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GLUEBALLS AND BEYOND†

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One of the most exciting developments in the physics of the 20th Century is the proposal that locally gauge invariant groups describe the strong, electromagnetic and weak interactions. $SU(2)_L \times U(1)$ the electroweak group has had enormous successes including the recent discovery of the $W^{\pm}$ and Z^0.

In the case of strong interactions, Quantum Chromodynamics is built upon the local gauge invariance of $SU(3)_{color}$ which gives rise to the eight massless spin 1 gauge bosons which carry color called gluons. The colored quarks are then added to yield Quantum Chromodynamics (QCD). Although there have been many dynamical and static successes of QCD, there has been one important missing link in QCD which casts a dark shadow over it and $SU(3)_{color}$. Let us assume the strong interactions are described by locally gauge invariant $SU(3)_{color}$ in a pure Yang Mills theory. Then if we consider the effects of confinement one is inescapably led to the existence of glueballs (multigluon resonant states). Yet experimentally we found vast numbers of $q\bar{q}$ states and qqq states but until recently no convincing evidence for glueballs.

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If this situation persisted I would conclude locally gauge invariant $SU(3)_{color}$ is in great trouble. Fortunately recent work has led to the discovery[1-4] of glueballs provided one assumes the following two simple input axioms

1. QCD is correct,

2. The OZI (or Zweig) Rule is universal for weakly coupled glue in disconnected Zweig diagrams where the disconnection is caused by creation or annihilation of new flavors of quarks.

Since these axioms merely represent modern QCD practice and agree with the experimental data very well, it is reasonable to assume that glueballs are discovered. In this case $SU(3)_{color}$ and QCD are in excellent shape and very probably correct.

There are other glueball candidates found in the radiative J/ψ decays[5-6,33] and some relatively weaker candidates from direct pattern recognition in hadronic spectroscopy, nonet + glueball → decuplet with characteristic mixing splitting. In this lecture I will discuss the evidence for glueballs. Then I will speculate somewhat on what lies beyond for the physics of the 21st Century.

How Does One Find a Glueball?

From the results of a vast number of experiments it is clear that if glueballs exist, they are masked in the vast collection of quark-built meson nonets in the mass range where one would expect to find glueballs ($\approx$ 1-3 GeV).

1. Pattern Recognition of a Nonet + Glueball → Decuplet

In this brute force method, one looks for a $q\bar{q}$ nonet with an extra singlet, a glueball with the same quantum numbers. If it is near enough to the singlets in the nonet it will mix with them giving Nonet + Glueball →

Decuplet, with characteristic mixing and splitting (and have other special characteristics of glueballs). Calculations have shown that the ideal mixing observed in a great deal of nonets would be affected in these decuplets, and pattern recognition would have to be used.[8] The $J^{PC} = 0^{++}$ $g_S(1240)$[9] is a glueball candidate of this type. This would make a 0^{++} decuplet with apparently the right characteristics. Of course one must realize that there are many other possible explanations for these states* and other candidates of this type.

2. Look in a Channel Enriched in Gluons

Glueball candidates of this type are the SLAC $J^{PC} = 0^{-+}$, iota (1440), which could be the tenth member of a ground state 0^{-+} decuplet,** and the SLAC $\theta(1640)$ which will be discussed later.[5-6,33] Of course one should realize that there are many other possible explanations for these states.

3. An OZI Suppressed Channel with a Variable Mass

Consider an OZI suppressed channel with variable mass for the disconnected part of the diagram which is composed of the hadrons involving only new flavors of quarks. Glueballs (i.e., strongly coupled multigluon states) with the right quantum numbers should break down the OZI suppression in the mass region where they exist and dominate the channel. Thus the OZI suppression can act as a filter for letting glueballs pass while suppressing other states. Furtherfore, the breakdown of the OZI suppression can serve as a clear signal that one or more glueballs are

* One could, for example, inadvertently mix states from the basic nonet with those of a radial excitation.

** The SLAC iota (1440) is thought to be in a channel where glueballs are enhanced since it is found in J/ψ radiative decay.

present in the mass region. According to present concepts in QCD, the OZI suppression is due to the fact that two or more hard gluons are needed to bridge the gap in a suppressed disconnected or hairpin(s) Zweig diagram involving new types (i.e. flavors) of quarks. The early onset of asymptotic freedom leads to a relatively weak coupling constant for these gluons, which then causes the OZI suppression. However, if the glue in the intermediate state resonates to form a glueball, the effective coupling constant (as in all resonance phenomena) must become strong, and the OZI suppression should disappear in the mass range of the glueball. This should allow hadronic states with the glueball quantum numbers to form with essentially no Zweig suppression. The author has made this argument previously.[10–12] Thus the Zweig suppression essentially is a filter which lets glueballs pass and suppresses other states. Incidentally this method which is the author's led to the first evidence for glueball candidates.[13,14,10] Subsequent work by the BNL/CCNY collaboration led to the conclusion[2–4] that there are indeed one or more glueballs if modern QCD practice is correct.

Except for the experiments which I cite as evidence for glueballs, The Zweig rule (or OZI suppression)[7,15,36] appears to be universally followed in disconnected diagrams in hadronic interactions where the disconnection is due to creation or annihilation of new flavor(s) of quark(s). Fig. 1 shows this clearly for the u,d,s quark system where the matrix element for the Zweig connected diagram is two orders of magnitude larger than for the corresponding Zweig disconnected diagram.[15–17]

That this occurs both in the decay and production processes is shown in Figures 1a and 1b. Figures 2a and 2b show that the J/ψ system exhibits even much greater Zweig suppression factors for Zweig disconnected diagrams. It should be noted in Fig. 2b that in addition to the well-known and striking Zweig suppression which occurs when the $c\bar{c}$ quarks annihilate there is a huge suppression in the Zweig disconnected diagram where $\psi(3685) \to J/\psi$ (3100 + 2π) which results in a width of the $\psi(3685)$ = 250 ± 40 kev even though the $\pi^+\pi^-$ case occurs in (33 ± 2)% of the cases and the $\pi^0\pi^0$ case occurs in (17 ± 2)% of the cases.

Figure 3 shows a similar and even more striking situation existing in the upsilon system since the $\Upsilon'(10,020) \to \Upsilon(9460)\pi\pi$ (30 ± 6)% of the time with the $\Gamma_{\Upsilon(10,020)}$ = (30 ± 10) kev whereas the $\Gamma_{\Upsilon(9460)}$ = 42 ± 15 kev. Thus the suppression in the first Zweig disconnection is strong enough to maintain the width of the Υ' consistent within errors with the width of the Υ. The same striking phenomena occurs in the process $\Upsilon''(10,020) \to \Upsilon(9460) + 2\pi$ which although it occurs ~ 10% of the time results in a width of the Υ'' which is consistent with the width of the Υ. Thus it is experimentally clear from the ψ and Υ systems that what I will call a double hairpin type of disconnection in a Zweig diagram is strongly suppressed.

Lipkin has argued[18] that what I call a double hairpin type of disconnected Zweig diagrams such as Fig. 6, $\pi^- p \to \phi\phi n$ (which is the process we are observing) should not be Zweig suppressed (or only suppressed by a very small factor) since it is related by crossing to $\phi + n \to \varphi + \pi^- + p$. He refers to this as a crossed pomeron diagram which is just elastic ϕ-nucleon scattering with additional pion production and states and there is no reason to believe this process is forbidden. Reference (18) has overlooked the fact that when you cross in that manner you get into different kinematic and physical regions and that you cannot simply relate the two reactions.[19] For example considering the kinematics only the crossed reaction (e.g. $\phi + n \rightarrow \phi\pi^- + p$) corresponds to very high momentum transfers and a very high mass for the $\pi^- + p$ system. Diffraction dissociation at very high momentum transfers and very high masses would be expected to be negligibly small and thus these processes would be expected to be suppressed much more than the Zweig suppression factors we are dealing with. The fact that $\sigma(\pi^- p) \rightarrow \phi\phi n \approx 20$ nanobarns whereas diffraction dissociation which Ref. 18 says is large (~ 10 mb) differ by a factor of 10^6 emphasizes that it is not justified to relate the two processes in the naive way Ref. 18 has.

One should note that Ref. 18 concludes the reaction $\psi(3685) \rightarrow J/\psi(3100) + 2\pi$ is Zweig allowed since it is also a crossed Pomeron diagram. Ref. 18 ignores the fact that the full width of $\psi(3685)$ is only $\approx$ 215 kev and thus this Zweig disconnected diagram (our Fig. 2b) is strikingly suppressed.

$\Upsilon(\Upsilon'') \rightarrow \Upsilon + 2\pi$ decays also impressively show that so-called crossed Pomeron diagrams (in the notation of Ref. 18) also exhibit very strong suppressions and thus this line of reasoning is obviously fallacious for the reasons I have already mentioned.

The reason why the $\psi' \rightarrow J/\psi + 2\pi$, and $\Upsilon'(\Upsilon'') \rightarrow \Upsilon + 2\pi$ have large branching ratios is probably at least partly due to the fact that these transitions can proceed by two relatively softer gluons compared to the direct three-gluon decays of the ψ', Υ' and Υ'', and also the kinematics of the decay favor the 2π channel, whereas there are many channels which compete for the three gluon partial decay width. One should note that observation indicates that all these Zweig disconnected diagrams show large suppressions. The exact reasons why quark and glue are so strongly decoupled in these diagrams cannot be arrived at until perhaps Lattice Gauge work attacks the problem.

Thus if we assume the OZI rule is universal for weakly coupled glue in Zweig disconnected diagrams where the disconnection is due to the creation or annihilation of new flavors of quarks, then the breakdown of the OZI suppression that we observe in $\pi^- p \to \phi\phi n$ must be due to strongly coupled glue. A glueball being a multi-gluon resonance would like in all hadronic resonance phenomena correspond to effectively strong coupling and thus the OZI suppression which in QCD can be viewed as due to weakly coupled multi-glue intermediate states would be broken down by a glueball. Thus in the reaction $\pi^- p \to \phi\phi n$, the multi-gluon system in the intermediate state which forms the $\phi\phi$ system would in the absence of glueballs lead to only Zweig suppressed $\phi\phi$ production. However the $\phi\phi$ system has a variable mass and all the possible glueball quantum numbers for C = +. Thus at those masses where the multigluon intermediate state forms a glueball with C = + the Zweig suppression should be broken down and the $\phi\phi$ system will contain the glueball resonance parameters and quantum numbers. Thus the $\phi\phi$ system in the reaction $\pi^- p \to \phi\phi n$ will act as a filter passing glueball states and rejecting the other $q\bar{q}$ states.

Other alternatives such as the possibility of more complicated hadronic states will be discussed later.

Figures 4, 5 and 6 show the reactions we have studied in three generations of experiments searching for glueballs. One should note that the situation has not changed appreciably since my Erice lecture last summer.[4]

The dramatic breakdown of the $\pi^- p \to \phi\phi n$ ($\approx$ 4,000) events OZI (or Zweig) suppression we saw in the earlier data[13-14,1-4] also occurs in the new sample as shown in Fig. 7. We see the general $\approx$ uniform background from the reactions a) $\pi^- p \to K^+K^-K^+K^-n$ which is OZI (or Zweig) allowed (Fig. 4) and the two ϕ bands representing b) $\pi^- p \to \phi K^+K^-n$ (Fig. 5) which is also Zweig allowed are evident. Where the two ϕ bands cross we have the Zweig forbidden reaction $\pi^- p \to \phi\phi n$. The black spot clearly shows a more-or-less complete breakdown of the Zweig suppression. This has been quantitatively shown[10b] to be so in these reactions, and also by comparing K^- induced ϕ and $\phi\phi$ production.[20–21] The black spot when corrected for double counting and resolution is ≈ 1,000 times the density of reaction (a) and ≈ 50 times the density of reaction (b). If by projecting out the ϕ bands, as shown in Fig. 8, one finds a huge $\phi\phi$ signal which is ≈ 10 times greater than the background from reaction (b) even with rather wide cuts. The recoil neutron signal is shown in Fig. 9 and is also very clean ≈ 97% neutron.

Figure 10 shows the acceptance corrected $\phi\phi$ mass spectrum in the ten mass bins which were used for the partial wave analysis. All waves with $J = 0 - 4$, $L = 0 - 3$, $P = \pm$ and η (exchange naturality) = $\pm$ were allowed in the partial wave analysis, leading to 52 waves. The incident π^- lab momentum vector and the lab momentum vectors of the four kaons completely specified an event. The Gottfried-Jackson frame angles β(polar) and γ(azimuthal) are shown in Fig. 11. These and the polar angles (θ_1,θ_2) of the K^+ decay in the ϕ rest systems relative to the ϕ direction and the azimuthal angles α_1 and α_2 of the K^+ decay direction in the ϕ_1,ϕ_2 rest systems (see Fig. 12) were also used to specify an event.

The same experimental arrangement as described earlier was used.[1,13] The results of the mass independent partial wave analysis are shown in Figs. 13 and 14. We had in 1982 determined that our 1200 $\phi\phi n$ event data contained two $J^{PC} = 2^{++}$ waves.[1,2] The predominant one being an S-wave with spin 2 peaked in the lower mass region and the other being a D-wave with spin 2 peaked at higher masses.

In this analysis (1983) of ≈ 4,000 events,[3-4] these two waves were again selected with a very high statistical precision >> 10σ. However the fit was totally unacceptable and required a third D-wave with spin 0 as shown in Fig. 13. The relative phase motion of the D waves using the S wave as a reference is shown in Fig. 14. The statistical significance of this third wave was ≈ 25σ. Although there was an indication for this third wave in the earlier 1200 event sample, it could not be considered statistically significant at that time. It should be noted that the 1200 event data sample and the new ≈ 4,000 event data sample agree very well with each other within statistical errors. One should note that the results of the partial wave analysis are quite insensitive to the acceptance and the detailed shape of the mass spectrum. We also found that for $t' < 0.3$ GeV^2, the t' distribution is consistent with $e^{(9.4 \pm 0.7)t'}$. If one looks at the quark structure of Fig. 6, one essentially has a pion exchange radiating several gluons (thought to represent a glueball) and thus one would expect a peripheral production mechanism, which is what we observe.

One might ask at this point why are we so incredibly selective - picking 3 waves out of 52 with the statistical significance of the third wave ≈ 25σ. The answer is that the background is small enough and incoherent and thus does not have a significant effect on the ϕϕ systems distinctive individual wave signals. The ϕϕ system wave signals are shown (roughly to scale) in Fig. 15 for M = 0 waves. The PWA clearly demonstrated that only M = 0 waves were significant in the fit, thus these are the most relevant. It is clear from Figs. 15a and 15b, that every wave has its own characteristic signature and thus the $\phi\phi$ system is an unusually selective wave content analyzer. This is in large measure due to the fact that each ϕ has spin 1 and thus the six angular variables and their correlations have large characteristic signatures which are very sensitive to the exact quantum numbers of each wave. Furthermore our very low incoherent background allows us to see the characteristics of the $\phi\phi$ system clearly.

The comparison of Monte Carlo generated events to the observed angular variables and their characteristic combinations are shown in Figs. 16a-c. It is clear the agreement is very good, and this is the case for all ten mass bins. The amplitudes and phase motion (see Figs. 13 and 14) of the waves relative to the S-wave clearly reveals resonance or Breit-Wigner behavior. The S-wave had to be used as a reference due to the fact that the background is both small and incoherent. It is important to note that the appropriate phase motion is the most sensitive test of resonant behavior, and we have clearly demonstrated that it occurs in just the required manner. In the analysis we actually employed the K-matrix method[22] which is approximately equivalent to but a somewhat more realistic approach to fitting with relativistic Breit-Wigner's. Nevertheless in this case either method would give results consistent with each other since the effects of other channels (taken into account in the K-matrix) are small.

Three resonant states (or K-matrix poles) were required to obtain an acceptable fit. Attempts to fit the results with two resonant states (or K-matrix poles) in which the three required waves were used were rejected by 13σ, whereas the three resonance fit was quite good. Table I lists the deduced Breit-Wigner parameters, quantum numbers and estimated content of the individual waves for the three states and the estimated errors. The Argand plot deduced from the K-matrix fit is shown in Fig. 17, and it clearly shows the characteristics expected of resonance behavior. By increasing the statistics from ≈ 1200 events to ≈ 4,000 events the upper of the two resonant $I^G J^{PC} = 0^+ 2^{++}$ states was resolved into two states with the same quantum numbers.

It should be noted that the mixing of waves is substantial in these three $J^{PC} = 2^{++}$ states and the exact wave content of each resonance or K-matrix pole is therefore sensitive to details and somewhat uncertain. However from the glueball physics point of view we are at present mostly interested in the quantum numbers and parameters of the resonant states and not very concerned about their exact wave contents.

If one assumes as input axioms:

1. QCD is correct;

2. The OZI rule is universal for weakly coupled glue in Zweig disconnected diagrams where the disconnection is due to the introduction of new flavors of quarks, then the states we observe must represent the discovery of 1-3 glueballs.[2-4]

Note that axiom (2) allows only resonating glue (i.e. glueballs) to break the Zweig suppression. One primary glueball could break down the Zweig suppression and possibly mix with two quark or other possible states.

Since these axioms strikingly agree with the data in the ϕ, J/ψ and Υ systems, and merely represent modern QCD practice, it is reasonable to consider this the discovery of glueballs.

The constituent gluon models[23-24] (i.e. gluon has effective mass) predict three low lying $J^{PC} = 2^{++}$ glueballs. The mass estimates from the MIT bag calculations and the lattice gauge groups[25-28] give the range ≈ 1.7 - 2.5 GeV for $J^{PC} = 2^{++}$ glueballs. Thus we are clearly in the right ballpark for agreement with present phenomenological mass calculations.

T.D. Lee has analytically calculated J = 2 glueballs in the strong coupling limit[29] by analytical means. He obtains three glueball states which correspond to our three states. His strong coupling calculation gives the mass differences between these three states in terms of two parameters, one being essentially the effective strength of the coupling and the second a mass scale parameter. In order to try to adjust his strong coupling calculation to the real world of intermediate coupling we took the mass of the 0^{++} glueball as ≈ 1 GeV from the Lattice Gauge calculations, and fit our three masses with the other parameter and found a reasonable fit.

TABLE I

Three Resonance Fit

$M_1 = 2.120^{+.020}_{-.120}$ $\Gamma_1 = .300^{+.150}_{-.050}$ ~ 40% data:

S-wave, S = 2 ~$30\%^{+70\%}_{-10\%}$ coupling sign (+) defined

D-wave, S = 2 ~$50\%^{+10\%}_{-50\%}$ coupling sign (−)

D-wave, S = 0 ~$20\%^{+30\%}_{-20\%}$ coupling sign (−)

$M_2 = 2.220^{+.090}_{-.020}$ $\Gamma_2 = .200 \pm .050$ ~ 40% data

S-wave, S = 2 ~$40\%^{+10\%}_{-20\%}$ coupling sign (+)

D-wave, S = 2 ~$50\%^{+20\%}_{-10\%}$ coupling sign (+)

D-wave, S = 0 ~$10\%^{+10\%}_{-10\%}$ coupling sign (+)

$M_3 = 2.360 \pm .020$ $\Gamma_3 = .150^{+.150}_{-.050}$ ~ 15% data

S-wave, S = 2 ~$25\%^{+25\%}_{-10\%}$ coupling sign (+)

D-wave, S = 2 ~ 0% + 25% coupling sign (−)

D-wave, S = 0 ~$75\%^{+15\%}_{-25\%}$ coupling sign (+)

A similar procedure was used by the author in the case of the Pauli-Dancoff strong coupling calculations of the nucleon isobars many years ago. In that case when I used the known f^2 and a reasonable value for the cut-off, the strong coupling calculation results gave reasonable agreement with the experimental observations on nucleon isobars.

<u>Expected Width for Glueballs</u>

In hadrons, the hadronization process consists of creation of one or more $q\bar{q}$ pairs. This must occur near the outer region of confinement involving strongly interacting soft glue, probably including collective interactions, if we are to have resonances decay with typical hadronic widths ($\Gamma_{hadronic}$ ~ 100 to several hundred MeV).

For example the $\rho(770) \rightarrow \pi\pi$ requires production of one quark pair. The width of the $\rho(770)$ is $\Gamma_\rho = 154 \pm 5$ MeV. The $\rho'(1600) \rightarrow 4\pi$ requires the production of three quark pairs. Yet $\Gamma_{\rho'} \approx 300 \pm 100$ MeV. Hence even though production of two additional quark pairs is required the $\Gamma_{hadronic}$ actually increases. This example clearly shows that hadronization easily occurs via collective soft glue effects and this is the basis of typical hadronic widths.

A glueball is a resonating multi-gluon system. The glue-glue coupling is stronger than the quark-glue coupling and thus it would be expected, via gluon splittings before the final hadronization, to have a similar hadronization process to a $q\bar{q}$ hadron. Hence a glueball would be expected to have typical hadronic widths. This is certainly to be expected for ordinary (non-exotic) J^{PC} states. In the case of exotic J^{PC} states, this arguement may not be relevant since no one yet knows what suppresses the unobserved exotic sector. Therefore Meshkov's oddballs[23] may be narrow.

I have previously discussed[10b,12b] some well-known peculiarities of the OZI rule. In particular if one introduces successive steps both of which are OZI allowed, one can on paper defeat the OZI rule.

For example, $\phi \rightarrow \rho\pi$ is OZI forbidden, but $\phi \rightarrow K^+K^+ \rightarrow \rho\pi$ represents two successive OZI allowed processes which appears to defeat the OZI rule. Similarly, $\pi^-p \rightarrow \phi n$ is OZI forbidden, but $\pi^-p \rightarrow K^+K^-n \rightarrow \phi n$ representing two successive OZI allowed processes which appears to defeat the rule. One can also introduce other complicated intermediate states or processes other than hard multi-gluons to join the disconnected part of the diagram and also appear to defeat the rule.

Thus the OZI rule is peculiar in that you can defeat it by two-step processes or in QCD language changing the nature of the multi-gluon exchange needed in the one-step diagram to a series of the ordinary OZI allowed gluon exchanges.* Thus based on the experimental validity of the rule, Zweig's diagrams are to be taken literally as one step processes and the multi gluon exchanges needed to connect disconnected parts of the diagram are not to be tampered with.

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* This may at least partly be explained by the fact that when you draw quark line diagrams for typical two-step allowed processes in a Zweig forbidden diagram, you are annihilating quark pairs after hadronization has occurred. Since annihilations occur at short distances, and hadronization, as I have discussed, occurs at large distances, these two-step processes are probably dynamically discriminated against. However it appears that why the OZI rule works so well in Zweig disconnected diagrams will only be understood when one has calculated the dynamics involved using QCD with intermediate and strong couplings.

If one does not accept axiom 2 and demotes the universal OZI rule to the improbable OZI accident could what we see be due to very non-ideally mixed radial excitations or 4-quark states containing $s\bar{s}$ pairs, etc.

Even in this event (for which there is no evidence) it would take a second striking accident for three $I^G J^{PC} = 0^+ 2^{++}$ resonant states and essentially nothing else to occur within the narrow high mass interval of ≈ 2120 to 2360 MeV. Since inventing enough unlikely accidents can destroy any theory I do not consider these possible explanations plausible.

Other Glueball Candidates

The radiative decay of the J/ψ is thought to occur as shown in Fig. 18 where one of the usual three gluons emitted in the annihilation of the $c\bar{c}$ pair is replaced by a photon. Thus it has been argued[8,30] that the two-gluon system could recoil from the photon and preferentially form a glueball. The first and most discussed glueball candidate of this type is the iota (1440).[5] The status of the iota (1440) with $J^{PC} = 0^{-+}$, M ≈ 1440^{+20}_{-15} and Γ ≈ 55^{+20}_{-30} was recently thoroughly reviewed in the Paris Conference.[6] Some concern was expressed that the ITHEP calculations on instanton effects would move a 0^{-+} glueball up to 2.0–2.5 GeV mass region. The possibility that the iota (1440) is a radial excitation rather than a glueball has also been discussed.

Another glueball candidate of this type is the θ(1640). $J^{PC} = 2^{++}$ was favored with a 95% C.L. The resonance parameters were M ≈ 1700 ± 50, Γ ≈ 160 ± 50. See Ref. 6 for a review of the status of these glueball candidates.

Recently at the Experimental Meson Spectroscopy Conference there were papers discussing them.[23b,31] Opinions differ strongly. The most recent and thorough review was made by Sid Meshkov.[23b] He concluded the iota (1440) and θ(1640) are not glueballs but also cited alternate explanations[31,32] in which they could be.

One can directly search for a nonet + glueball → decuplet with characteristic mixing splittings. The $g_S(1240)$ with $J^{PC} = 0^{++}$, M = 1240 ± 10 MeV, and Γ = 140 ± 10 MeV is one such a glueball candidate.[9] Of course other explanations such as the mixture of singlets from two nonets (one of which could be a radial excitation) are alternatives. The direct pattern recognition search for glueballs is a difficult and so far inconclusive program.

The Mark III collaboration reported new data in radiative J/ψ decay.[33] They observed the iota and the θ. For the iota, the $K_S^0 K_S^0 \pi^0$ mode was observed in addition to the previously seen $K^+K^-\pi^0$ and $K_S^0 K^{\pm}\pi^{\mp}$ modes. The Breit-Wigner fit parameters determined were M = 1.46 ± 0.01 GeV and Γ = 0.097 ± 0.0025 GeV. In the case of the θ the Breit-Wigner parameters were determined as M = 1.719 ± 0.006 GeV, Γ = .117 ± .023 GeV. The iota and θ situation did not appear to change substantially from the prior review.[6] The only essentially new development was the evidence for a new narrow structure (ξ(2200)).

It should be noted the question has often been raised as to whether φφ states are seen in radiative decay of the ψ. The new MK III results observe $\psi \to \gamma\phi\phi$.[33] Their detection efficiency for φφ is very low in the mass region of the $g_T(2120)$, $g_T'(2220)$ and $g_T''(2360)$. Thus they find only ~10 events in this mass region. However if one corrects their φφ mass spectrum for the detection efficiency it is not inconsistent with the shape of the mass spectrum seen by BNL/CCNY. However one should note we are comparing ≈ 4,000 observed events to ~ 10. It appears that the MK III can only observe strong signal, narrow, high mass φφ states such as the decay of the η_c, and thus is not likely to be able to observe the BNL/CCNY states.

Conclusions on Glueball States

If you assume as input axioms:

1. QCD is correct;

2. The OZI rule is universal for weakly coupled glue in disconnected Zweig diagrams where the disconnection is due to the creation or annihilation of new flavor(s) of quark(s), then the BNL/CCNY $g_T(2120)$, $g_T'(2220)$ and $g_T''(2360)$ are produced by 1-3 primary glueballs. One or two broad primary glueballs could in principle break down the OZI suppression and mix with one or two quark states which accidentally have the same quantum numbers and nearly the same mass. However the simplest explanation of the rather unusual characteristics of our data is that we have found a triplet of $J^{PC} = 2^{++}$ glueball states.

Since our input axioms are in good agreement with experiments and merely represent modern QCD practice, we have very probably discovered 1-3 $J^{PC} = 2^{++}$ glueballs.

The iota(1440) and the $\theta(1700)$ observed in J/ψ radiative decay are glueball candidates. The pros and cons of which have been discussed briefly here and more extensively in the references cited. Other recent glueball searches[3] have not yet led to candidates.

The Status of $SU(3)_c$ and QCD

The very probable discovery of glueballs has removed the most ominous cloud over $SU(3)_c$ and QCD. Therefore I now believe that locally gauge invariant $SU(3)_{color}$ and QCD are very probably correct.

However, it will be of great interest to compare the next generation of more realistic and complete Lattice Gauge Theory calculations and soft QCD observed phenomena.

Beyond

I now will address myself to the question of Beyond? and its relation to the physics of the 21st Century.

If $SU(3)_{color}$ is a locally gauge invariant group which describes hadronic interactions, it may be the most important of all the present gauge symmetries. This is so, in my opinion, for several reasons. First, $SU(3)_{color}$, like the electromagnetic U(1), is a gauge symmetry that is thought to be exact at low energies. However, unlike U(1), which combines with $SU(2)_L$ at energies ~ 100 GeV, $SU(3)_{color}$ is thought to continue as a good gauge symmetry until very high energies.

At present the most simple grand unification schemes have $SU(3)_c$ combining with the electroweak group at very high energies ~ 10^{12} GeV in the cms. Thus $SU(3)_{color}$ could remain as an exact symmetry from low energies to beyond the highest energies we can reach in the physics of the 21st Century. Of course the simple grand unification schemes suffer from a severe limitation. Namely, ultra-fine-tuning of the parameters are required to maintain the huge difference between the electroweak and grand unified mass scales which gives us the "Great Desert". This situation is alleviated if one introduces a series of nested gauge groups to cover the huge mass range.

The next step in nested gauge-gauge groups might be some new strong color interaction conceptually similar to hypercolor, or in particular extended technicolor. These schemes are essentially scaled up versions of QCD with typical mass scales ~ 1 GeV. Thus with a desertron these schemes would lead us to expect discovery of a new strong interaction described by a gauge theory similar to $SU(3)_{color}$. We would then find Technimesons, Technibaryons, and of direct interest to the present lecture, Hyperglueballs or Techniglueballs.

There are other nested gauge group schemes which would populate the great desert and place oases in them. I have not considered supersymmetry theories in this talk as they have been adequately considered recently.

Of course something very dramatic could occur at high enough energies. For example, there might be a fundamental length below which local commutativity and our present local field theory concepts fail.

In tests of the forward π-N dispersion relations in the latter sixties, we showed[34-35] that they are valid up to energies ~ 20 GeV (Lab) and concluded that if a fundamental length existed its value would be less than 10^{-15} cm and probably less than 10^{-16} cm. Recent work at higher energies is consistent with limits an order of magnitude lower. If one built a 40 TeV on 40 TeV desertron and allowed for pp as well as $\bar{p}p$ with polarized beam options, one could probably test the dispersion relations to cm energies ≈ 80 TeV.

Since if the forward dispersion relations hold for $|\ell| \gtrsim 1/E_{lab}$ where ℓ is the fundamental length, this would lead to tests of a fundamental length down to ~ 10^{-23} - 10^{-24} cm^2.

It should be noted that it was pointed out a long time ago that if local commutativity failed anywhere it would fail everywhere. This of course was based on a sharp boundary of the failure. R. Oehme, the speaker and others felt nature would temper such a boundary suitably.

T.D. Lee has recently shown[29] quantitatively that sufficient tempering can be achieved by eliminating exponential terms and keeping all power terms.

In conclusion the challenges of the 21st Century will include:

1) Do locally gauge invariant groups still determine the basic physical laws?

2) If so, will we find new gauge invariant groups or just a great desert?

3) Will our fundamental field theory concepts continue to hold? Or will we find a fundamental length?

4) Is there only a great desert or does it contain many new oases including unforseen phenomena?

It is clear that we need a desertron to explore the great desert and push forward our work on the physics of the 21st Century.

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Figure Captions

Fig. 1a Zweig connected (allowed reaction) diagrams for the u,d,s quark system.

Fig. 1b Zweig disconnected diagrams (suppressed reaction) for the u,d,s, quark system. The helixes represent gluons bridging the disconnection.

Fig. 2a A Zweig connected diagram for the $\psi(3685)$ decay.

Fig. 2b Zweig disconnected diagrams in the J/ψ and excited ψ states.[40]

Fig. 3 Zweig disconnected diagrams in the Υ system.[40–41]

Fig. 4 The Zweig quark line diagram for the reaction $\pi^- p \to K^+K^-K^+K^-n$, which is connected and OZI allowed.

Fig. 5 The Zweig quark line diagram for the reaction $\pi^- p \to \phi K^+K^- n$, which is connected and Zweig allowed.

Fig. 6 The Zweig quark line diagram for the reaction $\pi^- p \to \phi\phi n$ which is disconnected (i.e. a double hairpin diagram) and is OZI forbidden. Two or three gluons are shown connecting the disconnected parts of the diagram depending upon the quantum numbers of the $\phi\phi$ system.

Fig. 7 Scatter plot of K^+K^- effective mass for each pair of K^+K^- masses. Clear bands of $\phi(1020)$ are seen with an enormous enhancement (black spot) where they overlap (i.e. $\phi\phi$) showing essentially complete breakdown of OZI suppression.

Fig. 8 The effective mass of each K^+K^- pair for which the other pair was in the ϕ mass band.

Fig. 9 The missing mass squared for the neutral system recoiling from the $\phi\phi$.

Figure Captions (continued)

Fig. 10 The $\phi\phi$ mass spectrum corrected for acceptance. The solid line is the fit to the data with the three resonant states to be described later. The points at the bottom of the diagram are the acceptance for each mass bin to be read with the scale at the right.

Fig. 11 The Gottfried-Jackson frame with polar angle β and azimuthal angle γ.

Fig. 12 The ϕ_1 rest frame with the polar angle θ_1 of the decay K_1^+ (relative to ϕ direction) and the azimuthal angle α_1 of the decay K_1^+.

Fig. 13 The three $\phi\phi$, $J^{PC} = 2^{++}$ partial waves at production in 50 MeV mass bins (except ends). The smooth curves are derived from a K-matrix fit.

Fig. 14 D-S phase difference from the partial wave analysis vs. $\phi\phi$ mass. The smooth curves are derived from a K-matrix fit.

Fig. 15a Various pure waves from $J^{PC} = 0^{++}$ to $J^{PC} = 4^{++}$ with M = 0.

Fig. 15b Various pure waves from $J^{PC} = 0^{-+}$ to $J^{PC} = 3^{-+}$ with M = 0.

Fig. 16a Cos β and γ for three representative mass bins, where β is the polar angle and γ is the azimuthal angle of a given ϕ in the G.J. frame.

Fig. 16b α, $\alpha_1 - \alpha_2$, and $\alpha_1 + \alpha_2$ for three representative mass bins, where α is the azimuthal angle of the K^+ in the ϕ rest frame measured from the x-axis of the G.J. frame.

Fig. 16c $\cos\theta$, $\cos\theta_1' + \cos\theta_2'$, and $\cos\theta_1' - \cos\theta_2'$ for three representative mass bins, where θ is the polar angle of the K^+ in the ϕ rest frame measured from the other ϕ as the z-axis.

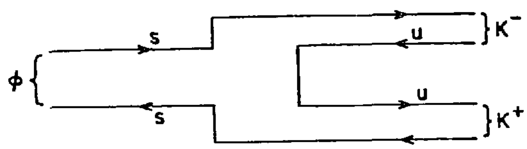
Figure Captions (continued)

Fig. 17 Argand plot from K-matrix.

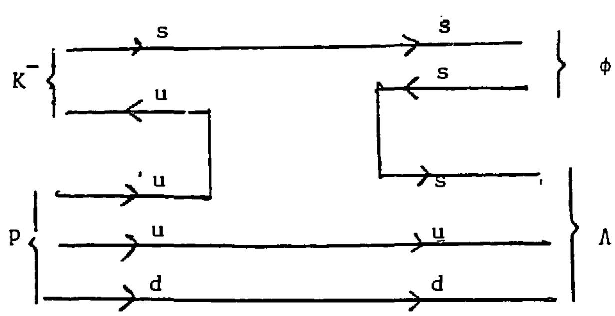
Fig. 18 The dominant diagram in radiative J/ψ decay.

CONNECTED

ALLOWED PROCESS



$\phi \rightarrow K^+K^-$

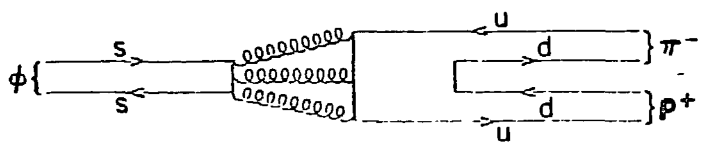


$K^-p \rightarrow \phi\,\Lambda$

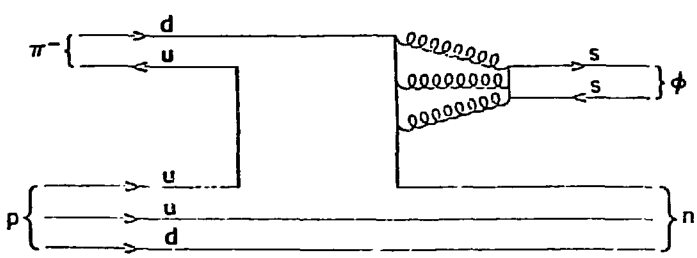
Figure 1a

DISCONNECTED

FORBIDDEN (SUPPRESSED)



$\phi \rightarrow \pi^- \rho^+$



$\pi^- p \rightarrow \phi n$

Figure 1b

CONNECTED

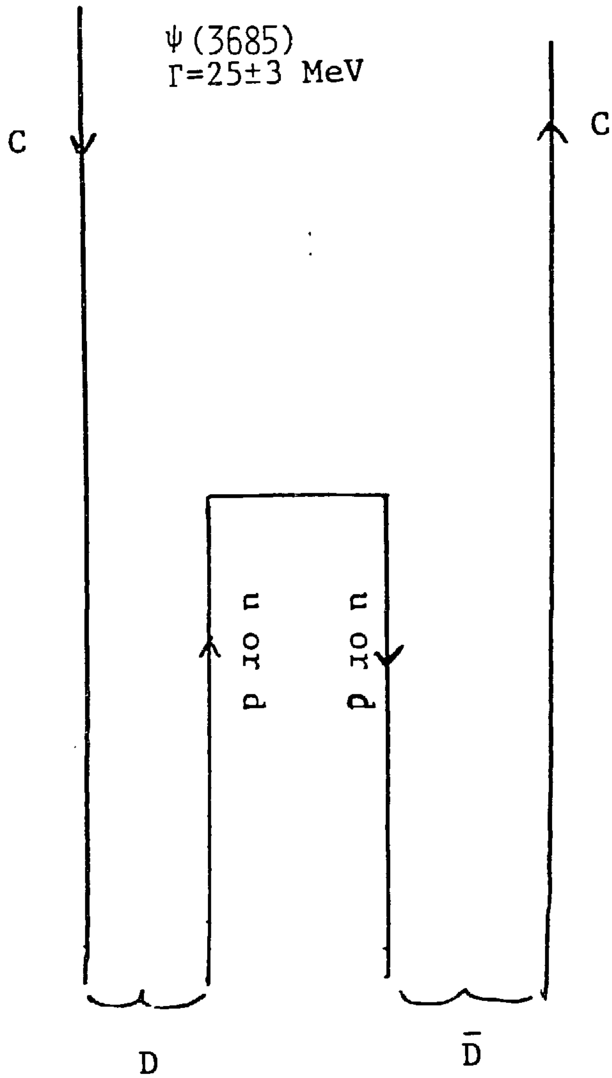


Figure 2a

DISCONNECTED

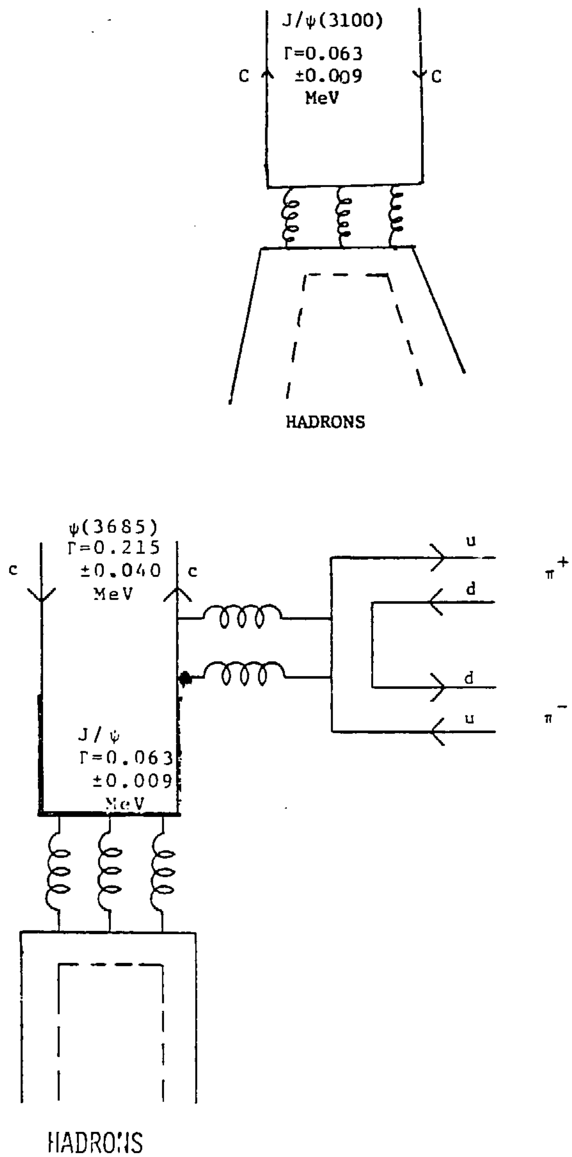


Figure 2b

DISCONNECTED ZWEIG DIAGRAMS IN ϒ SYSTEM

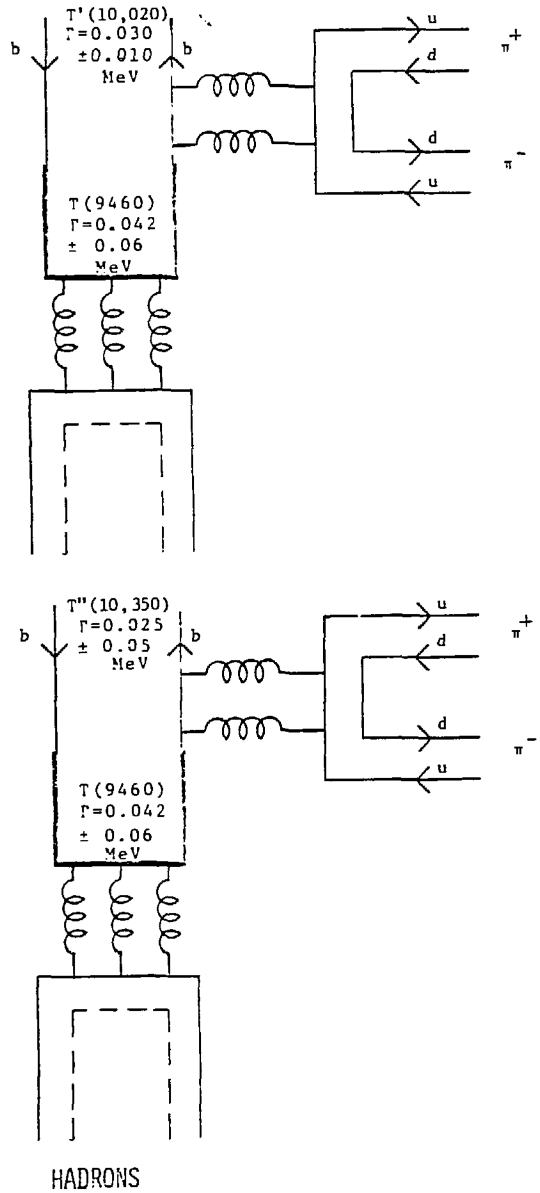


Figure 3

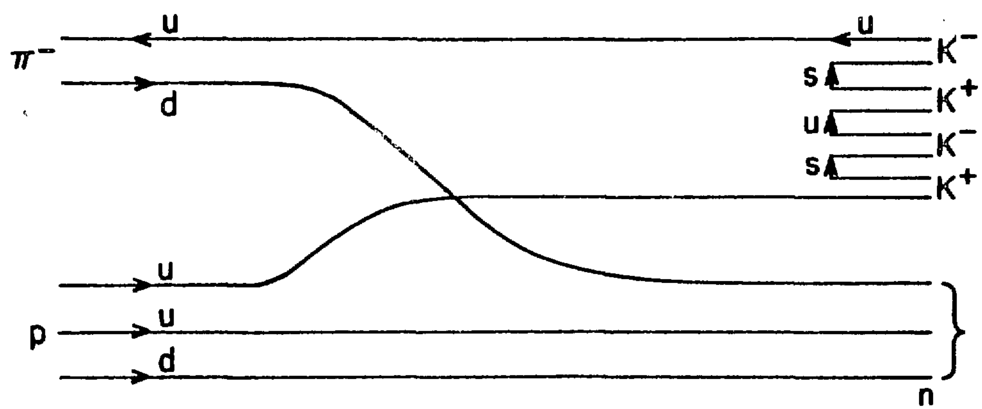


Figure 4

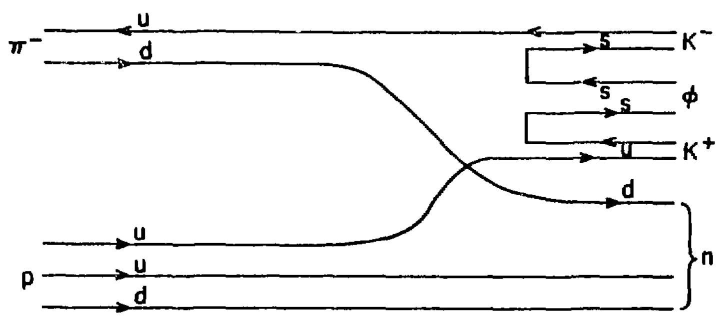


Figure 5

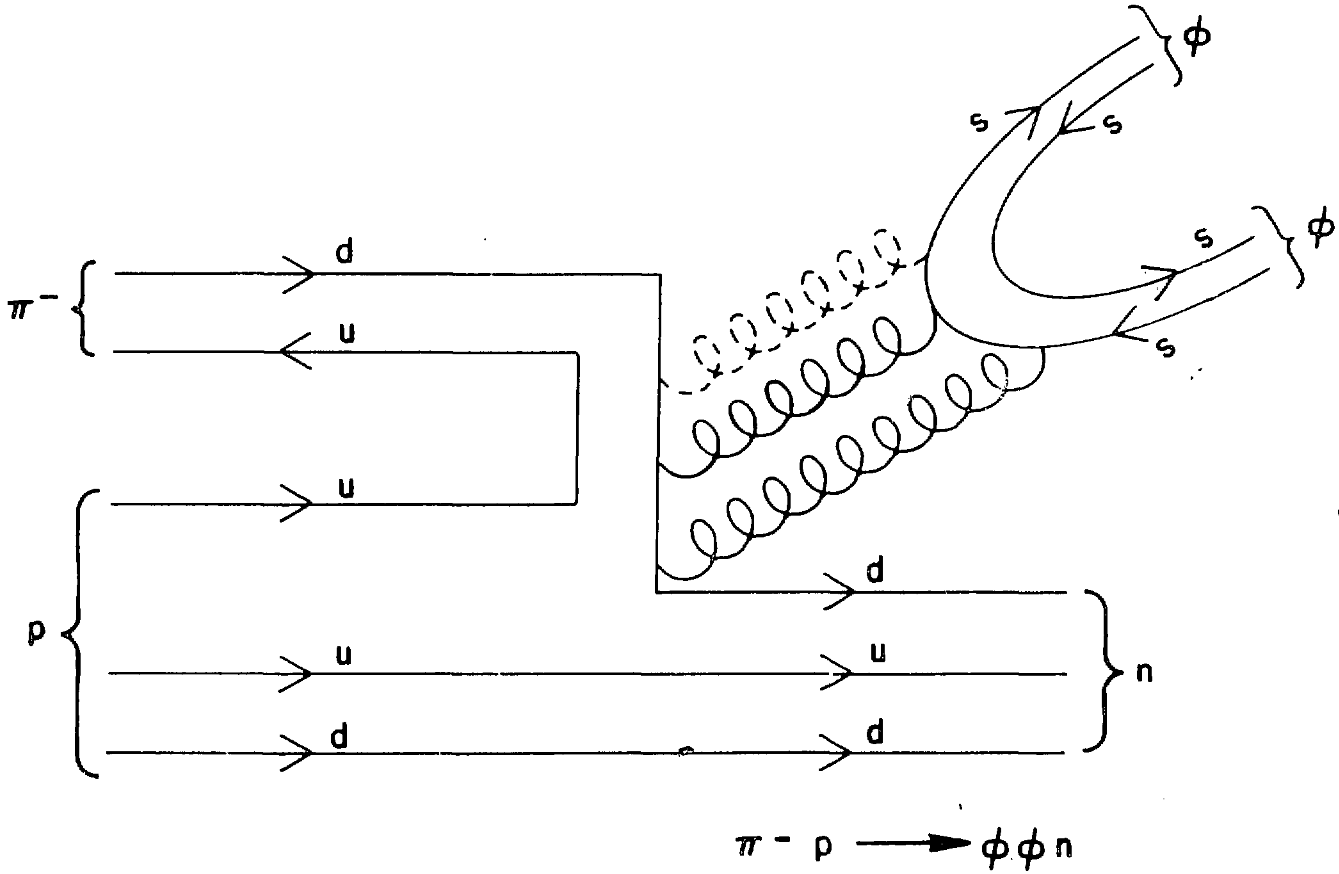


Figure 6

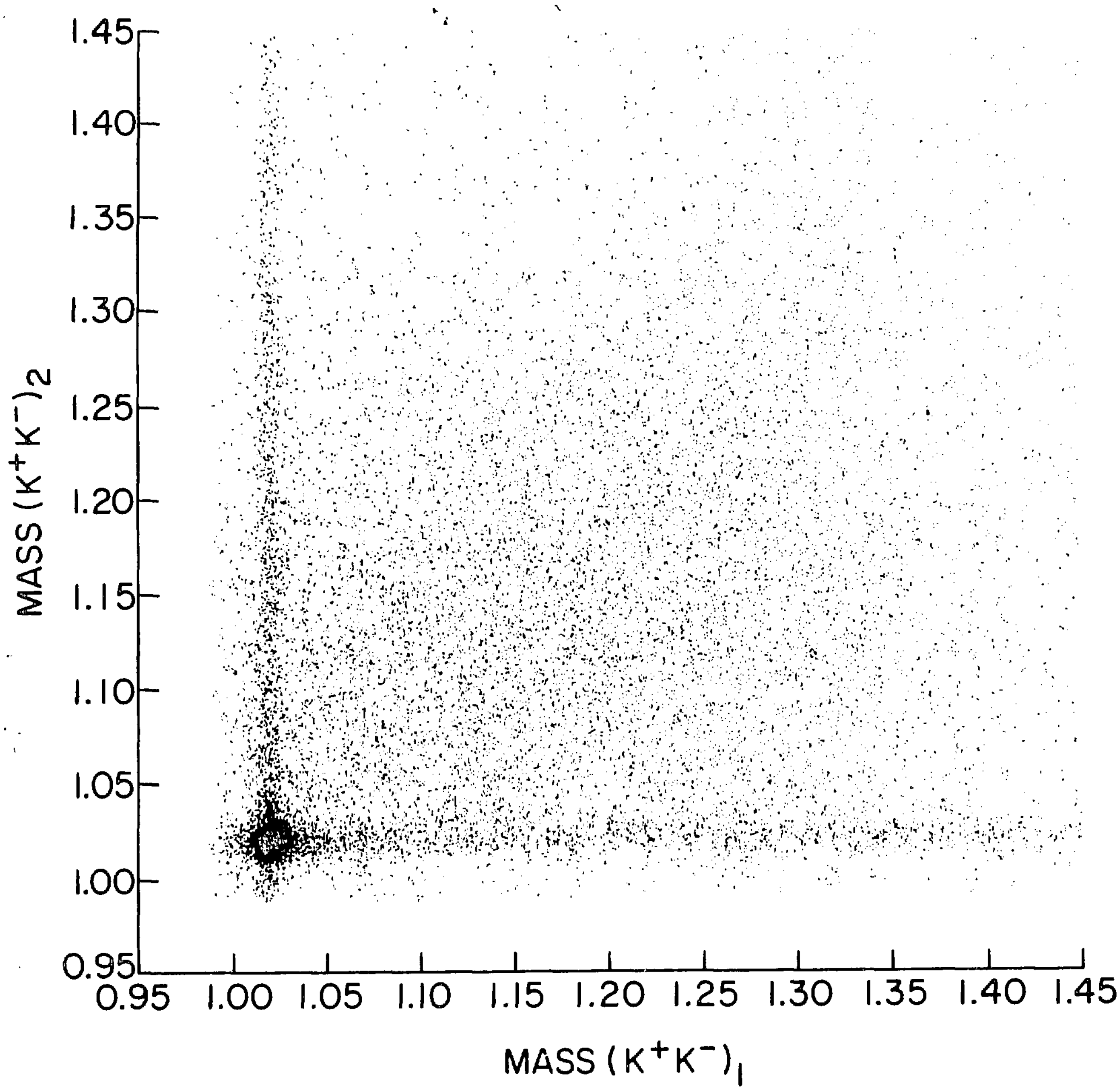
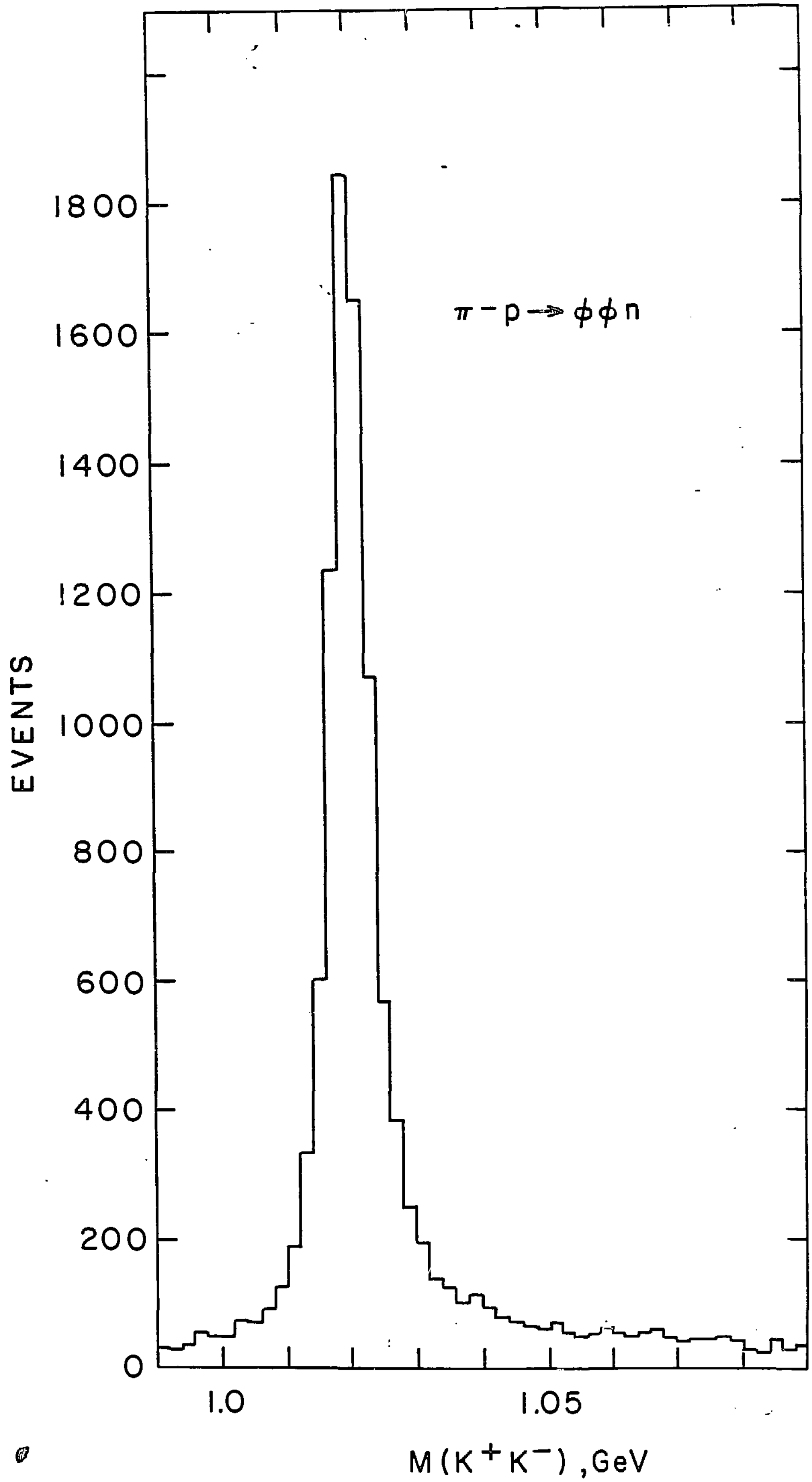

MASS $(K^+K^-)_2$
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.95 1.00 1.05 1.10 1.15 1.20 1.25 1.30 1.35 1.40 1.45
MASS $(K^+K^-)_1$

Figure 7



π⁻p → φφn
EVENTS
1800
1600
1400
1200
1000
800
600
400
200
0
1.0
1.05
M(K⁺K⁻), GeV

Figure 8

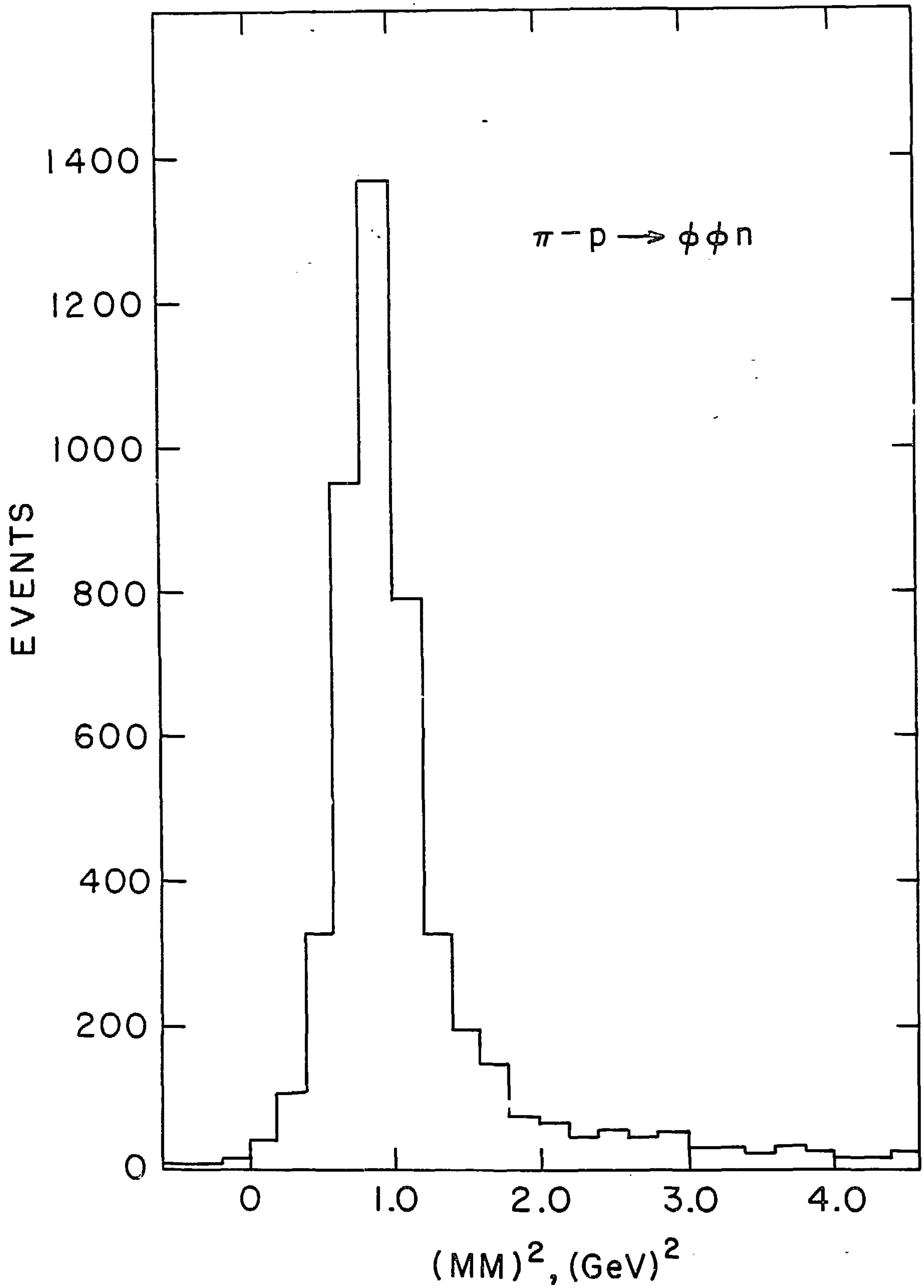
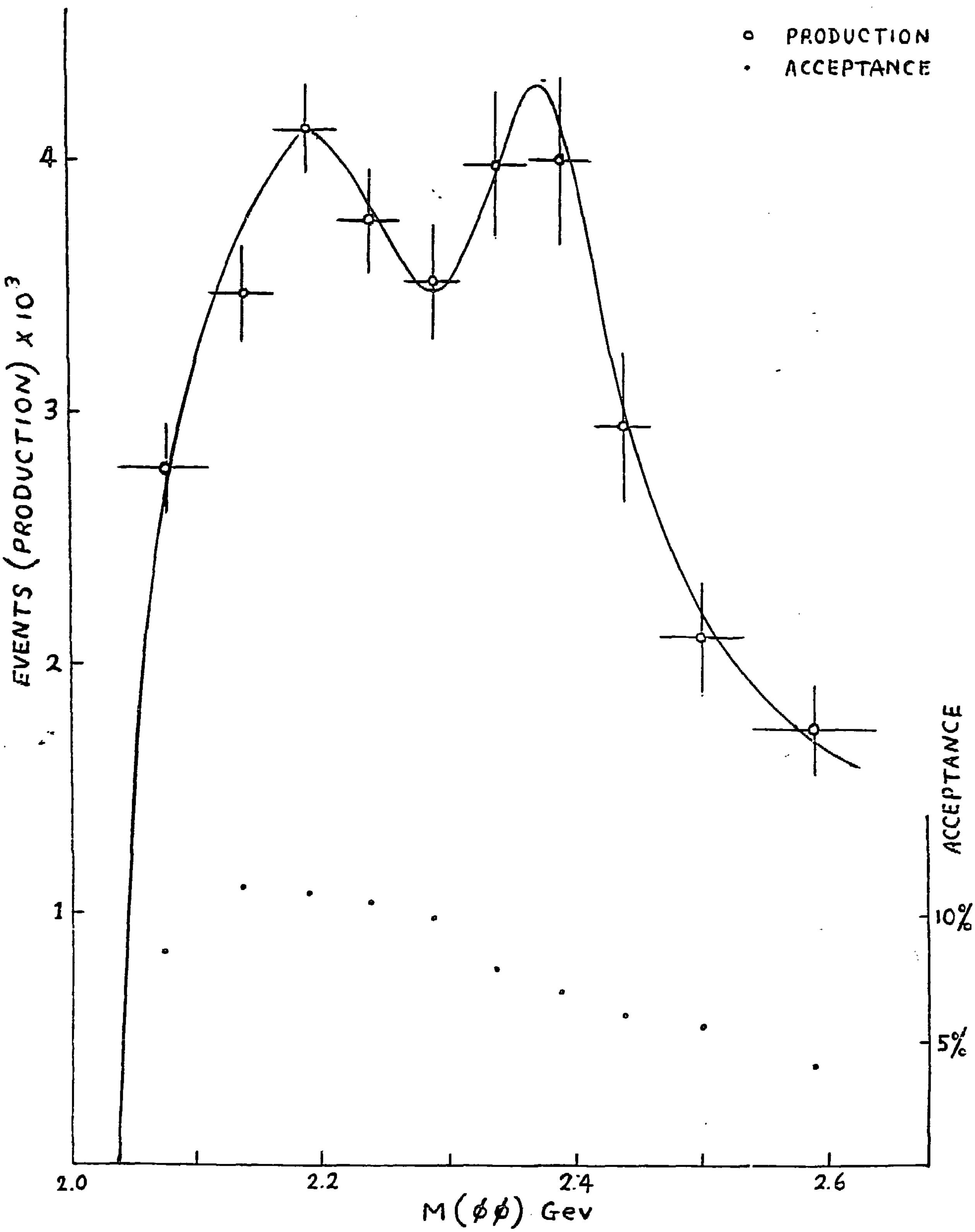
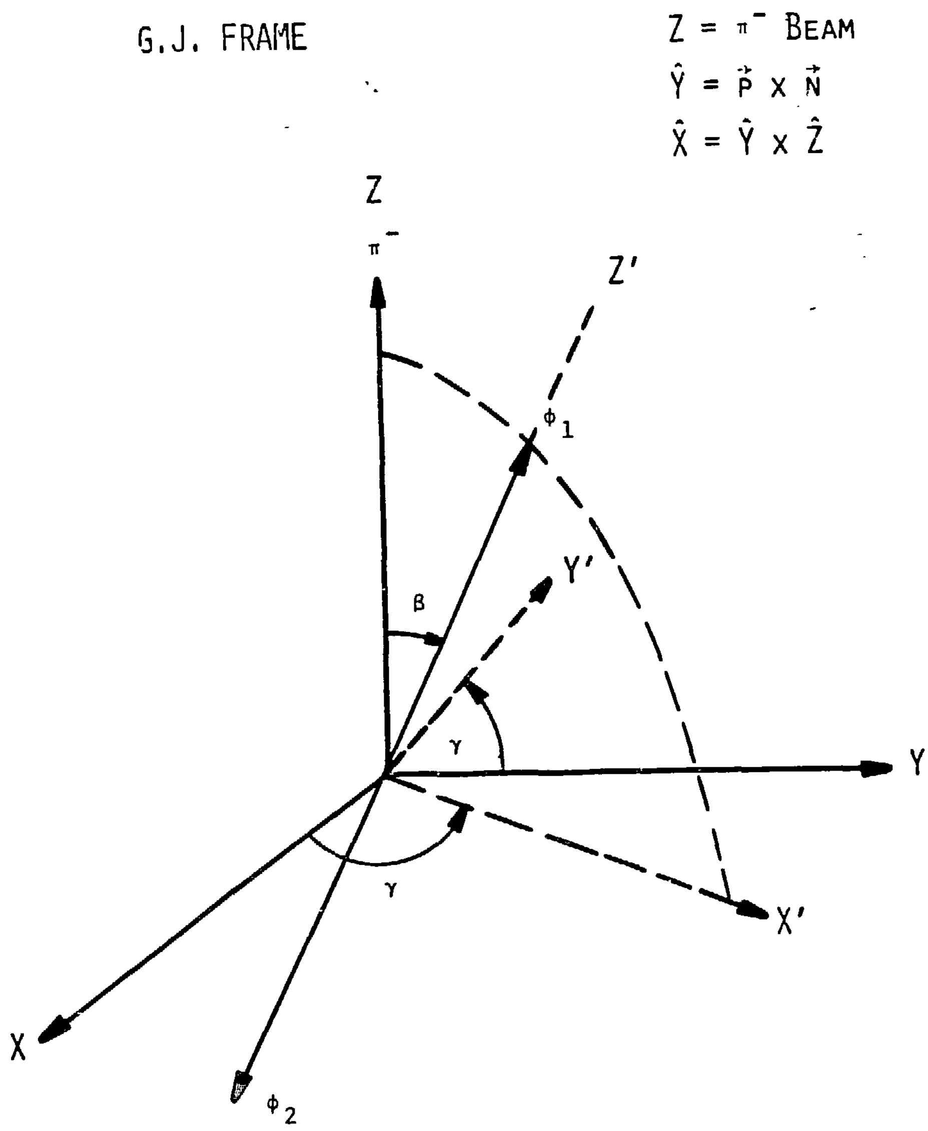

1400
1200
1000
800
600
400
200
0
EVENTS
$\pi^- p \to \phi\phi n$
0
1.0
2.0
3.0
4.0
$(MM)^2, (GeV)^2$

Figure 9



PRODUCTION
ACCEPTANCE
EVENTS (PRODUCTION) x 10^3
4
3
2
1
ACCEPTANCE
10%
5%
2.0
2.2
2.4
2.6
M($\phi\phi$) Gev

Figure 10



ϕ_1 AND ϕ_2 LIE IN (Z,X') PLANE

Figure 11

REST FRAME OF ϕ_1

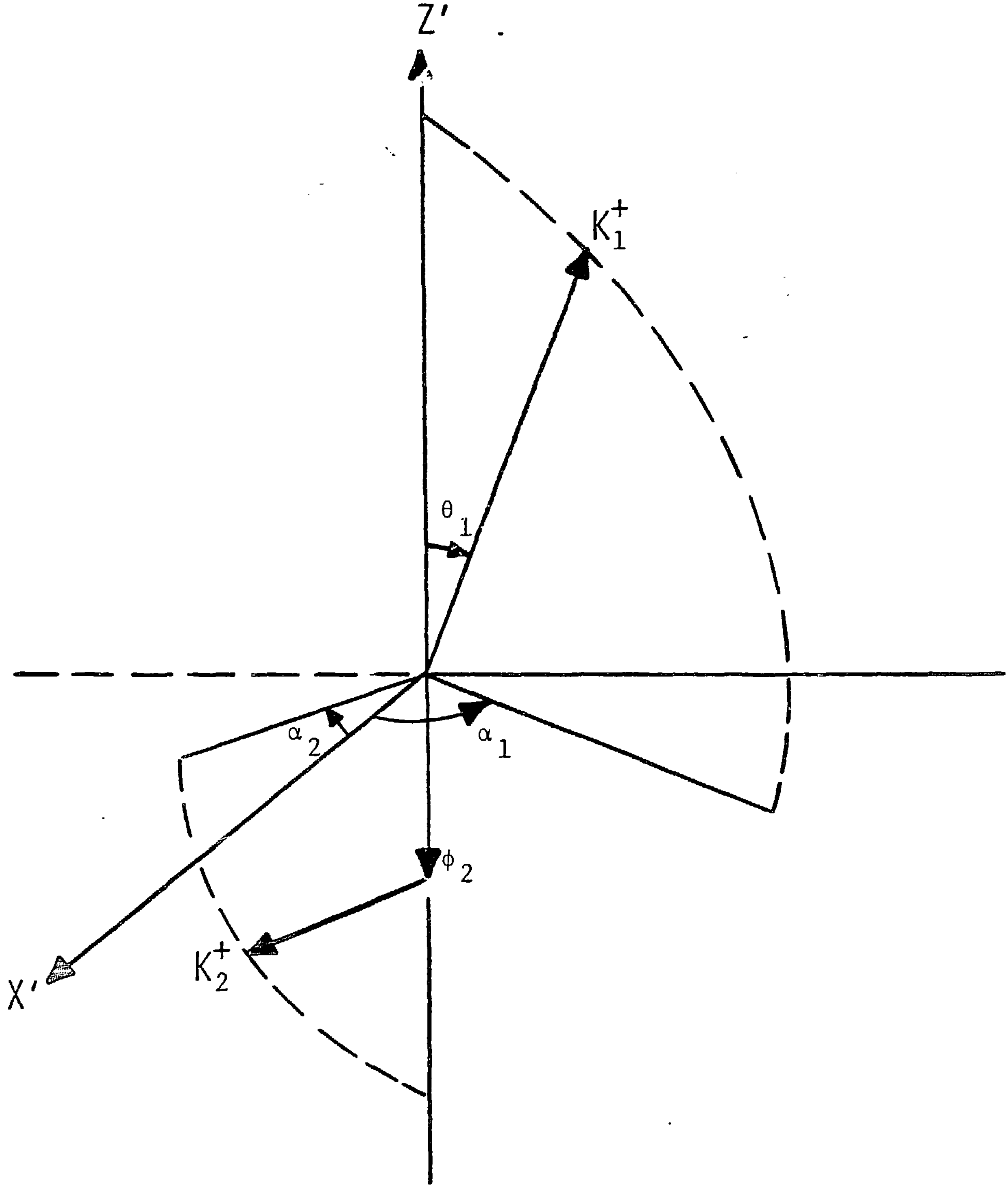


Figure 12

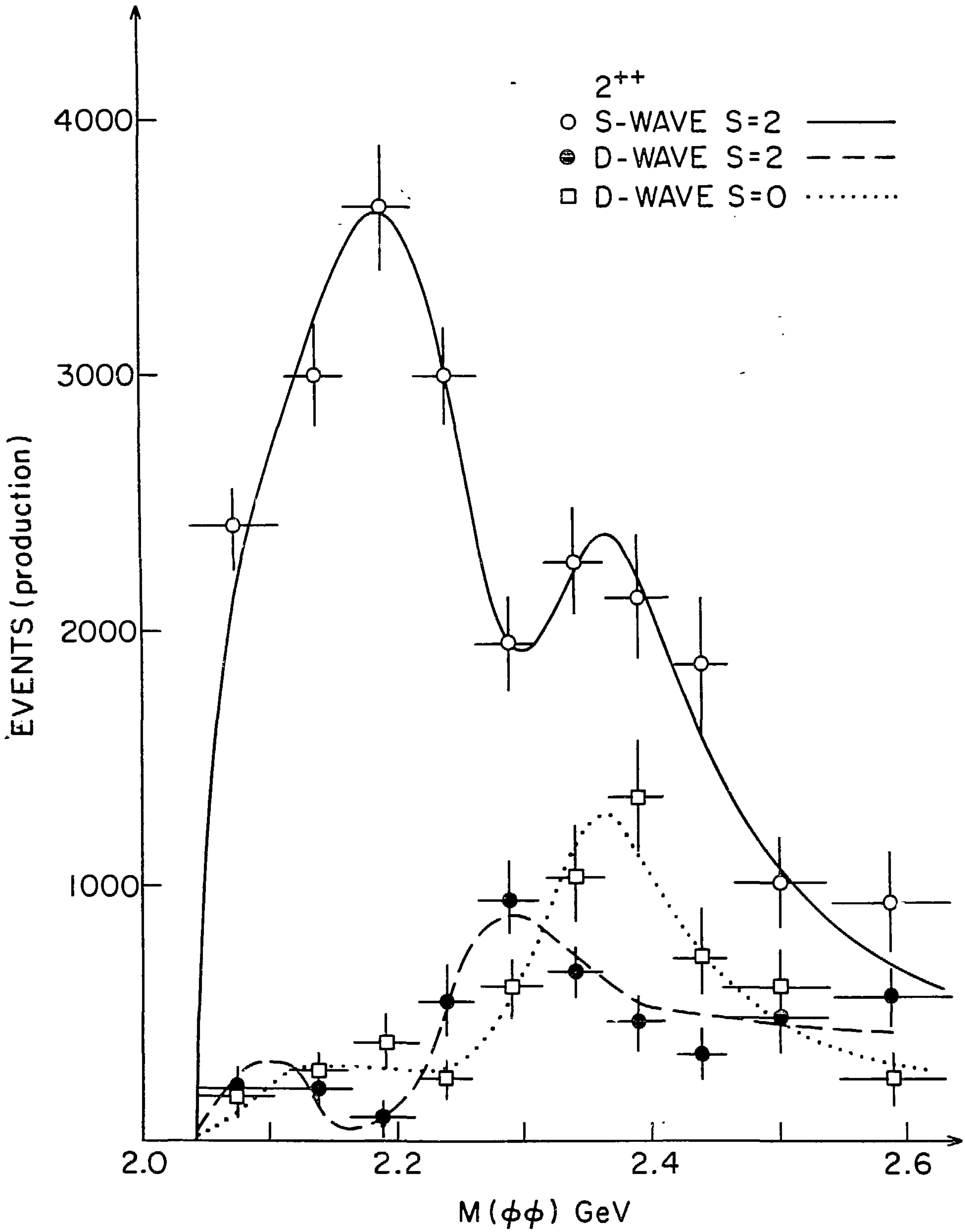
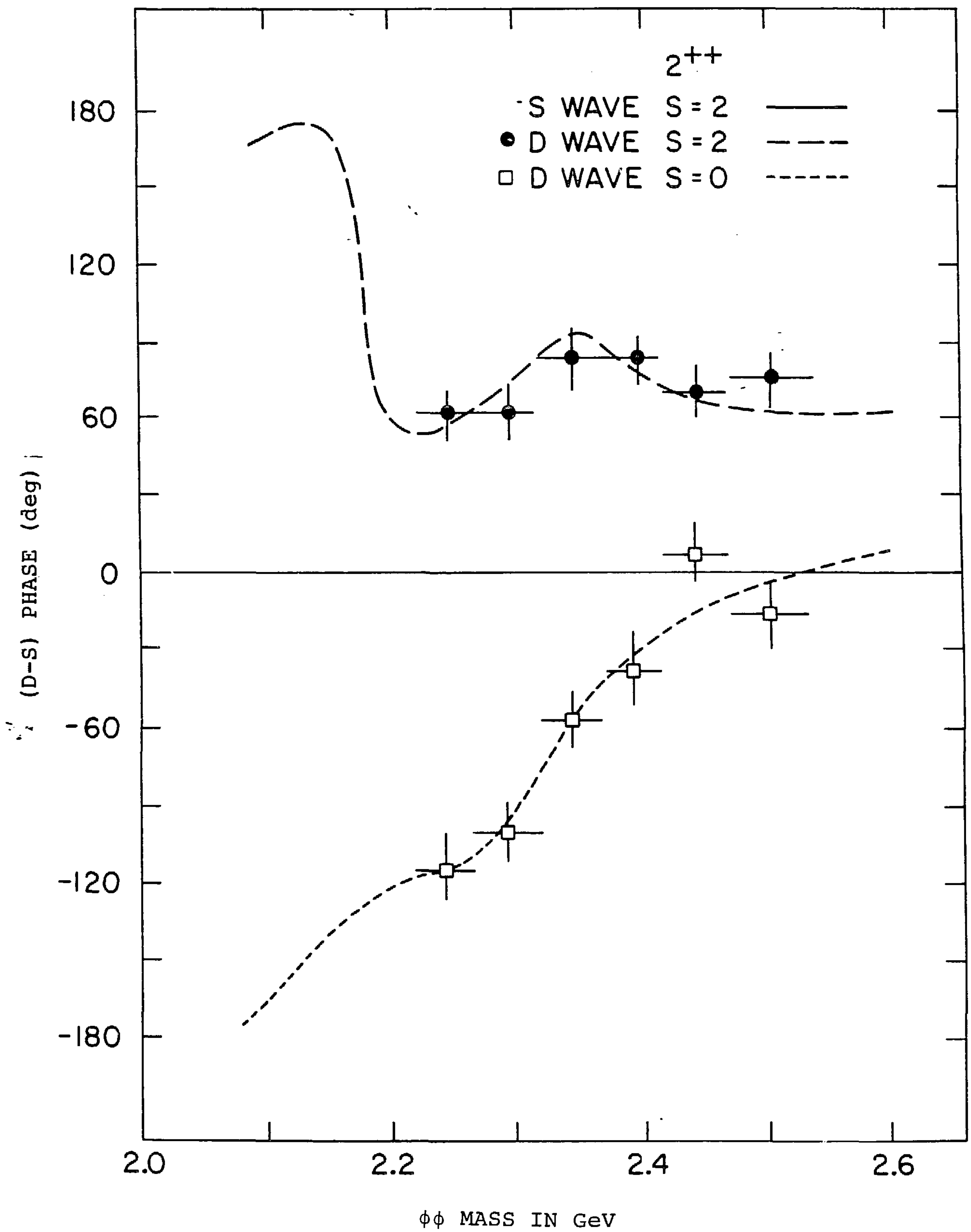

2++
S-WAVE S=2
D-WAVE S=2
D-WAVE S=0
4000
3000
2000
1000
EVENTS (production)
2.0
2.2
2.4
2.6
M(φφ) GeV

Figure 13



2^{++}
S WAVE S = 2
D WAVE S = 2
D WAVE S = 0
180
120
60
0
-60
-120
-180
(D-S) PHASE (deg)
2.0
2.2
2.4
2.6
ϕϕ MASS IN GeV

Figure 14

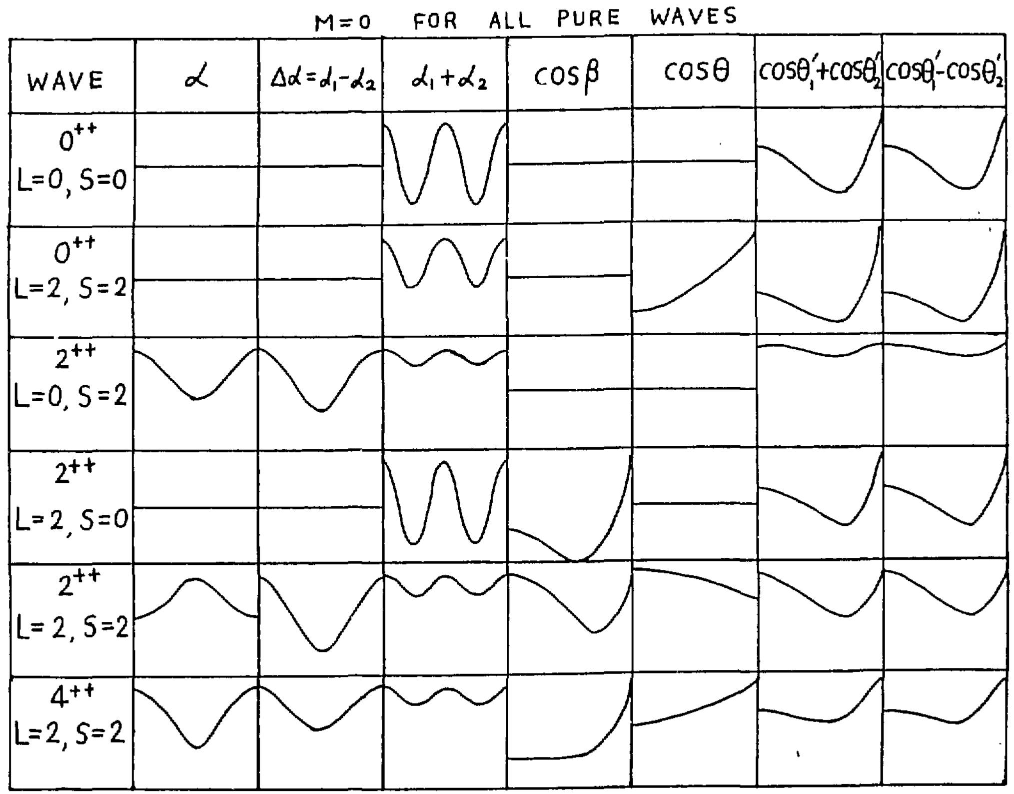
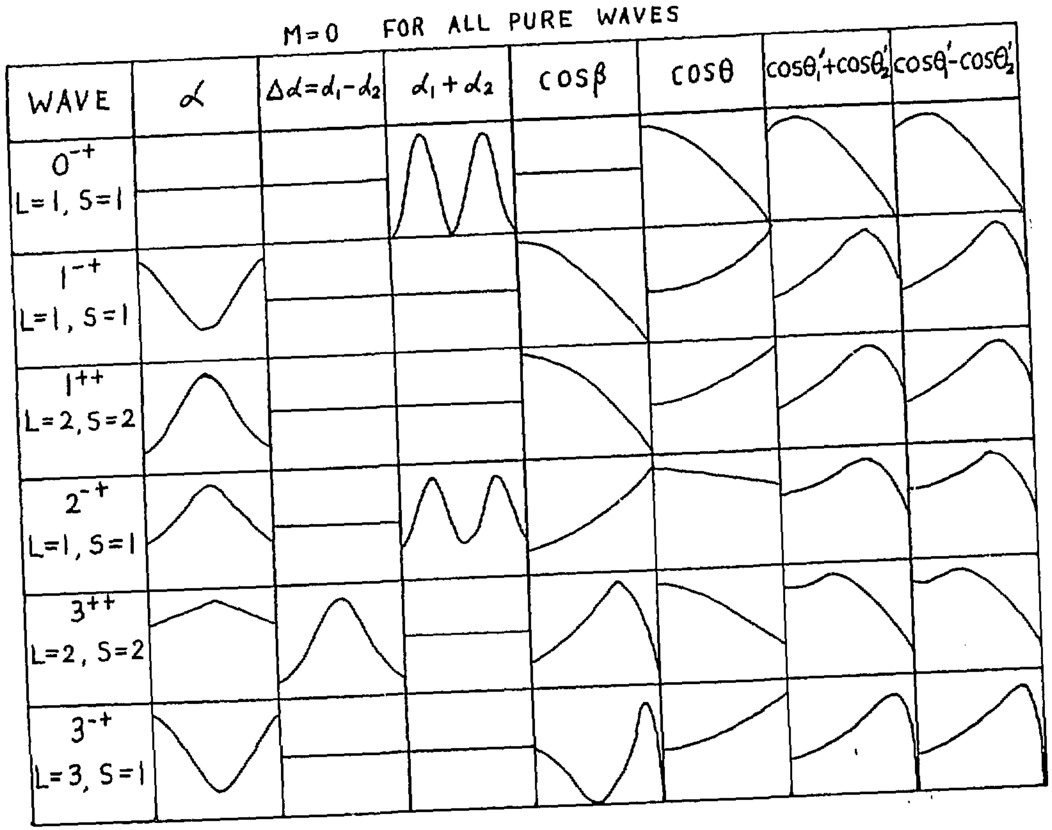

M=0 FOR ALL PURE WAVES
WAVE
α
$\Delta\alpha=\alpha_1-\alpha_2$
$\alpha_1+\alpha_2$
$\cos\beta$
$\cos\theta$
$\cos\theta_1'+\cos\theta_2'$
$\cos\theta_1'-\cos\theta_2'$
0^{++} L=0, S=0
0^{++} L=2, S=2
2^{++} L=0, S=2
2^{++} L=2, S=0
2^{++} L=2, S=2
4^{++} L=2, S=2

Figure 15a



M=0 FOR ALL PURE WAVES
WAVE
α
$\Delta\alpha=\alpha_1-\alpha_2$
$\alpha_1+\alpha_2$
$\cos\beta$
$\cos\theta$
$\cos\theta_1'+\cos\theta_2'$
$\cos\theta_1'-\cos\theta_2'$
0^{-+} L=1, S=1
1^{-+} L=1, S=1
1^{++} L=2, S=2
2^{-+} L=1, S=1
3^{++} L=2, S=2
3^{-+} L=3, S=1

Figure 15b

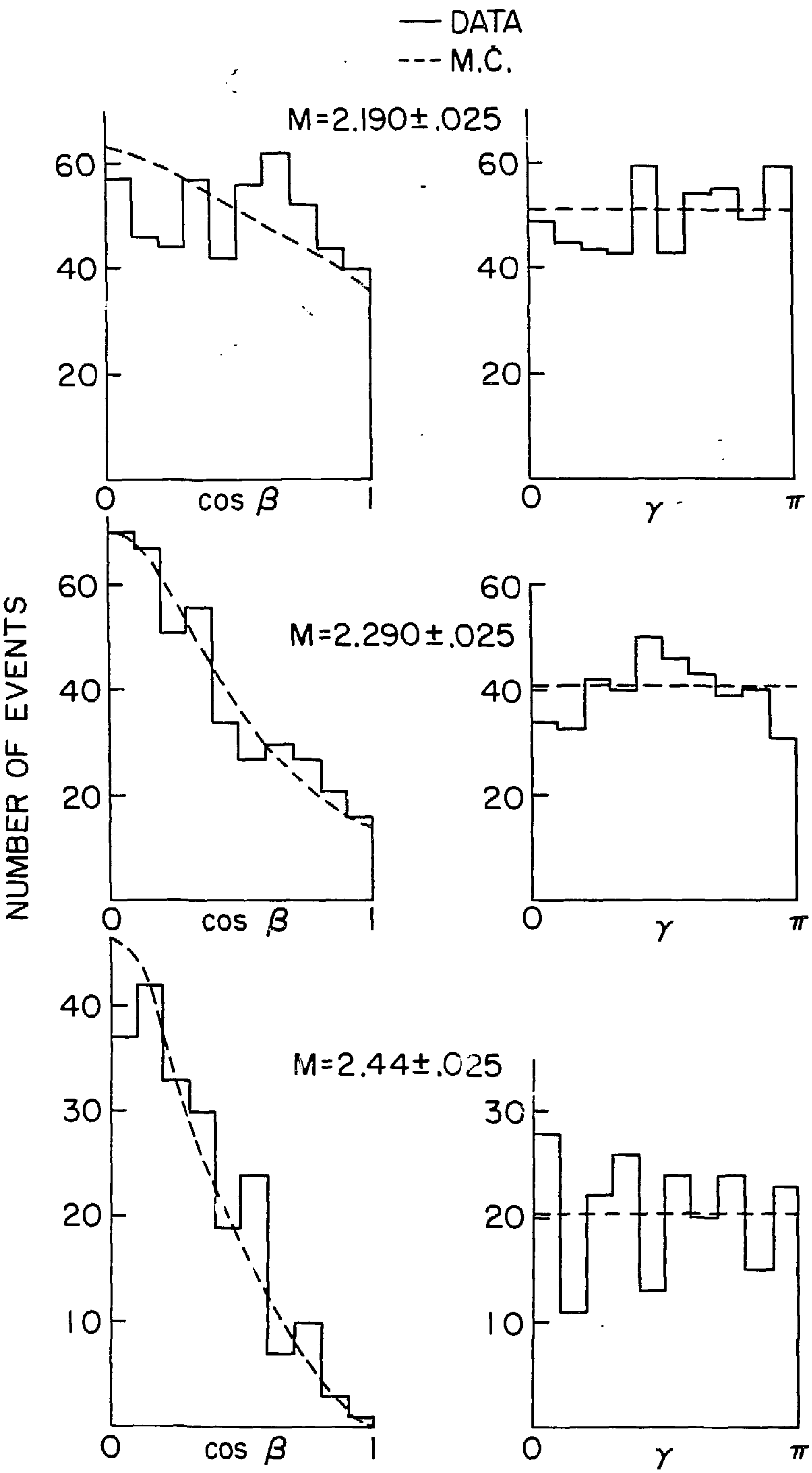


Figure 16a

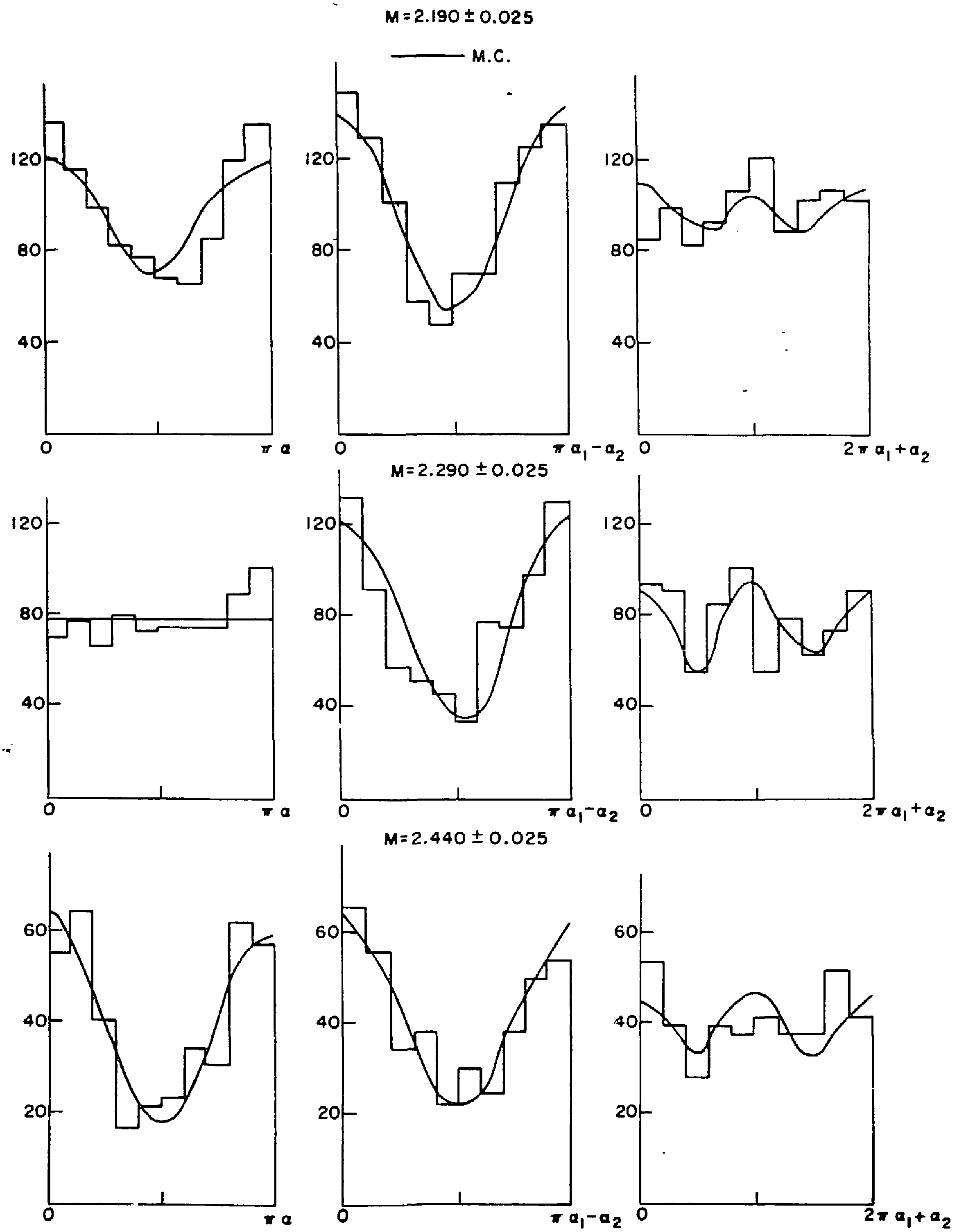

M=2.190 ± 0.025
M.C.
120
80
40
0
π α
π α1−α2
2π α1+α2
M=2.290 ± 0.025
M=2.440 ± 0.025
60
20

Figure 16b

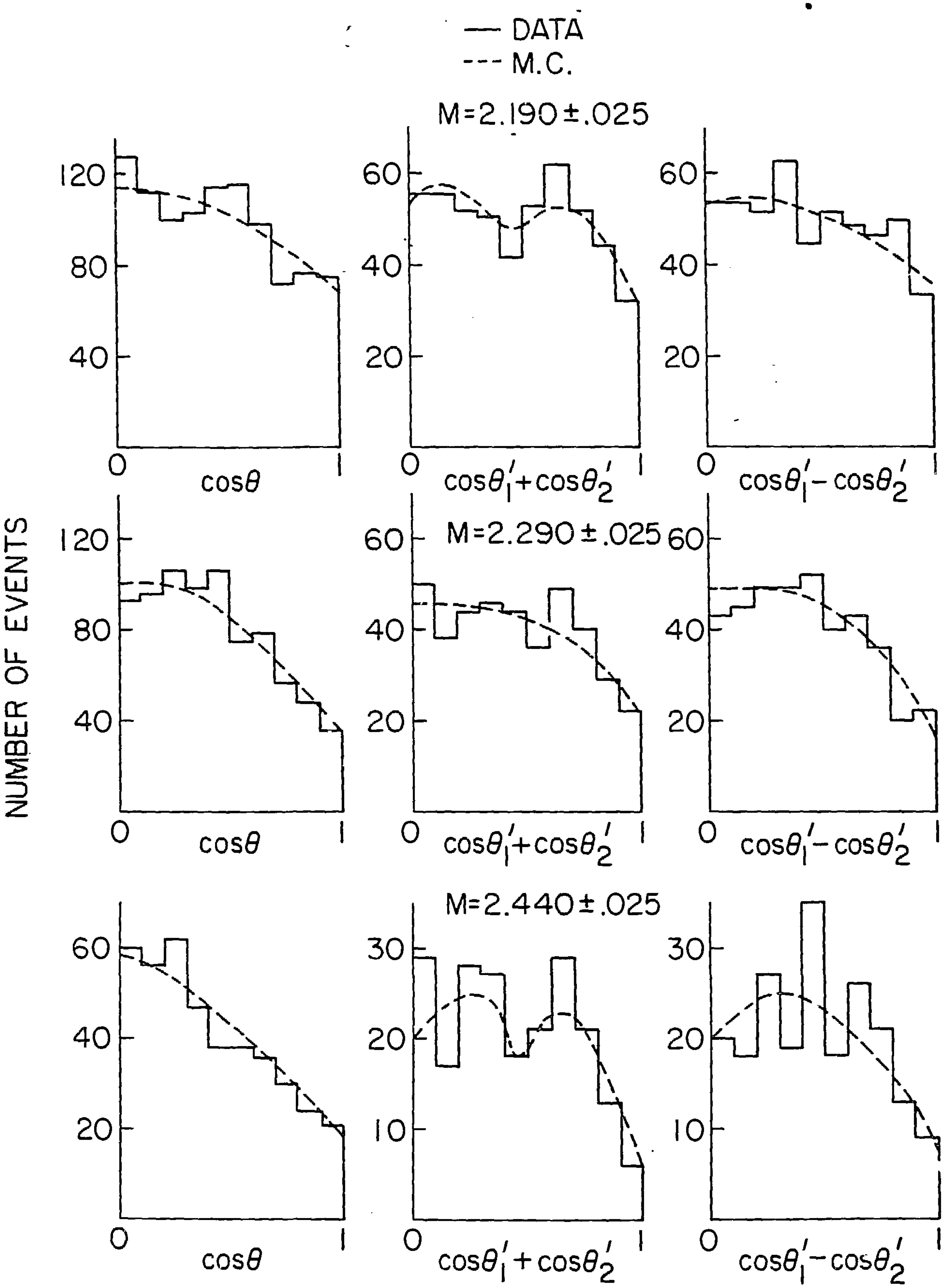

— DATA
--- M.C.
M=2.190±.025
M=2.290±.025
M=2.440±.025
NUMBER OF EVENTS
cosθ
cosθ'1+cosθ'2
cosθ'1−cosθ'2

Figure 16c

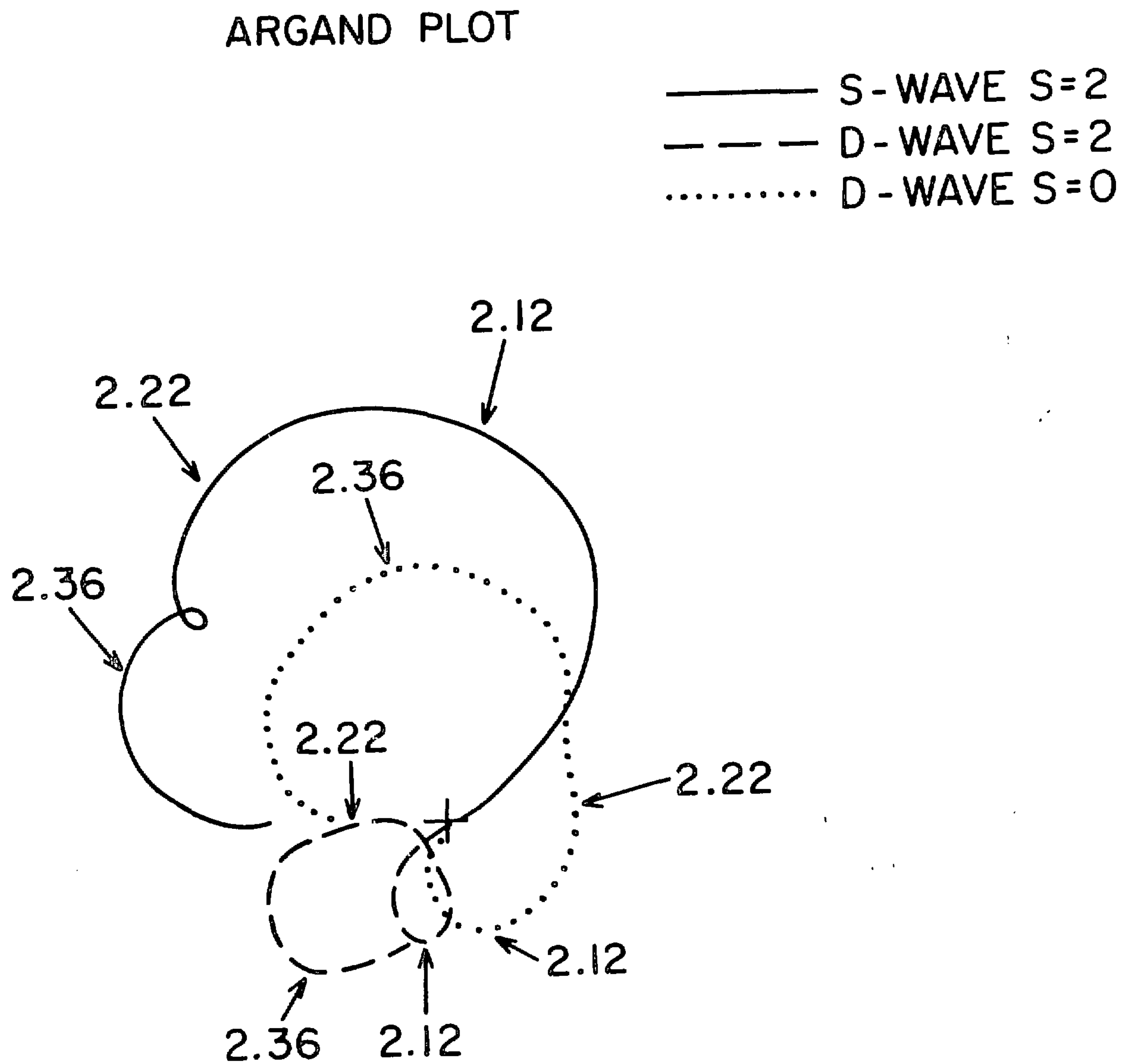

ARGAND PLOT
S-WAVE S=2
D-WAVE S=2
D-WAVE S=0
2.12
2.22
2.36
2.36
2.22
2.22
2.12
2.36
2.12

Figure 17

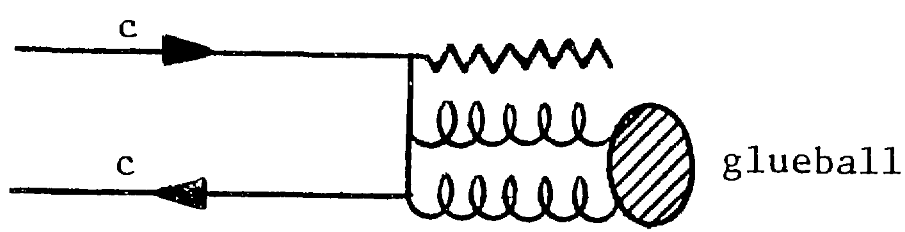


Figure 18