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A MODEL FOR TRANSIENT NATURAL CONVECTION IN A VERTICAL CYLINDER WITH SIDEWALL HEATING

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A simple two-region model has been developed to predict the transient velocity and temperature distribution in a vertical cylinder with sidewall heating. The two regions are a boundary layer region along the vertical walls and a central core region in the middle. The boundary layer behavior is analyzed by the local similarity method which has been modified to conserve energy and to include turbulence and mixed convection effects. The central core region is broken up into a number of vertical control volumes. The results of this model compare favorably with the limited available velocity and temperature data.

1. INTRODUCTION

A model is needed to predict the fluid velocities and temperature stratification in the crude oil in the more than 50 oil-filled Strategic Petroleum Reserve (SPR) caverns during long-term storage in order to understand processes such as fluid mixing and temperature stratification. Due to the large length scale encountered in these caverns (up to 600 m), highly turbulent natural convection will occur for the anticipated 30 year storage period with Rayleigh numbers up to 10^{16}. Finite difference techniques are impractical for these high Rayleigh numbers and a 30 year transient due to long estimated computing requirements.

In order to efficiently analyze the more than 50 caverns for a variety of operating scenarios for the 30 year lifetime, a rapid analysis technique is needed. Due to uncertainties in geometry and boundary conditions, approximate techniques are acceptable. For model development and evaluation purposes, a cavern can be assumed to be a vertical cylinder or enclosure. Since most of the heat addition to the caverns will come from the vertical walls, only sidewall heating is considered.

Limited data are available for natural convection in vertical cylinders. The only known velocity data were obtained by Hess and Miller (1979) for laminar conditions in a water-filled vertical cylinder with constant heat flux on the vertical walls; temperature data were not reported. Upward velocities were observed close to the vertical walls with a smaller, relatively uniform downward velocity in the center. Laminar and turbulent temperature data were obtained by Drake (1966) for a vertical cylinder filled with water, glycerine, or a water-glycerine mixture with a constant wall heat flux. Turbulent conditions were only obtained with water. Radial temperature gradients only existed near the walls; the radial gradient was negligible in the center. In contrast, the center region experienced a significant vertical temperature gradient. This work is summarized by Evans, et al. (1968). Other, more limited temperature data are also available which are similar to those of Drake.

Few models have been developed for a vertical cylinder. Drake (1966) formulated a three-region model with a boundary layer region near the wall, a mixing region at the top, and a core region in the center with plug flow. For the boundary layer behavior, the integral method was used assuming no entrainment, i.e., no mass or energy transfer between the boundary layer and the core region. The height of the mixing region was varied to fit the laminar and turbulent temperature data separately. Note that the results of Drake (1966) have an error in the inferred heights which was corrected by Evans, et al. (1968). This model predicted the data well as is shown later.

Finite differences have been applied to a cylindrical cavity including transient effects (for example, Sun and Oosthuizen (1989)). However, as discussed earlier, finite difference techniques are impractical for SPR cavern analysis. The present study develops a rapid approximate model to analyze transient natural convection in a vertical cylinder using a psuedo-steady-state approach for use when computing resources are limited.

2. MODEL DEVELOPMENT

The model developed in this study is patterned after Drake (1966). The Drake method has two major drawbacks - the empirical determination of the mixing zone height and the integral method of analysis for the boundary layer. The present model addresses these two problem areas and consists of a boundary layer region and a central core region; the mixing region is not used due to its empirical nature. The two regions are depicted in Figure 1.

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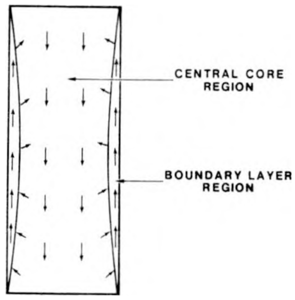


Figure 1 Model Regions

2.1 Boundary Layer Region

Due to the 30 year time frame of this analysis, the boundary layer region model must be fast running. In addition, the results only need to be approximate due to uncertainties in the cavern shape and in the initial and boundary conditions of the cavern fluids. Based on these constraints, the local similarity approach to natural convection was chosen. A number of difficulties were encountered in this application which are discussed below. Even with these problems, the local similarity approach has the best combination of speed and accuracy for the present application (Webb 1988).

Conservation of Energy. Local similarity generally depends only on the local conditions; no upstream history is used. The local similarity boundary layer equations for a vertical wall are:

$$f''' + (n+3)\, f\, f'' - 2\,(n+1)\, f'^{2} + \theta = 0 \quad (1)$$

$$\frac{\theta''}{Pr} + (n+3)\, f\, \theta' - 4\, n\, f'\, \theta - J\, f' = 0 \quad (2)$$

where n and J are similarity parameters, and f' and θ are similarity variables related to the local velocity and temperature values. In general, the local similarity method does not conserve momentum or energy as the boundary layer develops.

In a closed system such as an SPR cavern, energy conservation is critical to evaluate heat transfer between the fluid and its surroundings. To explicitly conserve energy, the local similarity method was modified resulting in the Modified Local Similarity (MLS) approach (Webb 1989a). The following relationships must be satisfied for energy conservation

$$n = \frac{1}{5}\, [4\, q'' / q''_{avg} - 3] \; . \quad (3)$$

Thus, the similarity parameter n is just a function of the ratio of the local to the average heat flux up to that point. The parameter J is determined by satisfying the following equation using the calculated boundary layer profiles

$$\dot{m}\; c_p\, (\bar{T} - T_f^{*}) = \frac{4}{(5n+3)}\, q''\, x \quad (4)$$

where T_f^* is the local fluid temperature. The MLS method has been tested for variable wall conditions and temperature stratification. The results are comparable or better than those obtained with the traditional local similarity method (Webb 1989a).

Turbulence. Turbulence is modeled through the eddy viscosity formulation. A zero-equation eddy viscosity model is appropriate for a local similarity boundary layer analysis. Cebeci and Khattab (1975) developed the most popular eddy viscosity model for natural convection, while Noto and Matsumoto (1975) present a local similarity model for natural convection. Other models are discussed briefly by Webb (1990). In the present study, the Cebeci and Khattab model is modified for local similarity. The Noto and Matsumoto approach seriously overestimates the velocity in the outer region of the boundary layer due to conservation of momentum problems as discussed by Webb (1990).

The Cebeci and Khattab model eddy viscosity is

$$\nu_t = l^2 \left| \frac{\partial u}{\partial y} \right| \quad (5)$$

$$l_i = 0.4y\, (1 - \exp(-y/A)) \quad \text{(inner region)} \quad (6)$$

$$l_o = 0.075\, \delta \quad \text{(outer region)} \quad (7)$$

where l is the minimum of l_i and l_o. In order to apply the model to local similarity, an expression for the boundary layer thickness is needed. According to George and Capp (1979), the thickness of the velocity boundary layer scales with the displacement boundary layer thickness, δ^*, or

$$\delta^* = \int_0^{\infty} \frac{u}{u_{max}}\, dy . \quad (8)$$

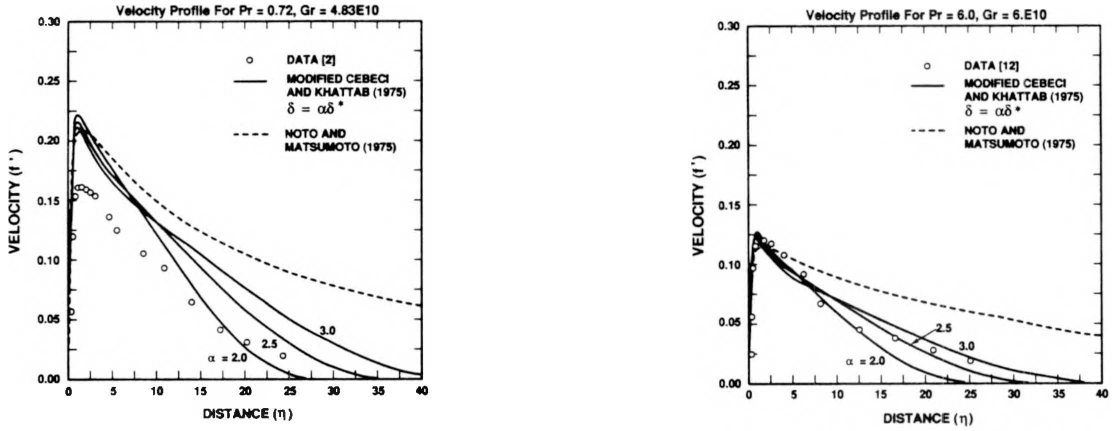
Based on this scaling, data-model comparisons were performed for air and water on vertical flat plates, and the boundary layer thickness is

$$\delta = 2.5\, \delta^* . \quad (9)$$

This relationships allows the use of the Cebeci and Khattab model in a local similarity mode. Some results are shown in Figure 2 for air and water. For air, reasonable agreement is achieved for the data of Cheesewright and Ierokiopitis (1982). For water, the model predicts the Vliet and Liu (1969) data well. In all cases, the Noto and Matsumoto model substantially overpredicts the data. Overall, the present model gives reasonable results as discussed in more detail by Webb (1990).

Mixed Convection. Mixed convection occurs in the boundary layer due to the upward boundary layer flow and the downward central core velocity. Most of the effort in mixed convection has been focused on the effect of buoyancy with forced convection dominated conditions. Mixed convection dominated by buoyancy, which is encountered in this application, has not been widely studied.

The relative strength of the buoyancy to forced convection contribution is measured by the ratio of the Grashof number to the Reynolds number squared. This ratio can be expressed in terms of the natural convection similarity variable f'_∞ as



Air (a) Cheesewright and Ierokiopitis (1982) Water (b) Vliet and Liu (1969)

Figure 2 Turbulence Data-Model Comparisons

$$\xi = \frac{Gr_x}{Re_x^2} = Gr_x \frac{\nu^2}{u_\infty^2 x^2} = \frac{1}{4 f'^2_\infty}. \quad (10)$$

Thus, the forced flow effect can be equivalently represented by either ξ or f'_∞ if the natural convection similarity variables are used. Note that the data of Hess and Miller (1979) as presented later have ξ values of ~ 1500. Typical values for SPR are in the range of 100 to 10,000.

Local similarity has never been applied to mixed convection problems using the natural convection local similarity equations since, in the limit as $\eta \to \infty$, the non-zero velocity boundary condition cannot be satisfied (see equations (1) and (2)). To handle this boundary condition, the following ad hoc modifications have been made

$$f''' + (n+3) f f'' - 2(n+1) \left[f'^2 - \Lambda f'^2_\infty \right] + \theta = 0 \quad (11)$$

$$\frac{\theta''}{Pr} + (n+3)\, f \theta' - 4n\, f' \theta - J\, (f' - f'_\infty) = 0 \quad (12)$$

where Λ is the sign of the local product of $f' f'_\infty$. The value of Λ is 1. if the two velocities are in the same direction and is -1. if they are not.

The above ad hoc model must be compared to experimental data to ascertain its usefulness. The comparison should be made to laminar and turbulent opposed mixed convection velocity and temperature profiles with flow reversal dominated by buoyancy. Unfortunately, such data are not available. The most applicable data are those of Ramachandran, et al. (1985) for laminar buoyancy dominated assisted mixed convection. The data-model comparison is shown in Figure 3 using natural convection similarity variables. A higher value of ξ indicates a larger buoyancy contribution. The velocity and temperature profiles are surprisingly well predicted considering the simple ad hoc modifications.

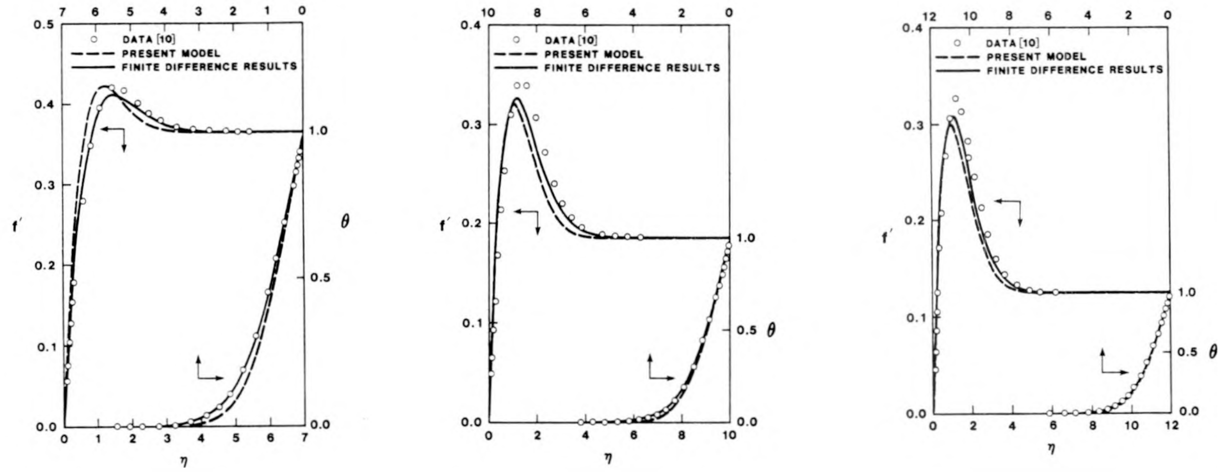
For turbulent mixed convection, the zero-equation eddy viscosity approach has been used by a number of investigators. All of these turbulence models are similar to the model developed by Cebeci and Khattab (1975) for pure natural convection flow. Oosthuizen (1974) proposed the only model for turbulent mixed convection flow for buoyancy dominated conditions which includes a variable to account for buoyancy effects. However, due to lack of data, no systematic attempt was made to determine the behavior of the variable. Velocity and temperature data for turbulent mixed convection conditions dominated by buoyancy are not available. In the absence of applicable data and models, the current turbulence model with the ad hoc mixed convection modifications will be employed.

For application to mixed convection, the following displacement boundary layer thickness, δ^*, definition has been used

$$\delta^* = \int_0^{y(u=0.)} \frac{u}{u_{max}}\, dy. \quad (13)$$

A number of questions remain to be answered about the applicability of the turbulent natural convection boundary layer thickness model to mixed convection. The scaling and the constant of 2.5 may not be appropriate for mixed convection conditions. However, since the application of the present model is to natural convection dominated conditions, the above boundary layer thickness model should be reasonable. At the present time, no data are available to evaluate these questions.

Numerical Solution. Calculation of the local similarity boundary layer profiles usually involves shooting methods. For laminar flow, such methods work well. However, for turbulent flow, shooting methods can and often do fail to converge. In order to overcome this difficulty, the finite difference Box method of Keller (1971) has been employed with variable mesh point spacing. This method is discussed further by Webb (1989b).



(a) ξ = 1.884　　(b) ξ = 7.324　　(c) ξ = 16.021

Figure 3 Mixed Convection Data-Model Comparison

With the above modifications, the local similarity method can be used to analyze the boundary layer region in a cylindrical cavity. The full local similarity equation set including turbulence and mixed convection effects is

$$(1+\frac{\nu_t}{\nu} f'')' + (n+3)ff'' - 2(n+1)\left[f'^2 - \Lambda f_\infty'^2\right] + \theta = 0 \quad (14)$$

$$((\frac{1}{Pr} + \frac{\nu_t}{\nu Pr_t})\theta')' + (n+3)f\theta' - 4nf'\theta - J(f'-f'_\infty) = 0 \quad (15)$$

using the modified turbulence model of Cebeci and Khattab (1975) as discussed above.

2.2 Central Core Region

Based upon the experimental data, the radial temperature and velocity gradients in the central core region are negligible, while significant vertical temperature gradients may exist. Therefore, the central core region is simply broken up into a number of vertically stacked control volumes. Due to the upward boundary layer flow, the flow direction is downward in the central core region. The velocity calculated in the central core region is used as the f'_∞ value in the boundary layer calculations. Heat and mass transfer between the central core and boundary layer region are based on boundary layer calculations.

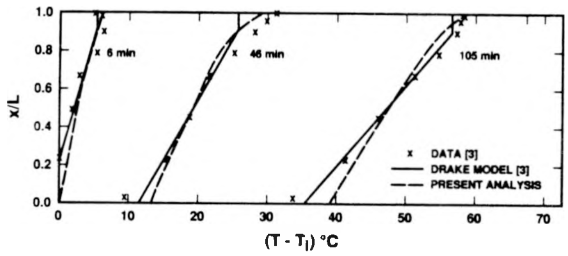
3. MODEL VALIDATION

The results presented in this section are based on boundary layer calculations performed at specific values of the Prandtl number, the similarity parameters n and J, the Grashof number, and the free stream velocity f'_∞. Multivariate interpolation is used to evaluate other conditions. In this scheme, only a limited number of boundary layer parameters are obtained such as the total mass flow rate, the boundary layer thickness, and the maximum boundary layer velocity; actual profiles are not calculated at each location. The central core velocities and temperatures are based on the boundary layer results as discussed earlier. The above approach yields rapid though approximate results which can be compared to data.

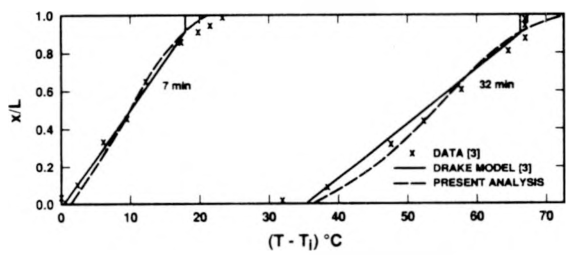
3.1 Drake Data

The Drake data were obtained for an initially isothermal fluid in a vertical cylinder with a constant wall heat flux. Temperature data in the central core region were taken at various times after the start of the test. Two tests for water were chosen for this comparison with different heat fluxes and aspect ratios. According to the criteria developed by Drake, both tests start out in the turbulent regime. At the end of the tests, the lower heat flux test is predicted to be laminar, while the higher heat flux test is in the transition region between laminar and turbulent flow.

The heat fluxes used are based on the average bulk temperature of the water and the time of the measurement considering conservation of energy. Figure 4 compares the temperature stratification predicted by the present model with the data and with the results from the Drake model. For the lower heat flux test, the model results compare reasonably well with the data except for the bottom portion of the test section. This discrepancy may be due to the heat losses to the bottom flange. According to Drake, large losses occurred to the bottom flange of the test section, and the temperature data in the bottom 10% of the test section were not used in Drake's model development effort. For the higher heat flux case, the agreement is slightly better than for the lower heat flux, possibly due to lower fraction of total heat input lost through the bottom flange. Overall, the results from the present model compare favorably with the data and the Drake model.



(a) L/D = 3, q" = 1450 W/m² (nominal)



(b) L/D = 2, q" = 5200 W/m² (nominal)

Figure 4 Drake Data-Model Results

3.2 Hess and Miller Data

Hess and Miller give fluid velocity data for the same geometry as Drake for three different heat flux values. The uncertainty in the velocities is estimated to be 10%, although errors of 10 to 15% in the upward and downward mass flows are noted. Velocities after a 30-minute transient are compared to the data using nominal heat flux values.

Velocity profile information predicted by the present model at the center of the cavity are compared to the data in Figure 5. The peak boundary layer velocity is underpredicted by about 15%, while the edge of the boundary layer region profile is well predicted. The central core region velocity predictions agree well with the data. Additional results are given by Webb (1988).

The data used in the above comparisons are limited in scope. The temperature data of Drake encompass laminar and turbulent conditions, although the maximum Rayleigh number is only about 10^{10}, well below the maximum value of 10^{16} expected for SPR caverns. The range of velocity data is even more limited. Hess and Miller obtained laminar velocity profiles; turbulent velocity profiles are not available for a vertical cylinder. Additional data for highly turbulent conditions with more general heating patterns (top, bottom, and sides) are needed to completely evaluate the present model for use in an SPR cavern. While the present model adequately predicts the available velocity and temperature data, its accuracy for SPR caverns cannot be fully assessed until more appropriate data become available.

All the calculations given in this paper were run on a VAX 8650 machine in single precision.

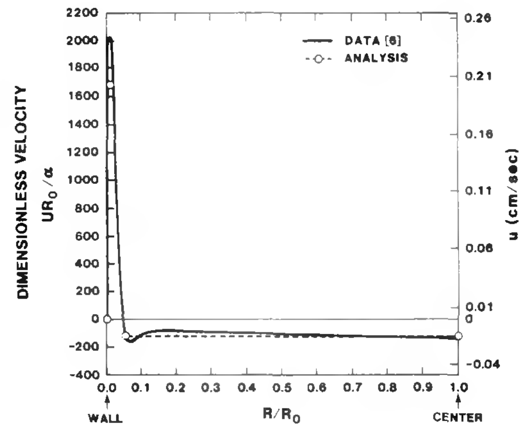


Figure 5 Hess and Miller Data-Model Comparison

Twenty control volumes were used, and the time step was restricted to 0.75 times the mass Courant limit for any control volume. The computer runs had warp factors (ratio of problem time to CPU time) of 28 to 35. Scoping calculations have also been run for an SPR cavern with five control volumes including transient 2-D heat conduction in the salt region. Heat conduction and convection were calculated for 5 years. The run took about 0.5 CPU hours for a warp factor approaching 10^5. More control volumes will be needed for the final calculations, but the above numbers indicate that the procedure is fast running as required for SPR cavern analysis.

4. SUMMARY AND CONCLUSIONS

A natural convection model has been developed for a vertical cylinder considering the constraints of a fast running method for a long term transient (~30 years) with reasonable accuracy for eventual application to SPR. The model consists of boundary layer and central core regions. The boundary layer behavior is analyzed by the local similarity method modified to account for conservation of energy, turbulence, and mixed convection. A plug flow model is used in the central core region.

The simple model compares favorably to the limited temperature and velocity data available for water in a cylindrical enclosure with uniform sidewall heating. Based on the accuracy and speed objectives, the present approach is feasible for the analysis of SPR caverns and is currently being used. However, the accuracy of this model to oil under highly turbulent conditions with a more complicated heating pattern as encountered in SPR caverns is uncertain until applicable data are available.

5. NOMENCLATURE

A	mixing length parameter
c_p	specific heat
f	natural convection similarity variable
f'	velocity similarity variable = $ux / 2\nu Gr_x^{1/2}$
Gr	Grashof number
J	stratification similarity parameter

l	mixing length
L/D	aspect ratio (height/diameter) of cylinder
$\dot{m}$	mass flow rate
n	temperature difference similarity parameter
Pr	Prandtl number
q"	heat flux
Re	Reynolds number
T	temperature
u	x-direction velocity
x	distance along plate
y	distance normal to plate

Greek

δ	boundary layer thickness
δ^*	displacement boundary layer thickness
η	similarity coordinate $= (y/x)\,(Gr_x/4)^{1/4}$
θ	dimensionless temperature
Λ	mixed convection parameter
ν	kinematic viscosity
ξ	mixed convection parameter

Superscript

′	derivative with respect to η

Subscripts

∞	at edge of the boundary layer
avg	average value
f	fluid
i	inner
max	maximum
o	outer
t	turbulent
x	based on x

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