

ENERGY DIVISION

COMMERCIAL SECTOR GAS COOLING TECHNOLOGY
FRONTIER AND MARKET SHARE ANALYSIS

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I. INTRODUCTION

The cooling profile for commercial buildings shapes the afternoon, summer peak for many electric utilities. Their high generating costs to meet expanding commercial sector and coincident loads are reflected in increasingly complex rate structures attempting to discriminate among, and value competing peak demands. Alternative fuels do not bear the generation-cost penalties embedded in these rates. As a result, cooling technologies higher in first cost than are competing electric options can be least in levelized or life-cycle cost, and socially preferred in terms of total efficiency of energy conversion and use.

Prudent commercial cooling R & D planning decisions require market assessment methods which realistically incorporate these factors. Far-sighted planning also looks at technology with initially estimated high first costs. Before rejecting these blueprints, the R & D planner would like to know what it would take to achieve economically viable market shares. Then, he or she can assess whether these *operating parameter, cost, and/or performance* goals are attainable.

Perhaps in the spirit of this assessment dilemma, Thomas Page, CEO and Chairman of the Board of San Diego Gas and Electric Co., stated to the 1984 IAEE North American Meeting participants,¹

We would prefer that every new commercial building in San Diego have either a gas air-conditioning unit that works (and there is a stiff caveat on that — “that works”); or that it have a thermal storage, electrically-driven unit that works. The last thing I want is a standard electrically-driven air-conditioning unit.

This paper describes a method, developed for the Gas Research Institute of the United States, that can assist planning for commercial sector natural gas cooling systems R & D. These systems are higher in first cost than conventional electric chillers. Yet, engine-driven chiller designs exist which are currently competitive in U.S. markets typified by high electricity *or* demand charges.

Section II describes a scenario analysis approach used to develop and test the method. Section III defines the technology frontier, a conceptual tool for identifying new designs with sales potential. Section IV describes a discrete choice method for predicting market shares of technologies with sales potential. Section V shows how the method predicts operating parameter, cost, and/or performance goals for technologies without current sales potential (or for enhancing a "frontier" technology's sales potential). Section VI concludes with an illustrative example for the Chicago office building retrofit market.

II. DEFINING SCENARIOS

Figure 1 illustrates the analysis framework developed for looking at commercial cooling markets and R & D goals. It combines notions of what different futures might occur, what technologies have sales potential (are "winners") in particular futures, and what might be done to develop sales potential for particular losers. The futures encompass notions of different energy price and rate schedule expectations, different equipment price and performance possibilities, and different predicted fuel-, equipment-, and maintenance- cost escalation rates. Rather than attempting to anticipate any possible future, our approach develops a few heuristics or stories of interest, as a way of learning about future possibilities. As R & D planners, we want to identify technical options with sales potential in many different futures, as well as the close seconds that might penetrate the market with small (and attainable) cost reductions or performance boosts. Table 1 lists seven scenarios (each incremental to the previous one beyond the first) developed for the Chicago market analysis described in Section VI.

Table 1. Stories of the Future

1.	Current Practice 5.05 COP base system — electric centrifugal compressor \$150/Ton capital cost premium on gas systems 1.69 COP — gas centrifugal compressor; 1.41 COP gas screw compressor; 0.65 COP gas absorption \$60/ton-hour cool storage cost
2.	Most Likely — \$100/Ton capital cost premium on gas systems
3.	Gas Systems capital cost premium Reduced to \$30/Ton
4.	Five Years Out with Less Favorable Gas Price Future
5.	Thermal Storage Cost Reduced to \$36/ton-hour
6.	Gas Cooling at higher COPS — 1.93 centrifugal compressor; 1.70 screw compressor
7.	Gas Cooling Higher COPS only possible at \$100/Ton capital cost premium

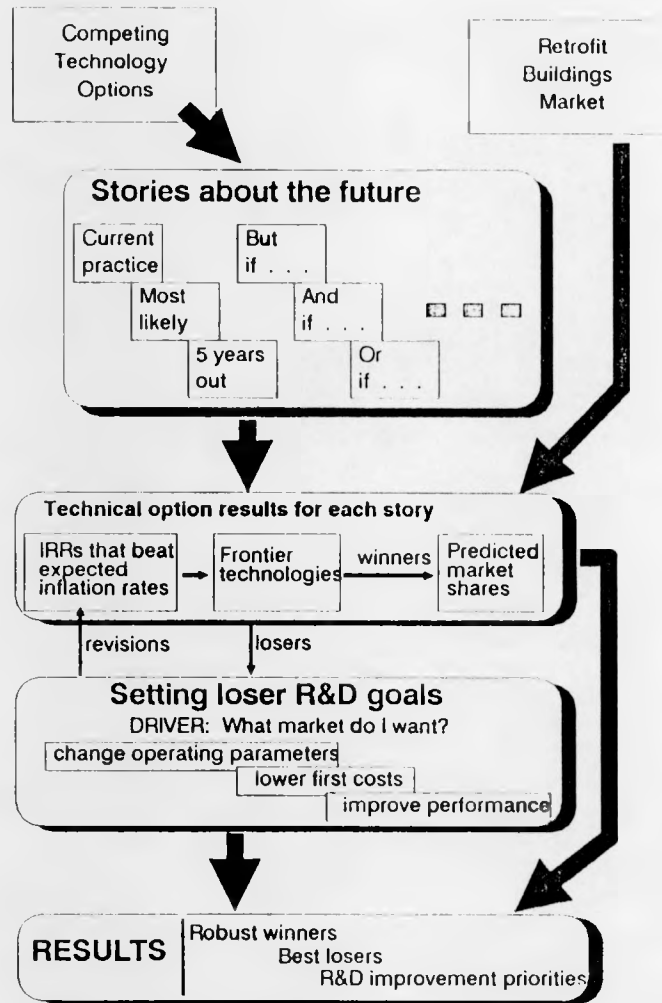


Figure 1. Commercial Sector Gas Cooling Analysis Framework

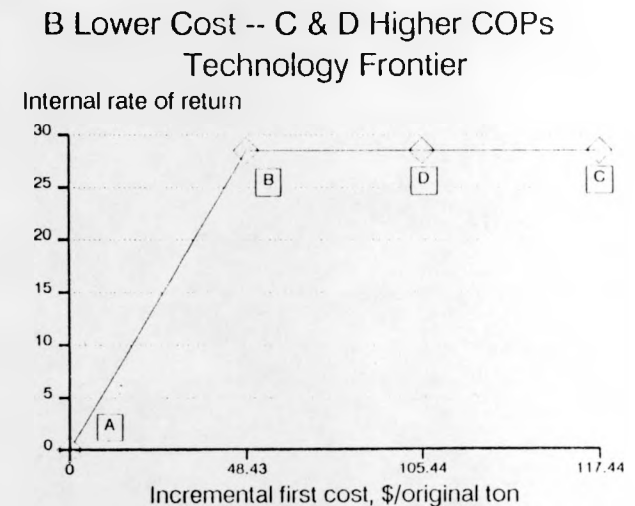
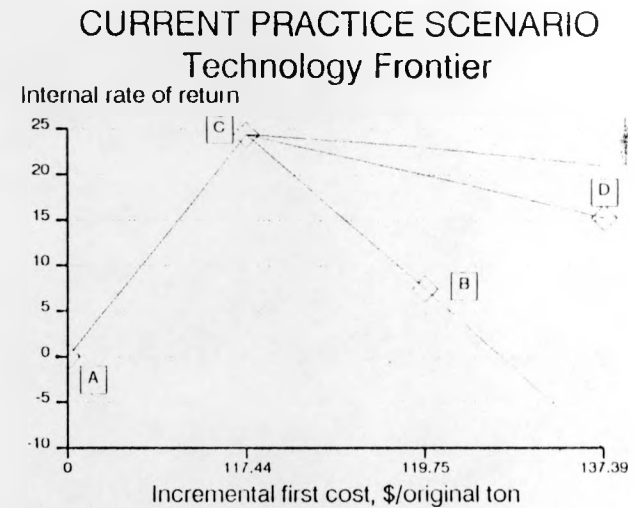


Figure 2. Technology Frontiers – Chicago Retrofit Office Buildings Market

III. ATTAINING THE TECHNOLOGY FRONTIER

For a commercial space cooling option to have sales potential, it must be lowest in life-cycle cost (LCC) for some building owner or operator. These decisionmakers differ in two respects which the LCC measure incorporates: (1) their costs of capital or implicit discount rates differ; and, (2) their cooling capacity expectations differ. Options with sales potential for low-discount-rate or long-payback decisionmakers are higher in first cost and lower in operating costs than are options with sales potential for high-discount-rate or short-payback consumers. Because commercial buildings such as offices and hospitals have schedule-dependent ("obligatory") space conditioning loads, we use conditioned space or area as a proxy for cooling capacity. (We assume use per square foot remains constant across buildings of different size.) For a given cost of capital or discount rate, decisionmakers with more conditioned space prefer or will accept higher first-cost/lower operating-cost systems than those preferred or accepted by owners or operators of smaller buildings.

We define the "technology frontier" as the locus of lowest-LCC options for the decisionmaker population. By our definition, every frontier option has sales potential. Any option not on the frontier has no sales potential, because for any implicit discount rate, another option available offers a lower LCC. We emphasize that this sales potential definition excludes many factors which may influence choice. Our "R & D planning" tool identifies and/or qualifies technical options for commercialization — on the basis of performance, cost, and operating parameters. Downstream packaging and marketing may add value with distinguishing attributes which significantly influence actual market penetration.

An option's internal rate of return (IRR) is the interest or earnings rate on its investment (first) cost which reduces the LCC difference between it and a base system to zero. The earnings result from operating cost reductions over the base system. For one or more options being compared to a base system, we graphically and computationally depict the "technology frontier" as the "highest" investment earnings schedule available to the market. Investment return will be greatest and LCC lowest on the highest investment earnings schedule. Figure 2 depicts two technology frontiers for the retrofit buildings market in Chicago. The option descriptions and frontiers conform to Table 3 and analysis steps (1) and (5), respectively, of Section VI. Note that, for the Current Practice Scenario, only the uppermost line connecting A, C, and an imaginary point above D defines the frontier. This is true because, when we discern the investment earnings rate which equates the LCC for option D and the base system, *option C offers yet a lower LCC*. Therefore, a decisionmaker with a cost of capital or implicit discount rate equal to D's IRR will get a higher return from C than from D.

Commercial cooling system *economic performance* forms the basis for sales potential (and frontier habitation). We measure economic performance by simulating each system serving load demands which vary for each hour in a typical climate year, in a typical building of a given type (e.g., large office building). System cost calculations use actual site-specific gas and electric rate schedules. The cost calculator is the Building Innovations Economic Analysis Code² (BIEA) developed for use with the U.S. Department of Energy's DOE2.1-C building load simulation model.³ DOE2.1-C also predicted the heating and cooling loads for the typical building. System-specific building hourly energy use

was simulated using the Gas Cooling Options Program (GCOP) developed by Howard A. McLain for this study.⁴

For any portfolio of gas and electric cooling options, the frontier is uniquely identified by a four-step screening process. BIEA assists frontier formation with an automated procedure which computes LCC's for all portfolio options at various discount rates. The screening process prohibits frontier definition obtained by connecting the "highest" points, whenever a "highest" point is not lowest in LCC for any decisionmaker. This was the case for option D in Fig. 2's Current Practice Scenario.

For the R & D planning goal-setting purpose, there are very definite better and worse parts of any frontier. Because of the many non-quantified (and in some instances, non-quantifiable) aspects of new-technology market acceptance, the R & D planner prefers sales potential up and to the left, over that down and to the right. This is because these non-measured aspects tend to raise "true" implicit discount rates above those used (in Section IV below) for predicting market shares along the frontier. On the other hand, technologies down and to the right may be socially preferred for energy-and-peak savings-based benefits accruing to gas and electric utilities and their ratepayers. Utilities may want to promote adoption through actions which subsidize building decisionmaker purchases.

IV. ESTIMATING MARKET SHARES

Our assessment of sales potential and R & D goals uses a PC-based market analysis tool (MEGAMETA), that combines a procedure invented by Glenn F. Roberts and David L. Greene⁵ with BIEA as a computational submodule. Roberts and Greene estimated sales potential for new automobile engine technologies distinguished by first cost and performance, among consumers distinguished by implicit discount rate and vehicle miles traveled or usage. MEGAMETA modifies and enhances the procedure in three principal ways:

1. It drops the static-price-expectations assumption, and allows different escalation rates for fuels, and for capital investments and operating expenses.
2. It drops the assumption that a higher first-cost technology must have a higher end-use efficiency. In fact, we focus our analysis on technology comparisons where the total efficiency of conversion (from energy resource such as coal, uranium, or natural gas) to useful cooling service is the relevant performance criterion. This efficiency is reflected in energy price regimes *and* in end-use efficiencies.
3. It explicitly includes site-specific gas and electric rate structures — with block rates, demand charges, and time-of-use rates appropriate for the commercial class of interest. (If applicable, it depicts resale of power at actual buy-back rates.)

MEGAMETA predicts sales potential for commercial cooling technologies distinguished by first cost and performance, among commercial sector building

owners and operators distinguished by implicit discount rate and cooling capacity need. It does so by making pairwise comparisons along the technology frontier. Each comparison examines the probability that a higher first-cost option is preferred to a lower first-cost option. Four primary factors influence the preference decision:

1. Relationship between Implicit Discount Rate and Building Size

For our study of the Chicago "HVAC retrofit" buildings market, a sample of 260 leased office buildings was used to develop distributions of building sizes and other physical characteristics which followed the actual profile for the city. Then, building-specific implicit discount rates were inferred as a function of building age, age since last renovation, square feet, number of stories, value per square foot, and occupancy rate. While individual rates vary with building characteristics, the rates considered together were constrained to give an average rate in real terms equal to the Electric Power Research Institute's COMMEND model 25% rate.⁶ (COMMEND is a commercial sector end-use forecasting model.) Table 2 summarizes the cumulative distribution of real rates developed for the Chicago market. In the table, cooling capacity is predicted from building size, with an assumed baseload requirement of one ton per each 312.5 sq. ft. of conditioned space.

Table 2. Real Discount Rates Inferred for the Chicago Retrofit Buildings' Market

Discount Rate	Percent of Buildings	Percent of Capacity
0 - 20%	32.69	9.01
20 - 30%	45.39	36.64
30 - 45%	18.84	38.37
above 45%	3.08	15.98

The Table 2 decisionmaker profile's practical conservatism for R & D goals analysis is noted in Section V.

2. Lower Cooling Capacity Bound

Options increase in first cost and improve in performance, as you move to the right along the frontier. Even without capital-cost scale economies (discussed below), investment earnings for improved performance increase as application size increases. Therefore, for a particular discount rate, the more expensive technology will only be preferred to the less expensive one (in the paired comparison) for applications above a certain cooling capacity. MEGAMETA solves for this lower cooling capacity bound, by simulating buildings of different size — through adjustments to the typical building results. These come from

GCOP, and are specific to the gas or electric cooling option simulated. Comparing adjacent *higher* to *lower* first-cost options along the frontier, it uses the Golden Step Search procedure,⁷ an optimum-speed convergence algorithm, to vary capital cost and GCOP hourly usage estimates until the BIEA internal rate of return falls arbitrarily close to an implicit discount rate observation. The solution cooling capacity describes a lower bound or smallest building, identified as that capacity application (for this particular discount-rate customer) for which the higher first-cost system's net present worth is zero. Hence, the customer is indifferent between the two systems, and will prefer the higher first-cost option for all larger applications.

3. Chiller Capacities and Scale Economies

The number and mix of compressors, depicting a particular gas or electric system configuration, are held constant at the number and mix whose performance GCOP simulated for the typical building (of 200,000 sq. ft. in the Chicago analysis). For example, several gas cooling options combined screw and centrifugal compressors in individual option configurations.

As noted above however, our method varies the cooling capacity, and concordantly, sizes of compressors. These are marketed within ranges specific to type. The compressor size ranges (in tons of capacity) assumed were:

Reciprocating — 0 to 250

Screw — 50 to 785

Centrifugal — 100 to 6000

Within its size range, a compressor's predicted cost benefits from scale economies specific to compressor type. For capacity applications below a compressor's size range, MEGAMETA simulates purchase of the minimum size available for that type. For capacity applications above a compressor's size range, MEGAMETA applies a "custom-application" penalty function which increases capital and replacement costs at an increasing rate as size increases beyond the maximum.⁸

4. Probability Distribution

Our buildings sample dimensions — discount rate and building size (cooling capacity) — vary multiplicatively rather than additively. That is, if the building manager's borrowing rate doubles, the size of building for which a particular innovation can recover its cost of capital may double or triple. Also, the logarithms of both discount rate and cooling capacity tend toward normality. These two features recommend the joint-log normal distribution⁹ for determining the probability that one frontier option is preferred to the option next less expensive in incremental first cost.

MEGAMETA uses two numerical quadrature algorithms to compute joint-log probabilities.¹⁰ This procedure solves a double integral on discount rate and cooling capacity. MEGAMETA finds a preference probability by partitioning the outside integral or discount-rate space into segments (permitting it to simulate the integral solution as the sum of areas in rectangles or polygons). And

as discussed above, it uses each candidate discount rate in Golden Step Search solutions of BIEA, looking for internal rates of return equal to candidate discount rates. The variation in IRR with each iteration of the Search is caused by adjusting (also, as noted above) capital costs and hourly energy use.

The sales potential for all frontier options is then determined recursively from the pairwise preference probability solutions along the frontier. The first paired comparison splits the market between the base system and least expensive innovation. The sales potential estimates for the next paired comparison (between the least and next least expensive innovations) divide the sales potential allocated to the innovation in the first paired-comparison. Similarly, each successive paired comparison produces preference probabilities which determine the market split of sales potential allocated to the more expensive option of the previous paired comparison.

V. SETTING COST AND PERFORMANCE GOALS

Many technologies investigated fail to exhibit sales potential. This study's market analysis method (MEGAMETA) assists the R & D planner in deciding if it's worthwhile to push a particular technology toward the frontier, and if it is, in contrasting cost reductions with performance improvements as a preferred method of gaining sales potential. Figure 3 depicts a possible R & D priority setting process assisted by MEGAMETA. Although the method is identified as assessing R & D goals for "losers" not on the frontier, it works equivalently and just as well for enlarging the market share of technologies currently on the frontier.

As illustrated, the user may also want to do specific things to enhance a technology's market appeal. If so, these so-called "changes in operating parameters" may or may not succeed in attaining the sales potential desired. If unsuccessful, the planner can increment the enhancement by non-specific (that is, not specific to particular components or processes) cost reductions or performance improvements.

Seeking non-specific R & D goals (*with or without* prior operating parameter improvements) calls into play a second dimension of frontier attainment, notably the market share or sales potential predicted for each "winner" technology. A winner's sales potential depends on its attractiveness or preferability for building sizes represented in the buildings sample, and on its appeal to decisionmakers with different implicit discount rates. MEGAMETA allows the planner to target a market segment of the frontier — *either in number of systems or in tons of capacity*, desired from a cost reduction or performance improvement. It assists the user in setting a so-called "vantage-point discount rate" associated with his or her sales potential goal, through interactive queries which look at discount rate and building size characteristics of the decisionmaker sample. MEGAMETA then attains the frontier for the "loser" technology by adjusting its cost or performance until it just equals in life-cycle cost the lowest LCC observed at the vantage-point rate.

Targeting is an approximate procedure because it relies on only one of two decisionmaker attributes — the implicit discount rate. Its accuracy also

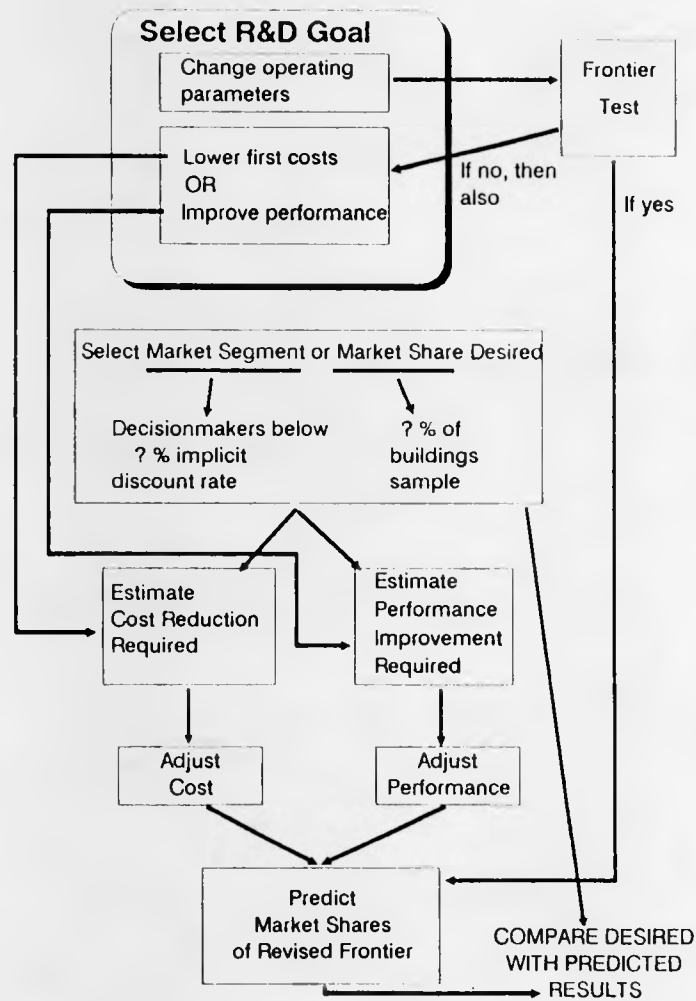


Figure 3. Setting Gas Cooling R&D Goals

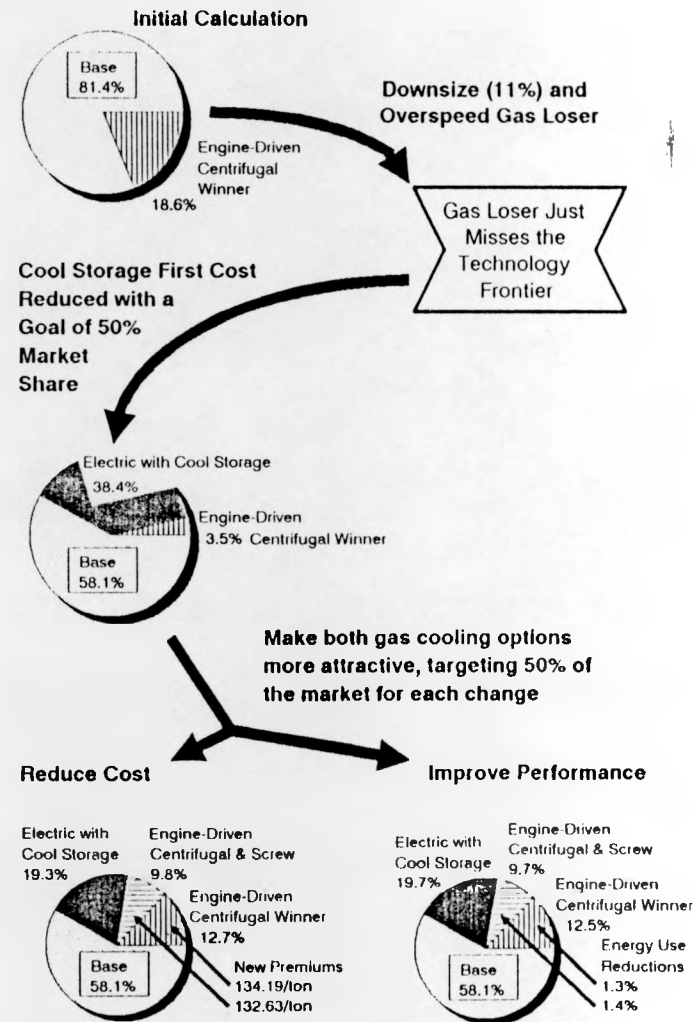


Figure 4. How Goal Attainment Changes Sales Potentials

decreases as the number of frontier options increases. However, successive applications of the method can attain a sales potential objective. Once any desired share is attained, MEGAMETA tells the planner the percentage cost reduction or percentage performance improvement required to attain the frontier with the sales potential sought.

For a technology not currently on the frontier, cost reductions move it up and to the left towards the frontier (See Fig. 2), whereas, performance improvements only move it up — holding investment first-cost constant. Because of frontier shape owing to diminishing marginal investment returns, it generally requires a smaller percentage performance improvement to attain a given sales potential than the percentage cost reduction required. However, the market segments differ, with a cost reduction required to achieve a particular sales potential goal (e.g., 25%) always attaining a shorter-payback frontier or market segment than does the performance improvement required to attain the same goal. Moreover, technologies pushed to the frontier affect sales potentials of the adjacent technology to the left and (ordinarily) all longer-payback options (to the right). Hence, a performance-improved option will not reduce sales potential for any shorter-payback option not adjacent to it on the frontier. For example, looking ahead to Section VI and Fig. 4, the base system's 58.1% sales potential remains unaffected by gas cooling option cost reductions or performance improvements, because the cool storage option remains between the gas options and the base system (as depicted for C & D higher COP's in Fig. 2).

The correlation between discount rate and building size critically affects the severity of cost and performance changes needed to boost technologies to the frontier. The Chicago buildings' sample correlation is +0.79, counter to the notion that building size embodies income and access-to-capital-market effects which should result in lower implicit discount rates as size increases (and in a negative correlation). However, in actual practice, "large building" investor access to more investment opportunities may dictate a higher required return for real estate "energy" investments than is required for the "small building" investor. Moreover, our inferred predictor of building-specific discount rates is multivariate, and separately includes such influential factors as site rental value and age since last renovation. If the retrofit market gives a worst case for "income effects" on the cost of capital, our correlation is truly conservative. The more positive the correlation, the harder it is to boost any particular technology to the frontier, and to attain any desired sales potential.

Because sales potential increases as first-cost decreases, "harder" means a lower required cost premium over the baseline electric chillers. For example, to attain a 25% systems (or buildings) sales potential, our most attractive gas cooling system requires a \$145 per ton cost premium. (That is, the first cost for this unit installed in a 200,000 square foot Chicago office building exceeds that of a "conventional" electric replacement by \$72,247.) Were the correlation - 0.38, the same buildings in physical characteristics require only a cost goal of \$168 per ton over the base system. Moreover, our estimate is additionally conservative because a 25% buildings sales potential translates into 37% of cooling capacity with our inferred correlation and 59% with the - 0.38 correlation.

VI. CHICAGO EXAMPLE

Our example uses a Chicago office building, eligible for retrofit with a “conventional” electric chiller or one of three alternative technologies, as depicted in Table 3. Equipment performance parameters and cost premiums for this example are listed under the “Current Practice Scenario” in Table 1. First, we calculate the frontier and sales potential for the selected technologies. Then we simulate goal setting. Our goal setting encompasses an action/reaction scenario, in which the electric-based option B targets a 50% (of buildings) sales potential, attainable through cost reductions or utility subsidies.

Table 3. Selected Chiller Technologies

Technology	Description
A (Base)	Two 320-ton electric centrifugal chillers
B	One 320-ton electric centrifugal chiller and one 3000-ton-h cool storage tank
C	Two 290-ton engine-driven centrifugal chillers
D	One 500-ton engine-driven centrifugal chiller and one 150-ton engine-driven screw chiller

Our analysis steps follow the bottom part of the Fig. 1 framework — setting “loser” R&D goals; first, through a change in the operating parameters, and, next, through non-specific cost or performance adjustment, each incremented to the initial parameter change. When this goal setting sequence was followed, the initial change failed the “Frontier Test” shown in Fig. 3.

Figure 4 illustrates the sales potentials (at each analysis step) predicted by the joint-log normal distribution method. The analysis steps are as follows:

- (1) Determine the current practice scenario frontier technologies and sales potentials.
- (2) Downsize and overspeed technology D. Perform the “Frontier Test.”
- (3) Anticipate possible cool storage (technology B) cost reductions or subsidies aimed at gaining or increasing market share.

(4) Reduce costs of gas cooling technologies C and D in an attempt to gain and improve market shares.

(5) *Alternatively*, improve performance of C and D.

In this example, we did not iterate to come closer to any target sought. The 50% cool-storage objective fell short by 11.6%. We fell further short of subsequent gas cooling targets because we had set the same "vantage-point discount rate" for the cool storage and two gas cooling technologies, implicitly portraying the real-life situation in which three vendors pursue the same sales objective (of a 50% share each).

In step 3, cool storage cost is reduced from \$60/ton-hour to \$45/ton-hour. Figure 4's alternative final market segmentations result from plausible reaction strategies. We emphasize that no specific market information led to hypothesizing this particular scenario. But we also note the assistance our method provides R & D planners in setting cost and performance objectives which anticipate competitor actions. Because we used actual Chicago electric and gas rate schedules and building characteristics, our method and results additionally suggest cost, performance, and operating parameters which make *economically* feasible shifting a significant portion of the replacement market's cooling load away from peak.

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