Master

Transient Two-Dimensional Flow In Porous Media

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ABSTRACT

The transient flow of an isothermal ideal gas from the cavity formed by an underground nuclear explosion is investigated. A two-dimensional finite element method is used in analyzing the gas flow. Numerical results of the pressure distribution are obtained for both the stemming column and the surrounding porous media.

Introduction

After an underground nuclear explosion, gas from the cavity may enter the stemming column or the surrounding porous material. The extent of penetration of cavity gas into the porous material is of interest in containment evaluation. Transient one-dimensional gas flow in a porous column has been treated by Morrison, e.g. see [1].

The purpose of this paper is to present the results of a numerical investigation of a transient, two-dimensional axisymmetric flow of an ideal gas in porous media. In particular, the stemming column of a proposed event in Nevada Test Site drill hole U10bg was modelled and the results were obtained assuming the surrounding porous medium to be uniform. Calculations were also performed with a highly permeable layer of tuff located within the otherwise uniform lithology.

Results were also obtained for the case in which the U10bg stemming column was modified. For the proposed stemming column, the first layer of porous material is Overton sand. In the modified model, the layer of Overton sand is replaced by a layer of pea gravel and the results were again obtained with and without the layer of highly permeable tuff. This modification permits examination of stemming effectiveness should open paths breach the Overton layer.

Formulation of the Problem

The basic relations governing the flow of gas in porous media are the continuity equation, the momentum equation, and the equation of state. The conservation of mass for axisymmetric flow in cylindrical coordinates in two dimensions is:

$$\varepsilon \frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r\rho u) + \frac{\partial}{\partial z} (\rho v) = 0 \tag{1}$$

where ρ is the gas density, ε is the porosity, t is the time, r is the radial position, z is the longitudinal position measured from the center of the cavity, u is the apparent radial velocity and v is the longitudinal velocity.

The momentum equation will be replaced by a constitutive equation, Darcy's law

$$u = \frac{-\hat{k}}{\mu} \frac{\partial p}{\partial r} \tag{2a}$$

$$v = \frac{-\hat{k}}{\mu} \frac{\partial p}{\partial z} \tag{2b}$$

where $\hat{K}$ is the permeability of the medium, μ is the fluid viscosity, and p is the fluid pressure. The viscosity will be assumed constant.

The gas is assumed to obey the ideal gas equation of state

$$p = \rho R\hat{T} \tag{3}$$

where $\hat{T}$ is the absolute temperature and R is the gas constant.

Assuming isothermal flow, equations (2a) and (2b) and equation (3) can be substituted into equation (1), yielding an equation with pressure as the sole dependent variable.

$$\varepsilon \frac{\partial p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\hat{k}}{\mu} p \frac{\partial p}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\hat{k}}{\mu} p \frac{\partial p}{\partial z}\right) \tag{4}$$

Defining a few dimensionless quantities permits easy comparison of the results. The dimensionless quantities are

$$x = \frac{z}{L} \qquad R = \frac{r}{L} \qquad T = \frac{p}{Po} \qquad \bar{K} = \frac{\hat{k}}{Ko} \qquad \tau = \frac{k_o p_o}{\mu L^2} t \tag{5}$$

where L is the length of the stemming column, 200 metres in the example presented here, p_o is the ambient pressure and k_o is some reference permeability and in this case is the permeability of Overton sand.

Using the dimensionless quantities defined above, equation (4) can be written as

$$\varepsilon \frac{\partial T}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\bar{k}T\frac{\partial T}{\partial R}\right) + \frac{\partial}{\partial x}\left(\bar{k}\, T \frac{\partial T}{\partial x}\right) \tag{6}$$

The transient conduction equation in two-dimensional cyclindrical coordinates is

$$\rho\, c_p \frac{\partial \hat{T}}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\, k \frac{\partial \hat{T}}{\partial R}\right) + \frac{\partial}{\partial Z}\left(k \frac{\partial \hat{T}}{\partial Z}\right) \tag{7}$$

where c_p is the specific heat, and k is the thermal conducitivity. If the quantities in quation (7) are dimensionless, then equation (7) is synonymous to equation (6) with ρ being identity, c_p being replaced by ε, and k, which can be a function of temperature, being replaced by $\bar{k}$ T.

The appropriate boundary and initial conditions for equation (6) are

$$T = 1 \text{ at } x = 1 \quad (8c)$$

$$T = T(\tau) \text{ along the cavity surface} \quad (8b)$$

$$\frac{\partial T}{\partial n} = 0 \text{ for all other boundaries} \quad (8c)$$

$$T = 1 \text{ at } \tau = 0 \quad (8d)$$

Impermeable plugs and a surface conductor were located in the stemming column. There is no flow along these boundaries. The plastic plugs and the surface conductor were omitted from the stemming column and the zero pressure gradient boundary condition was used. The cavity pressure versus time, that is, equation (8b), was derived using the Olsen-Knox Model [2] and is shown in Fig. 1. The relation roughly describes the pressure between the termination of cavity growth and the time of cavity collapse. For this two dimensional study, the cavity is assumed to remain intact and the boundary condition assumed valid for all time.

Numerical Solution

Equation (6), along with boundary and initial conditions has been solved using TACO [3], an implicit finite element code for heat conduction analysis. TACO has the capability to handle material properties that are time or temperature dependent. The materials may be either isotropic or orthotropic.

Shown in Fig. 2 is a typical material outline of the stemming column and the surrounding porous medium. The mesh, shown in Fig. 3, was generated using SLIC [4], an interactive graphics mesh generator. The output from this mesh generation code (SLIC) was used to provide the nodal point data and element data for TACO.

A transient solution was obtained for a dimensionless time of 0.0 to 0.004106; corresponding to 16 minutes. Results were obtained using $\alpha = 0.5$ (Crank-Nichslson) and a step size, for the time increment, of 0.00010265.

Graphical representation of data generated by TACO is provided by POSTACO [5], a graphics post processor for scaler two-dimensional finite element codes. POSTACO produces UXDD80 graphics files. These files were output with the utility routine UXRJ [6]. RJET plots were obtained with UXRJ.

Results

Fig. 4 shows the material outline of the stemming column and the surrounding rock for the U10bg model. All properties are shown in Table 1. Line A1-A2 is the distance along the edge of the stemming column. Line B1-B2 is the horizontal distance extending from boundary to boundary at a dimensionless height of 0.345. This dimensionless height corresponds to the interface between the first layer of LLL mix and the second layer of pea gravel. Fig. 5 through Fig. 12 show the dimensionless pressure versus dimensionless distance of lines A1-A2 and B1-B2 at times of 3.6, 7.2, 10.8, and 14.4 minutes after detonation, respectively. The pressure profiles of the first three layers of mixture are distinctly shown in the figures representing line A1-A2. Fig.13 through Fig.16 show isobars at 3.6, 7.2, 10.8, and 14.4 minutes respectively. Note that the isobars do not uniformly decrease as one gets farther from the cavity for larger times. This is true because the cavity pressure decreases with increasing time and the larger pressures at earlier times are propagating outwards. The contour levels shown in these figures are labeled, for example, A on one boundary and AA on the other boundary.

Table 1

Porosity and Permeability of Porous Media*

Material	ε	$\hat{k}$
Overton Sand	0.373	36D**
Pea Gravel	0.330	2500D
LLL Mix	0.319	8D
Surrounding Media	0.239	5D

* Griffiths, S. K., Morrison, F. A., Jr. "The Permeability of Grandular Beds Emplaced in Vertical Drill Holes", Lawrence Livermore Laboratory, UCRL-52674 (1979).

** $1D = 9.8 \times 10^{-13}\ m^2$

$\mu(air) = 2.09 \times 10^{-5}\ kg/m \cdot s$

Fig. 17 shows the material outline of the U10bg model with a layer of highly permeable tuff located at the position of the second layer of pea gravel and extending across the bed. Since no permeability measurements were made at the U10bg site, it will be assumed that the presumably permeable layer has properties identical to those of pea gravel. Fig. 18 through Fig. 21 show the dimensionless pressure versus dimensionless distance of line A1-A2 at the same times stated earlier. Note that the pressure is approximately one at all distances where the layer was inserted and beyond, that is, at a dimensionless distance of 0.2392 and greater. This was not the case before. A pressure of one was not obtained until the distance was approximately 0.35. Fig. 22 through 25 show lines of constant pressure at the same times given before. In general, these profiles look very similar to the isobars mentioned earlier, except that for later times the pressure does not propagate into the permeable layer. Flow into this layer is, of course, enhanced, but no high pressure exists to drive fluid higher in the hole.

Results were also obtained for a more conservative case, that is, the replacement of the layer of Overton sand with pea gravel. Fig. 26 and Fig. 27 show the pressure distribution along lines A1-A2 and B1-B2, respectively, at a time of 7.2 minutes. Here the pressure along line A1-A2 did not reach a value of one until the distance was approximately 0.38. This is verified in Fig. 27. The pressure along line B1-B2 deviates from one close to the stemming column. Isobars of this data are shown in Fig. 28.

Results were also obtained with a very permeable layer as before. Fig. 29 again show that the pressure reached a value near one at a dimensionless distance of approximately 0.2392. Minute deviations can be noticed up to a distance of approximately 0.38, but these deviations are small. Isobars are shown in Fig. 30.

Conclusions

A two dimensional axisymmetric flow of an ideal gas in porous media has has been investigated. A highly permeable lithologic layer proves useful for containment purposes in that the dimensionless pressure drops near a value of one at distances closer to the cavity. The pressure did not propagate as far into the porous media when this layer was used. The results further demonstrate that two layers of LLL mix with porous flow are sufficient to impede the flow.

References

1. Morrision, F. A., Jr., "Transient Gas Flow in a Porous Column", I & EC Fundamentals 11, 191-197 (1972).

2. Knox, R. J., "A Calculational Model for Condensation of Water Vapor During an Underground Nuclear Detonation", Lawrence Livermore Laboratory, UCRL-51901 (1975).

3. Mason, W. E., Jr., "TACO - A Finite Element Heat Transfer Code", Lawrence Livermore Laboratory, UCID-17980 (1978).

4. Gerhard, M., and Greenlaw, R., "SLIC - The Interactive, Graphic Mesh Generator", Lawrence Livermore Laboratory, UCID-18166 (1979).

5. Mason, W. E., JR. "POSTACO - A Post Processor for Scalar, Two-Dimensional Finite Element Codes", Lawrence Livermore Laboratory, UCID-17979 (1978).

6. Blair, M., "The UXD080 Graphics System", Lawrence Livermore Laboratory, UCID-30146 (1977).

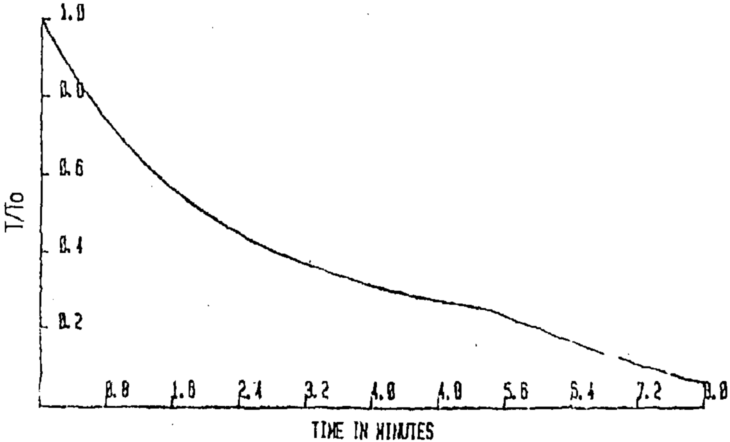


FIG. 1 OLSEN-KNOX MODEL: To = 30.756

Figure 2

Material Outline

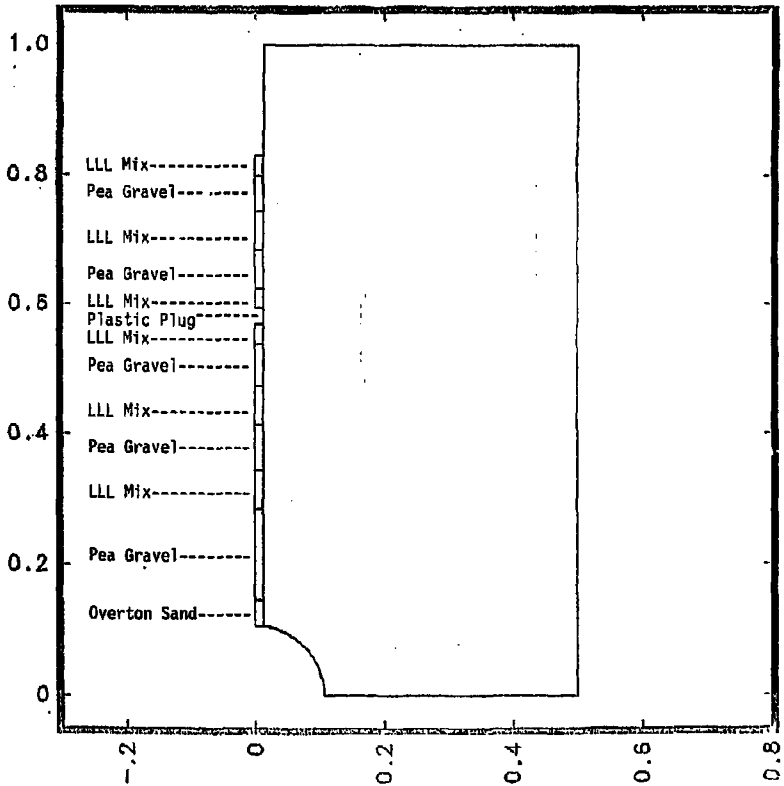


Figure 3

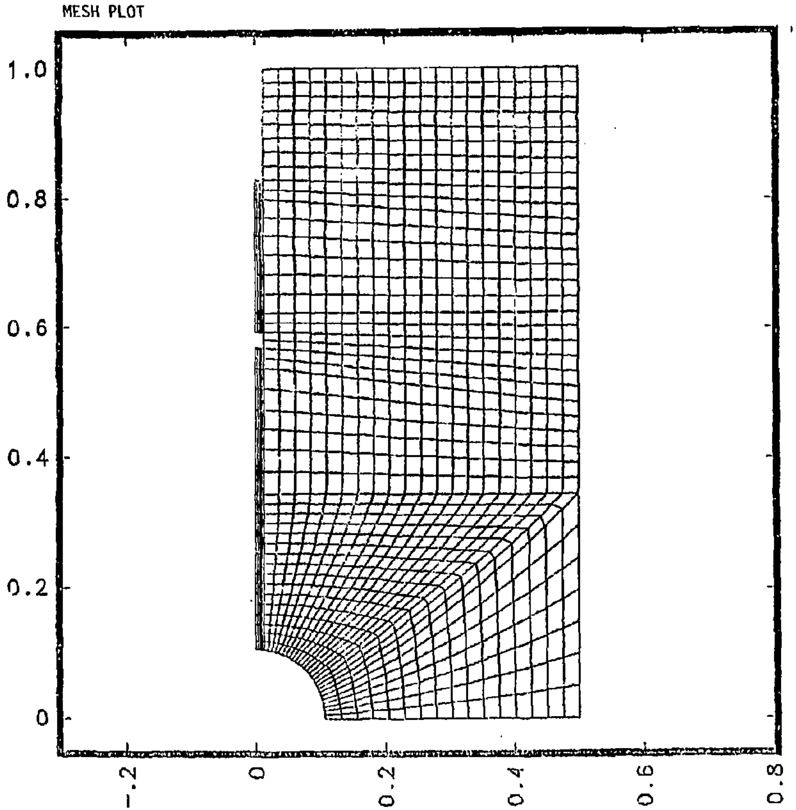


Figure 4

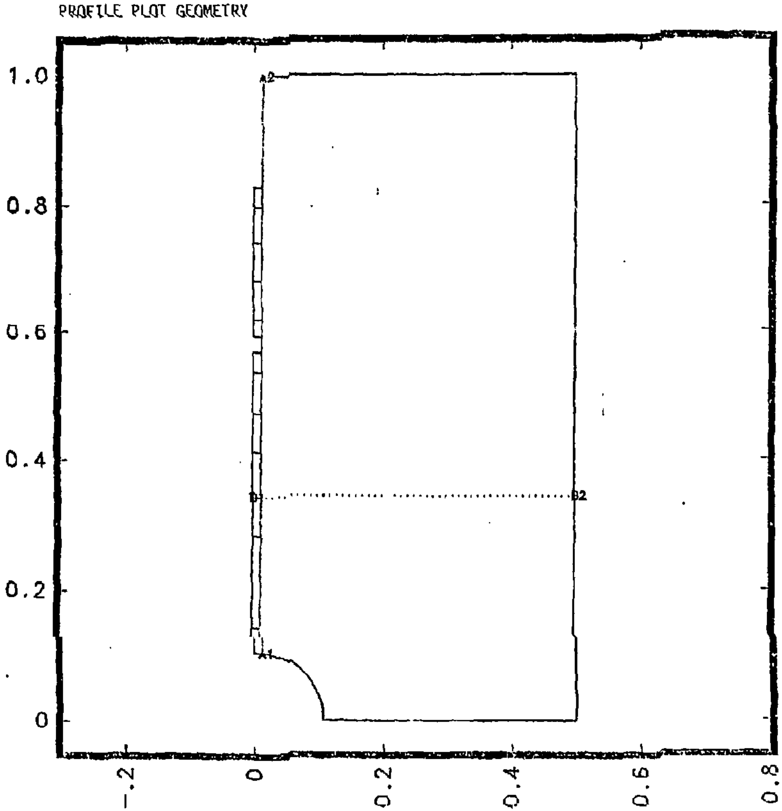
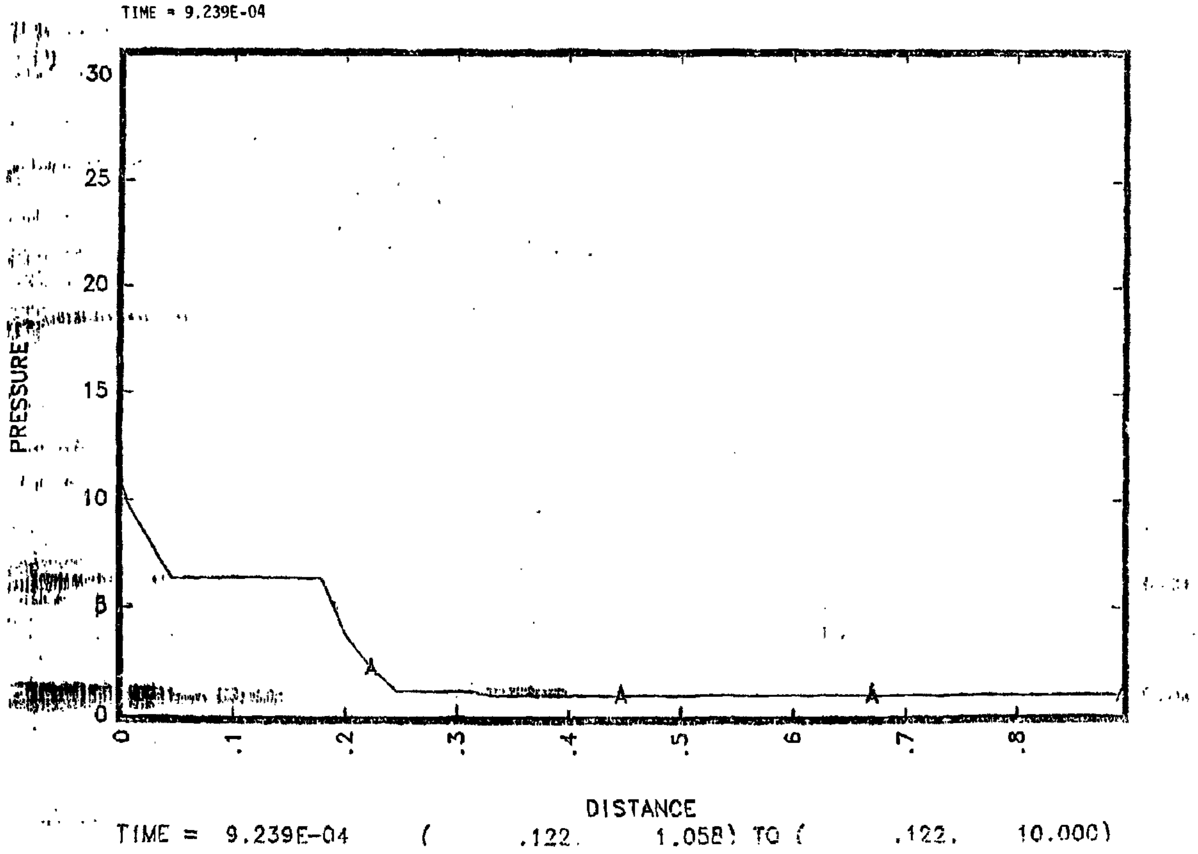


Figure 5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15
10
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.1
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.5
.6
.7
.8
DISTANCE
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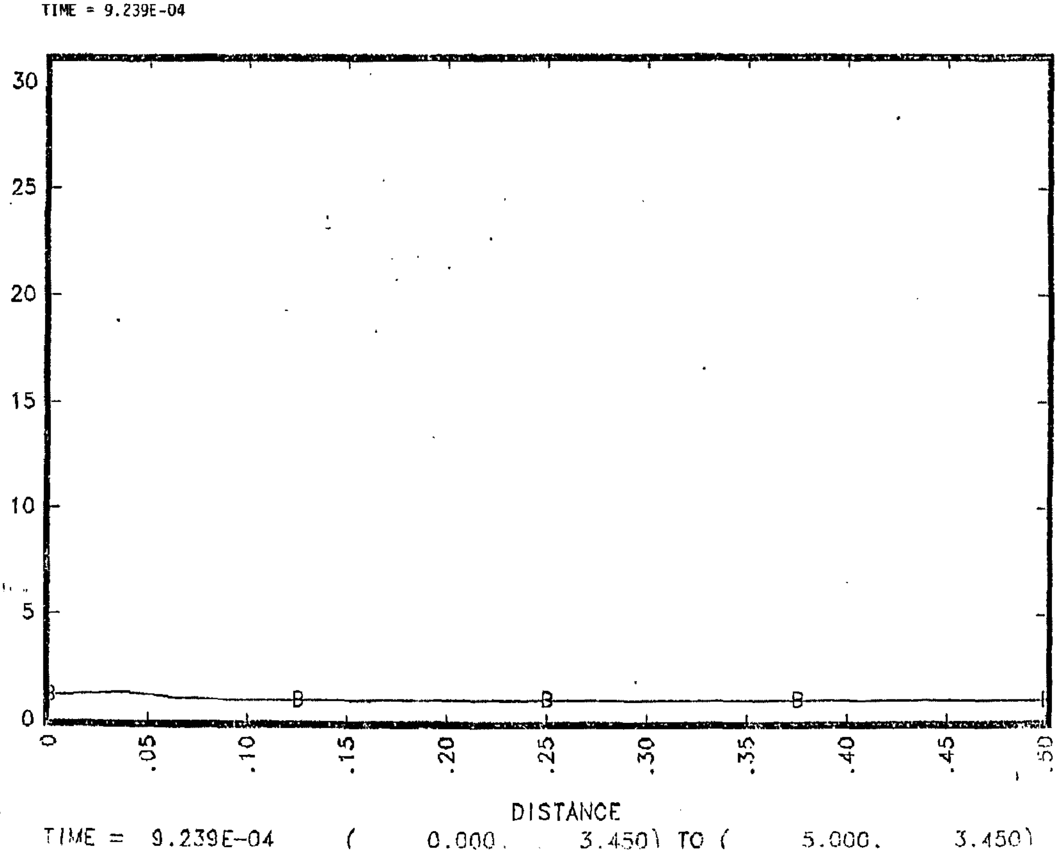
Figure 6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.20
.25
.30
.35
.40
.45
.50
DISTANCE
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Figure 7

TIME = 1.848E-03

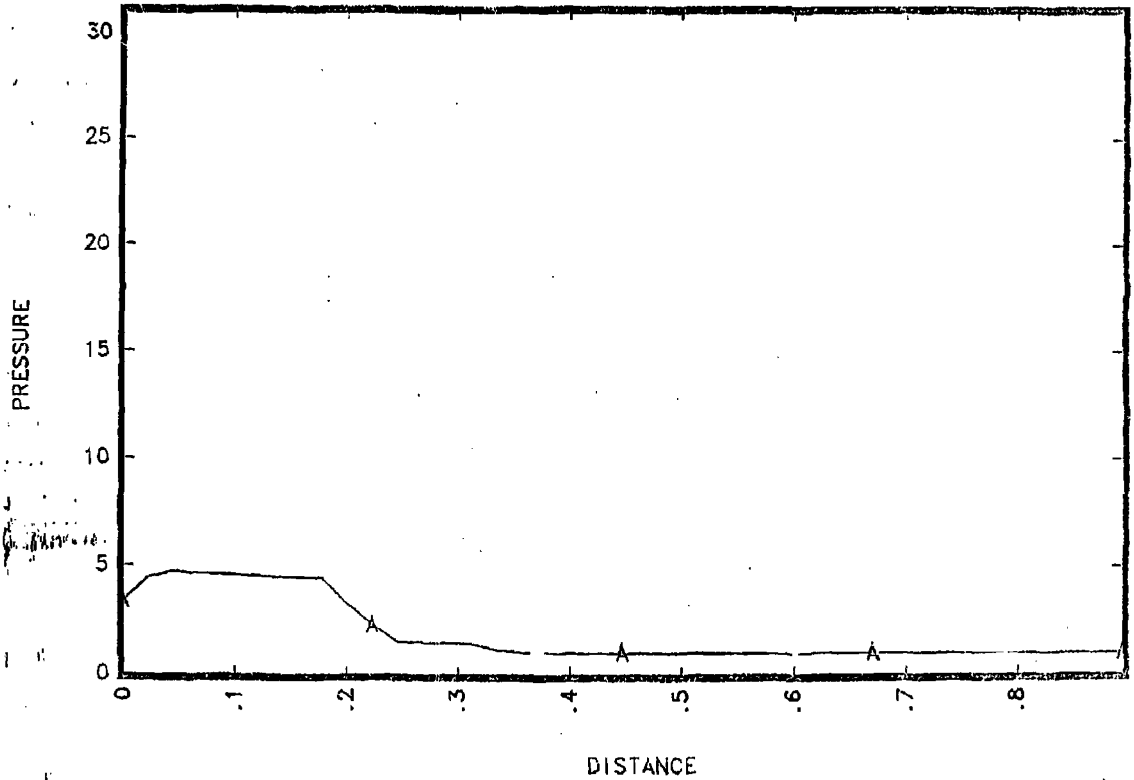

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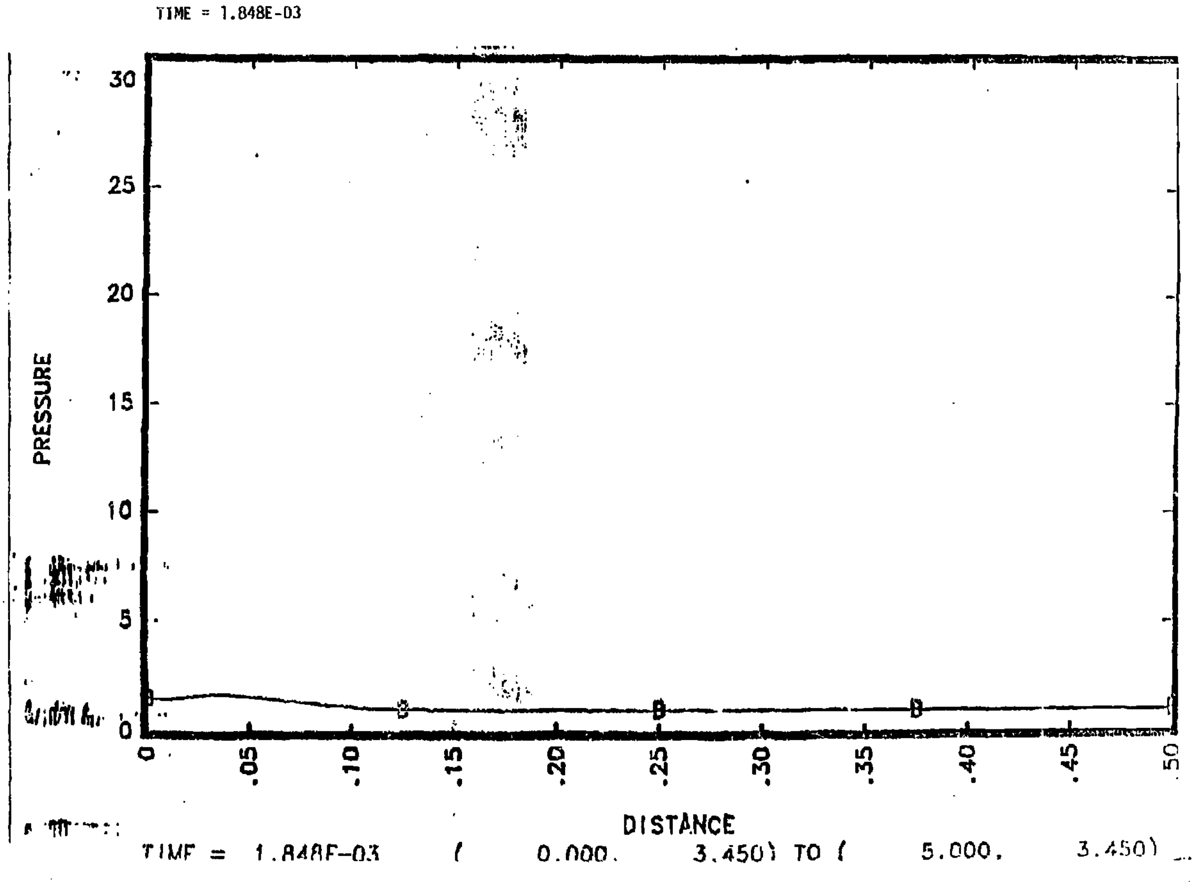
Figure 8



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.40
.45
.50
DISTANCE
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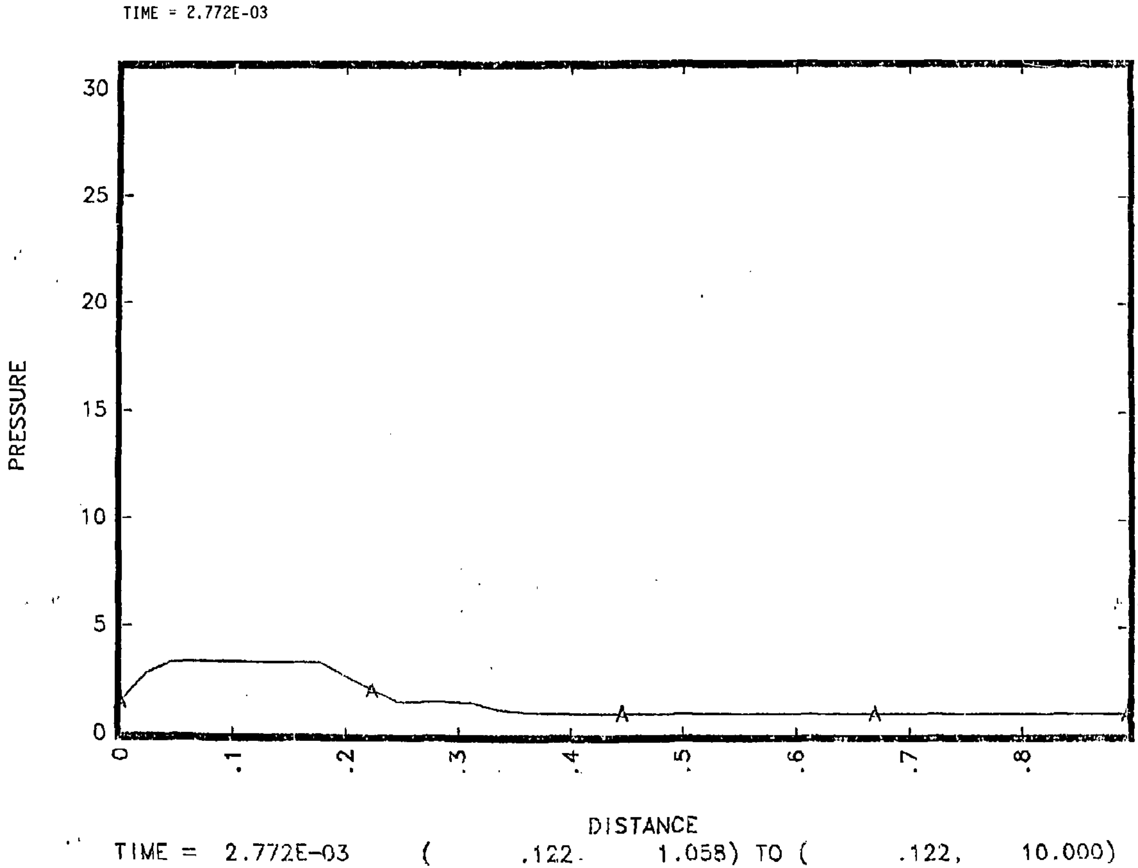
Figure 9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5
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0
.1
.2
.3
.4
.5
.6
.7
.8
A
DISTANCE
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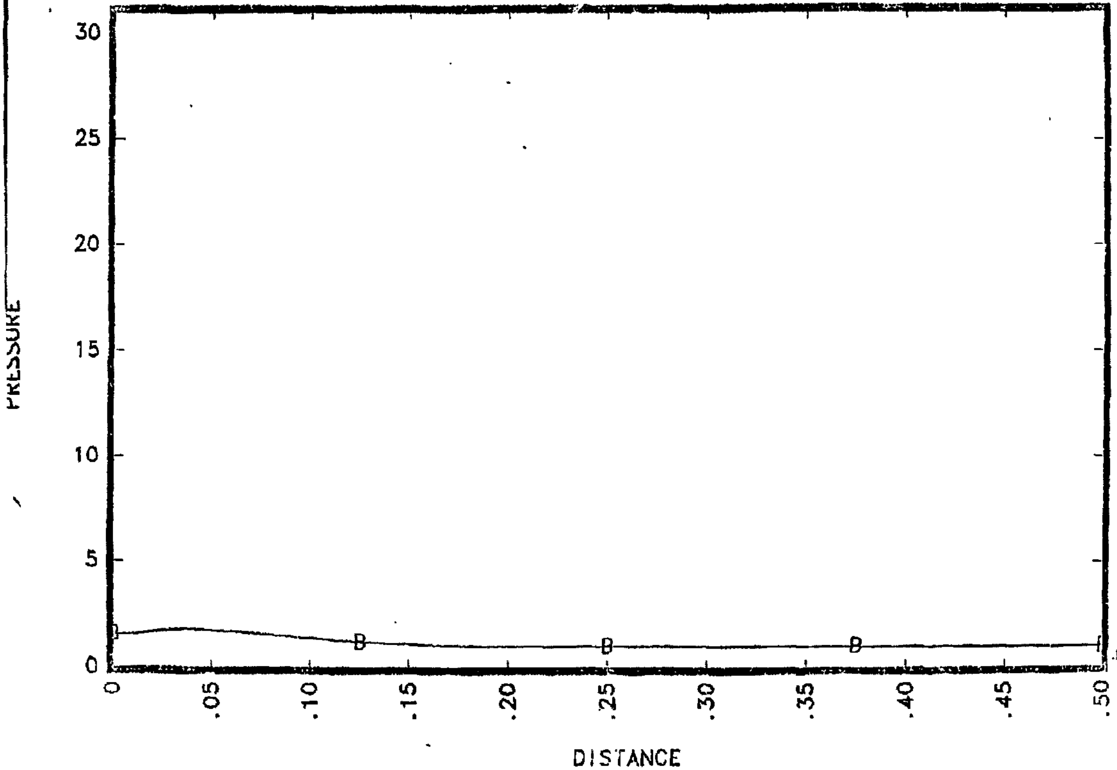
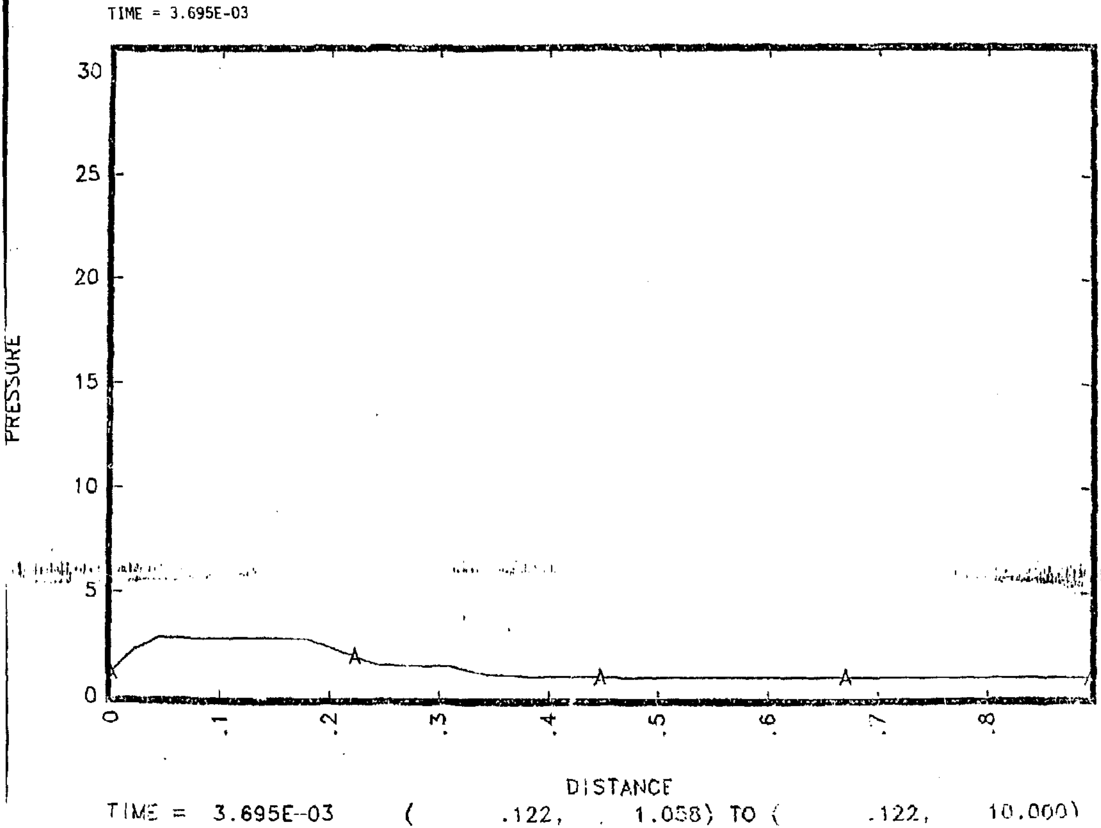
Figure 10

TIME = 2.772E-03



TIME = 2.772E-03 (0.000, 3.450) TO (5.000, 3.450)

Figure 11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15
10
5
0
PRESSURE
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.2
.3
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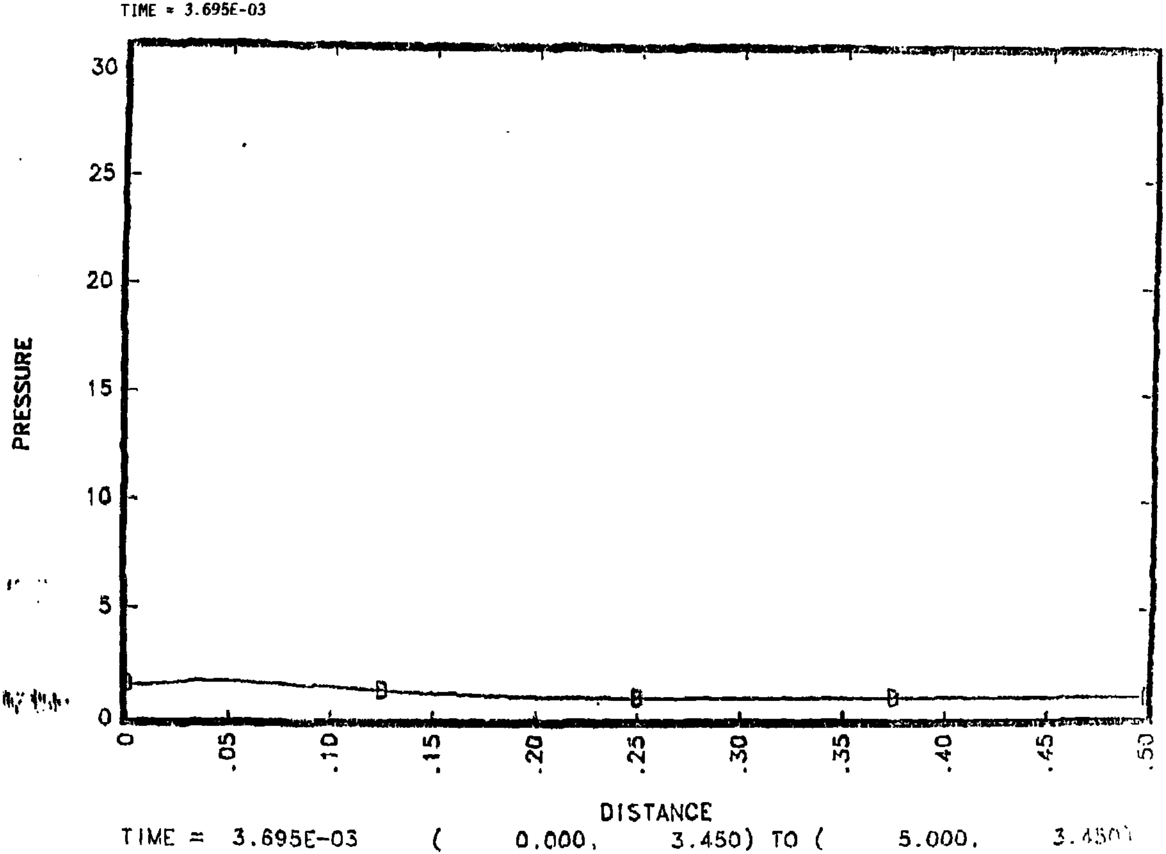
.5
.6
.7
.8
DISTANCE
TIME = 3.695E-03 (.122, 1.058) TO (.122, 10.000)

Figure 12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10
5
0
0
.05
.10
.15
.20
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.30
.35
.40
.45
.50
DISTANCE
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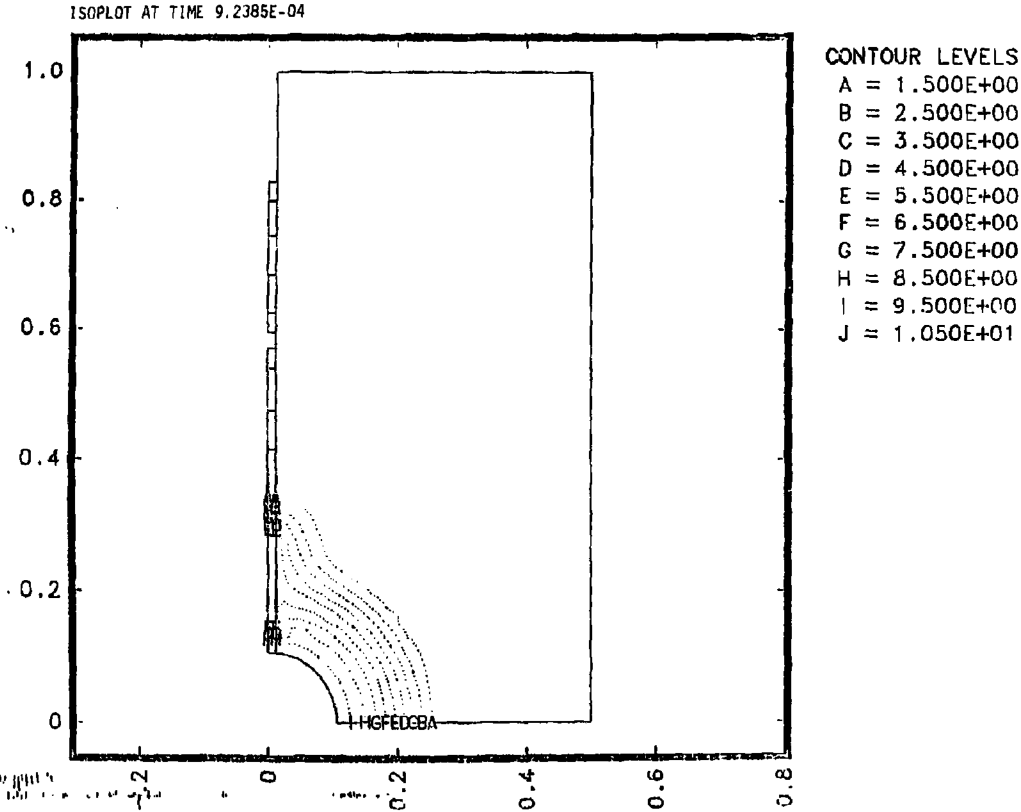
Figure 13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0.6
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8
IHGFEDCBA
CONTOUR LEVELS
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B = 2.500E+00
C = 3.500E+00
D = 4.500E+00
E = 5.500E+00
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G = 7.500E+00
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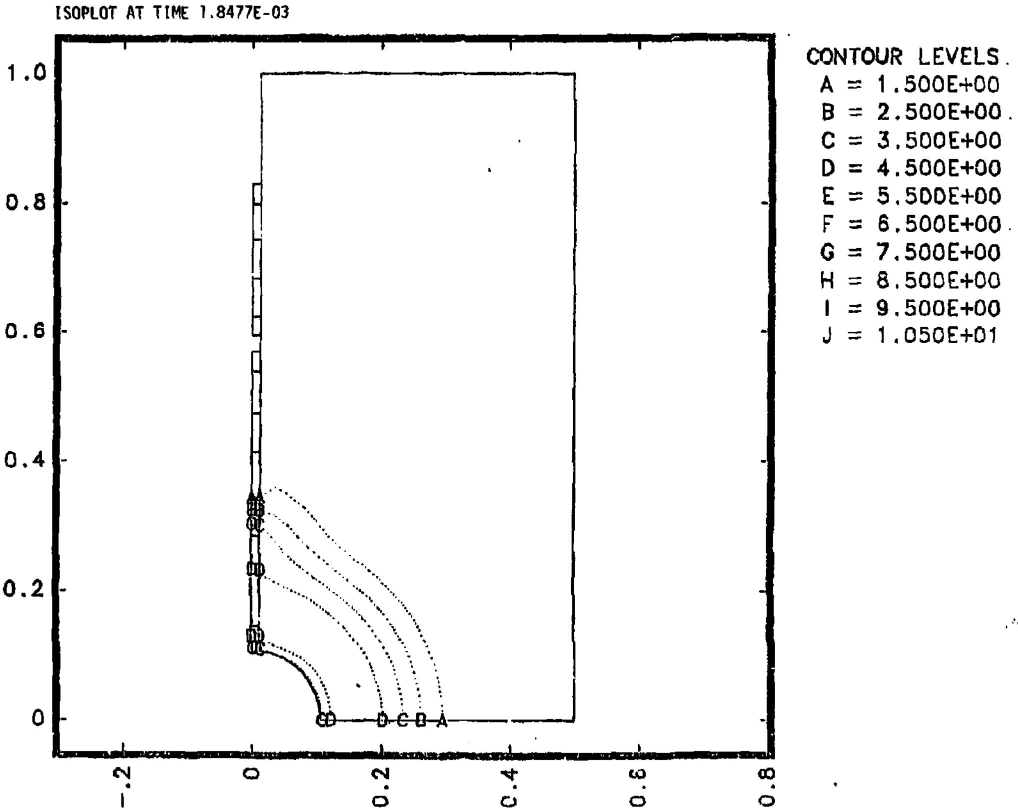
I = 9.500E+00
J = 1.050E+01

Figure 14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F = 6.500E+00
G = 7.500E+00
H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
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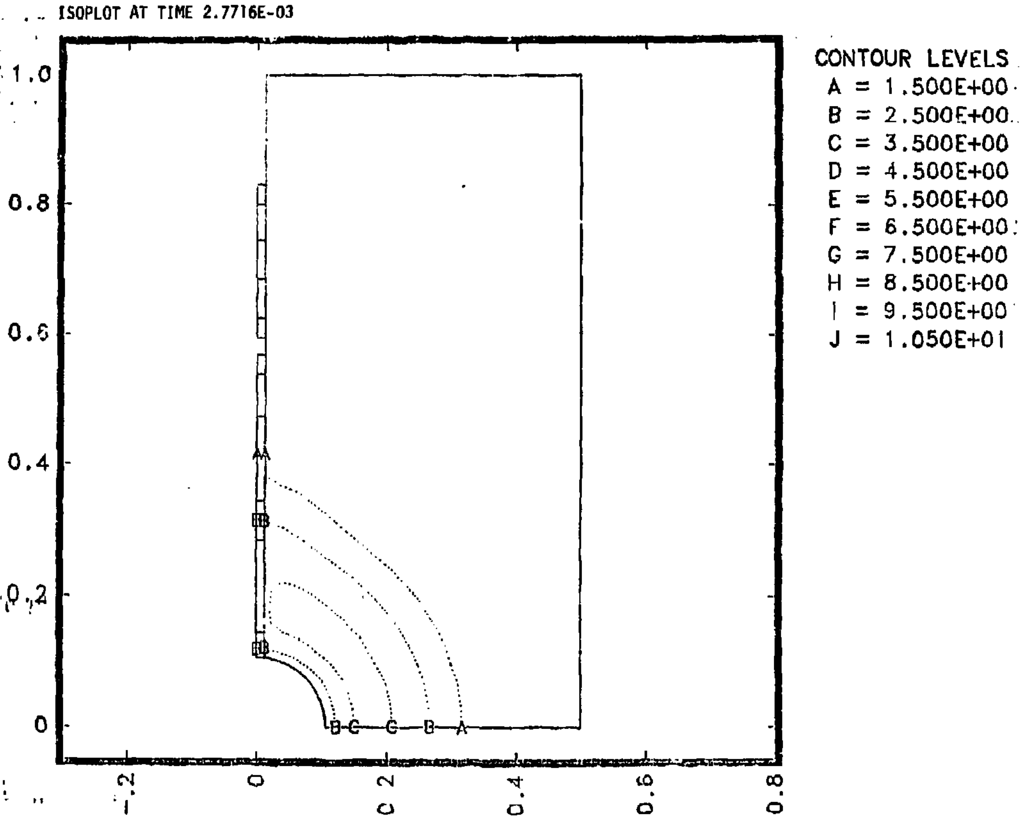
0.6
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8

FIGURE 15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0.2
0.4
0.6
0.8
A
B
C
C
B
A
CONTOUR LEVELS
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B = 2.500E+00
C = 3.500E+00
D = 4.500E+00
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Figure 16

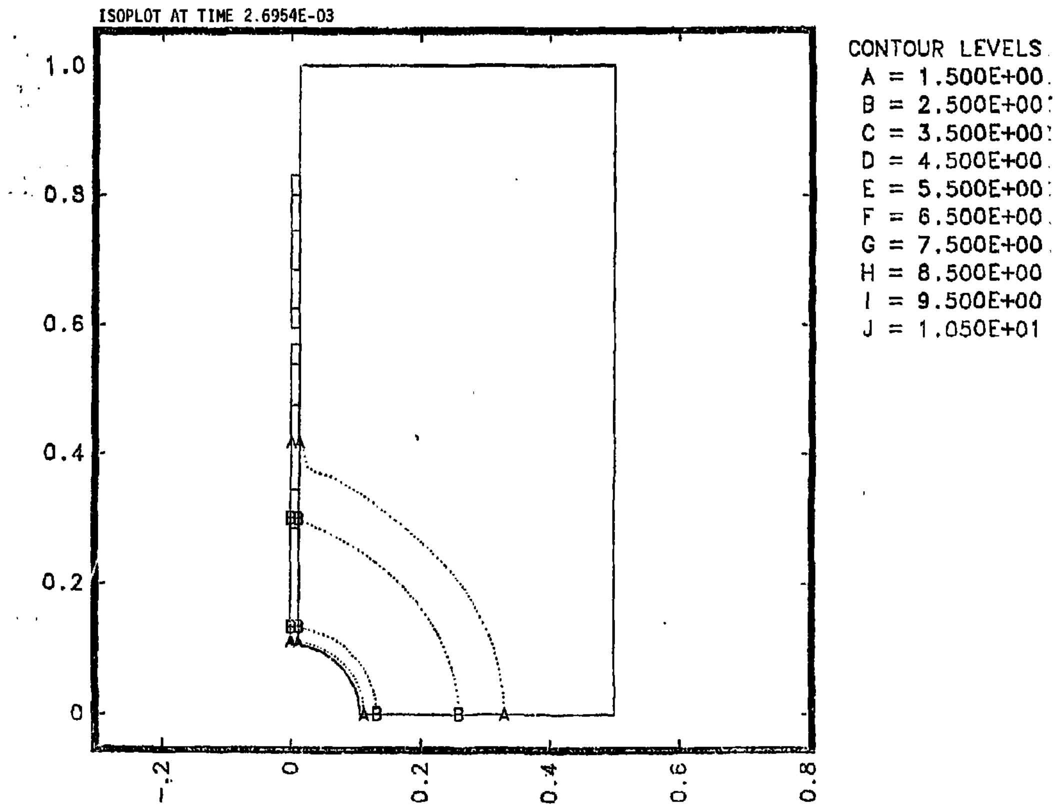
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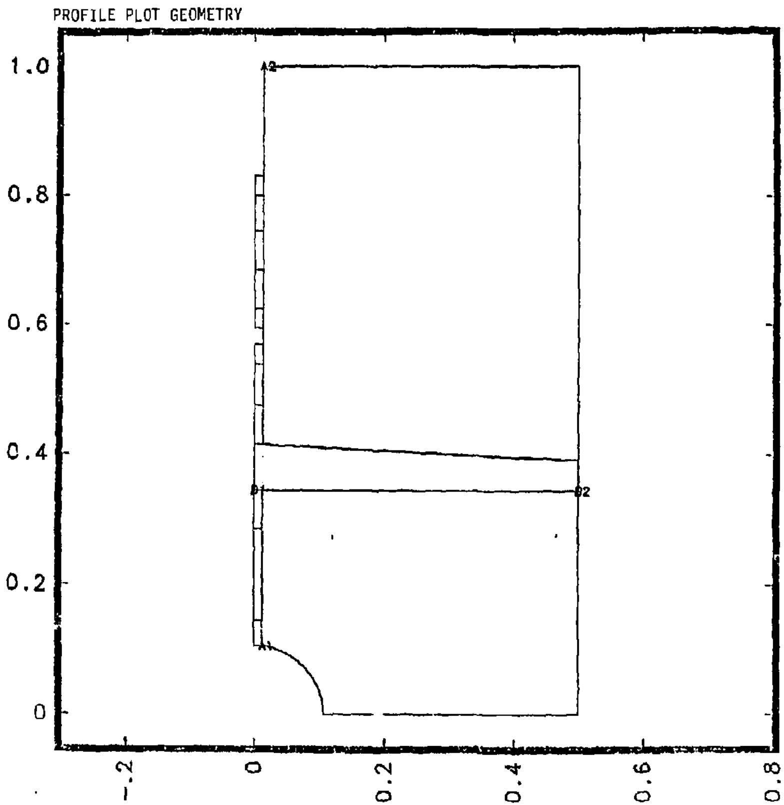
Figure 17



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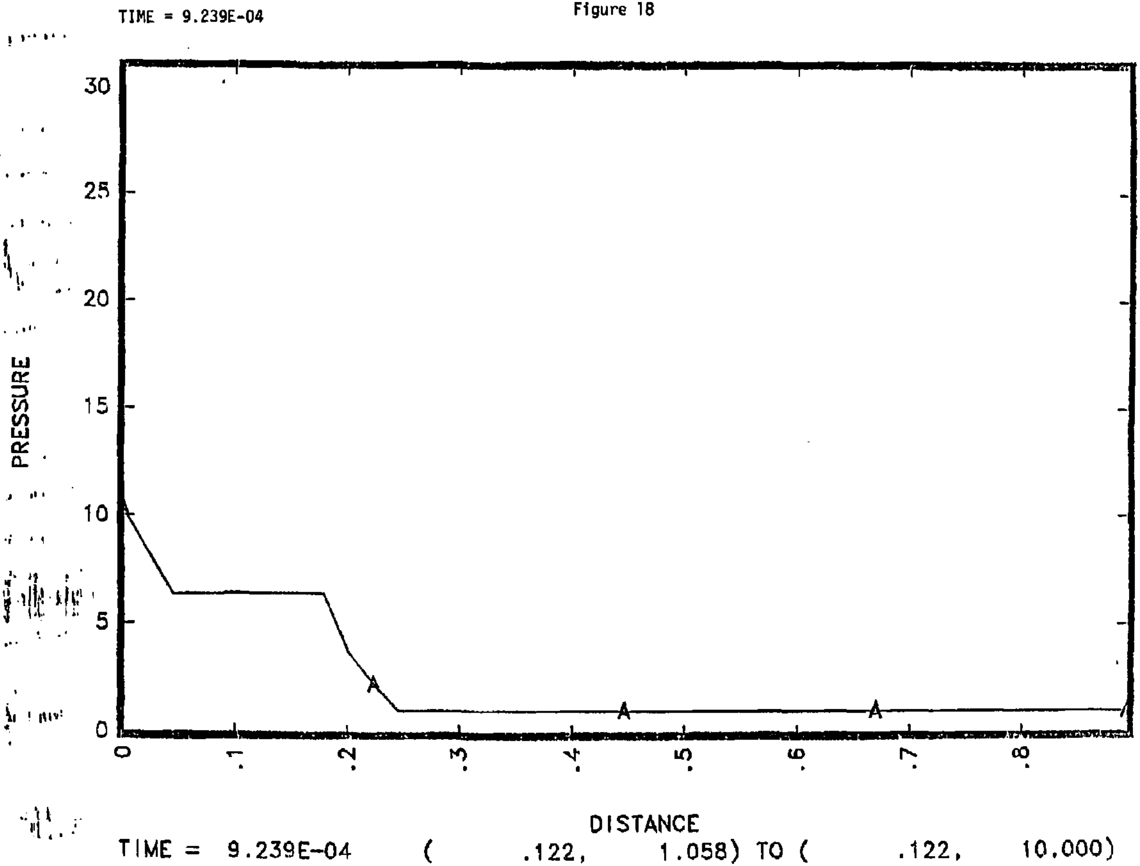
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PRESSURE
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.6
.7
.8
DISTANCE
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Figure 19

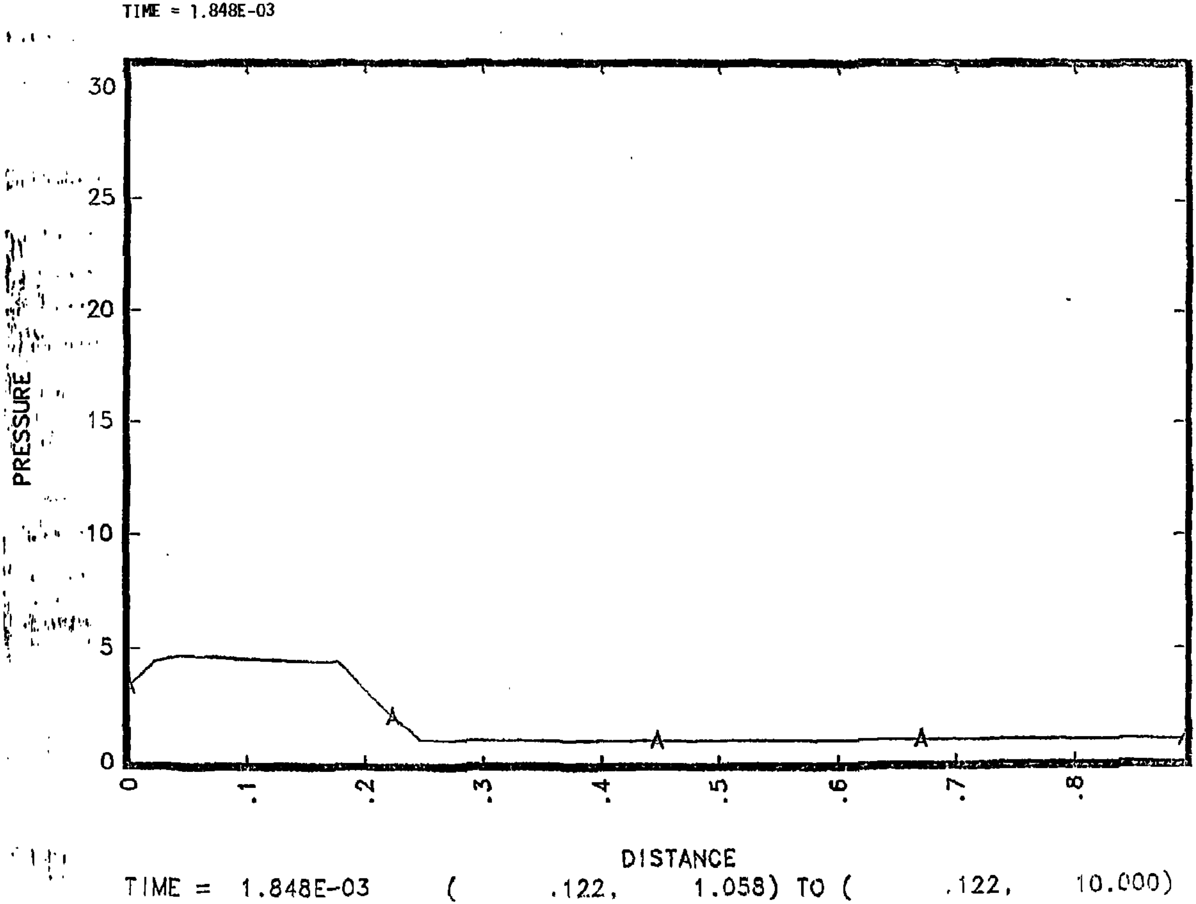
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Figure 20

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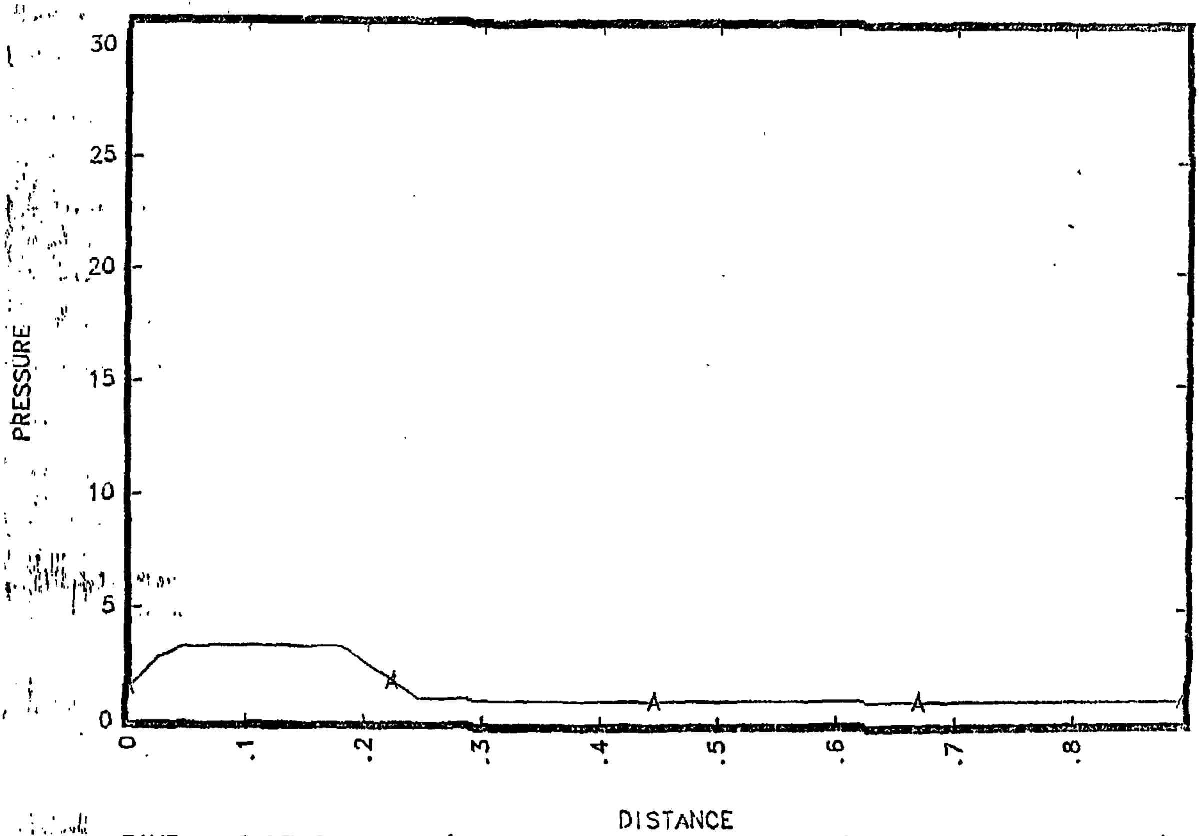

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Figure 21

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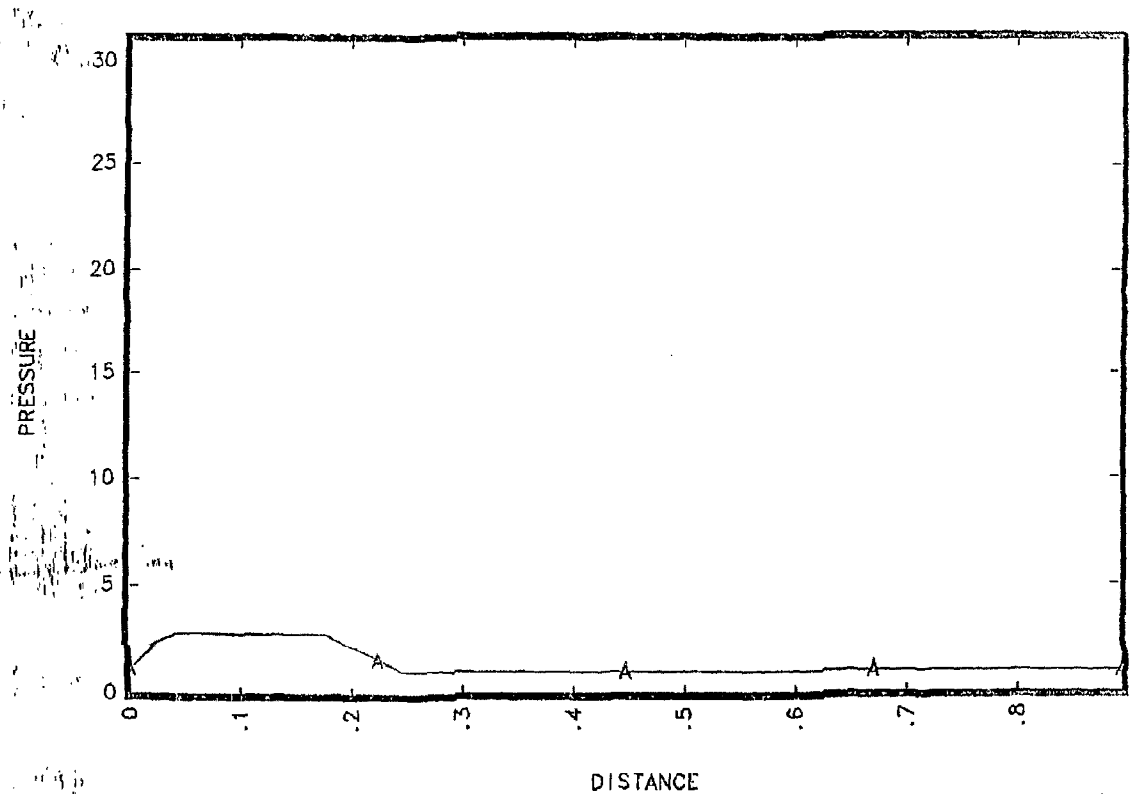

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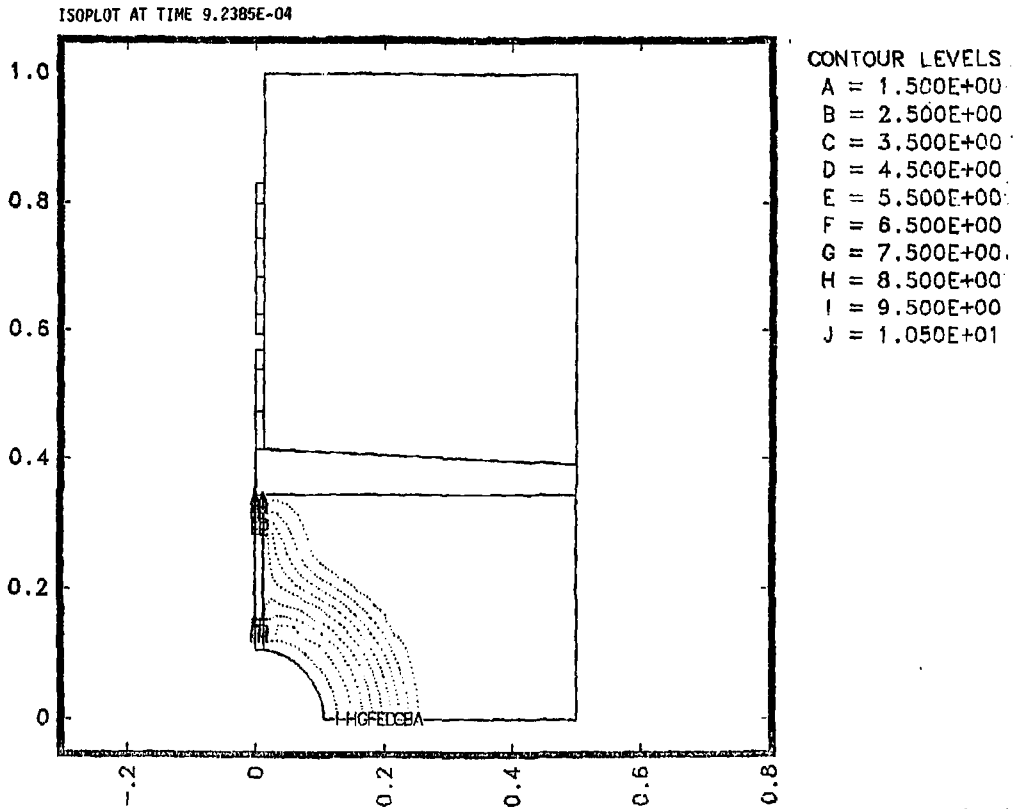
Figure 22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0.8
0.6
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-.2
0
0.2
0.4
0.6
0.8
I HGFEDCBA
CONTOUR LEVELS
A = 1.500E+00
B = 2.500E+00
C = 3.500E+00
D = 4.500E+00
E = 5.500E+00
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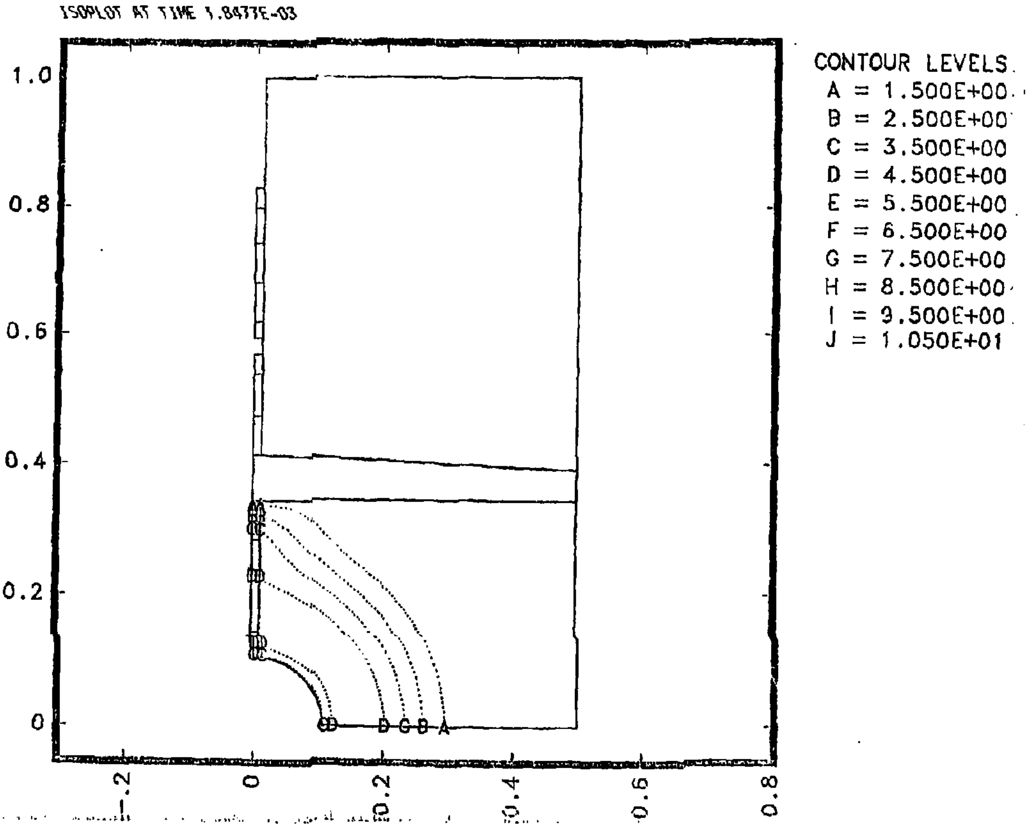
Figure 23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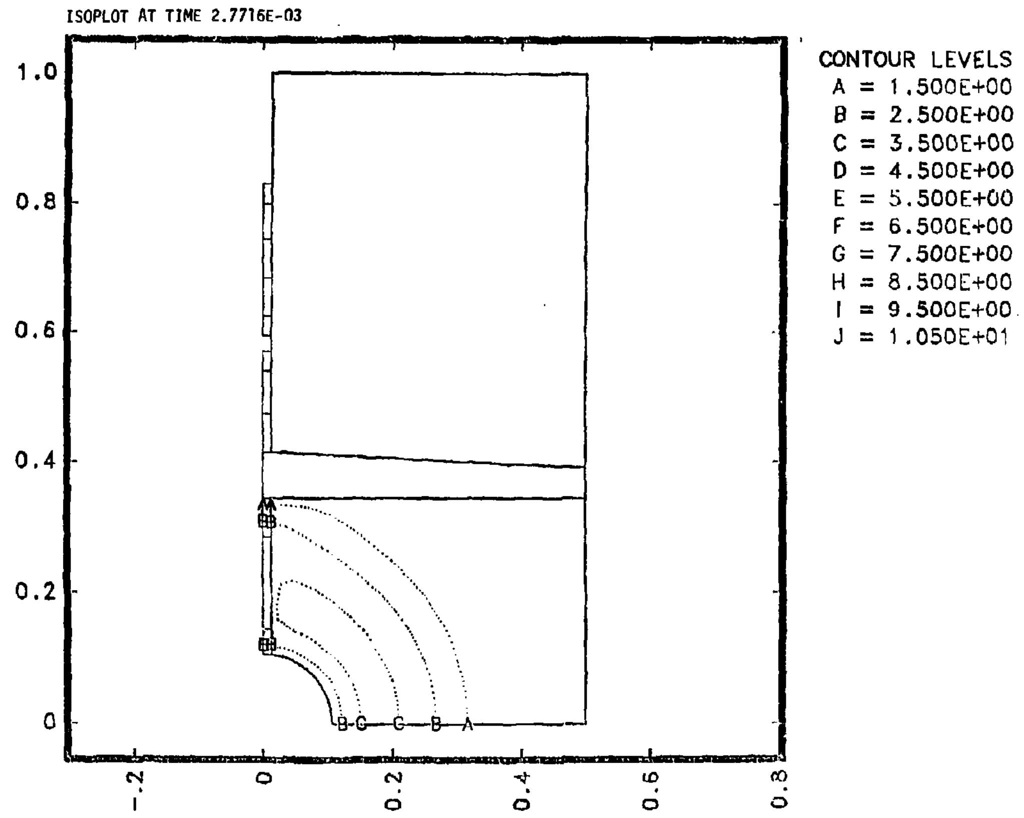
F = 6.500E+00
G = 7.500E+00
H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
1.0
0.8
0.6
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8
D G B A

Figure 24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H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
1.0
0.8
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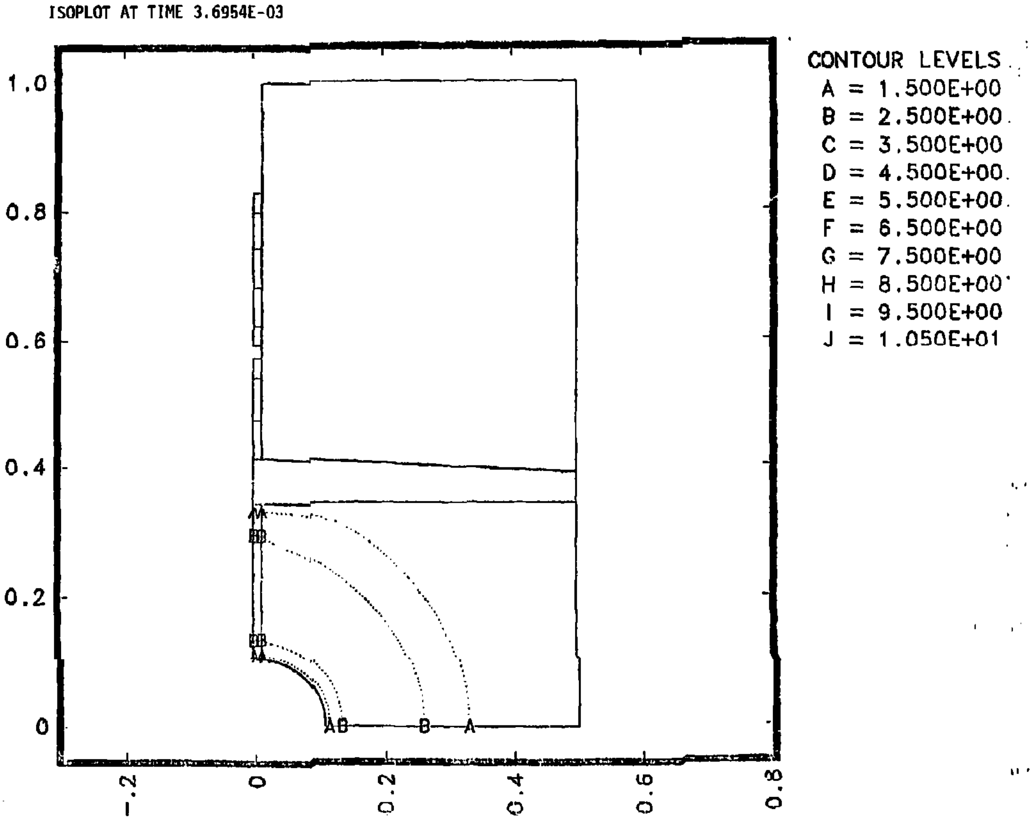
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8
B C C B A

Figure 25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CONTOUR LEVELS
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F = 6.500E+00
G = 7.500E+00
H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
1.0
0.8
0.6
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8
A
B
AB
B
A

Figure 26

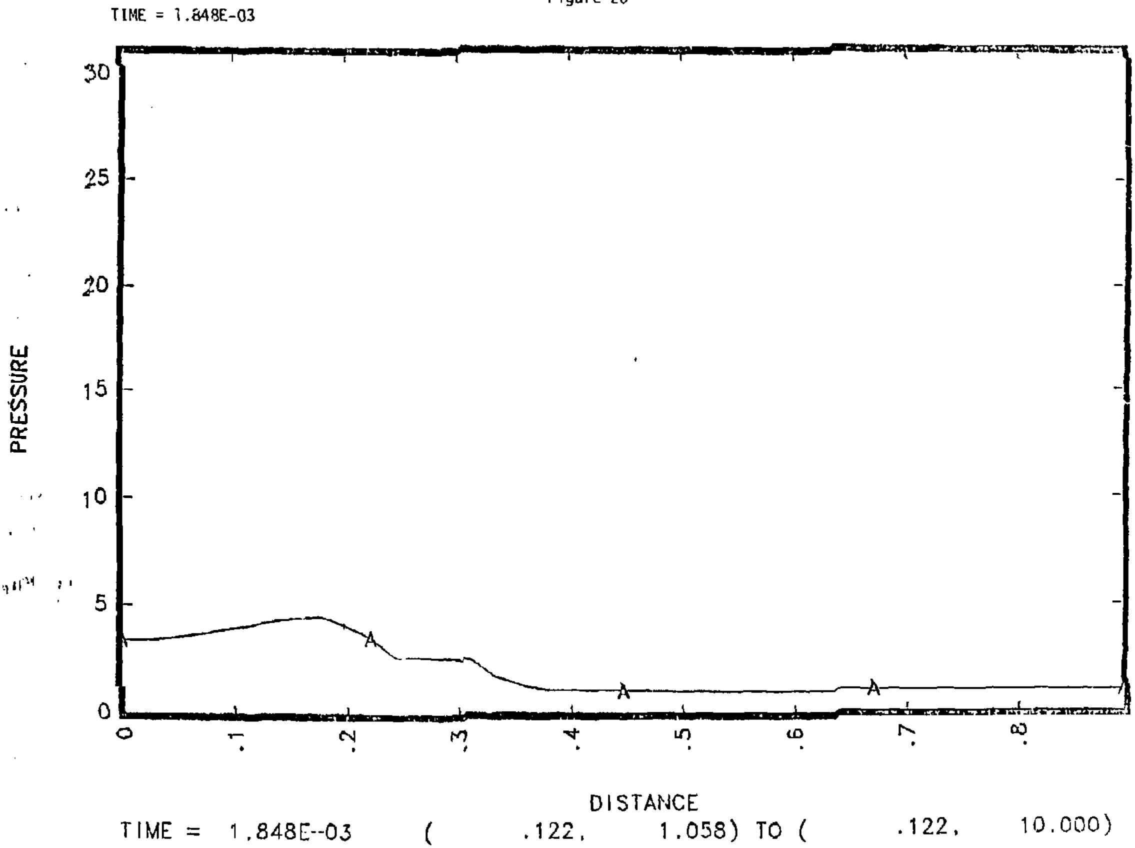
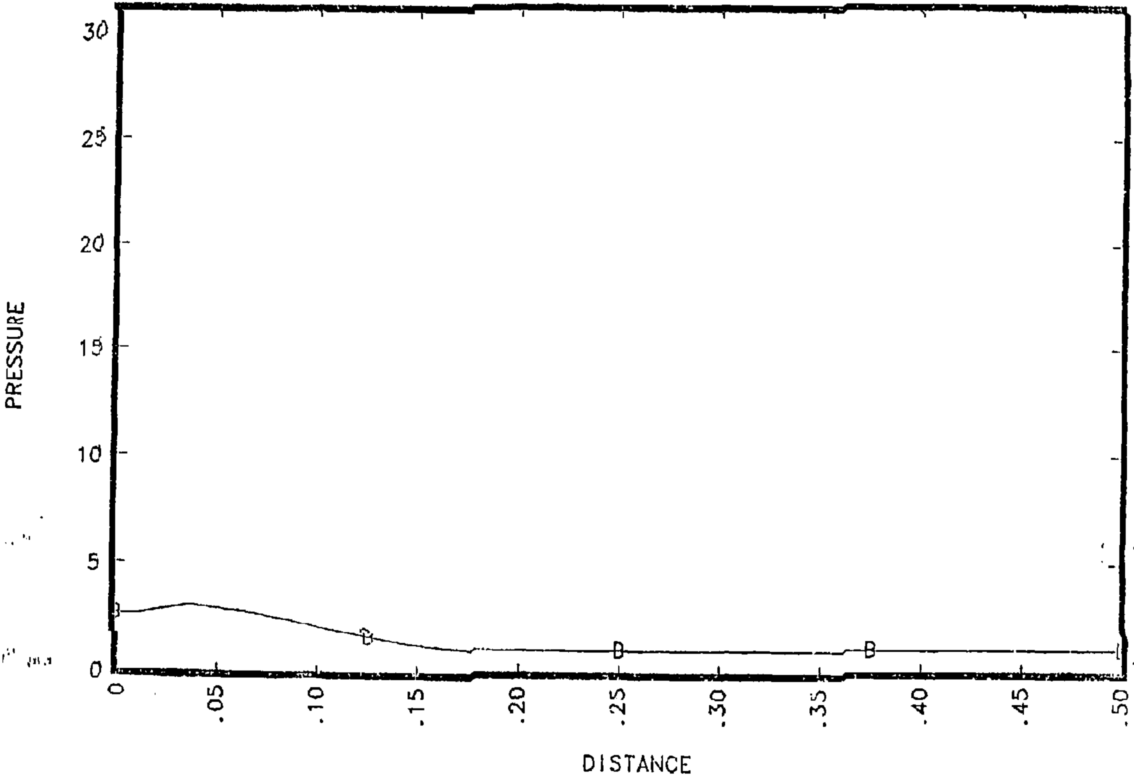

TIME = 1.848E-03
PRESSURE
30
25
20
15
10
5
0
0
.1
.2
.3
.4
.5
.6
.7
.8
A
A
A
DISTANCE
TIME = 1.848E--03 (.122, 1.058) TO (.122, 10.000)

Figure 27

TIME = 1.848E-03



TIME = 1.848E-03 (0.000, 3.450) TO (5.000, 3.450)

Figure 28

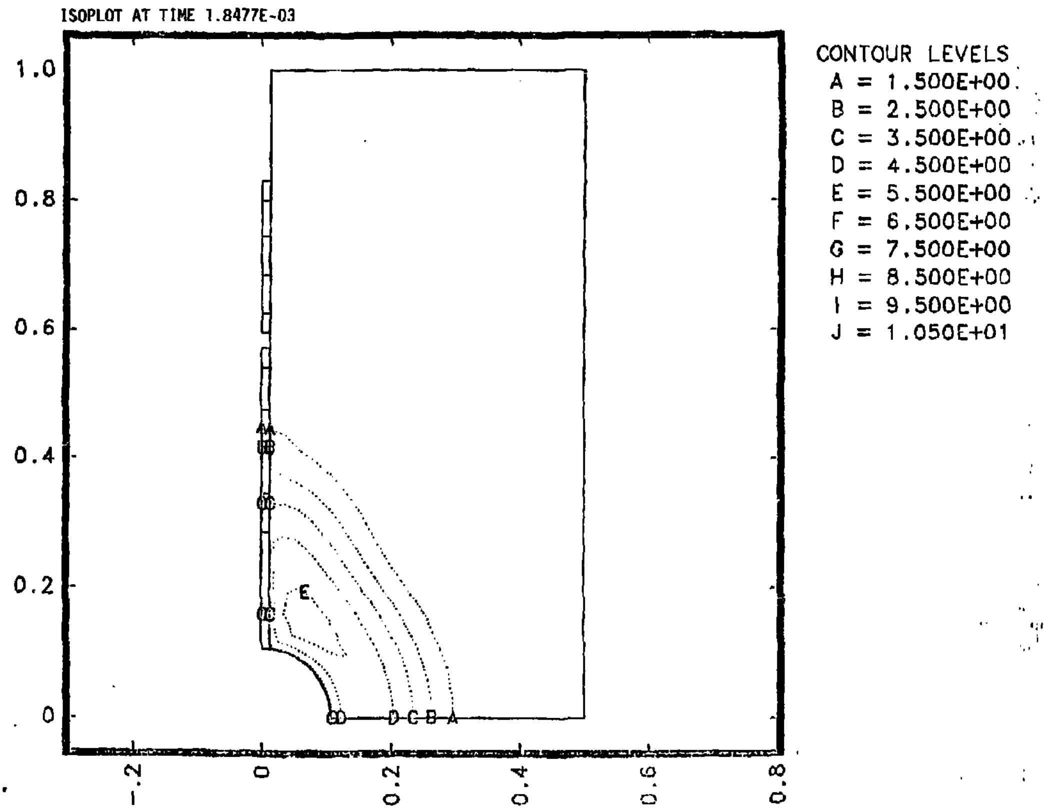
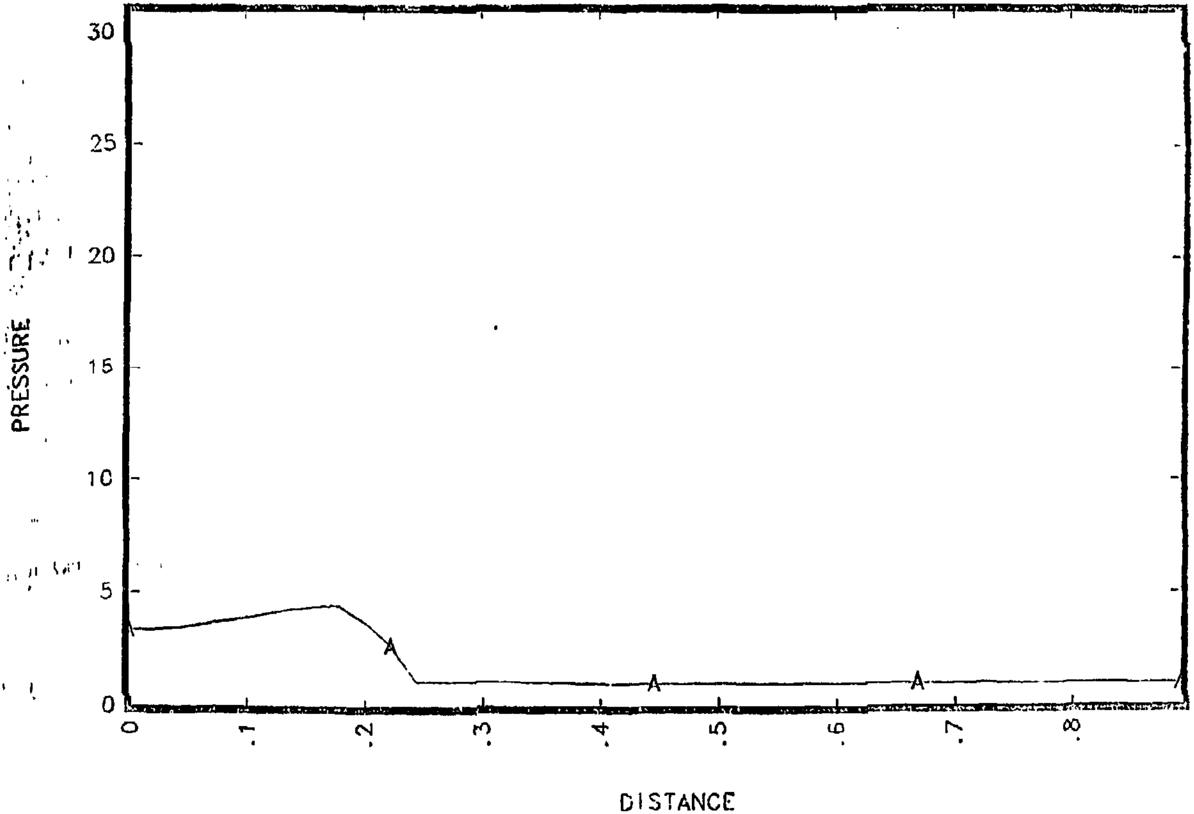

ISOPLOT AT TIME 1.8477E-03
CONTOUR LEVELS
A = 1.500E+00
B = 2.500E+00
C = 3.500E+00
D = 4.500E+00
E = 5.500E+00
F = 6.500E+00
G = 7.500E+00
H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
1.0
0.8
0.6
0.4
0.2
0
-.2
0
0.2
0.4
0.6
0.8

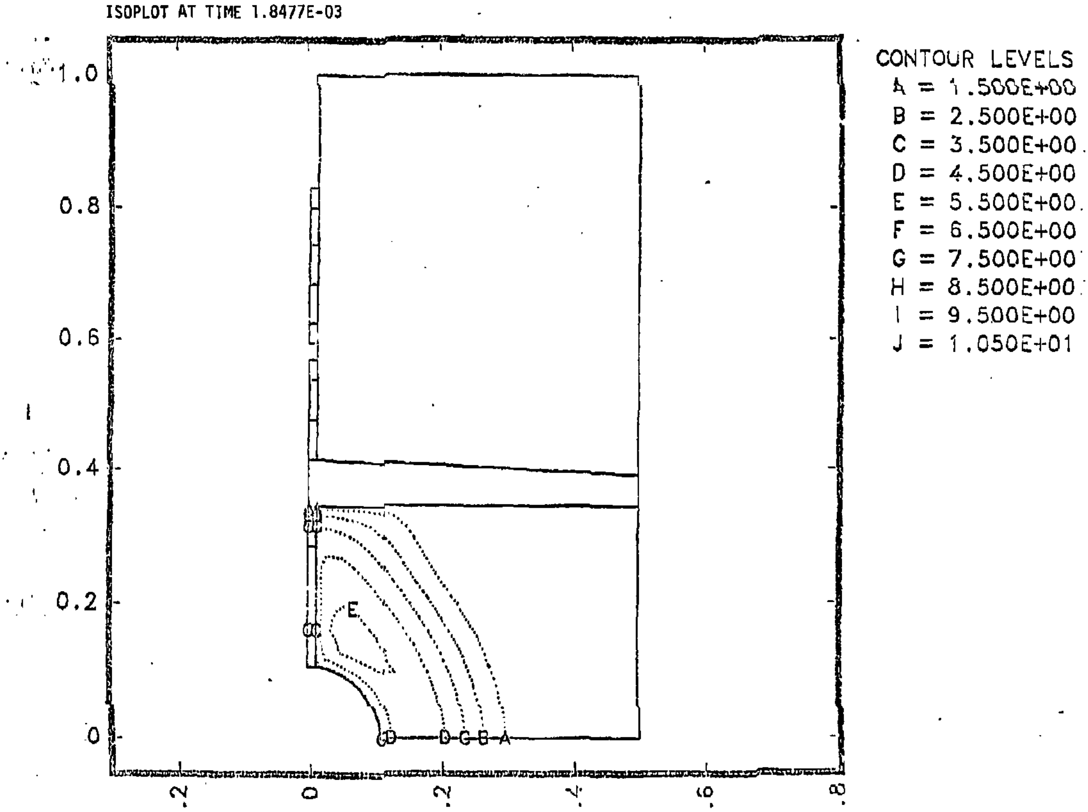
Figure 29

TIME = 1.848E-03



TIME = 1.848E-03 (.122, 1.058) TO (.122, 10.000)

Figure 30


ISOPLOT AT TIME 1.8477E-03
CONTOUR LEVELS
A = 1.500E+00
B = 2.500E+00
C = 3.500E+00
D = 4.500E+00
E = 5.500E+00
F = 6.500E+00
G = 7.500E+00
H = 8.500E+00
I = 9.500E+00
J = 1.050E+01
1.0
0.8
0.6
0.4
0.2
0
.2
0
.2
.4
.6
.8
E
D
G
B
A

[illegible] United States [illegible]
[illegible]
[illegible] Technical [illegible]
[illegible] Department [illegible]
[illegible] Road
Springfield [illegible]
[illegible] $ [illegible]

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