c

$R(E) = \sigma_\pi^{NN}(E) / \sigma_{TOT}^{NN}(E)$ → ratio of π production to total

$$\langle R \rangle = \int_0^{E_{max}} dE\, R(E)\, S(E)$$

Then

$$\boxed{\sigma_\pi^A(E) = 2\pi \langle R \rangle F_{be} \int_0^\infty b\,db \sum_{i=1}^{A} P(N_i, b) N_i}$$

total π prod. cross section

ratio of inelastic and total Pauli blocking factors

detectable pion flux strongly reduced by reabsorption of pions: calculated in isobar mode

small b → strongest absorption

for Ne + U, largest π multiplicities expected when $b \approx 5$ fm ; for C + C, small $b \approx 0$ corresponds to largest multiplicity

<u>typical reabsorption probability ~ 65%</u>

B. Jakobsson, J. Bondorf, G. Fai, Phys. Lett. 82B, 35 (1979)

Reaction	E/A	⟨Multiplicity⟩exp	⟨Mult⟩theor
$^{12}C + LiH$	400	0.02 ± 0.01	0.021 ± .004
$^{12}C + NaF$	400	0.038 ± 0.013	0.04 ± 0.008
$^{12}C + BaI_2$	400	0.078	0.062
$^{12}C + Pb_3O_4$	400	0.066	0.062
^{20}Ne + emulsion	250	< 0.13	0.03
$^{40}Ar + LiH$	400	0.043	0.03
$^{40}Ar + NaF$	400	0.034	0.075
$^{40}Ar + BaI_2$	400	0.107	0.161
$^{40}Ar + Pb_3O_4$	400	0.099 ± 0.02	0.124 ± .024

average pion multiplicity

<u>Pion Interferometry</u> → correlations of two identical pions

R. <u>Hanbury-Brown + R.Q. Twiss</u>, Nature 178, 1046 (1956)
→ $\gamma\gamma$ interference effects in astronomy used to measure stellar radii

G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, Phys. Rev. 120, 300 (1960) → $\pi^-\pi^-$ interference in $\bar{p}p$ annihilation (<u>GGLP effect</u>)

<u>application to heavy ion collisions</u>

S. E. Koonin, Phys. Lett. 70B, 43 (1977)
F. B. Yano and S. E. Koonin, Phys. Lett. 78B, 556 (1978)
M. Gyulassy and S. K. Kauffmann + L. Wilson, Phys. Rev. C20, 2267 (1979)
M. Gyulassy, Phys. Rev. Lett. 48, 454 (1982)

<u>Data</u>: B. Jakobsson, Physica Scripta, 17, 491 (1978)

J. J. Lu et al, Phys. Rev. Lett. 46, 898 (1981)

J. Bartke, Nucl. Phys. A335, 481 (1980)

W. A. Zajc et al, LBL-12652 (1981), p350
S. Y. Fung et al, Phys. Rev. Lett. 41, 1592 (1978

Pion Interferometry in particle physics : some caveats.

1) A. Eskreys et al, Nucl. Phys. B42, 44 (1972)
$K^+p \to K^0 2\pi^+\pi^- p$, $K^+ 2\pi^+ 2\pi^- p$, $K^0 3\pi^+ 2\pi^- p$ at 8.25 $\frac{GeV}{c}$

look for angular correlation $\gamma = B/F$ or $\epsilon = \frac{B-F}{B+F}$
B = pions with $\theta \geq 90°$, F = pions with $\theta \leq 90°$

typically $\gamma^{\text{like } \pi} < \gamma^{\text{unlike}}$

write $\epsilon = \int \epsilon(m)\, \rho(m)\, dm$
mass distribution of $\pi\pi$ pairs

Conclusion: differences in shape of $\rho(m)$ lead to diff. in ϵ $\to$ get simulated GGLP effect

also see apparent GGLP effect in $K^+\pi^+$ vs $K^+\pi^-$ from diff. in $\rho(m)$
exotic

expect true GGLP effect at low energy/particle (some region of m)

2) P. L. Jain et al, Phys. Rev. D7, 605 (1973)
$K^+ + p \to K^+ p \; 2\pi^+ 2\pi^-$ at 12.7 GeV/c

explain GGLP effect as due to effect of peripheral resonance production (N^*, K^*) on $\pi\pi$ correlations [peripheral mechanism induces strong angular correlations]

for this reaction, conclude that Bose-Einstei[n] statistics play a negligible rôle → no diffe[rence] in γ for like and unlike pairs when resona[nce] production taken into account (not true for $\bar{p}p$.

J. Schlesinger, Phys. Rev. D8, 2308 (1973) $\bar{p}p$
P. L. Jain et al, Phys. Rev. D8, 2309 (1973) K^+p

prescription to minimize influence of resonance production: <u>study angular correlations in a plane ⊥ to line of flight of resonance</u>

→ look at correlations in variable

$$q_t \text{ (or } P_T) = \text{projection of } \vec{p}_1 - \vec{p}_2 \text{ on plane} \perp \text{to } \vec{p}_1 + \vec{p}_2$$

Recent studies of pion interferometry

<u>M. Deutschmann et al</u>, Nucl. Phys. B103, 198

$\pi p \to p + 5\pi,\ p + 7\pi$ at 4 – 25 GeV/c (selec multi

of $\pi\pi$ pairs of like charge

look at $\dfrac{N_L - N_B}{N_B} = f(q_t, q_0)$; q_t = q_0 =

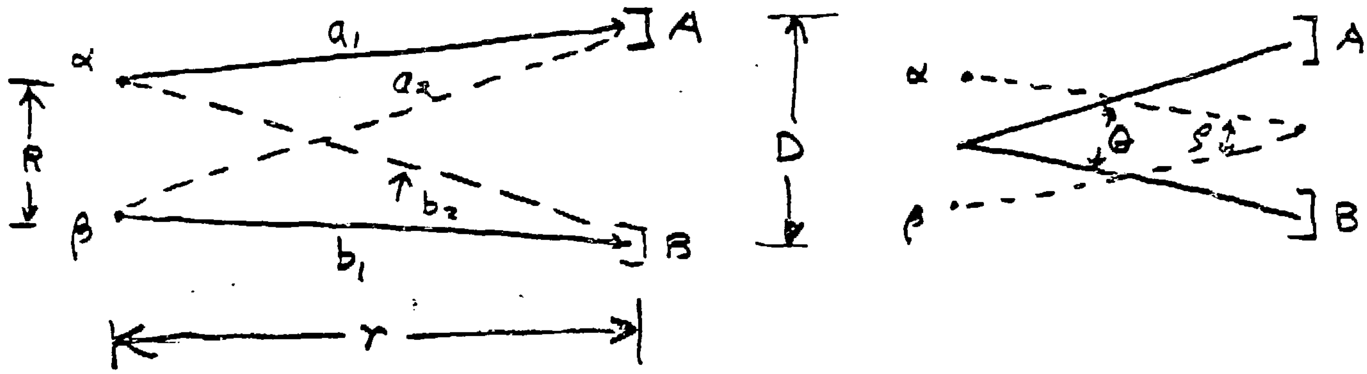
background

fit to form $\boxed{f(q_t, q_0) = \left(\dfrac{2J_1(q_t R)}{q_t R}\right)^2 \dfrac{\ }{1}}$

derived by Kopylov and Podgoretsky, Sov. J. N 15, 392 (1972); 18, 656 (1973); 19, 434 (1974); Lett. 50B, 472 (1974)

emission from disk source of radius R → "
form of q_0 dependence is model dependent

Simple derivation of interference pattern:
G. Cocconi, Phys. Lett. 49B, 459 (1974)



$\theta = D/r$, $\rho = R/r$, mesons of equal momentum k

amplitude of coincidence $\sim e^{ika_1} e^{ikb_1} + e^{ika_2} e^{ikb_2}$

rate of coinc. $= \left(e^{ik(a_1+b_1)} + e^{ik(a_2+b_2)}\right)\left(e^{-ik(a_1+b_1)} + e^{-ik(a_2+b_2)}\right)$

$$= 2(1+\cos kR\theta)$$

since $a_2+b_2-(a_1+b_1) \approx 2\rho\, D/2 = R\theta$

now integrate over source positions α and β

→ circle of radius R emitting π's uniformly

→ rate of coinc. $\propto \left(\dfrac{2J_1(kR\theta)}{kR\theta}\right)^2$ like Fraunhofer diffraction

for Gaussian distr. of luminosity, get

rate of coinc. $\sim e^{-(Rk\theta/2)^2}$

RESULTS FOR SIZE OF FIREBALL :

M. Deutschmann et al, Nucl. Phys. B 103, 198 (19

$\pi p \to p + 5\pi, p + 7\pi$

$$R \approx 1^{+0.4}_{-0.2} \text{ fm}, \quad \tau = 2.3^{+2}_{-1} \times 10^{-24}$$

$$c\tau = 0.7^{+0.6}_{-0.3} \text{ fm}$$

attempted to look at ellipsoidal shape of fireball → find evidence for sphero contracted in direction of collision axis

However, $c\tau$ varies with direction of obs

→ interpret $c\tau$ as depth of layer fro pions can escape (rather than τ as lifet

F. Grard et al, Nucl. Phys. B102, 221 (19

$K^+p \to K^+p\, 2\pi^+ 2\pi^-$ at 8.25 GeV/c

$$R \approx 0.8 \text{ fm}$$

$$\tau \approx 3-4 \times 10^{24} \text{ sec}$$

Conclusion: for elementary particle processes is essentially the bag radius

heoretical Approach to Hadron Interferometry:

levant to question of pion condensate:

G.N. Fowler, N. Stelte and R.M. Weiner, Nucl. Phys. A319, 349 (1979)

M. Gyulassy, S.K. Kauffmann and L.W. Wilson, Phys. Rev. C20, 2267 (1979)

efine a correlation function for $\pi^-\pi^-$ pairs:

$$R(k_1, k_2) = \frac{\langle n_{\pi^-}\rangle^2}{\langle n_{\pi^-}(n_{\pi^-}-1)\rangle} \sigma_{\pi^-} \frac{d^6\sigma(\pi^-\pi^-)/d^3\underset{\sim}{k}_1\, d^3\underset{\sim}{k}_2}{d^3\sigma(\pi^-)/d^3\underset{\sim}{k}_1\; d^3\sigma(\pi^-)/d^3\underset{\sim}{k}_2}$$

ormalization: $\int d^3\underset{\sim}{k}_1 \dfrac{d^3\sigma(\pi^-)}{d^3\underset{\sim}{k}_1} = \langle n_{\pi^-}\rangle \sigma_{\pi^-}$

$$\int d^3\underset{\sim}{k}_1\, d^3\underset{\sim}{k}_2 \frac{d^6\sigma(\pi^-\pi^-)}{d^3\underset{\sim}{k}_1\, d^3\underset{\sim}{k}_2} = \langle n_{\pi^-}(n_{\pi^-}-1)\rangle \sigma_{\pi^-}$$

.. $R(k_1, k_2) = 1$ if pions uncorrelated in momentum space, i.e. $d^6\sigma \sim f(\underset{\sim}{k}_1) f(\underset{\sim}{k}_2)$

Correlations between pions arise from

a) Conservation laws $\rightarrow$ energy-momentum, quantum numbers (isospin...

b) Bose-Einstein or Fermi statistics

c) Production or final state interaction dynamics (resonances, cluster model, optical distortions)

coherent vs. incoherent emission

standard heuristic derivation yields

$$R(k_1, k_2) \propto 1 + |\rho(\underset{\sim}{k}_1 - \underset{\sim}{k}_2, \omega_1 - \omega_2)|^2$$

$\uparrow$ Fourier transform of source density $\rho(x)$

Problems: 1) question of normalization unclear
2) assume pions produced independently at random space-time points
$\rightarrow$ cannot reveal coherent dynamics
3) not general enough even for chaotic pion fields

Examples: 1) <u>fully coherent pion field</u> $\phi(x)$

given by $(\Box + m_\pi^2)\phi(x) = J(x)$

c-number space-time fcn. (not coupled back to pion field)

final state produced by classical source is <u>coherent state</u>

$$|J\rangle = e^{-\bar{n}/2} e^{i\int d^3k\, J(k) a^\dagger(k)} |0\rangle$$

like a laser field in optics

where $$J(k) = \int \frac{d^4x}{(2\pi)^{3/2}} \frac{e^{i\omega_k t - i\underset{\sim}{k}\cdot\underset{\sim}{x}}}{(2\omega_k)^{1/2}} J(\underset{\sim}{x}, t)$$

$$\bar{n} = \int d^3k\, |J(k)|^2$$

physical picture same as for bremsstrahlung radiation

$|J\rangle$ involves arbitrary numbers of pions

<u>Properties</u>:

a) $\langle n_\pi\rangle^2 = \langle n_\pi(n_\pi - 1)\rangle = \bar{n}^2$

b) $a(k)|J\rangle = iJ(k)|J\rangle$

c) density matrix $\rho_\pi = |J\rangle\langle J|$

d) m-pion inclusive distribution

$$P_m(k_1 \cdots k_m) = \mathrm{Tr}\,(\rho_\pi\, a^\dagger(k_1)\cdots a_m^\dagger a_m \cdots a_1)$$

$$= |J(k_1)|^2 \cdots |J(k_m)|^2$$

no correlations! $\rightarrow$ e) $\boxed{R(k_1, k_2) = 1}$!!

Partially coherent pion fields:

$$J(k) = J_0(k) + J_{ch}(k)$$

collective current

chaotic current (pions from $NN \to NN\pi$)

$$J_{ch}(k) = J_\Delta(k) \sum_{i=1}^{N} e^{iQ_i} \psi_k^*(x_i)$$ → perform ensemble average over $\{N, Q_i, x_i\}$

correlation function

$$R(k_1, k_2) = 1 + \frac{n_{ch}(k_1)}{P_1(k_1)} \frac{n_{ch}(k_2)}{P_1(k_2)} |\rho(k_1 - k_2)|^2$$

$$+ 2\,\mathrm{Re}\left[\frac{(J_0^*(k_1) J_\pi(k_1))}{P_1(k_1)} \frac{(J_\pi^*(k_2) J_0(k_2))}{P_1(k_2)} * \langle N \rangle \rho(k_1 - k_2) \right]$$

where $n_0 = \int d^3k \, |J_0(k)|^2 = \int d^3k \, n_0(k)$

$n_{ch} = \int d^3k \, \langle |J_{ch}(k)|^2 \rangle_{\{N, Q_i, x_i\}} = \int d^3k \, n_{ch}(k)$

$P_1(k) = n_0(k) + n_{ch}(k)$

Partially coherent pion fields (cont.)

if we assume $J_0(k)$, $I_{\pi}(k)$ real, we get

$$R(k_1, k_2) = 1 + (1 - D(k_1))(1 - D(k_2))\,\rho^2(k_1, -k_2)$$

$$+ 2\left[D(k_1)D(k_2)(1 - D(k_1))(1 - D(k_2))\right]^{1/2}\rho(k_1, -k_2)$$

where the <u>degree of coherence</u> of mode k is

$$D(k) = n_0(k) / [n_0(k) + n_{ch}(k)]$$

Note $\boxed{R(k,k) = 2 - D^2(k)}$

If we have no coherent field, $n_0(k) = 0$, and $R(k,k) = 2$ as for chaotic Bose field

Note: for charged particles, must correct for Coulomb effects which are important for $\underset{\sim}{k_1} \approx \underset{\sim}{k_2}$

define $\hat{R}$ by $R(k_1, k_2) = G(k_1, k_2)\,\hat{R}(k_1, k_2)$

$G(k_1, k_2) = 2\pi\eta / (e^{2\pi\eta} - 1)$ = <u>Gamow factor</u>

where $\eta = \alpha m_\pi / |\vec{k}_1 - \vec{k}_2|$

Heavy Ion Data on Pion Interferometry:

) S.Y. Fung et al, Phys. Rev. Lett. 41, 1592 (1978)
1.8 GeV/A Ar^{40} beam

use two forms

$$R(q,q_0) = K \begin{cases} ⓐ\ 1 + e^{-\frac{R^2q^2}{2} - \frac{\tau^2 q_0^2}{2}} \\ ⓑ\ 1 + \left(\frac{2J_1(Rq/\sqrt{2})}{Rq/\sqrt{2}}\right)^2 * (1+\tau^2 q_0^2)^{-1} \end{cases}$$

Target	(a) R (fm)	(a) τ (10^{-24} sec)	(b) R (fm)	(b) τ (10^{-24} sec)
BaI_2	3.05 ± 1.1	5	3.09 ± 1.2	5
Pb_3O_4	3.3 ± 0.9	5	2.98 ± 0.8	5
Pb_3O_4	3.98 ± 0.8	$2^{+4}_{-1.8}$	3.88 ± 0.64	$3.2^{+4.6}_{-2.}$

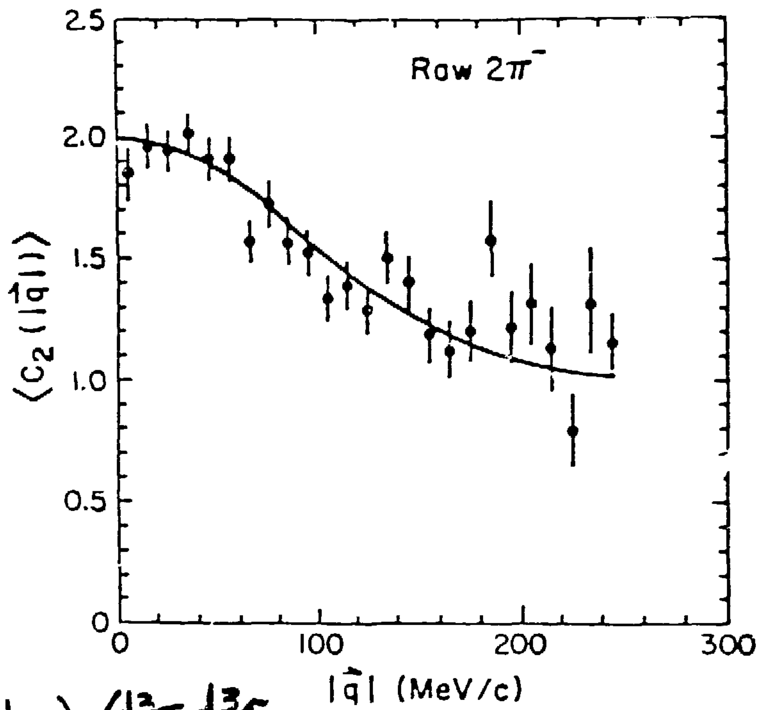
) J.J. Lu et al, Phys. Rev. Lett. 46, 898 (1981)
1.8 GeV/A Ar^{40} + Pb target → dependence of R on $\langle n_\pi \rangle$

$\langle n_{\pi^-} \rangle$	R (fm)	R ($\rho_c = \rho_0/3$)	R ($\rho_c = \rho_0$)
2 – 4	3.1 ± 1.1	5.7	4.2
5 – 8	4.0 ± 0.7	7.7	5.4
9 – 12	4.8 ± 0.7	8.36	5.8
13 – 15	5.6 ± 1.2	8.42	5.85

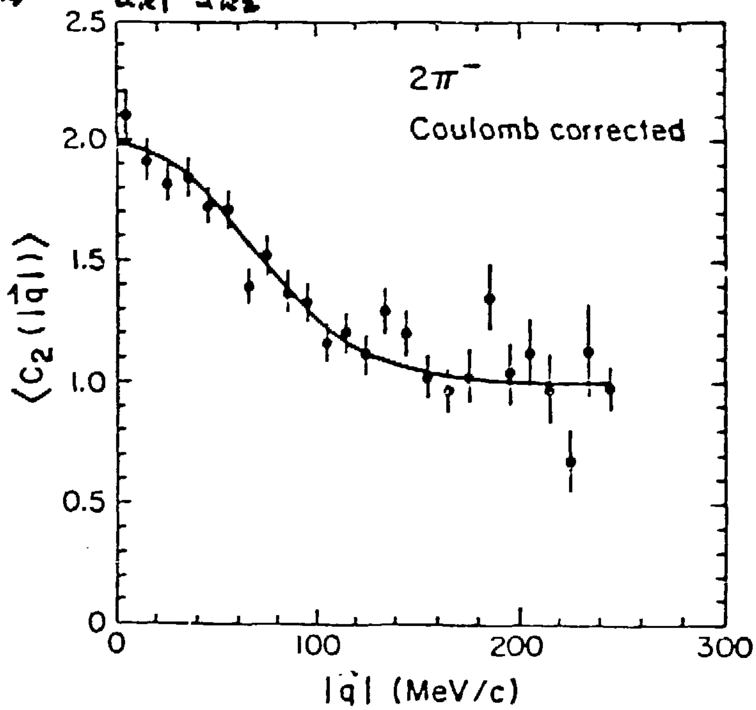
ρ_c = freezeout density

thermodynamic fireball model

$\pi^-\pi^-$ CORRELATION FUNCTION $\langle C_2(q)\rangle$



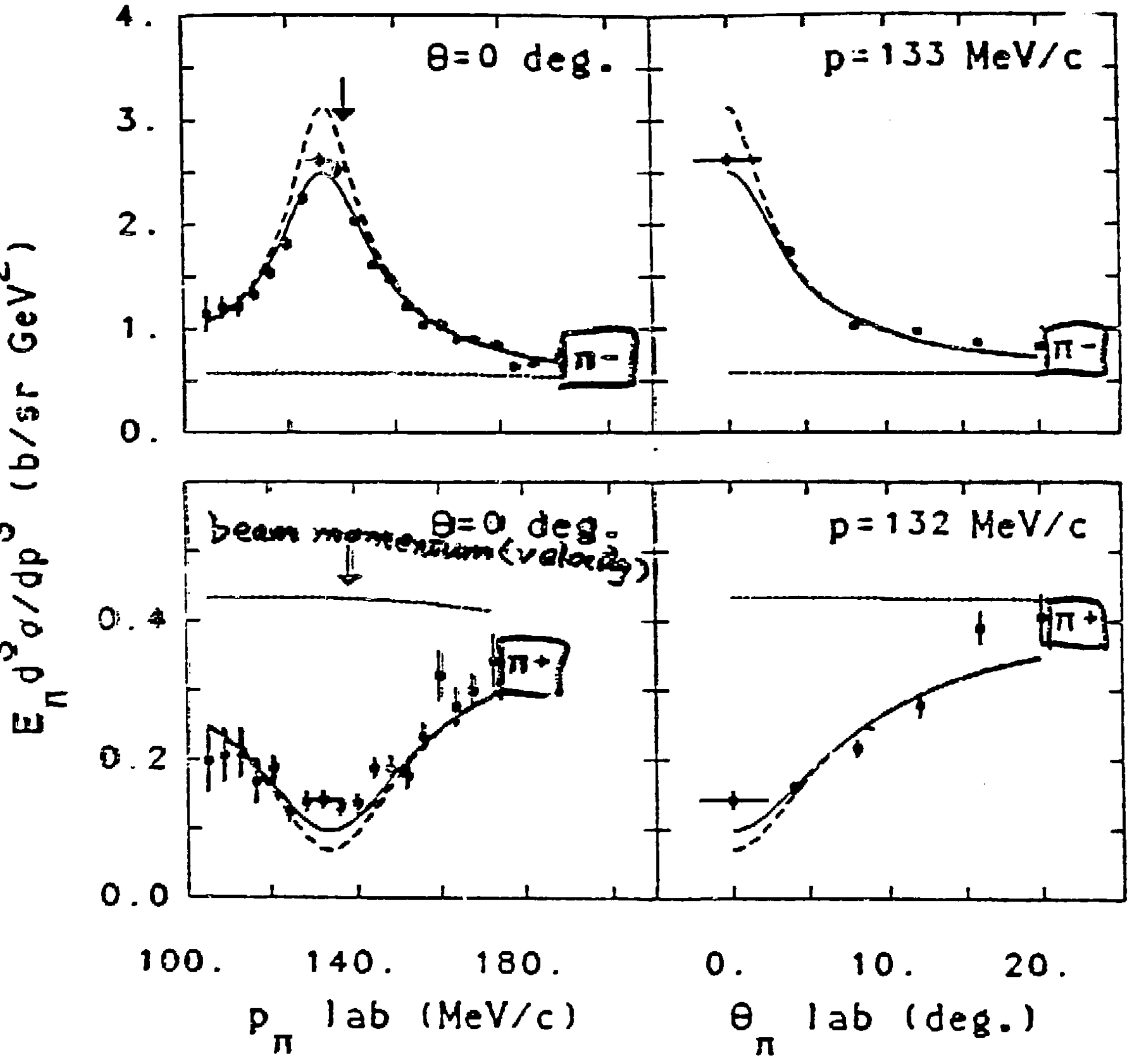
$$C_2 = \sigma_0 \left(d^6\sigma / d\underset{\sim}{k}_1 d\underset{\sim}{k}_2 \right) \Big/ \frac{d^3\sigma}{dk_1}\frac{d^3\sigma}{dk_2}$$



1.8A GeV Ar + KCl → R = 3 ± 0.3 fm

W. A. Zajc et al, Proc. 5th High Energy Heavy Ion Summer Study, LBL-12652 (May, 1981)

COULOMB EFFECTS IN π^+ AND π^- PRODUCTION



π^-/π^+ ratio $\approx$ 20–30 when $V_\pi \simeq V_{beam}$

W. Benenson et al, Phys. Rev. Lett. 43, 683 (1979)
Phys. Rev. Lett. 44, 54 (1980)

J.P. Sullivan et al, Phys. Rev. C (1982)

Coulomb Effects in Pion Production:

1) enhancement of π^+ yield in the mid-rapidity region
$\rightarrow$ peak at $\theta_{c.m.} = 90°$

K. L. Wolf et al, Phys. Rev. Lett. 42, 1448 (1979)
(peak at $p_\perp \approx 0.4\, m_\pi$ in Ar + Ca at 1.05 GeV/nucl.

J. Chiba et al, Phys. Rev. C20, 1332 (1979)
(^{20}Ne + NaF at 0.8 GeV/nucl.)

2) π^-/π^+ ratio large at $\theta_{c.m.} = 0°$ when $V_\pi \approx V_{beam}$

W. Benenson et al, Phys. Rev. Lett. 43, 683 (1979)
44, 54 (1980)

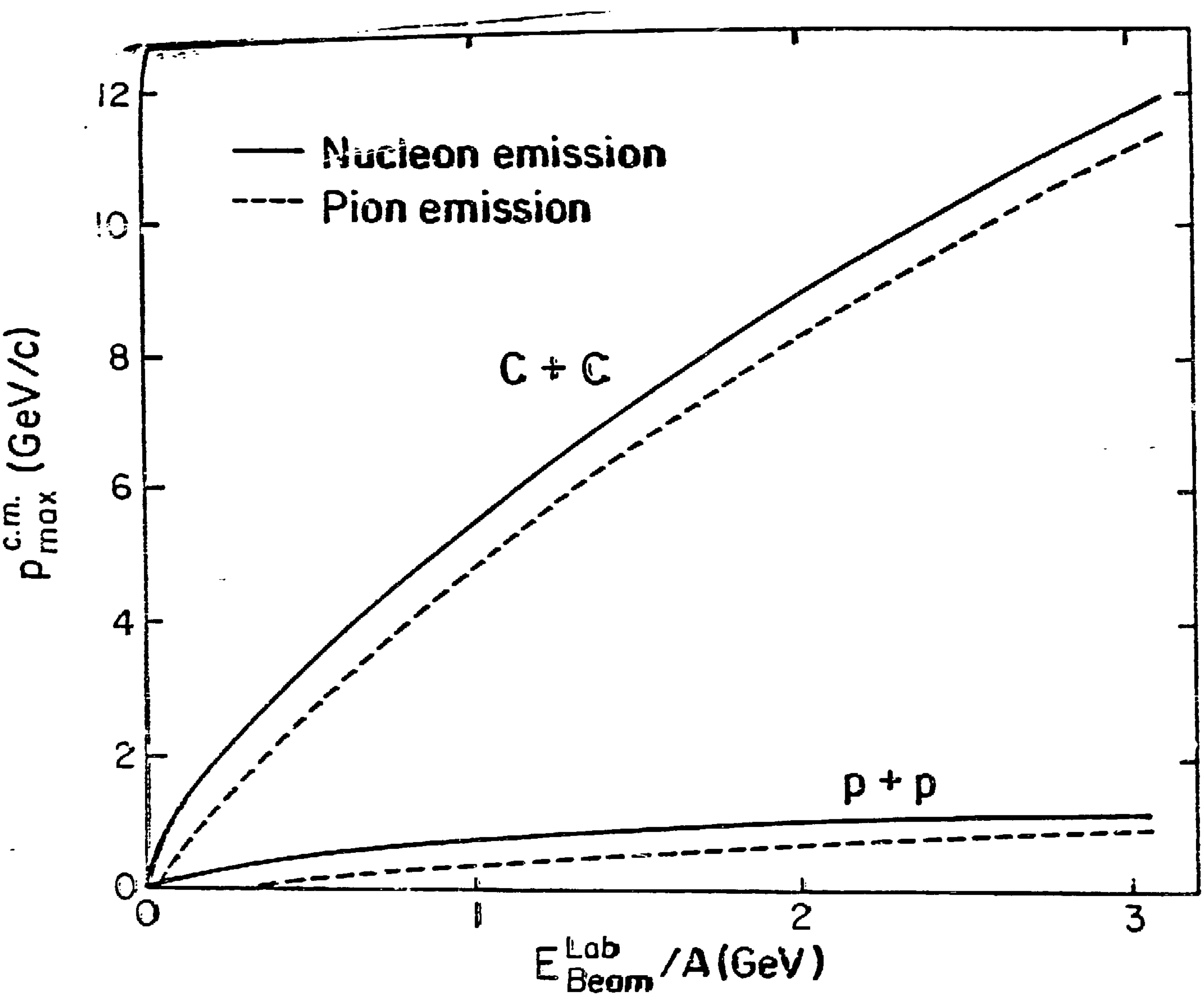
newest data from K. A. Frankel et al, Phys. Rev. C25, 1102 (1982)

^{40}Ar + ^{40}Ca at 1.05 GeV/c $\rightarrow$ do not see drop in cross section for $p_\perp < 0.4\, m_\pi$ of Wolf et al
π^-/π^+ ratio only 1.5

Theories of Coulomb effects

1) K. G. Libbrecht + S. E. Koonin, Phys. Rev. Lett. 43, 1581 ('7[illegible]
2) J. Cugnon + S. E. Koonin, Nucl. Phys. A355, 477 (1981)
3) M. Gyulassy + S. K. Kauffmann, Nucl. Phys. A362, 503 (1981)

ABSOLUTE KINEMATICAL LIMIT



$p_{max}^{c.m.}$ = maximum momentum of pion/proton in c.m. frame

→ much more phase space available for nucleon or pion in heavy ion collision than NN collision

PION EMISSION NEAR THE ABSOLUTE KINEMATICAL LIMIT

THEORY: I.A. Schmidt and R. Blankenbecler, Phys. Rev. D15, 3321 (1977) ; Phys. Rev. D16, 1318 (1977)

$$E\, d^3\sigma/d^3p \sim (1-x)^g \leftarrow$$

$$g = 2T(A-1) + H$$

A ↑ projectile mass

H ↑ 3 except for hydrogen target

R.H. Landau + M. Gyulassy, Phys. Rev. C19, 149 (197
(deviation from scaling due to cluster collisions)

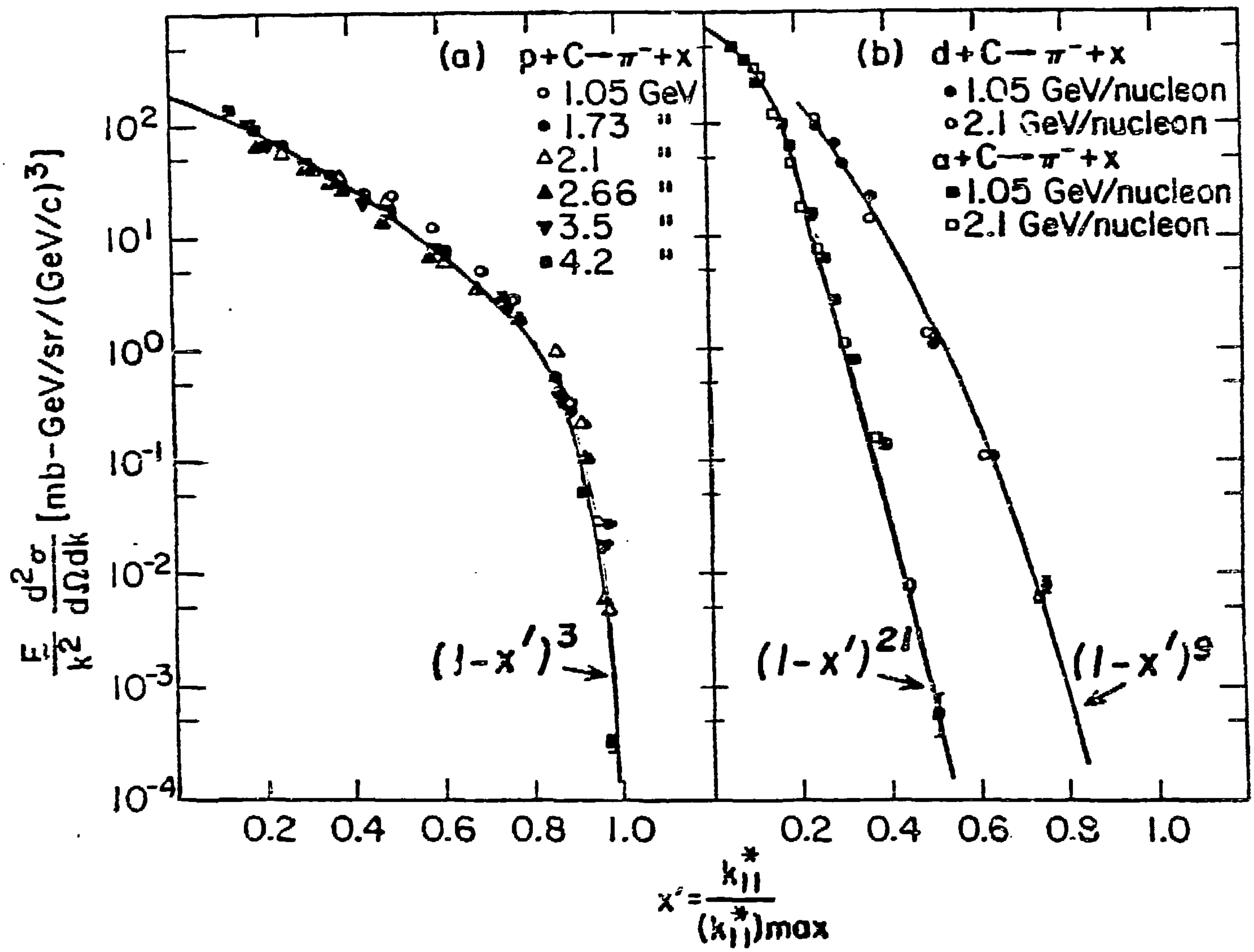
Data: a) π emission at 0°

A.M. Baldin et al, Sov. J. Nucl. Phys. 18, 41 (1974)
J. Papp et al, Phys. Rev. Lett. 34, 601 (1975)
E. Aslanides et al, Phys. Rev. Lett. 43, 1466 (1979)
E. Moeller et al, preprint (1982)

b) π emission at backward angles

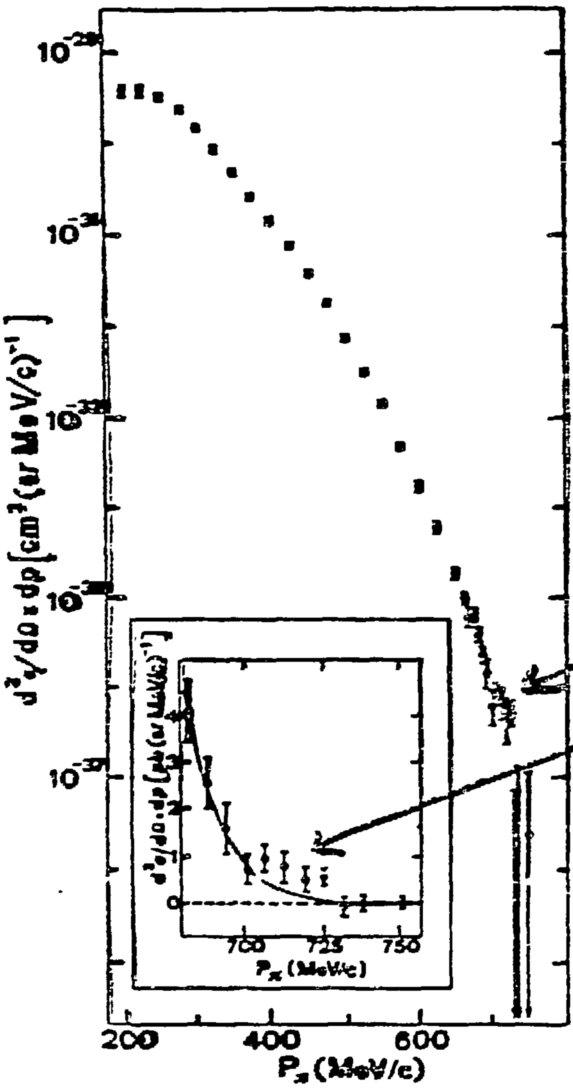
A.M. Baldin et al, Sov. J. Nucl. Phys. 20, 629 (1976)
C.F. Perdrisat et al, Phys. Rev. C18, 1764 (1978)
L.S. Schroeder et al, Phys. Rev. Lett. 43, 1787 (1979)

SCALING LAWS $(1-x)^n$ NEAR KINEMATICAL LIMIT



J. Papp et al, Phys. Rev. Lett. 34, 601 (1975)

PION MOMENTUM SPECTRA AT $0°$ IN $^{3}He + ^{6}Li$ AT 303 MeV/nuc



bump?

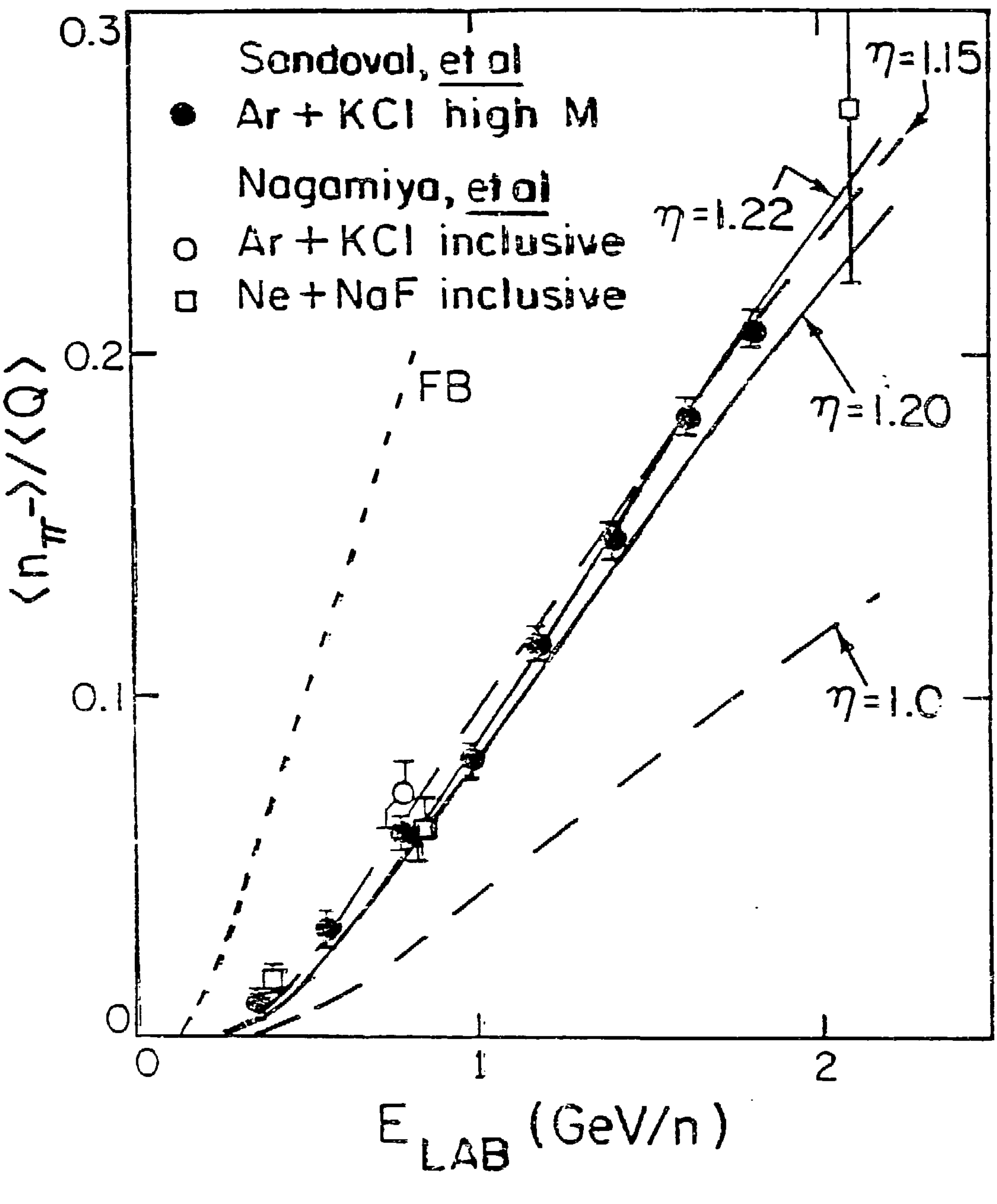
coherent enhancement?

look for structure near kinematical limit

E. Aslanides et al, Phys. Rev. Lett. 43, 1466 (1979)

$(1-x)^n$ deviates from data for $0.9 < x < 1$

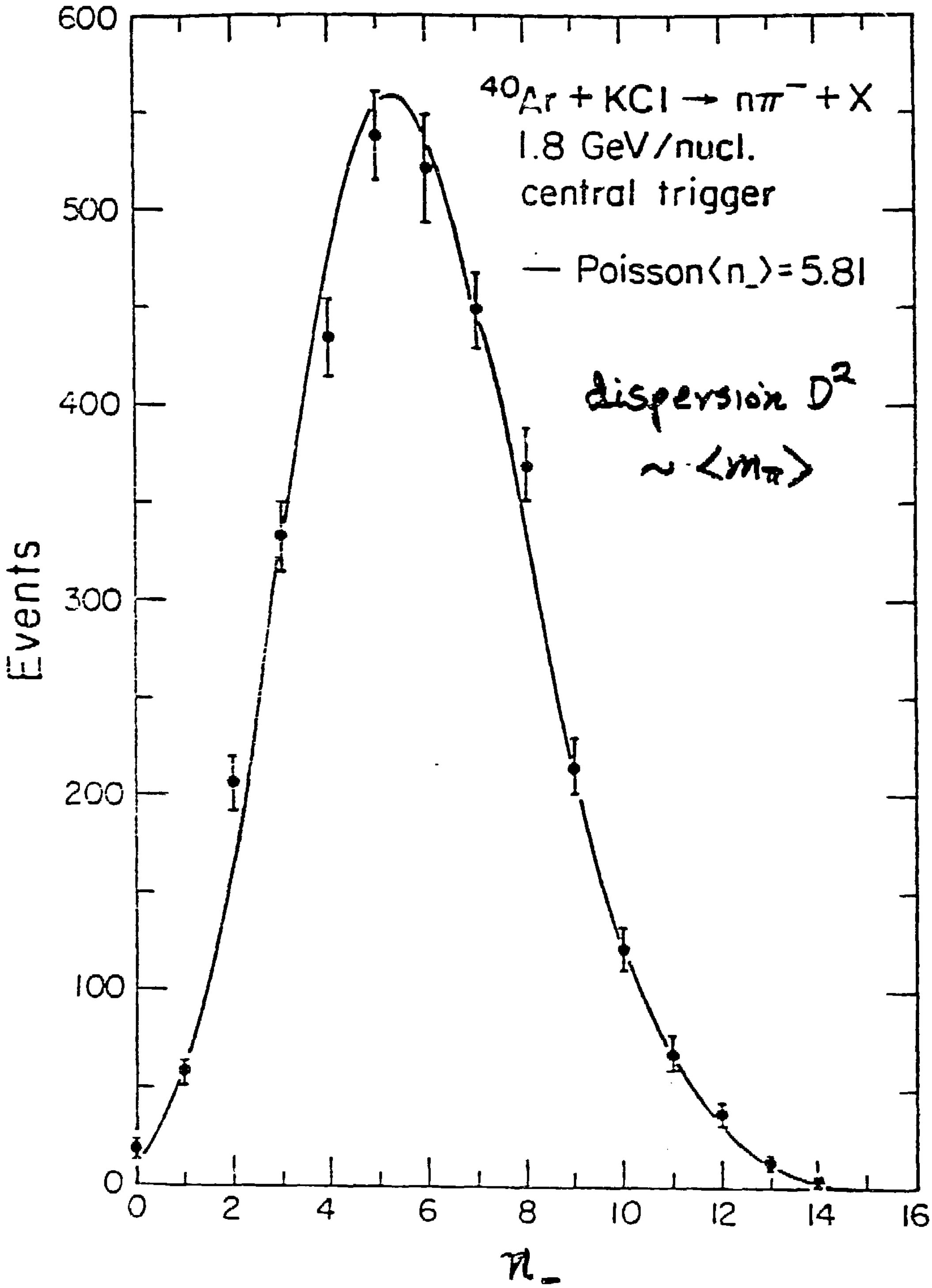
Energy Dependence of Average π^- Multiplicity



$\langle n_{\pi^-}\rangle$ = average multiplicity of π^-'s

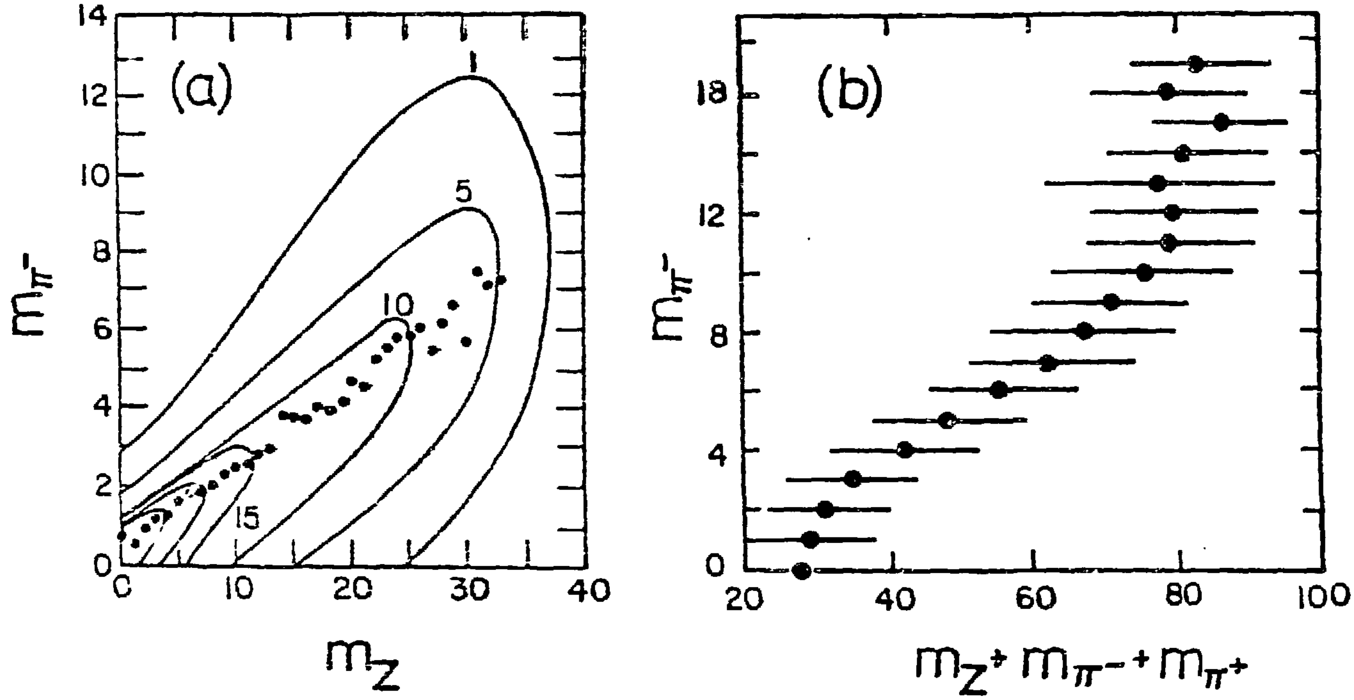
$\langle Q\rangle = \langle m_Z\rangle$ = multiplicity of charged nuclear fragments

PION MULTIPLICITY DISTRIBUTION



events with $m_Z \geq 30$ → almost head-on collision

Correlations between pion (m_{π^-}) and charged nuclear fragment (m_Z) multiplicities



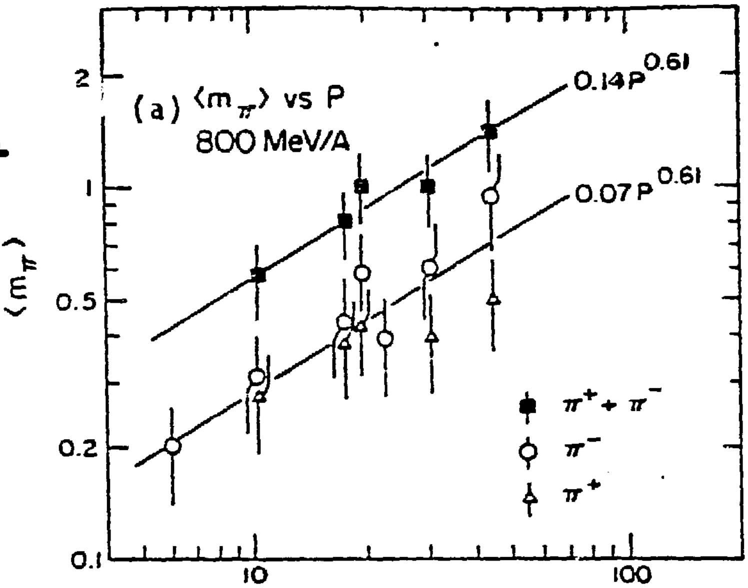
Ar + KCl

A. Sandoval et al, Phys. Rev. Lett. 45, 874 (1980)

Ar + Pb

J.J. Lu et al, Phys. Rev. Lett. 46, 898 (1981)

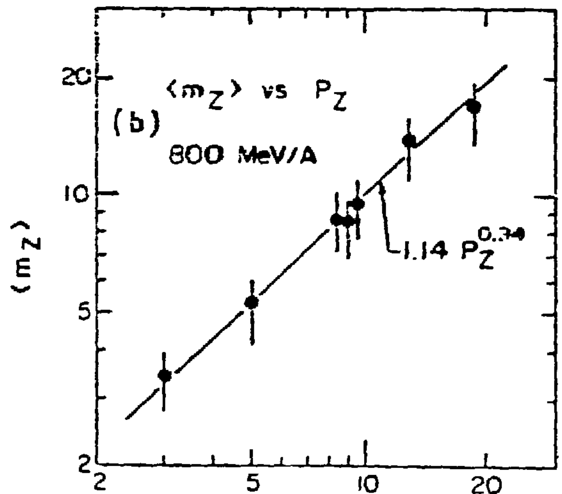
S. Nagamiya et al, Phys. Rev. C24, 971 (1981)



fit to form $\langle m_\pi \rangle = \alpha P^x$

$x \simeq 2/3 \rightarrow$ emission from surface of participant region

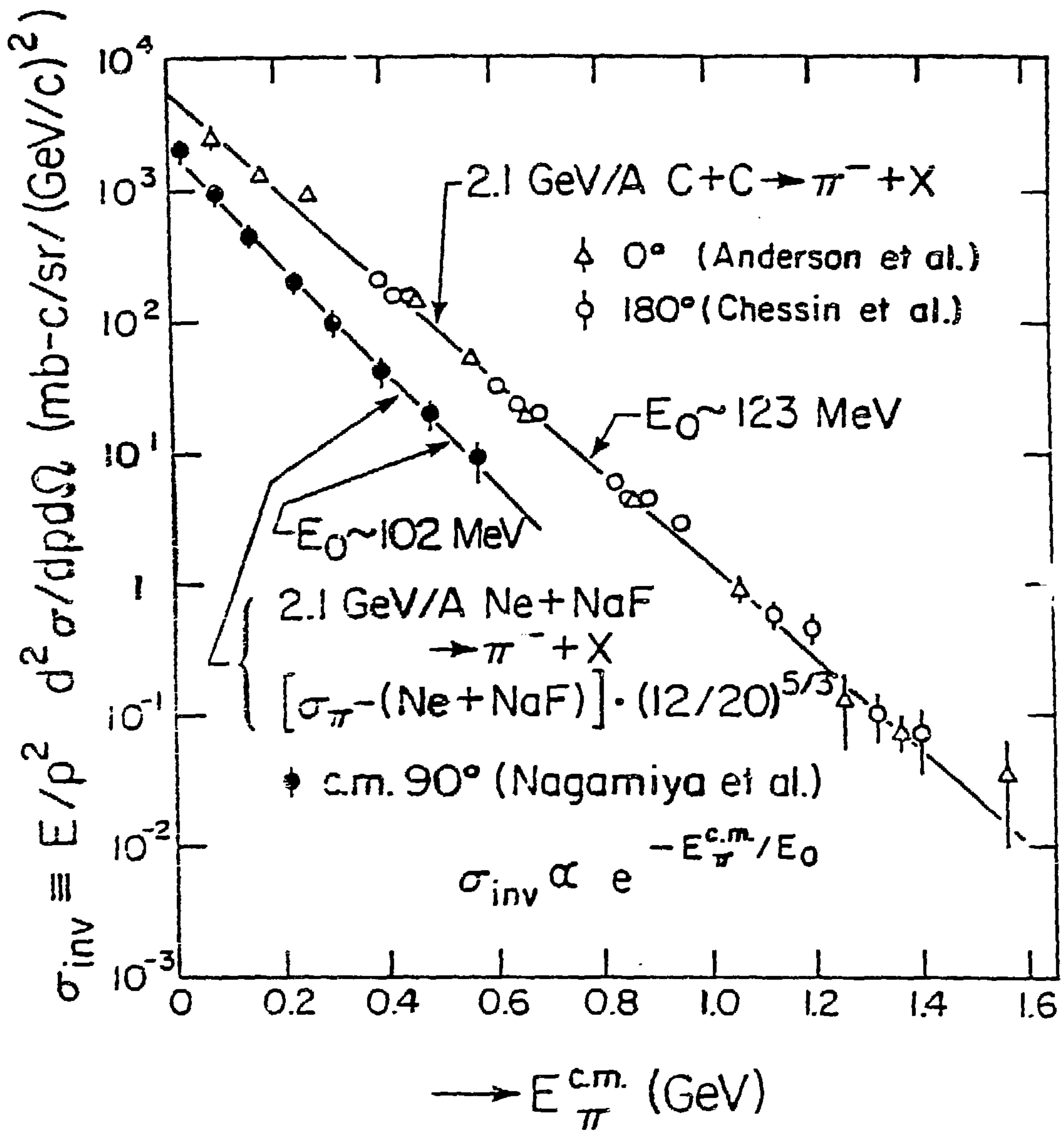
$$P = \frac{A_P A_T^{2/3} + A_T A_P^{2/3}}{(A_P^{1/3} + A_T^{1/3})^2}$$ = average participant nucleon number



$$P_Z = \frac{Z_P A_T^{2/3} + Z_T A_P^{2/3}}{(A_P^{1/3} + A_T^{1/3})^2}$$ = average nuclear charge in participant region

∴ effects of pion absorption cannot be neglected

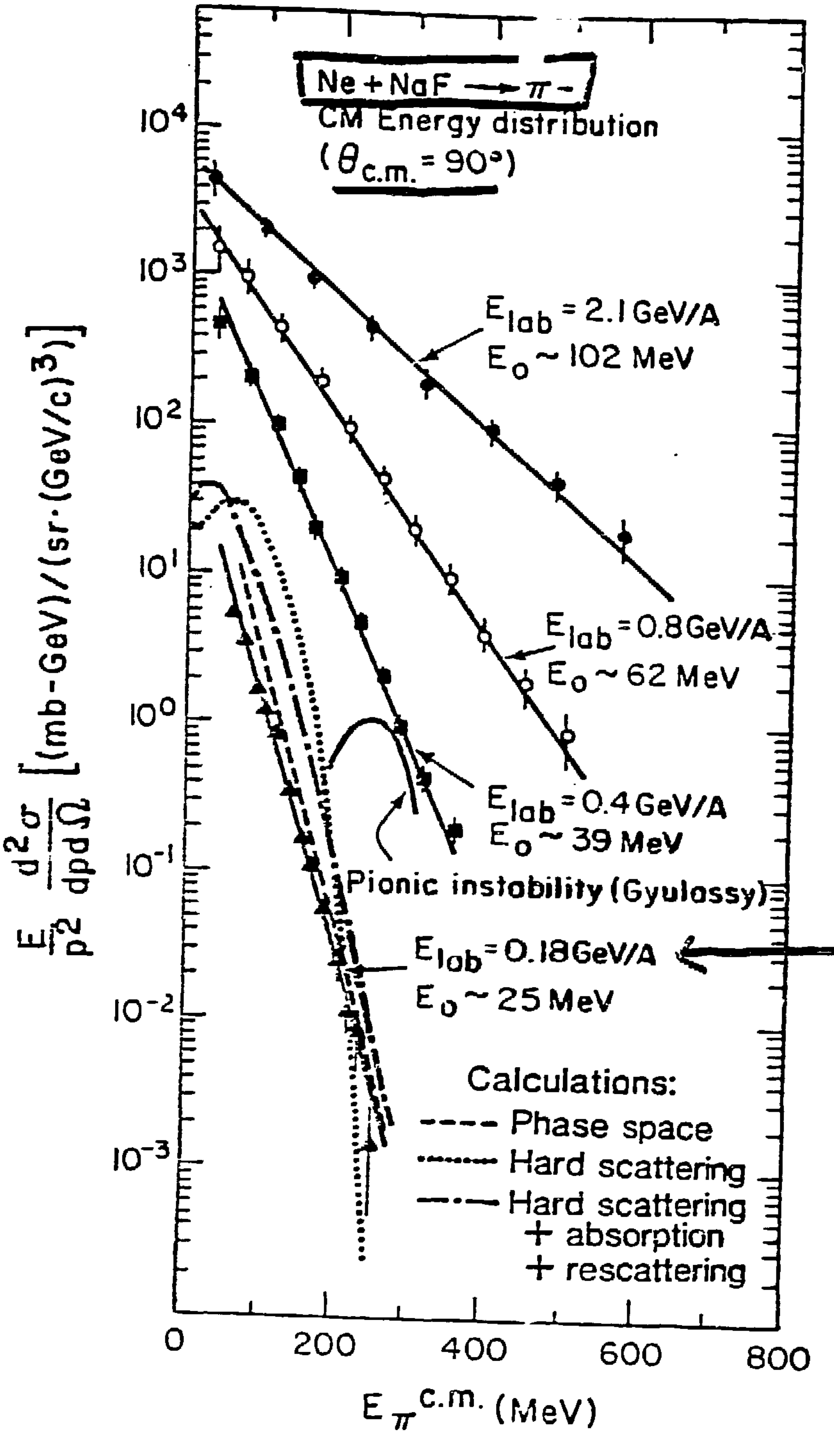
Energy Distribution of Pions at various angles



0° data: E. Moeller et al, preprint (1982)
90° data: S. Nagamiya et al, Phys. Rev. C24, 971 (1981)
180° data: S.A. Chessin et al, preprint (1982)

∴ energy distribution ~ exponential at any angle

Dependence of pion energy spectrum on beam energy

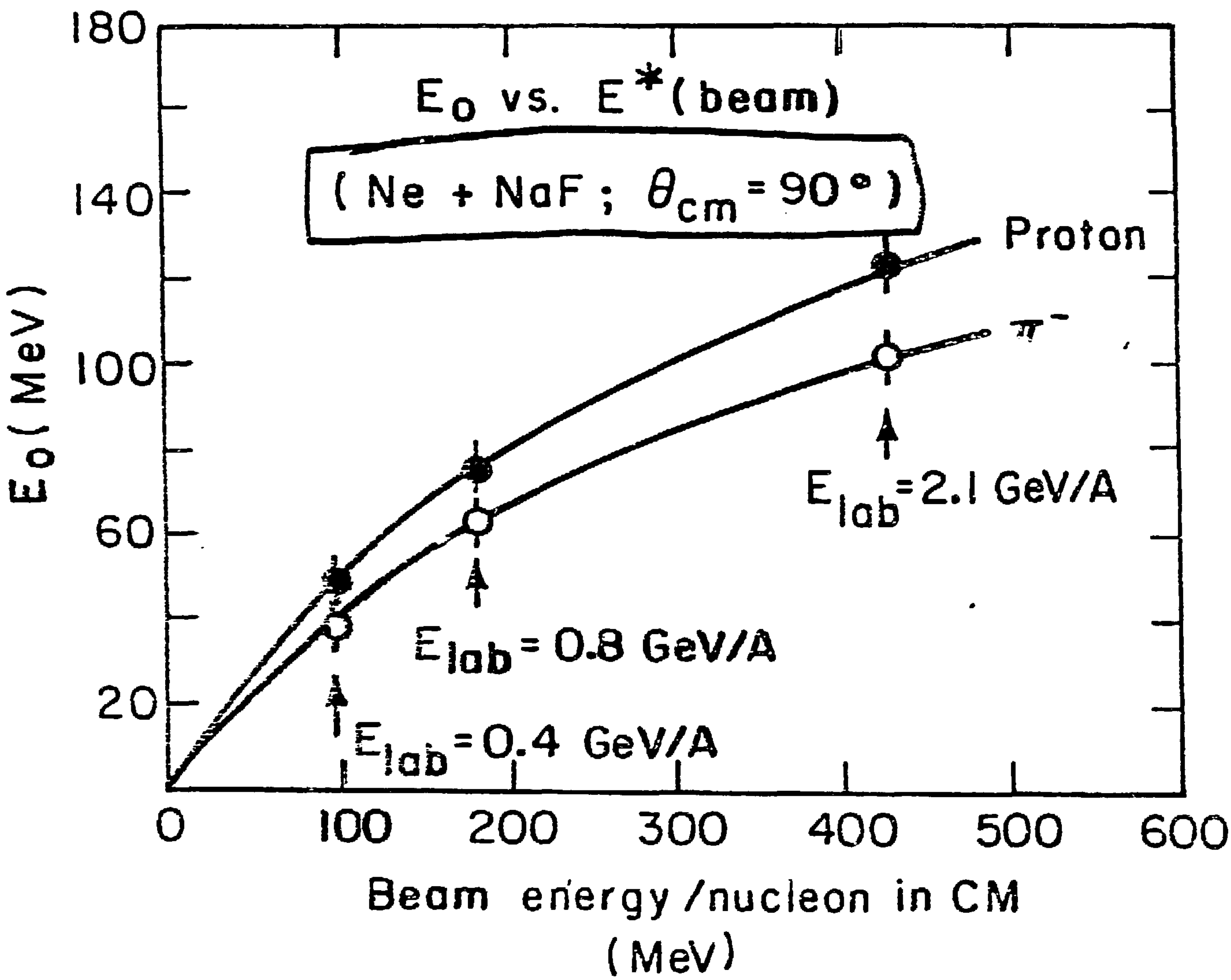


$$E\, d^3\sigma/d^3p \;\propto\; \exp\left(-E_\pi^{c.m.}/E_0\right)$$

slope factor

S. Nagamiya et al, PRC24, 971 (1981)
+ LBL-14033 (1982)

SLOPE E_0 OF PION ENERGY SPECTRUM AS FUNCTION OF BEAM ENERGY

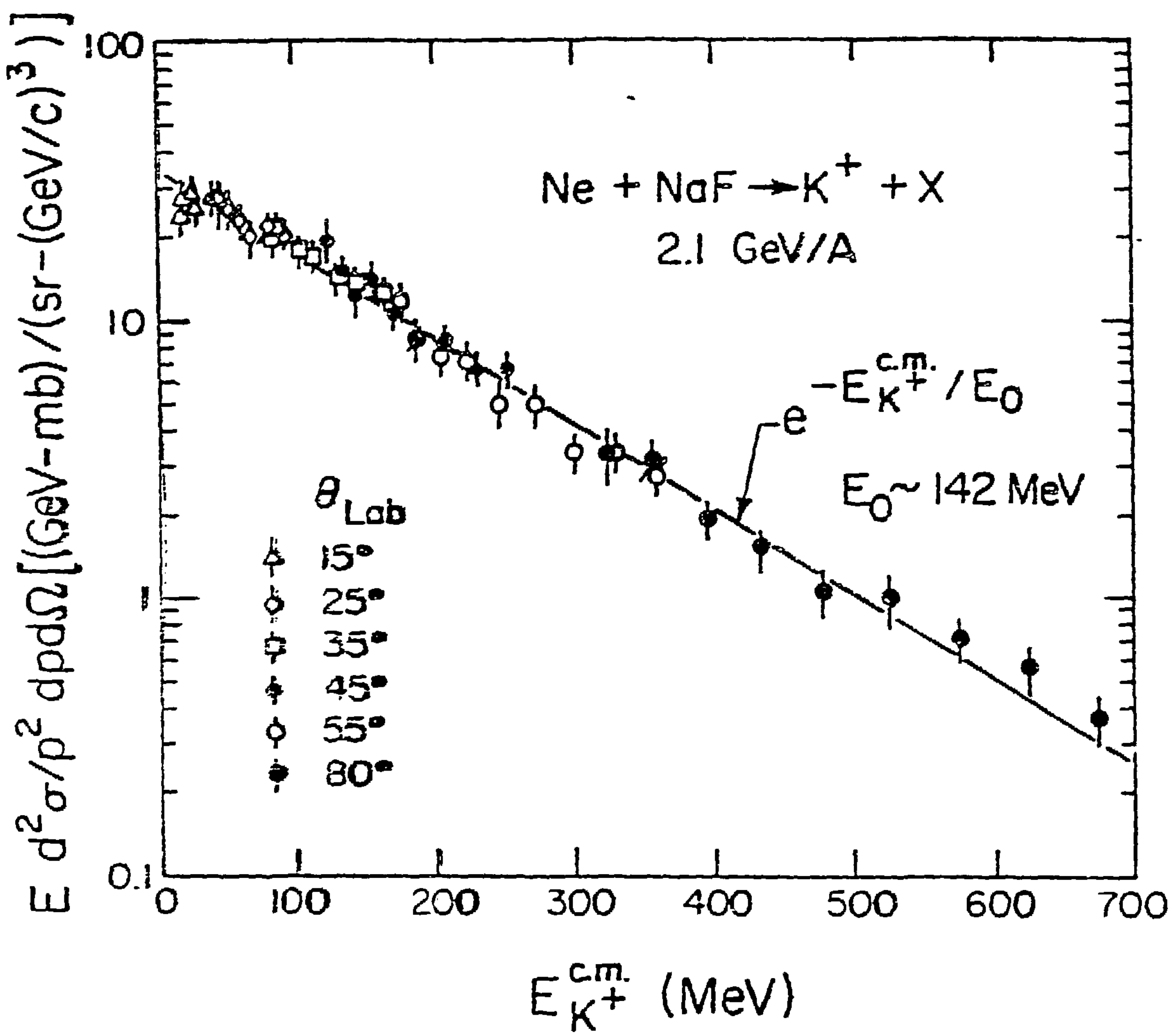


$$E\, d^3\sigma/d^3p \sim e^{-E_\pi^{cm}/E_0}$$

observation: $E_0(\pi) < E_0(p) < E_0(K^+)$

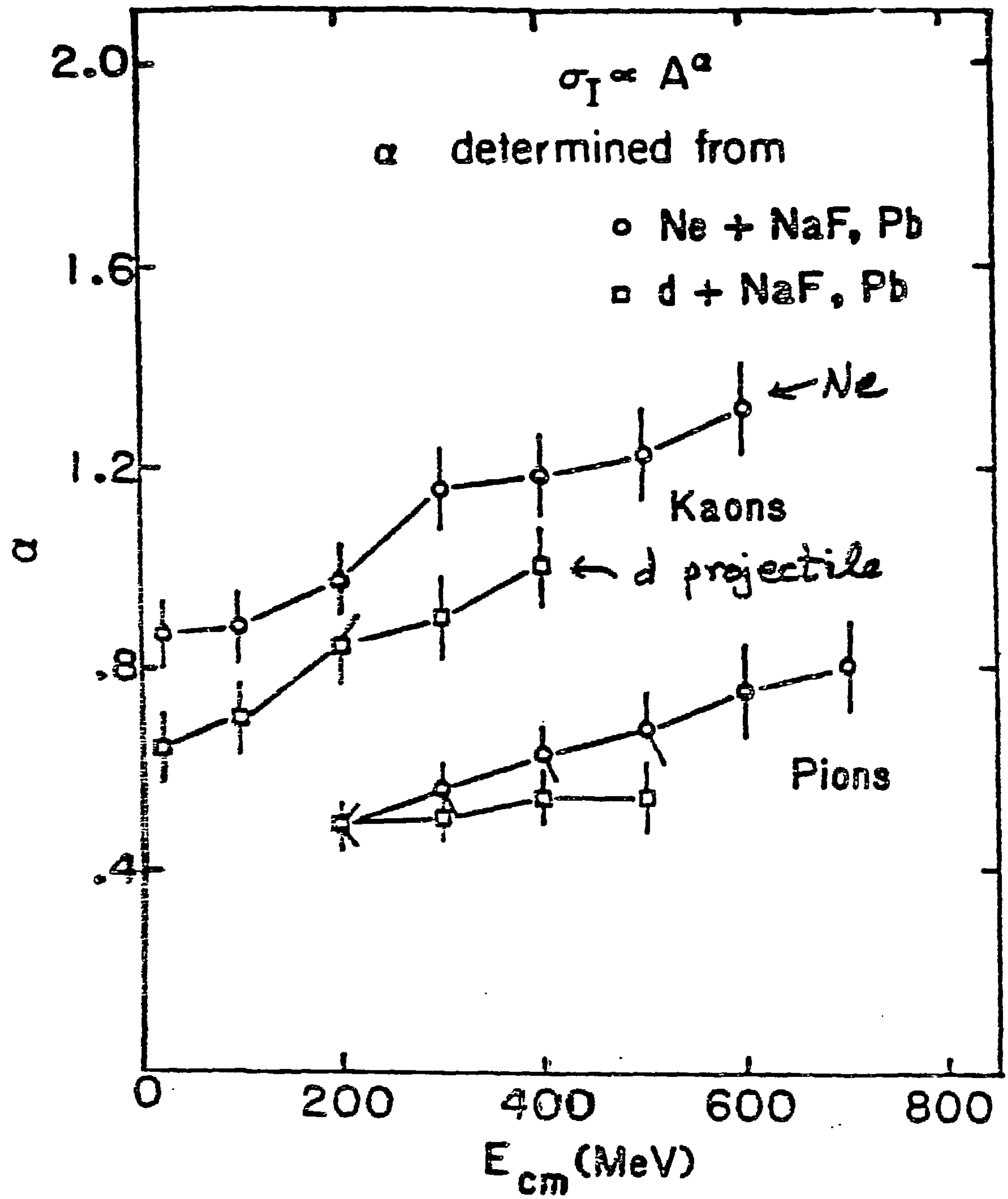
same as ordering of mean free paths

ENERGY SPECTRUM OF K^+ PRODUCTION



S. Schnetzer, Thesis, LBL-13727 (1981)

"A" DEPENDENCE OF PION AND KAON PRODUCTIC



<u>Theory for K^+ production:</u>

J. Randrup + C. M. Ko, Nucl. Phys. A343, 519 (1980)

J. Randrup, Phys. Lett. 99B, 9 (1981)

SLOPES E_0 OF ENERGY SPECTRUM

a) if E_0 reflects mean free path effects, then

$$E_0(\pi) < E_0(p) < E_0(K^+)$$

since $\lambda(\pi) < \lambda(p) < \lambda(K^+)$

weakest absorption

This agrees with experiment!

b) if E_0 reflects phase space effects, then

$$E_0(K^+) < E_0(\pi) < E_0(p)$$

highest threshold

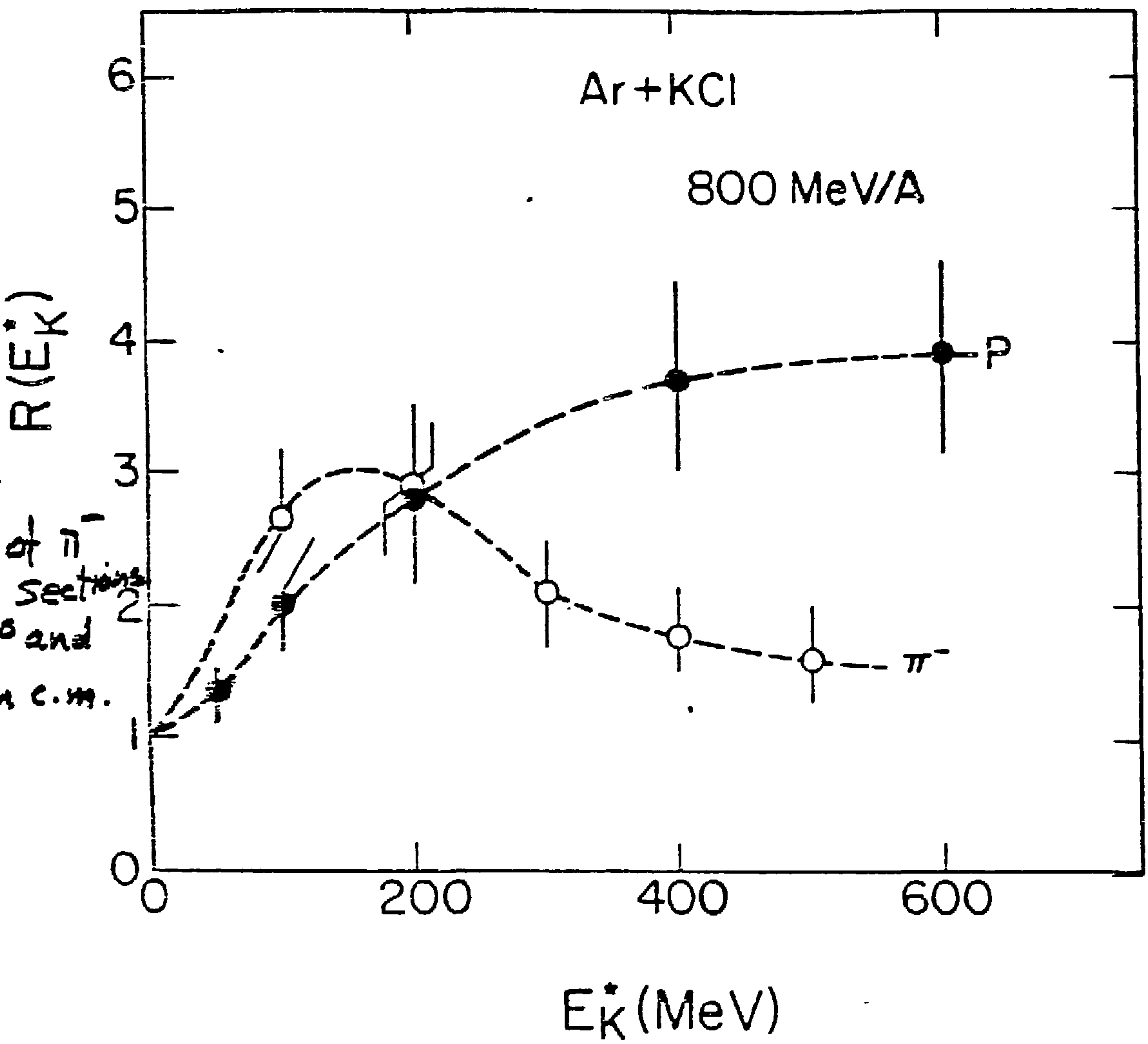
c) radial explosive flow (Siemens + Rasmussen, Phys. Rev. Lett. 42, 844 (1979))

$$E_0(\pi) < E_0(K^+) < E_0(p)$$

ordered like $m_\pi < m_K < m_p$

Angular Dependence of π^- Production

S. Nagamiya et al, Phys. Rev. C24, 971 (1981)

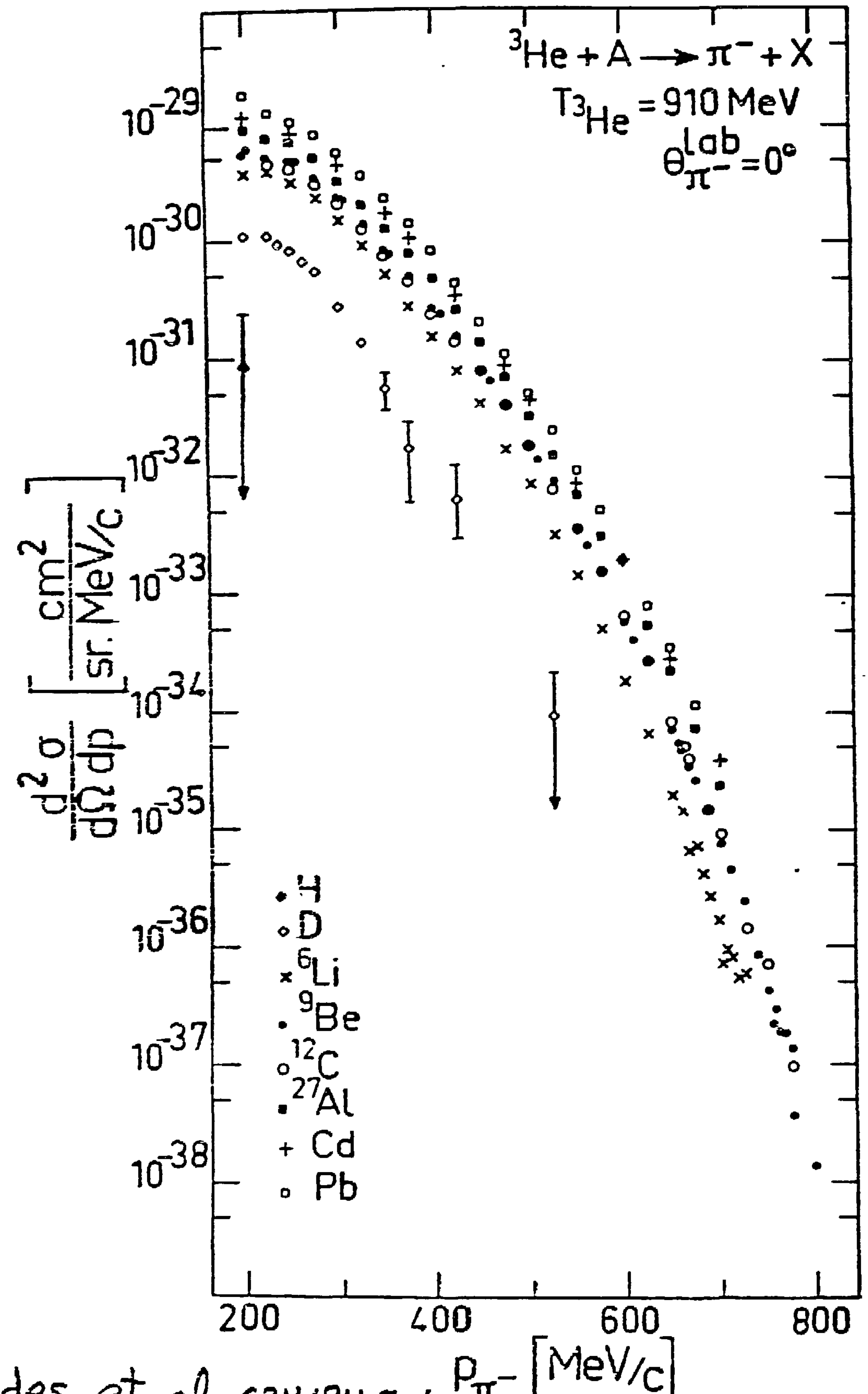


ratio of π^- cross sections at 30° and 90° in c.m.

pion energy in c.m. frame

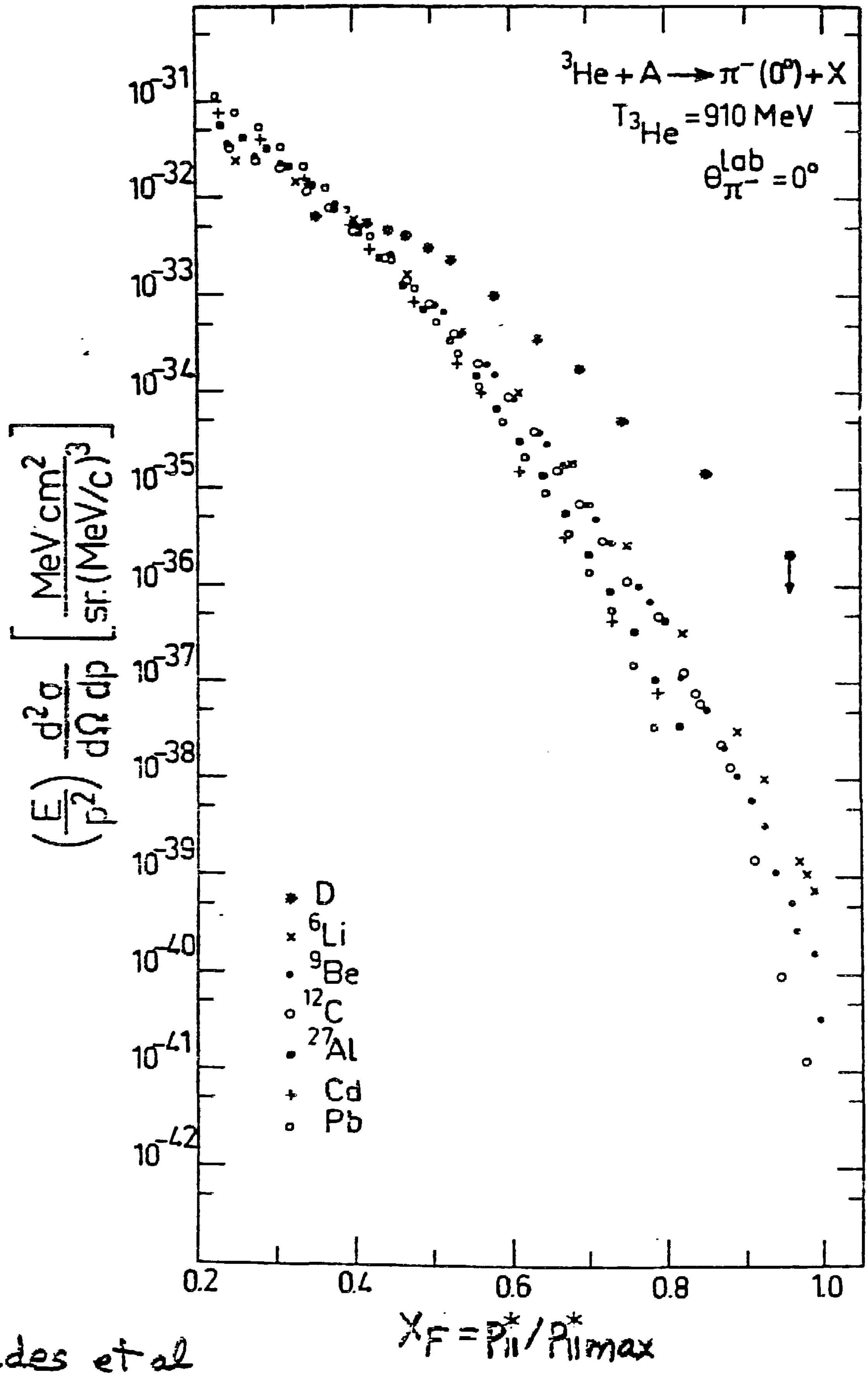
Consistent with expectation that most pions arise from $\Delta_{3,3}$ resonance in $NN \to NN\pi$

π⁻ Inclusive Spectra for various targets



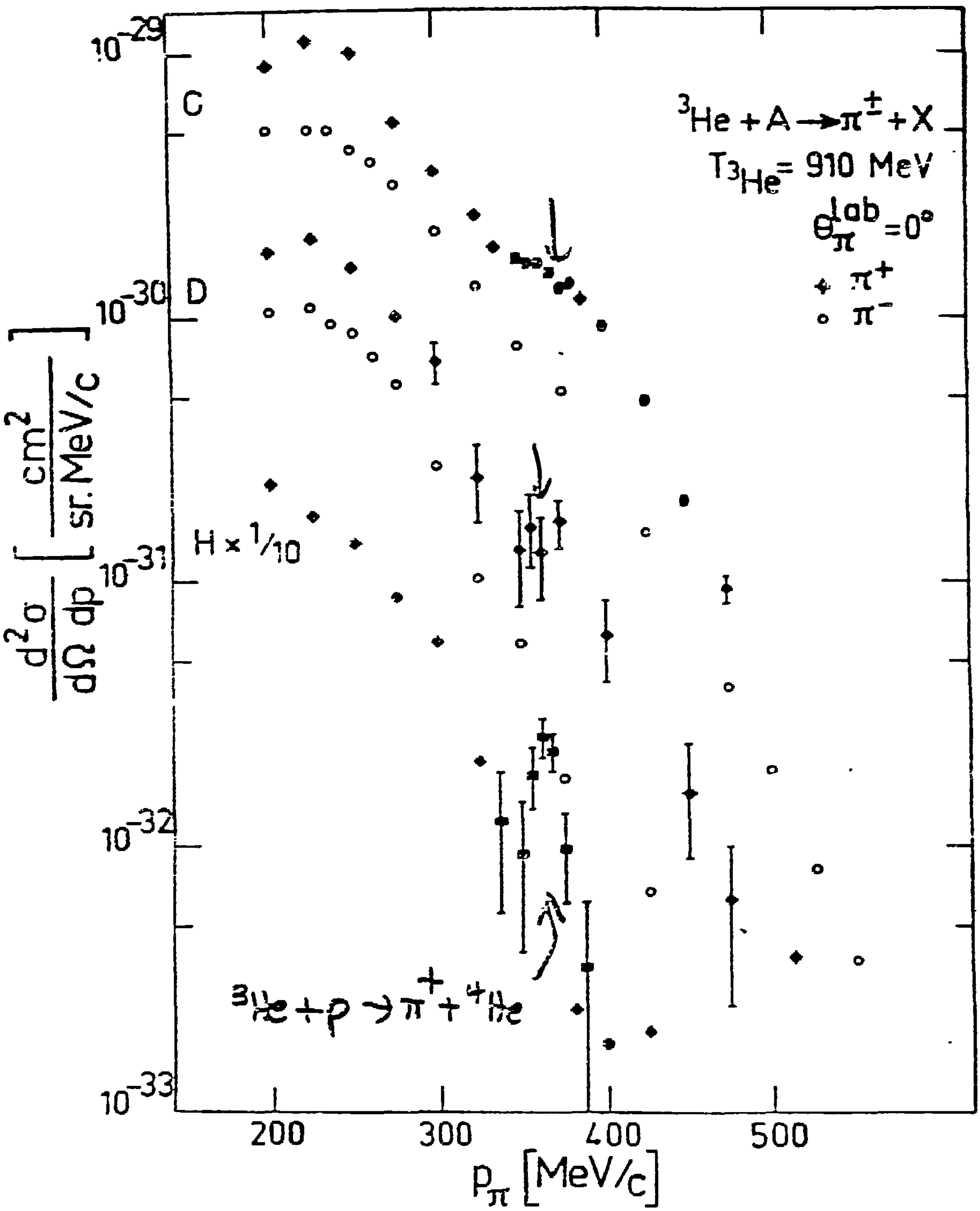
Aslanides et al, CRN/PN 82/16

Lorentz invariant cross section as fcn. of Feynman X_F



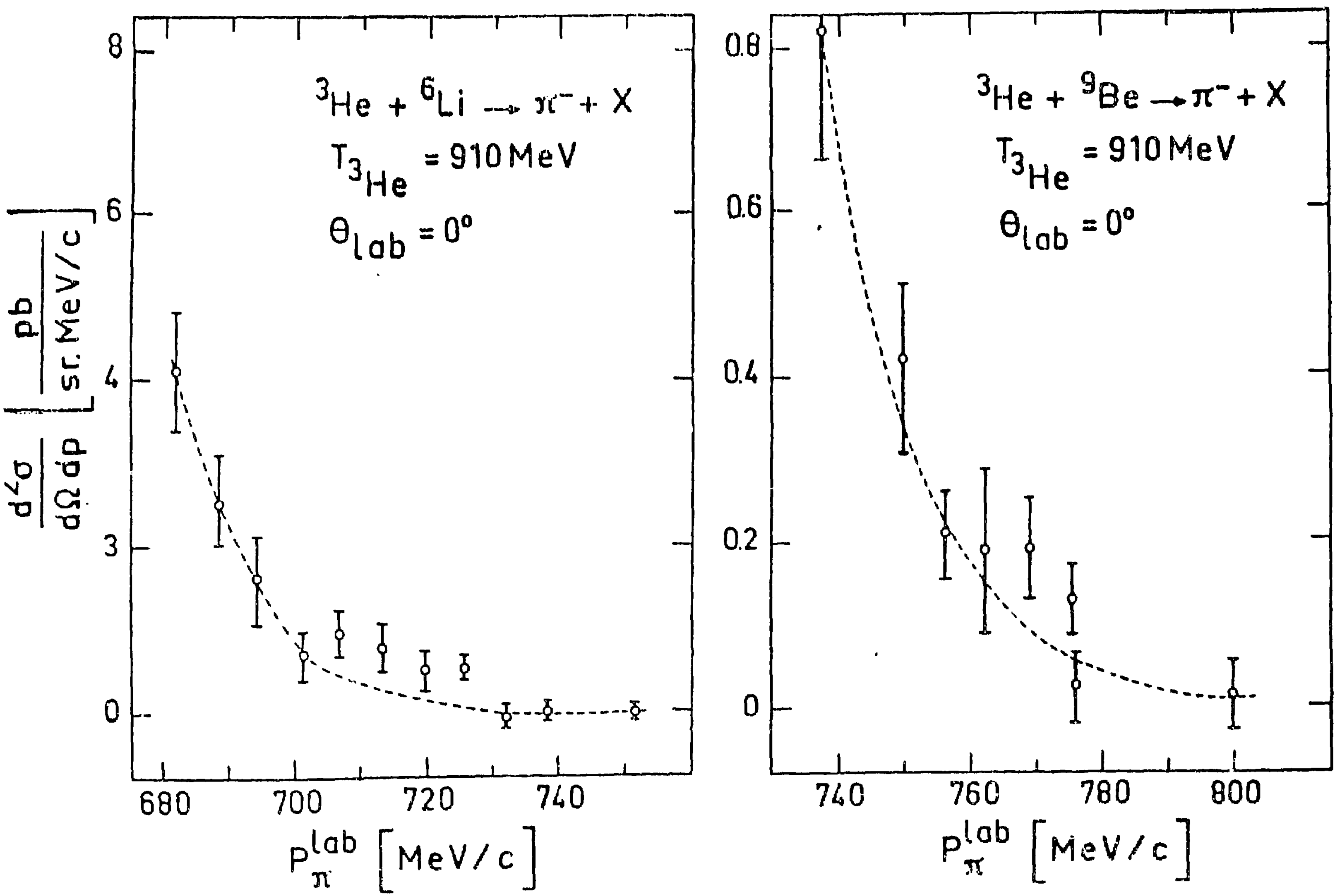
Aslanides et al

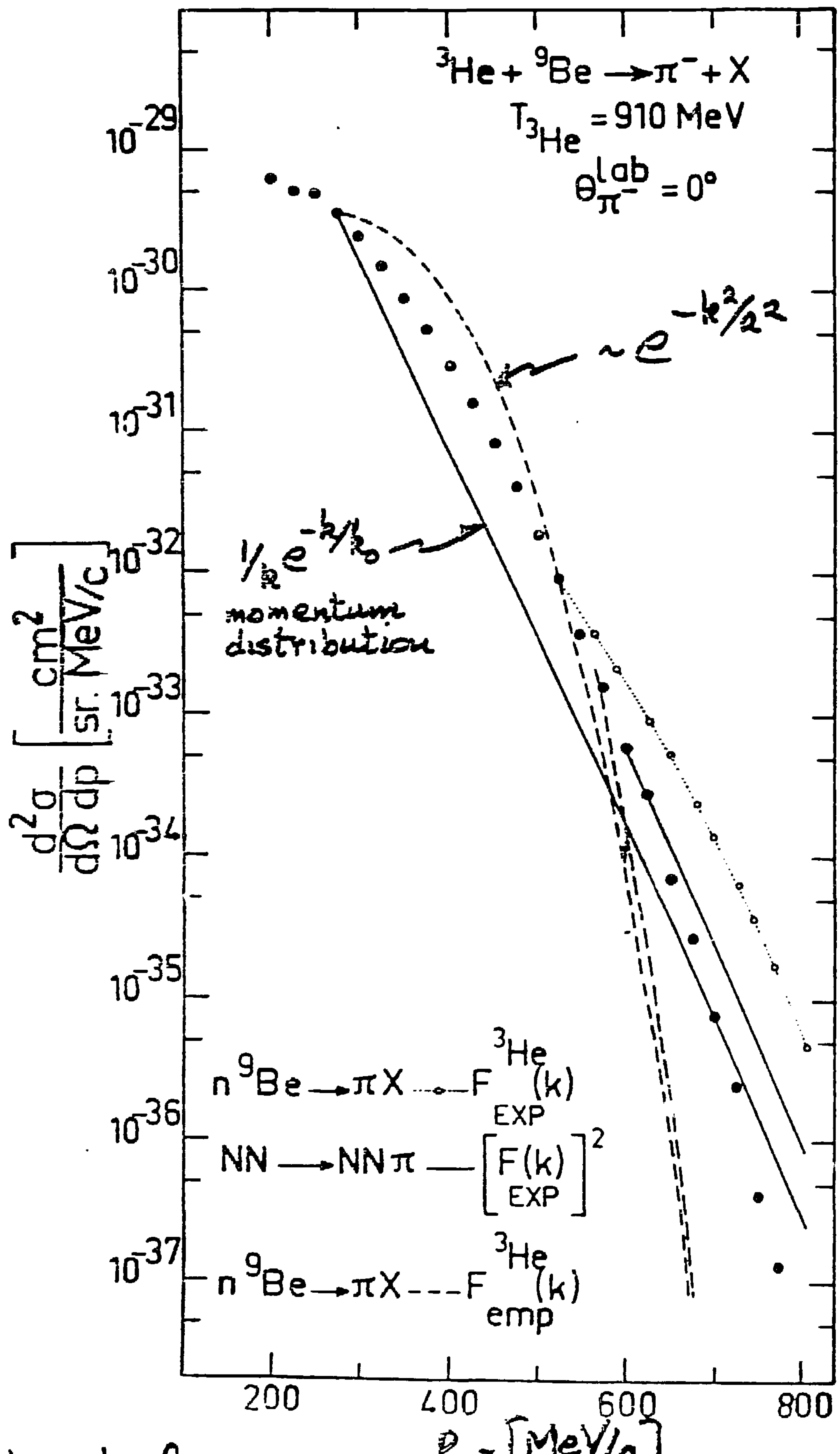
π± spectra on H, D, C targets



Aslanides et al

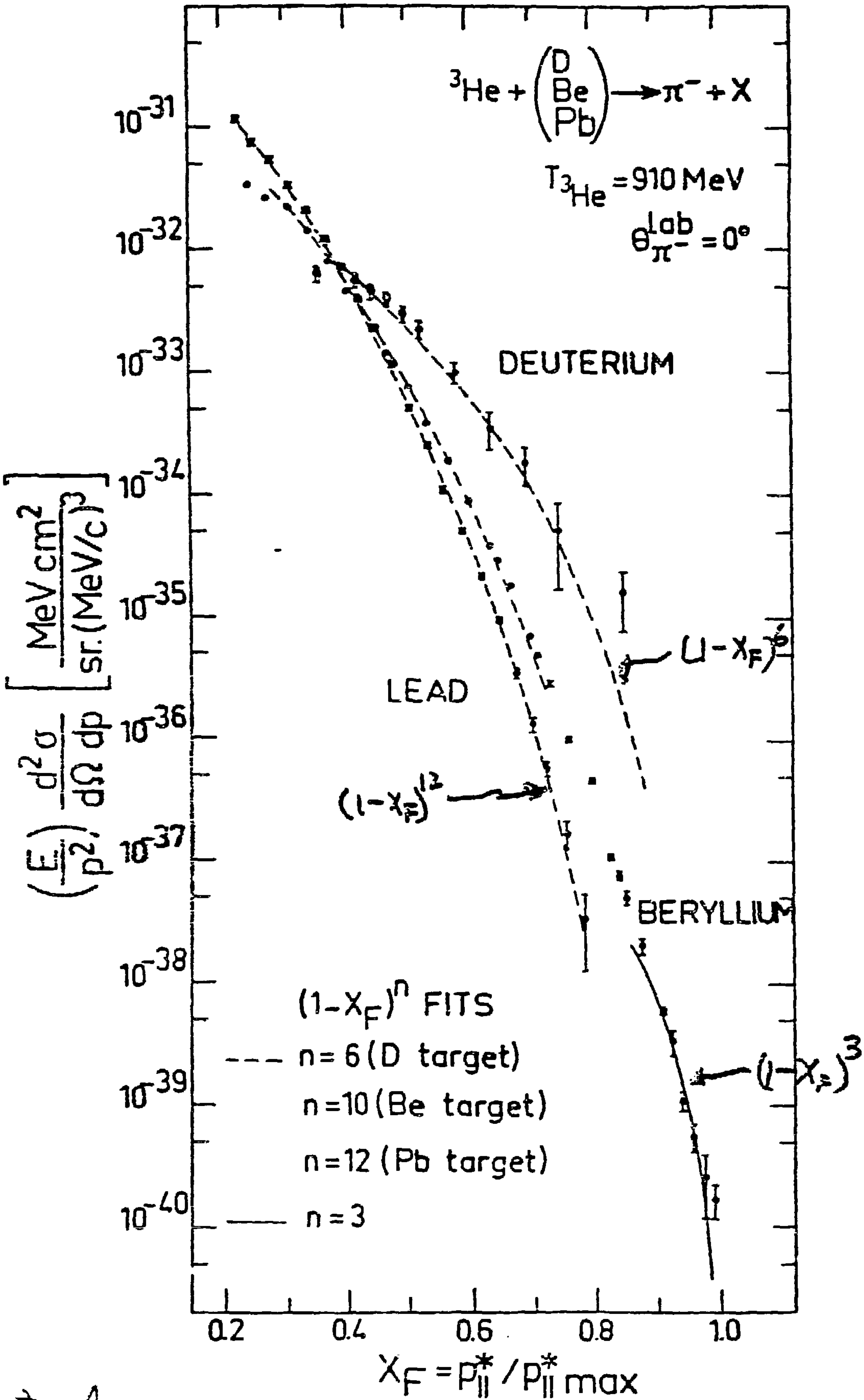
High Momentum Region of Inclusive Pion Spectra



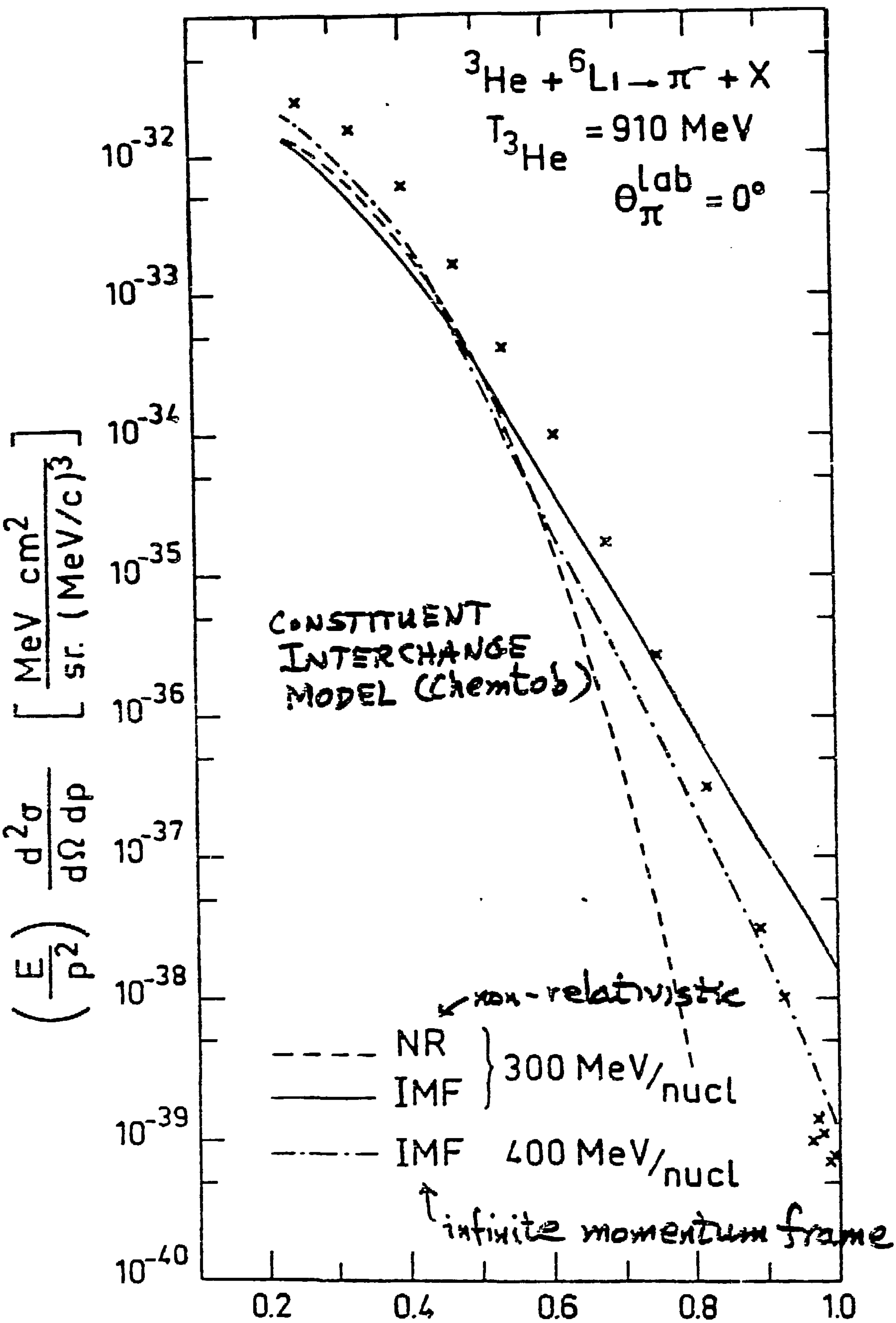


slanides et al

Invariant Cross Section as function of x_F



Aslanides et al



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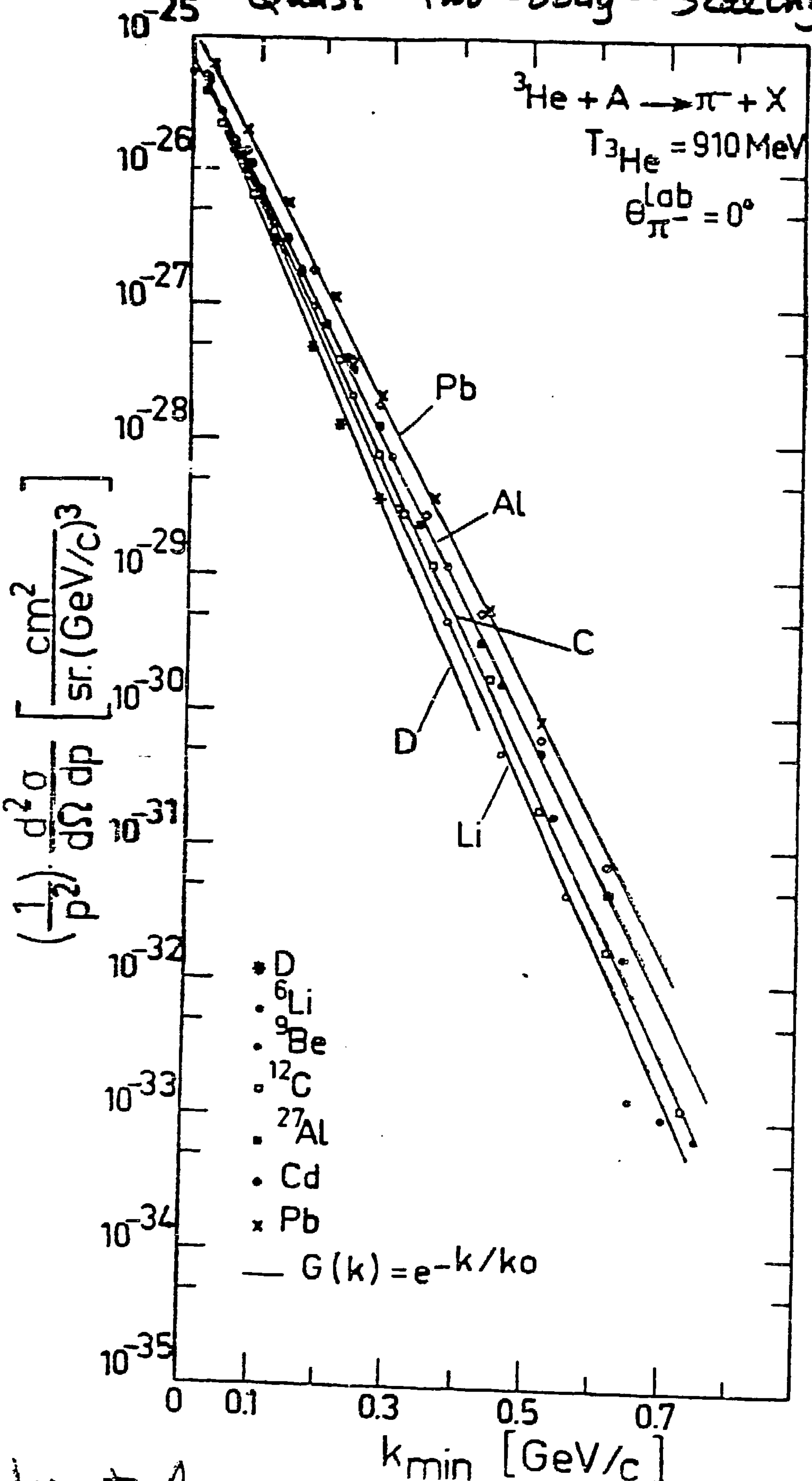
Quasi-Two-Body-Scaling



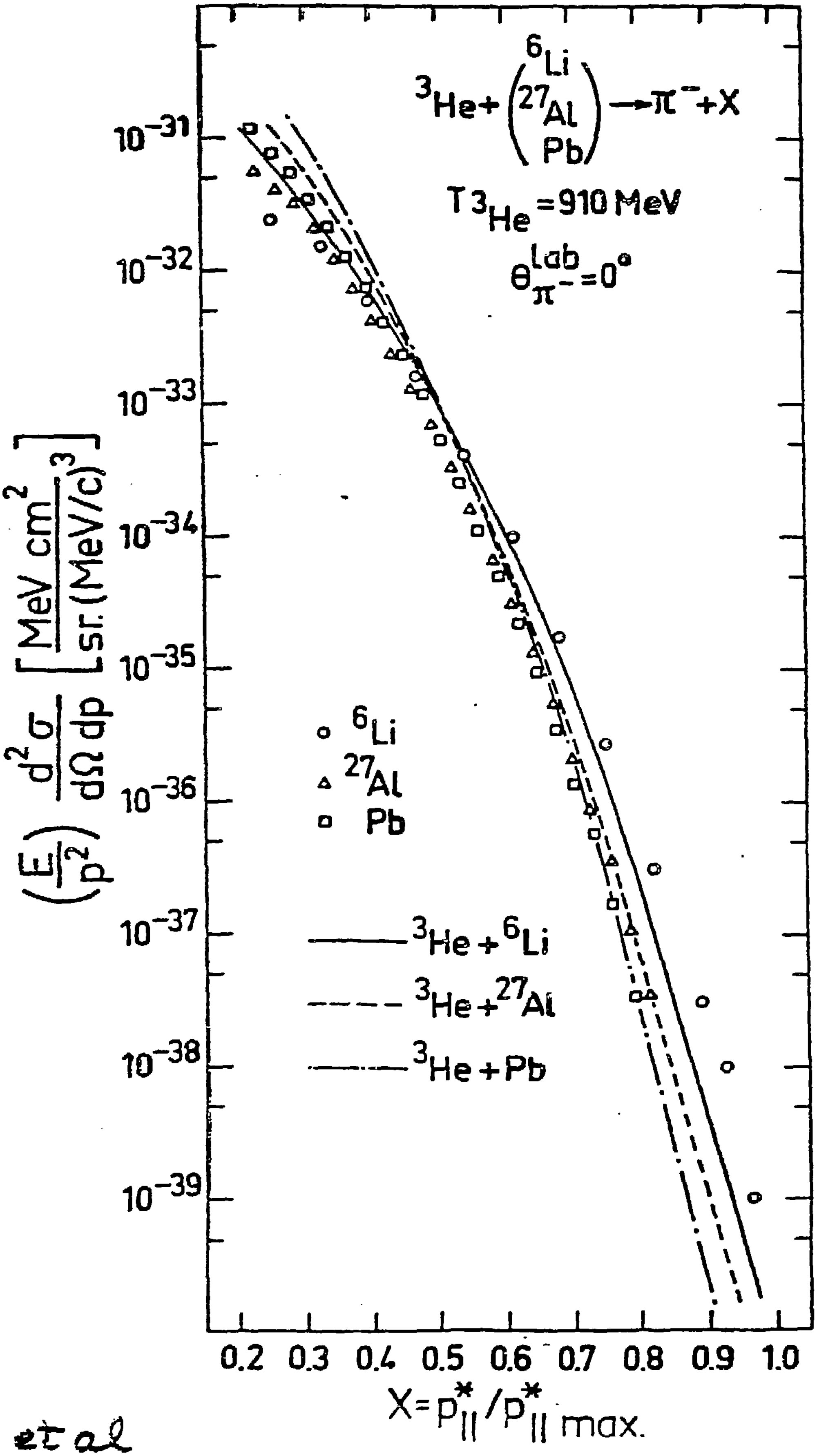
$\sim e^{-k/k_0}$

$k_0 \simeq 38\ MeV/c$ (D)

44 MeV/c (Pb)

Aslanides et al

STATISTICAL MODEL FOR π^- PRODUCTION



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