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Departure from Weinberg-Salam Model and Grand Unification

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ABSTRACT

The spontaneous symmetry breakdown of Grand Unified Groups like SO(10) can lead to an extra U(1) group beyond the SU(2) x U(1) of Weinberg-Salam. The neutral current data in such models could depend on three new parameters beyond the two in the standard model. However, grand unification imposes restriction on coupling constants limiting the analysis to two new parameters. Further, only one new parameter occurs in neutrino-scattering data. More accurate data can thus be used in the future to set limits on these parameters. The present data when used with the more general parametrization no longer determines the value of $\sin^2\theta_w$ as accurately. This leads to a greater uncertainty in the estimate of the proton lifetime.

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I. Introduction

The most popular model for grand unification, SU(5) of Georgi and Glashow,[1] leads to the unique low energy group $SU(3)_c \times SU(2) \times U(1)$. Weak neutral currents are then mediated by a single Z boson, and the resulting phenomenology is dependent on two parameters, ρ, the strength of the neutral current interaction compared to the charged interactions, and $x = \sin^2\theta_w$. The best fit from data for ρ and x results in[2] the values

$$\rho = .985 \pm .023 \tag{1.1}$$

$$x = .218 \pm .020 \tag{1.2}$$

The value of x so determined imposes a very stringent limit on the proton lifetime.[3]

There is however the possibility that the grand unified model is in fact not SU(5). There is some preliminary evidence for neutrino oscillations which suggests that right handed neutrinos may also exist.[4] A natural model that incorporates the right handed neutrino is the SO(10).[5] This model can undergo spontaneous breakdown to leave at low energies either of the two structures:

a) $$SO(10) \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1) \tag{1.3}$$

b) $$SO(10) \rightarrow SU(5) \times U(1) \rightarrow SU(3)_c \times SU(2)_L \times U(1) \times U(1) \tag{1.4}$$

The possibility (a) seems very unlikely in view of the bound in $\sin^2\theta_w$ that it would imply[6] of

$$\frac{1}{4} < \sin^2\theta_w < \frac{3}{8} \tag{1.5}$$

The possibility (b) can however be easily realized as it leads to the same bounds on $\sin^2\theta_w$ as in SU(5).

In this paper we shall obtain the general parametrization of the low energy neutral current interactions based on the group $SU(2)_L \times U(1) \times U(1)$. Constraints due to grand unification will be imposed on the coupling constants.

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The ten quantities that are experimentally accessible will be shown to depend on four parameters.

Further, the six coupling constants that enter neutrino scattering are found to depend on only one additional parameter. These parameters can be further reduced by a suitable choice of the Higgs bosons. We discuss two such cases and possible conclusions regarding mass of the heavier Z boson. Need for better measurements in parity violations in atoms is pointed out. When all four parameters are permitted, the value of $x \equiv \sin^2\theta_w$ is very poorly determined, thus the value could be much lower than hitherto supposed, and the life-time of proton not necessarily accessible to experiment.

II. Neutral Current Phenomenology

The low energy group that emerges is better characterized by $SU(2)_L \times (T_3)_R \times U_V(1)$ where $(T_3)_R$ refers the third component of the $SU(2)_R$ and $U_V(1)$ is a group whose generator is $\frac{B-L}{2}$, where B is the baryon number and L the lepton number. The charged interactions in this theory are the same as in the Weinberg-Salam[7] (W-S) model. The neutral interactions are described by the Lagrangian

$$\mathcal{L}_{int} = g_L J_L^\mu L_\mu + g_R R_\mu J_R^\mu + g_B B_\mu J_B^\mu \tag{2.1}$$

where

$$J_\mu^{L,R} = \frac{1}{4}\left[\bar{u}\gamma_\mu(1\pm\gamma_5)u - \bar{d}\gamma_\mu(1\pm\gamma_5)d + \bar{\nu}\gamma_\mu(1\pm\gamma_5)\nu - \bar{e}\gamma_\mu(1\pm\gamma_5)e\right] \tag{2.2}$$

and

$$J_\mu^B = \frac{1}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) - \frac{1}{2}\left(\bar{\nu}\gamma_\mu \nu + \bar{e}\gamma_\mu e\right) \tag{2.3}$$

From the normalization of currents it can be inferred that at grand unification scale

$$g_L \to g$$
$$g_R \to g$$
$$g_B \to \sqrt{3/2}\, g \tag{2.4}$$

Further, since $(T_3)_R$ and $U_V(1)$ are both U(1) groups, the relationship

$$g_B = \sqrt{3/2}\, g_R \tag{2.5}$$

is valid at all q^2. This is the constraint that we shall employ.

The neutral interactions can be written as[8]

$$\mathcal{L}_{int} = e A_\mu Q^\mu + (g_L/\cos\theta_w) Z_\mu J_Z^\mu + \sqrt{g_R^2 + g_B^2}\, C_\mu \left[J_R^\mu + (g_L \tan\theta_w/g_R)^2 (J_Z^\mu - \cos^2\theta_w Q^\mu)\right] \tag{2.6}$$

where

$$\tan\theta_w = \tilde{g}/g_L \equiv g_R g_B / g_L \sqrt{g_R^2 + g_B^2} \tag{2.7}$$

$$e = g_L \sin\theta_w$$

$$Q_\mu = J_\mu^L + J_\mu^R + J_\mu^B \quad \equiv \text{electromagnetic current}$$

$$J_\mu^Z = J_\mu^L - \sin^2\theta_w Q_\mu \quad = \text{neutral current of W-S model}$$

and the fields A_μ, Z_μ and C_μ are linear combinations of the three fields L_μ, R_μ, B_μ such that A_μ is the photon, and Z_μ corresponds to W-S Z boson in absence of the extra U(1). The general Z-C mixing matrix M^2 can now be considered, and we obtain for the low energy effective Lagrangian[8]

$$\mathcal{L}_{eff} = -\frac{8 G_F \rho}{\sqrt{2}} \left\{ J_Z^\mu J_\mu^Z + \alpha^2 \left[J_\mu^Z + \beta\left[J_\mu^R + \frac{3}{5}(J_\mu^Z - \cos^2\theta_w Q_\mu)\right]\right]^2 \right\} \tag{2.8}$$

here

$$\rho = M_{W^+}^2 / M_Z^2 \cos^2\theta_W \tag{2.9}$$

$$\alpha^2 = M_{Z-c}^4 / \det(M^2) \tag{2.10}$$

$$\beta = -\sqrt{g_R^2+g_\beta^2}\, M_Z^2 \Big/ \sqrt{g_L^2+\tilde{g}^2}\, M_{Z-c}^2 \tag{2.11}$$

The constraint on coupling constants has been imposed in (2.8). We have four parameters in the theory, ρ, $x = \sin^2\theta_w$, α^2 and β. The parameters ρ and β are in general of order 1, and can be determined theoretically provided the Higgs structure is sufficiently simple. In the appendix we present a few simple possibilities.

We shall now present the expressions for the ten quantities that can be measured in experiments. The standard definitions of these quantities are:

a) Neutrino-hadron scattering.

$$\mathcal{L}_{\nu-q} = -(G_F/\sqrt{2})\left[\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\right]\left[u_L \bar{u}\gamma^\mu(1+\gamma_5)u + u_R\bar{u}\gamma^\mu(1-\gamma_5)u + d_L\bar{d}\gamma^\mu(1+\gamma_5)d + d_R\bar{d}\gamma^\mu(1-\gamma_5)d\right] \tag{2.12}$$

b) Neutrino-electron scattering

$$\mathcal{L}_{\nu-e} = -(G_F/\sqrt{2})\left[\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\right]\left[\bar{e}\gamma^\mu(g_V + g_A\gamma_5)e\right] \tag{2.13}$$

c) Electron-hadron scattering

$$\mathcal{L}_{e-q} = +(G_F/\sqrt{2})\Big\{\left[\bar{e}\gamma^\mu e\right]\left[\varepsilon_{VA}(e,u)\,\bar{u}\gamma_\mu\gamma_5 u + \varepsilon_{VA}(e,d)\,\bar{d}\gamma_\mu\gamma_5 d\right] + \left[\bar{e}\gamma^\mu\gamma_5 e\right]\left[\varepsilon_{AV}(e,u)\bar{u}\gamma_\mu u + \varepsilon_{AV}(e,d)\bar{d}\gamma_\mu d\right] \tag{2.14}$$

The ten quantities thus defined are found to be related to the parameters in

Eq. (2.8) as follows:

$$u_L = \left(\tfrac{1}{2} - \tfrac{2x}{3}\right)\rho_1 - z \tag{2.15}$$

$$d_L = \left(-\tfrac{1}{2} + \tfrac{x}{3}\right)\rho_1 - z \tag{2.16}$$

$$u_R = -2x\rho_1/3 - z \tag{2.17}$$

$$d_R = x\rho_1/3 - 3z \tag{2.18}$$

$$g_V = \left(-\tfrac{1}{2} + 2x\right)\rho_1 + 4z \tag{2.19}$$

$$g_A = -\rho_1/2 + 2z \tag{2.20}$$

$$\varepsilon_{VA}(e,u) = -\varepsilon_{VA}(e,d) = \left(\tfrac{1}{2} - 2x\right)\rho_2 - z' \tag{2.21}$$

$$\varepsilon_{AV}(e,u) = \left(\tfrac{1}{2} - \tfrac{4x}{3}\right)\rho_2 \tag{2.22}$$

$$\varepsilon_{AV}(e,d) = -\left(\tfrac{1}{2} - \tfrac{2x}{3}\right)\rho_2 - z' \tag{2.23}$$

where

$$x = \sin^2\theta_w \tag{2.24}$$

$$\rho_1 = \rho\left[1 + \alpha^2(1 + 3\beta/5)\right] \tag{2.25}$$

$$z = \rho\alpha^2\beta(1 + 3\beta/5)/10 \tag{2.26}$$

$$\rho_2 = \rho\left[1 + \alpha^2(1 - 2\beta/5)\right] \tag{2.27}$$

$$z' = 2\rho\alpha^2\beta(1 - 2\beta/5)/5 \tag{2.28}$$

The parameters ρ_1, Z, ρ_2, Z' have one constraint since they depend on only three independent parameters α^2, β and ρ. The results go over to the standard model when

$$\rho_1 = \rho_2 = \rho \tag{2.29}$$

$$Z = Z' = 0 \tag{2.30}$$

This corresponds to $\alpha^2 \to 0$ or one of the Z bosons being extremely heavy. Conceivably $\beta \to 0$ could also accomplish this, but then it would be difficult to understand why $\rho_1 = \rho_2 = \rho(1+\alpha^2)$ is so close to unity.

If the Higgs boson with large vacuum expectation values transform as doublets or singlets under $SU(2)_L$, then $\rho \approx 1$. Further, there are two limiting values of β which can be realized by appropriate choices of the Higgs bosons.

(a) The case ρ = 1 and β = -5/3.

This requires the Higgs structure to satisfy Georgi-Weinberg[9] condition, i.e. the doublet with large expectation value is neutral under the group $(T_3)_R$ as is the left-handed neutrino. In that case

$$\rho_1 = \rho \qquad Z = 0 \tag{2.31}$$

and we recover the W-S model results in the neutrino-sector. An interesting consequence is that the asymmetry in electron-deuteron deep inelastic scattering turns out to be rather insensitive to the new parameters. We find (setting ρ = 1)

$$\frac{A^{eD}}{Q^2} = -(3G_F/10\sqrt{2}\,\pi\alpha)\left\{\left(\frac{3}{2} - \frac{10x}{3}\right) + \delta^2\left(\frac{1}{2} - 2x\right) + 3f(y)\left[\left(\frac{1}{2} - 2x\right) + \delta^2(7-12x)/10\right]\right\} \tag{2.32}$$

here

$$f(y) = \left[1 - (1-y)^2\right]/\left[1 + (1-y)^2\right] \tag{2.33}$$

where

$$y = \frac{E_e - E_e'}{E_e} \tag{2.34}$$

and

$$\delta^2 = 25\alpha^2/9 \tag{2.35}$$

For $x \approx .25$ we see that y independend part is the same as W-S theory. Further y dependent part is not very sensitive to δ^2 provided $\delta^2 < .4$. We display the fit to data for different values of x for $\delta^2 = .4$. The parity violation measurements in atoms however depend on Q_W, which is very sensitive to δ^2. We find for Bismuth[8]

$$Q_W = -126 + 192\,\delta^2 \tag{2.36}$$

this yields $Q_W = -49 \pm 3$ for $x \approx .25 \pm .02$ and $\delta^2 = .4$. In view of the large uncertainty in the value of Q_W, we feel that this solution is still acceptable. The mass of matrix of Z bosons can be diagonalized, and for $\delta^2 = .4$ we find

$$M_{Z_1} = .91\,M_Z \qquad M_{Z_2} = 1.78\,M_Z \tag{2.37}$$

(b) The case ρ = 1 and β = 5/2.

This case is realized if the HIggs structure in SO(10) is properly restricted as discussed in the appendix. A consequence is the electron-hadron sector is the same as W-S model. The neutrino sector is described in terms of two parameters

$$\rho_1 = 1 + \frac{5\alpha^2}{2} \tag{2.38}$$

$$Z = \alpha^2 \frac{5}{8} = \frac{\rho_1 - 1}{4} \qquad (2.39)$$

and

$$x = \sin^2\theta_w$$

In a recent analysis of the neutral current data Langacker et al[2] find the following values for the neutrino coupling constants

$$u_L = .351 \pm .034 \qquad u_R = -.180 \pm .028$$

$$d_L = -.415 \pm .028 \qquad d_R = -.011 \pm .046 \qquad (2.40)$$

$$g_V = .043 \pm .066 \qquad g_A = -.545 \pm .045$$

The values for $\sin^2\theta_w$ and α^2 from the above are

$$\sin^2\theta_w = .24 \pm .02$$

$$\alpha^2 = .016 \pm .012 \qquad (2.41)$$

The value of $\sin^2\theta_w$ is essentially the same as in the standard fit. From the central value of $\alpha^2 = .016$ we find the masses of the two Z bosons are

$$M_{Z_1} = .99\, M_Z$$

$$M_{Z_2} = 3.18\, M_Z \qquad (2.42)$$

(c) The general case.

In this case there are three parameters ρ_1, x and z to be determined from the neutrino data. The best fit is obtained with

$$\rho_1 = 1.02 \pm .067$$

$$x = .24 \pm .03 \qquad (2.42)$$

$$z = .014 \pm .01$$

This fit has $\chi^2 = 3.86$ for 3 degrees of freedom. This is comparable to W-S fit, but still not very reliable. A fit to one parameter x using identical data would yield $x = .255 \pm .02$, a value of x larger by 6%. To see what would happen if the value of $\sin^2\Theta_w$ was fixed at say $x = .2$, we fitted the data to ρ_1 and z.

$$\rho_1 = 1.0 \pm .045$$

$$z = .024 \pm .008 \qquad (2.43)$$

This fit has $\chi^2 = 4.8$ for 4 degrees of freedom, almost the same likelihood as the former solution. The conclusion we reach is that the data is not yet good enough to determine the values of the three parameters. The more general fit seems to give lower values of x than the W-S fit. We have not tried to determine the parameters from electron-hadron scattering because here the data is not sufficiently accurate.

III. Conclusions

The Weinberg-Salam model is a good approximation to the low energy neutral current phenomenology. Study of deviations from this model are however rewarding because of the light they throw on the grand unification group. Present data on parity violation in electron-hadron system is not able to rule out the presence of sizable deviations from W-S predictions. In particular experiments on parity violations in atoms should be pursued. An improvement in the neutral current neutrino data will enable a more precise measurement of $x = \sin^2\theta_w$. This parameter has a large uncertainty when the general three parameter fit is used. Consequently, the lifetime of the proton which is critically dependent of the value of x, is not as well determined as suggested.

Appendix

The low energy group $SU(2)_L \times (T_3)_R \times U_\pi(1)$ is not necessarily limited to SO(10). Another grand unified group that leads to the same low energy group is $SU(8)_L \times SU(8)_R$.[10] More general groups have been considered by Barr and Zee.[11] Within SO(10),[12] the choice of Higgs representation dictates the value of ρ and β. The case ρ = 1 and β = -5/3 will ensue if Higgs representations with large expectation values are confined to 16 and 45. The fermion get their masses through the 10 and 120 dimensional representation whose vacuum expectation values are small. The alternate solution ρ = 1 and β = 5/2 is obtained naturally if all the Higgs bosons have comparable expectation values. Prolification of Higgs bosons can of course lead to arbitrary values of ρ and β as considered in the general case.

In determining ρ, β and α^2, it is sufficient to know how the Higgs transform according to the subgroup $SU(2)_L \times T_3(R) \times (B-L)/2$. If the Higgs $\langle\phi_i\rangle$ with non-vanishing expectation value has the quantum numbers T^i_{3L}, T^i_{3R} and $\frac{(B-L)_i}{2}$, then the Mass matrix is given by

$$M_Z^2 = (g_L^2 + \tilde{g}^2) \sum_i (T_{3L}^i)^2 \langle\phi_i^0\rangle^2 \tag{A.1}$$

$$M_{Z-C}^2 = \sqrt{\frac{g_L^2 + \tilde{g}^2}{g_R^2 + g_B^2}} \sum_i T_{3L}^i \left(g_R^2 T_{3R}^i - g_B^2 \frac{(B-L)_i}{2}\right) \langle\phi_i^0\rangle^2 \tag{A.2}$$

$$M_C^2 = \frac{1}{(g_R^2 + g_B^2)} \sum_i \left(g_R^2 T_{3R}^i - g_B^2 \frac{(B-L)_i}{2}\right)^2 \langle\phi_i^0\rangle^2 \tag{A.3}$$

Further, if ϕ_i are only doublets or singlet under $SU(2)_L$, we have

$$M_Z^2 = M_W^2 / \cos^2\theta_W \tag{A.4}$$

The doublet that occurs in the 10 dimensional representation of SO(10) leads to $T_{3L} = -\frac{1}{2}$, $T_{3R} = +\frac{1}{2}$ and $(B-L)_i = 0$. This leads to ρ = 1 and β = 5/2, the special case discussed. Purely 16 dimensional representation has a doublet with quantum numbers $T_{3L} = -\frac{1}{2}$, $T_{3R} = +\frac{1}{2}$ and $\frac{(B-L)}{2} = \frac{1}{2}$. This would lead to β = -5/3.

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Figure Captions

Fig. 1. The asymmetries in the SLAC e – d deep-inelastic scattering as a function of y. The predictions of W-S model (dashed lines) for $\sin^2\theta_w = 0.224 \pm 0.020$ as well as the predictions of our model for $\sin^2\theta_w = 0.25 \pm 0.02$ and $\delta^2 = 0.4$ are shown.

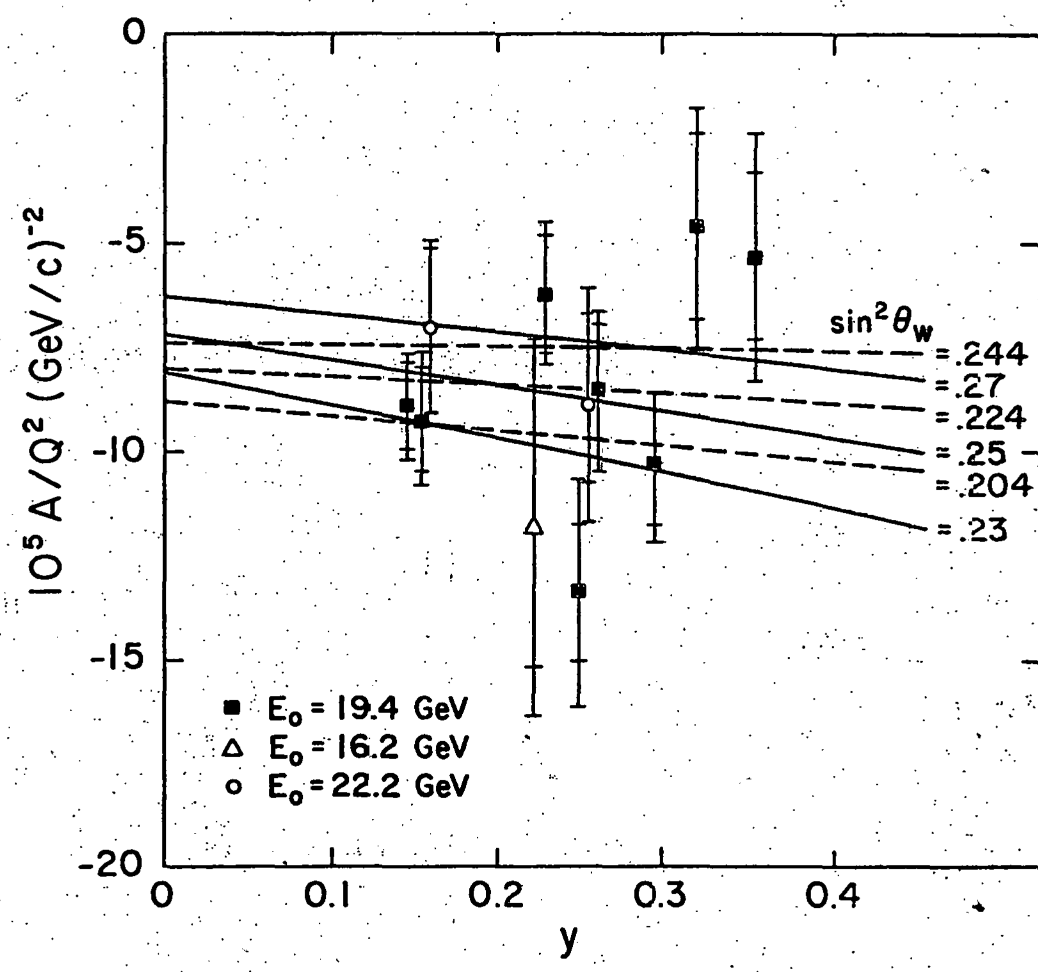

Figure 1