

## **Pool Daily Fuel Scheduling Volume 1: Technical Manual**

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Research Project 1048-5**

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## ABSTRACT

This report presents the results and efforts of research and development of methods for daily fuel scheduling performed under EPRI Project RP 1048-5 by Power Technologies, Inc. (PTI). The report is in three volumes: (1) Technical Manual, (2) Programming Manual and (3) Program Listings. Daily fuel scheduling involves the scheduling and dispatching of generating facilities to meet all system loads and operating requirements for periods ranging from a day to a week.

Daily fuel scheduling and computer requirements are defined. The scheduling problem is formulated as a mixed-integer linear programming (MILP) optimization problem in which the total system operating cost is minimized. A potentially practical scheduling procedure, based on a combination of search and MILP approaches, was proposed; these two approaches were investigated, coded in FORTRAN and tested individually. Tests using the New York Power Pool system show that the search approach may produce potential savings for fuel scheduling approaches. Additional efforts are needed to make the MILP approach practical. Finally, a number of special scheduling problems have been identified and recommended for future work.



## EPRI PERSPECTIVE

### PROJECT DESCRIPTION

In recent years, electric utilities have given careful attention to scheduling the use of fuel at each of their power plants. This was caused by the rapid rise in fuel costs and the uncertainties in fuel availability. Of particular importance is the daily scheduling of fuel use, since proper scheduling can result in substantial cost savings.

Prior to the issuing of the EPRI request for proposal for this project, Power Technologies, Inc., (PTI) had been developing a thermal unit commitment digital computer program for the New York Power Pool (NYPP). The PTI proposal to EPRI recommended expanding the PTI-NYPP work to include:

1. The scheduling of nonthermal units such as run-of-the-river hydro, ponded hydro, and pumped storage
2. An advanced optimization method called mixed integer linear programming (MILP) coupled with sparse matrix techniques

### PROJECT OBJECTIVES

This project under RP1048-5 had two objectives:

1. To complete the development of the daily fuel scheduling method for use at NYPP, which included the dynamic programming method for thermal unit commitment and the incremental search method for scheduling of hydro and pumped storage facilities
2. To expand the above method using the MILP and sparse matrix techniques

## PROJECT RESULTS

The work on this project did not produce all the results expected. Both of the computer programs that were developed were intended for daily use at a group of utilities that were organized into a power pool such as the NYPP. The dynamic programming and incremental search method (completed as part of this project) does show some promise for daily use in power pool fuel scheduling and may produce schedules with some cost savings. Feasible economic schedules for fuel use can be obtained for a power pool within acceptable computing time requirements. However, there is no assurance of finding the optimal schedule; that was why research on the second objective was performed.

The augmented method (including MILP and sparse matrix techniques) was able to find the optimum schedule for a very small number of generators. The method as presently programmed is not suitable for use by a power pool or even for use by large utilities. Further development of this approach is required to reduce the large number of integer variables required in finding the optimum schedule of fuel use. PTI found that when the number of integer variables is large, the computing time becomes extremely long and the data storage requirements become unreasonably high.

The computer program for the dynamic programming and the incremental search will be available through the Electric Power Software Center. The MILP computer program will not be available from the Center.

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## SUMMARY

This report presents the results and efforts of research and development of methods for daily fuel scheduling performed under EPRI Project RP1048-5 by Power Technologies, Inc. (PTI). The report is in three volumes: (1) Technical Manual, (2) Programming Manual and (3) Program Listings. Daily fuel scheduling involves the scheduling and dispatching of generating facilities to meet all system loads for periods ranging from a day to a week. There is a need for practical scheduling methods in view of the potential possible savings and improvements in fuel usage as presented in Section 1.

Daily fuel scheduling requirements are defined in Section 2. The scheduling problem is formulated as a mixed-integer linear programming (MILP) optimization problem in which the total system operating cost is minimized. However, a pure MILP solution is not practical since it requires extremely long computing time even with present day computers. A potentially practical scheduling procedure, based on a combination of search and MILP approaches, was proposed; these approaches were investigated, coded in FORTRAN and tested individually. Section 3 gives a description of this procedure and the computing requirements. Information and listings on the coded computer programs are contained in Volumes 2 and 3. These programs are used for the evaluation of the scheduling methods and are not designed for production run purposes.

As presented in Section 4, the search approach divides the scheduling problem into a number of sub-problems and uses dynamic programming and incremental search techniques to determine economic feasible schedules. Each sub-problem

represents the scheduling of a specific category of generating facilities such as thermal units with no fuel limits, pumped storage plants, conventional hydro plants and fuel limited thermal units. Also included in Section 4 are the tests using data on the New York Power Pool (NYPP) system which show that this search approach may produce potential savings for fuel scheduling applications. The NYPP system is considered as a large system with over one hundred thermal units, two pumped storage plants and a number of conventional hydro plants. However, there is no assurance of finding the optimal or near optimal schedules using the search approach alone.

The MILP approach can be used to improve the schedule obtained by the search method. Results from tests using small sample systems show that computing time for the MILP approach can be greatly reduced by taking advantage of certain special structures of the scheduling problem. These results are contained in Section 5. However, additional efforts are needed to make the MILP approach practical.

In Section 6, a number of special scheduling problems have been identified and defined, and some possible solutions have been proposed. Conclusions and recommendations for future work are presented in Section 7. All such future efforts will contribute towards the development of more efficient and effective practical scheduling methods and better representations for various aspects of the daily fuel scheduling problem.

## Section 1

### INTRODUCTION AND SUMMARY

#### 1.1 BACKGROUND

Most utility systems have grown in size and costs of fuels have increased to the point where significant savings in operating costs can be obtained with even a fraction of a percent improvements in operating efficiency. This situation together with the increasing uncertainties in the availability of energy resources and awareness of consumer groups and regulatory bodies have given rise to an urgency for the electric utility systems to operate as efficiently as possible. Of increasing importance in the electric utility system operations is the daily fuel scheduling problem. The term "fuel" is being used to include all energy resources: fossil, nuclear, geothermal steam, hydro and others.

Daily fuel scheduling involves the scheduling and dispatching of all generating facilities including thermal units, hydro and pumped storage plants, and various transaction contracts to meet all system loads. Scheduling period of interest ranges from one day to one week and is normally divided into hourly intervals so that generation is scheduled hour by hour. Daily fuel scheduling is complicated by a variety of operating requirements, varying load demands and differing characteristics of different generating units and plants.

Due to the complexity and size of the scheduling problem, early scheduling methods are based on certain rules-of-thumb which have evolved through years of experience. In general, most of these methods place more emphasis on satisfying load demands and various operating requirements or constraints. Optimization of

daily fuel schedules has not been given wide application.

With the advancement in computing capabilities (both computer memory size and speed) and applications of sophisticated computational techniques, some degrees of optimization are being done in daily fuel scheduling. In most cases, attempts are made to minimize the total system fuel costs while in some instances the fuel use efficiency is maximized.

The size and complexity of daily fuel scheduling problem have resulted in dividing or decoupling the problem into a number of sub-problems so that practical solution methods can be implemented. Generally, the division is determined by the limitation of fuel or energy resources. In particular, thermal generating units with no constraints on fuel use are scheduled and dispatched together, this is widely known as the unit commitment problem. On the other hand, energy limited generating facilities such as hydro and pumped storage plants are scheduled separately.

A variety of priority list techniques coupled with specially programmed logic are widely used in the unit commitment area. Dynamic programming approach is being applied to optimize daily fuel scheduling but this technique is not widely used as yet. Some form of gradient or search methods have been widely applied to the scheduling of hydro and pumped storage plants. Other optimization techniques such as linear and mixed-integer programming have been proposed for daily fuel scheduling purposes. However, their applications to date have been very limited and in those instances where they have been implemented, the sizes of the systems being scheduled are small.

Each of the scheduling methods has its merits and weaknesses in terms of system modeling capability, assurance of finding optimal and near optimal schedules and

computing requirements. There is a need to develop methods which are capable of handling the general daily fuel scheduling problem as a single entity, representing all system conditions as accurately as possible, and determining acceptable economic feasible schedules within reasonable computing time.

## 1.2 PROJECT OBJECTIVES

The major objective of the project was to develop an efficient and general technique for daily fuel scheduling purposes for a power pool. That is, the technique should be applicable to most systems having predominately thermal generation capability and some energy limited generating facilities, should require reasonable computing effort, and should determine schedules which are near optimal if not optimal. Systems with extensive and complex hydro systems generally require special and specific methods so that scheduling of these systems was outside the scope of the project.

The initial task was to define precisely the requirements for daily fuel scheduling and formulate the scheduling as an optimization problem in which a cost function is to be minimized. Reasonable computer and program requirements were also defined since they will influence the choice of scheduling methods.

When the methods with the potential of successful applications had been selected, prototype computer programs were coded for the testing and evaluation on the performance of the methods. These computer programs are not intended for production run purposes; to do so will require additional programming efforts. To provide meaningful and realistic results, a typical large system, the New York Power Pool system, was used for testing purposes.

### 1.3 PARTICIPATION BY NEW YORK POWER POOL

The New York Power Pool (NYPP) consists of the majority of the electric utility systems within the state of New York. It has a large number of thermal units (oil, coal and nuclear), combustion gas turbines, hydro plants and pumped storage plants. Its present annual peak load is about 20,000 MW and its weekly fuel costs are in the order of ten's of million dollars.

The operating staff at NYPP have extensive experience in daily fuel scheduling and are familiar with the scheduling problem. The participation of the NYPP has been invaluable in providing direct input as to the needs and requirements of the electric utility industry. Their roles were:

- To help in the definition of the scheduling problem and computer program requirements.
- To provide guidance as to the needs and applications in daily fuel scheduling.
- To supply typical test data based on the New York system.

### 1.4 ACCOMPLISHMENTS

The accomplishments of the project represent one step towards achieving the objective of developing efficient and practical methods for daily fuel scheduling. As with most research and development projects, budget constraints did not permit further investigation and development beyond the original scope of work. Nevertheless, specific project accomplishments are as follows:

- Daily fuel scheduling problem and computer requirements have been defined.
- Daily fuel scheduling has been formulated as an MILP optimization problem in which the total system operating cost is minimized.

- Scheduling procedure, based on search and MILP approaches, was investigated, coded in FORTRAN, and tested. The programs are prototype for evaluation purposes only and not for production runs.
- The search approach was tested using data on the New York State system. Results show that the search approach has potential application for large systems with present day computing technology. Significant savings can be obtained though the search approach does not assure the finding of optimal schedules.
- Tests on the MILP approach were conducted using simple test systems. Results demonstrated the significance in computing efficiency improvements when certain unique structures of the scheduling problem are incorporated. However, the MILP technique at the present state of development still requires excessive computing time and is yet to be practical for scheduling purposes.
- Several scheduling problems and modeling requirements have been identified. Also included are areas of the MILP approach requiring further work which will reduce the computing time.

## Section 2

### SCHEDULING PROBLEM AND REQUIREMENTS

This section discusses the daily fuel scheduling problem and its objectives. Scheduling requirements are presented and they are related to the scheduling constraints which are dictated by system operation, equipment limitations, availability of energy resources and environmental restrictions. Finally, daily fuel scheduling is formulated as a mathematical optimization problem in which the total system cost is minimized.

Daily fuel scheduling determines the hour by hour generation of all generating facilities such that all system loads, and the reserve and other operating requirements are satisfied. Generating facilities include thermal units, both fossil fired and nuclear, gas turbines, conventional hydro plants, pumped storage plants and external contracts. The scheduling period generally ranges from a day to a week.

#### 2.1 SCHEDULING OBJECTIVE

With the increasing cost of fuel, there is greater incentive to operate the electric power systems more efficiently. A reduction of a fraction of a percent of the system operating cost could produce significant savings [1] since fuel cost represents a major portion of the operating cost. The present fuel cost to operate the New York system over a typical week with a peak load of 20,000 MW is in the neighborhood of fifty million dollars. System operating cost also includes maintenance charges which are dependent on the degree of unit operation and other labor charges such as requirement for additional shift of operators.



Because of the complexity and size of the daily fuel scheduling problem, the scheduling is not normally optimized in actual operation.

In view of the potential savings, the major scheduling objective is to satisfy all load demands and operating requirements at minimum cost. Determining the optimum schedule for a large system is an enormous, if not impossible, task even with the availability of advanced modern computing facilities. Nevertheless, efforts are continually being made to develop computational methods which will lead us closer to achieving this particular objective of minimizing total cost in daily fuel scheduling. This project is an example of such efforts.

From the standpoint of some private utilities, a more desirable objective is to maximize profit. This is particularly attractive when these utilities have a number of sale and purchase contracts with other systems or large industrial customers. However, such situations are few and most regulatory bodies generally are not receptive to such practice by the utilities. Nevertheless, for most systems, minimizing the operating cost is equivalent to maximizing profit. Hence, this objective was not considered further in this project.

In the event of scarce fuel supply, such as an oil embargo, a coal strike and a rail strike, it may become more desirable to meet all the loads with minimum fuel. That is, the conservation of fuel takes precedent over the economics of operation. Since the cost of fuel is obtained by the product of fuel use and fuel price, any method used to minimize total cost can be used to minimize fuel use by eliminating all non-fuel related costs and fuel prices.

Finally, under an emergency situation in which there is inadequate fuel to meet all load demands, then a reasonable objective is to schedule the system such that load curtailments are minimized. This particular problem is not addressed

in this project. However, there has been significant amount of work on the minimization of load curtailments in the area of power system security control and evaluation.

## 2.2 CONSTRAINTS IN DAILY FUEL SCHEDULING

Daily fuel scheduling involves the operation of different generating facilities such that all operating requirements or constraints are satisfied [2,3]. It is these constraints which have made daily fuel scheduling a difficult and complex problem. The constraints may be divided into three broad categories: system constraints, facility constraints and fuel constraints.

### 2.2.1 System Constraints

The utmost important system constraint is the hourly system loads within the scheduling period. All attempts must be made to satisfy all loads within the system at all times. Since electricity is produced on a demand basis, the total generated power must be equal to the total system load and losses. Most system loads vary from hour to hour and exhibit fairly regular daily and weekly patterns.

To minimize the curtailments of loads due to the possibility of equipment (generating units, transformers and transmission lines) outages and fluctuation of loads, systems are required to carry adequate spinning and quick start reserve capabilities all the time. Commonly adopted reserve categories are 10-minute spinning or synchronous reserve, 10-minute non-synchronous reserve which includes capacities of quick start facilities such as diesel and gas turbine units, and 30-minute operating reserve. A reserve capability is always tied in with a time specification and it is the actual capacity which can be called upon within the given timeframe. Furthermore, reserve capabilities are to be distributed to different parts of the system so that the system can re-

spond to loss of capacity more effectively. Practices in allocating the amount and the distribution of reserve requirements vary from utility to utility and from pool to pool.

There exist situations in which the transmission network of a system imposes certain limits in the transfers of power between different parts of the system. Such constraints should be recognized in scheduling the level of generation by the various facilities so that overloading of transmission lines will be avoided.

Environmental requirements could also constrain the extent within which a system can be operated. For example, the atmospheric condition may be such that certain emissions be kept to a minimum which in turn limits the generation levels of certain plants. Another example is the prohibition of starting coal fired plants near urban areas during daylight hours because of soot and other particulates. This project does not address the effects of environmental constraints on daily fuel scheduling. However, many such constraints can be reformulated as limits on the generation level or fuel use for the affected generating plants.

#### 2.2.2 Facility and Operating Constraints

The design and the construction of the various generating facilities (thermal units, hydro plants, pumped storage plants and others) impose a variety of constraints on the scheduling of these facilities to meet the system load. Each type of generating plant or unit has its characteristics and special operating requirements for minimizing the outage or downtime of the facility.

All generating units are designed to operate within a minimum and a maximum capacity limits. Thermal units invariably have minimum up time and minimum down time requirements as specified by the manufacturers. These two constraints min-

imize any undue thermal stresses which can be detrimental to the unit. Furthermore, they prevent the unit from cycling too frequently, otherwise its useful life span may be shortened. The rate of change in loading of some thermal units, also known as response or ramping rate, could be relatively low so that it is not possible to move its generation from one level to another different level within the normally used scheduling interval of an hour. For such units, the response rates become important in scheduling their generation levels from hour to hour.

Certain plants may have limited crew size which prohibit the simultaneous starting up and/or shutting down of two or more units in the same plant. Such constraints could be specified by the times required to bring a unit on-line and to shut down a unit.

For those hydro plants located on river systems which are also used for other purposes, like recreation, irrigation and navigation, there are specific requirements on the rate and the amount of discharge through the plants. As far as daily fuel scheduling is concerned, constraints due to other than for power generation will be treated as specifications for maintaining minimum generation, and as additional limits on the rate of discharge and plant capacities.

The units in pumped storage plants are normally designed for best pumping efficiency at the rated capacity so that pumping is frequently done at full load. Another reason for full load pumping is that partial load pumping could produce undesirable mechanical vibrations.

Operation of hydro and pumped storage plants are also constrained by the finite storage capacity of the pond or reservoir. This directly influences the degree of freedom in the release of the stored water or "fuel."

### 2.2.3 Fuel Constraints

Hydro plants are examples of generating facilities with fuel constraints, the fuel is the available water. The schedule of the hydro plants are thus dependent not only on the total inflow over the scheduling period but also on the rate of the inflow. As discussed in the previous section, the capacity of the storage pond also affects the schedule.

Similar situations also occur for some thermal units for which the amount of fuel to be used within the scheduling period must fall within minimum and maximum limits. The minimum limit could very well be zero. Recent events such as the oil embargo and coal strike, have shown that conditions with limited available fuel for power generation do occur and the frequency of these conditions may be on the increase in the future.

There are some gas fired plants for which the rate of flow through the pipelines becomes so low that it is impossible to dispatch the generation plants at the rated capabilities. This situation is due to unseasonably cold weather in most instances. The limit on the gas flow rate is analogous to the rate of water inflow into hydro plants. Such problems can be modeled by limiting the generation to the levels corresponding to the gas flow rates.

## 2.3 SCHEDULING PROBLEM FORMULATION

Daily fuel scheduling determines when to start and shutdown generating facilities and how to dispatch the on-line generation such that all load demands and operating requirements are satisfied at minimum cost. The scheduling period ranges from a day to a week and is usually divided into hourly intervals. Since most generating plants have non-zero minimum generating capacity and minimum up and down time of more than an hour, the scheduling problem lends itself into a

integer programming problem in which some of the variables are integers [4,5]. The dispatching of generation entails the use of continuous variables so that both integer and continuous variables will be needed to describe the scheduling problem mathematically.

Most of the generation characteristics, such as fuel rate curves, can be represented by linear or piecewise linear functions. Hence, it is possible to formulate daily fuel scheduling as a mixed-integer linear programming (MILP) problem in which the total system operating cost is minimized. There are a number of advantages to use linear programming (LP) for the non-integer part of the problem [6]:

- LP always finds the optimal feasible solution if the solution exists.
- LP is computationally very efficient.
- LP has been widely used for solving problems with very large number of variables and constraints.
- There are efficient LP program packages on the market.

The MILP formulation given below incorporates most of the constraints encountered in daily fuel scheduling, the majority of these have been discussed in the previous section. It may be pointed out that for a given system, some of the constraints represented could be ignored while new constraints may be needed.

Formulating the daily fuel scheduling problem as an MILP optimization problem does not necessarily imply that an MILP solution technique will be used. However, it is an important initial step in describing the problem mathematically before appropriate solution methods can be applied.

In the formulation, both thermal units and hydro plants are treated the same way recognizing that the inputs are fuel and water for the respective type of generation. Also, generating unit and plant are used interchangeably so that a plant with two or more units will be represented as an entity. Nevertheless, individual units in multiple unit plants can be formulated in exactly the same manner with the use of another set of subscripts to identify these units. Formulation does not include coupled hydro plants since the required equations depend entirely on the configuration of the hydro system. However, given the hydro system configuration and specifications the appropriate representation can be expressed in the same MILP format.

### 2.3.1 Definitions of Variables

$C_{fi}$  = Price of fuel for unit  $i$

$C_{di}$  = Fixed shutdown cost for unit  $i$

$C_{di}^t$  = Shutdown cost for unit  $i$  at hour  $t$

$C_{si}^t$  = Startup cost for unit  $i$  at hour  $t$

$C_{csi}$  = Cold startup cost for unit  $i$

$C_{hsi}$  = Hot startup cost for unit  $i$

$\bar{E}_i$  = Total maximum fuel or water for unit or plant  $i$

$\underline{E}_i$  = Total minimum fuel or water for unit or plant  $i$

$F_{gi}$  = Piecewise linear function on fuel rate versus output for unit or plant  $i$

$I_i^t$  = Rate of inflow at unit or plant  $i$  and at hour  $t$

$K_{ij}$  = Conversion coefficient for the  $j$ -th segment of function  $F_{gi}$

$K_{gi}$  = Power versus discharging coefficient in generating mode for pumped storage plant  $i$

$K_{pi}$  = Power versus discharging coefficient in pumping mode for pumped storage plant  $i$

$K_{sij}$  = j-th cost increment for the startup cost of unit i  
 $M_i^d$  = Minimum down time for unit i  
 $M_i^u$  = Minimum up time for unit i  
 $NPH$  = Number of pumped hydro plants  
 $NS$  = Number of segments in fuel rate function  $F_{gi}$   
 $NU$  = Number of units or plants (thermal and hydro but not pumped hydro)  
 $P_{gi}^t$  = Generating output (MW) of pumped storage plant i at hour t  
 $\bar{P}_i$  = Maximum MW output for unit or plant i  
 $\underline{P}_i$  = Minimum MW output for unit or plant i  
 $P_i^t$  = MW output for unit or plant i at hour t  
 $\bar{P}_{ij}$  = MW size of segment j of unit i fuel rate function  $F_{gi}$   
 $P_{ji}^t$  = MW output from segment j of unit or plant i at hour t  
 $P_L^t$  = Load (MW) at hour t  
 $P_{pi}^t$  = Pumping load (MW) of pumped storage plant i at hour t  
 $\bar{Q}_{gi}$  = Maximum discharge rate in generating mode for pumped storage plant i  
 $\underline{Q}_{gi}$  = Minimum discharge rate in generating mode for pumped storage plant i  
 $Q_{gi}^t$  = Discharge rate during generation of pumped storage plant i at hour t  
 $\bar{Q}_i$  = Maximum fuel or discharge rate for unit or plant i  
 $\underline{Q}_i$  = Minimum fuel or discharge rate for unit or plant i  
 $Q_i^t$  = Fuel or discharge rate for unit or plant i at hour t  
 $Q_{pi}$  = Fixed charging rate in pumping mode for pumped storage plant i  
 $Q_{pi}^t$  = Charging rate in pumping mode for pumped storage plant i at hour t  
 $R^t$  = Reserve requirement in MW at hour t



- $S_i^t$  = Startup status (variable) for unit or plant i at hour t: 1 - startup, 0 - no startup  
 $T$  = Number of hours in the scheduling period  
 $U_i^t$  = Status for unit or plant i at hour t: 1 - on-line, 0 - off-line  
 $U_{gi}^t$  = Generating indicator for pumped storage plant i at hour t: 1 - generating, 0 - not generating  
 $U_{pi}^t$  = Pumping indicator for pumped storage plant i at hour t: 1 - pumping, 0 - not pumping  
 $\bar{V}_i$  = Maximum fuel or water storage at unit or plant i  
 $\underline{V}_i$  = Minimum fuel or water storage at unit or plant i  
 $V_i^0$  = Initial fuel or water in storage at unit or plant i  
 $V_i^t$  = Fuel or water in storage at unit or plant i at hour t  
 $Y_i^t$  = Shutdown status for unit or plant i at hour t: 1 - shutdown, 0 - no shutdown  
 $Z$  = Objective function  
 $Z_i^t$  = Non-zero startup cost slack variable for unit i at hour t

### 2.3.2 Objective Function

The objective of daily fuel scheduling is to meet all system loads and to satisfy all operating requirements and constraints at minimum cost. Hence the objective function is the total system cost which is to be minimized:

$$Z = \sum_{t=1}^T \sum_{i=1}^{NU} [C_{fi} F_{gi} (P_i^t) + C_{si}^t + C_{di}^t] \quad (2.1)$$

By removing the cost parameters,  $C_{fi}$ ,  $C_{si}^t$ , and  $C_{di}^t$ , the objective function becomes the total system fuel usage.

### 2.3.3 Constraint Representations

The constraint equalities and inequalities must be satisfied for each hour t of the scheduling period and for each generating facility i.

### System Load Requirement:

The total generation must be equal to the system load at each hour  $t$ :

$$\sum_{i=1}^{NU} P_i^t + \sum_{i=1}^{NPH} (P_{gi}^t - P_{pi}^t) = P_L^t \quad (2.2)$$

### Unit or Plant Fuel Rate Characteristic:

The fuel rate characteristic of a unit or plant is represented by a convex piecewise linear function, an example is shown in Figure 2-1. In practice, up to seven segments are normally used. Fuel rate could be in BTU or volume of water discharged per unit energy (KWH) generated, depending on the type of generating facility.

$$\begin{aligned} Q_i^t &= F_{gi}(P_i^t) \\ &= U_i^t F_{gi}(P_i) + \sum_{j=1}^{NS} K_{ij} P_{ij}^t \end{aligned} \quad (2.3)$$

$$P_i^t = U_i^t P_i + \sum_{j=1}^{NS} P_{ij}^t \quad (2.4)$$

$$P_{ij}^t \leq \bar{P}_{ij} \quad (2.5)$$

### Operating Limits:

The generation level of a generating unit or plant must be within specified minimum and maximum limits. These limits can be defined by the MW generation or the fuel rate.

$$U_i^t P_i \leq P_i^t \quad (2.6)$$

$$U_i^t \bar{P}_i \geq P_i^t \quad (2.7)$$

or

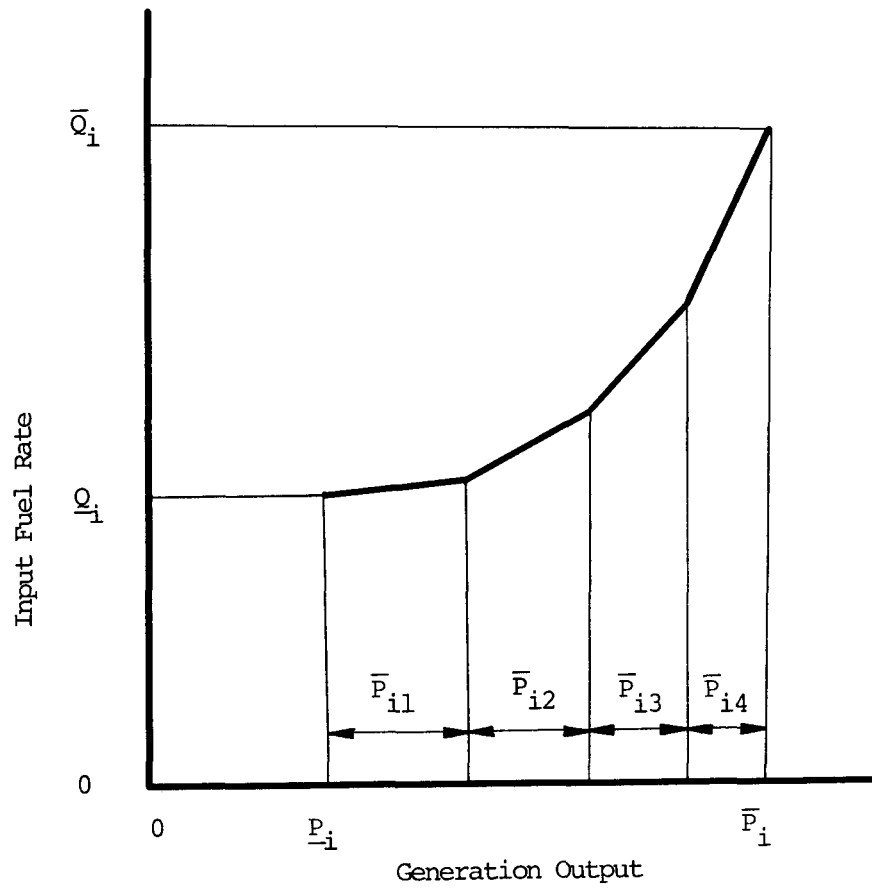


Figure 2-1. Fuel Rate Characteristic of Unit  $i$

$$U_i^t \underline{Q}_i \leq Q_i^t \quad (2.8)$$

$$U_i^t \bar{Q}_i \geq Q_i^t \quad (2.9)$$

#### Energy, Fuel or Water Limits:

Certain unit or plant may have constraints on both the quantity of fuel and the rate fuel use depending on the nature of fuel supply. An example is a conventional hydro plant with weekly cycle. Most thermal plants will not have these sets of constraints since they normally have fuel stockpile lasting more than 60 days. However, there do exist thermal plants for which fuel constraints are active.

$$\sum_{t=1}^T Q_i^t \leq \bar{E}_i \quad (2.10)$$

$$\sum_{t=1}^T Q_i^t \geq \underline{E}_i \quad (2.11)$$

$$V_i^t \leq \bar{V}_i \quad (2.12)$$

$$V_i^t \geq \underline{V}_i \quad (2.13)$$

$$V_i^t = V_i^{(t-1)} + I_i^t - Q_i^t \quad (2.14)$$

#### Startup and Shutdown Costs:

Generally, costs are incurred with the starting of generating units. For thermal units, these starting costs are a function of the unit downtime just prior to startup. Figure 2.2 shows a typical startup cost function and the startup cost at any time  $t$  is given by equation (2.15) in which  $M$  is the time for which cold start cost applies minus the minimum downtime.

$$C_{si}^t = S_i^t C_{csi} - \sum_{j=1}^M U_i^{(t-M_i^d-j)} K_{sij} + Z_i^t \quad (2.15)$$

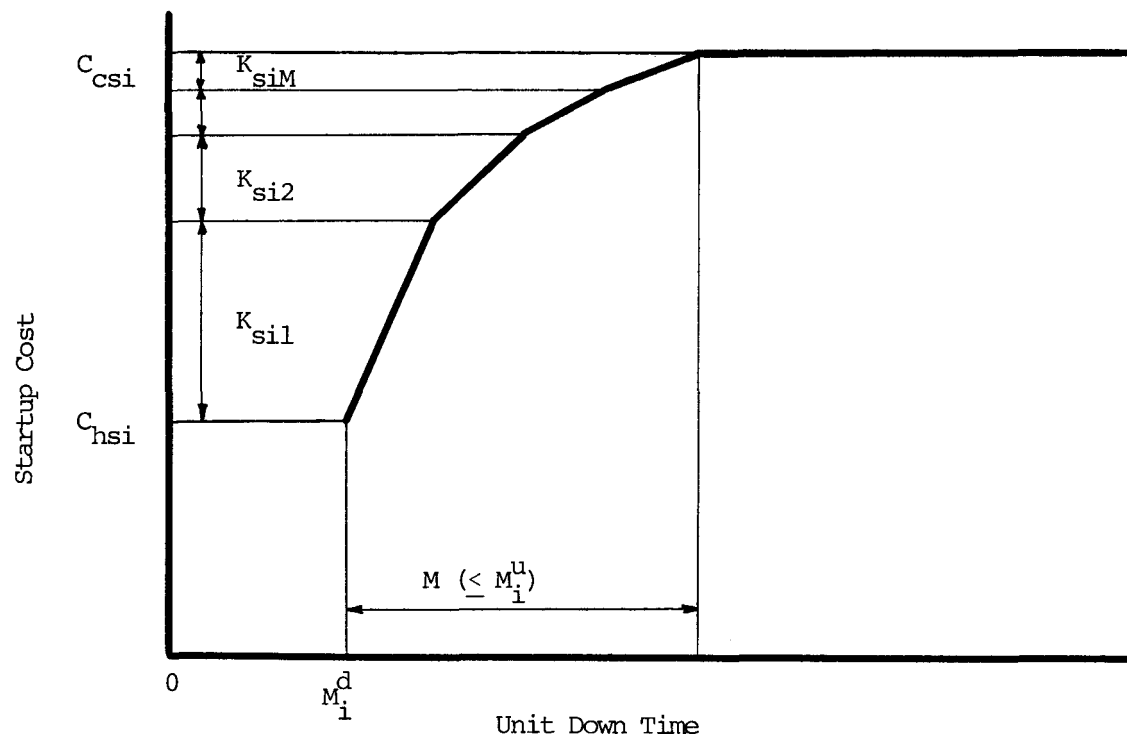


Figure 2-2. Time Dependent Startup Cost Function for Unit  $i$

A fixed cost may be associated with the shutdown of a unit, however, in most instances no shutdown costs are represented.

$$C_{di}^t = Y_i^t C_{di} \quad (2.16)$$

#### Unit Status, Startup and Shutdown Variables:

Integer variables are needed to indicate the unit status (on-line or off-line) and whether a startup or shutdown occurs at a given hour. Both the startup and shutdown variables are used for the minimum up and down time constraint expressions.

$$S_i^t = U_i^t - U_i^{(t-1)} + Y_i^t \quad (2.17)$$

#### Unit Minimum Up and Down Times

Most thermal units are required to be up for a minimum length of time before they can be shutdown. Similarly they must remain down for a minimum length of time before they can be started up again.

$$\sum_{\tau=t}^{t+M_i^u-1} U_i^\tau \geq S_i^t M_i^u \quad (2.18)$$

$$\sum_{\tau=t}^{t+M_i^d-1} (1 - U_i^\tau) \geq Y_i^t M_i^d \quad (2.19)$$

#### Operation of Pumped Storage Plants:

Pumped storage plants are not permitted to operate in both the generating and the pumping modes simultaneously. Generally, pumping is done at full load. In addition to the operating limits, the capacity of the storage reservoir also constrains the operation. Most pumped storage plants do not have natural inflow into the storage reservoir.

$$U_{gi}^t + U_{pi}^t \leq 1 \quad (2.20)$$

$$U_{gi}^t \bar{Q}_{gi} \geq Q_{gi}^t \quad (2.21)$$

$$U_{gi}^t Q_{gi} \leq Q_{gi}^t \quad (2.22)$$

$$P_{gi}^t = K_{gi} Q_{gi}^t \quad (2.23)$$

$$U_{pi}^t Q_{pi} = Q_{pi}^t \quad (2.24)$$

$$P_{pi}^t = K_{pi} Q_{pi}^t \quad (2.25)$$

$$\bar{V}_i \geq V_i^t \quad (2.26)$$

$$\underline{V}_i \leq V_i^t \quad (2.27)$$

$$V_i^t = V_i^{(t-1)} + Q_{pi}^t - Q_{gi}^t \quad (2.28)$$

#### Reserve Requirements:

The following constraint applies to the spinning reserve requirement for each hour and all generating facilities are assumed to be capable of being loaded to their maximum.

$$\sum_{i=1}^{NU} (U_i^t \bar{P}_i - P_i^t) + \sum_{i=1}^{NPH} (U_{gi}^t \bar{P}_{gi} - P_{gi}^t + P_{pi}^t) \geq R^t \quad (2.29)$$

#### 2.3.4 General Observations

Daily fuel scheduling problem as formulated above is actually a special class of MILP problem in that all the integer variables ( $U_i^t$ ,  $U_{gi}^t$ ,  $U_{pi}^t$ ,  $S_i^t$  and  $Y_i^t$ ) take on values of either 0 or 1. There are available special efficient solution techniques for this class of problems and they require much less computational effort than those methods designed for a general MILP problem in which an integer may take on a range of integer values.

Another observation is that the size of the problem is approximately proportional to the product of the number of time steps and the number of units or plants in the system. With the scheduling period of a day to a week and typical systems having 20 to 100 units, the problem size increased 35 (7x5) fold from the smallest to the largest problem.

An examination of the various equalities, and inequalities indicates that there is very limited coupling of variables through time so that the constraint matrix for the problem is extremely sparse. This is a desirable feature in that computer requirements will be much less than a similar problem whose constraint matrix is not sparse. There are available different sparsity techniques for solving various types of problems. Furthermore, there is a specific pattern in the coupling of the integer variables, such as specified by unit minimum up and down times (2.18 and 2.19), unit status, startup and shutdown variables (2.17), so that by taking advantage of this particular structure or pattern, the formulated problem can be computationally simplified.



## Section 3

### SCHEDULING PROCEDURE AND COMPUTING REQUIREMENTS

The previous section presented the daily fuel scheduling problem with a variety of constraints imposed by system requirements and the operation of the different types of generating facilities in a system. This section discusses the next and important step of daily fuel scheduling: to determine economically feasible schedules using practical computing resources and within reasonable time. A brief survey of current scheduling practices is given, followed by the selection of scheduling procedures for evaluation and testing. Finally, this section presents the necessary computing and program requirements for the implementation of any scheduling procedure.

#### 3.1 CURRENT SCHEDULING PRACTICES

Most utility systems have grown in size and costs of fuels have increased to the point where significant savings in operating costs can be obtained with even fractions of a percentage improvements in operating efficiency [1]. This situation has provided an added incentive for the electric utility systems to operate as efficiently as possible.

Daily fuel scheduling is complicated by a variety of requirements and factors which include the following:

- Large number of units or plants
- Differences in unit cost and operating characteristics
- Fuel or energy limitation on certain units
- Variation in cost of fuel for power generation
- Varying power demand within the scheduling time periods
- Operating requirements such as spinning and non-spinning reserves
- Transmission limitations within the system
- Environmental considerations.

Due to the complexity and size of the scheduling problem, early scheduling methods were based on certain rules-of-thumb which have evolved through years of experience. In general, most of these methods placed more emphasis on satisfying the demand and various operating constraints. Optimization of daily fuel schedules has not been given wide application.

With the advancement in computing capabilities and applications of sophisticated computational techniques, some degrees of optimization are done in daily fuel scheduling. A variety of priority list techniques are widely used in the thermal unit commitment area [7-9]. Dynamic programming approach [10] is being applied to optimize daily fuel scheduling but this technique is not widely used as yet [11-14]. A domestic survey [2], conducted in July, 1975 on unit commitment scheduling practice, shows that out of 61 utilities which responded, 55 utilities were using some form of priority list and only four utilities had implemented the dynamic programming method. Since the time of the survey, a number of systems (companies and pools) have also begun the use of unit commitment programs based on dynamic programming.

The size and the complexity of the daily fuel scheduling problem have resulted in dividing the problem into a number of sub-problems [2,9,15,16]. Generally, the division is determined by the limitation of fuel or energy resources. In particular, thermal generating units with no constraints on fuel use are scheduled together while energy limited generating facilities such as conventional hydro plants and pumped storage hydro plants are scheduled separately. Some form of gradient or search methods have been widely applied to the scheduling of energy limited facilities [17-20].

Recently, a domestic power pool implemented a daily fuel scheduling procedure which minimizes the total operating cost. This procedure uses dynamic programming for thermal unit commitment and gradient or search techniques to schedule energy limited plants. A foreign utility is implementing a procedure which uses dynamic programming for both hydro thermal scheduling and unit commitment. It includes a hierarchy of scheduling activities for preparing monthly and weekly energy or fuel schedules, daily power schedules, hydro generation and thermal unit commitment.

Other optimization techniques such as linear programming [21] and mixed-integer programming [4,5,22] have been proposed for daily fuel scheduling purposes. Their applications to date have been very limited, mainly due to long computing times. In those instances where they have been implemented, the size of the systems are small. Nevertheless, they do have potential applications in the area of daily fuel scheduling and there exist certain structures which can be exploited thereby enhancing the applicability of these methods.

### 3.2 SELECTION OF SCHEDULING PROCEDURE

There are a number of factors which must be considered when selecting or choosing practical procedures for the determination of daily fuel schedules. The

schedules obtained must be at least near optimal in that the total system cost is close to minimum. This is to maintain efficient operation of the electric power systems. The rising fuel costs and the increasing awareness by the consumers and regulatory bodies of the need to operate efficiently have added greater urgency and incentive to establish economic schedules.

To warrant meaningful and realistic schedules, the models for generating facilities, load demands, and various operating requirements must be representative of the actual situation. Furthermore, the necessary computing effort to determine optimal or near optimal schedules should not be excessive. Computing effort can be measured in terms of time, computer requirements and computing cost.

It is important that the scheduling procedures should produce consistent results and that they should be acceptable to the operators who will be using them regularly. In most cases, careful and proper implementation will achieve this goal. All too often, sophisticated and elegant scheduling computer programs have been written and most of them are left unused because they are too cumbersome and too difficult to use.

In summary, the following forms a reasonable and acceptable basis for the selection of practical scheduling procedures:

- Realistic models
- Feasible and economic schedules
- Reasonable computer and program requirements
- Consistent results
- Acceptable by operators or users.

It can be seen that the selection of scheduling procedures includes an evaluation on the computer and program requirements, however, discussions on such specific requirements will be given in the next section. Remarks on this matter in the section will be both general and qualitative in nature.

References 2 and 16 provide comprehensive surveys of different scheduling procedures which have been proposed and some of these are still being used. In general, each procedure has certain desirable features and some weaknesses. Each of the procedures has been applied to a particular aspect of the daily fuel scheduling problem at a time, such as hydro scheduling or thermal unit commitment. Even when a system has only thermal units, there is normally no assurance that the schedule obtained will be optimal or even near optimal. Nevertheless, they are still very valuable in establishing reasonable economic schedules automatically and fairly quickly.

The selected scheduling procedure for this project consists of a combination of two types of techniques: the search approach and the MILP approach. The search approach will be used to establish a good feasible schedule while the MILP approach will start with this schedule and make successive improvements to arrive at the optimal one. Figure 3-1 shows the setup of the procedure.

The term search approach is used in a very general and broad sense to cover simple search methods based on incremental cost (or incremental search) to more sophisticated gradient methods and dynamic programming. Dynamic programming is also commonly known as recursive search by virtue of the way in which the search for optimal paths progresses from one stage to another. Gradient methods require that all variables be continuous and hence their applications in scheduling have been mainly restricted to the establishment of hydro schedules. Dynamic programming, on the other hand, is both versatile and flexible in its

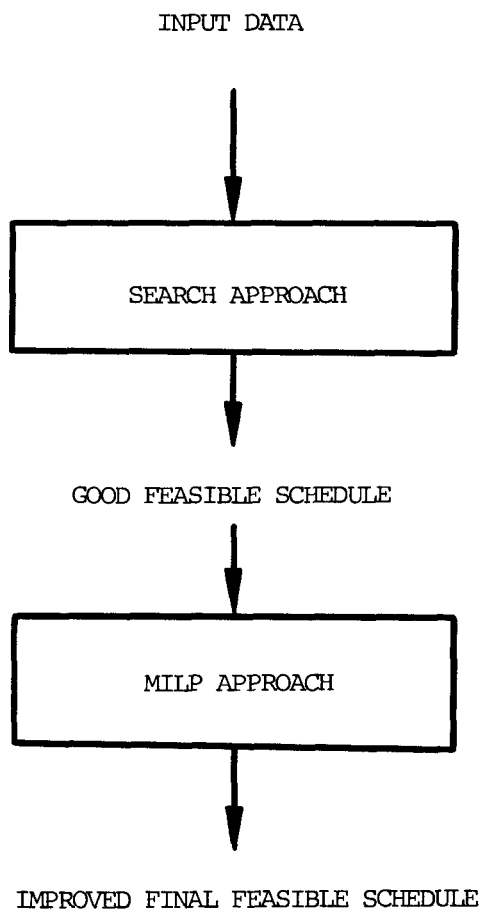


Figure 3-1. Proposed Procedure for Daily Fuel Scheduling

ability to handle both continuous and integer variables, complex constraints, and multi-stage problems. Thermal unit commitment falls into this framework and, therefore, it is not surprising to find that most applications of dynamic programming to scheduling is in the area of thermal unit commitment.

Search techniques have been successfully used in a variety of scheduling problems. But currently, there is no one technique which can solve the daily fuel scheduling problem efficiently. Hence, the search approach, as proposed in this project, uses both incremental search and dynamic programming. The daily fuel scheduling problem will be decoupled into a number of sub-problems which are then solved by the appropriate methods; the incremental search will schedule energy limited plants such as conventional hydro and pumped storage plants and dynamic programming will schedule the thermal units with no fuel restriction.

The overall structure of the search approach is given in Figure 3-2 which shows a main scheduling program with a number of different scheduling modules. Each module is designed to use a specific method which is suited to the particular aspect of the scheduling problem. In this way, the advantages of different methods can be exploited. The main program coordinates the data and schedules from the different modules such that overall system operating requirements are satisfied and economical schedules are determined. There would be a certain amount of iterations among the various modules in order to obtain schedules which are more economical. A tradeoff can be reached between the amount of computing effort and the optimality of the schedule.

The proposed search approach by itself does not assure that the optimal schedule will be obtained. To do so will require further investigation on the coupling between the scheduling of energy limited facilities and thermal unit commitment as represented by the thermal cost curves which do not model any effect due to

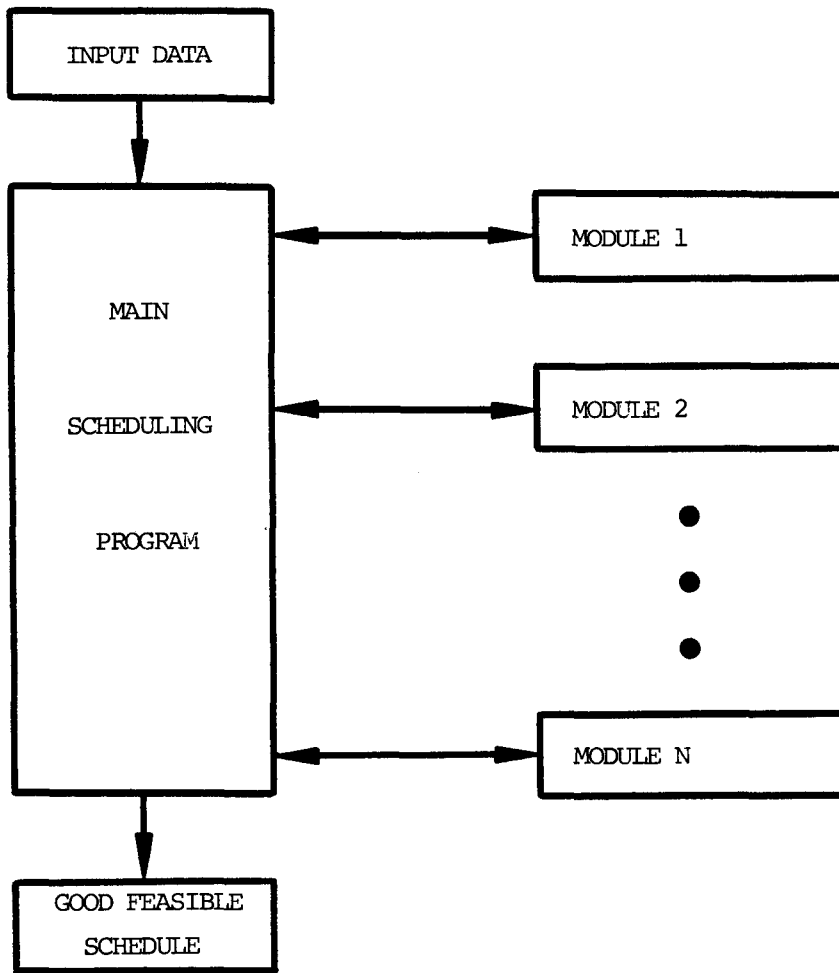


Figure 3-2. Overall Structure of the Search Approach



reserve requirements. This is not the intention of the approach. Its objective is to determine a feasible economical schedule within reasonable computing effort. The task of getting the optimal schedule lies with the MILP approach. With a given feasible economical schedule, one would expect the MILP approach to have greater success in finding the optimal schedule within reasonable time than the case for which there is no feasible starting schedule. When computing time is not a constraint, MILP will find the optimal schedule regardless of any starting schedule.

Integer and mixed integer programming have received much attention in recent years. The objective has been to overcome the "curse of dimensionality" or, more specifically, the "curse of exponentiality," that is, the objective has been to devise algorithms whose solution time does not increase exponentially with the number of problem variables.

In the excellent survey by Geoffrion and Marsten [23], integer programming algorithms were broken down by their method of separation, relaxation and fathoming criteria. The list of algorithms includes enumerative, penalty, heuristic and priority case selection, penalty bounds, surrogate constraint, decomposition, cutting plane, and group theoretic algorithms. The results presented by Geoffrion and Marsten clearly show that some of the above methods do show significant improvement over others for some types of problems.

Similarly the survey written by Rose and Willoughby [24] shows the tremendous advances in the speed of running linear programming (LP) algorithms obtained through the use of sparsity techniques. Again, a good deal of research has been done to overcome the effects of problem size increases through the use of product form and elimination form codes which are most applicable to sparse LP constraint matrices. Advances upon these techniques by Forest and Tomlin [25] and

Hellerman and Rarick [26] show significant reductions in running time for very large LP problems (1,000 to 10,000 rows).

The daily fuel scheduling problem is one which combines integer programming (unit on/off status) with the time optimal dispatching of units which is quite amenable to linear programming. There are certain aspects of the fuel scheduling problem which are unique, and if taken advantage of, will reduce algorithm run time to a practical limit. Certain combinations of integer variables are known prior as either infeasible or always economically disadvantageous. Examples are the minimum up and down time constraints in the operation of most thermal units. Further, the linear programs which result when fathoming the integer programming solutions have a special structure which can benefit from either LP decomposition or product form inverse techniques.

Due to the rapid increase in computing time with problem size for the MILP approach, the extent of the system to be scheduled by MILP at any one time should be arranged so that computing time does not become excessive. In fact, MILP approach should be used only for marginal generating facilities and those portions of the schedule for which the operators are not confident. This reduces the size of the scheduling problem for the MILP approach and thereby decreases the computing effort requirements.

### 3.3 COMPUTER AND PROGRAM REQUIREMENTS

A scheduling procedure is useful only when the computer requirements are not excessive. In the past, there were two basic computer requirements: memory size and running time. Present day computer technology has introduced the capability of virtual memory systems even for the so-called "mini" computers. Virtual memory systems can provide literally more than adequate computer memory for most practical problems. Hence, the size of computer memory is not an important factor in implementing a scheduling procedure.

However, time is still a crucial factor in the application of any scheduling procedure. Daily fuel scheduling covers a scheduling period of one day to a week. Therefore, one can expect the scheduling program to be run at least once a day. The program will be required to re-establish the schedule when changes occur, such as outages of generating units and load deviations. Thus, the system operators may run the program two or three times a day. Furthermore, scheduling program will be executed in the background mode since top priority is usually devoted to system security and control functions. An acceptable computing time is thirty minutes central processing unit (CPU) time with a total elapsed or turn around time of no more than two hours. Of course, such computing time requirements will depend on the available computer.

As an example, the CPU time required to schedule the New York State system should not be much more than thirty minutes on an IBM 370/155 computer. There are about one hundred and ten (110) thermal units, two pumped storage plants and a number of hydro plants in the New York State system.

Both the computer memory and running time requirements are very much dependent on the efficiency of the program coding. There are also other aspects of program coding which can make a program better and more useful.

Flexibility is a very important feature in a program since operating conditions vary from system to system and from time to time for a particular system. A flexible and well-designed program will require minimal modifications when conditions or requirements change. A lot of data required for fuel scheduling are also used by other utility programs. Therefore, consideration must be given to the compatibility of data requirements and the structure of various data files in designing a program.

Another very important task in program design is the definition of output requirements. Essentially, the output is tied in with the information which can be obtained and the applications of the available information. Therefore, the type and quantity of output information would depend on the setup and scope of the operating or control center. Nevertheless, the basic outputs from any daily fuel scheduling program should include the following:

- Total system operating cost
- Startup and shutdown of units
- Hourly loading of all generating units or plants
- Hourly reserve contributions
- Hourly costs by categories
- Total fuel consumption by types.

An essential program feature is that the program should be easy to use. This will put less burden on the users and minimize any possible errors in the execution of the program.

The main concern in the coding of prototype programs in this project is to evaluate the computing requirements, running time in particular, and the performance of the selected scheduling procedure. Hence, no special attention is directed to meeting all the program requirements discussed above. To do so will be premature at this stage of the investigation. However, when it is time to code a production type or user-oriented program then all attempts should be made to take into consideration all the above program requirements and others which may be appropriate.

The proposed procedure has been structured to be flexible in that it can accommodate new generating facilities with different characteristics or special interchange contracts. For the search approach, this is accomplished by develop-

ing additional modules to schedule such new facilities and contracts. One could expect the scheduling of these new additions can be formulated in the framework of the MILP problem.

## Section 4

### SEARCH APPROACH TO FUEL SCHEDULING

Various search techniques have been used for the scheduling of different generating facilities. Search techniques include the simple search based on incremental cost, heuristic search, gradient search and recursive search or dynamic programming.

The search approach described in this section is used to determine a good feasible schedule within reasonable computing time. Though the setup of this approach attempts to minimize the total system operating cost, there is no guarantee that the final schedule will minimize the system cost. While the search approach is not designed to find the minimum cost schedule, the obtained schedule is expected to be near optimal. Starting with this schedule, the MILP approach presented in Section 5 will be used to determine an improved, if not the optimal schedule.

The search approach consists of a number of different scheduling modules; the overall structure of the approach and descriptions on these modules are given below. Results using the search approach on sample and New York test systems are included at the end of this section.

#### 4.1 OVERALL STRUCTURE

The overall structure of the search approach is given in Figure 4-1 which shows a main scheduling program or Master Module (MASTER) and the following modules:

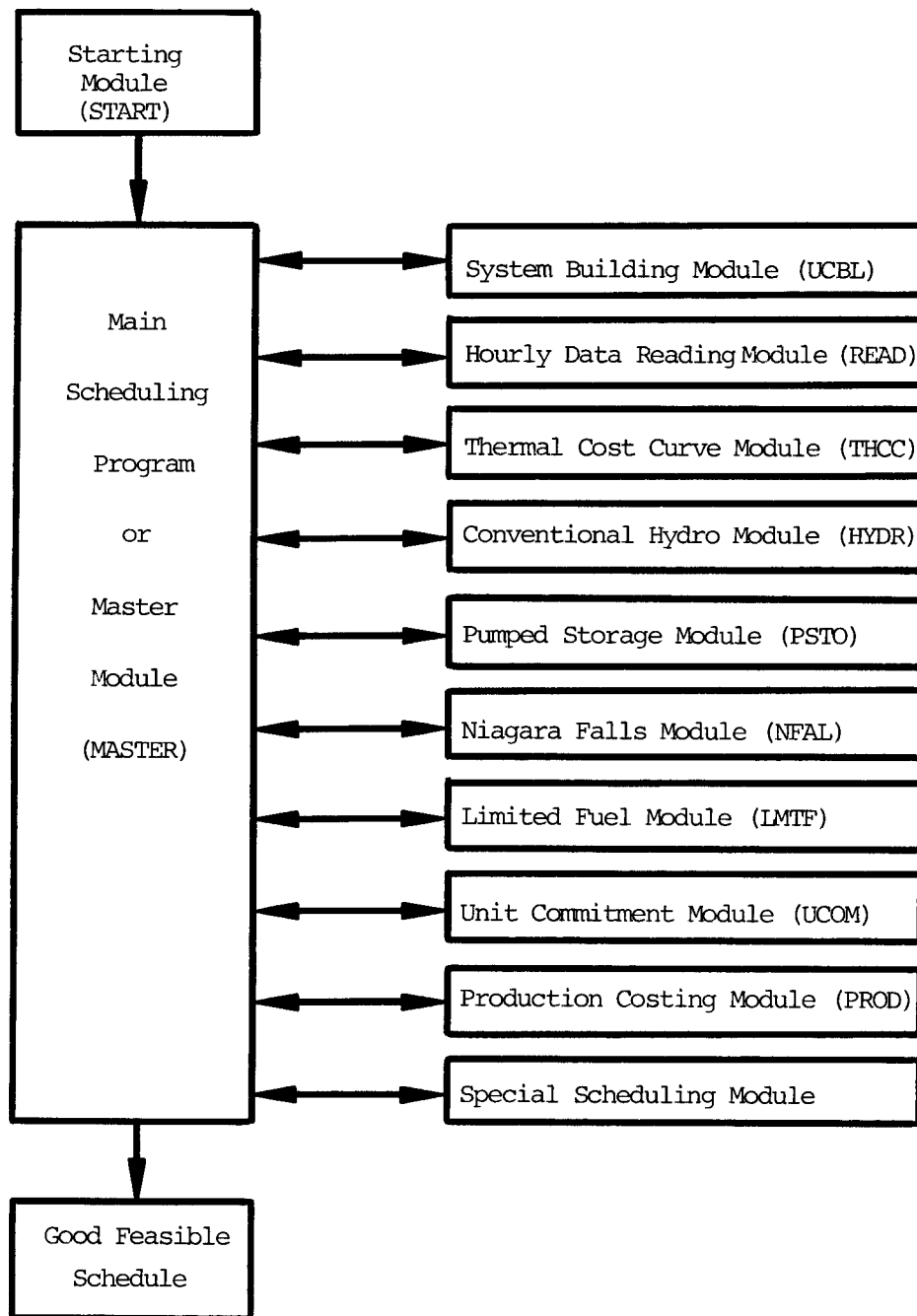


Figure 4-1. Overall Structure of the Search Approach

- Starting Module (START)
- System Building Module (UCBL)
- Hourly Data Reading Module (READ)
- Thermal Cost Curve Module (THCC)
- Conventional Hydro Module (HYDR)
- Pumped Storage Module (PSTO)
- Niagara Falls Module (NFAL)
- Limited Fuel Module (LMTF)
- Unit Commitment Module (UCOM)
- Production Costing Module (PROD)
- Special Scheduling Module.

The setup of this approach is such that other modules may be included for the scheduling of specific generating facilities or contracts as denoted by the Special Scheduling Module. For example, the Niagara Falls Module is a special module to be used in the scheduling of the Niagara Falls Project which is a combination of a conventional hydro plant and a separate pumped storage plant.

Each module uses a specific method which is suited to the particular aspect of the scheduling problem. In this way, the advantages of different methods can be exploited. The Master Module coordinates the data and schedules from the different modules such that overall system operating requirements are satisfied and economical schedules are determined. A number of iterations among the various modules will be required in order to improve the overall schedule. Thus, a trade-off must be made between the amount of computing effort or time and the optimality of the final schedule.



The coding of the modules is such that each module is an independent program and the execution of the successive modules is by the chaining process. Therefore, only the active module resides in the computer memory at one time.

## 4.2 MODULE DESCRIPTIONS

Functional descriptions on all the modules are presented in this section. Summaries of preliminary results on modules HYDR, PSTO, NFAL and LMTF are included; these results were used to verify the program logic for the four modules. Modules UCBL, READ, UCOM and PROD are part of PTI's proprietary unit commitment program package which was supplied to NYPP prior to the start of this project.

### 4.2.1 Starting Module - START

Module START is used to initiate the execution of the daily fuel scheduling process. It reads from an input file the setup for the execution of the various scheduling modules and the corresponding data files. Figure 4-2 shows the overall flowchart for the module.

The module also initializes certain variables and logical unit (device) numbers. When Module START is completed the scheduling proceeds to Module MASTER.

### 4.2.2 Master Module - MASTER

This is the main scheduling module which coordinates the schedules among the different modules which may be used for the daily scheduling of various generating facilities within a system. Figure 4-3 gives the overall flowchart for Module MASTER.

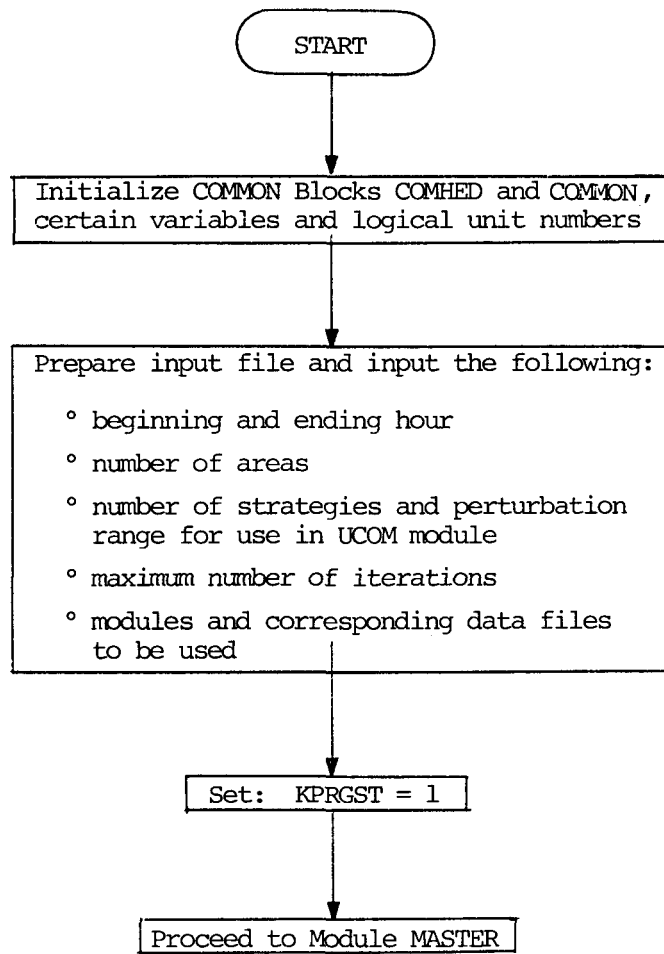


Figure 4-2. Overall Flowchart for Module START

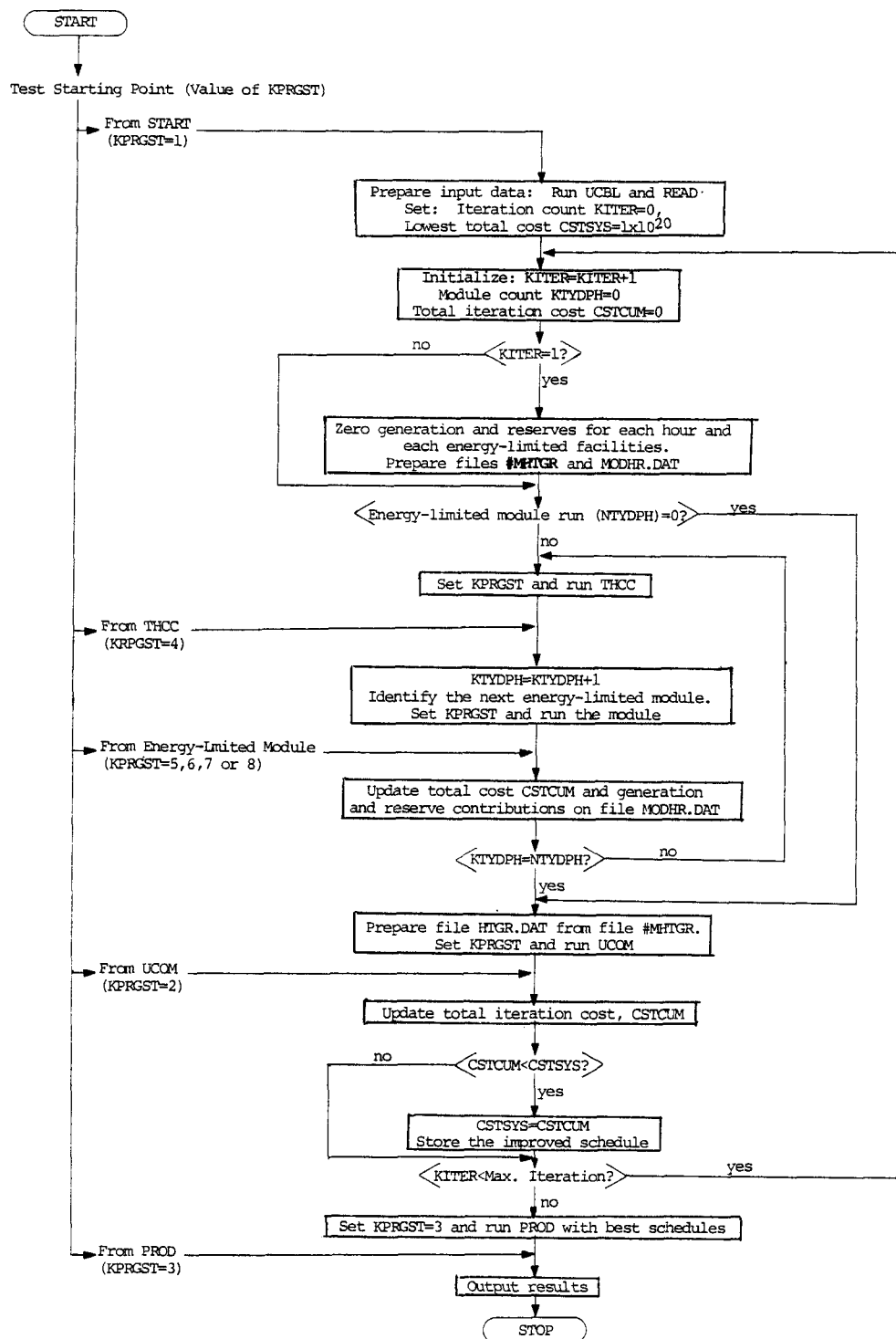


Figure 4-3. Overall Flowchart for Module MASTER

Modules UCBL and READ are called upon to prepare the thermal system data and the hourly system data such as the loads, availability of thermal units, changes in area transfer limits and reserve requirements. Two files, #MHTGR and MODHR.DAT, are prepared and initialized for the storing of generation and reserve contributions by energy limited facilities. File #MHTGR contains the total generation and reserve contributions for each hour while file MODHR.DAT contains the hourly generation and reserve contributions for each set of energy limited facilities scheduled by each execution of any one module. These two files are modified and updated as each energy limited module is executed.

Prior to the execution of an energy limited module (HYDR, PSTO, NFAL and LMTF), Module THCC is called upon to prepare the thermal cost curve characteristics based on the assumption that all units in the thermal system are available for the period or that a unit commitment schedule is available. These thermal cost curve characteristics are then used to schedule the energy limited facilities.

After all the energy limited facilities have been scheduled, file HTGR.DAT is prepared by duplicating the results in file #MHTGR. File HTGR.DAT contains the total generation and reserve contributions from all energy limited facilities for each hour; and the hourly data in this file can be read sequentially by the proper routines in Modules UCOM and PROD.

Module UCOM is used to determine an economic unit commitment schedule which satisfies all load and reserve requirements for all the hours in the period. This completes one iteration of the scheduling process. If the total iteration cost is lower than any previous total cost, the schedules for the energy limited generating facilities and the unit commitment schedule are stored in files HTGR.OLD and UCOMT.OLD respectively.

When the maximum number of iterations have been performed, Module PROD is used to determine the production cost of the thermal units using the best schedules obtained. Hourly system output summary, dispatches of thermal units, and a plot of the unit commitment schedule may be requested. The output of the above results marks the end of the scheduling process which is then terminated.

#### 4.2.3 System Building Module - UCBL

Module UCBL is used to read all non-hourly system data from a source file UCSYS.DAT and to build a new binary system data file GUNDT.DAT for use by Modules THCC, UCOM and PROD. Binary data files allow rapid and easy reading of data and thus they are suitable for the search approach in which specific set of data may be used a number of times in the scheduling process.

Non-hourly data read by Module UCBL include generating unit data and transmission area interconnection data. Generating unit data are represented as sets of unit characteristics consisting of generating limits, reserves contribution capabilities, incremental heat rate table, startup cost table, maintenance costs, fuel cost, minimum up and down times. Interconnection data give the connected area pairs and the corresponding transfer capabilities. The system network is represented by a linear flow or transportation network in which Kirchoff's current law is observed and the voltage law is ignored. Furthermore, the transfer capabilities between areas are absolute limits on the flow of power within the network.

#### 4.2.4 Hourly Data Reading Module - READ

System and unit operating data are read from source file UCOP.DAT by Module READ. The data include the changes in reserve requirements, transfer capabilities, initial unit status and unit availabilities for the scheduling period. Module READ also reads the hourly area loads from a separate source file UCLF.DAT.

All the hourly data are then stored in a new binary file `HRDT.DAT` for sequential reading and each record contains all the required data for each hour of the scheduling period.

#### 4.2.5 Thermal Cost Curve Module - THCC

4.2.5.1 Problem Definition. Thermal Cost Curve Module is one of the modules to be used in the search approach to daily fuel scheduling. The overall structure of the proposed search approach consists of a main scheduling program or Master Module and a number of different modules designed for scheduling specific generating facilities. Each module uses a specific method which is suited to the particular aspect of the scheduling problem. In this way, the advantages of different methods can be exploited. The Master Module coordinates the schedules from the different modules such that overall system operating requirements are satisfied and economical schedules are obtained.

The coordination of the schedules from different modules requires the consideration of the effect on the overall total system cost when scheduling or dispatching a specific generating facility such as a conventional hydro or pumped storage plant. The cost characteristics of the thermal system, which is normally not limited by energy resource and is scheduled by unit commitment, are used for the above coordinating purpose. These cost curves should be representative of what it would actually cost if the generation is to be produced by the thermal system.

4.2.5.2 Method. The fuel rate curve of each unit in the thermal system is represented by a convex piecewise linear function so that the incremental fuel rate

or cost for the unit will be in the form of increasing steps; that is, the incremental fuel rate for each step is constant and the step sizes in MW or blocks may be different.

The procedure to build the thermal system cost curves is shown by the flowchart in Figure 4-4. All system and input data are read, and a master thermal cost curve is built assuming all units are available. This cost curve is the aggregation of the incremental cost curves of all the units arranged in non-decreasing value; associated with each MW block or step are the size in MW and the corresponding thermal unit number. Hence, the thermal system cost curve resembles a large thermal unit with many steps.

Hourly thermal system cost curves are obtained to reflect the different commitment of the units in different hours of the scheduling or dispatching period. When building the curve for a given hour, the blocks in the master thermal cost curve for all the off-line non-peaking units are deleted. To reduce the size of the resulting thermal system cost curves, consecutive blocks with small or negligible cost differences are merged into one block with size equal to the sum of all the previous blocks and cost equal to the weighted average value. Since these hourly thermal system cost curves are to be used for scheduling other generating facilities having smaller capacity, only the useful portion of the cost curves are retained and stored. It is common to have the same units committed for a number of consecutive hours, so that only one thermal system cost curve will be needed for those hours.

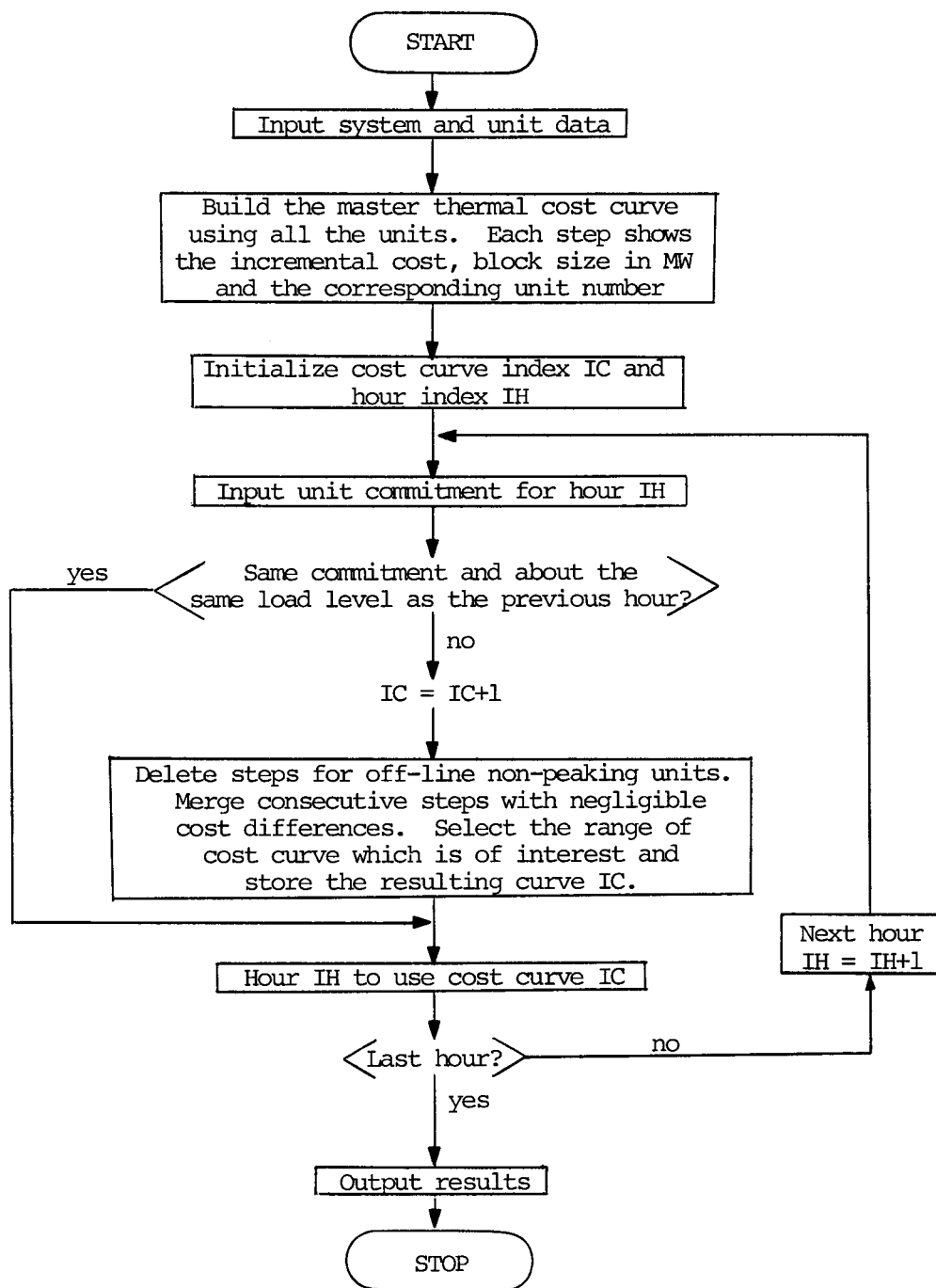


Figure 4-4. Overall Flowchart for Thermal Cost Curve Module - THCC



#### 4.2.6 Conventional Hydro Module - HYDR

4.2.6.1 Problem Definition. Conventional hydro plants use hydro energy to generate the electrical power. They are different from pumped storage hydro plants in that they have no pumping capability and all their hydro energy comes from upstream water inflow. The amount of pondage or storage capacity varies widely: (1) Little or no storage - such plants are commonly known as run-of-river plants in that the plant generation strictly follows the total inflow, (2) Moderate storage - these plants normally operate on a daily or weekly cycle by storing up water during the off-peak periods and releasing for generation during the peak periods, and (3) Large storage - generally these plants have sufficient storage for seasonal operation, typically in the U.S., most of the water storage is built up during the spring for use through the year.

The overall objective is to determine the generating schedule such that the total system operating cost is minimized over the scheduling period. Since the scheduling period for daily fuel scheduling ranges from a few hours to a week, the long term schedule for large storage hydro plants is outside the scope of the present project. Thus, for our purposes, the Conventional Hydro Module should schedule all hydro plants with moderate storage given their respective inflows and all plants with large storage given the amount of water to be released during the scheduling period.

4.2.6.2 Assumptions. Below are the assumptions made on hydro plants and their operation for use in the Conventional Hydro Module:

- Water conversion factor is constant for the plant.
- Storage of water must be within the maximum and minimum limits.

- Hourly inflows are given.
- Starting and ending storage levels are specified.
- Required minimum plant release is specified.
- The plant may have up to ten identical units.
- Unit generation must be within the minimum and maximum capacities.
- Maximum unit generating capacity is constant.
- Change in hourly generation must satisfy the allowable plant response rate.
- Time delay for flow between cascading plants is ignored.

4.2.6.3 Method. Hydro scheduling is assumed to be only part of the overall problem of daily fuel scheduling and details in the scheduling of complex hydro systems would not be considered. Nevertheless, the schedules obtained by the Conventional Hydro Module should take into consideration various essential constraints and requirements.

The hydro system is represented by a number of composite plants and each composite plant may correspond to one actual plant or a group of cascading plants in a river system. An upstream plant with fairly large storage capacity and all the downstream plants with much smaller capacities are chosen to form a composite plant. Such selection minimizes the spillage of downstream composite plants. The average discharge rate to be used for a composite plant will be equal to the smallest plant maximum discharge rate within the group so that spillage does not occur. The storage of the composite plant is equal to the sum of the equivalent energy storage capacities for all the represented plants.

An incremental search technique is used to determine the most economic schedule of composite hydro plants. The plants are scheduled in the sequence according

to the order in the input data. Figure 4-5 shows the flowchart on the overall scheduling procedure. First, all study and system data are input. For each plant, the schedule proceeds from the starting hour. At each hour, the storage is increased by the amount of inflow and the plant is scheduled to meet any specified minimum release requirement. When the amount of water in storage is greater than the storage capacity, the excess is used up by generating during the best or most economic hours within the period starting from the initial hour to the current hour. If there is surplus water in storage after scheduling the last hour, this surplus water is used up for generation during the most beneficial hours, that is, those hours for which the generation worth is highest. Having determined the complete schedule for a plant, its reserve contributions are computed and the next plant is scheduled.

4.2.6.4 Preliminary Results. A number of preliminary sample cases were used to test and verify the logic of the Conventional Hydro Module. Results for these cases are presented in Volume 2. The following sets of studies were carried out:

- Variation in plant inflow
- Increase in the number of units
- Effects of outages
- Increase in the number of plants
- Change in the number of storage targets
- Effects of plant response rate
- Effects of minimum plant generation.

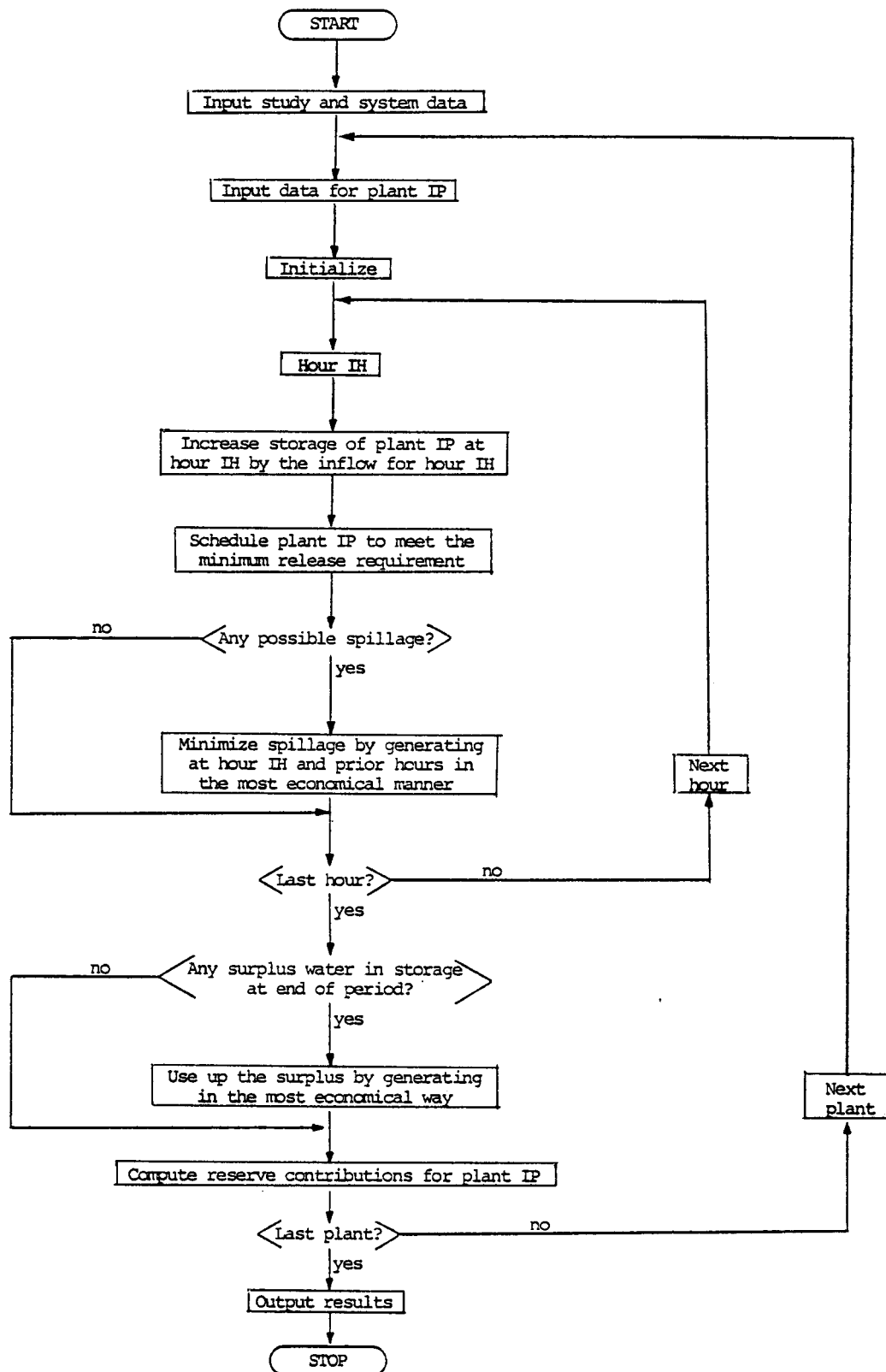


Figure 4-5. Overall Flowchart for Conventional Hydro Module

#### 4.2.7 Pumped Storage Module - PSTO

4.2.7.1 Problem Definition. Pumped storage plants are widely used to reduce the total system production cost. Generally, a pumped storage plant has an upper and a lower reservoir. Normally there is no natural inflow into the upper reservoir. Practically all pumped storage plants are equipped with reversible pumping/generating units.

The economy in the operation of pumped storage plants is the result of available cheap off-peak pumping power and relatively high generating cost during peak load periods. As long as the cost differential exceeds the losses in the pumping-generating cycle, it will be economical to operate the plant. The cycle efficiency is equal to the generated energy by a certain amount of water divided by the energy required to pump the same amount of water back into the upper reservoir. Typical cycle efficiencies for most pumped storage plants are in the range of 60 percent to 75 percent.

To achieve economical operation, it is necessary to consider simultaneously both the pumping and the generation. The extent of the operation will be determined by cost and plant operating constraints such as pumping and generating capacities, and reservoir storage limits. A general practice is that the upper reservoir should be full at the beginning and the end of the scheduling period.

The overall objective is to determine the pumping and generating schedule such that the total system cost is minimized over the scheduling period.

4.2.7.2 Model and Assumptions. Below are the model and assumptions for the pumped storage plants and their operation as used in the Pumped Storage Module.

- Lower reservoir has adequate storage capacity and surplus water so that all constraints on the lower reservoir are neglected.
- Storage of water in the upper reservoir must be within the maximum and minimum limits or elevations.
- Plant cycle efficiency is constant.
- There is no natural inflow into the upper reservoir.
- The plant may have any number of units which are identical.
- Generation must be between the maximum and minimum capacities of the unit. Also the maximum capacity may vary with the elevation of the upper reservoir.
- Pumping is to be at full load.
- Elevations of the upper reservoir at the beginning and the end of the period must be specified.
- Thermal system incremental cost curves are represented in non-decreasing steps; this is to assure the optimality of the dispatch of pumped storage plants.

4.2.7.3 Method. An incremental search technique is used to determine the most economic schedule of pumped storage plant. Energy (MWh) dispatch is used and conversion of storage quantities from and to elevation are carried out at the beginning and end of the dispatch respectively.

Figure 4-6 shows the flowchart on the overall scheduling procedure. If the starting reservoir elevation is greater than the ending elevation, generation is scheduled until the target is reached. When the situation reverses, pumping will be scheduled so as to meet the specified ending elevation. From this point on, the ending elevation is preserved by pumping back whatever water used for generation within the period.

A search is made for the hour with the highest generation worth and the hour with the least pumping cost. Both generation and pumping will be scheduled in

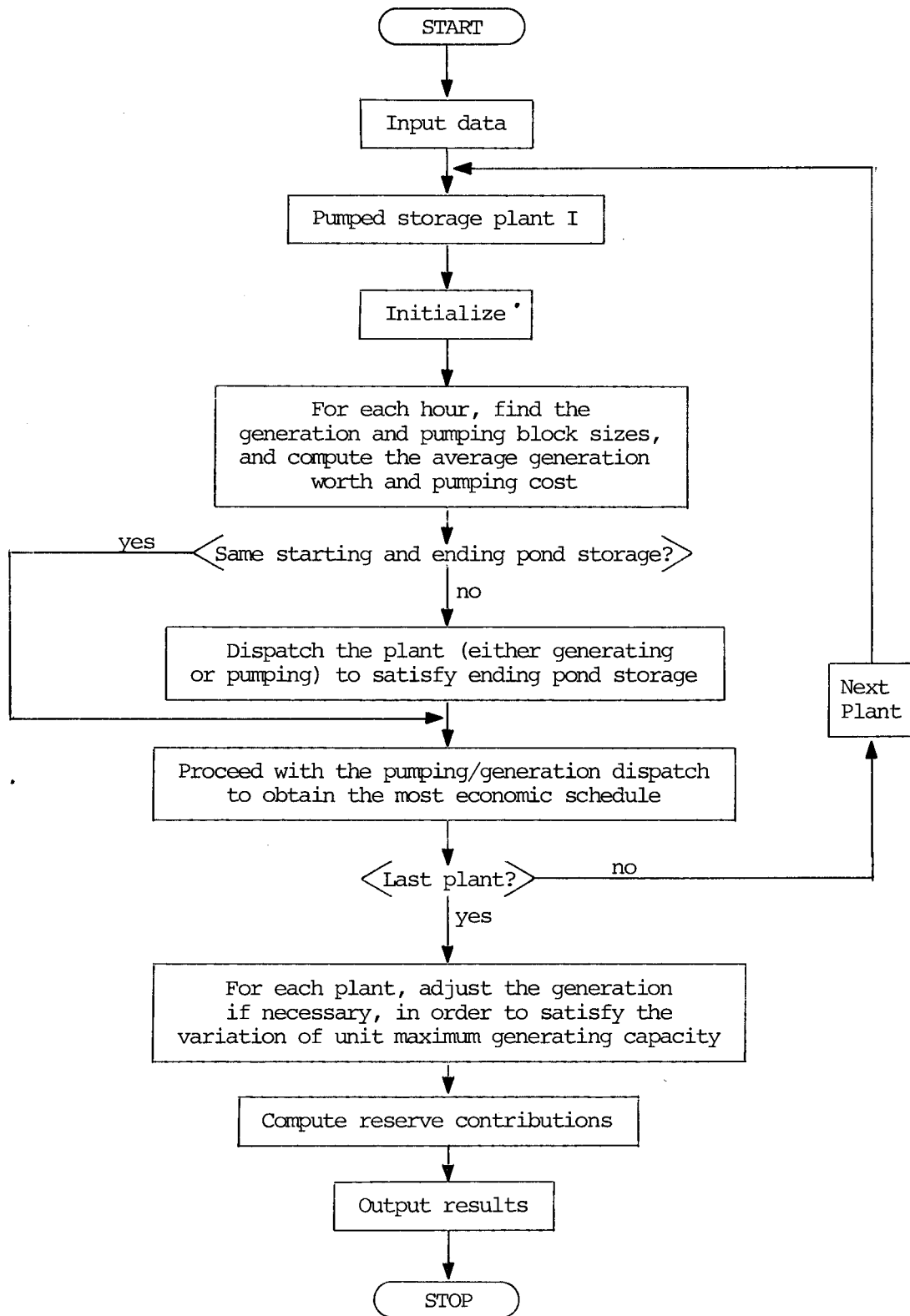


Figure 4-6. Overall Flowchart for Pumped Storage Module

the respective hours as long as this operation is economical and permissible. An operation is economical if the ratio of the pumping cost to the generation worth is not less than the plant cycle efficiency. The feasibility of an operation is dictated by the plant storage constraints and available unit capacities. As the scheduling progresses, the pumping cost increases while the generation worth decreases. The scheduling terminates when further operation becomes uneconomical or would violate plant constraints.

For those plants with varying maximum unit generation capacity, the nominal maximum capacity is used initially. After the schedule has been determined, a check is made on the scheduled generation, any violation of the maximum capacity will be corrected by moving the excess generation into other hours. Finally reserve contributions are computed and the scheduling procedure is complete.

4.2.7.4 Preliminary Results. A number of preliminary sample cases were used to test and verify the logic of the Pumped Storage Module. Results for these cases are presented in Volume 2. The following sets of studies were carried out:

- Change in plant cycle efficiency
- Increase in the number of units
- Effects of outages
- Change in reservoir storage capacity
- Variation in unit maximum generating capacity

#### 4.2.8 Niagara Falls Module - NFAL

4.2.8.1 Problem Definition. The Niagara Falls Project is a combination of a hydro plant and a pumped storage plant. Neither the Conventional Hydro Module described in Section 4.2.6 nor the Pumped Storage Module in Section 4.2.7 can



adequately schedule the project. Hence a special module is developed specifically for scheduling this project.

The layout of the Niagara Falls Project is shown in Figure 4-7. Water intake ( $Q_I$ ) takes place upstream from the falls. This water could be channeled directly to Robert Moses hydro plant for generation or be pumped up by Lewiston pumped storage plant for use later on. The limit on the amount of water that can be pumped is constrained by the plant pumping capacity and the intake,  $Q_I$ . The latter constraint is due to the absence of lower pond which exists in most pumped storage projects. Water released from the storage pond is used for generation at the Lewiston Pumped Storage Plant and again at the Robert Moses Plant. Thus the flow through the Robert Moses Plant ( $Q_H$ ) is given by the sum of the intake ( $Q_I$ ) and the flow through the Lewiston Plant ( $Q_P$ ), that is,

$$Q_H = Q_I + Q_P,$$

where,

$$Q_I \geq 0,$$

$$Q_H \geq 0,$$

$$Q_P \geq -Q_I.$$

The constraint,  $Q_I \geq 0$ , is established on the basis of economics, as it may be physically possible for the intake flow ( $Q_I$ ) to be negative.

Based on the forecast flow on the Niagara River and the requirements of Niagara Falls, the forecast on the intake can be determined. Given the intake forecast and the storage pond conditions, the objective is to schedule both the Robert Moses Plant and the Lewiston Pumped Storage Plant such that the total system cost is minimized.

**4.2.8.2 Assumptions.** The following assumptions used in the Niagara Falls Module are basically the same as those used in the Conventional Hydro and the Pumped Storage Module.

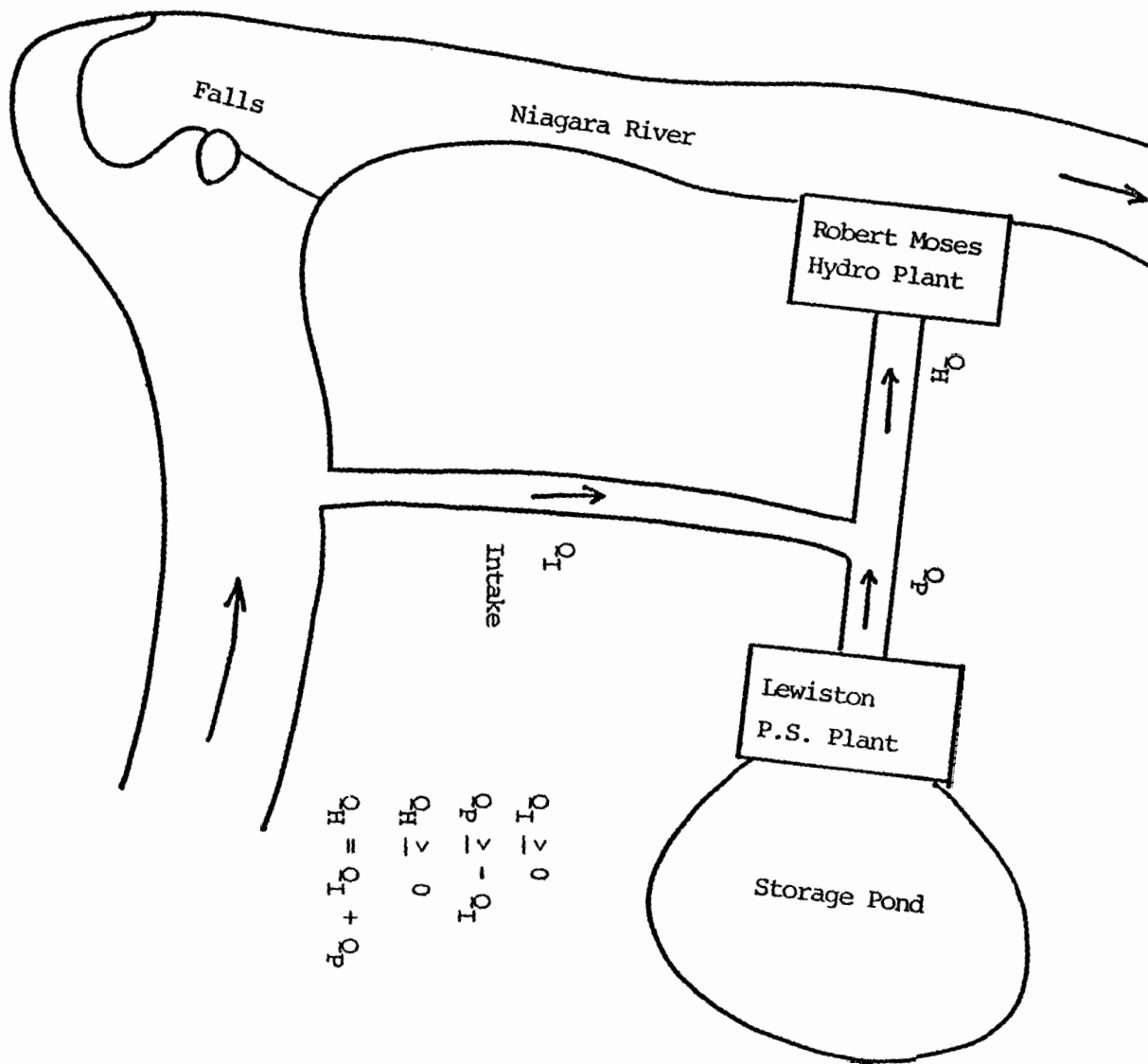


Figure 4-7. Layout of Niagara Falls Project

- Water in the storage pond must be within the minimum and maximum limits.
- Cycle efficiency of the pumped storage plant is constant.
- Water conversion factors are constant for both plants.
- Generation at each plant must be within the corresponding limits.
- Maximum generating capacities of both plants are constant.
- All units within a plant are identical.
- Pumping is to be at full unit loading.
- Partial outage is permitted for generating capacity but not for pumping. Hence, a pumping unit is either fully available or completely out of service.
- Pond storage at the beginning and the end of the scheduling period are specified.

4.2.8.3 Method. The scheduling method used in the Pumped Storage Module is used for the scheduling of the Niagara Falls Project. Certain additions and modifications to the method are needed to satisfy the special requirements of the project. Actual water schedule is used instead of energy schedule since the two plants have different water conversion factors.

Figure 4-8 shows the overall flowchart for the Niagara Falls Module. An initial schedule which minimizes spillage is first determined. At each hour, as much of the intake as possible is used for generation by the Robert Moses Plant. Any surplus water will be pumped up for storage. If the pond is full, then sufficient water is used up for generation in prior hours to make room in the pond. Spillage will occur if the above is not possible.

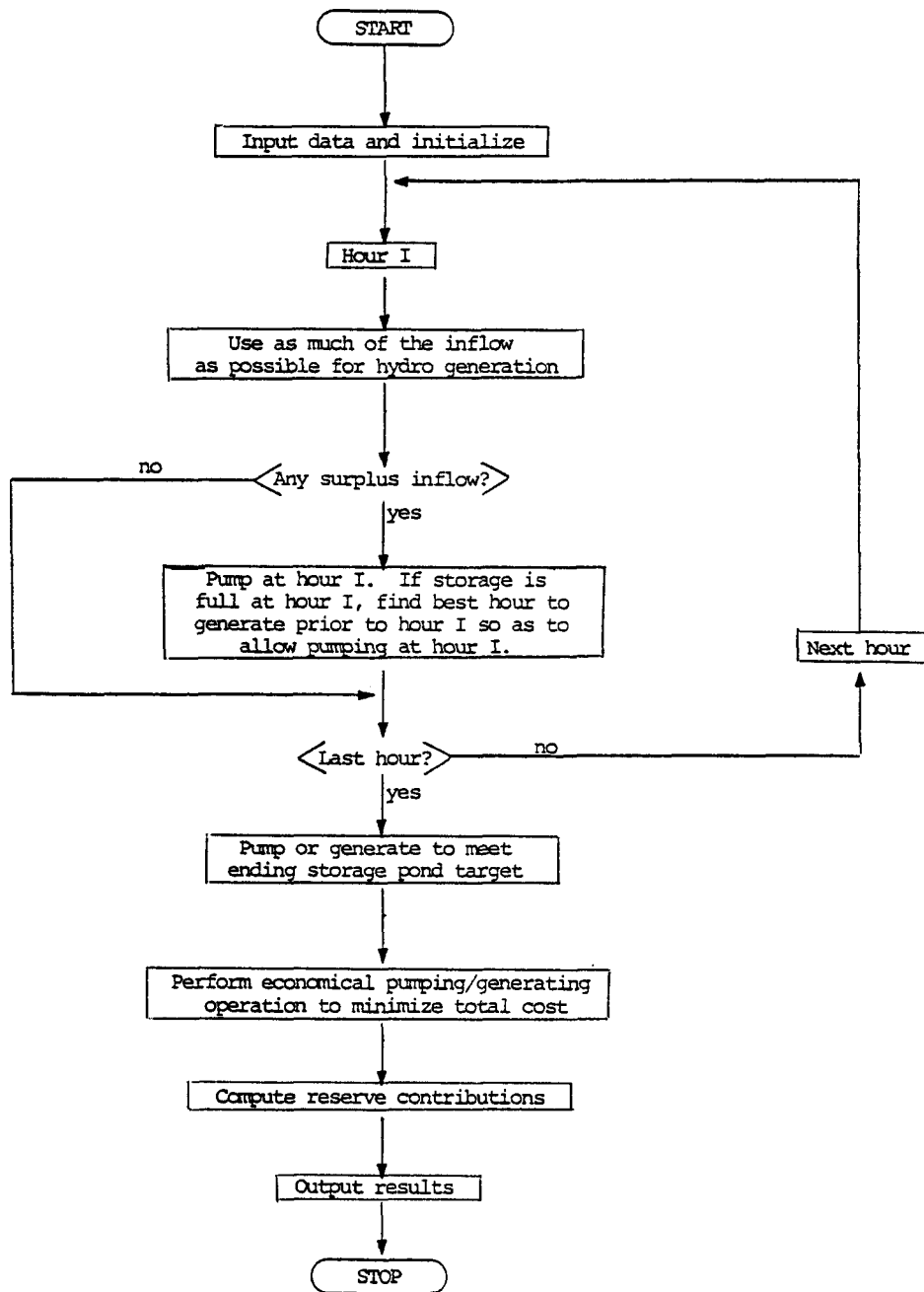


Figure 4-8. Overall Flowchart for Niagara Falls Module

After establishing the initial schedule, the method proceeds in exactly the same manner as in the Pumped Storage Module. However, the availability of intake will be an additional constraint on the pumping and in scheduling additional generation, checks must be made on the conditions of both plants.

4.2.8.4 Preliminary Results. Several sample test cases were performed to verify the program logic in the Niagara Falls Module. The model test plant has the same set of operating constraints and characteristics as the Niagara Falls Project but the plant and storage capacities are about one-tenth the actual values. Results for these test cases are reported in Volume 2.

#### 4.2.9 Limited Fuel Module - LMTE

4.2.9.1 Problem Definition. When determining the commitment of thermal units, it is generally assumed that the fuel supply is adequate for the scheduling period. The oil embargo and the recent coal strike have shown that shortages of fuel for thermal units can arise. Normal unit commitment procedures are not capable of handling the constraints on the availability of fuel. Techniques used for scheduling hydro plants appear to be suitable since the fuel or resource (amount of water) is limited. However, hydro plants do not have such constraints as minimum up and down times. Thus, a separate method is needed to schedule fuel constrained or limited fuel thermal units.

There are basically two sets of constraints, namely (i) fuel constraints which consist of the minimum and maximum amount of fuel to be used, and (ii) Unit constraints which include operating limits, unit minimum up and down times. For fuel constraints, the minimum could be zero or both minimum and maximum are equal. Normally such fuel constraints are for individual plants which may have any number of generating units.

The objective in scheduling fuel constrained thermal plants is to minimize the total system cost while satisfying both fuel and operating constraints on these plants. The Limited Fuel Module is set up to schedule such plants so as to achieve the preceding objective.

4.2.9.2 Model and Assumptions. Fuel constrained thermal plants are basically energy limited plants, like conventional storage hydro plants. But the characteristics of the units are different from those of hydro units. The model and assumptions for the Limited Fuel Module are as follows:

- The fuel limitations are specified by a minimum amount of fuel that the plant must use within the time period and a maximum amount that the plant can use.
- There can be any number of generating units in a plant.
- Units need not be identical, so that each unit has its own characteristics; namely, fuel rate curve, generating limits, minimum up and down times, time dependent startup cost, and response rate.
- Incremental cost curves for the thermal system and individual units are in increasing steps. Thermal system incremental cost curves are derived from the commitment of thermal units which have no fuel constraints.

4.2.9.3 Method. Scheduling of fuel constrained thermal units are performed in three steps as shown in the three boxes of the flowchart in Figure 4-9. Units in a plant are scheduled in the order in which the units are entered. Instead, a criterion such as average full load cost can be used to establish the scheduling order of the units within a plant.

Minimization of total system cost is based on the comparison of fuel constrained unit and thermal system incremental costs. It is possible that the method gives local or sub-optimal schedules. Determination of optimal schedules will be the effort of the mixed-integer linear programming approach.

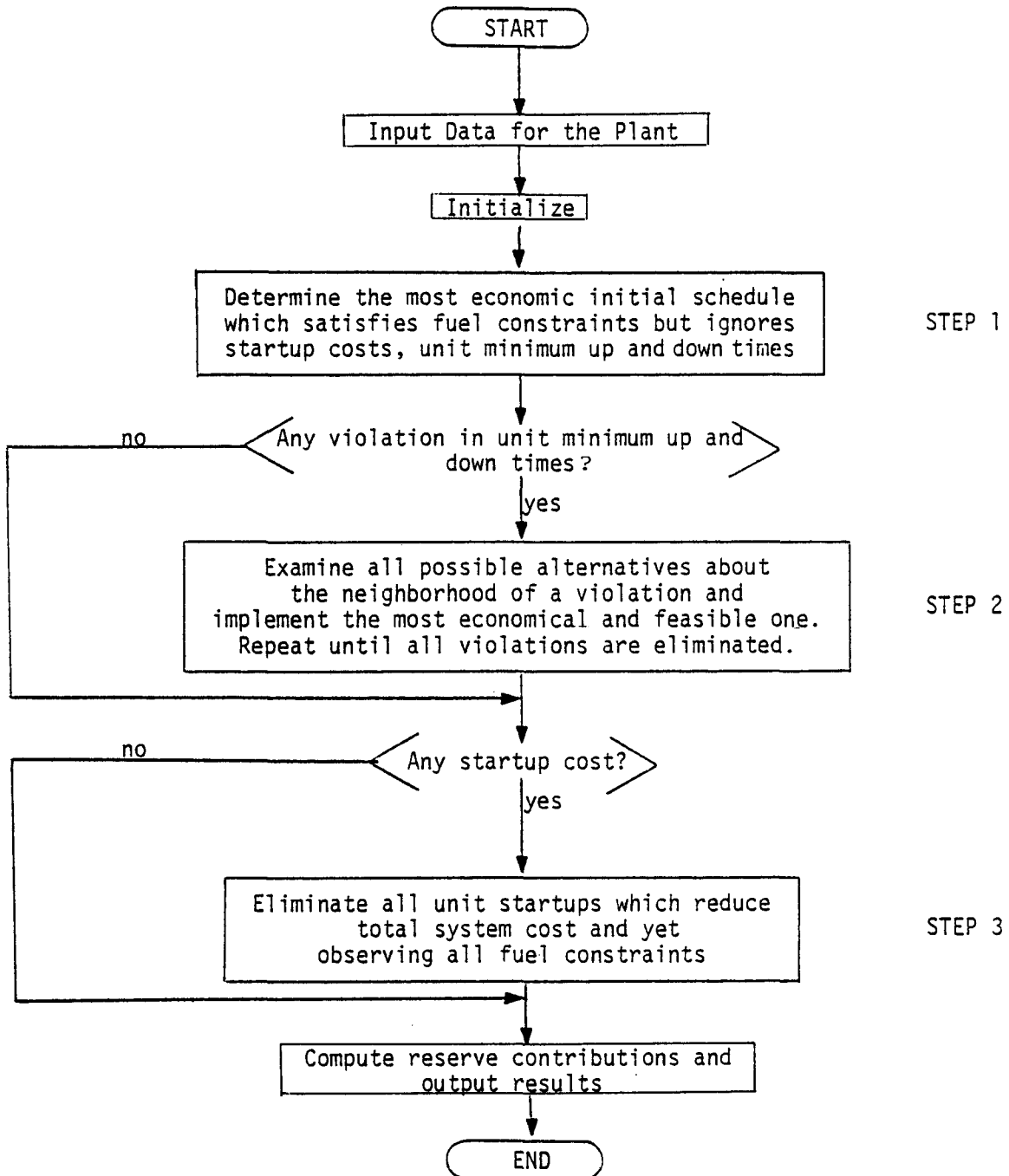


Figure 4-9. Overall Flowchart for Limited Fuel Module

In the first step of scheduling the fuel constrained thermal plants, unit startup cost and the minimum up and down times are ignored. An initial schedule is obtained using the incremental search, peak or cost shaving method which is widely used in the scheduling of storage hydro plants. In determining this schedule, fuel constraints are observed at all times. A simplified flowchart on the preceding scheduling is shown in Figure 4-10.

The scheduling proceeds to the third step if there is no violation of unit minimum up and down times. Otherwise, Step 2 will be used to eliminate all such violations in the most economical manner. A flowchart on Step 2 is given in Figure 4-11. The longest stretch (in terms of time) of violation is eliminated first. A neighborhood is defined about a violation such that the interval of this neighborhood is not less than the unit minimum up and down times. All possible commitments of the unit within the interval are examined and the most economically feasible one is implemented. Unit startup costs are included in this cost evaluation. The above procedure is repeated until all violations of minimum up and down times for all the units in the plant are eliminated.

Step 3 may be bypassed when the units have no startup cost. The main motive of this step is to evaluate the increase in savings from reducing the number of individual unit startups. Reduction may be achieved through either keeping the unit up or down. The former corresponds to the elimination of a shutdown/startup process while the latter avoids a startup/shutdown. After the execution of Step 3, the resulting schedules generally have lower total costs, but not higher. Figure 4-12 shows a flowchart on the procedure for Step 3.

Finally, the reserve contributions by the individual units are computed. The amount of reserves is limited to the amount of fuel remaining which is capable of sustaining the operation for a specified time period. For example, when all fuel is used up, the plant will have zero reserve for the remaining period.



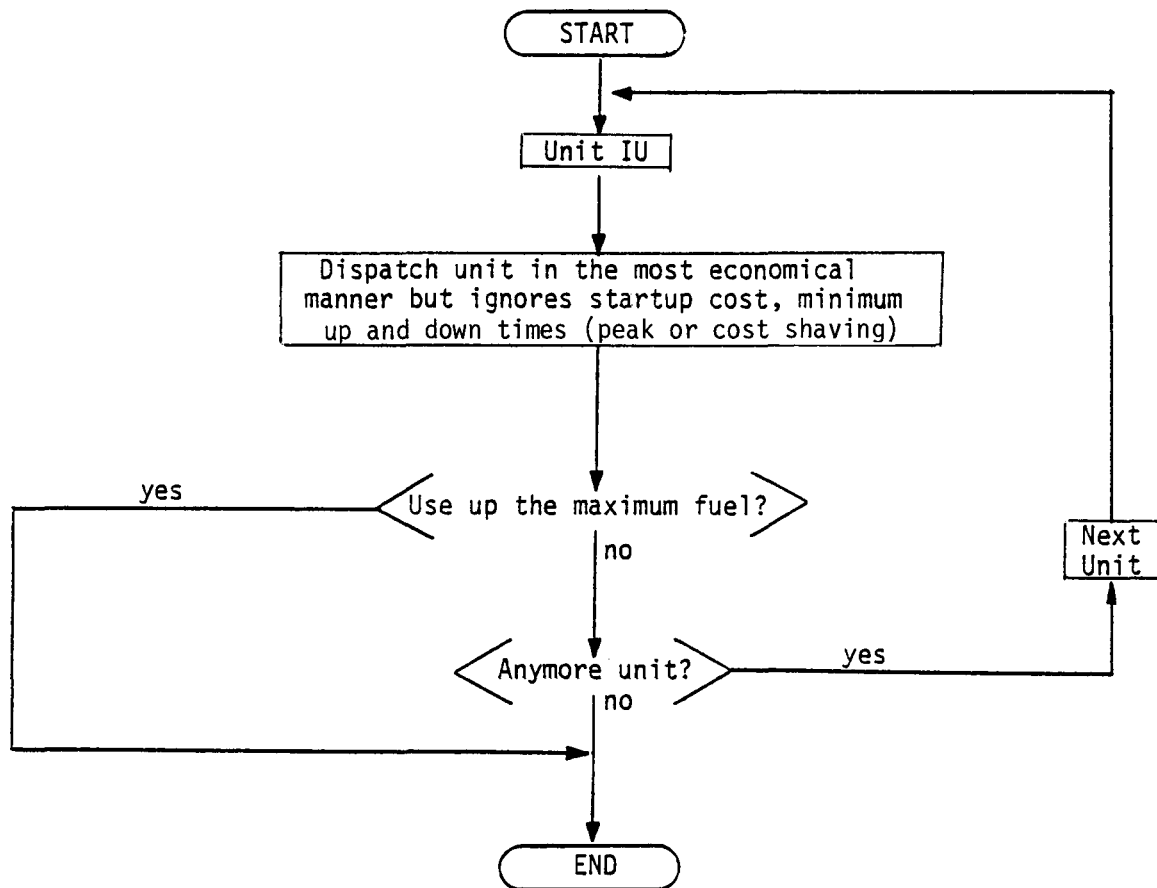


Figure 4-10. Flowchart for Step 1 in Limited Fuel Module

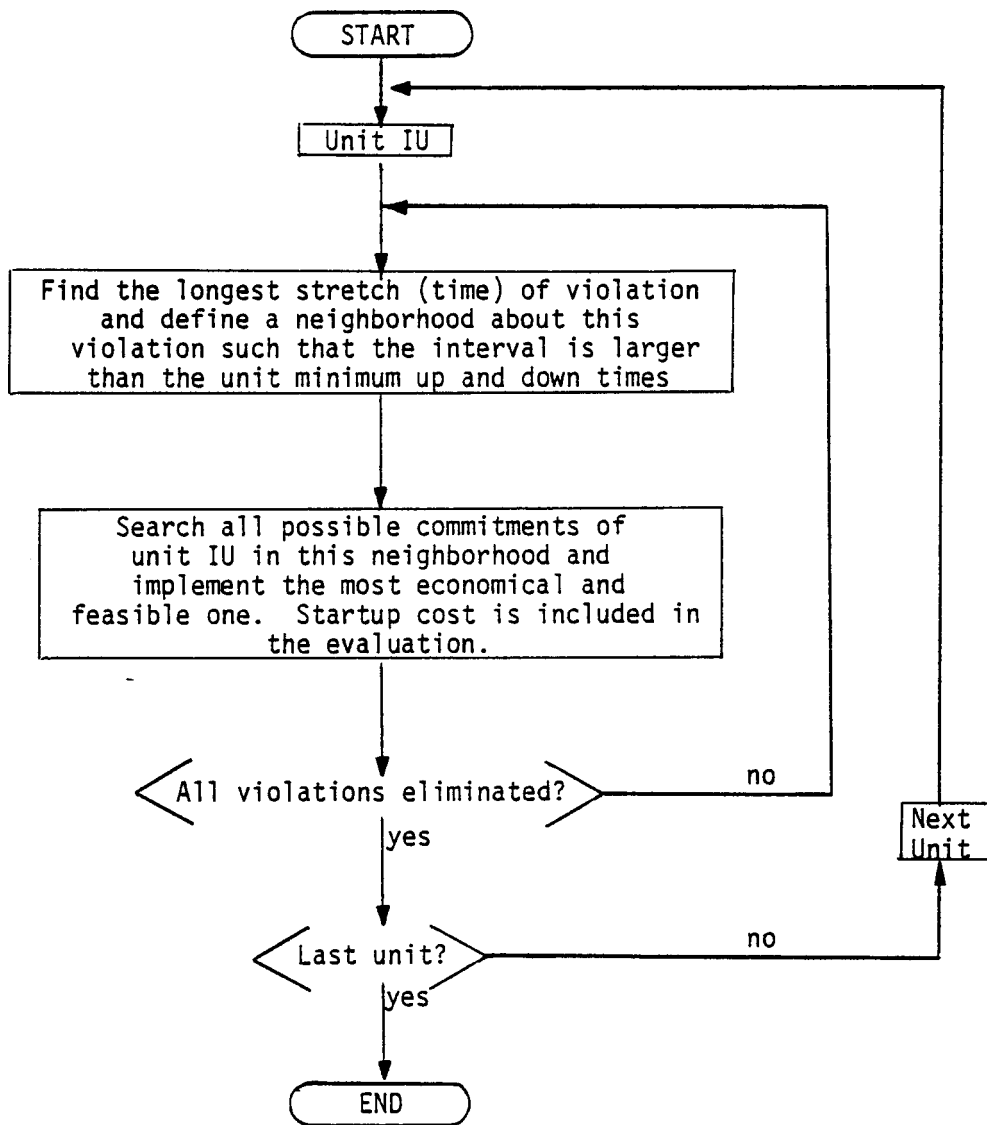


Figure 4-11. Flowchart for Step 2 in Limited Fuel Module

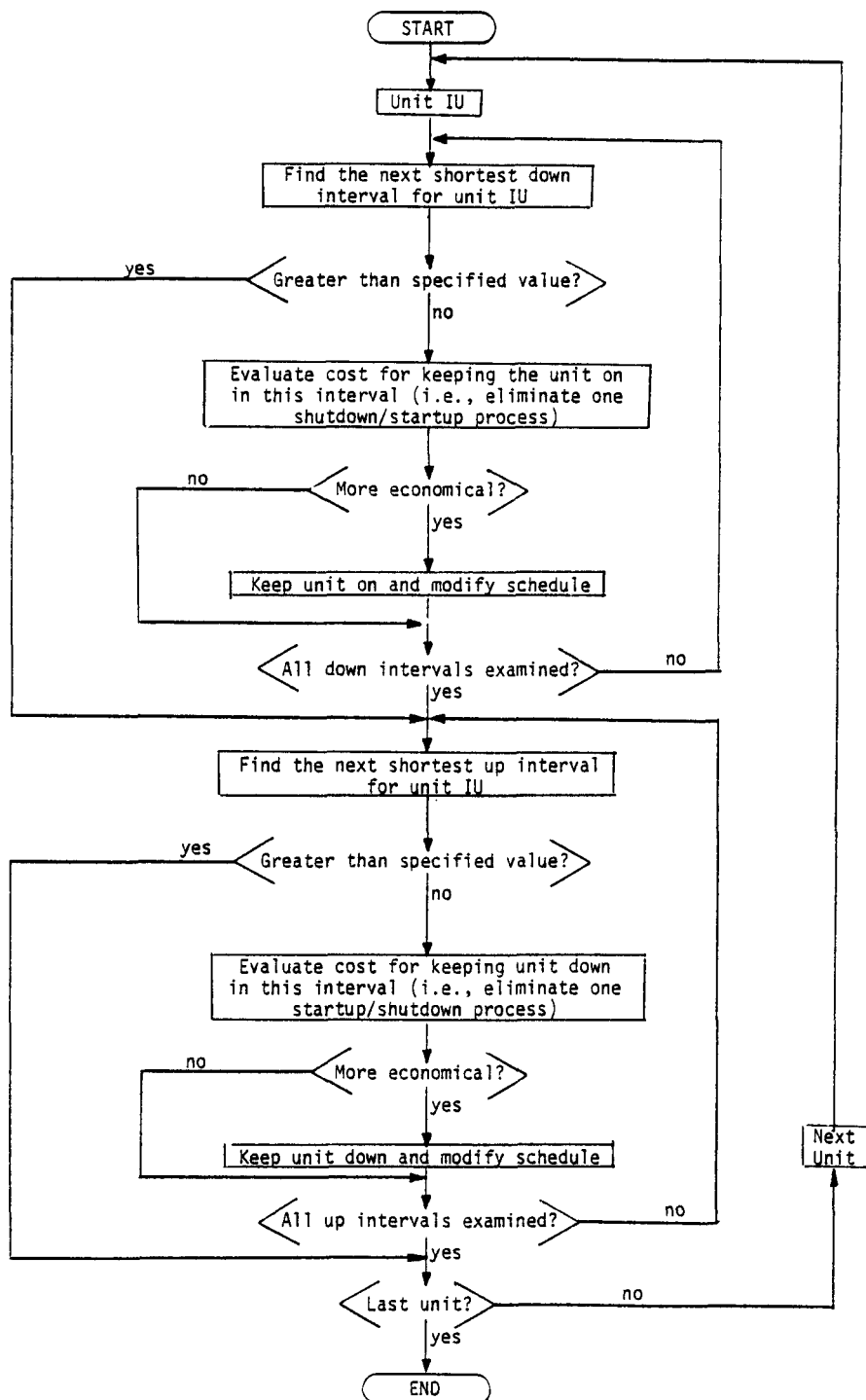


Figure 4-12. Flowchart for Step 3 in Limited Fuel Module

4.2.9.4 Preliminary Results. Several preliminary sample cases were used to test and verify the logic of the Limited Fuel Module. Results for these cases are reported in Volume 2. Test studies undertaken are as follows:

- Change in maximum fuel
- Increase in the number of units
- Effects of unit outages
- Variation in fuel costs
- Change in minimum fuel

#### 4.2.10 Unit Commitment Module - UCOM

4.2.10.1 Problem Definition. Unit commitment determines when to start and shutdown generating units so that the loads and all operating constraints for a specified time period are satisfied. Commitment periods normally vary from a day to a week and the periods are usually divided into hourly intervals during which the loads and reserve requirements are assumed to be constant. Operating constraints include spinning and non-spinning reserve requirements, availability of fuel or energy, unit minimum and maximum capacities, unit minimum up and down times, crew constraint on the startup and shutdown of units, and environmental requirements.

Generating units which may be committed generally consist of thermal units (both nuclear fueled and fossil fired), storage hydro units, pumped storage units, and gas turbine units. Storage hydro and pumped storage units are usually energy limited. These generating units are to be scheduled by the appropriate Conventional Hydro Module (HYDR) and Pumped Storage Module (PSTO). Any thermal units for which fuel availability is restricted will be scheduled by the Fuel Limited

Module (LMTF). Unit commitment as defined in this section excludes the scheduling of any generating units with energy or fuel constraints. Therefore, it will be assumed all the units to be scheduled by the Unit Commitment Module will have surplus available fuel for the commitment period.

Differences in the fuel costs and unit characteristics result in different total costs for different possible unit commitment schedules. Therefore, it is desirable to determine the schedule with a minimum total cost and at the same time satisfies all operating constraints. This is particularly important because of increasing fuel costs and even a small percent reduction in the total cost could result in significant cost savings.

To summarize, unit commitment determines when to start and shutdown generating units which have no fuel constraints such that the total operating cost is minimized and the following constraints are satisfied:

- All system loads
- Various reserve requirements
- Unit operating limits
- Unit minimum up and down times
- Crew constraints
- Transfer limitations between areas
- Environmental requirements
- Contractual agreements
- Other operating requirements.

For systems with energy-limited generating facilities, the above system loads and reserve requirements to be satisfied have been adjusted to account for the generation and reserve contributions by the energy-limited generating facilities. In addition, when observing the transfer limits between areas, the generation for all generating facilities are considered.

4.2.10.2 Method. Module UCOM uses a truncated forward dynamic programming technique to determine economic feasible unit commitment schedules. A description of this method is also presented in Reference 14. Dynamic programming has been used successfully for determining the optimal solutions of many types of discrete multi-stage decision problems such as unit commitment. Figure 4-13 shows a simplified flowchart on the method used in Module UCOM. Commitment of units progresses one hour at a time and a number of feasible combinations of units to be on-line are stored for each hour. Finally, the most economical unit commitment schedule is obtained by backtracking from the unit combination with the least total cost at the final hour to the initial hour.

The unit commitment procedure may be divided into two major parts. The first involves the formation of a unit priority selection list while the second part consists of the dynamic programming or recursive search which determines optimal (least total operating cost) or near optimal feasible schedules for a given time period.

The unit priority selection list reduces the number of unit combinations to be examined at each hour so that computational time does not become too excessive; this is particularly critical for systems with fifty or more units. All available units are grouped into three categories. Category A consists of the base-loaded units and must run units which have been prescheduled or must be kept on-line by virtue of the minimum up time constraints. These are the must run units which are on-line for all combinations. Combustion or gas turbines and quick start units without minimum up and down time constraints form the second category (B). These are the peaking units and are arranged in the order of increasing average operating costs. Such units are added to any combination

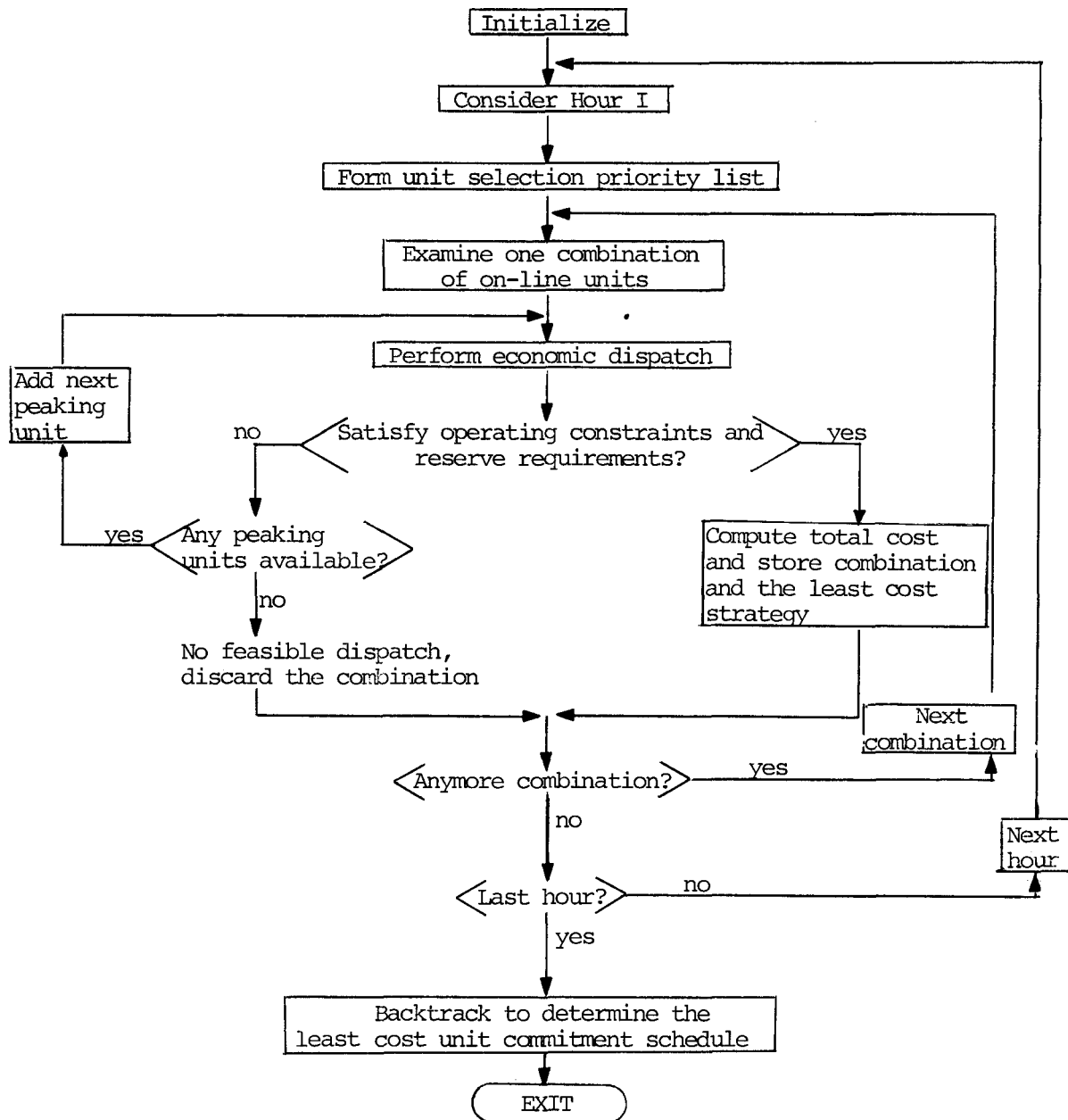


Figure 4-13. Simplified Flowchart of Unit Commitment Procedure

when necessary. The last category (C) is formed by all the remaining available units which can be cycled and they are generally the marginal units. These units are also arranged in a priority order determined by their average operating costs and their status in the saved combinations for the preceding hour.

After completing the priority selection list, generation of unit combinations begins by assuming all marginal units in category C are on-line. When necessary, appropriate peaking units from category B are added to satisfy area reserves and load including the available transfer from other areas. Other combinations of units are then obtained by removing category C units one at a time starting from the last (usually less economical) unit. This generation of unit combinations is stopped when a deficiency in capacity is encountered even after adding all available units in category B (peaking units) or when all category C units have been removed.

Having defined the unit combinations for an hour in the scheduling period, the next step is to determine economic and feasible unit commitment schedules. This is achieved by the use of a recursive search method which is generally known as forward dynamic programming. The attractive features of such an approach are that the search procedure is performed hour-by-hour, it proceeds forward in time, only the information on the last evaluated hour has to be available and it can handle complex constraints.

Let  $COMB(J,K)$  denote the K'th unit combination at hour J.  $COMB(J,K)$  is rejected if no feasible dispatch is possible. A feasible dispatch is one for which the on-line units can satisfy the load, reserves and operating requirements. When  $COMB(J,K)$  yields a feasible dispatch, let  $PCOST(J,K)$  be the production cost to meet the load at hour J, this cost includes all the fuel cost and any operation and maintenance charges.



The overall objective of the unit commitment problem is to minimize the overall total cost, hence the production cost should be minimized also. Numerous methods of economic dispatch are available to determine the minimal production cost. In view of the large number of economic dispatches that would be performed, a simple, non-iterative, feasible and fast economic dispatch procedure is used.

Figure 4-14 shows the overall flowchart of the dispatch method. All units are assumed to have convex piecewise linear generation cost curves, that is, the incremental cost curves are represented by increasing steps. In Module UCOM, no more than two steps or segments are used to represent the incremental cost curve of a unit. This is to keep the computational time to a minimum and results have shown that such representation is suitable for unit commitment purposes. After the commitment schedule has been established, the actual unit incremental cost curves are then used to determine the correct operating cost in the Production Cost Module.

All the incremental cost segments for all the on-line units are arranged in the order of increasing cost. The loading of the units is carried out beginning with the segment having the lowest incremental cost. The dispatch continues by loading succeeding segments as much as possible but without violating any operating constraints. The process stops when the desired generation is met or when all the segments have been dispatched. Whenever feasible dispatches exist, the method will always find the one with the minimum production cost.

Having determined the production cost  $PCOST(J,K)$  for  $COMB(J,K)$ , the search method proceeds to find the least cost strategy to arrive at  $COMB(J,K)$  by examining the transitions from all combinations for the previous hour ( $J-1$ ). A transition from  $COMB(J-1,L)$  to  $COMB(J,K)$  could involve the startup and/or shut-

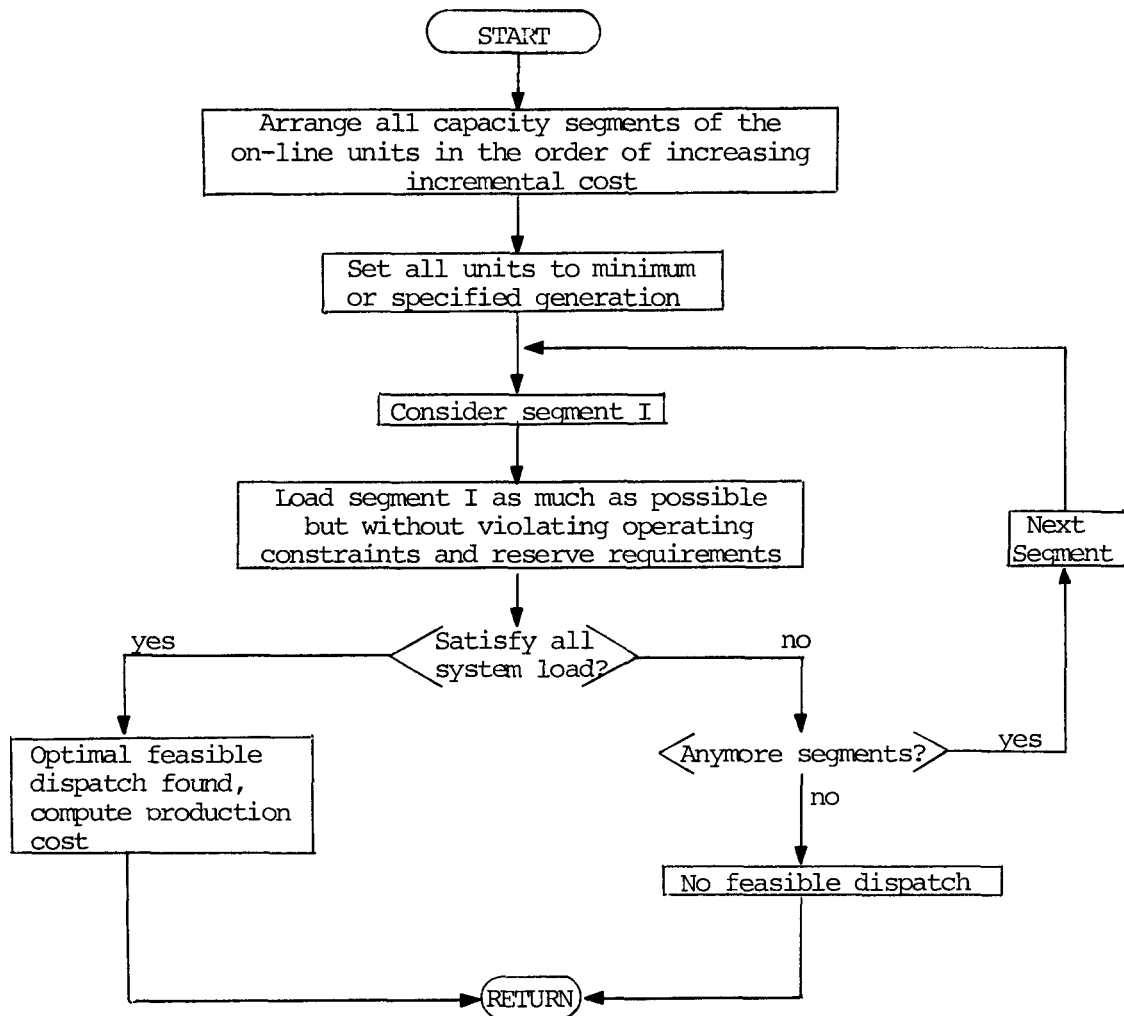


Figure 4-14. Flowchart of the Dispatching Method

down of certain units and costs may be incurred with these changes; let this transitional cost be  $SCOST(J-1,L:J,K)$ . If the least total cumulative cost incurred to arrive at  $COMB(J-1,L)$  is  $TCOST(J-1,L)$ , then the least total cumulative cost required to arrive at  $COMB(J,K)$  through  $COMB(J-1,L)$  is equal to the sum of  $TCOST(J-1,L)$ ,  $PCOST(J,K)$  and  $SCOST(J-1,L:J,K)$ . Therefore, by examining this sum for every combination at hour  $J-1$ , (i.e., over all  $L$ ), the least cumulative total cost,  $TCOST(J,K)$ , required to reach  $COMB(J,K)$  can be found. The equation for the determination of  $TCOST(J,K)$  is given below:

$$TCOST(J,K) = \min_{\{L\}} \{TCOST(J-1,L) + SCOST(J-1,L:J,K) + PCOST(J,K)\}$$

where  $\{L\}$  is the set of combinations at hour  $J-1$ . In this way, the least cost strategies to reach the various combinations at hour  $J$  are determined.

After the least total cumulative cost for each combination at hour  $J$  has been found, the search procedure continues with the next combinations at the next hour, as shown in Figure 4-13. The search procedure terminates at the final hour of the scheduling period. The most economical unit commitment determined by the search is obtained by back-tracking the strategies from the combination with the least cumulative cost at the final hour to the initial hour of the scheduling period.

The speed of execution of Module UCOM is very sensitive to the number of unit combinations examined and saved at each hour, especially when scheduling systems with more than fifty units. To ensure reasonable computer running time, a maximum number of unit combinations which are credible candidates for the economic unit commitment are saved at each hour while those combinations which apriori can be said to be non-economical are discarded before going to the next hour.

#### 4.2.11 Production Costing Module - PROD

Module PROD is used after the final unit commitment schedule has been established and it computes a more precise value of the total production cost. The economic dispatch method is exactly the same as that in Module UCOM. However, the fuel rate characteristics of a unit can now be represented by up to seven segments of incremental fuel rates rather than two segments. The dispatch observes the same operating constraints and reserve requirements as those in Module UCOM.

### 4.3 RESULTS ON SAMPLE TEST SYSTEM

Prototype computer programs on the search approach have been coded in FORTRAN for testing purposes. Preliminary tests were performed using a sample test system to verify program logic before any test was done on the New York system. Data on the sample test system and the final output for one of the test cases are given in Volume 2.

#### 4.3.1 Description of Sample Test System

The sample test system consists of four distinct areas connected in a chain. Transfer or tie capacities between the areas are assumed to be large so that there is no constraint on the transfer of power from one area to another. This is not a necessary requirement since the unit commitment module, UCOM, is capable of recognizing any transfer limits between areas in the scheduling and dispatching of generating units. The identity of the areas is still needed since there are reserve requirements associated with the areas.

Total system hourly loads are available for the period Monday, March 26, 1979, to the following Monday, April 2, 1979, and the system peak load is 1820 MW. It is assumed that each area hourly load is proportional to the total system hourly load, but this is not a necessary condition since individual area hourly loads

can be used instead. The proportions on the system load for the four areas are 0.3762, 0.0619, 0.3437 and 0.2182.

There are three categories of reserves: 10-minute spinning or synchronous reserve, 10-minute non-synchronous reserve and 30-minute operating reserve. All areas except Area 2 have all the above three reserve requirements. Area 2, with the smallest load, has no reserve requirements at all. These reserve requirements are fixed for the whole scheduling period.

The test system has 25 thermal units without any fuel constraints and these shall be scheduled by the unit commitment module. The units are of two types: fossil fired steam and gas turbine. There are no minimum up and down time constraints on the gas turbines. Furthermore, they can be started and loaded to full capacity within ten minutes. Unit size ranges from 40 MW to 235 MW and units are located at the four areas.

There are two conventional storage hydro plants coupled together and both are located in Area 1. The upstream plant has two 50 MW units while the downstream plant has two 30-MW units. It is assumed that the upstream plant has a constant inflow of 15 KCFS\* and all the discharge from this plant flows into the other plant which has no natural inflow.

Located in Area 3 is the system's only pumped storage plant with two identical reversible units having both pumping and generating capacities of 50 MW. The plant cycle efficiency of the plant is 74%. Operation of the plant is such that the upper storage reservoir must be full at the beginning and the end of the scheduling period.

\*KCFS: Thousand cubic feet per second)

There exists a special plant, a scaled-down version of the Niagara Falls Project, which has a pumped storage plant and a hydro plant (see Section 4.2.8). Thus, this plant will be scheduled by Module NFAL. The capacities of the hydro and pumped storage facilities are 200 MW and 30 MW respectively. Natural inflow can either be pumped directly into the storage reservoir or be allowed to pass through the hydro plant for power generation. Any release of water from the reservoir will be used twice for power generation: the first time at the pumped storage plant and the second time at the hydro plant. This special plant is located in Area 2.

There are two separate thermal units in a plant which has maximum and minimum limits on the total amount of fuel which can be used within the scheduling period. This plant is in Area 4 and is to be scheduled by Module LMTF. The capacities of the two units are 100 MW and 90 MW. The larger unit is the more efficient one and therefore will be loaded first before starting the smaller unit.

#### 4.3.2 Results for Sample Test Cases

Results for three test cases are presented in this section. All three cases include the scheduling of the 25 thermal units and two of the energy limited facilities. The scheduling period for Cases 1 and 3 is 24 hours while that for Case 2 is 168 hours.

For all the cases, the scheduling process terminates when the same unit commitment schedule is obtained for consecutive iterations. One iteration consists of the scheduling of all energy limited facilities followed by the commitment of thermal units with no fuel limits. Results show that the total system cost is reduced for each successive iteration until the process finds the same unit commitment schedule.

4.3.2.1 Results for Sample Test Case 1. The scheduling period is 24 hours and it begins from 8:00 a.m. on the first Monday to 7:00 a.m. the following Tuesday. Generating facilities scheduled are the 25 thermal units, the two coupled hydro plants and the pumped storage plant. The remaining two energy limited facilities are assumed to be unavailable.

The scheduling process terminates after three iterations since the second and third iterations produced identical schedules. Table 4-1 shows the total system operating cost for each scheduling iteration. For this particular case, a reasonable schedule was obtained even at the first iteration in that the final schedule resulted only \$287 savings compared to the total cost of \$387,950. Outputs for the final schedule for this case are presented in Volume 2.

Table 4-1  
RESULTS FOR SAMPLE TEST CASE 1

Iteration	Total System Cost
1	\$388,237
2	\$387,950
3	\$387,950

4.3.2.2 Results for Sample Test Case 2. The generating facilities to be scheduled for Case 2 are exactly the same as those for Case 1, namely, all 25 thermal units, the hydro plants and the pumped storage plant. However, the scheduling period is extended to 168 hours (1 week) instead of 24 hours. This time period is from 8:00 a.m. on the first Monday to 7:00 a.m. the following Monday.

The same unit commitment schedule was obtained for the second and third scheduling iterations, which have practically the same total system cost. This represents a reduction of \$2,100 from the first iteration as can be seen in Table 4-2.

Table 4-2

## RESULTS FOR SAMPLE TEST CASE 2

Iteration	Total System Cost
1	\$2,584,100
2	\$2,581,980
3	\$2,582,000

4.3.2.3 Results for Sample Test Case 3. The scheduling period for Case 3 is the same 24 hours as that for Case 1, i.e., from 8:00 a.m. on the first Monday to 7:00 a.m. the following Tuesday. All the 25 thermal units are scheduled by Module UCOM but the energy limited plants to be scheduled are the hydro-pumped storage plant which is a scaled down version of the Niagara Falls Project and the two-unit thermal plant with limits on fuel use.

Table 4-3 gives the total system cost for each iteration. Scheduling stopped at the fourth iteration which has the same unit commitment schedule as that for the preceding iteration. This case shows a significant saving in the total system cost from the first to the final iteration, being a sum of \$5,331 compared to the total system cost \$347,157 or an improvement of 1.54%. The main bulk of this saving occurs at the second iteration. From the results for the three test cases, it appears that two iterations will be adequate to achieve most of the savings in the scheduling of the sample test system. More test cases should be carried out to establish any such trend when scheduling any specific systems since the occurrence of this type of predictable phenomena could reduce the number of insignificant scheduling iterations.



Table 4-3

## RESULTS FOR SAMPLE TEST CASE 3

Iteration	Total System Cost
1	\$352,488
2	\$347,415
3	\$347,192
4	\$347,157

#### 4.4 RESULTS ON NEW YORK TEST SYSTEM

To provide more realistic performance evaluation of the search approach, additional tests were carried using the New York system which is representative of a large system with predominately thermal units and a smaller proportion of energy limited facilities comprising of hydro and pumped storage plants. The data used in the tests are essentially the same as those for the New York system, but not completely identical. For example, the reserve requirements for the actual system generally depend on the load and operating condition while the tests assume fixed reserve requirements through the scheduling period. However, such differences do not in any way affect the performance evaluation of the scheduling method though the resulting schedules and total system costs may be different. No attempt is made to duplicate the operating conditions of the actual New York System and the test results bear no relation whatsoever to the actual schedules or costs.

##### 4.4.1 Description of New York Test System

The New York system consists of the following seven major utilities within New York State and the Power Authority of the State of New York (PASNY):

- Long Island Lighting Company (LILCO)
- Consolidated Edison Company (CONED)
- Orange and Rockland Utilities (OR)
- Central Hudson Gas & Electric Corporation (CHGE)
- New York State Electric and Gas Corporation (NYSEG)
- Niagara Mohawk Power Corporation (NMPC)
- Rochester Gas and Electric Corporation (RGE)

PASNY operates a number of major generation facilities including the Niagara Falls Project, St. Lawrence Hydro Project, Gilboa Pumped Storage Project and a few larger thermal units. However, PASNY does not have any load and it sells bulk power to various utility companies and municipal agencies.

The system is divided into five transmission areas which form a chain as shown in Figure 4-15. LILCO and CONED are located downstate and they represent the first two areas. The other three areas stretch across upstate New York. NMPC is divided into three geographical regions: eastern (NME), central (NMC) and western (NMW). CHGE, OR, and NME together form the third area. The fourth area is made up of NYSEG and NMC while the remaining fifth area is comprised of RGE and NMW.

Actual hourly loads from July 17 to July 31 of 1978 are used for the series of tests. The period covers two complete weeks and scheduling is performed for one week at a time, starting from 8:00 a.m. on a Monday to 7:00 a.m. the following Monday. This period is chosen because it resembles a fairly typical condition in which there is no acute shortage or surplus of capacity and water. Figures 4-16 and 4-17 show the plot of the system hourly loads for the two weeks. Within this period the system peak load is 20,107 MW and the individual area peak loads are given in Table 4-4. The system load is divided fairly equally

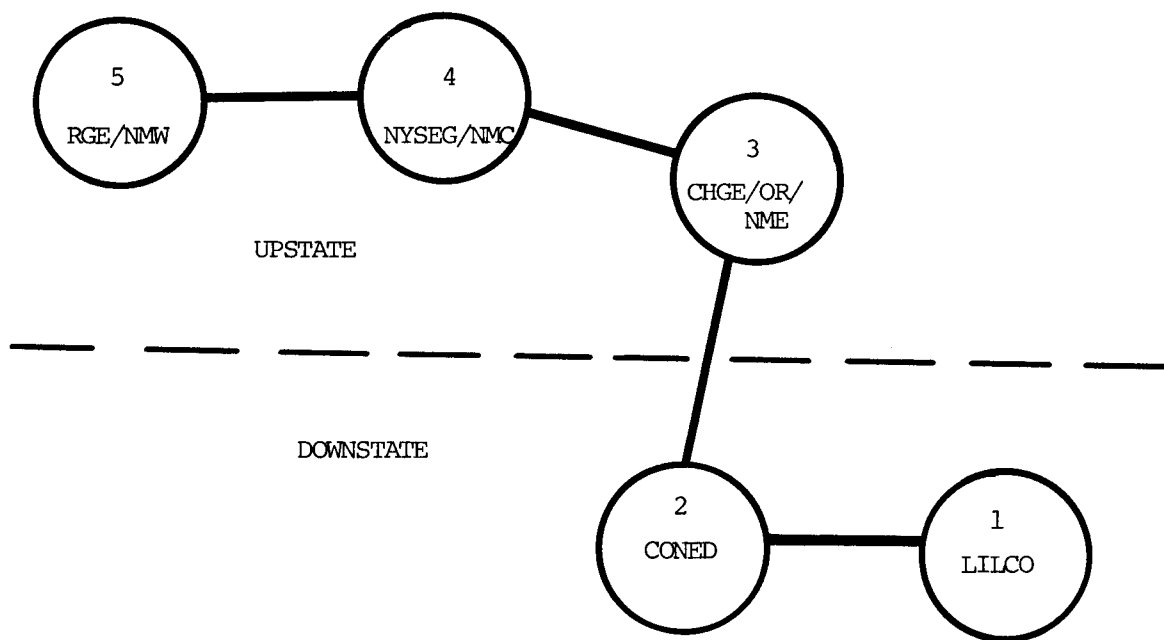


Figure 4-15. Transmission Areas of New York System

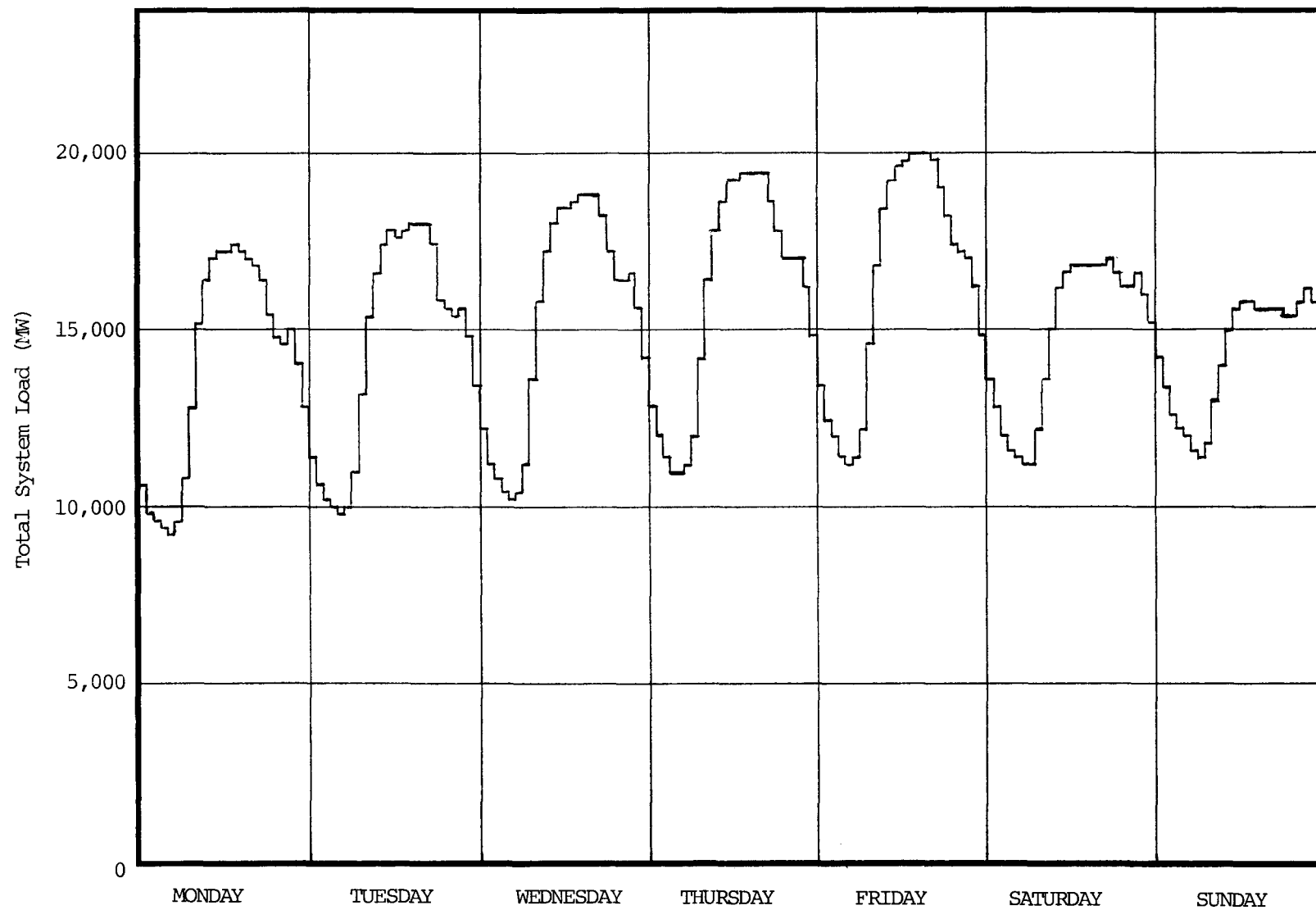


Figure 4-16. Total New York System Load for Week of July 17, 1978

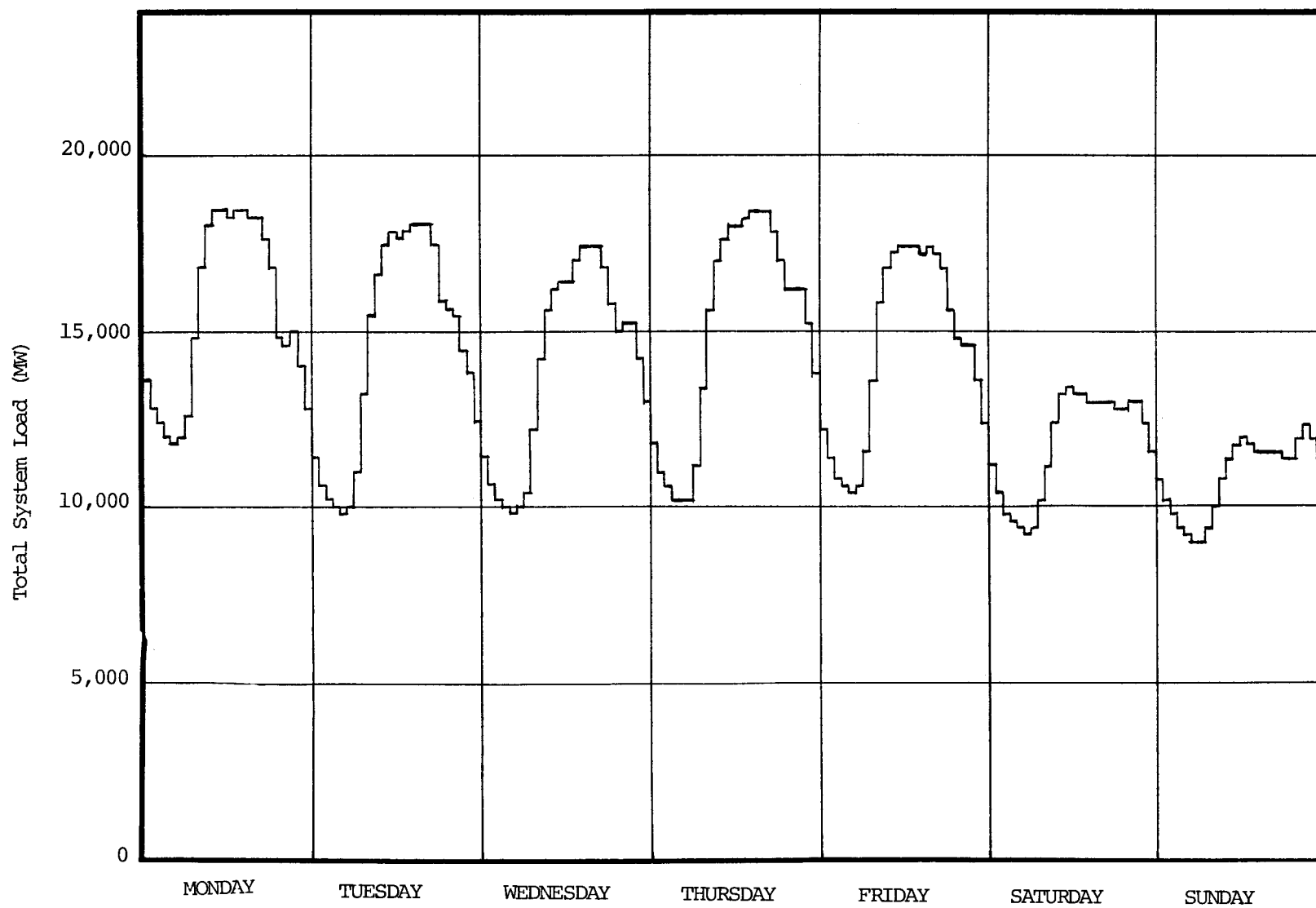


Figure 4-17. Total New York System Load for Week of July 24, 1978

between the downstate and upstate regions. All area loads, except the load for Area 2, are of about the same magnitude. Area 2 load is slightly more than double the load of any other area.

Table 4-4  
AREA AND SYSTEM PEAK LOADS

Area	Peak Load (MW)
1	2,691
2	7,657
3	2,806
4	3,480
5	3,554
System	20,107

General descriptions of the generating facilities are given below while detailed data on each facility are presented in Volume 2. Table 4-5 shows the system capacity distribution by area and by type. Most of the fossil steam units are oil-fired. There are some coal-fired steam units all of which are located in upstate. The bulk of the gas turbine units, which include all quick start diesel and jet units, are in downstate. Furthermore, gas turbine capacity represents almost 20% of the downstate total capacity. There are altogether 111 thermal units which include the 5 nuclear units and all the gas turbine units. All the hydro and pumped storage capacities are in the three upstate areas. Most of these capacities are from three major projects: Niagara Falls Project with 2635 MW, St. Lawrence Project with 912 MW and Gilboa Pumped Storage Project with 1000 MW. The remaining 853 MW hydro capacity are represented by fifteen equivalent hydro plants with capacity sizes ranging from 20 MW to 144 MW. Each equivalent plant is characterized by a water conversion coefficient, mini-

minimum and maximum generating capacities and a reservoir with minimum and maximum storage capacities. Except for the Niagara Falls Project inflows, the average water flow through each hydro plant is obtained from the actual log on the river flow and/or generation for the period July 17 to July 31, 1979.

Table 4-5  
SYSTEM CAPACITY DISTRIBUTIONS

Generating Capacity (MW) by Type					
Area	Fossil Steam	Nuclear	Gas Turbine	Hydro *	Total
1	2,718	0	977	0	3,695
2	5,419	1,770	1,388	0	8,577
3	3,626	0	346	1,317	5,289
4	2,317	1,499	8	1,368	5,192
5	1,724	415	29	2,715	4,883
System	15,804	3,684	2,748	5,400	27,636

\*Includes pumped storage generating capacity

In general, the more economical generating units are found in upstate so that normally there is a transfer of economical power to downstate. The transmission capabilities between the areas are sufficiently large to permit such transfer of economical power.

A fuel limited plant with two 100 MW thermal units is added to the New York test system so that Module LMTF could be used in this series of tests. Though this plant does not exist physically, it is possible that a situation could arise in which the operation of certain generating plants is also constrained by the availability of fuel.

Two large fictitious units (2500 MW each) have been included in the New York test system. These are introduced as possible sources of emergency power which are available to the system under conditions of inadequate capacity. This is a

realistic model since the New York system is interconnected with other utility systems in the Northeast and Canada. As far as the results of the test cases are concerned, these two fictitious units have no influence whatsoever since the system has adequate capacity and these units were not scheduled in all the cases. For actual applications, all contracts with outside systems should be modeled correctly.

In scheduling the New York test system, it is required to maintain a fixed total reserves of 1500 MW distributed equally among the three categories of reserves: 10-minute synchronous reserve, 10-minutes non-synchronous reserve and 30-minute reserve. Each category of reserve requirement is again divided between the five areas in the system. In actual operation of the system, the amount of reserve requirements varies from hour to hour depending on the operating condition.

Currently, there exists a number of contracts among the utilities themselves, and with utilities outside of New York State. All these contracts are ignored in the performance of the tests; that is, the system is to be scheduled as a single isolated system with the objective of minimizing the total system cost. This is in line with the motive of using the New York system for the sole purpose of evaluating the performance of the scheduling method which has been implemented on a set of prototype computer programs.

#### 4.4.2 Results for New York Test System

Results on the scheduling of the New York test system for four test cases are presented below. The scheduling period for all cases is one week or 168 hours covering from 8:00 a.m. on a Monday to 7:00 a.m. the following Monday. Two cases assume the average water flow rate for the period while the other two cases assume two-thirds water flow rates for all the hydro plants in the system. The four test cases are:



1. Week of July 17, 1978 and average water flow rates.
2. Week of July 17, 1978 and two-thirds average water flow rates.
3. Week of July 24, 1978 and average water flow rates.
4. Week of July 24, 1978 and two-thirds average water flow rates.

All the scheduling modules are used in each test case.

All the tests were performed on the PTI PRIME 650 time-sharing computer whose computational speed is about 40% that of an IBM 370/155 computer. Due to the long computing time, the maximum number of scheduling iterations is limited to four, even though more economical schedules could be obtained by executing additional iterations. The scheduling process is also terminated when the iteration produces a schedule with a higher total system cost than that for the preceding iteration.

Tables 4-6 to 4-9 give the total system cost at each iteration for the four cases. Cases 1 and 4 show successive cost reduction with each iteration with the largest reduction or about 90% of the total reduction occurring at the second iteration. Such behavior has also been observed for the sample system test cases. Cases 2 and 3 show a decrease of the total system cost at the second iteration but there was a slight increase in the cost at the third iteration after which the scheduling process was terminated. Such a behavior could be attributable to the significant reserve contribution capability of the hydro facilities in the system and the thermal cost curves used for scheduling these facilities do not reflect any effect of the reserve requirements. This particular problem is discussed in greater detail in Section 6.4. Nevertheless, the lowest total cost obtained from schedules resulted in savings of \$151,600, \$50,300, \$300,200 and \$210,000 for Cases 1, 2, 3 and 4 respectively in relation to the first iteration schedules.

Table 4-6

RESULTS FOR NEW YORK TEST CASE 1

Iteration	Total System Cost
1	\$49,796,700
2	\$49,654,800
3	\$49,648,900
4	\$49,645,100

Table 4-7

RESULTS FOR NEW YORK TEST CASE 2

Iteration	Total System Cost
1	\$54,125,200
2	\$54,074,900
3	\$54,090,800

Table 4-8

RESULTS FOR NEW YORK TEST CASE 3

Iteration	Total System Cost
1	\$41,117,600
2	\$40,817,400
3	\$40,870,600

Table 4-9  
RESULTS FOR NEW YORK TEST CASE 4

Iteration	Total System Cost
1	\$44,616,100
2	\$44,441,400
3	\$44,421,000
4	\$44,406,100

When less water is available for the hydro plants, the total system cost will increase since more costly generation will be used to cover the reduction in hydro energy which is essentially free. For the test cases, a reduction of one-third hydro energy resulted in about \$4,000,000 additional expense per week.

The amount of available hydro energy for the two weeks is about the same, however the total system cost for the first week (Cases 1 and 2) is considerably much higher than that for the second week (Cases 3 and 4). Such difference is essentially due to the higher load demand during the first week as can be seen in Figures 4-16 and 4-17.

The average computational time for each scheduling iteration was 2639 CPU seconds on the PRIME 650 computer. Scheduling of all the energy limited generating facilities (hydro, pumped storage and fuel limited plants) average 254 CPUS while the remaining 2385 CPUS was taken up in establishing the commitment schedule of the 113 thermal units. Thus, for the New York test system, about 90% of the computing time is used in Module UCOM.

Had an IBM 370/155 computer been used, one would expect the average computing time for each of the scheduling iteration reduced to 1055 CPUS or almost 18 CPU-minutes. By improving the coding of the computer program, computing time

will be reduced since no special effort was directed to make the prototype programs to execute as efficiently as possible. The above computing time performance is well within the acceptable limit for using such programs on a daily basis. With the availability of faster and more powerful computers, computing time will become less and less of a constraint on the use of the presented search approach for daily fuel scheduling of large systems, such as the New York system.

## Section 5

### SOLUTION TO THE FUEL SCHEDULING PROBLEM BY MIXED INTEGER LINEAR PROGRAMMING

This section covers the efforts to achieve scheduling solutions using mixed integer-linear programming (MILP). Thermal unit commitment and dispatch, hydro and pumped storage unit scheduling, together with startup/shutdown, spinning reserve, and fuel-energy limitation effects can be set up as an MILP problem. However, few investigations have been made into the structure of an adequate algorithm to solve the extremely complex MILP's required for a real power system.

To start, an extremely simple unit commitment problem was set up with startup/shutdown constraints and a solution was attempted by a standard one-zero integer programming solution algorithm. It took almost an hour of computer time to reach a solution. We were, however, able to make dramatic solution time reductions by adapting the algorithm to our problem.

The standard one-zero integer programming algorithm was eventually enhanced to the point where it could utilize a linear programming algorithm to solve (relaxed) sub-problems thus forming a specialized MILP algorithm. The final research then concentrated on enhancing the linear programming code to take advantage of constraint matrix sparsity.

Ultimately the MILP might be able to solve the entire fuel scheduling problem by itself. However, a more practical and promising scheme is to couple it to the search approach presented in Section 4. The MILP algorithm could also be used

in place of the incremental search method in the individual scheduling modules for the search approach.

#### 5.1 REVIEW OF MILP SOLUTIONS TO POWER SYSTEM SCHEDULING PROBLEMS

The earliest applications of integer programming and mixed integer-linear programming to unit commitment problems were the papers of Garver [22] and Muckstadt and Wilson [4]. More recent papers by Harhammer [29] and Dillon et al. [5] have extended the MILP formulation to cover the following objectives:

- Optimal thermal operating cost
- Hydro thermal coordination
- Hydro coordination
- Energy interchange contracts

While accounting for the following effects and constraints:

- Piecewise linear thermal operating cost functions and hydro plant characteristic curves
- Rate of change limits
- Energy limits
- Minimum up and minimum down times
- Variable startup and shutdown costs
- Reserve constraints
- Security constraints
- Hydro plant storage constraints
- Pumped hydro modeling with reservoir constraints.

Harhammer stresses the advantage of the MILP approach because solutions can be obtained using standard computer codes. Dillon et al. have indicated that special adaptations are required and include:

- Derived constraints using knowledge of the unit commitment problem
- Time decoupling
- Priority lists and penalty factors in branching decisions
- Forcing initial feasible solutions
- Scaling of integer variables
- Merit ordering of units for large commitment problems.

Harhammer solved only very small sample problems of the order of five thermal units and two pumped storage plants for seven time periods. Dillon et al. solved a problem with 16 units for 24 time steps. Neither of these examples is representative of a solution of a practical sized system. For example, the New York Power Pool unit commitment program which uses a dynamic programming technique, is capable of solving problems with 200 units and 50 interchange "blocks" for 180 time steps and includes checks of internal power transfers for security purposes, it does not include scheduling of hydro units or pumped storage units.

## 5.2 SOLUTION BY ONE-ZERO PROGRAMMING

Three different approaches were taken for the solution of a basic unit commitment problem using the powerful enumerative one-zero method of Balas [28]. All test cases were performed on the PTI in-house PRIME 400 time-sharing computer and all computing times quoted below are for this computer.

The problem was to commit thermal units to meet a sequence of hourly MW loads while minimizing total fuel load costs. For this particular example, the cost associated with each unit was equal to the maximum capacity of the unit. Also

included in the problem definition are the enforcement of unit minimum up and down time rules and the effects due to the initial conditions of the units. The problem can be formulated as follows:

$$\text{Minimize } \sum_{i=1}^N \sum_{t=1}^T C_i^t \cdot U_i^t$$

Subject to:

1.  $U_i^t = K_i^t$  for some  $i, t$  {initial constraints}
2.  $S_i^t = U_i^t - U_i^{t-1} + Y_i^t$  for all  $i, t$  {relationship constraints}
3.  $\sum_{\tau=t}^{t+M_i^u-1} U_i^\tau \geq M_i^u \cdot S_i^t$  for all  $i, t$  {minimum uptime constraints}
4.  $\sum_{\tau=t}^{t+M_i^d-1} (1-U_i^\tau) \geq M_i^d \cdot Y_i^t$  for all  $i, t$  {minimum downtime constraints}
5.  $\sum_{i=1}^N \bar{P}_i \cdot U_i^t \geq P_L^t$  for all  $t$  {load constraints}
6.  $U_i^t \in I$  for all  $i, t$  (1)
7.  $S_i^t \in I$  for all  $i, t$
8.  $Y_i^t \in I$  for all  $i, t$
9.  $K_i^t \in I$  for some  $i, t$

(1) Read  $U_i^t$  is a member set  $I$



All the variables in the above expressions have been defined in Section 2.3.1 except for the following four variables:

$C_i^t$  = cost associated with unit  $i$  at hour  $t$

$I = \{0,1\}$

$K_i^t$  = variable for initial condition constraints associated with unit  $i$   
at hour  $t$

$N$  = number of units

The Balas algorithm has been used extensively as a solution method of binary variable problems due to its versatility in adapting the nature of the problem into the solution formulation. Figure 5-1 shows the flowchart of the algorithm. Because unit commitment essentially implies the determination of each unit's on/off status, the Balas algorithm appears to be a likely solution method candidate. Three implemented and analyzed adaptations of Balas are summarized. Four problems of different sizes were tested (problem size is given by the product of the number of units and the number of hours: 2 units for 2 hours, 3 units for 4 hours, 3 units for 5 hours, and 5 units for 10 hours).

The first approach can be characterized as the straight-forward approach in that all aspects of the general problem formulation are explicitly modeled. Unit hourly startup and shutdown variables as well as unit on/off variables are incorporated into the objective function. The constraints include the effects due to each unit's initial conditions, the relationships among unit hourly startup, shutdown and on/off variables and the load constraints. The startup, shutdown and on/off variables are free variables which are systematically changed one per iteration until the optimum solution is achieved. The only advantage to this first approach is the ease with which the algorithm can be implemented.

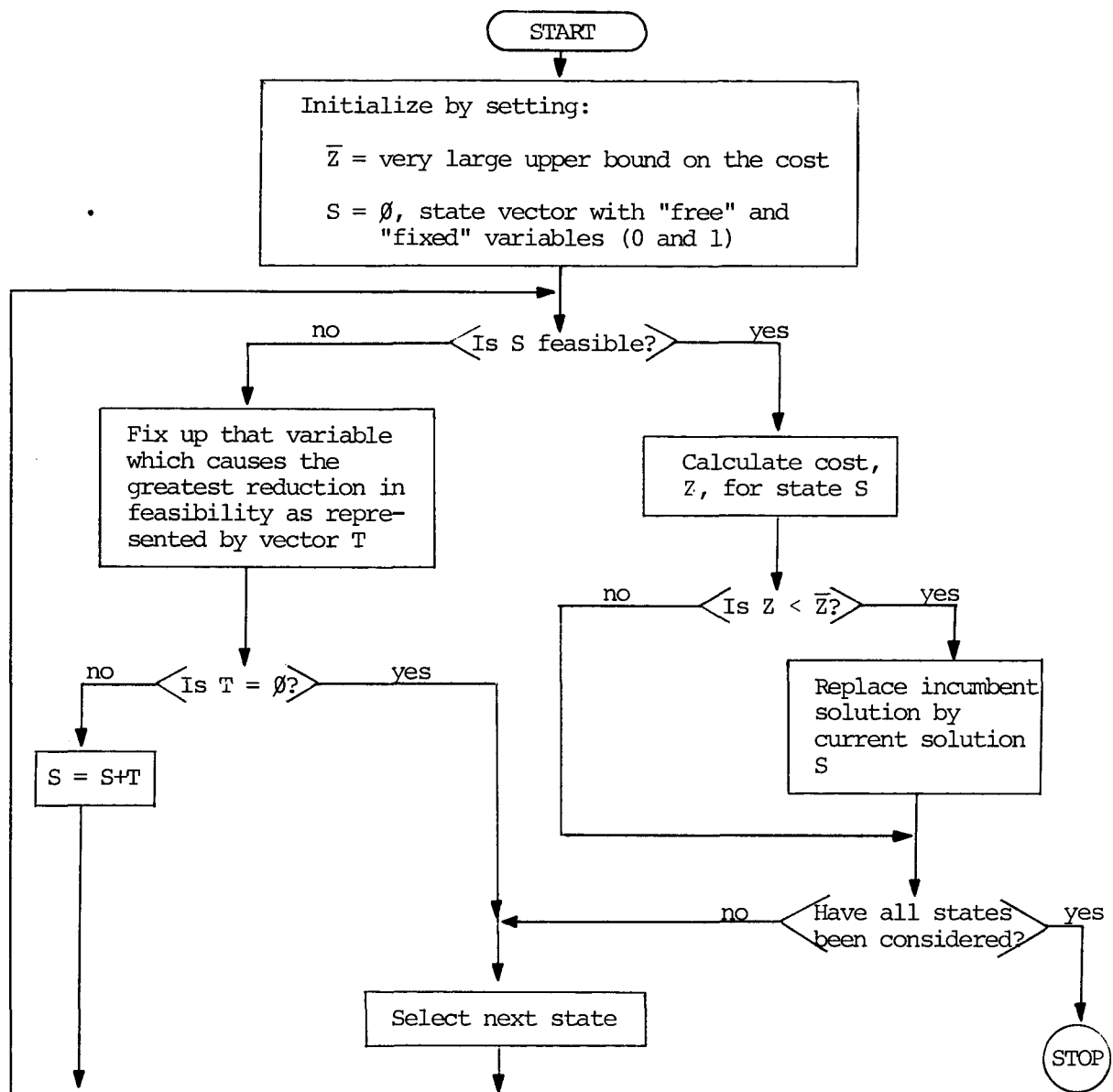


Figure 5-1. Flowchart on the Balas Algorithm

The disadvantage of the first approach results from the lengthy execution times as exemplified by the 3000 CPU seconds required to solve a unit commitment problem of 3 units and 4 hours (problem size of 12). Several factors contribute to these unreasonable execution times. These include the presence of explicitly defined unit hourly minimum up and down time constraints and initial condition constraints. However, the major contributors to the time expended are the inclusion of startup and shutdown variables into the objective function and the restriction that only one variable change per iteration. The additional variables increase the total number of possible enumerations to two raised to the power of three times the number of units times the number of hours. Changing one variable per iteration implies that each branch is examined one node at a time which is unreasonable when considering the number of possible enumerations.

The unattractive execution times resulted in the revision of the algorithm. In this second approach the minimum up and down time constraints are implicitly defined thus removing the startup and shutdown variables from the objective function and deleting unit hourly minimum up and down time constraints. This is accomplished at each iteration by checking for unit minimum up and down time violations along with unmet load constraints. Then if a violation is detected, the free variable which most aids the load constraints and secondly aids the violations is selected as the next node for branching. In this approach the objective function to be minimized is  $\sum_{i=1}^N \sum_{t=1}^T C_i^t \cdot U_i^t$ . The only constraints explicitly defined are the load constraints.

The advantages of this method is the removal of startup and shutdown variables from the objective function. This reduces the number of possible enumerations to two raised to the power of the number of units times the number of hours and hence reduces execution times. The disadvantage of this approach is that the

execution times though lower in CPU seconds than the first method are still lengthy as exemplified by about 3300 CPU seconds required to commit 5 thermal units for 10 hours (problem size of 50). This again reflects the tedious nature of selecting one variable setting per node.

Unfortunately, the second approach also required lengthy execution times, hence a third approach to the algorithm was developed. This approach is similar to the second except that each node may consist of several settings of variables. This is accomplished by selecting the first/next unit with lowest average full load costs. Then this unit is chronologically committed in blocks of hours which conform to the unit's minimum up and down time rules. If it is determined uneconomical to commit a particular block of hours then the unit is left uncommitted during the first hour of the block through as many hours as are required to satisfy the minimum down time rule. Then the block starting with the next hour is examined for commitment. The process continues until all load constraints are met or all hourly blocks are exhausted. Thereafter, backtracking occurs and the procedure is reiterated.

This method offers the advantage of reasonable execution times, for example, 305 CPU seconds are required to commit 5 units for 10 hours. This is because each node represents multiple unit on/off variables and thus each iteration is feasible from minimum up and down time rules.

The computer running time for each of the three methods is compared on the graph in Figure 5-2. The plots clearly illustrate tremendous reduction in computing time when the special structure of the problem has been incorporated into the algorithm. For all three methods, the computing time increases exponentially with the size of the problem.

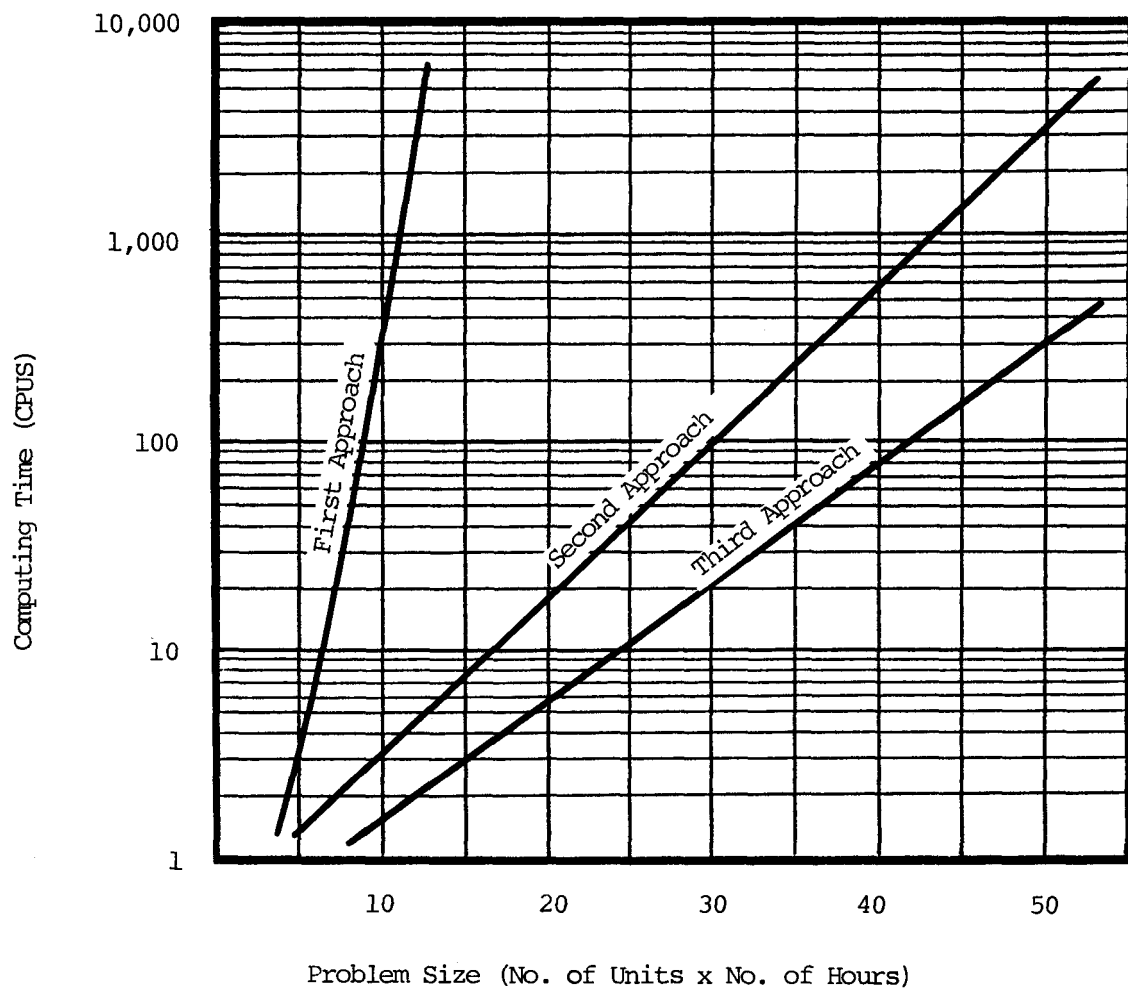


Figure 5-2. Computer Execution Times for Balas Algorithm

In summary, the reductions shown were obtained by imbedding the characteristics of the simple unit commitment problem into the algorithm logic. It is our firm belief that only by following this path can a truly optimal scheduling program be written which has sufficient capacity for realistic problems.

### 5.3 SOLUTION BY MILP METHOD

The MILP algorithm determines the optimum solution to a mathematical problem containing both continuous and integer variables. It does this by a process of "fathoming" wherein a standard LP algorithm is used to solve a series of LP problems each having some of the integer variables fixed at one or zero and the remaining variables treated as continuous variables where they are allowed to become any value between zero and one. Just how this process is carried out will be explained using the example in Table 5-1 and Figures 5-3 and 5-4.

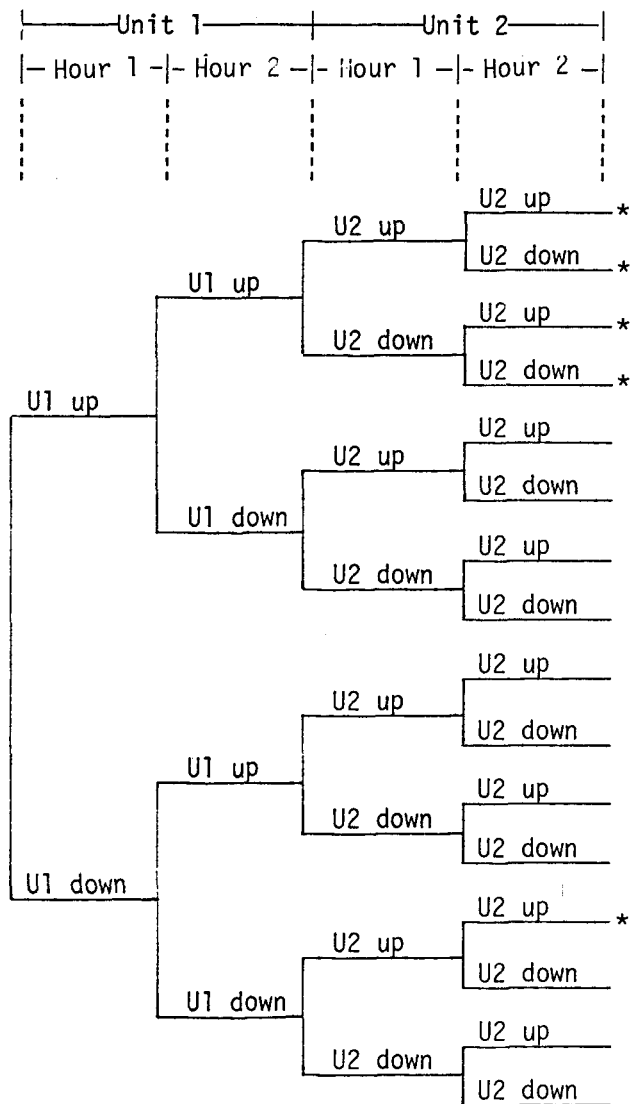
In Table 5-1 we have set up a sample system consisting of two units to meet the load profile spanning a two-hour period. Either unit can supply the load in either hour but certain unit combinations are not allowed due to minimum up and down time restrictions. Figure 5-3 shows the complete "tree" with all the possible combinations or solutions with the two units for two hours. Out of the sixteen solutions, only five are feasible and these are shown with an asterisk in Figure 5-3.

The entire optimization technique can be summed up as one of finding the least cost feasible combination. The difficulty in finding the optimal combination arises because there are usually so many combinations. It would be futile to simply test them all and record their cost. Therefore, some systematic procedure must be used which searches through the tree until the optimum combination is found. This search procedure is illustrated in Figure 5-4. At the start of the procedure we have all integer variables free (i.e., we have not assigned any

Table 5-1

A SAMPLE SCHEDULING PROBLEM WITH TWO UNITS FOR TWO HOURS

	Unit 1	Unit 2
Maximum capacity	$\bar{P}_1 > P_L^1$	$\bar{P}_2 > P_L^1$
Minimum capacity	$\underline{P}_1 < P_L^2$	$\underline{P}_2 < P_L^2$
Minimum uptime	2 hours	1 hour
Minimum downtime	2 hours	2 hours
Operating cost	$C_1(P_1^t)$	$C_2(P_2^t)$
Maximum fuel limit	$E_1$	$E_2$
Starting condition	Down 2 hours	Up 2 hours
Ending condition	Down after 2 hours	Free



U1 = Unit 1

U2 = Unit 2

\* = Feasible solution

Figure 5-3. Complete Unit Commitment Tree for Sample Problem



units on or off in either hour of the sample problem). To start the problem off, we commit unit 1 for hour 1 and then run an LP with all other integer variables free. If this first LP gives a feasible solution, we continue down that "path" (or "branch"). If the LP gave an infeasible solution, we would know that fixing any integer variables at 1 or 0 (i.e., going further down that path) would also be infeasible so the algorithm would go back up the tree and follow a different path. In Figure 5-4 we see the program running an LP at each "vertex" or "node" of the tree until it finds a feasible solution. From then on the algorithm will only pursue solutions further down from a vertex if that vertex's LP gives a solution with lower cost than the already established optimum (see step N+2 in Figure 5-4).

This procedure is the standard "branch-and-bound" algorithm of solving an MILP problem. However, to speed up this search we coupled the linear programming code to the Balas one-zero block method described in Section 5.2. Figure 5-5 shows a simple flowchart of this method wherein the vertices which will be tested by the LP are selected by the Balas block method logic. At the top of Figure 5-5 we have sketched the fathoming procedure. Instead of running an LP at each vertex as in the standard branch-and-bound procedure, the Balas block logic simply finds the first feasible vertex and hands it to the LP (step 1). It then backs up several vertices and starts down another path before handing the LP another problem to solve. The Balas method thus eliminates running LP's at vertices which do not meet up/down time and loading restrictions. Hopefully, the LP will eliminate futile search by showing cost relationships indicating which paths are not going to give a lower cost than the optimum already found.

An algorithm was built according to the logic of Figure 5-5 which was capable of solving small fuel limited dispatching problems. Two sample solutions are given in Figures 5-6 and 5-7. In the sample on Figure 5-6 a four hour load curve is

Solve the Commitment and Dispatch as one problem:

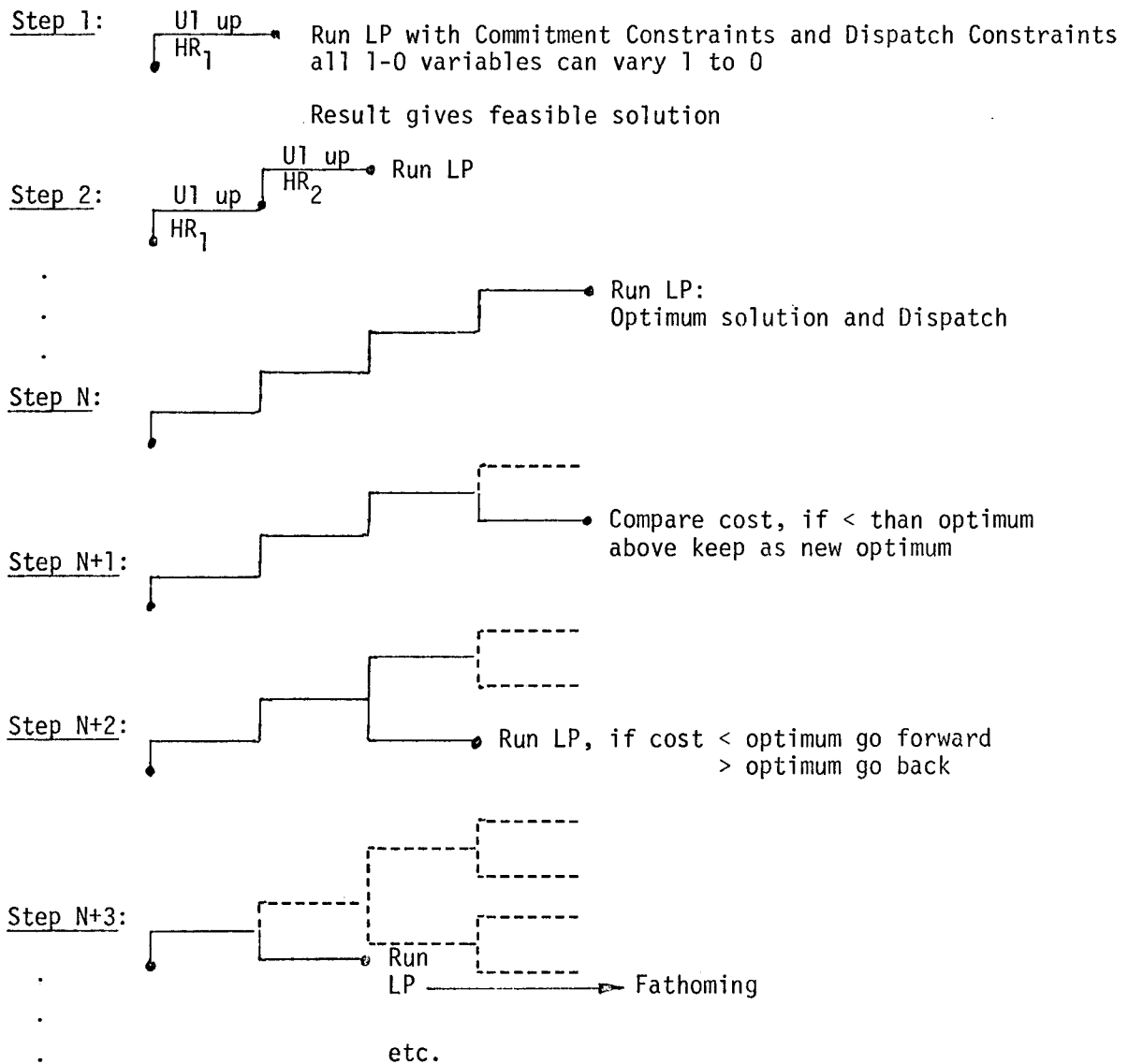


Figure 5-4. MILP Method Without Balas Block Method

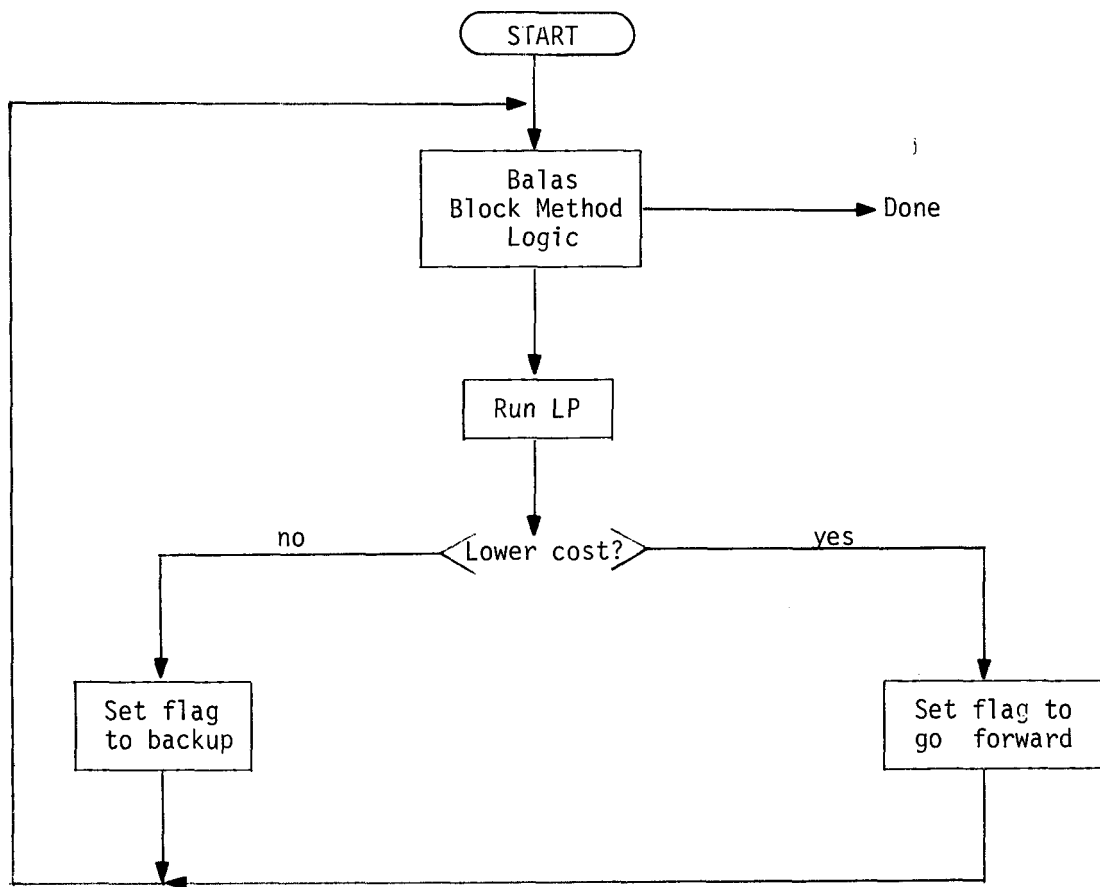
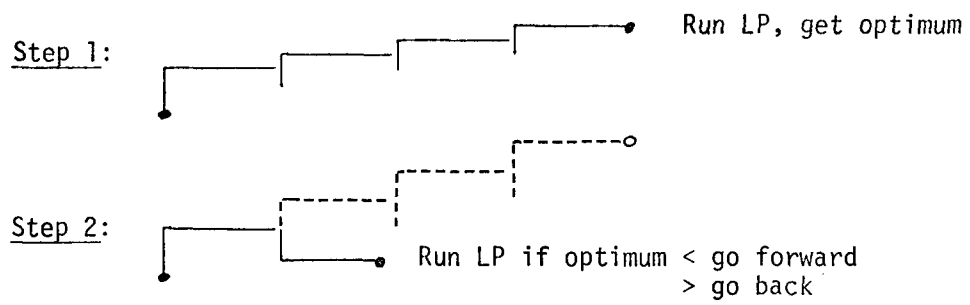
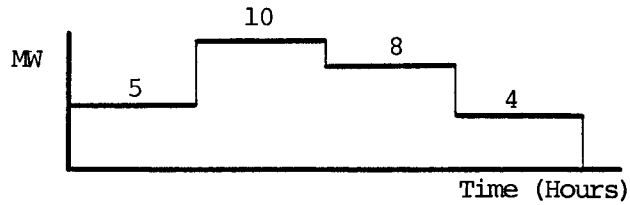


Figure 5-5. Combining Balas Block Method with MILP

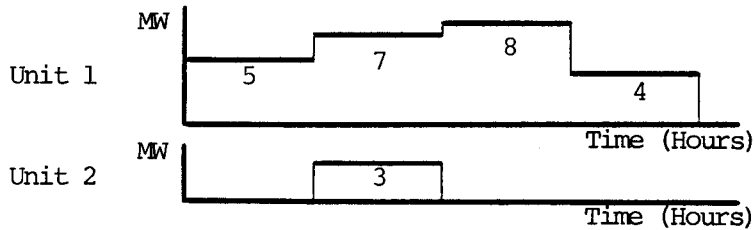
Loads:



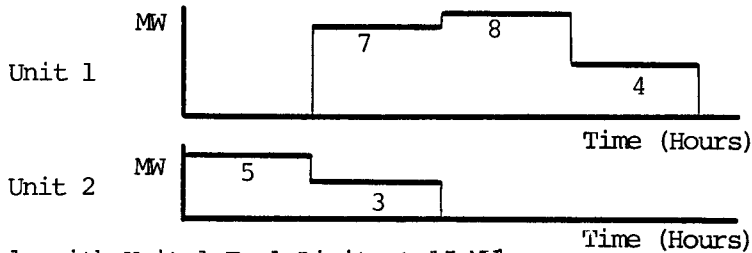
Generating Units:

	Unit 1	Unit 2
Capacity Range:	4 - 8 MW	3 - 6 MW
Minimum up and down times:	1 hour	1 hour
Price of fuel:	low	high

Optimum Schedule with No Fuel Limit:



Schedule with Unit Fuel Limit at 20 MWh:



Schedule with Unit 1 Fuel Limit at 15 MWh:

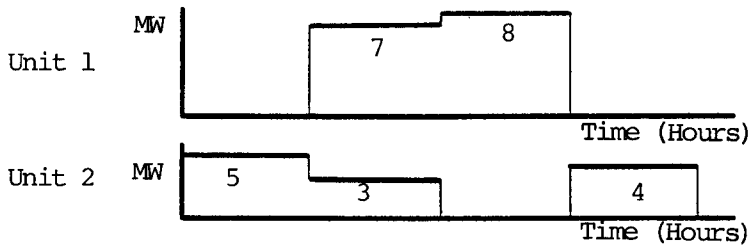
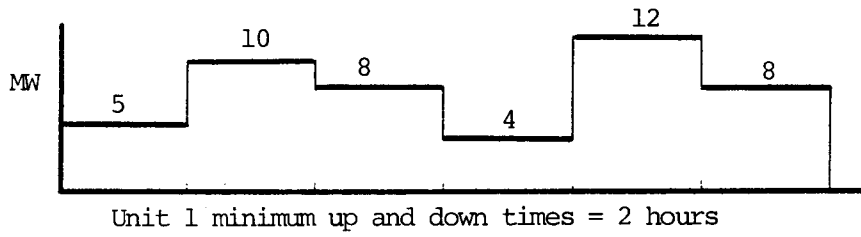
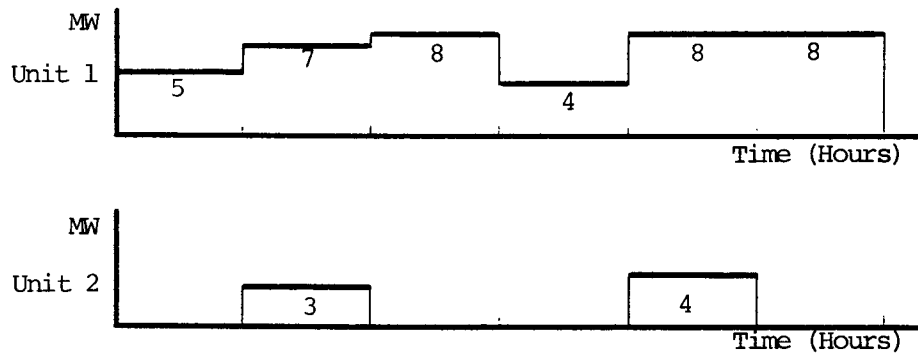


Figure 5-6. Sample Fuel Constrained Problem with Four Hours

Load:



Optimum Schedule with No Fuel Limit:



Schedule with Unit 1 Fuel Limit at 36 MWh:

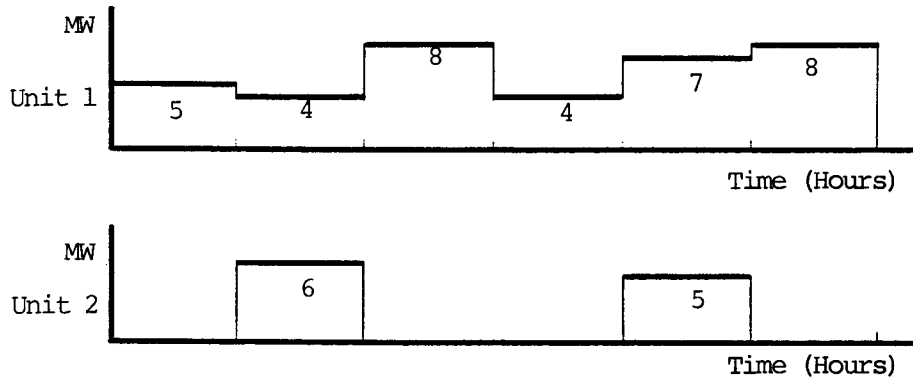


Figure 5-7. Sample Fuel Constrained Problem with Six Hours

to be supplied by two units whose characteristics are given at the top. With no restriction on the fuel usage unit 1 will supply all the load except when the load exceeds its maximum output. As the fuel limitations are progressively tightened on unit 1, the algorithm schedules more generation from unit 2. The example in Figure 5-7 is an extension of that in Figure 5-6 with an additional restriction on unit 1 which eliminates shutting it down for less than two hours. As fuel is restricted to unit 1 it must stay on line the second and fourth periods because it is needed in the third and fifth periods respectively.

#### 5.4 EXTENSION OF THE MILP APPROACH

A difficulty which arises when using the MILP-Balas Block method is the large LP's which must be solved. The equations shown in Section 5.2 must be expanded, in the fathoming LP, to the larger set given in Section 2.3. In general, over the solution of the MILP, the largest possible LP could have the following constraints:

- Load constraints:           1 per hour               = NHR
- Unit capacity limit  
  constraints:               2 per unit, per hour =  $2 * NHR * NUNIT$
- Minimum up and  
  down time and unit  
  status constraints:       3 per unit, per hour =  $3 * NHR * NUNIT$
- Fuel constraints           1 per unit               = NUNIT

Hence, the largest possible LP could have  $(5 * NHR * NUNIT + NUNIT + NHR)$  rows, where NUNIT is the number of units and NHR the number of hours.

For the 2 unit, 4 hour problem, an LP of up to 46 rows could be generated; for a 10 unit, 168 hour (1 week) problem, there could be up to 8578 rows.

Fortunately, the constraint matrix is extremely sparse. Figure 5-8 shows the constraint matrix for the 2 unit, 4 hour problem (fewer than 46 constraints are required because unit 2 has minimum up and down times of 1 hour and is not fuel constrained). Only 8.1% of the matrix is non-zero. For larger problems, the fraction of non-zeros would be even less.

To exploit this sparsity, an LP was programmed to solve by the revised simplex method using the elimination form of the basis inverse. Several techniques were applied to minimize fill-ins and maximize the overall efficiency of the solution algorithm, and are described next.

#### Updating the Basis Inverse

Saunders describes a technique [29] for updating the elimination form of the basis inverse. Through the use of permutations and Gaussian eliminations, the update can be accomplished with a minimum amount of fill-ins and in such a way as to preserve numerical stability. The method given in reference [29] assumes that the basic factors will be stored on disc; since in the current application they have been stored in main memory, the algorithm may be simplified slightly.

#### Scaling of Constraint Matrix

An examination of Figure 5-8 reveals that many elements of the constraints matrix are  $\pm 1$ . The number of such elements can be further increased by scaling some of the equations. In fact, it is possible to obtain a constraint matrix in which only the columns associated with the "U" variables have non-unity elements ( $\approx 15\%$  of the columns). Less than 50% of the non-zero elements in the resulting matrix are different from  $\pm 1$ .

The large number of ones in the constraint matrix can be exploited by replacing many costly multiplications using specialized solution subroutines. This is ac-

UNIT 1																UNIT 2																			
Hour 1				Hour 2				Hour 3				Hour 4				Hour 1				Hour 2				Hour 3				Hour 4							
$s_1^1$	$u_1^1$	$y_1^1$	$p_1^1$	$s_1^2$	$u_1^2$	$y_1^2$	$p_1^2$	$s_1^3$	$u_1^3$	$y_1^3$	$p_1^3$	$s_1^4$	$u_1^4$	$y_1^4$	$p_1^4$	$s_2^1$	$u_2^1$	$y_2^1$	$p_2^1$	$s_2^2$	$u_2^2$	$y_2^2$	$p_2^2$	$s_2^3$	$u_2^3$	$y_2^3$	$p_2^3$	$s_2^4$	$u_2^4$	$y_2^4$	$p_2^4$				
1				1				1				1				1				1				1				1							
$\bar{p}_1$ -1				$\bar{p}_1$ -1				$\bar{p}_1$ -1				$\bar{p}_1$ -1				$\bar{p}_1$ -1				$\bar{p}_2$ -1				$\bar{p}_2$ -1				$\bar{p}_2$ -1				$\bar{p}_2$ -1			
$\underline{p}_1$ -1				$\underline{p}_1$ -1				$\underline{p}_1$ -1				$\underline{p}_1$ -1				$\underline{p}_1$ -1				$\underline{p}_2$ -1				$\underline{p}_2$ -1				$\underline{p}_2$ -1				$\underline{p}_2$ -1			



complied by a special storage structure for the constraint matrix which segregates the unity and non-unity elements.

In addition, several specialized coding techniques were employed to improve the efficiency of solving LP's, and are described in the MILP Program Manual. For example, both forward and backward pointers rather than just forward pointers are used when manipulating sparse vectors. This minimizes the time spent in locating a given element of the vector.

Table 5-2 and Figure 5-9 give the data and solution for another sample 2 unit, 6 time step problem in which the first unit is limited to a total of 30 MWh generation. Table 5-3 shows the computer times required for the four LP solutions required to solve this problem. Comparisons are made between the non-sparsity coded LP ("old program") and the sparsity coded LP ("new program").

Additional timing comparisons are shown in Figure 5-10. Even at modest LP sizes the sparsity coded LP reduces the time required to solve by greater than a factor of ten.

## 5.5 FUTURE WORK

The current prototype sparse LP receives its input data from the MILP through a "coupling routine" (Subroutine DPASS) in non-sparse format. Under this format, LP's of up to 72 rows and 121 columns have been solved. The next increase in program size, probably to the 750 to 1000 row range, would require elimination of the non-sparse format entirely, with the LP input data being built in sparse format directly by the MILP.

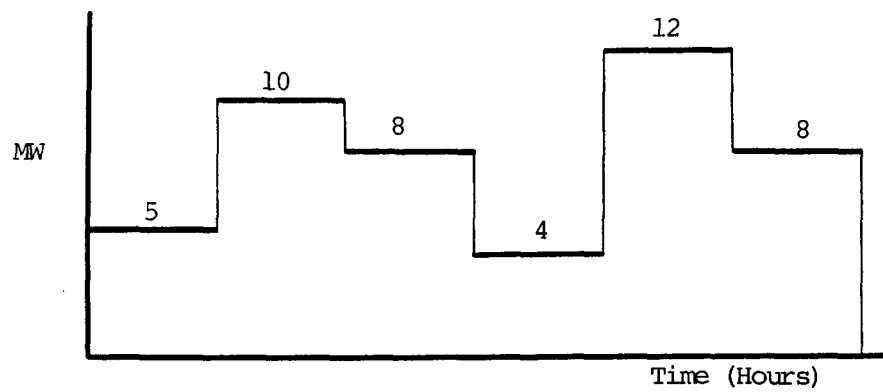
When the problem size reaches these dimensions, basis matrix refactorization becomes critical (for LP's even up to 72x121 refactorization is not required).

Table 5-2

## UNIT DATA FOR SAMPLE PROBLEM

	Unit 1	Unit 2
Minimum capacity (MW)	4.0	3.0
Maximum capacity (MW)	8.0	6.0
Minimum up time (HR)	2	1
Minimum down time (HR)	2	1
Cost function	$10.0 + 1.1 P_1$	$11.0 + 1.5 P_2$
Startup cost	0.5	1.0
Shutdown cost	0.0	0.0
MBTU/MW	1.0	1.0
Maximum MBTU	30.0	No limit

Load:



Schedule with Unit 1 Fuel Limit at 30 MWh:

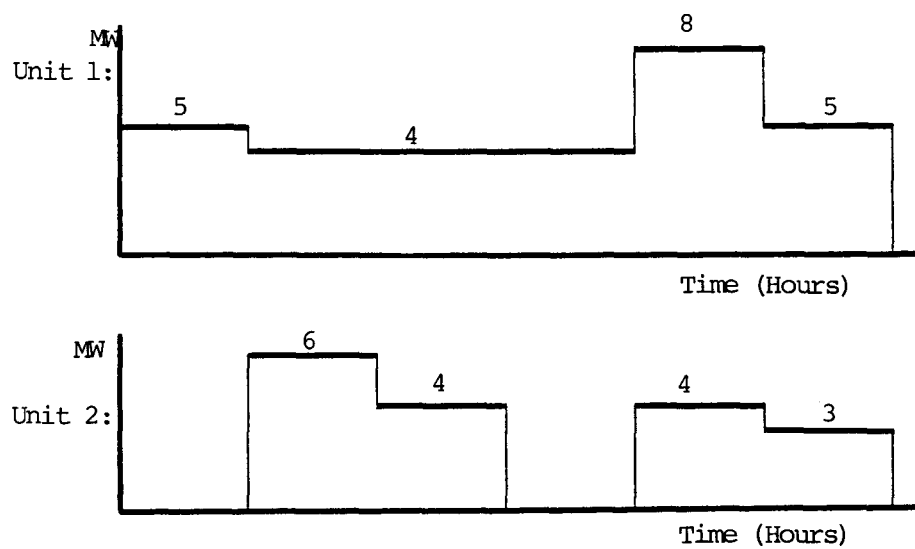


Figure 5-9. Sample Problem Solution

Table 5-3

## COMPUTING TIMES FOR SAMPLE PROBLEM

No.	Phase	LP Problem		Result	Old Program		New Program	
		Size*	Iteration		CPUS	Average	CPUS	Average
1	1	7x16	11	Not feasible	0.652		0.215	
					0.661	0.658	0.215	0.213
					0.661		0.209	
2	1	7x17	11	Optimal	0.648		0.197	
	2	7x11	0	Optimal	0.648	0.648	0.197	0.198
					0.648		0.200	
3	1	7x16	11	Not feasible	0.658		0.209	
					0.655	0.655	0.206	0.209
					0.652		0.212	
4	1	24x40	13	Not feasible	2.461		0.424	
					2.442	2.446	0.424	0.427
					2.436		0.433	

\*Rows x columns

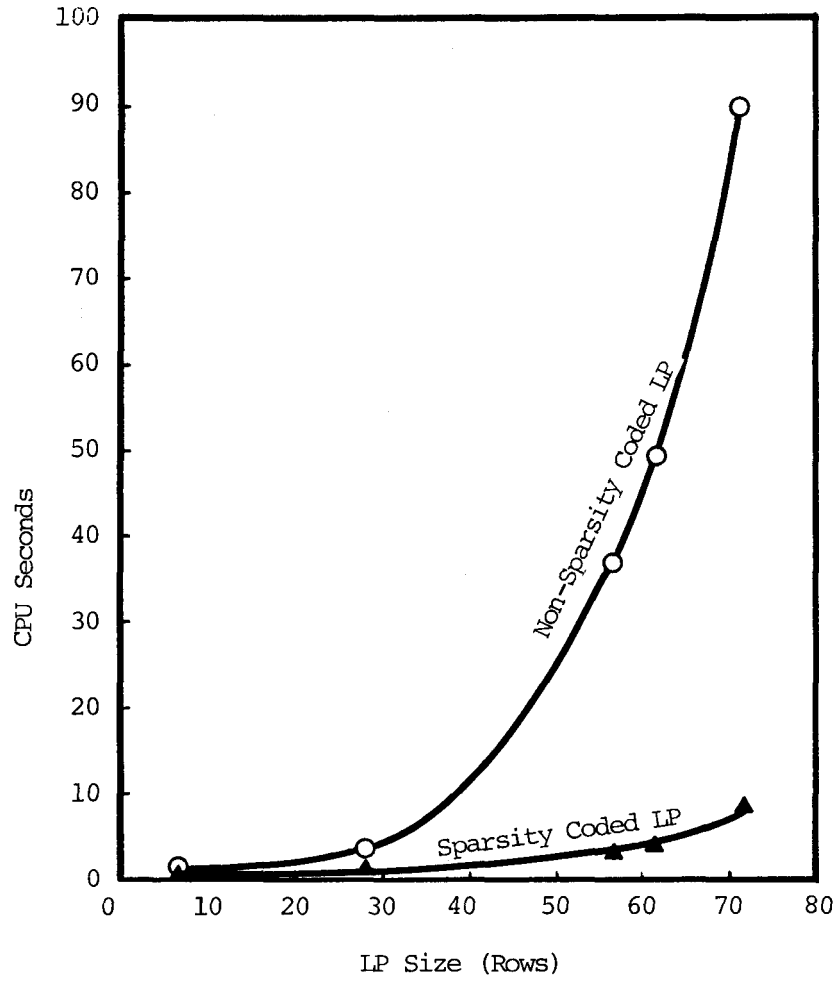


Figure 5-10. Computing Time Versus LP Size in Rows

References [30] and [31] provide the techniques required to reorder the basis rows and columns and factor the basis in an efficient and numerically stable manner.

Additional enhancements which will reduce the computing time for the MILP include: 1) adding the logic required to handle multi-break-point unit cost functions, and 2) logic to have the capability of going from one LP solution to another instead of using some arbitrary solution at the start of each LP problem.

When the MILP approach has been improved to the point at which computing requirements become reasonable, the next logical step is to couple the MILP with the search approach.

## Section 6

### SPECIAL PROBLEMS IN DAILY FUEL SCHEDULING

This section discusses the following five special problems in daily fuel scheduling which are normally ignored or treated in very simplistic manner in practice:

- Multiple fuel plants
- Transmission network representation
- Transmission constraints on reserves
- Reserves from energy limited plants
- Unit response rates

The above problems are not represented or modeled in the formulation given in Section 2. Except for the representation of transmission network by linear flow or transportation network, none has been incorporated into the prototype program either in the search or the MILP approach. However, these problems could have significant effect on the schedules and the cost for certain systems in which such problems are important. The discussions below outline the nature of the problem and suggest approaches which appear to be most reasonable and practical at the present time. Before attempting to implement any of the suggestions, the specific problem should be investigated in further details and a thorough evaluation be made on the selected approach.

#### 6.1 SCHEDULING OF MULTIPLE FUEL PLANTS

There are generating plants which have been designed to burn two or more types of fuel simultaneously. For example, a plant may be fueled by a combination of gas, oil and coal. In view of the cost differential of such fossil fuels and the varying uncertainties associated in the availability of these fuels, it may be desirable or even advantageous for the utilities to plan on

more multiple fuel plants rather than designing the plants to burn only one specific type of fuel.

In general, the lower cost fuel is limited in quantity so that it will not be possible for a plant to generate at full load using the lower cost fuel alone. Environmental constraints can also dictate the rate at which specific fuel may be used at designated plants. Normally these constraints apply to a plant as a whole and there may exist two or more generating units in the plant. All these constraints complicate the scheduling of multiple fuel plants. The scheduling becomes even more complex when the efficiency of a plant depends on the composition of the fuel mixture.

The modeling of multiple fuel plants in daily fuel scheduling is presented below as a Mixed-Integer Linear Programming (MILP) problem in the same manner as that in Section 2 for single fuel plants. The constraints given do not represent a model of any particular multiple fuel plant, rather, they represent a list of most conditions which may be encountered in practice. Normally, one could expect only a subset of these constraints to apply to any specific plant at one time period.

#### 6.1.1 Independent Variables

A multiple fuel generating unit can produce a given power output using different composites of fuels. Hence, it is not possible to use the unit power output as an independent variable as normally done with single fuel units. Instead, the independent variables in the modeling of multiple fuel plants are the input rates of the available fuels.



### 6.1.2 Unit Heat Rates

The amount of useful heat energy which can be extracted when fuel is burned or the efficiency of the fuel burning process depends on a number of factors. There is the inherent loss associated with the hydrogen content in a given fuel. Higher hydrogen content results in greater stack losses in the form of superheated steam which is formed when the hydrogen is oxidized in the fuel burning process. Thus, the burning of gas will have significantly higher such losses compared to the burning of coal which has little or no hydrogen content. Moisture content in the fuel also has the same effect in the loss of efficiency due to the resulting superheated steam.

Different types of fuel require different air/fuel ratio and higher ratios result in larger stack losses and hence lower fuel burning efficiency. Generally, gas requires lower air/fuel ratios than oil which in turn requires lower ratios than coal.

It is computationally beneficial to formulate the multiple fuel scheduling problem such that linear solution techniques can be used. This goal can be realized when the efficiency of the plant is represented as a piecewise linear function of the input fuel rates:

$$\eta = \text{piecewise linear function of } (F_1, F_2, \dots) \quad (6.1)$$

where:  $\eta$  = efficiency of fuel burning,

$F_i$  = rate of burning fuel  $i$ .

Simplifications of this model which will result in computational savings include:

$$\eta = \text{linear function of } (F_1, F_2, \dots) \quad (6.2)$$

$$\eta = \text{piecewise linear function of } (F_1 + F_2 + \dots) \quad (6.3)$$

$$\eta = \text{linear function of } (F_1+F_2+\dots) \quad (6.4)$$

$$\eta = \text{constant} \quad (6.5)$$

To ensure that appropriate fuel efficiency function is used, it will be necessary to perform tests on the generating unit so that the model is representative of the actual operation. Such tests are the same as those frequently conducted to establish the fuel rate curves for single fuel units.

### 6.1.3 Definition of Variables

$\eta_{if}$	= efficiency of unit i.
$\bar{E}_{si}^t$	= maximum rate of $SO_2$ emission from unit i at time t
$\bar{E}_{si}$	= maximum total emission of $SO_2$ by unit i over time period of study
$\bar{E}_{sp}^t$	= maximum rate of $SO_2$ emission from plant p at time t
$\bar{E}_{sp}$	= maximum total emission of $SO_2$ from plant p over time period of study
$E_{sif}$	= emission conversion constant as $SO_2$ /MBTU of fuel f burned in unit i
$F_{if}^t$	= MBTU of fuel f burned by unit i during time period t
$\bar{F}_{if}^t$	= minimum MBTU of fuel f to be burned by unit i during period t
$\bar{F}_{if}^t$	= maximum MBTU of fuel f to be burned by unit i during period t
$\bar{F}_{if}$	= minimum MBTU of fuel f to be burned by unit i over time period of study
$\bar{F}_{if}$	= maximum MBTU of fuel f to be burned by unit i over time period of study
$\bar{F}_{pf}^t$	= maximum MBTU of fuel f to be burned by plant p during period t
$\bar{F}_{pf}$	= minimum MBTU of fuel f to be burned by plant p over time period of study
$\bar{F}_{pf}$	= maximum MBTU of fuel f to be burned by plant p over time period of study
K	= 0.2931 MW/MBTU/HR, a constant

$L^t$	= system load during period t
$N_g$	= number of multiple fuel generating units
$N_{if}$	= number of fuel types that unit i can burn
$N_{ifr}$	= maximum number of times that burning fuel type f can begin and end at unit i during each time interval of $N_{itr}$ periods
$N_{ir}$	= number of time periods required for unit i to go from startup to rated power output
$N_{itr}$	= see definition of $N_{ifr}$
$N_t$	= number of time periods in study
$\underline{P}_i$	= minimum generation for unit i
$\overline{P}_i$	= maximum generation for unit i
$r_{if}^t$	= $\begin{cases} 1 & \text{if unit i starts burning fuel at time t} \\ 0 & \text{otherwise} \end{cases}$
$s_i^t$	= $\begin{cases} 1 & \text{if unit i goes on line at time t} \\ 0 & \text{otherwise} \end{cases}$
$T_i^U$	= minimum uptime for unit i
$T_i^D$	= minimum downtime for unit i
$T_{iff}^U$	= minimum "uptime" for fuel f in unit i
$T_{iff}^D$	= minimum "downtime" for fuel f in unit i
$T_{ifh}^D$	= minimum "switching time" from fuel f to fuel h in unit i
$u_i^t$	= $\begin{cases} 1 & \text{if unit i is committed during hour t} \\ 0 & \text{otherwise} \end{cases}$
$v_{if}^t$	= $\begin{cases} 1 & \text{if unit i is burning fuel f during period t} \\ 0 & \text{otherwise} \end{cases}$
$w_{if}^t$	= $\begin{cases} 1 & \text{if unit i stops burning fuel f at time t} \\ 0 & \text{otherwise} \end{cases}$
$y_i^t$	= $\begin{cases} 1 & \text{if unit i goes off-line at time t} \\ 0 & \text{otherwise} \end{cases}$

#### 6.1.4 Constraints on Scheduling of Multiple Fuel Plants

All the constraints are represented as integer-linear equalities or inequalities and they are divided into three categories below:

##### I. System constraints:

###### A. Power balance:

$$\sum_{i=1}^{N_g} \sum_{f=1}^{N_{if}} K \eta_{if} F_{if}^t = L^t - (\text{other generation}) \quad (6.6)$$

###### B. Capacity constraints:

$$\sum_{i=1}^{N_g} \bar{P}_i u_i^t \geq L^t - (\text{other generation}) \quad (6.7)$$

$$\sum_{i=1}^{N_g} \bar{P}_i u_i^t \leq L^t - (\text{other generation}) \quad (6.8)$$

##### II. Unit and plant constraints:

###### A. Generator limits:

###### 1. Non-rate-limited units:

$$u_i^t \bar{P}_i \leq \sum_{f=1}^{N_{if}} K \eta_{if} F_{if}^t \leq u_i^t \bar{P}_i \quad (6.9)$$

$$i = 1, 2, \dots, N_g; \quad t = 1, 2, \dots, N_t$$

###### 2. Rate-limited units:

$$P_i^\tau \leq \bar{P}_i + S_i^t \bar{P}_i \left[ \frac{\tau - t + 1}{N_{ir}} - 1 \right] \quad (6.10)$$

$$i = 1, 2, \dots, N_g; \quad t = 1, 2, \dots, N_t; \quad \tau = t, t+1, \dots, t+N_{ir}-2$$

$$P_i^t \geq u_i^t \bar{P}_i \quad (6.11)$$

$$i = 1, 2, \dots, N_g; \quad t = 1, 2, \dots, N_t$$

B. Emission constraints:

1. Constraint on unit rate of emission:

$$\sum_{f=1}^{N_{if}} E_{sif} F_{if}^t \leq \bar{E}_{si}^t \quad (6.12)$$

2. Constraint on total emission by unit:

$$\sum_{t=1}^{N_t} \sum_{f=1}^{N_{if}} E_{sif} F_{if}^t \leq \bar{E}_{si} \quad (6.13)$$

3. Constraints on plant rate of emission:

$$\sum_i \sum_{f=1}^{N_{if}} E_{sif} F_{if}^t \leq \bar{E}_{sp}^t \quad (6.14)$$

units in  
plant P

4. Constraint on total emission by plant:

$$\sum_{t=1}^{N_t} \sum_i \sum_{f=1}^{N_{if}} E_{sif} F_{if}^t \leq \bar{E}_{sp} \quad (6.15)$$

unit in  
plant P

C. Unit minimum uptime and minimum downtime constraints:

$$u_i^t - u_i^{t-1} - s_i^t + y_i^t = 0 \quad (6.16)$$

$$\sum_{\tau=t}^{t+T_i^U-1} u_i^\tau \geq \frac{T_i^U}{T_i} s_i^t \quad (6.17)$$

$$\sum_{\tau=t}^{t+T_i^D-1} u_i^\tau + \frac{T_i^D}{T_i} y_i^t \leq \frac{T_i^D}{T_i} \quad (6.18)$$

III. Fuel constraints:

A. Fuel quotas:

1. Constraint on fuel rate:

$$\underline{F}_{if}^t \leq F_{if}^t \leq \bar{F}_{if}^t \quad (6.19)$$

2. Constraint on unit total fuel consumption:

$$\underline{F}_{if} \leq \sum_{t=1}^{N_t} F_{if}^t \leq \bar{F}_{if} \quad (6.20)$$

3. Constraint on plant rate:

$$\sum_i F_{if}^t \leq \bar{F}_{pf}^t \quad (6.21)$$

units in  
plant P

4. Constraint on plant total fuel consumption:

$$\underline{F}_{pf} \leq \sum_i \sum_{t=1}^{N_t} F_{if}^t \leq \bar{F}_{pf} \quad (6.22)$$

units in  
plant P

B. Unit up/down status vs. number of fuel types being burned:

1. Units that can burn only one fuel type at a time:

$$u_i^t \leq \sum_{f=1}^{N_{if}} v_{if}^t \leq u_i^t \quad (6.23)$$

2. Units that can burn multiple fuel types simultaneously:

$$u_i^t \leq \sum_{f=1}^{N_{if}} v_{if}^t \leq u_i^t N_{if} \quad (6.24)$$

C. Designated fuel - i.e., constraint that fuel type h will always be burnt at a rate sufficient to generate  $\underline{P}_i$ :

$$v_{if}^t \leq v_{ih}^t \quad (f \neq h) \quad (6.25)$$

$$v_{ih}^t \underline{F}_{ih}^t \leq F_{ih}^t \leq v_{ih}^t \bar{F}_{ih}^t \quad (f=h) \quad (6.26)$$

$$\emptyset \leq F_{if}^t \leq v_{if}^t \bar{F}_{if}^t \quad (f \neq h) \quad (6.27)$$

D. Fuel minimum up and minimum down times:

$$v_{if}^t - v_{if}^{t-1} - f_{if}^t + w_{if}^t = \emptyset \quad (6.28)$$

$$\sum_{\tau=t}^{t+T_{iff}^U-1} v_{if}^{\tau} \geq T_{iff}^U r_{if}^t \quad (6.29)$$

$$\sum_{\tau=t}^{t+T_{iff}^D-1} v_{if}^{\tau} + T_{iff}^D w_{if}^t \leq T_{iff}^D \quad (6.30)$$

E. Minimum switching time from fuel f to fuel h:

$$\sum_{\tau=t}^{t+T_{ifh}^D-1} v_{ih}^{\tau} + T_{ifh}^D w_{if}^t \leq T_{ifh}^D \quad (6.31)$$

F. Constrain the number of times that the burning of fuel type f can begin and end at unit i during each time interval of  $N_{itr}$  time periods:

$$\sum_{\tau=t}^{t+N_{itr}-1} r_{if}^{\tau} \leq N_{ifr} \quad (6.32)$$

#### 6.1.5 Discussion on Multiple Fuel Plant Scheduling

The modeling of multiple fuel plants for fuel scheduling results in many more constraint equalities and inequalities compared to the modeling of single fuel plants. Hence, it will require much more computational effort to schedule multiple fuel plants. For systems with multiple fuel plants, the appropriate constraints given in the preceding section and the cost on fuel use can be added to the scheduling formulation presented in Section 2. Therefore, the scheduling of all generating plants, both single and multiple fuel plants, can be modeled as a single MILP problem.

Since it is not practical to use only the MILP approach to daily fuel scheduling, a reasonable alternative to establish initial multiple fuel plant schedule is to use a modified version of the Limited Fuel Module (LMFF) sug-

gested in the search approach (Section 4.2.9). Generally, constraints on fuel use in multiple fuel plants are for the lowest cost fuel. Therefore, Module LMTF can be modified to schedule the lowest cost fuel first and when this fuel is exhausted, the module will proceed to schedule the use of the next lowest cost fuel in an economical manner.

## 6.2 TRANSMISSION NETWORK REPRESENTATION

The flow of power from the generating plants to the loads is dependent on the transmission network of the system. An ac load flow is usually used to determine not only the flow pattern or line flows but also the voltage levels at all the buses. However, the transmission network is normally ignored in daily fuel scheduling; this assumes that the network is capable of handling any resulting schedule. Such assumptions may be acceptable for certain systems which cover small geographical areas and have more than adequate transmission capabilities. However, there exist conditions in which transmission network should be represented; the New York system is a good example in that there is generally an upper limit on the amount of economical power which can be transmitted from upstate to downstate despite the surplus of economical power in upstate.

Theoretically, it is possible to include ac load flows in daily fuel scheduling, but this will require the use of optimal load flow techniques in the economic dispatch of the generating units. Such a direct application will increase the computational time to such an extent as to render this scheme impractical since many dispatches are performed.

A proven practical approach is to represent the transmission network as a linear flow or transportation network [33] in which Kirchoff's current law is observed while Kirchoff's voltage law is neglected, and the line or link ca-



capacities are treated as hard constraints. This has been implemented in the Unit Commitment Module (UCOM) of the search approach presented in Section 4. An efficient Ford and Fulkerson labeling algorithm [33] is used in establishing the flow pattern. Actual tests in Section 4 have shown that such representation of the transmission network does not require excessive computational time even for large system such as the New York system. Furthermore, this representation permits the distribution of reserve requirements among the various areas or nodes thereby eliminating the possibility of having most or all of the reserve requirements concentrated in one part of the system, this being one of the problems when transmission network is neglected.

The network flow model can be incorporated into the MILP formulation of the daily fuel scheduling problem. For each of the hours within the scheduling period, this requires one variable (flow) and two inequality constraints (limits on flow) for each link or line represented, and one equality constraint (Kirchoff's current law) for each node or area. If the scheduling period has  $T$  hours, and the system consists of  $N$  nodes and  $L$  links, then the network representation will require  $LT$  variables,  $2LT$  inequality constraints and  $NT$  equality constraints.

To overcome the inadequacy of the linear flow network model, the approach shown in Figure 6-1 is suggested and this appears to be a practical method with our present computational technology. An economical schedule is first obtained based on the linear flow network model of the transmission network. After this, the dispatch for each hour is checked by the use of a dc load flow model of the transmission network to assure that transmission constraints are satisfied. When any such constraints are violated, a linear program can be used to determine a new dispatch for which all transmission constraints are satisfied. One would expect only a small number of hours for

which the linear program will be needed thereby rendering this approach practical. An ac load flow model can be used instead of the suggested dc load flow model, however, this will require much more computational efforts.

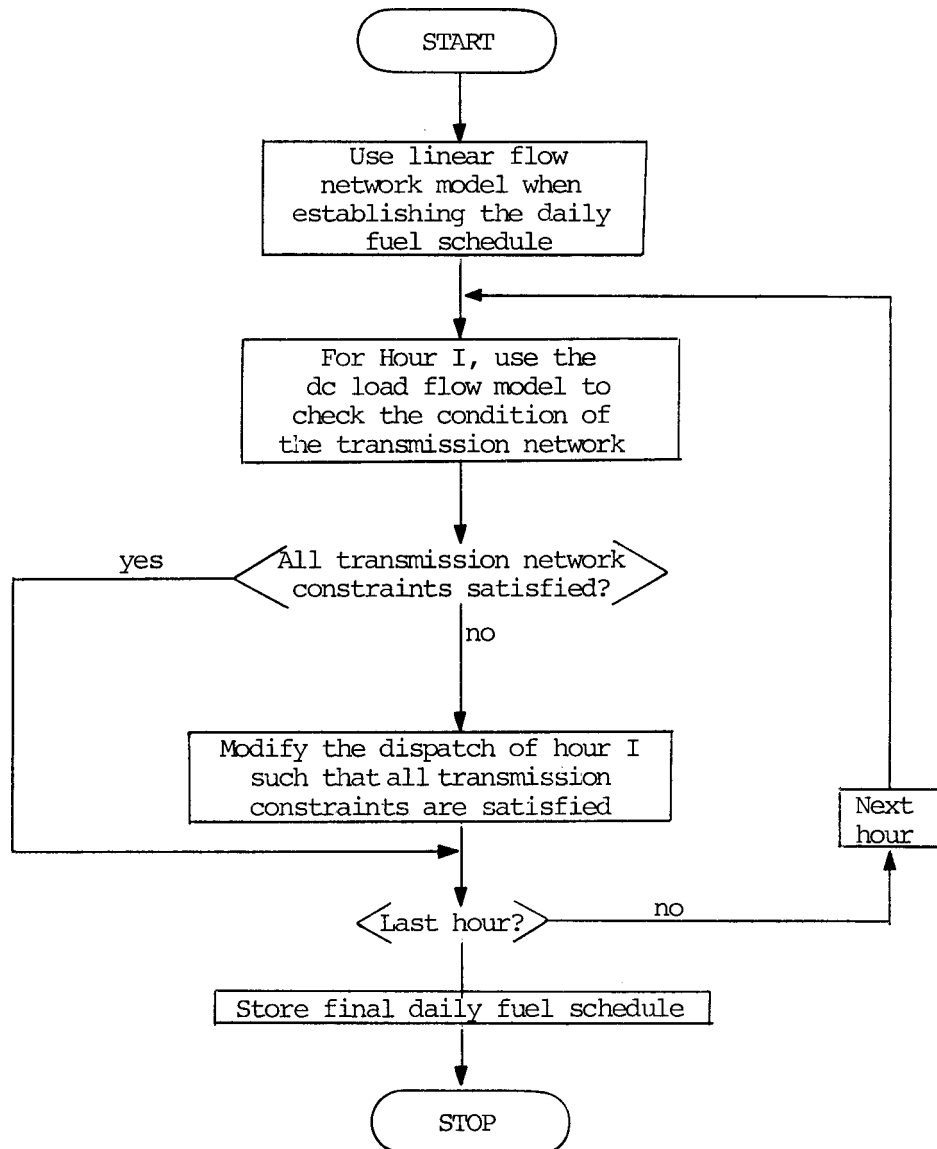


Figure 6-1. A Practical Approach for Transmission Network Representation in Daily Fuel Scheduling

### 6.3 TRANSMISSION CONSTRAINTS ON RESERVES

To minimize the curtailment of load due to loss of generating units or transmission lines, a system always schedules a certain amount of ready reserve according to specified operating requirements. Generally, the total reserve must be greater or equal to the loss of available power due to the worst single or double contingency. Categories of reserve include: (1) on-line, spinning or synchronous reserve, (2) quick-start off-line reserve for units which can be started within five or ten minutes, and (3) other off-line reserve, normally for units which are not in the quick start category but can be started within thirty minutes.

There are rules covering the distribution of reserve among the units and in various parts of the system. Such distribution avoids concentration of reserve to a few locations and it helps to assure the availability of the reserve when needed. The Unit Commitment Module has the capability of meeting a variety of reserve distribution requirements.

Despite the step taken to distribute the reserve, there can arise situations for which the transmission system is unable to transfer the available power to the deficient region in the system. Very little work has been done in this area of transmission reserve and most proposed approaches use a sequence of empirical checks on the import and export capabilities of individual regions, one region at a time.

A more logical approach is to dispatch the system such that there is not only adequate generation reserve but also sufficient transmission reserve. With such an approach, it may be possible to eliminate heuristic rules governing distribution of reserve and furthermore, the resulting schedule could be more economical in that there is greater freedom in dispatching the available generating units.

This section concerns the modeling and algorithm needs to properly reflect and evaluate reserve capabilities in a multi-area representation of a bulk power system. The reserve capabilities are in addition to the actual transmission capabilities presented in the previous section.

One multi-area flow model proposed for the bulk power system is the linear flow or transportation network studied by Ford and Fulkerson [33]. This model has the property of satisfying power continuity at nodes, and recognizing limitations in transfers between nodes, and has available a very efficient algorithm to solve for the maximum transfer capability in the interconnected network. Disadvantages of the transportation network as applied to electric systems are that branch impedances are not recognized by the model and that for transfers less than maximum, the model does not provide a unique flow solution.

A practical problem encountered with the application of the transportation model is how to determine the transfer capacities assigned to the inter-area links. The problem may become especially difficult in on-line dispatch and in daily scheduling applications. The difficulty is in maintaining up-to-date capabilities in the presence of changes in the network.

Most preferable, from the viewpoint of circuit and equipment modeling, is to use linear power flow models with generation shift distribution factors, or a factorized bus reactance matrix. Generation shift distribution factors may be aggregated to model area exports as, for example, with the multi-area dispatch model [34]. Use of generation shift distribution factors requires factor updating with network changes and base power flows for the point of reference. Line flow changes following generator contingencies can be calculated very efficiently with these factors.

The problem, briefly, is to check if, with a given commitment of units, a dispatch can be found which will minimize an operating cost function and satisfy normal transfer limits, satisfy reserve pickup requirements of each area under capacity contingency conditions, and area reserve requirements under line contingency conditions.

The reserve requirements for each area are to be specified in terms of a reserve pickup capability under conditions of specified contingency and time span. That is, the reserve requirement must cover specified unit outage and provide for additional pickup or regulating capability as a function of system load, generation of the most heavily loaded unit, or specified value. This requirement will be regarded as an obligation to be met by each area.

#### 6.3.1 Definition of Variables

$\Delta I_A^C$	Change in $I_A$ under contingency condition
$\Delta T_{AB}^C$	Change in $T_{AB}$ under contingency condition
$D_A$	Area A reserves contingency obligation (usually a percentage of the dispatch of the unit with the greatest load)
$I_A$	Area A interchange, and $I_A = \sum_A P_{GA} - L_A$
$L_A$	Total load in Area A
$P_G$	Unit generation
$P_{GA}$	Total generation in Area A
$R_A$	Area A reserve pickup capability determined from the unit dispatches
$R_{A-1}$	Area A generation reserve pickup under loss of the largest or most heavily loaded unit
$T_{AB}$	Transfer from Area A to Area B
$T_{AB}^E$	Emergency transfer limit from Area A to Area B

$T_{AB-1}^E$  Line contingency emergency limit transfers from Area A to Area B

$T_{AB}^N$  Normal transfer limit from Area A to Area B

### 6.3.2 Illustration in Using a Three-Area System

The three area sample system shown in Figure 6-2 will be used to illustrate the formulation of the transmission reserve problem in which the transmission network is represented by a linear flow network model. The following contingency limits must be interpreted as applicable to specific time spans such as 10-minute and 30-minute reserves.

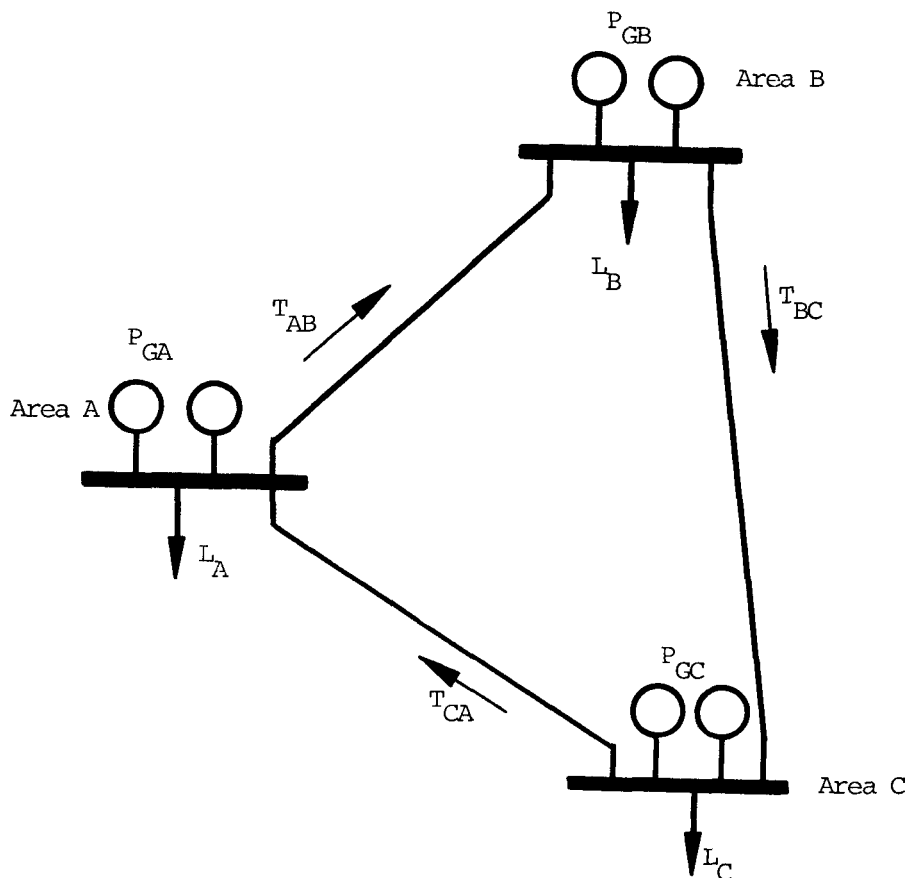


Figure 6-2. Loading Condition for a Three-Area System

For a given unit commitment, the objective is to determine if a feasible dispatch  $\{P_G\}$  can be found which minimizes the operation cost,  $COST$ , the sum of the generation costs:

$$1. \quad \Sigma COST = \sum_{GA} F(P_{GA}) + \sum_{GB} F(P_{GB}) + \sum_{GC} F(P_{GC}) \quad (6.33)$$

2. Subject to the normal constraints:

- All loads are satisfied:

$$I_A + I_B + I_C = 0 \quad (6.34)$$

- All units within loading ranges:

$$P_{GMIN} \leq P_G \leq P_{GMAX} \quad (6.35)$$

- Area transfers within limits:

$$I_A = T_{AB} + T_{AC} \quad (6.36)$$

$$I_B = T_{BA} + T_{BC} = -T_{AB} + T_{BC} \quad (6.37)$$

$$I_C = T_{CA} + T_{CB} = -T_{AC} - T_{BC} \quad (6.38)$$

$$- T_{BA}^N \leq T_{AB} \leq T_{AB}^N \quad (6.39)$$

$$- T_{CB}^N \leq T_{BC} \leq T_{BC}^N \quad (6.40)$$

$$- T_{AC}^N \leq T_{CA} \leq T_{CA}^N \quad (6.41)$$

And subject to the generation and line contingency constraints:

3. Area capacity contingency:

This is illustrated for Area A and on only one time span or one category of reserve. Area A has lost generation and must depend upon own remaining reserves,  $R_{A-1}$ , plus reserves from other areas within limits of reserve pickup and inter-area emergency transfer capabilities:

- Load pickup requirements in test area:

$$L_A + D_A = \sum_{G_{A-1}} P_G + \Delta I_{A-1}^C + (I_B + \Delta I_B^C) + (I_C + \Delta I_C^C) \quad (6.42)$$

- Limits on pickup:

$$\Delta I_{A-1}^C \leq R_{A-1} \quad (6.43)$$

$$\Delta I_B^C \leq R_B \quad (6.44)$$

$$\Delta I_C^C \leq R_C \quad (6.45)$$

- Limits on inter-area transfers:

$$I_B + \Delta I_B^C = T_{BA} + \Delta T_{BA}^C + T_{BC} + \Delta T_{BC}^C \quad (6.46)$$

$$I_C + \Delta I_C^C = T_{CA} + \Delta T_{CA}^C + T_{CB} + \Delta T_{CB}^C \quad (6.47)$$

$$\sum_K (I_K + \Delta I_K^C) = 0 \quad (6.48)$$

$$- T_{BA}^E \leq T_{AB} + \Delta T_{AB}^C \leq T_{AB}^E \quad (6.49)$$

$$- T_{CB}^E \leq T_{BC} + \Delta T_{BC}^C \leq T_{BC}^E \quad (6.50)$$

$$- T_{AC}^E \leq T_{CA} + \Delta T_{CA}^C \leq T_{CA}^E \quad (6.51)$$

#### 4. Line contingency check:

This is illustrated for the case of a contingency affecting a line outage between Areas A and B, that is,  $T_{AB}^E$ . Note that in this case, each area needs to be checked as the transfer limit between two areas may interfere with reserves wheeling between other areas. For check on Area "J:"

$$L_J + D_J = \sum_{G_J} (P_G + \Delta P_G^C) + \sum_{K \neq J} (I_K + \Delta I_K^C) \quad (6.52)$$

where,

$$\Delta I_K^C = \sum_{G_K} \Delta P_G^C \leq R_K \quad \text{for all areas.} \quad (6.53)$$



New "Power Balance"

$$\sum_{G \in J} \Delta P_G^C + \sum_{K \notin J} \Delta I_K^C = D_J \quad (6.54)$$

Changes in "flows" on the network:

$$\sum_{G \in J} (P_G + \Delta P_G^C) - (L_J + D_J) = \sum_{K \notin J} (T_{JK} + \Delta T_{JK}^C) \quad (6.55)$$

$$(I_K + \Delta I_K^C) = \sum_{K \notin L, J} (T_{KL} + \Delta T_{KL}^C) \quad (6.56)$$

Line flow limits under Area J test:

$$- (T_{AB} + T_{BA-1}^E) \leq \Delta T_{AB}^C \leq (T_{AB-1}^E - T_{AB}) \quad (6.57)$$

$$- (T_{BC} + T_{CB}^E) \leq \Delta T_{BC}^C \leq (T_{BC}^E - T_{BC}) \quad (6.58)$$

$$- (T_{CA} + T_{AB}^E) \leq \Delta T_{CA}^C \leq (T_{CA}^E - T_{CA}) \quad (6.59)$$

The above formulation includes the representation of the transmission network. Hence, the problem on transmission reserves automatically includes the transmission network representation problem given in the preceding section.

Instead of the linear flow representation, a more accurate method, but requiring much longer computing time, is to use a dc load flow in conjunction with distribution factors to check for adequate transmission reserves. Distribution factors have been used successfully for the evaluation of both generation and line outage contingencies on a regular basis [35,36]. However, when the check reveals a deficiency in transmission reserve, there arises a problem to reschedule the generating schedule which will eliminate such deficiency. This may be accomplished through the application of a linear program.

#### 6.4 RESERVES AND ENERGY LIMITED PLANTS

The search approach to daily fuel scheduling decomposed the problem into the scheduling of thermal units with no fuel limits and energy limited plants such as the hydro plants. The coupling between these two categories of generation is provided by the thermal cost curves which represent the cost of thermal generation at various load levels. Such curves do not reflect the effect on the cost of maintaining the specific reserve requirements. Since most energy limited plants also provide reserve contributions, it is logical that the costs of generation and reserves be considered simultaneously to arrive at economic schedules. This particular problem does not arise in the MILP approach since it can schedule all the generating plants simultaneously.

This section presents a formulation of the problem and suggests a method of solution. Since hydro plants are the most commonly encountered energy limited plants, the following treatment has been directed to hydro plants though it is equally applicable to other types of energy limited plants.

A spinning reserve rule requiring that the system shall be capable of increasing total generation from on-line units by a specified amount (MW) within a specified time period such as 5 or 10 minutes, may result in thermal unit commitments and dispatches with increased operating cost. If all or some of this reserve requirement can be covered by hydro-plants, the cost impact may be reduced.

Neglecting the effects of spinning reserve requirements, the hydro-thermal scheduling algorithm contains an iterative loop as shown in Figure 6-3 in which  $P$ ,  $P_T$  and  $P_H$  are the total, thermal and hydro generations respectively.

Each iteration in the process indicated covers all hours in the study interval. The hydro scheduling routine requires as input the system incremental

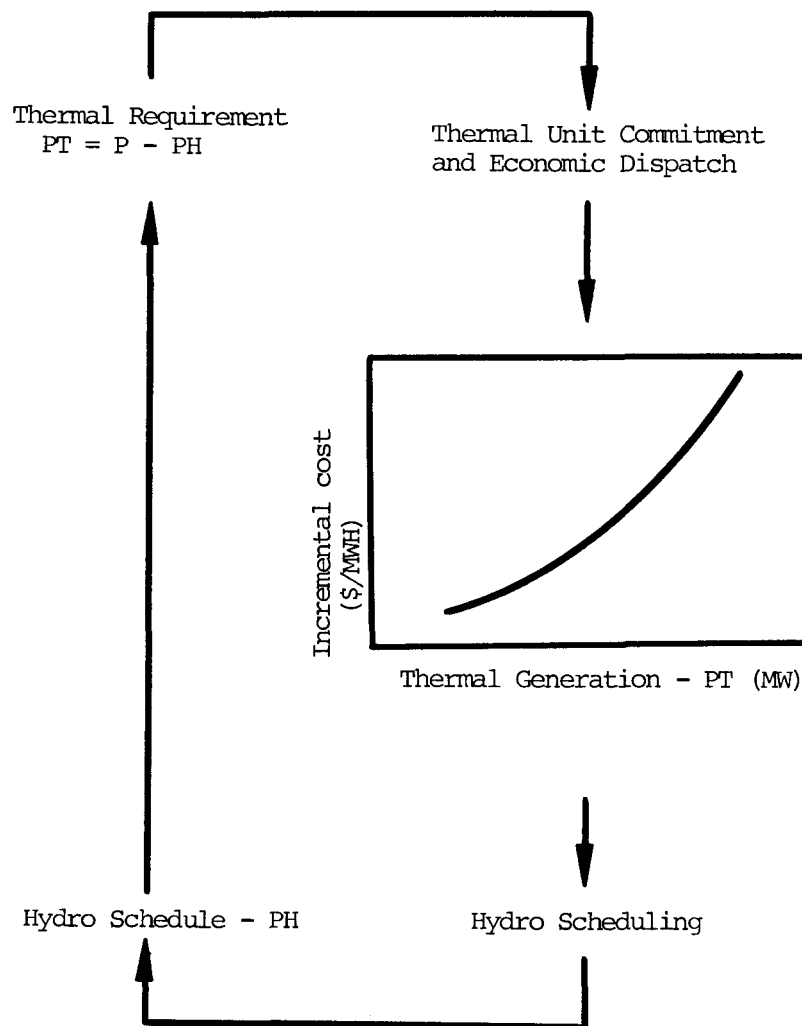


Figure 6-3. Hydro-Thermal Scheduling Neglecting Spinning Reserve Requirements

cost versus thermal requirement. Hours with different unit combinations will have different system incremental cost curves. The thermal unit commitment routine requires the hourly hydro schedule as input to establish the hourly thermal requirement.

Imposition of a spinning reserve requirement may change the system incremental cost curve for certain hours both through unit commitment and through higher cost dispatches. For a specific combination of generator units, a set of curves with different reserve requirements may be developed as shown in Figure 6-4. This figure illustrates the nature of the information that need to be made available to the hydro scheduling routine. It can be seen that as the spinning reserve requirement increases the cost of thermal system generation also increases. Keep in mind though, that for a particular hour, the combination of units may also be affected by the reserve requirement.

#### 6.4.1 Definition of Variables

$EH$  = total hydro energy available for study

$F_i$  = system thermal operating cost at hour  $i$

$i$  = hour

$N$  = number of hours in study interval

$P_i$  = desired generation at hour  $i$

$PH_i$  = hydro generation at hour  $i$

$PT_i$  = thermal generation at hour  $i$

$RH_i$  = spinning reserve contribution from hydro plant at hour  $i$

$R_{reg,i}$  = total reserve requirement at hour  $i$

$RT_i$  = spinning reserve contribution from thermal plants at hour  $i$

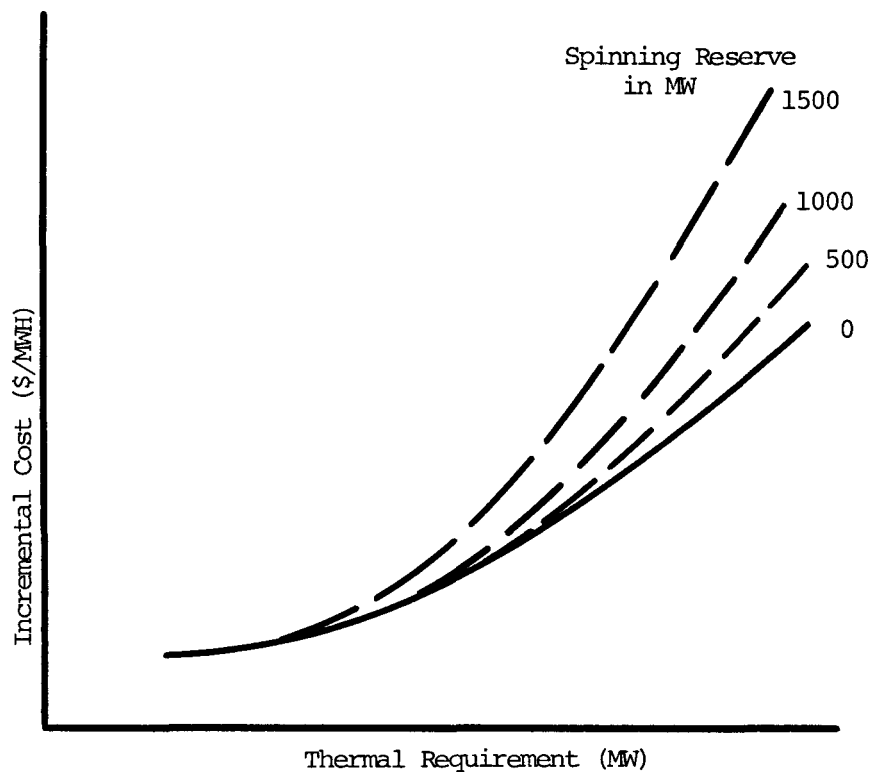


Figure 6-4. Effect of Spinning Reserve Requirements on System Incremental Cost Curves

- $N_h$  = number of hydro units  
 $PMX_{j,i}$  = maximum output of unit j at hour i  
 $PHMX_i$  = maximum output of hydro system at hour i  
 $P_{j,i}$  = output of unit j at hour i  
 $SRCAP_j$  = maximum increase in unit j output over time period allowed by spinning reserve rule

#### 6.4.2 Problem Formulation for Hydro-Thermal Scheduling

The hydro-thermal scheduling problem with spinning reserve constraints may be presented as an optimization problem in which the sum of the hourly system thermal operating costs ( $F_i$ ) in the study interval is minimized:

$$\text{Minimize } \sum_{i=1}^N F_i(P_{T,i}, R_{T,req,i}) \quad (6.60)$$

subject to the following constraints:

$$P_{T,i} + PH_i = P_i \text{ for } i = 1, 2, \dots, N \quad (6.61)$$

$$\sum_{i=1}^N PH_i = EH \quad (6.62)$$

$$RH_i + R_{T,i} \geq R_{req,i} \text{ for } i = 1, 2, \dots, N \quad (6.63)$$

The reserve contributions by all the hydro units can be expressed as follows:

$$RH_i = \sum_{j=1}^{N_h} \text{Min} (PMX_{j,i} - P_{j,i}, SRCAP_j) \quad (6.64)$$

As a rule, and as a working assumption for the following development, the spinning reserve contribution from hydro will not be rated constrained, i.e.,  $SRCAP_j$  is not limiting in equation (6.64). In this case:

$$RH_i = \text{Max} \left[ \sum_{j=1}^{N_h} (PMX_{j,i} - P_{j,i}), 0 \right] = \text{Max} (PHMX_i - PH_i, 0) \quad (6.65)$$

The reserve requirement for the thermal system ( $RT_{req}$ ) may then be written as a function of PH:

$$RT_{req,i} = \text{Max} [R_{req,i} - \text{Max} (PHMX_i - PH_i, 0), 0] \quad (6.66)$$

Since  $PT_i = P_i - PH_i$ , it is possible to express the hourly thermal operating cost of equation (6.60) as:

$$\begin{aligned} F_i(PT_i, RT_{req,i}) &= F_i(P_i - PH_i, \text{Max} [R_{req,i} - \text{Max} (PHMX_i - PH_i, 0), 0]) \\ &= F_i(PH_i) \end{aligned} \quad (6.67)$$

Associated with  $F_i$  is a function of incremental cost versus  $PH_i$  as illustrated in Figure 6-5. Portion A-B of the curve is not affected by reserve considerations; i.e., all reserve is covered by the hydro system. Therefore, this portion of the curve contains the same information (complementary) as the thermal system incremental cost curve used without reserve constraints. Determination of portion B-C requires a series of dispatches of the thermal system varying  $PH_i$  as specified through corresponding thermal requirement ( $PT_i$ ) and thermal spinning reserve requirement ( $RT_{req,i}$ ). The dispatch routine used must be capable of minimizing operating cost within the spinning reserve constraint imposed.

Since PH and PT are complementary, the incremental cost curves as a function of PH in Figure 6-5 may replace the incremental cost curves as a function of PT in the iteration scheme shown in Figure 6-3. Therefore, spinning reserve may be included in the same framework of analysis as used earlier with the following modification:

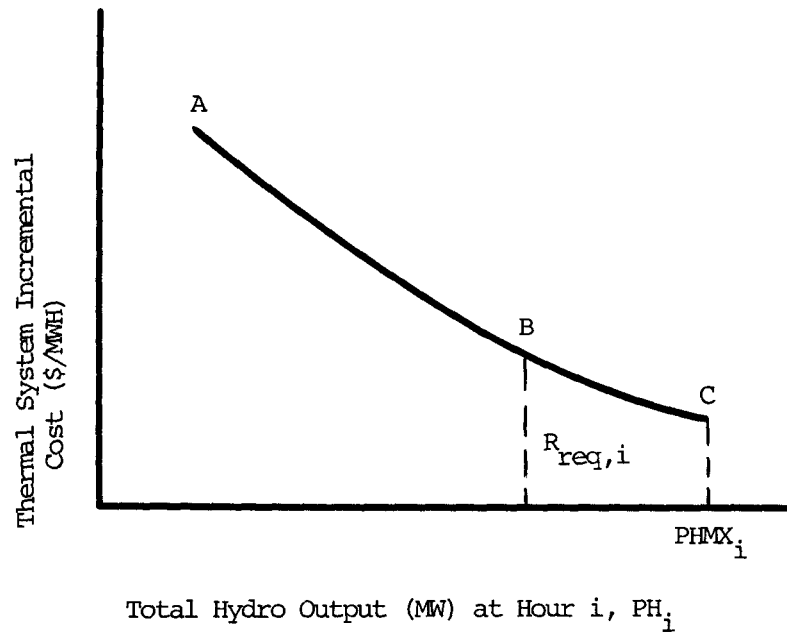


Figure 6-5. Thermal System Incremental Cost as a Function of Hydro Output



1. The hourly thermal system incremental cost curves input to the hydro scheduling routine is expressed as a function of PH rather than PT.
2. A series of dispatches within reserve constraints are required to determine the portion of the thermal system increment cost curves that are affected by reserve constraints.
3. The hourly information transferred from the hydro scheduling routine to the thermal unit commitment/economic dispatch routine must be augmented by the thermal reserve requirement corresponding to the hydro schedule.

A summary of these modifications can also be obtained by comparing Figures 6-3 and 6-6.

#### 6.4.3 Limitations of Proposed Method

The method proposed is strictly valid only when the spinning reserve contribution of hydro units can be expressed as a function of PH; i.e., it is not dependent on the relative dispatch of individual hydro units. If the spinning reserve contribution should be limited by response rate of individual units, the thermal incremental cost can no longer be described precisely as a function of PH alone.

One runs into the same problem in case one or more hydro units that are to be scheduled by the program cannot be counted on for spinning reserve at all. Note, however, that the problem is limited to the precise representation of the system incremental cost curve. The thermal unit commitment and the associated operating point dispatches performed at each iteration can correctly reflect the current hydro schedule spinning reserve contribution:

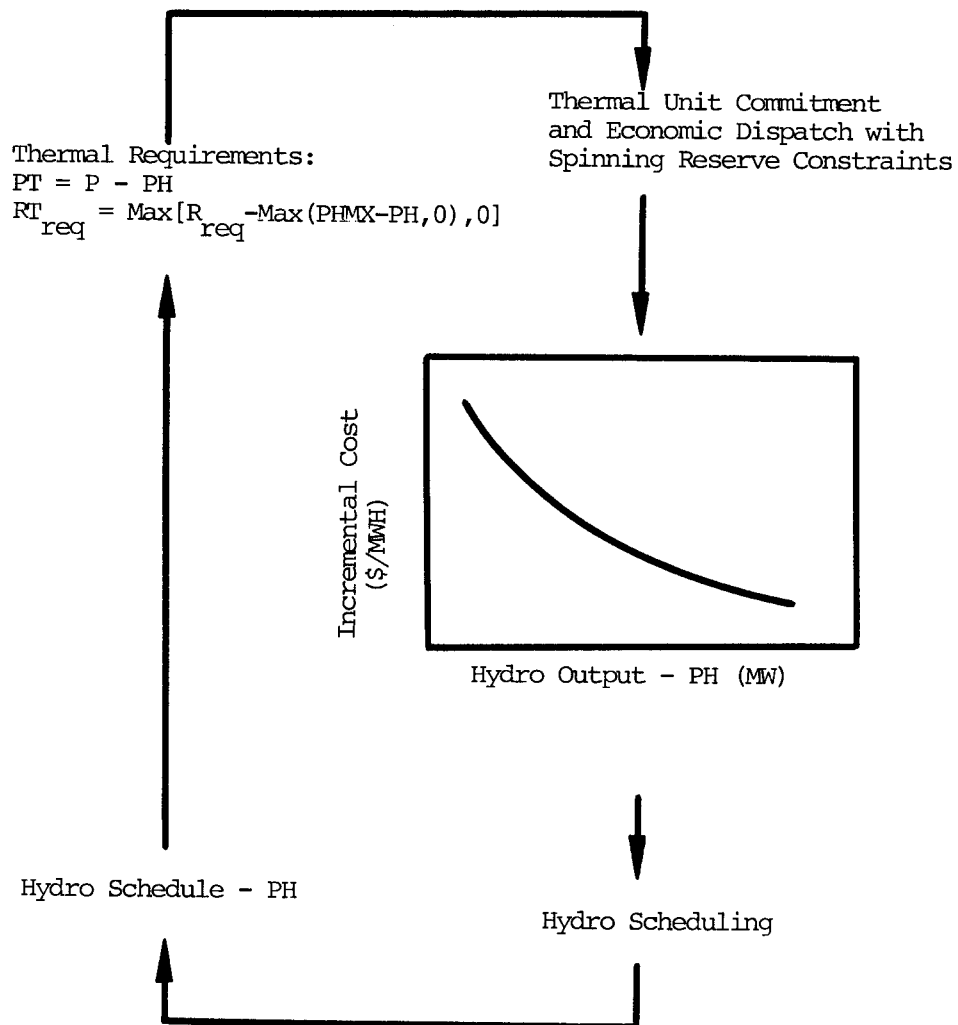


Figure 6-6. Hydro-Thermal Scheduling Incorporating Spinning Reserve Requirements

$$RT_{req,i} = \text{Max} [R_{req,i} - \sum_{j=1}^{N_h} \text{Min} (PMX_{j,i} - P_{j,i}, SRCAP_j), 0] \quad (6.68)$$

where

$SRCAP_j = 0$  for units not contributing to spinning reserve

Therefore, the correct solution is obtained provided the hydro thermal scheduling iteration process converges. The problem cited is thus limited to potential convergence difficulties that can only be explored by testing.

#### 6.5 EFFECTS OF UNIT RESPONSE RATES ON DAILY FUEL SCHEDULING

In the past, most thermal units have the capability of sustained loading change rates in the range from 2% to 5% per minute. Loading change rates are also known as response rates or ramping rates. In the hour-by-hour scheduling of these thermal units, the above response rates permit the dispatching of the units at any loading level without much consideration to the system load following capability. However, there has been an increased use of large base-loaded units, nuclear units in particular, which have response rates of less than 1% per minute. Such units will require more than an hour to increase its loading from zero to full output. Hence, it has become necessary in some cases to include constraints due to unit response rates in daily fuel scheduling so that the system is capable of following the increasing load demand normally occurring in the morning hours.

Since low response rates are generally associated with thermal units, unit response rate constraints need only be considered in the thermal unit commitment process. Limited unit response rates affect both the reserve capability of the units and the system load pickup capability which is also dependent on the dispatch of the units; thus, they may also influence the system operating cost.

Reference [37] illustrated a sample case in which the use of economic dispatch resulted in a deficiency in generation during a morning pickup period. This deficiency is strictly due to the systems' inability to respond to the increasing load demand despite the availability of adequate capacity. On the other hand, by dispatching the same units in proportion to their remaining capabilities (i.e., unit high limit less the current dispatch loading), the system maintains sufficient response capability to meet the load during the morning pickup period. However, such departure from economic dispatch means higher operating cost.

The constraints on reserve contributions imposed by unit response rates have been modeled in the unit commitment module. This is done by limiting the reserve contribution of a unit to the amount determined by the units response rate. Suppose a unit has a response rate of 2 MW per minute. Then its maximum ten and thirty minute spinning reserve capabilities are 20 MW and 60 MW respectively. This particular aspect of unit response rate can also be modeled in the MILP approach using the same concept.

Modeling of unit response rate for load pickup and economic dispatch is a straightforward matter for the MILP approach to daily fuel scheduling. This requires two additional constraints for each unit at each hour of the scheduling period. These two constraints set the upper and lower generation levels for a unit at an hour based on the dispatched generation at the previous hour and the unit response rate.

In the search or dynamic programming approach used in the unit commitment module, a direct solution to the problem of unit response rate constraint on economic dispatch is to perform a dispatch for a combination of units at an hour for each feasible transition from a saved combination of units at the

preceding hour. Therefore, when the average number of saved combinations at each hour is fifty, then there will be fifty times as many dispatches to be performed in the scheduling process. This could result in undue computing burden on the approach since dispatches already constitute a major proportion of the computing time. A more practical solution is to perform an initial economic dispatch without regard to unit response rates, and then to redispatch the units only for those cases in which response constraints are active and at the same time the feasible transition has the potential of resulting in a lower cost schedule. Since the incidents of such cases will be relatively few, the increase in computing time will not be excessive.

The above solution does not necessarily assure that a system will be dispatched to have the required load following capability especially during the morning pickup hours. When the scheduling process encounters an hour in which the system is unable to meet the load demand despite having surplus capacity, the scheduling should backup one hour and perform a dispatch which maximizes the system pickup capability in place of the economic dispatch. Since the number of hours in which system pickup capability is critical will be very limited, the economic penalty of using a non-economic dispatch during these hours will not be very significant on the total system cost for a scheduling period.

## Section 7

### CONCLUSIONS AND RECOMMENDATIONS

#### 7.1 CONCLUSIONS

Daily fuel scheduling is a complex problem. It involves the scheduling of various generating facilities with different characteristics; there are various types of operating requirements; and the number of variables involved is usually large, particularly for large systems such as a pool level.

Significant fuel savings can be achieved by computerized scheduling techniques such as the search approach which was developed and tested as part of this EPRI project. The proposed search approach uses both dynamic programming and incremental search procedures. The structure of this approach is modular; it consists of a number of different modules, each is designed to schedule a specific type of generating facility, and a master module which coordinates the schedules from all the modules such that all system operating requirements are satisfied and economical schedules are determined. Such modular structure is desirable since it is flexible; other subroutines could be added with minimal alterations and any module could be modified or improved upon without affecting others.

Prototype computer programs on the search approach were coded in FORTRAN for testing and evaluation of the performance of the scheduling procedure. Initial results were obtained using relatively small sample test systems. To provide meaningful results, a large test system based on the New York system was used. This test system is divided into five transmission areas, each with its generating facilities and loads. The generation system was represented by 111 thermal

units, 16 hydro plants, 2 pumped storage plants and 1 fuel limited thermal plant with 2 units. Scheduling periods were 168 hours (1 week) and actual hourly loads for two weeks of July 1978 were used. The peak load during that time period was about 20,000 MW. Test results show that the search approach is applicable to large systems, such as the New York system, in that economical schedules are obtained within reasonable computing time using present day computers. However, there is no assurance of finding the most economic or optimal schedule. Therefore, the MILP approach is also proposed for daily fuel scheduling.

MILP method can be used to improve the schedule obtained by the search approach and it can also be incorporated into some of the modules to provide more economical schedules. Again, prototype FORTRAN computer programs were coded for testing and evaluation of the MILP approach using simple sample test systems. The first set of tests was performed on the Balas zero-one programming algorithm (integer part). Results demonstrated the need to take advantage of the problem structure to improve computing time. For example, incorporating the unit minimum up and down times of the unit commitment problem into the Balas algorithm reduced the computing time by a few orders of magnitude compared to a straightforward general Balas algorithm. The second set of tests was conducted on the fathoming and LP part of the scheduling problem. Results showed that computer running times using sparsity programming were roughly the square root of computing times for those cases in which sparsity was not employed. Despite the above significant improvements in the computing times, the MILP approach still requires excessive computing times for practical application to daily fuel scheduling. There are areas of the MILP approach in which further research and development efforts will be needed to render the approach practical for scheduling purposes.

To summarize, the following conclusions may be drawn from the efforts of this project:

- Daily fuel scheduling problem has been well defined and formulated as an optimization problem in which the total system operating cost is minimized.
- Significant savings may be possible in daily fuel scheduling through the use of techniques such as the proposed search approach.
- The proposed search approach is modular in structure, and is flexible in that new modules can be added for special generating facilities or contracts, and each module may be modified without affecting other modules. Test results using prototype programs show this approach is applicable to the determination of economic daily fuel schedules for large systems such as the New York system.
- Though daily fuel scheduling can be formulated as an MILP optimization problem, a complete MILP solution is not yet practical with present day computer technology and techniques because of extremely long computing times for most actual systems. Test results demonstrated that by taking advantage of some of the special structures in the daily fuel scheduling problem computing time can be reduced significantly.
- The MILP approach has potential applications in improving economic schedules obtained by the search approach and in providing more economical schedules for certain modules of the search approach.
- Areas of MILP in which additional effort will improve running time have been discussed and presented.
- Special scheduling problems concerning multiple fuel plants, transmission network representations, transmission reserves, contribution of reserves by energy limited plants such as hydro plants, and plant or unit response rates are presented and some possible approaches have been suggested.
- The advances in computer technology, especially the increasing computing speed of newer computers, will enhance the use of both the search and MILP approaches for daily fuel scheduling and will increase the assurance of determining the optimal schedules within acceptable computing time.



## 7.2 RECOMMENDATIONS FOR FUTURE WORK

A number of problems related to daily fuel scheduling have been identified during the course of this project. There are also some problems which have not been investigated due to budget and time constraints. Such recommendations often arise from research and development projects, this project being an example. However, these problems deserve further attention as all such future efforts will contribute towards the development of more efficient and effective methods for daily fuel scheduling and better representations or models for the various aspects of the scheduling problem.

There exist generating plants using two or more types of fuel. The presence of such plants increases the size of the scheduling problem as a multiple fuel plant requires more variables and constraints than a single fuel plant. Thus, this will demand even longer computing times to arrive at economic schedules. With the increasing uncertainties in the availability of various fuel supplies, it may be advantageous to design future fossil plants with the capability of burning a few types of fuel. Therefore, modeling of multiple fuel plants in daily fuel scheduling may become an essential and routine requirement.

Modeling of transmission network has been largely ignored in daily fuel scheduling. However, there are situations in which transmission network do affect the transfer of power between parts of the system and hence the dispatch and schedule of the generation system. To date, only the transportation or linear flow network model has been used to represent the transmission network for daily fuel scheduling. This representation does not observe the dependence of power flows between parallel paths. A related problem is the effect of transmission constraints on the distribution of reserves in the system. Ability to handle such constraints could eliminate the use of reserve rules and yield more economic and secure dispatches and schedules. The applicability of the dc load flow model and distribution factors should be investigated.

In the search approach which decouples the scheduling problem, the reserve contributions by energy limited plants are treated as by-products in the scheduling of power generation. Since there is a cost penalty associated with the maintenance of reserve by thermal plants, it is necessary to consider to cost effects of reserves from energy limited plant so as to establish economic schedules.

In the hour-by-hour scheduling, unit or plant response rates of less than 1% per minute could lead to schedules in which a system cannot meet its load despite adequate capacity. This is particularly critical during the fast load pickup periods and for systems with high proportion of base loaded units with low response rates, like the nuclear units. In such situations, it is essential to be capable of modeling unit response rates.

It is essential that the computing times for MILP methods be reduced. One major step is by including the ability to proceed from one LP solution to another rather than starting from an arbitrary solution for every LP problem in the scheduling process.

Finally, the MILP approach should be coupled to the search approach, as has been proposed for daily fuel scheduling. Such combinations of diverse methods hold promise of determining optimal daily fuel schedulings, even for large systems.

## Section 8

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