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Final Report

POWER SYSTEM PLANNING AND RELIABILITY

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ABSTRACT

The recent literature on power system reliability has emphasized the importance of sound planning to satisfy future loads. In view of the extremely high investment costs of electric power systems, it is imperative to have procedures for adding the right kind of equipment at the right time in the right location to achieve the desired level of reliability and quality of service at an acceptable cost.

These procedures are very complex because of the large number of design alternatives that need to be reviewed from the points of view of economy and technical performance (including reliability). In this report, power system planning is viewed as a problem of mathematical optimization in which the sum of long-term investment and operating costs is minimized subject to reliability constraints. This mathematical optimization provides the desired procedures for system design and gives the most economical schedule for system expansion.

Digital computers have been used extensively during the past decade for simulating the expansion of a power system. Direct optimization, however, constitutes a new development; its feasibility is proven and illustrated in this report.

In addition to outlining new methods for direct optimization, this report develops efficient methods for computing the operating and capital costs and for assessing the reliability properties (including transient stability) associated with each design alternative.

Finally, the problem of expanding a power system in the presence of uncertainty about future load, technology, financing costs, etc., is viewed as one of stochastic optimization, and procedures for computing the optimum decision policy are developed and illustrated.

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I INTRODUCTION

The present final report summarizes the results obtained in the course of Contract 14-03-72910 during the period of 24 May 1967 to 25 May 1968.

A. Contract Objectives[1]*

The initial contract objectives were stated as follows:

Item 1: In conjunction with BPA, SRI will provide a detailed formulation of the generation planning problem via direct methods, with particular emphasis on the combined operation of nuclear and hydro-electric generation. The output of this study item will be a digital computer program encompassing a sufficient number of generation alternatives to make it a useful and practical planning tool for the BPA. The relative advantages of stochastic optimization over deterministic optimization will be assessed in the course of this work item.

Item 2: In conjunction with BPA, SRI will further develop Jamoulle's concept of placing a quantitative value upon service reliability and will attempt to include this quantitative value into the objective function to be maximized by the power system planner. To accomplish this result, it will be necessary to identify the exact statistical description of component failures and to determine the relation of service reliability at a given point in the system to this statistical description, the dynamics of the power system, and the operating policy (maintenance, reserve schedule, etc.) to which it is subjected.

Item 3: In conjunction with BPA, SRI will provide a detailed formulation of the transmission planning problem via direct methods with particular emphasis on the logical inclusion of the reliability of service aspects studied under Item 2. The output of this study item will be a digital computer program encompassing a sufficient number

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*References are listed at the end of this report.

of transmission alternatives to make it a useful and practical planning tool for the BPA.

Item 4: The relations previously established with other organizations active in power system research, notably Electricité de France, EBES, etc., will be further cultivated with the aim of deriving mutual benefit from the exchange of ideas, personnel, and programs.

In the course of numerous subsequent discussions with BPA staff, it was decided to emphasize and expand transmission system planning (Item 3) and transmission system reliability (Item 2), and to de-emphasize generation planning.

B. Purpose and Organization of the Report

In view of the numerous technical memoranda produced in the course of the project, the purpose of this report is not to repeat in detail the contents of these memoranda but to provide an overall view of the problem statements, the assumptions made, the solution techniques developed, and the results obtained. With this aim in mind, the various factors that influence transmission system planning are carefully reviewed in Sec. II, and the results obtained, together with suggestions for further work, are given in detail in Sec. XI.

The approaches taken in the technical memoranda and the results obtained are summarized in Secs. III-X. In order to avoid lengthy expositions, frequent reference is made to these memoranda, and flow charts are used extensively.

The titles of the technical memoranda are listed below:

- J. C. Kaltenbach and J. Peschon, "Transmission System Planning," Technical Memorandum 1, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (September 1967).[2]

- J. C. Kaltenbach, "Determination of a Nominal Expansion Schedule by a Second-Order Gradient Method," Technical Memorandum 2, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (10 December 1967).[3]

- J. Peschon, "Optimal Planning of Transmission Subsystems in the Presence of Uncertainty," Technical Memorandum 3, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (December 1967).[4]

- M. W. Siddiqee, "Stability Considerations in Transmission System Planning," Technical Memorandum 4, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (January 1968).[5]

- L. P. Hajdu and M. W. Siddiqee, "Economic Considerations of Series Compensation in Transmission System Planning," Technical Memorandum 5, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (February 1968).[6]

- M. W. Siddiqee, "Line Outage Analysis in Transmission System Planning," Technical Memorandum 6, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (February 1968).[7]

- M. W. Siddiqee, "Network Reduction in Power Systems," Technical Memorandum 7, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (March 1968).[8]

- Patrice H. Hénault and L. P. Hajdu, "Transmission Subsystem Planning Via Stochastic Dynamic Programming. An Example: The BPA Olympia-Port Angeles Expansion," Technical Memorandum 8, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (March 1968).[9]

- Patrice H. Hénault and L. P. Hajdu, "Line Outage Analysis with Emergency Load Curtailment in Transmission System Planning," Technical Memorandum 9, SRI Project 6619, prepared for Bonneville Power Administration, Portland, Oregon (April 1968).[10]

C. State of the Art in Computer-Aided System Planning

This discussion will be limited to long-term planning of systems, with special emphasis on power systems.

The first class of techniques involving the digital computer in long-term planning of power systems is referred to as simulation techniques. The planner specifies the expansion schedule (nature and size of equipment, time of installation), and the resulting system characteristics (notably operating cost, capital cost, and reliability properties) are computed. The obvious drawback of the simulation is that the determination of satisfactory planning schedules, not to speak of optimum schedules, requires much intelligent experimentation on the part of the system planner. The overwhelming percentage of computer-aided, long-term planning work falls into the category of simulation.[11-16]

In other industries, system expansion problems have sometimes been solved successfully by linear programming, which yields an optimum (least costly) solution without any need for experimentation. However, the classes of systems whose expansion can be put into a linear programming format are severely limited and generally exclude power systems. This fact was ascertained in the course of discussions with Prof. M. Kurz of the Economics Department of Stanford University, who acted as a consultant on this project. Further evidence of the failure of linear programming approaches to power system, and comparable system, expansion problems was obtained in the course of a survey of applicable techniques.

A significant step forward was taken by E. Jamoulle and colleagues, who applied continuous variational techniques to generation planning.[17-19] Although this approach adequately answers simple planning questions, such as "should nuclear generation be purchased right away, or can it be delayed?," it is not applicable to the problems of concern in this report. Somewhat similar variational techniques were developed in Ref. 20 for the planning of simple system models to exploit regional load diversity.

In the course of discussions at SRI with E. Jamoulle and M. Cuénod, who acted as consultants during the definition phase of this project, a dynamic programming approach was formulated and tested on a simple generation expansion problem.[21-22] A direct optimization was obtained, and no experimentation on the part of the planners was required.

The application of dynamic programming to generation expansion was also suggested in Ref. 23 and 24, but no results were given. To the best of our knowledge, none of these dynamic programming techniques are presently used in the electric power industry.

In the references discussed so far, the assumption of accurately predicted future loads is always made. In a related application, involving the expansion of a weapon system in response to uncertain changes of future military requirements,[25-26] a stochastic planning procedure based on dynamic programming was evolved by the authors. This procedure, which was adapted to the needs of the present project, also appears to be unique in the power industry.

D. Acknowledgments

We wish to acknowledge the guidance and suggestions received from the staff of the Branch of System Engineering of the Bonneville Power Administration. Their guidance was instrumental in identifying the major technical and economic factors and eliminating those of secondary importance. Their suggestions, derived from extensive experience with traditional common-sense planning methods, led to the development of new computer algorithms that mechanize and optimize these same methods.

II PRELIMINARY PROBLEM DISCUSSION

A. Introduction

In view of the multiplicity of factors that had to be accommodated in the course of the project, we will first give an overall view of the problems to be solved, the models used in subsequent parts of the study, and an outline of the optimization approach we developed. We will also briefly discuss related work in planning, and we will outline the applicability of the procedures developed for transmission planning to generation planning and to simultaneous generation-transmission planning.

B. Transmission System Planning and Reliability

The problem of finding the least costly long-term transmission system expansion schedule or policy can be formulated as a mathematical optimization problem encompassing a number of special features, most of which complicate the mathematics. These special features are reviewed below.

1. Discrete Expansion Alternatives

The capacity of a branch* can only be increased (or decreased) by adding (or removing) lines of standard type, such as 345 kV or 500 kV. As a consequence of this discrete feature, the continuous techniques of mathematical programming[27-28] (linear and nonlinear programming, gradient methods) are ruled out, and discrete techniques must be employed. The best known of these techniques are integer programming, the branch and bound technique, the exhaustive search, and dynamic programming.[29-30] The exhaustive search can be eliminated at the outset, because of the exorbitant computer time it requires. Integer programming, branch and bound, and dynamic programming in their commonly known forms are not directly applicable either, because of their excessive computation time

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*In this report, a branch is defined as an assortment of one or more lines connecting the same two nodes in the system.

and/or memory requirements in all but the simplest transmission system configurations. Suitable modifications of at least one of these techniques, hence, had to be developed. Largely as a result of SRI's great familiarity with dynamic programming and its preliminary knowledge of how to make the required modifications, dynamic programming was retained. Integer programming and the branch and bound technique were abandoned after a brief review.

2. The Reliability Constraint

One of the overriding reasons for increasing transmission capacity is reliability. Although we do not know the exact reliability criterion that will be imposed in the future, it may be safely assumed that it will be of the following form: "If one (or more) lines, possibly the highest capacity line(s), is (are) subjected to an unscheduled outage, no other lines shall be overloaded resulting in their loss at any time of the year, including peak periods."[27,31,32] From a mathematical point of view, the resulting constraint is somewhat awkward, since it cannot be stated under the usual form

$$h(x,u) \le 0 \quad , \qquad \text{(II-1)}$$

but it implies a test for each proposed addition. This test ensures that the worst single (or uncorrelated multiple) outage does not cause any overloads.

Since the reliability constraint is of a logical rather than an algebraic nature, the continuous techniques of mathematical programming (linear and nonlinear programming, gradient methods) are again ruled out, and discrete techniques must be employed.

3. The Transient Stability Constraint

Another overriding reason for increasing transmission capacity is transient stability. If a generator is connected to a system by transmission lines of insufficient capacity, then a transient, such as the loss of one line connecting this generator to the system, may cause this generator to fall out of step and as a result, the generator's protections will disconnect it from the system.

The previously stated reliability criterion also encompasses transient instability, which can be viewed as an overload leading to the loss of equipment as a result of the single (or multiple uncorrelated) unscheduled outage.

There are various ways of improving the transient stability properties of a power system other than by the strengthening of transmission, namely, the switching in of series capacitors or shunt resistors, overexcitation, and governor action. In all cases, however, one can define a maximum steady-state post-fault power flow that must not be exceeded. Consequently, to ensure transient stability, it suffices to impose an upper bound on steady-state post-fault power flow. As a result of this observation, the reliability and transient stability constraints can be tested by similar computational procedures, and the cumbersome transient stability programs need not be used in planning, except possibly to establish the upper bound of the post-fault steady-state power flow. This simplified approach to transient stability is absolutely necessary, in view of the very large number of planning configurations to be tested.

4. Time-Varying Loads

In a given year, the loads and generations vary from minute to minute. A characterization of the loads and generations (the injections) is required for two reasons:

(1) The reliability constraint must be satisfied for all injection patterns that may occur during the year,

(2) The yearly energy lost has a bearing upon the expansion of the transmission system and therefore must be calculated.

To test the reliability for <u>all</u> injection patterns involves a very considerable amount of computation time. In accordance with standard practice, this test will only be carried out for a small number of injection patterns (e.g., winter peak and summer peak) that are believed to create maximum stresses. More accurate procedures for determining the most vulnerable injection patterns could be derived from game theory, but this refinement was not pursued during the present contract.

The traditional way of computing yearly losses by means of the estimated load factor is quite acceptable. In the course of the study, a more accurate and more versatile method of computing system losses was developed; this is discussed in Appendix B.

5. Economy of Size

A plot of transmission line investment cost in dollars per mile versus transmission capacity is shown in Fig. II-1. The relation

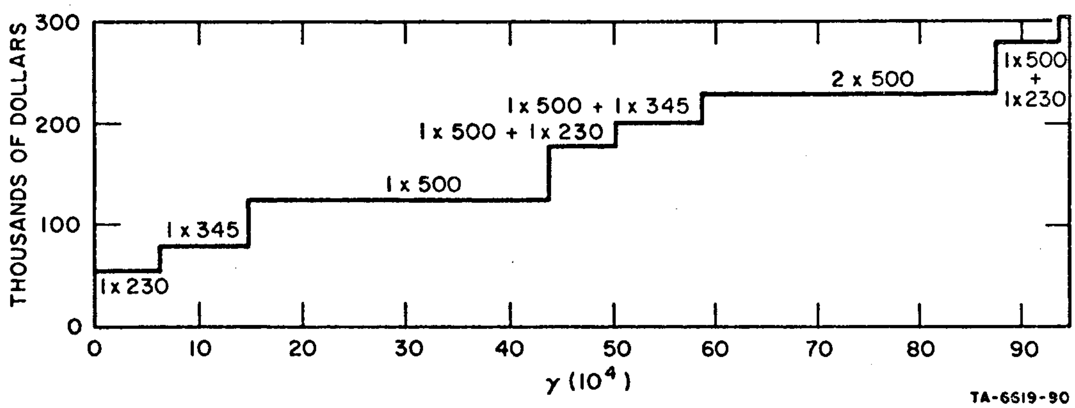


FIG. II-1 REPRESENTATIVE RELATION BETWEEN INVESTMENT COST AND CAPACITY.
The notation 1 × 500 + 1 × 345 means that there is one 500 kV line in parallel with one 345 kV line.

shown in Fig. II-1 is discrete, because standard transmission lines have been assumed. This figure displays two important factors:

(1) The cost per unit capacity tends to decrease as capacity increases; this is referred to as "economy of size" and is one of the main reasons for the trend to high-capacity lines. From a

mathematical point of view, an assumed continuous relation between cost and capacity is hence concave and not convex as is required by many of the continuous-optimization procedures.

(2) The cost for a specified capacity depends significantly on the composition of the branches; to be specific, a single high-capacity line is always cheaper than several parallel lower-capacity lines with equal admittance. This emphasizes the fact that branch capacity (or branch admittance) is not a sufficient description, but that branch composition must be retained for planning studies. Branch composition also has a determining effect upon reliability, since several parallel lines are more reliable (in the sense discussed in Sec. II-B-2) than a single line of equal admittance. This points to a trade-off between investment cost and system reliability.

6. The Geographical Constraint

The limited right-of-way of certain mountain passes restricts the number of parallel lines that can be built. Generally speaking, a small number of high-capacity lines occupy less space than a large number of low-capacity lines of equal admittance. These geographical considerations introduce another complex constraint [unlike the constraint of Eq. (II-1)], which again precludes the use of continuous optimization procedures.

Another effect of the geographical constraint is that, under certain circumstances, old but healthy low-capacity lines may need to be torn down to make room for required high-capacity lines. This possibility introduced further complications, to be discussed in Sec. II-D-2-b.

A fortunate simplifying factor is that the cost of demolition of transmission equipment is approximately equal to the salvage value of the materials of which it is made.

In addition to the hard geographical constraints discussed above, a softer constraint related to beautification and land use has recently arisen. This constraint also prefers a small number of high-capacity lines to a large number of lower-capacity lines.

7. Series Compensation

Series capacitors are being extensively used not only to temporarily increase branch admittance to enhance the system's transient stability properties, but also to be permanent line elements that reduce the investment cost per unit capacity. Series compensation thus allows a low-capacity line to carry large amounts of power, while keeping the phase angle differences (which influence the transient stability characteristics) within acceptable limits. On the other hand, the losses correspond to those of a low-capacity line.

Series compensation thus appears to be another design parameter that allows trade-offs among investment cost, reliability, and losses. It also introduces accounting complications, because, unlike transmission lines, they could be moved around in a system at the cost of moving and installation.

8. Nonmonotonic Growth of Power Flows

Although the general trend of power consumption in an area is to increase with time, this same general trend does not necessarily hold for the required branch capacity. The reason for this is that, after a sufficiently large consumption has been reached in an area, local generation may become justified, in which case the previous power flow is reduced or even reverses sign.

The nomonotonic growth of power flows eliminates simple economic rules for transmission system design. In particular, the marginal laws (incremental yearly investment cost should be equal to incremental yearly cost of losses) are of doubtful value. The same objection holds for most of the "cost-effectiveness" procedures of recent fame.

9. Generation Planning

In this study, it was assumed that the loads and the generations (the injections) were known throughout the planning period. Planning of future generation independent of future transmission may well correspond to an existing administrative division in many utilities, although there exists a strong technical and economic interrelation between these two classes of major power system equipments. Transmission influences reliability of supply and thus allows economy of size in generation planning, and generation, when properly located, reduces the transmission requirements.

10. Uncertainty

The planning process is complicated by numerous imperfectly known facts about the future, notably

- Future loads
- Future capital costs and interest rates
- Future technology, e.g., economic schemes of underground transmission
- Reliability criteria imposed in the future
- Development of new operating policies and techniques to augment the system's reliability and stability properties, such as load curtailment, temporary system-wide frequency reduction, acceptance of islanding, use of fast-response emergency generation, etc.

The presence of these and other uncertainties significantly complicates the optimization mathematics. A proper statement of the planning objectives in the presence of uncertainty is: to minimize the expected long-term cost, given the fact that more information about the future will be acquired as time proceeds (rather than to minimize this cost, assuming perfect information now). From a practical point of view, uncertainty implies that the planner must build into the system sufficient <u>flexibility</u> to accommodate unlikely, but possible, future events. From a mathematical point of view, consideration of uncertainty rules out all known optimization procedures except dynamic programming.

C. Breakdown into Overall System and Subsystem Planning

From the previous discussion of planning phenomena, it becomes clear that the optimum planning of a complete transmission system is an exceptionally difficult problem, the successful solution of which requires numerous plausible and temporary simplifications.

Besides separating generation from transmission, the main simplification was to break the planning of the transmission system into an <u>overall system</u> and <u>subsystems</u>.

The overall system model consists of the main transmission arteries (230 kV and higher) terminating at a suitable number of nodes, of the order of 30 to 40. Each of these nodes may represent a single or a small number of geographically adjacent substations. The details of these substations (number and rating of the transformers and of the switchgear) are not considered, and an estimated substation cost can be added to the transmission lines.

The subsystems, of which there are a large number, consist of the medium- and low-voltage circuits contained in a small geographical area. The details of each subsystem, such as the type and rating of the transformers and switchgear, are considered from the point of view of both cost and technical performance.

This division into an overall system and subsystems corresponds exactly to the division that presently exists among the transmission system planning departments in most utilities.

D. Model for Overall Transmission System Planning

Since the aim is to find a long-term expansion schedule that minimizes the sum of the discounted capital and operating costs, subject to reliability constraints, it is necessary to create mathematical models for

(1) The network and the operating cost

(2) The investment cost

(3) The load, both long-term and short-term

(4) The reliability of operation.

These four fundamental models, together with the simplifying assumptions made, will be reviewed below.

1. The Network Model

The topological model of the network used throughout this part of the study was obtained from the Bonneville Power Administration and is shown, together with the peak node loads predicted, in Fig. II-2.

In this model, a branch is defined as a set of one or more transmission lines connecting two nodes. The detailed composition of the network is shown in Fig. II-3, where all the major lines, together with their voltage classes, are shown.

The most accurate mathematical model relating branch flows, node voltages, node voltage phase angles, and system losses to the node injections is the well-known AC power flow model[33,34]

$$g_1(V, \theta, \mathcal{J}) = 0$$

$$g_2(V, \theta, K) = 0 \quad , \qquad \text{(II-2)}$$

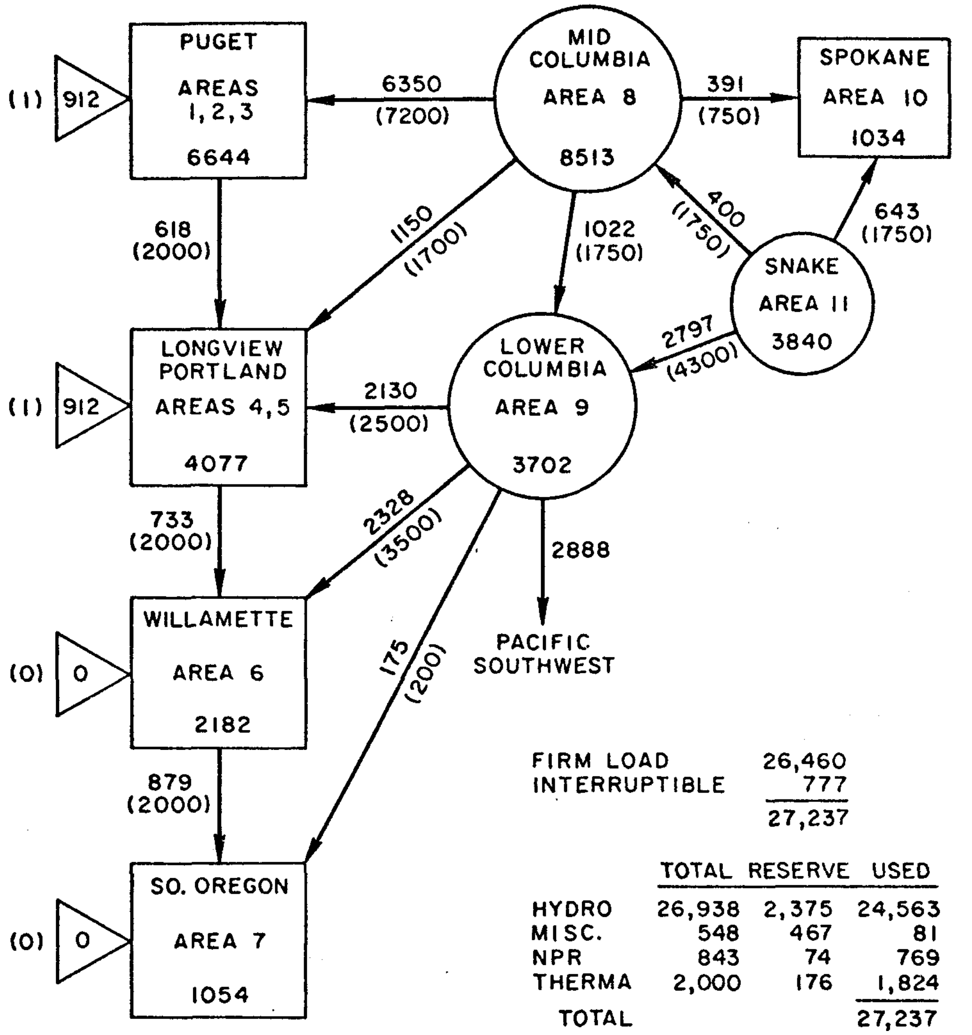
where

V_i = voltage at node i

θ_i = voltage phase angle at node i

$\mathcal{J}_i$ = active injection at node i

K_i = reactive injection at node i.



FIRM LOAD	26,460
INTERRUPTIBLE	777
	27,237

	TOTAL	RESERVE	USED
HYDRO	26,938	2,375	24,563
MISC.	548	467	81
NPR	843	74	769
THERMA	2,000	176	1,824
TOTAL			27,237

(2000) 1976 transmission capacity based on economic loading

□ NET AREA REQUIREMENT
○ NET AREA SURPLUS
▷ THERMAL

(2) Number of new thermal units

NOTE: All numerical values are in megawatts

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FIG. II-2 TOPOLOGICAL MODEL USED FOR LONG-TERM TRANSMISSION SYSTEM PLANNING

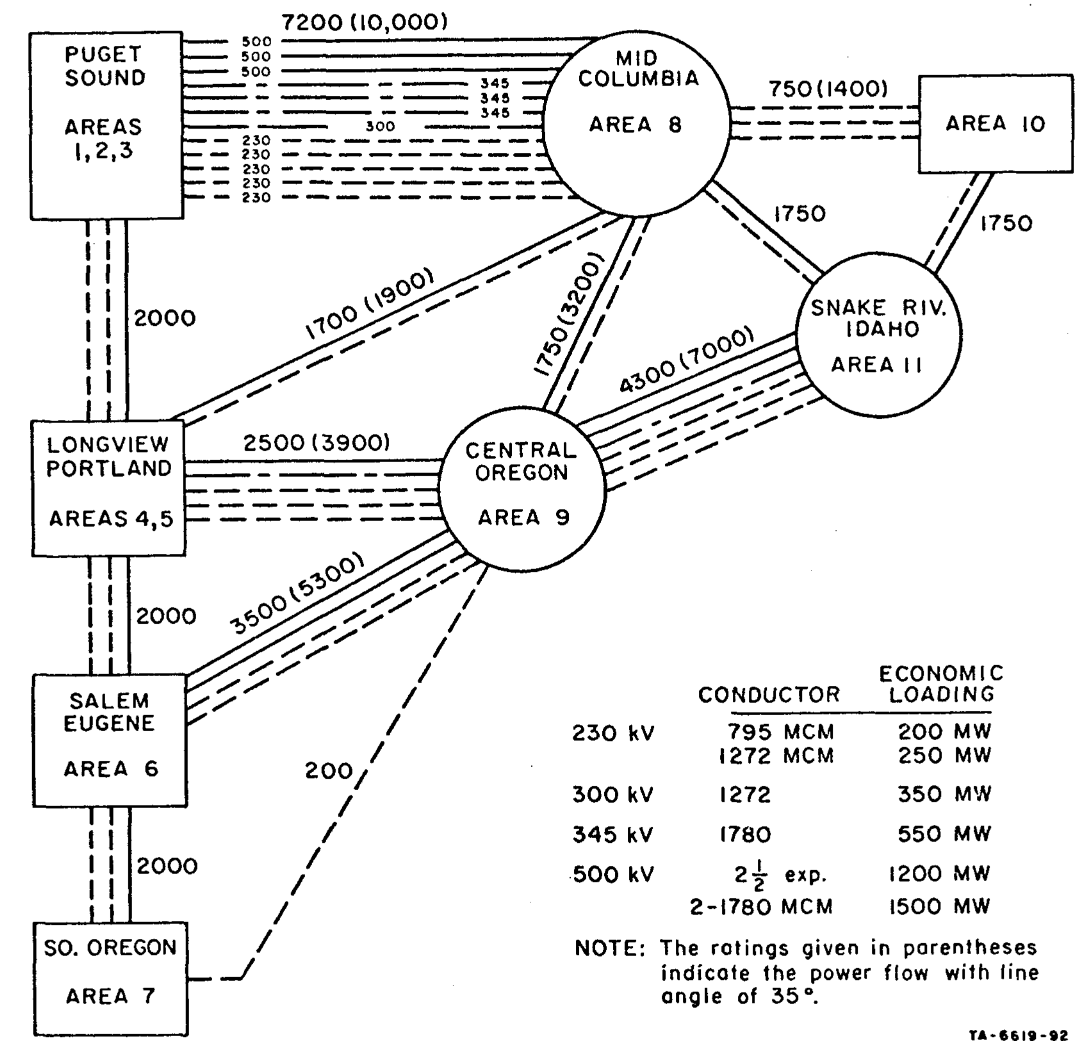

FIG. II-3 DETAILED COMPOSITION OF THE TOPOLOGICAL MODEL OF FIG. II-2

The parameters entering into the 2n Eqs. (II-2) are the resistances and reactances of the r branches. The branch flows and system losses are easily calculated, once V and θ are known.

For long-term planning purposes, all voltage and reactive power considerations have been omitted from the study. This is a simplifying assumption that is justified for two reasons: voltage control equipment costs much less than the lines, and the reduction in losses obtainable by a reactive flow optimization is relatively small. In the future, these voltage and reactive power considerations should be reentered into the planning process, since they affect the design of the network. In the absence of voltage and reactive power considerations, the set of equations g_2 can be omitted. For further simplicity, the g_1 set of the AC power flow equations can be linearized,[33] in which it becomes

$$\sum_{j \in K(i)} \frac{V_{ij}^2 X_{ij}}{Z_{ij}^2} \left(\theta_i - \theta_j\right) = \vartheta_i \qquad \text{(II-3)}$$

$$\begin{cases} i = 1, \ldots, n-1 \\ K(i) = \text{set of all nodes connected with node } i. \end{cases}$$

The system losses given by this linearized model are

$$L = \sum_{i,j} \frac{V_{ij}^2 R_{ij}}{Z_{ij}^2} \frac{\left(\theta_i - \theta_j\right)^2}{2} \qquad \text{(II-4)}$$

ij = all pairs of connected nodes.

Let us assume that the ratio X_{ij}/R_{ij} is large and the same for all lines. Therefore:

$$X_{ij} \cong Z_{ij} \quad ,$$

and we define

$$\mu = \frac{X_{ij}}{R_{ij}}$$

$$\gamma_{ij} = \frac{V_{ij}^2}{X_{ij}} \quad .$$

Equations (II-3) and (II-4) then become

$$\sum_{j \in K(i)} \gamma_{ij} \left(\theta_i - \theta_j\right) = \vartheta_i$$

$$\begin{cases} i = 1, \ldots, n-1 \\ K(i) = \text{set of all nodes connected with node } i \end{cases}$$

$$L = \frac{1}{2\mu} \sum_{i,j} \gamma_{ij} \left(\theta_i - \theta_j\right)^2 = \frac{1}{\mu} \sum_{k=1}^{r} \gamma_k \left(\theta_i - \theta_j\right)^2 \quad .$$

2. The Investment Cost Model

In a discussion of the effects of investment cost upon network planning, two factors need to be considered, namely

(1) The price of a transmission line as a function of its capacity, length, amount of series compensation, etc.

(2) The accounting procedures that specify the yearly cost of owning a line and keep track of its age and value at the end of the planning period.

a. Transmission Line Price

In this study, it was assumed for simplicity that only four types of transmission lines were available and their cost per mile was invariant. The properties of each line type are given in Table II-1.*

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*Data obtained from the Bonneville Power Administration.

Table II-1

PROPERTIES OF LINE TYPES STUDIED

Type	Voltage (kV)	R (Ω/mile)	X (Ω/mile)	$Y = \frac{V^2}{X}$	Maximal Flow (MW)	Cost per Mile ($1000)
1	230	1.29×10^{-1}	8.120×10^{-1}	6.515×10^{4}	160	53
2	230	8.50×10^{-2}	7.997×10^{-1}	6.615×10^{4}	200	64
3	345	6.10×10^{-2}	8.030×10^{-1}	1.482×10^{5}	440	78
4	500	3.02×10^{-2}	5.706×10^{-1}	4.381×10^{5}	1200	123

The assumption that the cost per mile is independent of terrain does not constitute a restriction of the method, since terrain-dependent costs could be used.

The cost of substations was not considered in this part of the study. Substation costs, however, can be added easily.

Series compensation has become an accepted means for reducing the cost per unit capacity of EHV lines. The optimum amount of series compensation and the cost of optimally compensated transmission lines were considered in the study;[6] compensation, however, was omitted from the examples programmed to show feasibility of the method.

In a first approach, to be discussed in Sec. III, it was assumed that possible transmission lines to be added could be selected from a continuous capacity range. The resulting continuous model, relating cost per mile and resistance per mile, was found by least squares fitting[2] of the discrete data contained in Table II-1. The resulting model, together with the appropriate algebraic relations to be used in Sec. III, is shown in Figs. II-4 and II-5.

b. Accounting Model

An expansion schedule is a set of decisions about building certain lines at certain times. Disbursements that take place must be accounted for in the dynamic programming procedure.

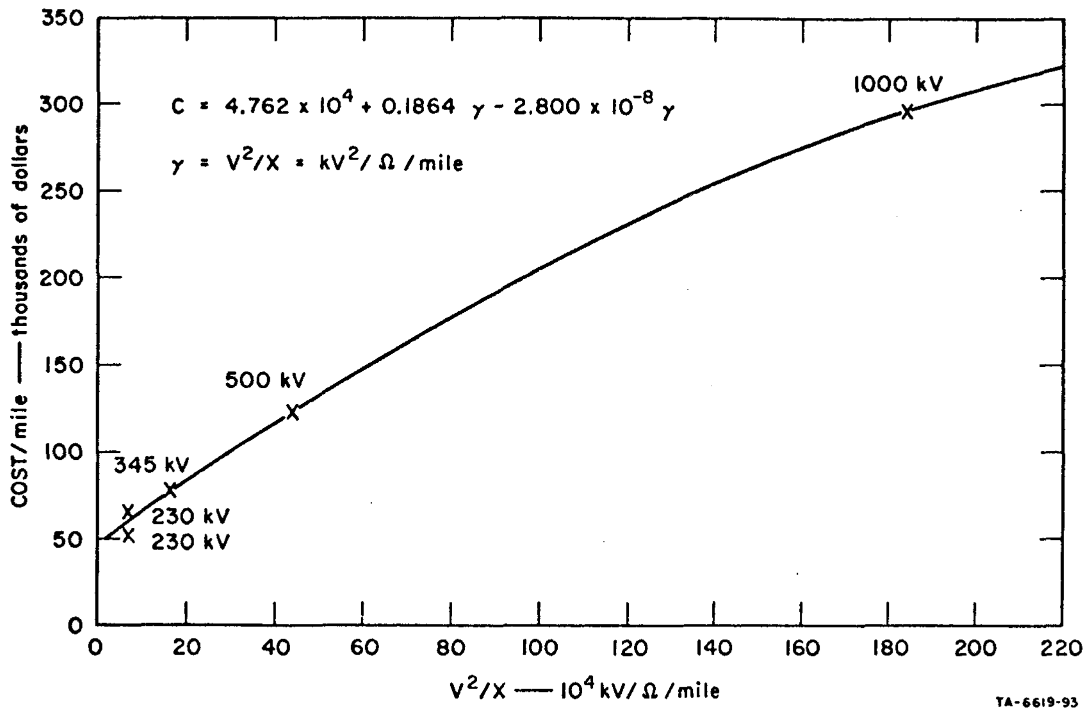

FIG. II-4 COST PER MILE AS A FUNCTION OF CAPACITY V^2/X

A new line is never fully paid for in cash at the time of its construction. Instead, we assume that a series of equal installments are due every year during the lifetime of the line (e.g., to the bank that finances the expansion). These installments cover both depreciation expenses and interest charges.

Let

t = time when the line is built

c = cash value of the line at time t

Q = lifetime of the line (usually 50 years)

ξ = interest rate

ν = annual payment factor corresponding to ξ and Q

T = end of the planning period.

In the dynamic programming procedure, we only consider the expenditures that take place during the planning period, i.e., only

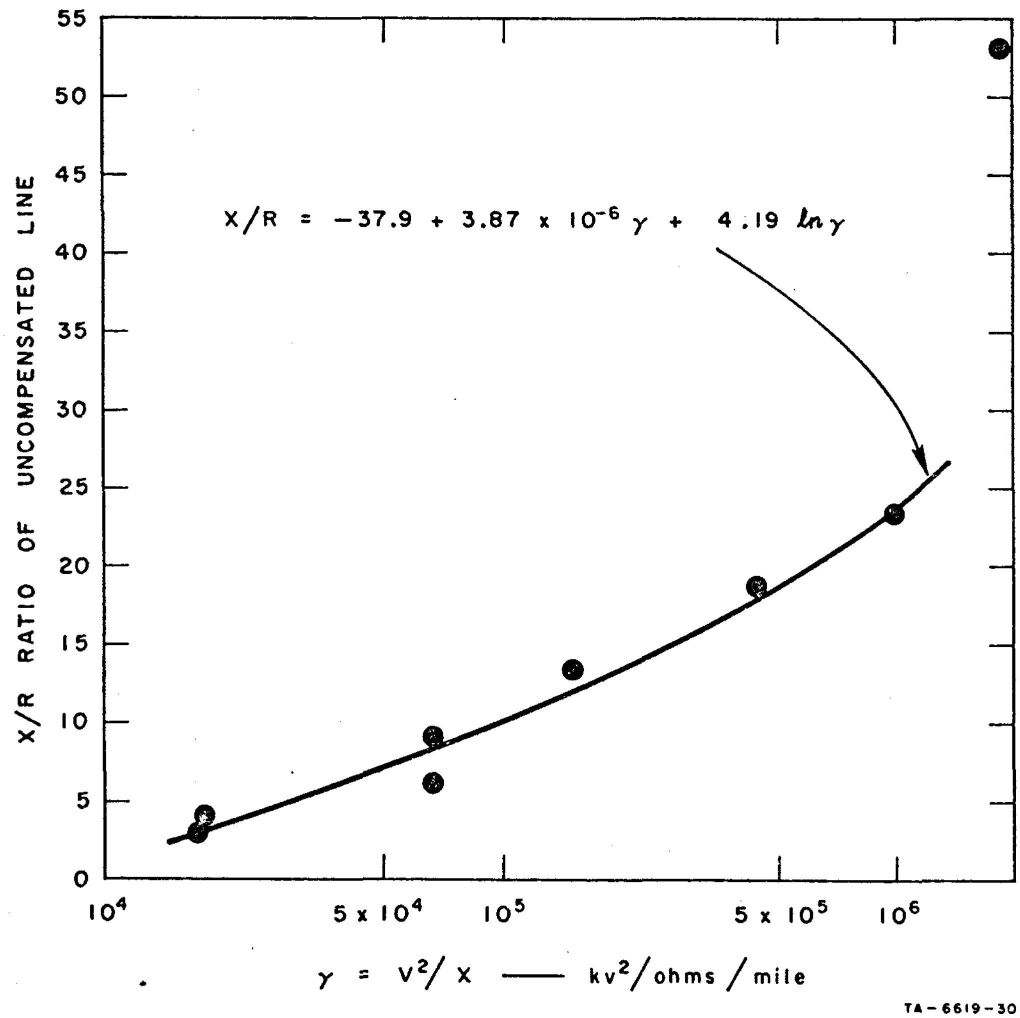

FIG. II-5 RESISTANCE PER MILE AS A FUNCTION OF CAPACITY V^2/X

the first T - t + 1 installments. The "investment cost" corresponding to the decision of building the line at time t is thus given by the formula:

$$C = \sum_{\alpha=t}^{\alpha=T} \frac{c\ \nu}{(1 + \xi)^{\alpha}} \quad .$$

The compounded value of the remaining installments represents the residual value of the line at the end of the planning period. An equivalent point of view is that the residual value of a line at the end of the planning period is to be recovered (e.g., in a sale to the next owner of the system), and thus must be subtracted from the initial construction cost.

The installments corresponding to lines built before the beginning of the planning period are <u>not</u> accounted for in the dynamic programming procedure, although these disbursements actually take place during the planning period. In fact, they correspond to a previously taken decision, and these commitments cannot be affected by any decision taken during the planning period (even if one of these lines is torn down, the commitment to pay the installments still holds).

A special attention must be paid to the case in which a line has to be torn down before the end of its useful life. This can be caused by geographical constraints (Sec. II-B-6). The cost of demolition is usually equal to the salvage value of the materials of which the line is constructed. However, the residual "resale" value of the deleted line will not be recovered at the end of the planning period. Therefore, a penalty equal to this "resale" value must be incurred.

In the computational examples presented in this report, geographical constraints have been left aside. Consequently, no line is ever torn down.

3. The Load Model

The expansion schedule of the network obviously depends on the location and magnitude of production and demand, not only at the present time, but for every year of the planning period. Network reliability is checked most conveniently during extreme conditions, such as winter peak and summer peak, whereas losses are integrated over the whole year.

Both peak branch flows and losses depend on the injections

$$\mathcal{J}_i = P_i - C_i \quad . \tag{II-5}$$

It is therefore necessary to know the productions $P_i(t)$ and the demand $C_i(t)$ at all nodes i and at all times t during the planning period. In accordance with current practice, it was assumed that the injections $\mathfrak{J}_i(t)$ corresponding to the peak conditions to be checked had been obtained from the generation planning and load forecasting departments. This, obviously, is a simplifying assumption, since a good expansion schedule for generation must take into account projected transmission facilities, and future load is a random rather than deterministic quantity.

To calculate yearly losses in a given network, it is necessary to know either the RMS branch power flows (or load factors) or, preferably, the injection matrix Φ defined as[2]

$$\Phi = \frac{1}{\mathfrak{T}} \int_0^{\mathfrak{T}} \mathcal{J}(t)\ \mathcal{J}'(t)\ dt \quad , \tag{II-6}$$

where the planning interval $\mathfrak{T}$ is customarily taken to be one year.

If the demands $C_i(t)$, were known accurately for all the planning intervals contained in the planning period and if the exact characteristics of future planned generation had been given, then the injections $\mathcal{J}_i(t)$ could all be determined from an optimum dispatching program[28] for each network expansion alternative. In the absence of such precise information, it is best to estimate the load factors or the elements of the injection matrix from past operating data and to assume that these estimates will not change significantly in the future.

4. The Reliability Model

In accordance with current practice in Europe[27,32] and probable standards to be imposed by the FPC[31] in the U.S., a network is considered to be reliable if the loss of one line, possibly the biggest, in any branch will not cause any excessive branch flows or transient stability problems that would trigger the loss of additional equipments. This definition of reliability was used throughout the study, with the following exceptions:

(1) To establish a coarse, continuous expansion schedule of the network (see Sec. III), an upper bound $\bar{\psi}_k$ of the order of ±36° was imposed upon the angular difference

$$\psi_k \triangleq \theta_i - \theta_j$$

across branch k, originating at i and terminating at j. $\bar{\psi}_k$ is the maximum allowable angular difference under no-fault conditions; it is justified purely on historical grounds, since past experience seems to indicate that systems operating with $|\psi_k| \leq 36°$ under normal conditions do not experience overloads or transient instabilities subsequent to a single fault.

(2) To penalize a system design that is barely capable of surviving a single fault, procedures for calculating the expectation of lost load (in MWH/year) at each node were established.[7,10]

(3) To ensure that a system designed to survive a single failure will not experience cascading degradation as a result of a subsequent and uncorrelated second fault, the emergency load curtailment procedures worked out by the authors

in Ref. 35 were further refined, and possible connections with under-frequency protections were explored.[10]

5. Outline of the Optimization Approach

Dynamic programming has been shown to be the best suited method here. However, a straightforward application of it is ruled out because of the high dimensionality of the problem.

In similar cases of high dimensionality, a method of successive approximations in dynamic programming has proved to be successful.[36,37] The principle of it is first to find an approximate solution that satisfies all the constraints but is not yet optimal with respect to total cost. This starting solution is called the "nominal" expansion schedule.

The nominal schedule is, in fact, a set of r expansion schedules, one for each branch of the system. In the next phase, one of these branch-expansion schedules is optimized through a dynamic programming procedure, while the (r - 1) often are left temporarily unmodified. The process is then repeated throughout all the branches until convergence occurs.

The above steps are shown on the general flow chart of Fig. II-6.

E. Model for Subsystem Planning in the Presence of Uncertainty

The aim again is to find the long-term expansion that minimizes the sum of the discounted investment and operating costs, subject to reliability constraints. Though the planning of a subsystem seems easier than the planning of the overall system, because of the smaller size of the network, this simplicity is only apparent, for two reasons:

(1) The details of each subsystem (e.g., the type, the rating, the location of lines, transformers, and switchgears) are all variables of the problem. Therefore the reduction in problem dimension may be fictitious.

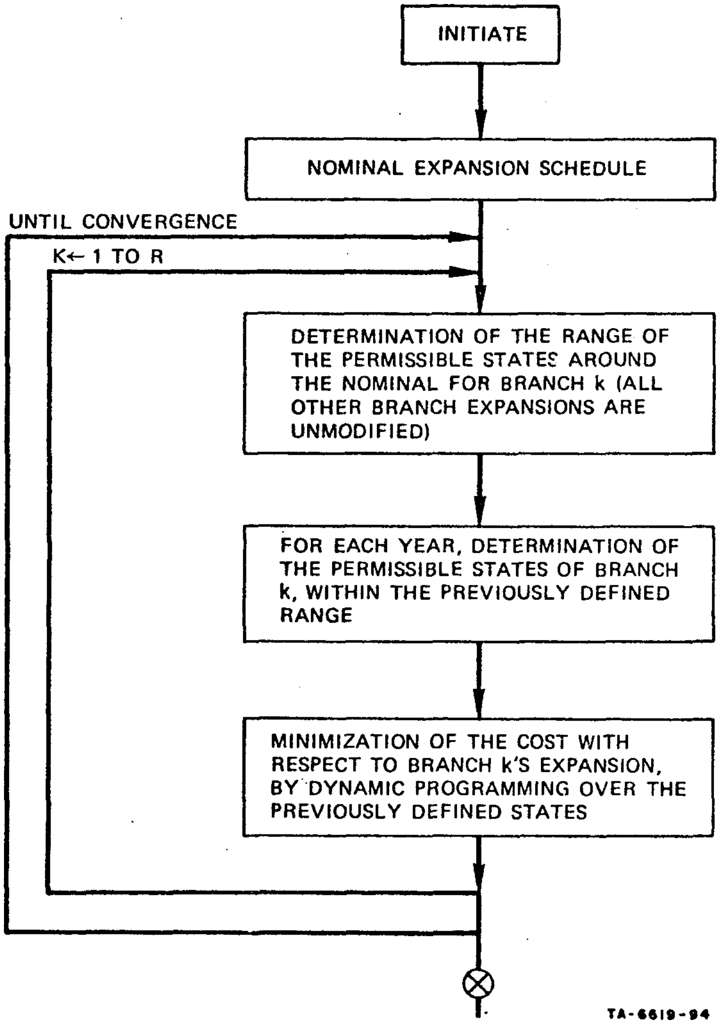

FIG. II-6 GENERAL FLOW CHART

(2) The uncertainty of future exogenous requirements (e.g., the loads) should be taken into account. Because of the wide variations from the estimates in load forecasts caused by local events (e.g., an industrial plant moving in) a load cannot be described as confidently by its expected value. Load probability distributions should be considered to obtain meaningful results.

Thus, the planner is faced with the problem of making a good decision though his information about the future is not very precise. The best the decision maker can do is to develop flexible decision rules, improving them as his knowledge of the future improves. The main output in this study has been to obtain feedback solutions (decision policies); the experience of the laboratory in control theory was used extensively to this end. Dynamic programming, properly formulated, appeared to be a most efficient tool. This implied that suitable models for the following had to be developed:

- The network
- The investment costs
- The operating costs
- The load.

1. The Network Model

In our formulation the network is always considered as a whole. Its configuration is referred to by an identification number. Two configurations differing by one or more pieces of equipment have a difference identification number. The fundamental state variable for subsystem expansion planning is the identification number of a complete network configuration that is reasonable from the point of view of electrical engineering. A particular configuration contains all the information required to:

- Actually build the system

- Determine its performance (for instance, the yearly power losses as a function of load).

The planner thus can choose from among a finite number of configurations proposed to him by electrical engineers.

With this formulation, where the individual network components are not treated as independent decision variables, the necessary reduction in the problem dimension has been achieved.

It is believed that computer aids could be used very efficiently in the design of the feasible alternatives. It appears to have become a very effective tool for electric circuit design; its application to power circuit design does not appear to have been fully exploited to date.

In order to determine the performance of a given configuration, in the face of given exogenous requirements, the accurate mathematical model described by Eq. (II-2) in Sec. II-D-1 could be used. However, for the reasons given above, the "DC model" described by Eq. (II-3) was used in the subsystem expansion program.

The reliability constraint was expressed as in the overall expansion problem, namely:

A configuration is said to be reliable if the connected loads can be satisfied without any overloading when the most critical equipment fails.

2. The Investment Cost Model

The investment cost is here the sum of the transition costs the planner must pay when he decides to abandon a configuration in favor of another one. Its components are

(1) Cost of the new equipment installed

(2) Value of the old equipment removed

(3) Cost of installation of the new equipment and removal of the old equipment.

We assumed that the full cash price associated with a transition, properly discounted, was paid at the time of the transition.

3. The Operating Cost Model

This is the cost of the losses and maintenance incurred for a given configuration during a planning subperiod.

a. Losses

For given loads and a given configuration the losses are easily computed, using the simplified "DC model" described above, with Eq. (II-4). The cost of losses was assumed to be constant ($2/MWH).

b. Maintenance Costs

These costs are not included at this time, but could easily be incorporated in the program.

4. The Load Model

Uncertainty about future loads is characterized by two aspects:

(1) There exists zero-mean deviation about a predictable trend. For instance, the variation of load due to weather conditions averages out over time. An averaging effect may also be observed geographically as the load may increase in one area and decrease in another. Though it would be more satisfactory to describe this type of uncertainty by a probability distribution, the mean value of the forecast can often be used meaningfully. Such a forecast was used in overall system planning.

(2) There exists a possibility for sudden, discrete, local variations, occurring at sparse and discrete intervals of time so that no averaging effect is observed over time. For instance, in a subsystem, a large load may be encountered suddenly, because of an industrial plant being moved in. Such a load will not be counterweighted by a load decrease somewhere else in the network.

In order to simplify the computations, it was assumed that uncertainty of type (1) could be represented by a uniform distribution. Furthermore, the deviations every year were supposed to be uncorrelated. Uncertainty of type (2) was approximated by a sequence of uncorrelated random loads occurring every year, according to reasonable statistics.

The resulting model, used throughout the study is formulated mathematically in Sec. X and in Ref. 4.

It is not claimed at this time that this model is the one that should be retained. To obtain an acceptable statistical model for demand prediction, past records must be analyzed, and future technological and sociological changes must be correctly interpreted.

It was also assumed that uncertain future loads could be known perfectly some time in advance, for instance a year. Thus the planner is certain that an investment made at time t to satisfy the demand at time t + 1 will not be obsolete then, because he *knows* the demand at time t + 1.

It has been shown in Refs. 4 and 9 that uncertainty about the future can be taken into account simply by means of the *conditional probabilities* that load will be d_t at time t, given that it was d_{t-1} at time t - 1. These are the only statistics required for the optimization.

F. Generation Planning and Reliability

1. Introduction

Some of the problems of optimum long-term generation planning have been discussed in Refs. 11-17, 19, 21-24, 38. With reference to the optimum long-term transmission system planning discussed in the present report, the following three observations are in order:

(1) Many of the factors that enter into long-term generation planning, including the dependence of power system reliability upon it, are identical with the factors considered in long-term transmission system planning.

(2) In order for the complete power system to be planned optimally, it is necessary to consider the effects of generation upon transmission, and vice versa. This does not necessarily imply that planning procedures established independently for transmission and generation are of little value. It does imply, however, that independently established procedures must ultimately be coupled together by a suitable algorithm, most likely successive approximation in dynamic programming.

(3) Since the investment and operating costs of generation are usually much higher than those of transmission, the dollar savings obtainable by optimum procedures for generation planning should also be expected to be much higher.

2. Technical Discussion

The aim of an optimum generation planning procedure is to find the size, type, geographical location, and year of implementation of generation that minimizes the total investment and operating costs over the planning period for a given regional load, given the transmission system expansion schedule, and given reliability constraints. This procedure must take into account existing generation, the utilization of which may change in the future (for example, existing base hydro units may be used as peaking units in the future, when the base load is carried by nuclear units). As in transmission system planning, the following factors affect generation planning:

(1) The capital and operating costs

(2) The reliability constraint that requires that the load will be satisfied in the event of one or more unscheduled outages.

In addition to these, the following factors must also be considered:

(3) To assess operating costs for each investment alternative, a fixed dispatching policy (which should preferably be the optimum policy) must be assumed. This implies that the planning procedure includes an optimum dispatching procedure, which need not be extremely accurate, but must take into consideration the existence of hydro units.

(4) To assess the reserve capacity required for reliability, the existence of a maintenance policy (preferably the optimum maintenance policy) must be assumed for each investment alternative.

3. Approach to Optimum Generation Planning

Considering the similarities and dissimilarities between transmission and generation planning, we would suggest the following approach to generation planning, the ultimate aim being the simultaneous optimum expansion of generation and transmission:

Step 1: Optimum expansion schedule of generating units of different types and sizes, but excluding hydro and pumped storage and without transmission. The aim would be to understand the following factors:

(1) Optimum mix of generation (i.e., peaking vs. base) and optimum dispatching

(2) Economy of size vs. overinvestment required by reliability

(3) Optimum maintenance.

Step 2: Optimum expansion schedule of generating units of different types and sizes, but excluding hydro and pumped storage, and with a given transmission system whose expansion schedule is known. The aim would be to understand the following factors:

(1) Optimum location of generation

(2) Effect of the reliability constraint when both transmission and generation failures are considered

(3) Optimum dispatching and maintenance, with transmission included.

Step 3: Optimum expansion schedule of generating units of different types and sizes, including hydro, with a given transmission system whose expansion schedule is known. The aim would be to understand the following factors:

(1) Optimum dispatching

(2) Optimum maintenance when hydro reserve capacity is available.

Step 4: Optimum simultaneous expansion schedules for generation and transmission. The aim would be to develop computational procedures that produce an overall optimum by perturbing the transmission schedule obtained in the present project and the generation schedule suggested in Step 3 above.

III THE CONTINUOUS NOMINAL

A. Introduction

It was shown in Sec. II that the only rigorous approach to long-term transmission system expansion planning is dynamic programming. In view of the high dimensionality of the optimization problem, it is necessary to apply the dynamic programming algorithm sequentially, that is, to use successive approximations.[36,37] A prerequisite to the application of the successive approximation technique is that the optimum solution--the schedule of branch capacities as a function of time--be known approximately. This approximate solution, which should be relatively easy to obtain, is referred to as the nominal solution in what follows.

Finding an optimal expansion schedule is a variational problem, i.e., an optimization with respect to a set of functions. The high dimensionality of this problem is greatly reduced by breaking the total cost optimization into a sequence of separate optimizations, one for every year, assuming the results just obtained for the preceding year. This is the way the determination of the nominal is performed. The resulting schedule will be optimal for each successive year, but probably not optimal for the whole planning period. It will just serve as a starting solution for the dynamic programming procedure that will follow.

One way to obtain this nominal is to assume that continuous capacity additions can be made to each branch as time proceeds. The resulting solution, which is the subject of this section, is referred to as the continuous nominal.

Another possibility is to monitor the reliability constraint each year--see Sec. II-B-2--and to add lines whenever required for the increase in loads. This approach, which was actually used in the optimization algorithm, will be discussed in Sec. VI. The resulting nominal is referred to as the discrete nominal.

Apart from the requirement for a nominal to initiate the sequential dynamic programming algorithm, it is useful for coarse long-term planning studies to know approximately how the capacity of each branch in the system should increase with time. An efficient program capable of generating this nominal is hence a valuable analysis tool in its own right.

The development of such a program was pursued at the beginning of the project; this work was documented in detail in Refs. 2 and 3. An experimental program based on the penalty function method[28] was established; although it works, it is very slow, because of the convergence difficulties we encountered. After trying various known modifications to improve the convergence properties of this program, we concluded that the penalty function method, which was shown to work well in Ref. 28, was not appropriate here, and that a combination of gradient procedures and linear programming had to be developed. Since the primary aim of the project was to develop optimum techniques for transmission system planning, this improvement has not been completed at this time. In addition, the optimization work performed subsequently pointed out two important weaknesses of the continuous nominal, which further reduced its importance in the framework of the overall project. The first weakness relates to the reliability constraint, to be discussed in Sec. V. The second weakness relates to economy of size, the accurate consideration of which requires that a branch be defined not only by its capacity, but also by its composition.

B. Formulation of the Continuous Nominal Problem

1. General

The continuous nominal is obtained by minimizing in each consecutive year t the sum of the yearly capital cost C and the yearly cost of losses E. Since both C and E are functions of the transmission system design parameters p, the optimality condition to be satisfied in each year t is simply

$$\frac{\partial C}{\partial p} + \frac{\partial E}{\partial p} = 0 \qquad \text{all } p \text{ and } t \quad . \qquad \text{(III-1)}$$

The simple optimality condition of Eq. (III-1) is written down only to indicate the principle of the method.

2. The Design Parameter p

For the reasons indicated in Sec. II, the branch capacity

$$\gamma = \frac{V^2 R}{X^2} \cong \frac{V^2}{X} \qquad \text{MW} \quad , \qquad \text{(III-2)}$$

was chosen as the basic design parameter. For given injections $\mathcal{J}$ and given γ, the voltage phase angles θ can be readily computed and related, at least approximately, to the reliability properties of the system.

3. The Capital Cost

The capital cost C of the branch is approximately related to the capacity γ, as shown in Fig. II-4. The least-squares fit used for this approximation has been explained in Ref. 2. This relation assumes that the branch of capacity γ consists of a single line.

4. The Cost of Losses

The cost of losses depends on the parameter

$$\frac{V^2 R}{Z^2} \cong \frac{V^2 R}{X^2} \quad ,$$

which is related to γ as shown in Fig. II-5. Again, this relation assumes that the branch of capacity γ consists of a single line. In terms of this parameter, the instantaneous system losses are given by the equation

$$L = \frac{1}{2} \sum_k \frac{V^2 R}{Z^2} \left(\theta_i - \theta_j\right)^2 \qquad \text{MW} \quad , \qquad \text{(III-3)}$$

where the index k refers to the branch connecting nodes i and j. Since the θ_i vary throughout the year with time of day, season, etc., it is necessary to compute the yearly losses $\bar{L}$ in MWH. These yearly losses are given by

$$\bar{L}(\gamma) = \frac{1}{2\mu} \int_0^{1\ \text{year}} \sum_{i,j} \gamma_{ij} \left(\theta_i - \theta_j\right)^2 dt \quad , \qquad \text{(III-4)}$$

where $\mu = X/R$. (This ratio is assumed to be the same in all branches.)

5. Total Yearly Cost

The total yearly cost F in dollars is the sum of the yearly capital cost

$$D = \nu C = D(\gamma) \quad , \qquad \text{(III-5)}$$

where ν is the discounting factor--of the order of 0.05 and the yearly cost of losses

$$E = \eta \bar{L} = E(\theta, \gamma) \qquad \text{(III-6)}$$

where η is the cost of lost energy--here assumed to be of the order of \$2/MWH.

Thus

$$F = D(\gamma) + E(\theta, \gamma) = F(\theta, \gamma) \quad . \qquad \text{(III-7)}$$

6. The Reliability Constraint

Since branch composition is deliberately omitted from the continuous nominal, the reliability constraint can only be stated under the form

$$-\bar{\psi}_k \leq \theta_i - \theta_j \triangleq \psi_k \leq \bar{\psi}_k \qquad \text{(III-8)}$$

where $\bar{\psi}_k$ is usually of the order of 36°.

7. The Equality Constraints

The variables $\mathcal{J}$, θ, and γ are related by the power-flow equations, which can be summarized by the set of n algebraic equations*

$$g\ (\theta,\ \gamma,\ \mathcal{J}) = 0 \quad . \tag{III-9}$$

Since the injection vector $\mathcal{J}$ is a known function of time, it can be omitted from Eq. (III-9), which is then rewritten as

$$g\ (\theta,\ \gamma) = 0 \quad . \tag{III-10}$$

8. The Minimization Problem

With the definitions and functional relations given above, the continuous nominal is obtained each year by solving the classical nonlinear programming problem below.

$$\min_{\gamma} F\ (\theta,\ \gamma) \quad . \tag{III-11}$$

Subject to the equality constraints

$$g\ (\theta,\ \gamma) = 0 \tag{III-10}$$

and the inequality constraints

$$-\bar{\psi}_k \le \theta_i - \theta_j \le \bar{\psi}_k \quad . \tag{III-8}$$

9. Output

For uniformly increasing injections, that is

$$\mathcal{J}_i\ (t + 1) \ge \mathcal{J}_i\ (t) \qquad \text{all i and t} \tag{III-12}$$

---

*If the linear approximation of Eq. (II-3) is used, one of these n equations is redundant and can be omitted.

and assuming that initially no system characterized by the design parameter $\gamma(0)$ exists, the minimization problem defined by Eqs. (III-8), (III-10), and (III-11) should give in year year t the least costly transmission system $\gamma(t)$ capable of satisfying the reliability constraint of Eq. (III-8).

For a particular branch k, the optimum capacity variation $\gamma_k(t)$ would be as shown in Fig. III-1.

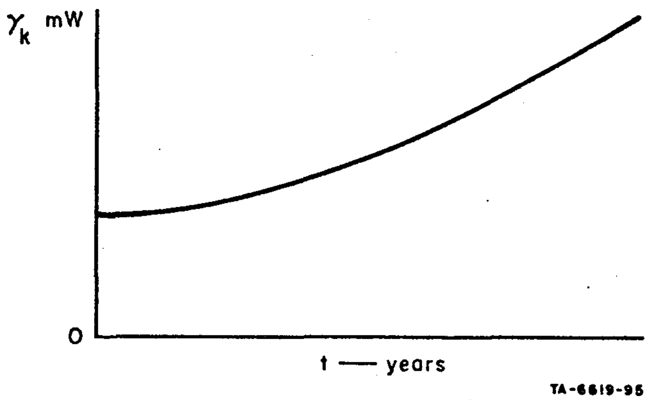

FIG. III-1 OPTIMUM CONTINUOUS CAPACITY VARIATION FOR BRANCH k

10. Additional Constraints

Since customarily a system characterized by $\gamma(0)$ is already in existence, one may impose the additional constraint

$$\gamma(t) \geq \gamma(0) \quad , \tag{III-13}$$

which states that no lines will be removed. More generally, if the injections are not uniformly increasing, one may impose that

$$\gamma(t + 1) \geq \gamma(t) \quad \text{all } t \quad . \tag{III-14}$$

On the other hand, there may be geographical constraints that impose an upper bound on transmission capacity in a branch k. This can be stated as follows:

$$\gamma_k(t) \leq \bar{\gamma} \quad \text{all } t \quad . \qquad \text{(III-15)}$$

Since the natural independent variable of the design optimization is γ, and not θ, these additional inequality constraints do not introduce any computational difficulties.

11. Note

It is clear that this approach only finds the optimum capacity of branches whose existence has been specified in the network model of Eq. (III-10). If the system planner suspects that a branch k that does not exist now may be economically justified later, this possibility must be included in the function g. If this branch reduces the total cost F, γ_k will come out positive; if not, it will remain zero.

C. Solution Methods

In view of the success reported in Ref. 28 with the penalty function method in accommodating indirect constraints, the constrained minimization problem of Eqs. (III-8), (III-10), and (III-11) was converted into the following minimization problem without inequality constraints on the dependent variables.

$$\min_{\gamma} [F(\gamma, \theta) + P(\theta)] = \min_{\gamma} J(\gamma, \theta) \qquad \text{(III-16)}$$

subject to

$$g(\theta, \gamma) = 0 \quad . \qquad \text{(III-10)}$$

The penalty function P (θ) chosen in Eq. (III-16) has been described in Ref. 3.

In principle, the unconstrained minimization of Eqs. (III-10) and (III-16) can be solved computationally by first- and second-order gradient procedures, both of which were tested in the course of the project.

1. First-Order Gradient Method

Using the standard computational approach, as discussed, for example, in Ref. 28, a first-order gradient program was established. Its convergence monitored by computing total cost J at each iteration, was found to be extremely slow, because of the shape of the cost function. Whereas in well-behaved optimization problems the solution follows the trajectory of Fig. III-2(a), we found the very slow trajectory shown in Fig. III-2(b).

Certain known remedies to the situation shown in Fig. III-2(b), such as PARTAN[28] or conjugate gradients, have not been applied to date.

2. Second-Order Gradient Method[28]

The second-order gradient method was also applied and found to cause difficulties, partly because of the concave shape of the capital cost function $C(\gamma)$, the second derivative of which is discontinuous at the constraint $\bar{\psi}$. These difficulties are discussed with greater detail in Ref. 3.

3. Elimination of the Penalty Function

Rather than applying PARTAN or conjugate gradients, we decided to omit the penalty function approach and to search for methods capable of solving the constrained minimization problem of Eqs. (III-8), (III-10), and (III-11) directly.

One such method combines the first-order gradient with linear programming; it was originally suggested to the Bonneville Power Administration in Ref. 39 and was then applied successfully in Ref. 25.

The essence of this method is to linearize the cost function F and the equality constraints g about some point (γ^o, θ^o) that satisfies Eq. (III-8). For sufficiently small deviations $\Delta\gamma$, $\Delta\theta$ about this point, one may thus write

$$\Delta F = F_\gamma \, \Delta\gamma + F_\theta \, \Delta\theta \qquad \text{(III-17)}$$

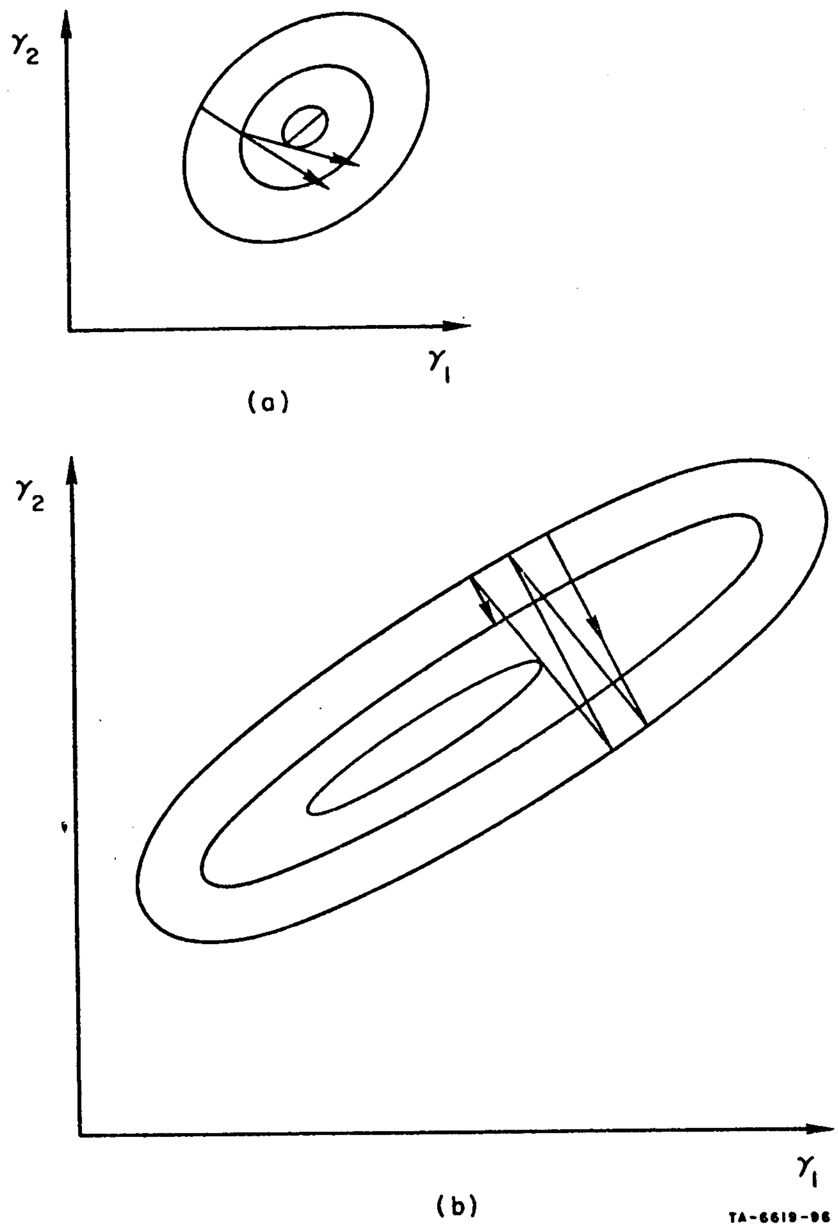

FIG. III-2 SOLUTION TRAJECTORY FOR FIRST-ORDER GRADIENT METHOD.
The closed contours are contours of equal cost J.

and

$$g_\theta \, \Delta\theta + g_\gamma \, \Delta\gamma = 0 \qquad . \tag{III-18}$$

It is desired to minimize ΔF subject to the equality constraint (III-18) and subject to two kinds of inequality constraints, namely

$$\Delta\underline{\psi}_k \leq \Delta(\theta_i - \theta_j) \leq \Delta\overline{\psi}_k \qquad , \tag{III-19}$$

where the bounds $\Delta\overline{\psi}_k$ and $\Delta\underline{\psi}_k$ are simply

$$\begin{aligned} \Delta\overline{\psi}_k &= \overline{\psi}_k - \left(\theta_i^o - \theta_j^o\right) \\ \Delta\underline{\psi}_k &= \left(\theta_i^o - \theta_j^o\right) + \overline{\psi}_k \qquad . \end{aligned} \tag{III-20}$$

The second kind of inequality constraint is of the form

$$\underline{\Delta\gamma} \leq \Delta\gamma \leq \Delta\overline{\gamma} \qquad , \tag{III-21}$$

and its purpose is simply to limit the search to a sufficiently small region in which the linearizations in Eqs. (III-17) and (III-18) hold.

The problem of minimizing ΔF subject to the constraints (III-18) and (III-19) is in the standard linear programming (L.P.) format. The L.P. algorithms would need to be applied about successive nominal points

$$\gamma^i + \Delta\gamma_*^i \qquad ,$$

where $\Delta\gamma_*^i$ is the optimum solution found in the i^{th} iteration.

Another less straightforward approach based on the Dantzig-Wolf algorithm has been proposed as part of a doctoral dissertation to the Operations Research Department of Stanford University by one of the authors.

Neither of these two approaches has been tested yet on electrical networks.

D. A Fundamental Shortcoming of the Continuous Nominal

In addition to the coarse model used for reliability, the continuous nominal, which does not consider branch composition but only total capacity, has a fundamental shortcoming related to the "economy of size" consideration.

Indeed, the cost of two lines $\gamma_k^{(1)}$ and $\gamma_k^{(2)}$ is always larger than the cost of a single line γ_k of equal capacity

$$\gamma_k = \gamma_k^{(1)} + \gamma_k^{(2)} \quad . \tag{III-22}$$

The inaccuracy in estimating cost may be quite substantial, as evidenced by the example shown in Fig. III-3.

In the course of the project, the expansion of a single branch was optimized using first the continuous nominal and dynamic programming that

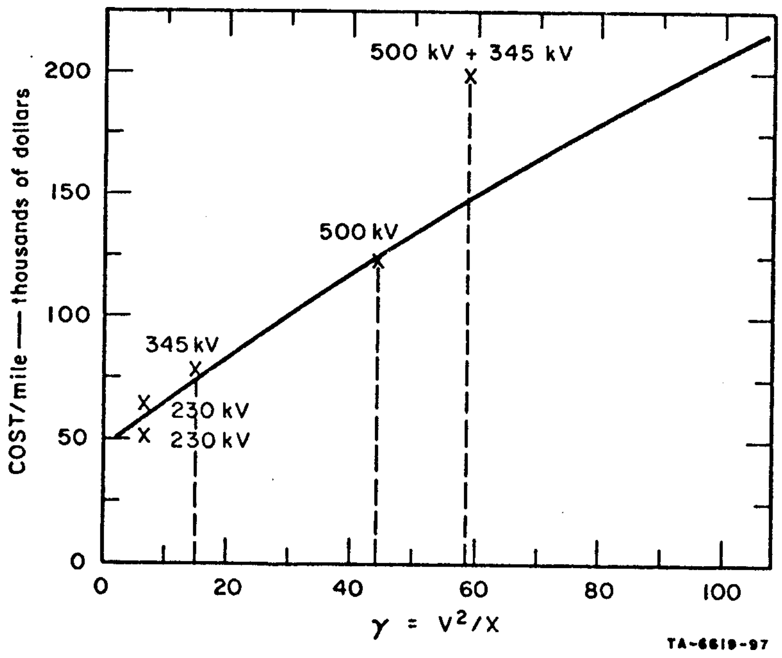

FIG. III-3 COST ERROR INTRODUCED BY TREATING TWO PARALLEL LINES AS A SINGLE LINE OF EQUIVALENT CAPACITY accommodates branch composition. The detailed assumptions pertaining to load growth, capital cost, and loss cost are given in Ref. 2. The resulting schedules are shown in Fig. III-4.

We expected the staircase trajectory of the rigorous dynamic programming solution to straddle the continuous nominal. The dynamic programming solution always calls for lower capacity because the true capital cost is higher than the capital cost assumed in the nominal solution.

E. Conclusions and Recommendations

The continuous nominal solution can be used to introduce sequential dynamic programming and to provide some preliminary planning information with a relatively minor computational effort.

The discrete nominal solution, to be discussed in Sec. VI was preferred to the continuous nominal to introduce sequential dynamic programming. The computational effort is also moderate, and the reliability and economy of size considerations are taken into account in a much more satisfactory manner. This discrete nominal, it is also felt, provides much better preliminary planning information, because it gives not only branch capacities, but also branch compositions.

It is for these reasons that the suggested improvements to the nominal solution were not completed in the course of the project, and it is for the same reasons that we do not recommend an urgent or significant effort to develop efficient nominal programs. On the other hand, considering that the major part of the work has been accomplished and would be of some value, we suggest a low-level effort, possibly in conjunction with a thesis project, to bring about the suggested improvements in such programs.

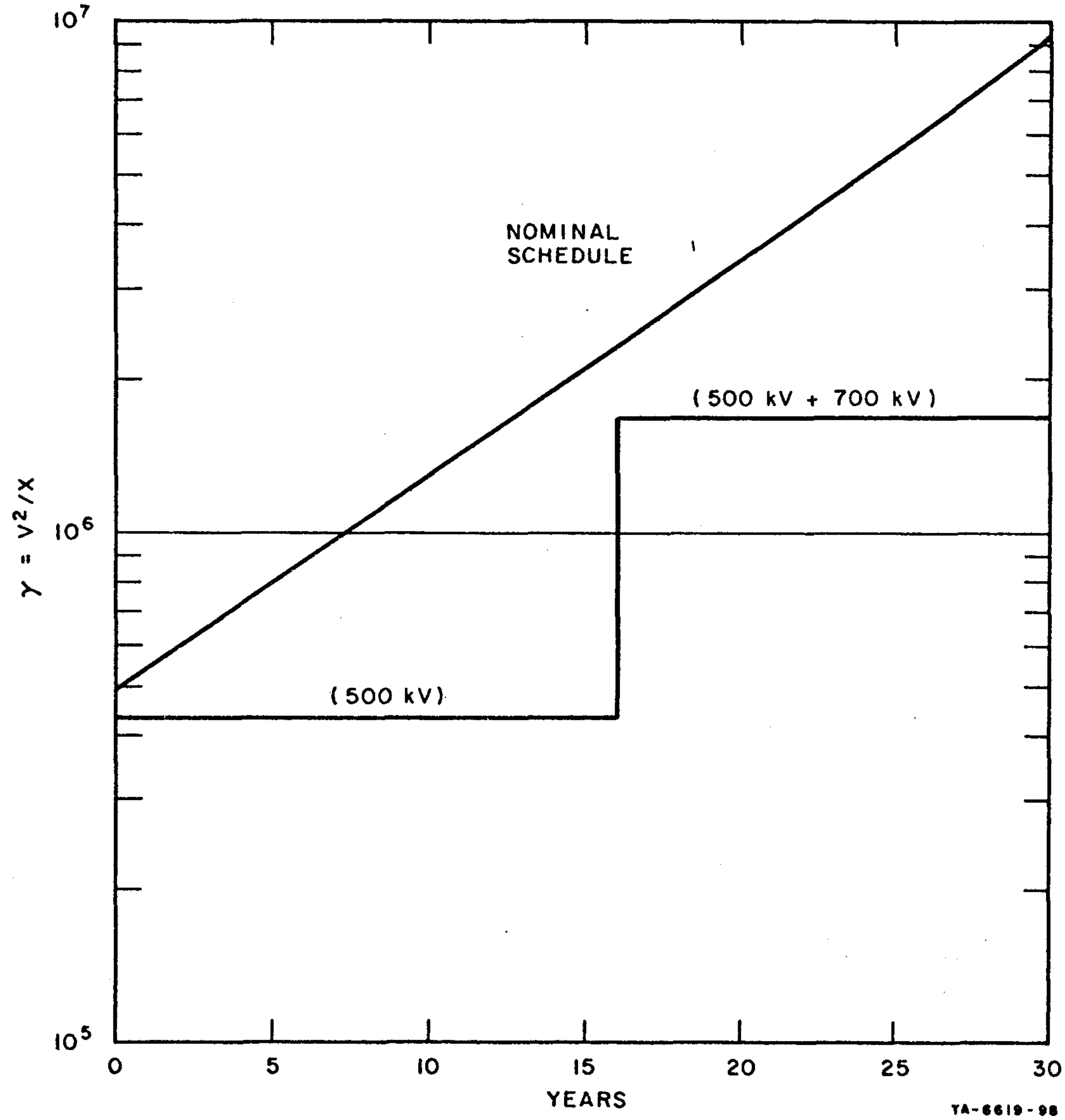

FIG. III-4 DYNAMIC PROGRAMMING SOLUTION AND CONTINUOUS NOMINAL SOLUTION

IV SERIES COMPENSATION

A. Introduction

It has become customary to increase the capacity of EHV transmission lines by means of series capacitors, which, in effect, reduce the inductive reactance of the line and therefore the voltage phase angle difference $\theta_i - \theta_j = \psi_k$.

Some of the technical and economic implications of series compensation have been discussed in the literature.[40,41] Since these studies were not aimed at the objectives of this project, a fairly complete and original analysis of series compensation was performed (reported in detail in Ref. 6).

The main purpose of this analysis was to determine to what extent series compensation would affect the technical and economic models needed for long-term transmission system expansion.

The main technical difficulties that series compensation could introduce are:

(1) Reduction of transmission capacity in the event of overcompensation.

(2) Loss of synchronizing torques, and hence transient instability, in the event of overcompensation.

(3) Excessive line currents caused by overcompensation.

The economical aspects of series compensation can be summarized as follows: Series compensation adds transmission capacity at a capital cost that is lower than that of an equivalent parallel line, but the transmission losses are higher, since the resistance of the compensated line remains unchanged. One would hence expect that there exists a most economical degree of compensation; this optimal compensation factor should be used, if the technical constraints discussed above permit this.

The question to be answered is illustrated in Fig. IV-1 in terms of the compensation factor s defined by

$$s = \frac{\text{capacitive reactance}}{\text{inductive line reactance}} = \frac{X_c}{X} \qquad \text{(IV-1)}$$

and the sum F of yearly capital and loss costs.

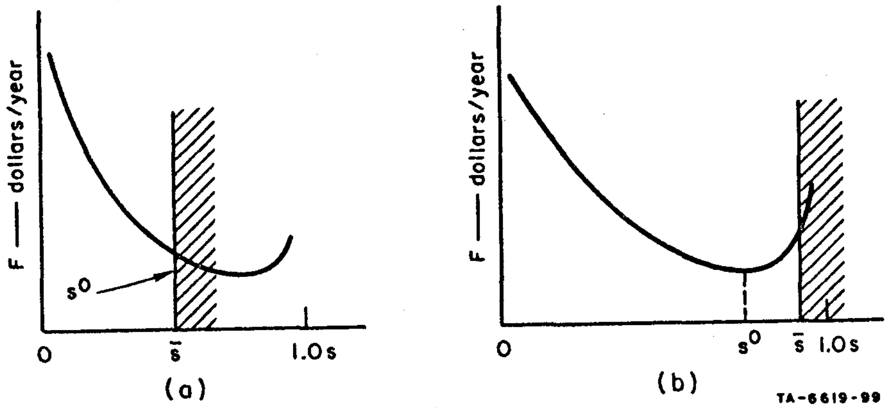

FIG. IV-1 YEARLY TOTAL COST F AS A FUNCTION OF THE COMPENSATION FACTOR s; IN (a) THE TECHNICAL CONSTRAINT $\bar{s}$ PREVENTS A PURELY ECONOMIC MINIMIZATION, WHEREAS IN (b), $\bar{s}$ ALLOWS IT

If the situation shown in Fig. IV-1(b) holds, then the effect of series compensation upon optimum expansion planning can be taken into account (subject to certain simplifying assumptions to be reviewed later) very simply, and the compensation factor s is removed from the optimization as an independent design parameter. For a detailed proof of this statement, see Sec. 2 of Ref. 6. If, on the other hand, the technical constraint $\bar{s}$ prevents a purely economic minimization, this simplification does not hold.

In our work, the two technical constraints were reviewed in detail. It was found that for EHV lines, where the ratio X/R is large, the technical constraint $\bar{s}$ lies well to the right of the economic minimum s^o. Hence, the optimum compensation factor is determined purely by economic considerations. The results of these technical and economic studies are summarized below.

B. Transmission Capacity as a Function of Compensation s

The results of this study are shown in Fig. IV-2, where the maximum steady-state power flow through a line is related to the compensation factor s. This relation is given for various X/R ratios corresponding to typical lines of different kV ratings. For instance, X/R = 3 corresponds to a 115 kV line, whereas X/R = 24 corresponds to a 700 kV line.

The main conclusion to be drawn from Fig. IV-2 is that the transmission capacity of EHV lines continues to increase unless compensation factors in excess of 75 percent are contemplated.

C. Synchronization Torque as a Function of Compensation s

The related question of the effect of series compensation upon the transient stability properties of a line connected on one end to a generator and on the other to an infinite bus was studied in detail in Ref. 6. The relative stability of this line was defined by the incremental torque $dT/d\psi$ where

T = torque

ψ = phase angle difference between line terminations.

It is desirable to have a large value of $dT/d\psi$, since this prevents excessive generator angle transients.

The results of this study are summarized in Fig. IV-3, where $dT/d\psi$ and ψ are related to the compensation factor s, the X/R ratio, and the line power flow.

The main conclusion to be drawn again is that the relative stability continues to increase unless compensation factors in excess of 75 percent are contemplated.

In some cases involving relatively short lines, the thermal constraint may limit the permissible compensation to a lower value.[6]

D. Economic Optimization

Finally, a program was written to minimize the total yearly cost F (capital and operating) for given line length and RMS power flow. The

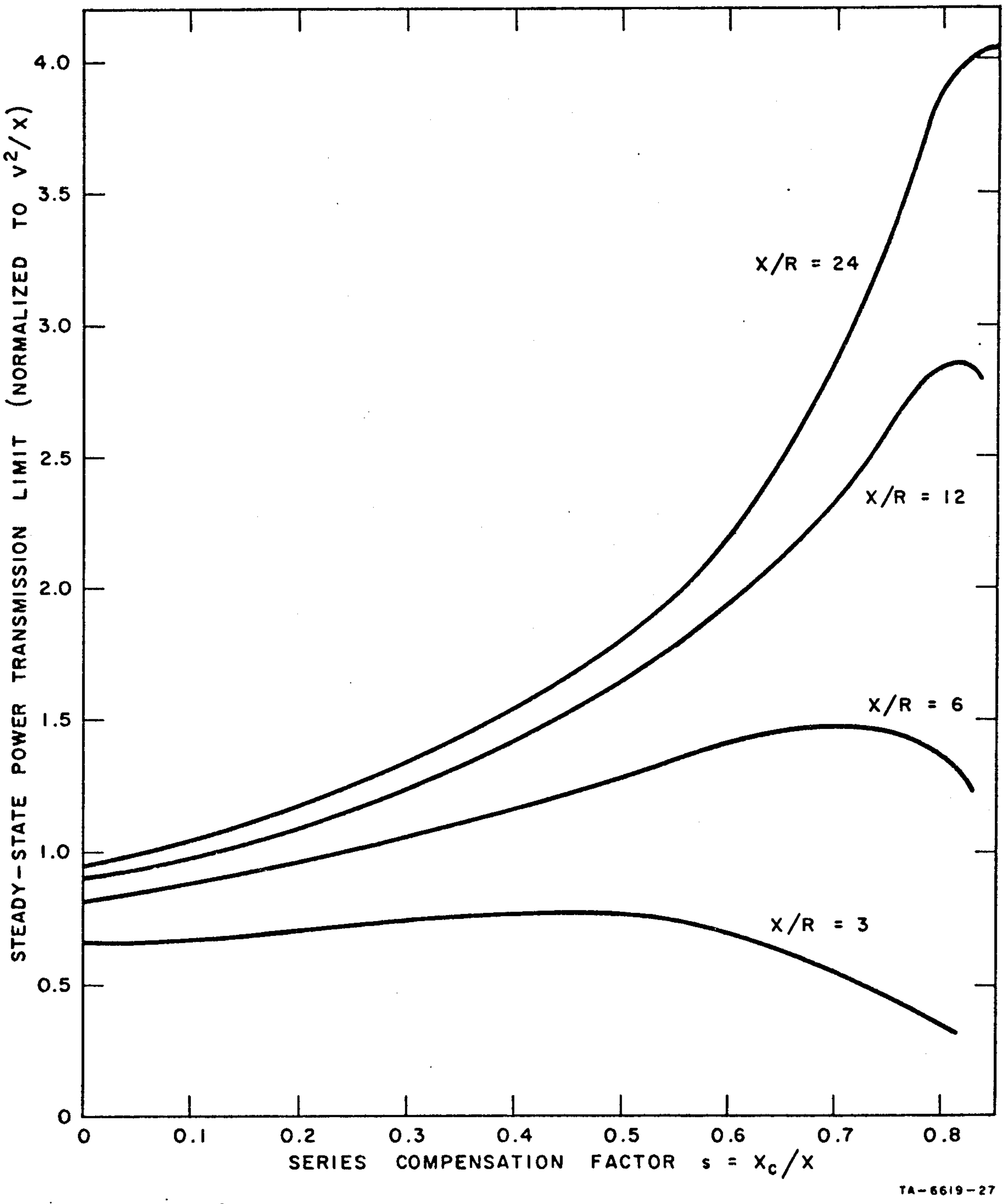

FIG. IV-2 STEADY-STATE REAL POWER TRANSMISSION LIMIT OF A SINGLE LINE AS A FUNCTION OF SERIES COMPENSATION AND X/R RATIO (Thermal Limits Omitted)

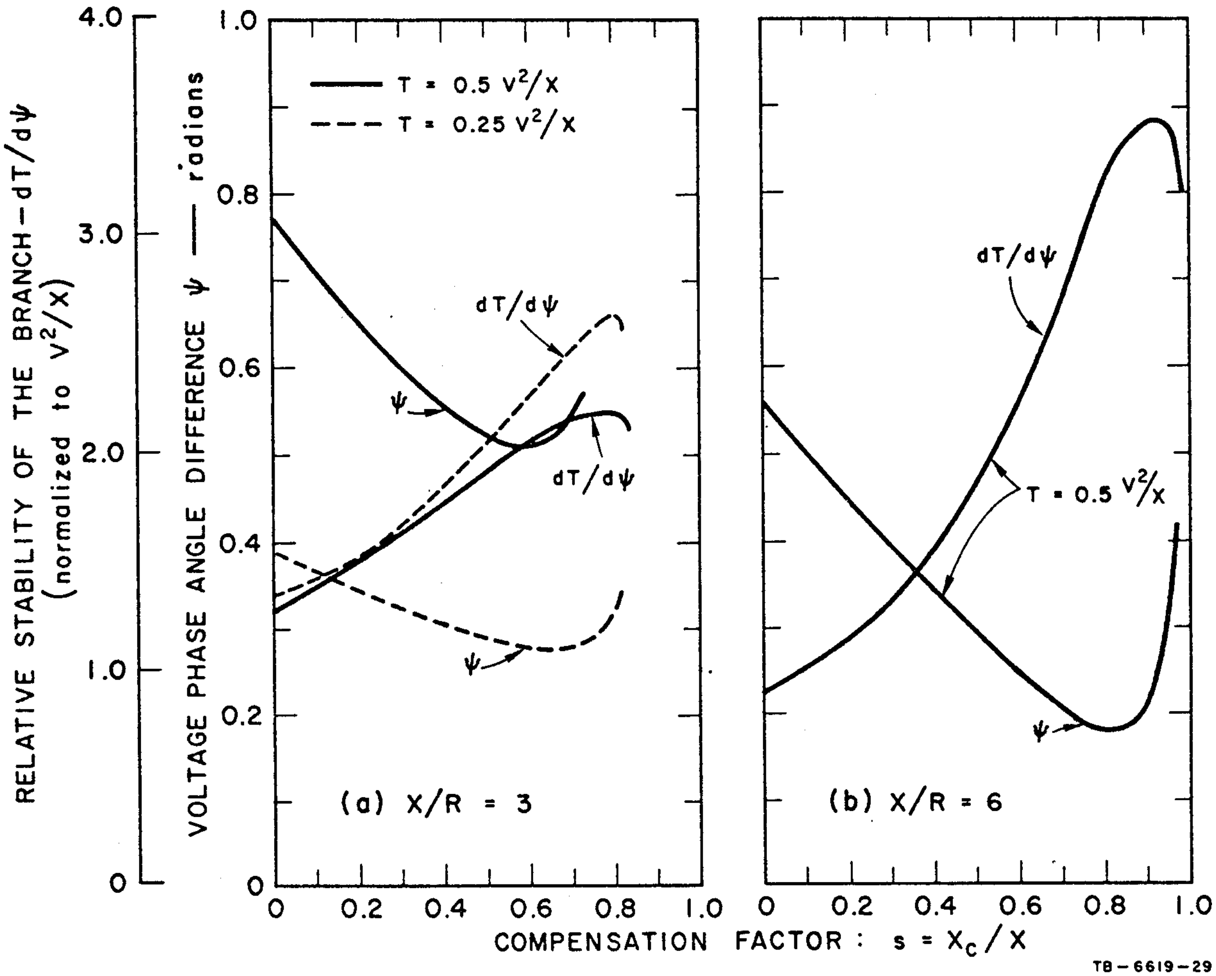

FIG. IV-3 VOLTAGE PHASE ANGLE DIFFERENCE ψ AND RELATIVE STABILITY $dT/d\psi$ AS FUNCTIONS OF COMPENSATION FACTORS s, STEADY-STATE POWER FLOW T, AND LINE X/R RATIO results provided by this program for an example involving a 100-mile-long line are given in the design chart of Fig. IV-4, which can be interpreted as follows:

Suppose that 1500 MW are to be transmitted at minimum yearly cost. This is accomplished by a line of capacity

$$y = \frac{V^2}{X(1 - s)} \qquad \text{(IV-2)}$$

of ≅12,000 MW with a compensation factor s of ≅0.6. The resulting phase angle difference ψ is ≅0.14 radians and the yearly cost is ≅\$2 million/year. From Eq. (IV-2), the capacity $\gamma = V^2/X$ of the uncompensated line is found to be 4800 MW.

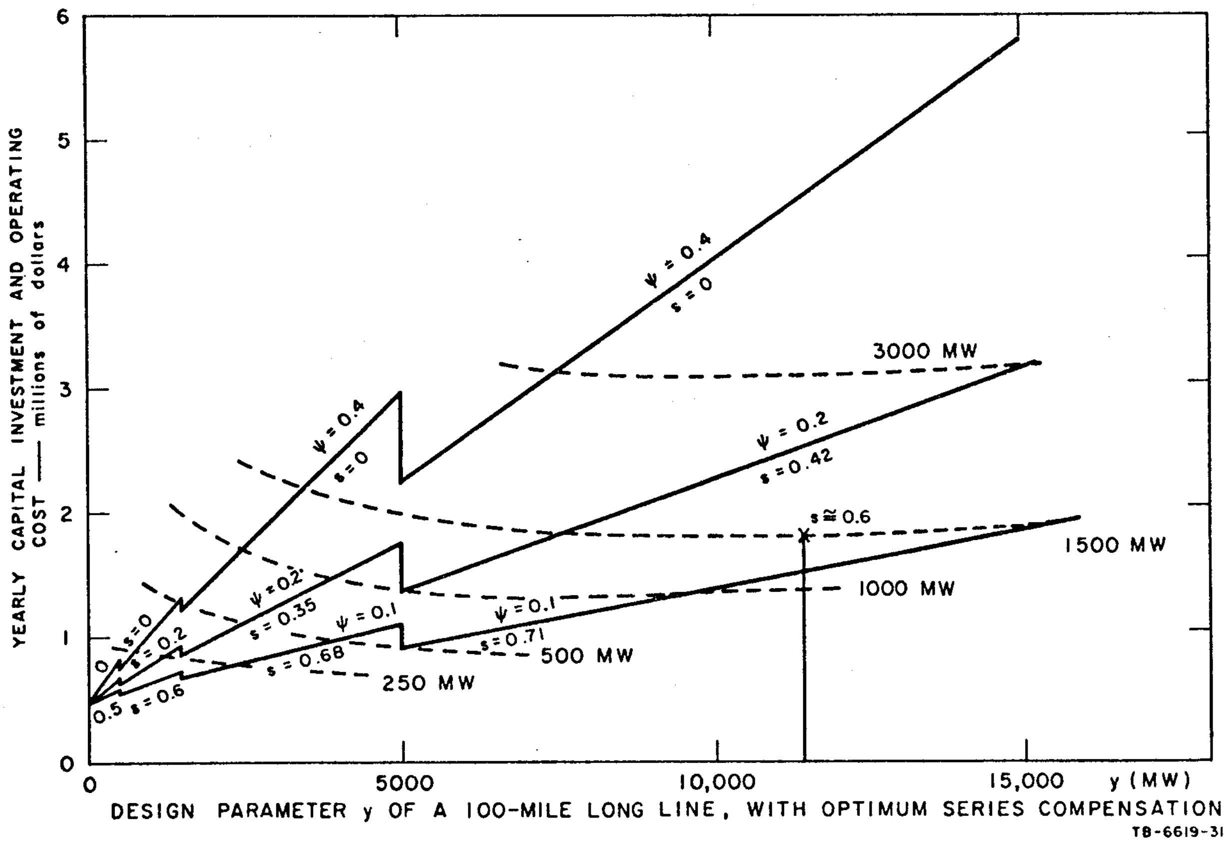

FIG. IV-4 YEARLY COST OF A 100-MILE-LONG LINE WITH THE OPTIMAL DEGREE OF SERIES COMPENSATION, AS A FUNCTION OF LINE PARAMETERS $y = V^2/X(1-s)$, s, AND PHASE ANGLE DIFFERENCE ψ

It is also useful to note that the minimum cost curve is very flat, which implies that the optimum compensation factor s^o need not be chosen with great accuracy and that the commonly accepted figure of $\cong$50 percent for EHV lines is usually quite reasonable.

E. Simplifying Assumptions

In the previous discussions, certain simplifying assumptions were made; it is useful to review these assumptions at this point.

(1) The detailed effect of series compensation upon reliability has been omitted; indeed, the reliability criterion to be discussed in Sec. V may lead the designer to prefer the addition of a parallel line to the strengthening of existing lines by series compensation, although this second alternative is less costly.

(2) Throughout the economic part of the study summarized in Sec. IV-D above, it was assumed that the uncompensated line capacity γ can be selected arbitrarily and need not belong to a discrete set. In order to use the design chart of Fig. IV-3, it is hence necessary to select the nearest discrete line and to find the optimum compensation for this line.

(3) The economic optimization performed above is purely static and does not consider the expansion of the network. To obtain an optimum solution to the long-term (variational) problem, the capacitors now installed would constitute constraints (initial conditions); also, the progressive addition of capacitors to existing lines would be a design option.

(4) One might even consider the temporary strengthening of existing lines by series compensation to delay the addition of the next line. After installation of this next line, the capacitors could be moved to another bottleneck in the system.

F. Effect Upon Long-Term Expansion Optimization

To take into account the effect of series compensation upon the dynamic programming optimization of Sec. VII, the following approaches of increasing complexity and accuracy are available:

(1) Assume that a line of given kV-rating always has a given amount of compensation, e.g., 50 percent, regardless of length and power flow. Under these circumstances, the capital cost and the cost of losses of the line are easily calculated for use in the expansion optimization by dynamic programming.

(2) Assume that the optimum degree of compensation and, hence, the capital and operating costs of a line depend on its capacity γ and the RMS power flow through it; this is precisely the situation shown in the design chart of Fig. IV-3, corresponding to the example of a 100-mile line. Under these circumstances, the optimum degree of compensation s^o and the resulting yearly minimum capital and operating costs F^o can be retrieved from a chart such as shown in Fig. IV-3 for every branch composition alternative γ_k tested on branch k in the course of the dynamic programming procedure of Sec. VII.

(3) Assume that the degree of compensation is a free decision variable and treat it as such in the dynamic programming procedure of Sec. VII. This implies that the search performed at each stage in time on branch k ranges over the two independent decision variables γ_k and s_k; it also implies that the dimension of the dynamic programming problem is two instead of one. The costs of adding and removing capacity from a branch can be included; in fact, all the simplifications pointed out in the previous section can be taken into account rigorously with this last approach.

For the remainder of the project, series compensation was not considered any further, since it was known how it could be accommodated, either approximately or rigorously. The recommended approach toward further refinements of the expansion optimization with dynamic programming to be discussed in Sec. VII would be to use initially the results of a design chart, such as shown in Fig. IV-3 (approach 2) and thereafter to proceed to the rigorous problem statement (approach 3), if sufficient savings could be obtained to justify the somewhat greater complication.

V THE RELIABILITY CONSTRAINT

Additional investments in transmission are required for two reasons, namely:

(1) Reduction of transmission losses

(2) Reliability, or security of transport.

While reduction of losses usually constitutes the main incentive for strengthening low voltage (≤230 kV) systems, reliability is the dominating consideration for EHV transmission system expansion. Also, while losses enter directly into the cost function, reliability enters into the optimization by means of inequality constraints, which specify lower and sometimes upper bounds on branch capacity.

In Sec. II, a heuristic model of the reliability constraint was used. This model, which is widely used in the power industry, stipulates that the angular difference

$$\theta_i - \theta_j \triangleq \psi_k \leq \bar{\psi}_k$$

between connected nodes (ij) must not exceed some upper bound $\bar{\psi}_k$, of the order of 36°, under no-outage condition and for peak loads. The bound $\bar{\psi}_k$ is justified mainly by historical reasons; indeed, experience seems to indicate that a normally designed system will carry a line outage if $\psi_k \leq \bar{\psi}_k$. Although this model of the reliability constraint is relatively easy to accommodate in transmission system design, it does not reflect accurately the reliability criteria presently used in Europe and apparently favored in the U.S. by the FPC. As a result, the constraint $\psi \leq \bar{\psi}$ is either too conservative (thus leading to overinvestment) or else not conservative enough (in which case, cascading faults may occur under certain load conditions).

For EHV systems, the reliability criterion to be imposed upon the system is given by the following conditions:

(1) In the event of any single (and possibly multiple and uncorrelated) outage occurring at the worst time of the year, there must not follow any overloads causing outages, and there must not be any need for curtailing load.

(2) In the event of a second, third, ..., uncorrelated outage (i.e., not triggered by the first outage, in view of condition 1), load may be curtailed to prevent further deterioration of the system.

For low voltage systems, these two conditions would also apply. However, these systems would in addition be assigned a statistical and quantitative figure of merit related to the average yearly curtailed demand caused by multiple uncorrelated outages.

At this point, it is timely to review the reasons whereby a first outage causes a second, third, ..., outage. There are two main reasons, namely:

(1) Excessive power flows due to the first outage that cause the protection system to remove additional lines and/or terminal equipments.

(2) Transient instability, which causes the system to fall out of synchronism; loss of synchronism will thereafter trigger the overload and other protections.

Whereas the steady-state post-fault line flows can be calculated (assuming the availability of a good load-flow program) to determine if the protections will or will not respond, the transient stability properties are much more difficult to assess. In our opinion, it is hopeless to expect that explicit stability criteria, such as Lyapunov's second method, can ever be extended to systems of more than three or four generators. On the other hand, direct simulation--the transient stability program--is impractical for long-term expansion studies, where thousands and possibly millions of design alternatives must be tested. It is for these reasons that we have developed an approximate criterion for transient stability, to be discussed below. This approximate approach to transient stability not only meets the need for the long-term planning procedures of concern here, but can be put to use (after some further refinement and verification) in system operation and present-day planning.

In the optimal expansion procedures to be discussed in the remainder of this report, the reliability properties of a projected system will be tested in the following manner, and a projected system will be accepted only if it passes all these tests:

(1) Steady-State Overload: During peak conditions, the loss of any line (or any small number of lines) must not create power flows anywhere else in the system that would cause the overload protections to respond. This test, in effect, consists of specifying maximum angular differences $\bar{\psi}_k$ that, instead of being set arbitrarily equal to 36°, depend on branch composition, system topography, injections, etc. This test has been applied throughout the optimizations performed in the course of the project.

(2) Transient Instability: During peak conditions, it is ensured that the loss of any line will not create transient instability. This test, in effect, consists of relating the transmission capacity and composition between each node and the remainder of the system to the injection at that node. This test has not been applied in the optimizations performed in the course of the project, but would be treated computationally in the same manner that the steady-state overload test is treated.

(3) Limits on Short-Circuits Currents: Toward the end of this study, it was brought to our attention* that

*Private communication with R. B. Shipley of the Tennessee Valley Authority.

some projected systems may be unacceptable because presently existing switch-gear could not handle the excessive short-circuit currents caused by certain faults. These short-circuit limits were not taken into account in this study, but could be accommodated by techniques very similar to those developed for steady-state overloads.

To summarize, it can be said that the reliability constraint in all three cases becomes a logical constraint. By this, we mean that the feasibility of a projected system is not determined from a simple functional relation of the classical form $x \in X$ or $u \in U$, but depends on the outcome of an exhaustive review of the consequences of all possible outages. With a view on computational efficiency, this exhaustive review draws heavily on the sensitivity relations developed in conjunction with BPA in Ref. 42. The resulting test procedures are required for the optimum long-term investment approach developed in the course of the project, but in addition constitute new and efficient tools that can be used as a matter of routine in conjunction with present-day planning methods and, in our opinion, will constitute, in a slightly different form, the first new element in the forthcoming automatic dispatching centers. Indeed, a major function of these centers consists of monitoring in real time the system's ability to survive outages and to redistribute injections until the system is found to be reliable in the sense defined above.

A. Sensitivity Approach

Considering a network of n nodes and r branches, the linearized power-flow equations are:

$$\mathcal{J}_i = \sum_j \gamma_{ij} \left(\theta_i - \theta_j\right) \quad , \tag{V-1}$$

where

$\mathcal{J}_i$ = Net injection at node i

$\gamma_{ij} = V^2/X_{ij}$.

The linearized power-flow equations and the assumption $r \ll X$ (implying $X_{ij} \simeq Z_{ij}$) have been used for simplicity. The following discussion will be valid also for exact power-flow equations, including line resistance.

If line losses are neglected, then $\sum_i \mathcal{J} = 0$ and (V-1) represents (n - 1) equations, which can be written in compact form as

$$G\,(\theta,\ \gamma) = 0 \quad . \tag{V-2}$$

If θ_1 is selected as the reference angle, then there remain (n - 1) angles $\theta_2, \ldots, \theta_n$ to be fixed. Suppose the injections $\mathcal{J}_i$ are all specified and we are considering some feasible design vector γ^* and the corresponding θ^*, then

$$G\,(\theta^*,\ \gamma^*) = 0 \quad . \tag{V-3}$$

Here,

G is an (n - 1) vector with elements $G_1, G_2, \ldots, G_{n-1}$,

θ^* is an (n - 1) vector with elements $\theta_2, \theta_3, \ldots, \theta_n$,

γ^* is an r vector with elements γ_k where k denotes the k^{th} branch.

To study the effect of variations $\Delta\gamma$ in γ on θ, we write the Taylor Series expansion of Eq. (V-3) as

$$G\,(\theta^* + \Delta\theta,\ \gamma^* + \Delta\gamma) = 0 \simeq G\,(\theta^*,\ \gamma^*) + G_\theta\,\Delta\theta + G_\gamma\,\Delta\gamma + \text{H.O.T.} \quad .$$

Considering only the first-order terms, we get, since $G\,(\theta^*,\ \gamma^*) = 0$,

$$G_\theta\,\Delta\theta = -\,G_\gamma\,\Delta\gamma \quad . \tag{V-4}$$

Assuming that the inverse of G_θ exists, we can write

$$\Delta\theta = -\,G_\theta^{-1}\,G_\gamma\,\Delta\gamma \quad . \tag{V-5}$$

Note that

G_θ is an $(n - 1) \times (n - 1)$ matrix

G_γ is an $(n - 1) \times r$ matrix

$\Delta\theta$ is an $(n - 1)$ vector

$\Delta\gamma$ is an r vector.

We now define an r vector ψ whose components are of the form ψ_k and denote the voltage angle differences $(\theta_i - \theta_j)$ of all connected nodes i, j $(j > i)$. The vector ψ and the vector θ can be related as follows:

$$\psi = M\,\theta \quad , \tag{V-6}$$

where M is an appropriate $r \times (n - 1)$ matrix whose elements are either 0, +1, or -1, as illustrated in the following example.

<u>Example</u>:

Consider the 4-node network shown in Fig. V-1. There are 5 branches. The relation (V-6) for this network is given by:

$$\underset{\substack{\psi \\ \text{5 vector}}}{\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \end{bmatrix}} \quad \underset{\substack{M \\ 5 \times 3 \text{ matrix}}}{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}} \quad \underset{\substack{\theta \\ \text{3 vector}}}{\begin{bmatrix} \theta_2 \\ \\ \theta_3 \\ \\ \theta_4 \end{bmatrix}}$$

Now

$$\Delta\psi = M\,\Delta\theta = -\,M\,G_\theta^{-1}\,G_\gamma\,\Delta\gamma$$

or

$$\Delta\psi = B\,\Delta\gamma \quad , \tag{V-7}$$

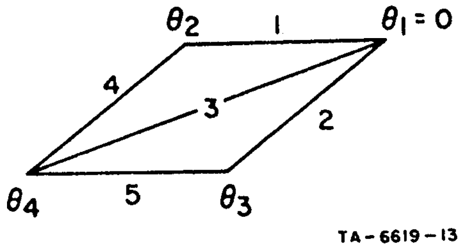

FIG. V-1 A 4-NODE 5-BRANCH NETWORK USED TO EXPLAIN THE RELATION $\psi = M\theta$,
$\psi_1 = (\theta_1 - \theta_2)$
$\psi_2 = \theta_1 - \theta_3, \ldots$, etc.

where

$$B \equiv - M G_\theta^{-1} G_\gamma .$$

Note that B is an r × r matrix.

Equation (V-7) gives the variation in the voltage angle difference as the design parameter γ of the branch is changed. The elements of the matrix B will indicate which γ's probably have the most significant effect on the line angles. We now introduce another useful matrix, $\hat{B}$.

A typical expression for $\Delta\psi_k$ will be as follows:

$$\Delta\psi_k = \sum_\ell b_{k\ell}\, \Delta\gamma_\ell \quad .$$

Let $\hat{\Delta}\gamma_\ell$ denote the biggest loss in the admittance of line ℓ. We then define the matrix $\hat{B}$ as a matrix whose elements are given by

$$\hat{b}_{k\ell} = b_{k\ell}\, \hat{\Delta}\gamma_\ell \quad .$$

The elements $\hat{b}_{k\ell}$ of $\hat{B}$ matrix thus give the change in the voltage angle of the line k due to the maximum loss of admittance in the line ℓ. A study of the elements of the $\hat{B}$ matrix in combination with the knowledge of initial operating angles of the lines will indicate whether and which of the branches are critical.

The matrix $\hat{B}$ and the vector $\Delta\psi$ can be related for convenience in the following way:

$$\Delta\psi = \hat{B}\,\delta \quad . \qquad \text{(V-8)}$$

δ is an r vector whose components are zero if there is no loss of admittance in any line. If the effect of the biggest loss in admittance of line k is to be determined, we let the k^{th} component of the δ vector equal to one, with other components being zero. The product of $\hat{B}$ and δ then gives an r vector indicating the $\Delta\psi$ in the corresponding branches due to loss of admittance in branch k.

The most negative number in the i^{th} row of $\hat{B}$, say $\hat{b}_{ik}$, gives the largest negative change in ψ_i, resulting from the biggest loss of admittance in branch k. Similarly the largest positive number $\hat{b}_{i\ell}$ gives the largest positive change in the ψ_i resulting from loss of admittance in branch ℓ. If two faults are to be considered, then the sum of the largest negative element $\hat{b}_{ik}$ and the next largest negative element $\hat{b}_{ir}$ gives the largest negative change in ψ_i. Similar remarks apply to positive change. A study of the elements of the $\hat{B}$ matrix is thus helpful in locating the probable critical branches of a network.

1. Important Note

The aforementioned first-order sensitivity relation could be employed to locate only approximately the probable critical branches. We could then solve Eq. (V-3) accurately for critical cases indicated by the approximate sensitivity relation to find the exact angle deviations and compare them with the allowable deviations. As will become apparent later, the first-order sensitivity relation discussed above is suitable only if small $\Delta\gamma$s are considered. For larger $\Delta\gamma$s the relation does not give reliable indications about the critical branches, as will be shown by the results of a numberical example. A second iteration of Eq. (V-2) will be used in Sec. VI to improve the accuracy.

2. Application of the Sensitivity Relation to an 8-Node Model

The above sensitivity relations and matrices B and $\hat{B}$, Tables V-1 and V-2, were computed for the 8-node model shown in Fig. V-2. The composition of the transmission network was assumed to be that given by

Table V-1

B MATRIX FOR 8-NODE MODEL

(Angles in Microdegrees)

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	24.5	-1.1	0.78	1.3	1.5	0.73	0.96	9.33	3.5	-0.46	-0.21	-0.15	-0.004
2	15.8	-3.0	-2.3	-3.9	-4.5	-2.1	-2.8	-27.8	-10.6	1.3	0.64	0.47	0.013
3	-3.1	-0.67	-22.9	2.5	14.5	1.39	1.8	-13.4	-11.1	4.6	-20	-0.3	-0.008
4	2.9	0.62	-1.3	9.8	-2.6	5.5	0.72	12.3	-6.2	0.81	0.38	-1.2	-0.034
5	2.06	0.44	-4.8	-1.6	26	-0.91	-1.2	8.8	7.3	2.8	1.3	0.19	0.005
6	-1.3	-0.28	0.62	-4.5	1.2	-13	-17	-5.6	2.8	-0.37	-0.17	-1.3	0.08
7	-1.2	-0.28	0.61	-4.4	1.2	-12	-41	-5.5	2.8	-0.36	-0.17	-1.3	-6
8	-8.7	-1.87	-3.1	-5.2	-6.1	-2.9	-3.8	-37	-14	1.8	0.86	0.63	0.02
9	-5.8	-1.25	-4.5	4.6	-8.7	2.5	3.3	-24	-20	2.6	1.2	-0.55	-0.015
10	-3.7	-0.8	-9.3	2.9	17	1.6	2.1	-15	-13	5.5	2.5	-0.35	-0.01
11	-0.58	-0.12	13	0.46	2.6	0.25	0.33	-2.4	-2.05	0.86	22	-0.05	-0.001
12	1.57	0.34	-0.74	5.3	-1.4	-7.5	-9.8	6.7	-3.4	0.44	0.2	-2.5	0.046
13	0.02	0.005	-0.001	0.08	-0.02	0.24	-23	0.1	-0.05	0.06	0.03	0.02	-6

Note: 1 Microdegree = 10^{-6} degrees.

Table V-2

$\hat{B}$ MATRIX FOR 8-NODE MODEL

(Angles in Degrees)

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	-6.86	0.28	-0.02	-0.55	-0.32	-0.23	-0.01	-1.67	-0.89	0.09	0.06	0.07	0.002
2	-4.43	0.75	0.07	1.64	0.95	0.68	0.04	5.00	2.67	-0.28	-0.20	-0.23	-0.008
3	0.88	0.17	0.71	-1.05	-3.05	-0.44	-0.02	2.42	2.80	-0.98	6.36	0.15	+0.005
4	-0.81	-0.15	0.04	-4.13	0.56	-1.73	-0.09	-2.22	1.57	-0.17	-0.19	0.59	0.02
5	-0.57	-0.11	0.15	0.68	-5.45	0.28	0.02	-1.58	-1.83	-0.60	-0.42	-0.04	-0.003
6	-0.37	+0.07	-0.02	1.88	-0.25	4.1	0.23	1.016	-0.72	0.07	0.054	0.68	-0.05
7	0.36	0.07	-0.02	1.85	-0.25	4.02	0.55	0.99	3.57	0.07	0.053	0.67	3.78
8	2.43	0.47	0.09	2.19	1.28	0.92	0.05	6.68	5.15	-0.38	-0.27	-0.31	-0.01
9	1.62	0.31	0.14	-1.93	1.84	-0.81	-0.04	4.45	3.317	-0.55	-0.39	0.27	0.01
10	1.04	0.20	0.29	-1.24	-3.6	-0.52	-0.03	2.86	0.52	-1.16	-0.81	0.18	0.006
11	0.16	0.03	-0.42	-0.19	-0.56	-0.08	-0.004	0.44	0.85	-0.18	-7.17	0.028	0.001
12	-0.44	-0.08	0.02	-2.24	0.31	2.37	0.134	-1.21	0.81	-0.09	-0.06	1.28	-0.003
13	-0.006	-0.001	0.0003	-0.034	0.004	-0.075	0.32	-0.018	0.01	-0.001	-0.001	0.012	3.83

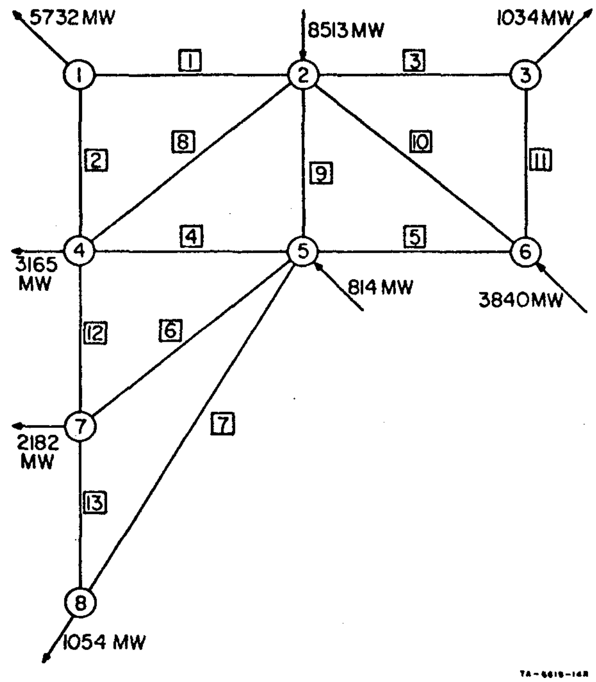

FIG. V-2 AN 8-NODE MODEL SHOWING INITIAL NET INJECTIONS AND NET CONSUMPTIONS AT VARIOUS NODES

BPA shown in Fig. V-3. Voltage angles between various connected nodes with the given transmission network composition are indicated in Table V-3. These were computed by solving Eq. (V-3) for the model. It is seen that the network in its present form meets the 36°-angle-difference criterion. In fact, except for branches 1 and 8, when the angles are -24.01° and 25.74° respectively all other angles are quite small. In Table V-4 we give various branches of the network in the first column; in the second column we locate those two branches (in order of significance) having the greatest effect on the voltage angles of the branch in column 1. This table was prepared from a study of the various elements of the B matrix.

In Table V-3 we also give the maximum allowable positive and negative deviations for each line using ±36° as the limit.

Table V-3

INITIAL OPERATING ANGLES AND MAXIMUM ALLOWABLE VARIATIONS IN VARIOUS BRANCHES OF THE 8-NODE MODEL

Branch	Initial Operating Angle Degrees ψ	Maximum Allowable Variation (rounded off)	
		Maximum $+\Delta\psi_M = 36^\circ - \psi$	Maximum $-\Delta\psi_m = -36^\circ - \psi$
1	(-24.01)	60	-12
2	(1.74)	34	-37
3	(5.98)	30	-41
4	(-10.91)	46	-25
5	(-17.8)	53	-18
6	(13.33)	22	-49
7	(17.80)	19	-53
8	(25.74)	10	-61
9	(14.83)	21	-50
10	(-2.98)	38	-33
11	(-8.96)	45	-27
12	(2.42)	33	-38
13	(4.47)	31	-40

Table V-4

CRITICAL BRANCHES OF THE 8-NODE MODEL

Voltage Angle of Branch	Influenced Most Significantly by a Change in γ of the Branch
1	1 and 8
2	8 and 1
3	3 and 11
4	8 and 4
5	5 and 8
6	7 and 6
7	7 and 6
8	8 and 9
9	8 and 9
10	5 and 8
11	11 and 3
12	7 and 6
13	7 and 13

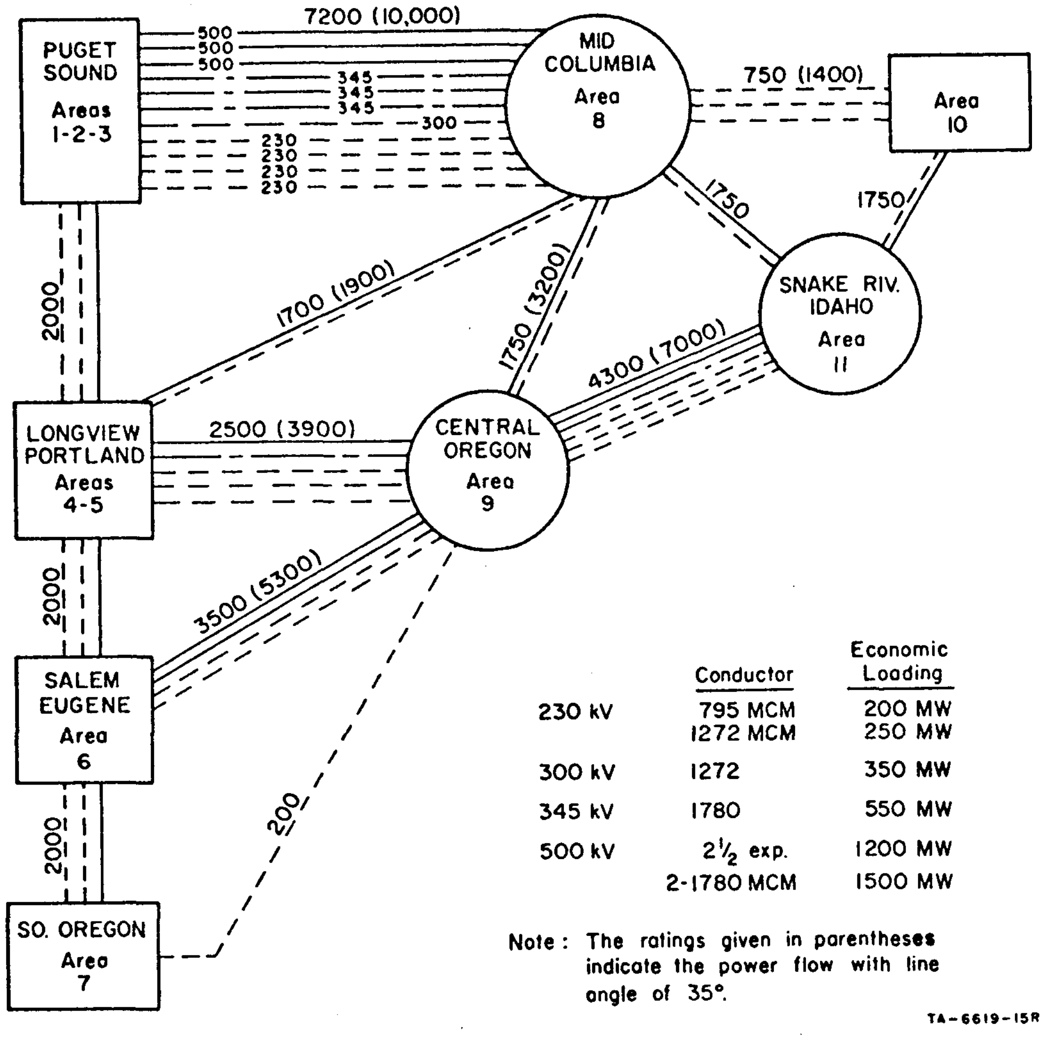

FIG. V-3 COMPOSITION OF VARIOUS BRANCHES OF THE 8-NODE MODEL

Matrix $\hat{B}$ was computed considering the disconnection of the following lines from the respective branches as indicated below in Table V-5.

A study of the elements of matrix $\hat{B}$ indicates that the changes in various voltage angles due to loss of the strong lines in various branches are small and in most cases much less than the 36° limits. For example considering branch 8, the largest change in its angle when the branch loses 2 X 500 kV lines will be 6.68°. According to Table V-3 the maximum allowable change is 10°. The largest negative change in the same branch is -0.38° when line 10 loses 2 X 500 kV lines. The allowable

Table V-5

LISTING OF LINES ASSUMED DISCONNECTED TO COMPUTE $\hat{B}$ MATRIX

Branch	Lines Assumed Disconnected
1	2 × 500 kV lines
2	2 × 500 kV lines
3	2 × 230 kV lines
4	2 × 500 kV lines
5	2 × 500 kV lines
6	2 × 500 kV lines
7	1 × 230 kV lines
8	2 × 500 kV lines
9	2 × 500 kV lines
10	2 × 500 kV lines
11	2 × 500 kV lines
12	2 × 500 kV lines
13	2 × 500 kV lines

change is -61°, which is much larger than -0.38. Thus the system in its present form fulfills the 36°-angle-difference criterion not only in normal operation but even when any of the branches lose the biggest line. It thus appears that the system in its present form is overdesigned as far as the 36° limit is concerned. It is felt that the safe operating angles may actually be more than 36° for some branches, depending on the configuration of the network. In Sec. V-B we present a simple approximate method to test the transient stability power limits.

3. Accuracy of the Linear Sensitivity Relation

In Tables V-6, V-7, and V-8, we have indicated the line angles resulting as a change in $\Delta\gamma$ for three different cases. These line angles were calculated by using the sensitivity relation as well as by exact solution of Eq. (V-3). The accuracy of the calculations by sensitivity relation is also indicated.

Table V-6

ACCURACY OF SENSITIVITY RELATION FOR SMALL $\Delta\gamma$

γ_3 Changes from 1085 to 1585 (i.e., $\Delta\gamma_3$ = 500). All Other γs Remain Unchanged.

Branch	ψ (degrees) Reference State	ψ (degrees) By Sensitivity Relation	By Exact Solution	Percent Error
1	-24.01	-23.98	-23.98	0
2	1.74	1.67	1.68	-0.6
3	5.98	5.33	5.39	-1.1
4	-10.91	-10.95	-10.95	0
5	-17.81	-17.95	-17.94	+0.06
6	13.33	13.35	13.35	0
7	17.80	17.81	17.81	0
8	25.74	25.65	25.66	-0.04
9	14.83	14.70	14.71	-0.07
10	-2.98	-3.25	-3.22	-0.93
11	-8.96	-8.58	-8.62	0.47
12	2.42	2.40	2.40	0
13	4.47	4.47	4.47	0

Table V-7

ACCURACY OF SENSITIVITY RELATION FOR A MODERATE $\Delta\gamma$

Branch 8 Loses Just 1 X 500 kV Line (i.e., γ of line 8 changes from 3362 to 1792: i.e., $\Delta\gamma_8$ = - 1570). All Other γs Remain Unchanged.

Branch	By Sensitivity Relation $\Delta\psi = B\ \Delta\gamma$	By Exact Solution of Eq. (2)	Percent Error
1	-24.84	-24.97	0.52
2	4.24	4.61	-8.0
3	7.19	7.37	-2.44
4	-12.03	-12.19	1.3
5	-18.61	-18.72	0.6
6	13.84	13.91	-0.54
7	18.29	18.37	-0.4
8	29.08	29.58	-1.68
9	17.06	17.39	-1.94
10	-1.55	-1.34	-15.7
11	-8.74	-8.71	-0.37
12	1.81	1.72	-5.25
13	4.46	4.45	-0.22

Table V-8

ACCURACY OF SENSITIVITY RELATION FOR LARGE $\Delta\gamma$s

Branch 1 Loses 4 X 500 kV and 3 X 345 kV Lines; Branch 8 Loses 2 X 500 kV Lines; Branch 9 Loses 2 X 500 kV Lines

Branch	By Sensitivity Relation	ψ By Exact Solution	Percent Error
1	-48.75	-126.06	62
2	-1.69	-21.16	93
3	13.40	43.08	-70
4	-13.59	-21.78	36.6
5	-22.68	-42.16	45.2
6	14.55	18.29	-22.2
7	18.99	22.66	-17.8
8	42.07	104.90	-61
9	28.48	83.12	-65
10	5.80	40.96	-85
11	-7.60	-2.12	-260
12	0.96	-3.49	130
13	4.44	4.37	0.75

B. An Approximate Method of Estimating the Transient-Stability Power Limits at Various Nodes of a Power System

In this section we present a simple, yet reasonably accurate, method for quickly testing the transient stability of a power system and/or estimating the transient stability power limits at its various nodes. The essence of the method lies in regarding the node under consideration as a single machine and treating the rest of the system as an infinite bus, so that the classical theory of the stability of a single machine connected to an infinite bus can be used. It will be shown that this method gives, with much fewer computations and, therefore, much faster, the stability limits at various nodes that are close to the limits calculated by computing the so-called swing curves obtainable by numerical integration of the dynamic equations at various nodes.

We shall consider one node at a time and estimate the transient stability power limit at this node for the case when the most heavily loaded line connected to it drops out because of some faulty condition.

For simplicity, we shall make the following assumptions:

(1) The voltages at all nodes will be assumed to be equal to their nominal values, and line resistances will be neglected. The method, however, can also include resistance and non-nominal voltage conditions.

(2) It will be assumed that the transient period is of short duration compared to the time constants of governor and voltage regulators, so that the net injections and voltages at various nodes may be assumed to remain unchanged during the transient period. This assumption makes the analysis considerably simpler and is on the safer side, since the governor and voltage-regulator actions are designed to improve stability. If, however, it is desired to include these effects, an extension of Lyapunov's method is possible.[43]

(3) It will be assumed that machine reactances connected to various nodes can suitably be accounted for in the reactance X_{ij} between various nodes i, j.

1. <u>Derivation of the Approximate 2-Node Model</u>

Considering now a particular node, say the node i, the dynamic behavior of the angle θ_i at this node immediately after tripping of the faulty line is given by

$$M_i \frac{\left(d^2\theta_i\right)}{dt^2} = \mathcal{J}_i - \sum_j \gamma_{ij} \sin\left(\theta_i - \theta_j\right) \quad , \qquad \text{(V-9)}$$

where

$\gamma_{ij} = V^2/X_{ij}$

M_i = Inertia constant of the machine at node i.

In Fig. V-4, the node voltages V_1, V_2, ..., are shown schematically with respective angle differences. If we draw vectors A_1, A_2, ... along voltage vectors V_1, V_2, ... respectively such that $|A_1| = \gamma_{i1}$, $|A_2| = \gamma_{i2}$, ..., etc., then the expression $\sum_j \gamma_{ij} \sin\left(\theta_i - \theta_j\right)$ is simply the sum of the projections of vectors A_1, A_2, ... on an axis P′OP perpendicular to the axis of V_i.

If we add A_1, A_2, ... vectorially to get an equivalent vector y_i, then the expression $\sum_j \gamma_{ij} \sin\left(\theta_i - \theta_j\right)$ can also be regarded as the projection of y_i on P′OP. The magnitude of y_i is given by

$$|y_i| = \sqrt{\left[\sum_j \gamma_{ij} \cos\left(\theta_i - \theta_j\right)\right]^2 + \left[\sum_j \gamma_{ij} \sin\left(\theta_i - \theta_j\right)\right]^2}$$

$$= \sqrt{\sum_j \left(\gamma_{ij}\right)^2 + \sum_{\substack{j,k \\ j,k \neq i}} 2\gamma_{ij}\gamma_{ik} \cos\left(\theta_j - \theta_k\right)} \quad . \qquad \text{(V-10)}$$

Note that the magnitude of y_i is a function of γ_{ij} and the relative angle differences between other nodes not involving node i.

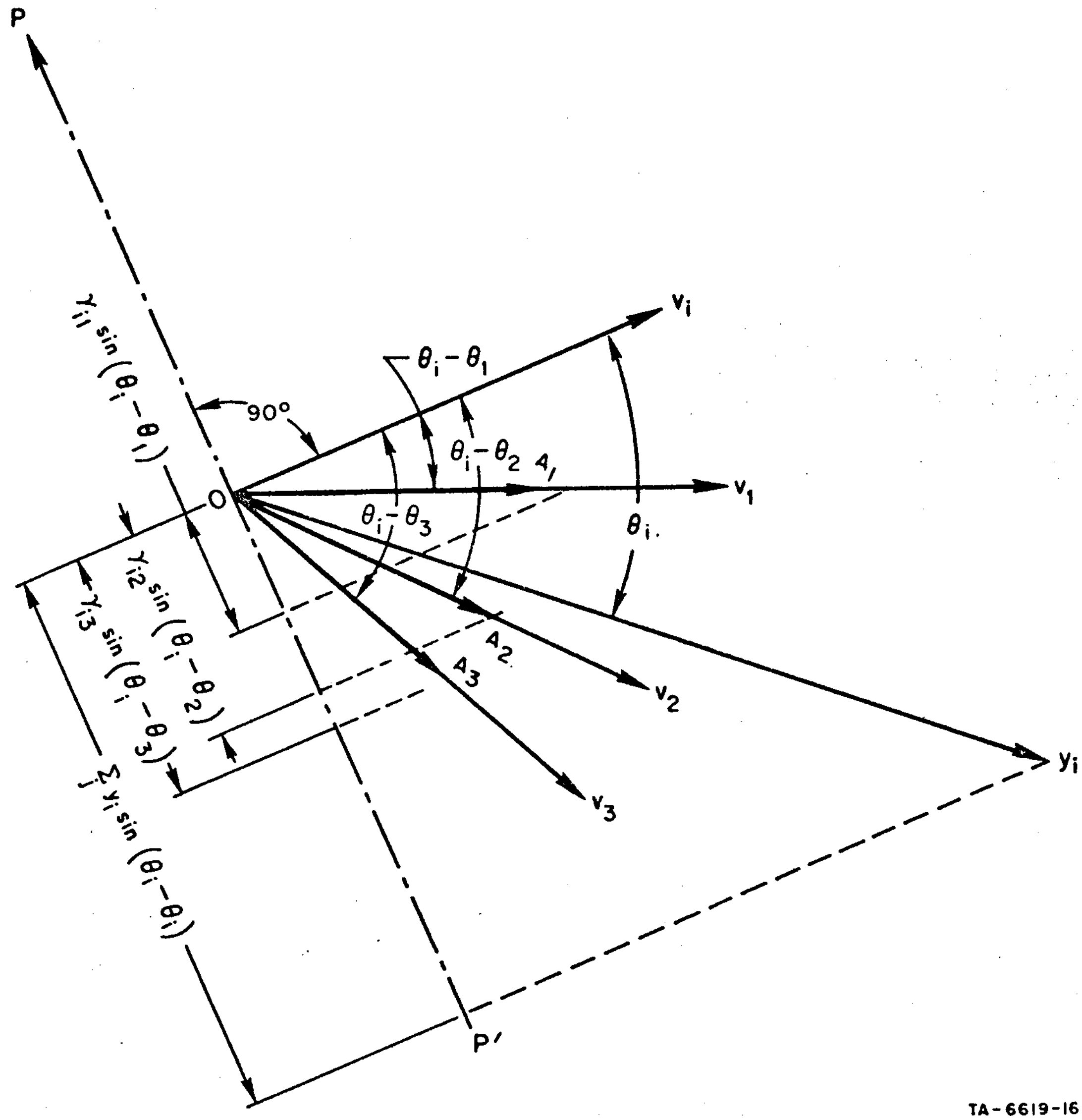

FIG. V-4 VECTOR REPRESENTATION OF THE EXPRESSION $\sum_i \gamma_{ij} \sin(\theta_i - \theta_j)$. $|A_1| = \gamma_{i1}$, $|A_2| = \gamma_{i2}$, etc.

If we take the axis of y_i as the reference axis, then Eq. (V-9) can be written as

$$M_i \frac{\left(d^2\theta_i\right)}{\left(dt^2\right)} = \mathcal{J}_i - y_i \sin\left(\theta_i\right) \quad , \qquad \text{(V-11)}$$

where θ_i is the angle of voltage V_i relative to the axis of y_i.

Equation (V-11) is similar to the dynamic equation of a single machine connected to an infinite bus, with the difference that during the transient period the amplitude y_i in the present case is strictly not a constant, but will vary as angles $\left(\theta_j - \theta_k\right)$ vary during this period. However, referring to Eq. (V-10), it is seen that the value of y_i consists of two parts: one depending on γ_{ij}, which during transient period remains constant at post-fault value, and the other depending on the cosines of the relative angles between nodes other than node i. Now in a realistic case, it is reasonable to assume that the primary effect of a change in the γ of a branch connected to node i will be on the angles of the branches connected to the node i, whereas the angles $\left(\theta_j - \theta_k\right)$ when $j,k \neq i$ will not be affected drastically. Thus, the change from pre-fault to post-fault values in the relative angles between nodes <u>not involving node i</u> can be assumed small, so that the amplitude y_i during transient period can be regarded essentially a constant whose value can be calculated using the post-fault values of γ_{ij}, assuming that the angles $\left(\theta_j - \theta_k\right)$ adopt the post-fault steady-state values immediately after tripping of the line. With this assumption, the expression $y_i \sin\left(\theta_i\right)$ represents the familiar sinusoidal power angle curve of a single machine connected to an infinite bus, so that the transient stability of the node i can be analyzed using the familiar equal-area criterion without requiring the solution of the dynamic Eq. (V-11). If we calculate y_i using the post-fault values of γ_{ij} and the post-fault steady-state values of angles $\left(\theta_j - \theta_k\right)$, and denote this value of y_i as y_i^*, then $y_i^* \sin\left(\theta_i\right)$ will represent approximately the post-fault power-angle curve at node i. Similarly, if we use the pre-fault values of γ_{ij}, and pre-fault steady-state values of $\left(\theta_j - \theta_k\right)$ to calculate y_i, and denote it with y_i^o, then $y_i^o \sin\left(\theta_i\right)$ will represent approximately the pre-fault power-angle curve at node i. Note that our interest in the pre-fault values is only to establish the initial conditions. Knowing the values y_i^o and y_i^*, the maximum transient-stability power limit $\mathcal{J}_{i_{max}}$ at node i can be estimated or alternatively if $\mathcal{J}_i$ is known, the stability at node i can be tested using the equal-area criterion as discussed below.

2. Relation Between $\mathcal{J}_i$, y_i^o, and y_i^*

In Fig. V-5 we have shown the equivalent pre-fault and post-fault power-angle curves for node i. Stability at node i will be maintained for a pre-fault input $\mathcal{J}_i$ if area A_1 is less than or equal to area A_2.

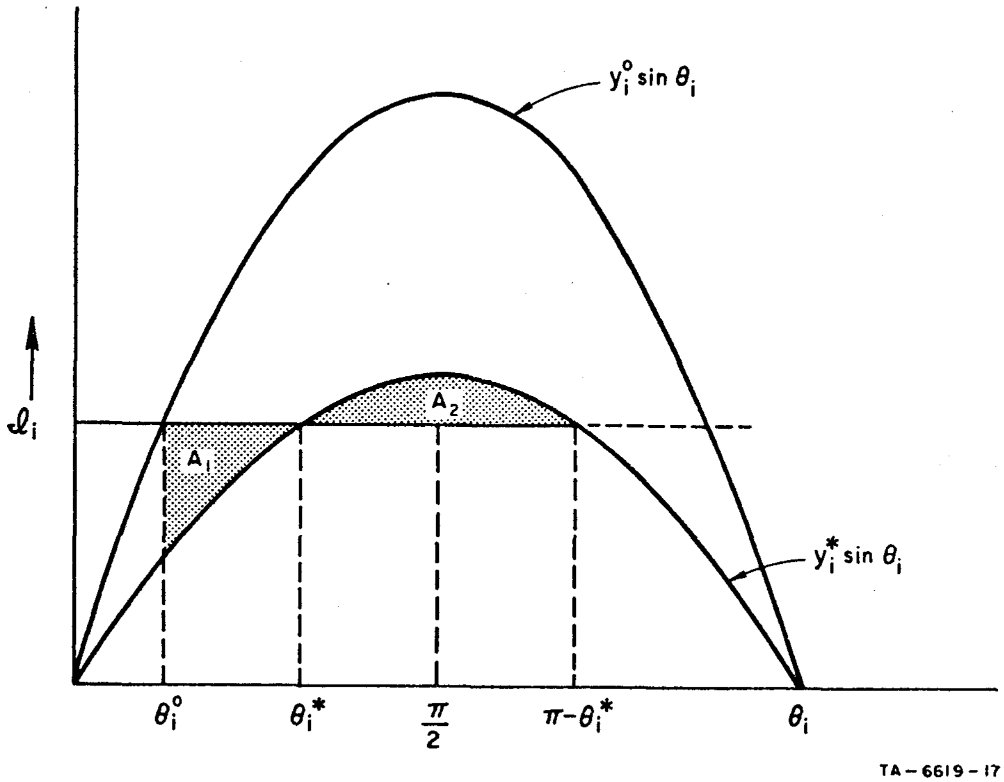

FIG. V-5 CALCULATION OF TRANSIENT-STABILITY POWER LIMIT AT NODE i USING EQUAL-AREA CRITERION

θ_i^o and θ_i^* are the pre-fault and post-fault steady-state values of θ_i and are given by

$$\theta_1^o = \sin^{-1}\frac{\vartheta_i}{y_i^o} \quad ; \quad \theta_1^* = \sin^{-1}\frac{\vartheta_i}{y_i^*} \quad .$$

We now find expressions for A_1 and A_2 to get the desired relation between ϑ_i, y_i^{o}, and y_i^*. We have

$$A_1 = \left(\theta_1^* - \theta_1^o\right)\vartheta_i - \int_{\theta_1^o}^{\theta_1^*} y_1^* \sin\theta \, d\theta$$

$$= \left(\theta_1^* - \theta_1^o\right)\vartheta_i + y_i^* \cos\theta \Big|_{\theta_1^o}^{\theta_1^*}$$

$$\therefore \quad A_1 = \left(\theta_1^* - \theta_1^o\right)\vartheta_i + y_i^*\left(\cos\theta_1^* - \cos\theta_1^o\right) \qquad \text{(V-12)}$$

$$A_2 = \int_{\theta_1^*}^{\pi-\theta_1^*} y_i^* \sin\theta \, d\theta - \left(\pi - 2\theta_1^*\right)\vartheta_i$$

$$= y_i^* \cos\theta \Big|_{\pi-\theta_1^*}^{\theta_1^*} - \left(\pi - 2\theta_1^*\right)\vartheta_i$$

$$\therefore \quad A_2 = y_1^*\left[\cos\theta_1^* - \cos\left(\pi - \theta_1^*\right)\right] - \vartheta_i\left(\pi - 2\theta_1^*\right) \quad .$$

For stability $A_1 \leq A_2$, or

$$\left(\theta_1^* - \theta_1^o\right)\vartheta_i + y_i^*\left(\cos\theta_i^* - \cos\theta_i^o\right) \leq 2y_i^*\cos\theta_i^* - \vartheta_i\left(\pi - 2\theta_i^*\right) \quad . \qquad \text{(V-13)}$$

Expressing θ_1^* and θ_1^o also in terms of ϑ_i, y_1^o, and y_1^* we get:

$$\left(\sin^{-1}\frac{\vartheta_i}{y_i^*} - \sin^{-1}\frac{\vartheta_i}{y_i^o}\right)\vartheta_i + y_i^*\left[\cos\left(\sin^{-1}\frac{\vartheta_i}{y_i^*}\right) - \cos\left(\sin^{-1}\frac{\vartheta_i}{y_i^o}\right)\right]$$

$$\leq 2y_i^*\cos\left(\sin^{-1}\frac{\vartheta_i}{y_i^*}\right) - \vartheta_i\left(\pi - 2\sin^{-1}\frac{\vartheta_i}{y_i^*}\right) \qquad \text{(V-14)}$$

or

$$\left(\sin^{-1}\frac{\vartheta_i}{y_i^*} - \sin^{-1}\frac{\vartheta_i}{y_i^o}\right)\vartheta_i - y_i^* \cos\left(\sin^{-1}\frac{\vartheta_i}{y_i^o}\right) \le y_i^* \cos\left(\sin^{-1}\frac{\vartheta_i}{y_i^*}\right)$$

$$- \vartheta_i\left(\pi - 2\sin^{-1}\frac{\vartheta_i}{y_i^*}\right) \quad . \qquad \text{(V-15)}$$

The relation (V-15) contains only the terms ϑ_i, y_i^o, and y_i^*. The largest value of ϑ_i that satisfies the relation (V-15) gives the desired estimate of transient stability power limit at node i. Alternatively, if ϑ_i is given and it is desired to find out if stability of the system will be maintained when a given branch loses one or two lines, relation (V-15) can be tested for the nodes directly connected with the branch losing the lines as well as other nodes in its immediate vicinity.

3. A Useful Relation

We can rearrange Eq. (V-15) in a more useful form as follows:

Define:

$$\frac{\vartheta_i}{y_i^o} = y \quad ; \quad \frac{y_i^*}{y_i^o} = x \quad ; \quad \text{and} \quad \frac{y_i^o - y_i^*}{y_i^o} = 1 - x = z \quad ,$$

then Eq. (V-15) can be written as:

$$\left(\sin^{-1}\frac{y}{x} - \sin^{-1}y\right) y - x\cos\left(\sin^{-1}y\right) \le x\cos\left(\sin^{-1}\frac{y}{x}\right) - y\left(\pi - 2\sin^{-1}\frac{y}{x}\right) \qquad \text{(V-16)}$$

or

$$\left[\sin^{-1}\frac{y}{(1-z)} - \sin^{-1}y\right] y - (1-z)\cos\left(\sin^{-1}y\right) \le (1-z)\cos\left(\sin^{-1}\frac{y}{1-z}\right)$$

$$- y\left(\pi - 2\sin^{-1}\frac{y}{1-z}\right) \quad . \qquad \text{(V-17)}$$

For any given power system the pre-fault value y_i^o (i.e., the amplitude of pre-fault power-angle curve) may be regarded as fixed and indicates the steady-state stability power limit at node i based on the approximate method. The ratio $y_i^*/y_i^o = x$ is the amplitude of the post-fault power-angle curve expressed as a fraction of the amplitude of the pre-fault power-angle curve and indicates indirectly the reduced admittance of the equivalent 2-node model. The value $\left(y_i^o - y_i^*\right)/y_i^o = z$ represents the percent reduction in the equivalent admittance. The ratio ϑ_i/y_i^o is the injection at node i as a fraction of steady-state stability power limit. Thus the Eq. (V-16) can be regarded as a relation between transient stability power limit and the post-fault admittance both expressed as a fraction of the amplitude of pre-fault power-angle curve. Similarly, Eq. (V-17) can be regarded as a relation between transient stability power limit expressed as a fraction of steady-state stability limit versus loss of admittance expressed as a fraction of pre-fault admittance. The relation (V-17) has been plotted in Fig. V-6. This figure is very useful in finding the transient stability limit at various nodes for any given percentage admittance loss at that node.

In the next section we introduce another useful concept, the "Measure of Stability."

4. Measure of Stability

To compare the stability under different conditions, a useful concept is that of the measure of stability. Referring to Fig. V-5, it is clear that the bigger area A_2 is than area A_1, the more stable node i is under the assumed conditions. We can define in quantitative terms the measure of stability S_m as:

$$S_m = \frac{A_2 - A_1}{A_2} \quad .$$

If $A_2 = A_1$, S_m will be zero, indicating that the stability is critical, i.e., the net accelerating torque is just balanced by the net restoring torque, and the transient stability power limit has been reached. For values of $S_m > 0$, the greater the value of S_m, the more stable the system is and the more the system operates below the transient stability limit. A negative value of S_m would indicate that the system will not be transient stable under the presumed conditions. The relation for S_m in

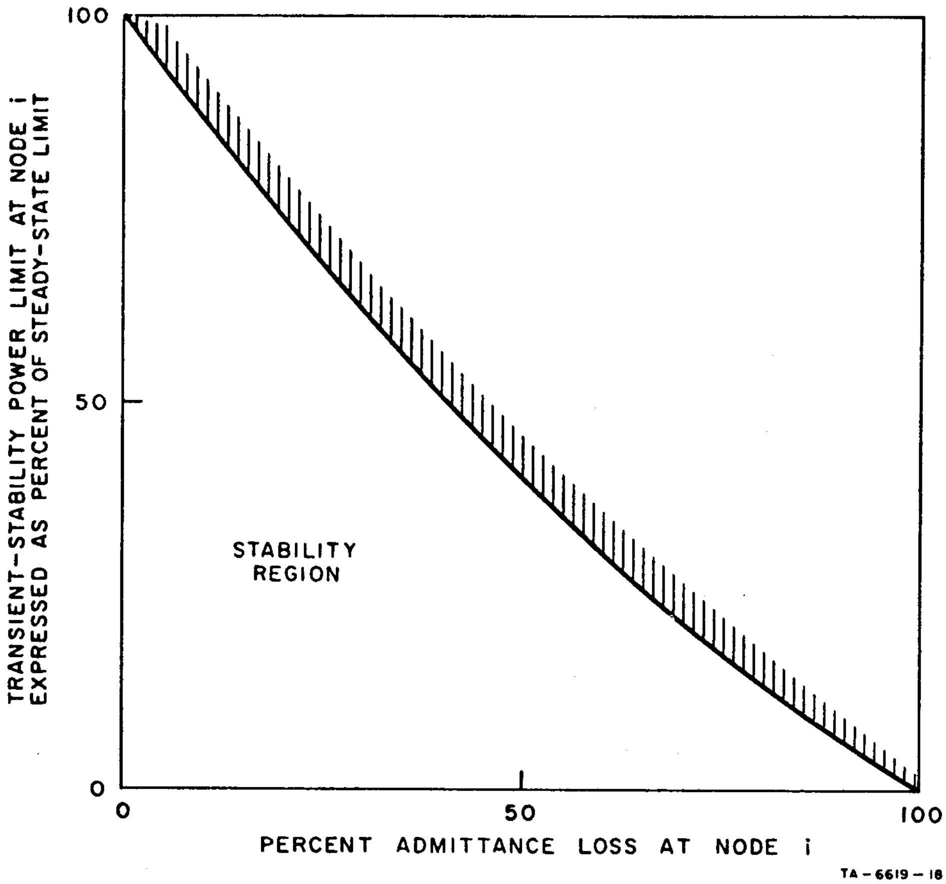

FIG. V-6 TRANSIENT-STABILITY POWER LIMIT vs. PERCENT LOSS OF ADMITTANCE AT NODE i

terms of y and z (as defined above) can be written for the present case as:

$$S_m = \left[\sin^{-1}\frac{y}{(1-z)} - \sin^{-1}y\right]\frac{y}{(1-z)} - \cos\left[\sin^{-1}\frac{y}{(1-z)}\right] - \cos\left(\sin^{-1}y\right) + \frac{y}{(1-z)}$$

$$\cdot \left[\pi - 2\sin^{-1}\frac{y}{(1-z)}\right] \Big/ \left\{2\cos\left(\sin^{-1}\frac{y}{x}\right) - \frac{y}{x}\left(\pi - 2\sin^{-1}\frac{y}{x}\right)\right\} \quad . \quad \text{(V-18)}$$

In Fig. V-7 we have plotted the relation between measure of stability S_m and net injection expressed as a percentage of steady-state power limit for various fixed values of percentage loss in admittance at a node. This figure is useful in quickly finding the measure of stability for any given input power at a node and a given percent loss of admittance.

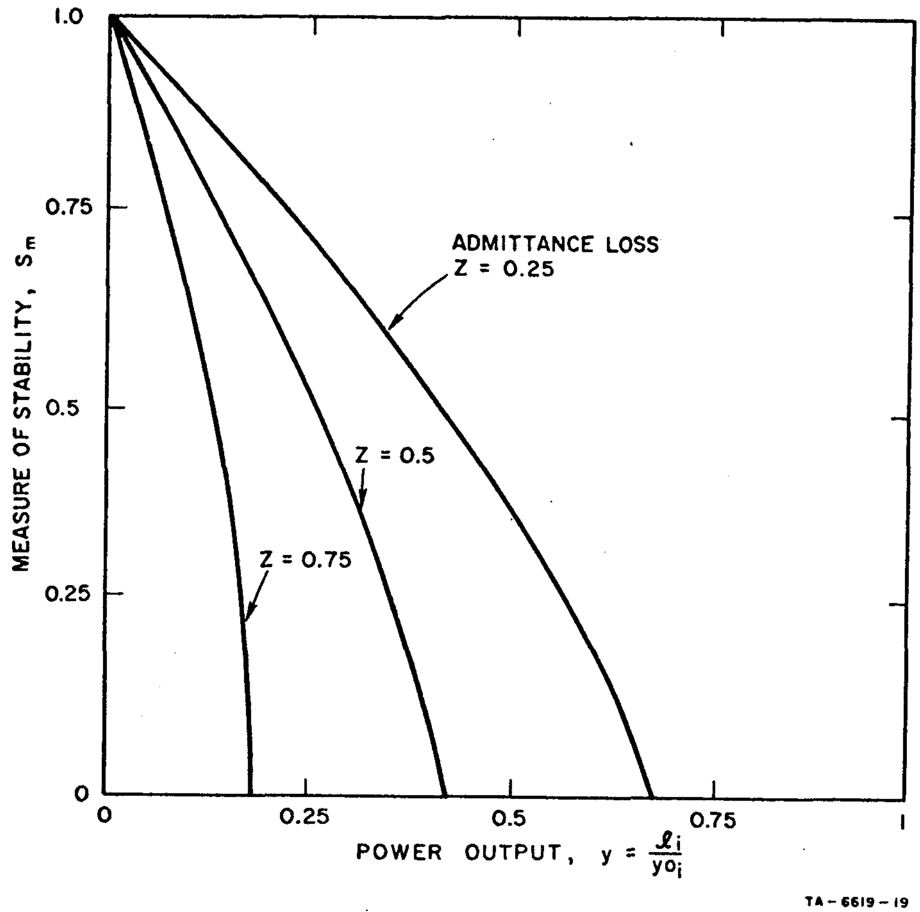

FIG. V-7 RELATION BETWEEN POWER INPUT AT NODE i (Expressed as a fraction of steady-state power limit) AND MEASURE OF STABILITY FOR VARIOUS FIXED ADMITTANCE LOSSES

We present now an example to test and further clarify the various concepts discussed above.

5. <u>An Example</u>

We consider the 8-node model shown in Fig. V-2. Considering the various initial power inputs and outputs at various nodes as shown in this figure, we wish to find out if the system will remain stable if branch 1 loses 2 × 500 kV lines. Furthermore, we wish to find the transient stability power limit at, say, node 2 for the above mentioned loss of 2 × 500 kV lines. The pre-fault and post-fault values of γs and the operating angles were first computed and are tabulated in Table V-9. It is interesting to note that the variation in the angles between nodes

Table V-9

PRE-FAULT AND POST-FAULT VALUES OF OPERATING ANGLES AND ADMITTANCES FOR THE 8-NODE MODEL

Branch	From Node i	to j	Pre-fault Values γ	Pre-fault Voltage Angles	Post-fault Values γ	Post-fault Voltage Angles
1	1	2	14,021	-24.01	9,142	-34.6
2	1	4	4,707	1.74	4,707	1.73
3	2	3	1,085	5.98	1,085	7.5
4	4	5	10,166	-10.91	10,166	-11.9
5	5	6	8,734	-17.81	8.734	-18.85
6	5	7	11,767	13.33	11.767	13.7
7	5	8	236	17.80	236	18.3
8	2	4	3,362	25.74	3,362	29.6
9	2	5	4,707	14.83	4,707	17.7
10	2	6	3,922	-2.98	3,922	-1.15
11	3	6	5,883	-8.96	5,883	-8.65
12	4	7	10,065	2.42	10,065	1.8
13	7	8	12,582	4.47	23,582	5.4

not involving node 2, e.g., branch 4 or 5 or 6 from pre-fault to post-fault is not very big, as was assumed in the derivation of the 2-node model.

Using the values indicated in the above table we first calculate y_2^o.

$$y_2^o = \sqrt{A_o^2 + B_o^2} \quad ,$$

where

$$A_o^2 = [14021 \sin(24.01) + 3362 \sin(25.74) + 4707 \sin(14.83) + 3922 \sin(-2.98) + 1085 \sin(5.95)]^2$$

$$B_o^2 = [14021 \cos(24.01) + 3362 \cos(25.74) + 4707 \cos(14.83) + 3922 \sin(-2.98) + 1085 \cos(5.95)]^2 .$$

This is gives

$$y_2^o = 26,800 \quad .$$

y_2^* was similarly calculated using post-fault values and was found to be 21,000.

The pre-fault and post-fault power-angle curves with corresponding amplitudes are shown schematically in Fig. V-8.

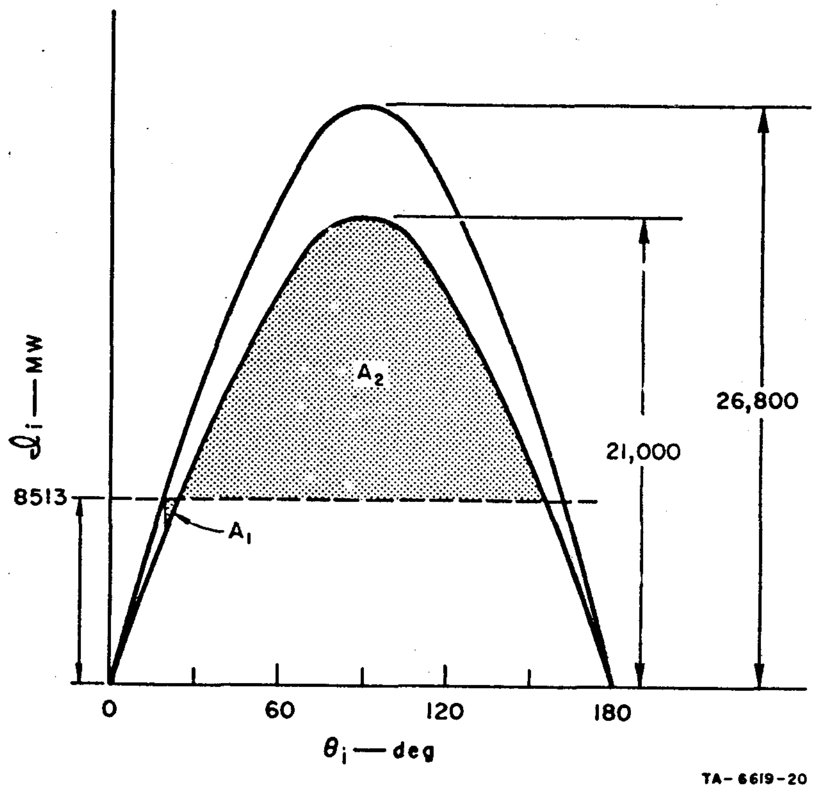

FIG. V-8 PRE-FAULT AND POST-FAULT POWER-ANGLE CURVES AT NODE 2

The net injection $\mathcal{J}_2$ (8513) at node 2 is also indicated. With this value of $\mathcal{J}_2$ at node 2, we now calculate areas A_1 and A_2.

$$A_1 = \left(\sin^{-1}\frac{8513}{21,000} - \sin^{-1}\frac{8513}{26,800}\right) \times 8513$$

$$+ 21,000\left[\cos\left(\sin^{-1}\frac{8513}{21,000}\right) - \cos\left(\sin^{-1}\frac{8513}{26,800}\right)\right]$$

$$= (0.42 - 0.32) \times 8513 + 21,000\ [0.913 - 0.949]$$

$$= 95$$

$$A_2 = 2 \times 21,000\ (0.913) - 8513\ [3.14 - 2 \times 0.42]$$

$$= 18,400 \quad .$$

Thus $A_2 \gg A_1$ and node 2 will remain stable when 2 × 500 kV lines are tripped from branch 1.

The measure of stability $S_m = (18,400 - 95)/18,400 = 0.994$. We can also estimate the transient stability power limit at node 2 as follows:

$$Z = \frac{y_1^o - y_1^*}{y_1^o} = \frac{26,800 - 21,000}{26,800} = 21.6 \text{ percent} \quad .$$

Referring to Fig. V-6, we see that for an admittance loss of 21.6 percent, the transient stability limit is 68 percent of the steady-state power limit, i.e., the transient stability power limit at node 2, according to approximate method, is 0.68 × 26,000 = 18,200 MW.

The stability at node 1, connected at the other end of the branch losing the line and any other node likely to be affected by such a loss, can be tested in a similar way. The stability at node 1 was also tested, and this node was also found to be stable, with a measure of stability of about 0.99.

6. Verification of the Results by Computation of Swing Curves

In order to test the accuracy of the results obtained by the approximate method, a computer program was written to numerically integrate the dynamic equations of the form (V-9), giving the variations of the angles $(\theta_2 - \theta_1)$, $(\theta_2 - \theta_3)$, ..., $(\theta_2 - \theta_8)$, during the transient period immediately after line 1 loses 2 × 500 kV lines (i.e., swing curves). The inertia constants at various nodes were assumed proportional to the power at the node. As an example, the dynamic equation for $(\theta_2 - \theta_1)$ is given by

$$\frac{d^2(\theta_2 - \theta_1)}{dt^2} = \frac{1}{M_2}\Big[\mathcal{I}_2 - Y_{12}\sin(\theta_2 - \theta_1) - Y_{24}\sin(\theta_2 - \theta_4) - Y_{25}\sin(\theta_2 - \theta_5)$$
$$- Y_{26}\sin(\theta_2 - \theta_6) - Y_{23}\sin(\theta_2 - \theta_3)\Big]$$
$$- \frac{1}{M_1}\Big[\mathcal{I}_1 - Y_{12}\sin(\theta_1 - \theta_2) - Y_{14}\sin(\theta_1 - \theta_4)\Big] .$$

Similar equations were written for angle $(\theta_2 - \theta_3)$, $(\theta_2 - \theta_4)$, ..., $(\theta_2 - \theta_8)$, and all angles were expressed in terms of the angular differences $(\theta_2 - \theta_1)$, $(\theta_2 - \theta_3)$, ..., $(\theta_2 - \theta_8)$.

The swing curves were obtained for various inputs at node 2 with corresponding adjusted powers at other nodes. Two sets of these swing curves are shown in Figs. V-9 and V-10. In Fig. V-9 all the angles, $(\theta_2 - \theta_1)$, $(\theta_2 - \theta_3)$, ..., $(\theta_2 - \theta_8)$, oscillate about an average value indicating stability. The oscillations do not damp out because no damping has been included in computing these curves. The input at node 2 for this case is 16,513. The increased input from 8513 MW to 16,513 MW at node 2 was distributed at consumption nodes roughly in proportion to original ratios. The values actually used are indicated on the top right side of Fig. V-9.

In Fig. V-10 the swing curves indicate instability for an input power of 17,513 at node 2. Thus, according to the calculation based on swing curves, the transient stability limit at node 2 lies between 16,500 MW and 17,500, while the approximate method had indicated the limit to be about 18,200 MW, which is about 9 percent higher than the lower value of 16,500, based on computation of the swing curves. Two other examples, one with 3 nodes and the other with 4 nodes, were also tested in a similar way, using approximate method as well as computation

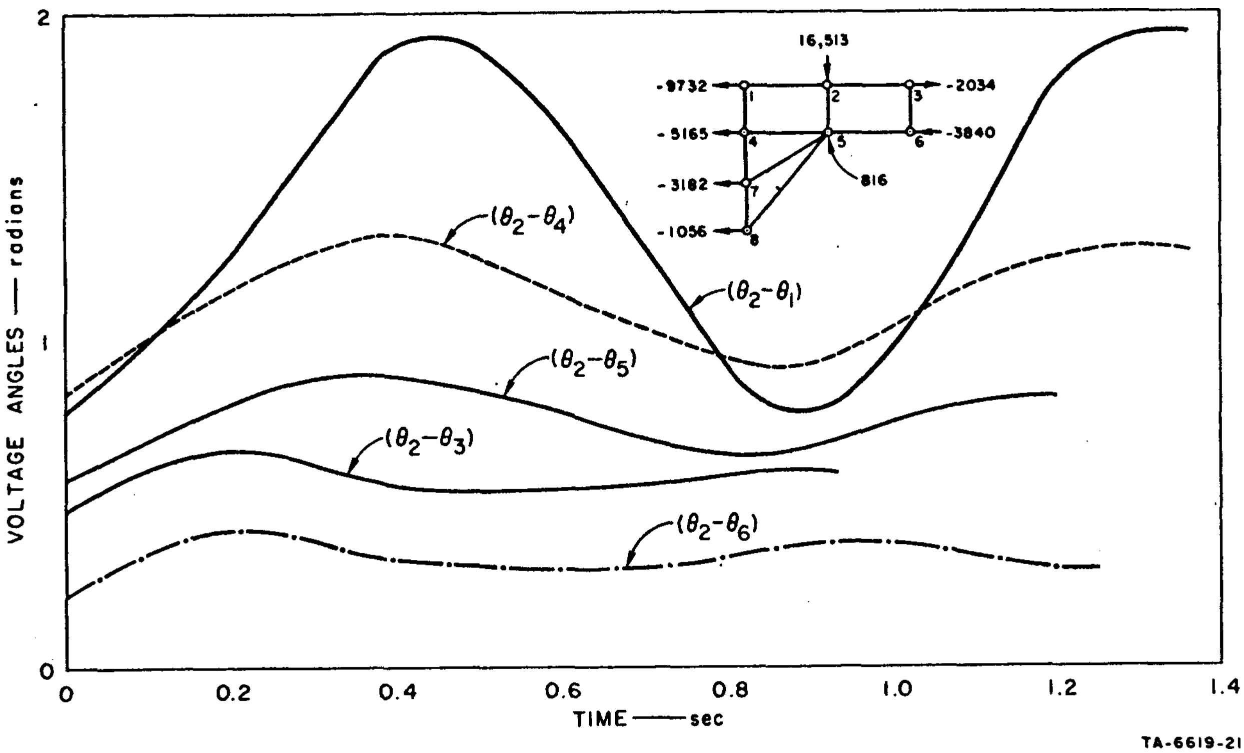

FIG. V-9 SWING CURVES FOR NODE 2 WITH INITIAL INPUT AT THIS NODE EQUAL TO 16,513, WHEN BRANCH 1 LOSES 2 × 500 kV LINES

of swing curves. In both these cases, the results given by the approximate method were found to be correct within 10 percent of those given by swing curves.

A brief summary of the 3-node example is given below. The pre-fault and post-fault values of γs are indicated in Fig. V-11.

It was assumed that $\mathcal{J}_3 = 0$ in both pre-fault and post-fault conditions so that $\mathcal{J}_1 = -\mathcal{J}_2$ in each case. Voltages at all nodes were assumed constant at unity. Considering various values of $\mathcal{J}_1$, at node 1, the results obtained by approximate method and by computation of swing curves were

$\mathcal{J}_1$	Results by approximate method	Results by swing curve computation
1	stable	stable
1.15	stable	stable
1.2	stable	unstable
1.3	unstable	unstable

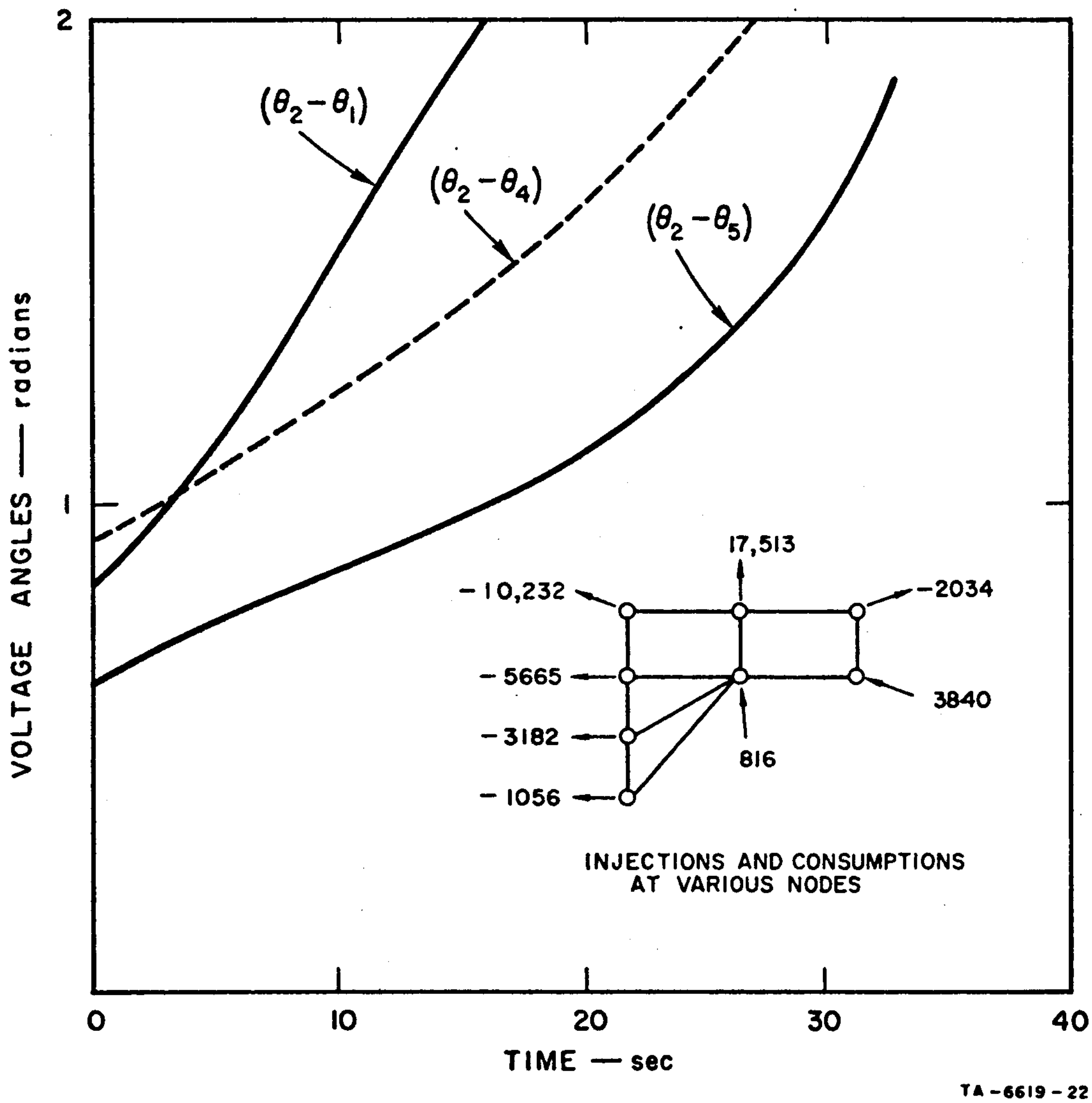

FIG. V-10 SWING CURVES AT NODE 2 WITH INITIAL INPUT EQUAL TO 17,513 AT THE NODE

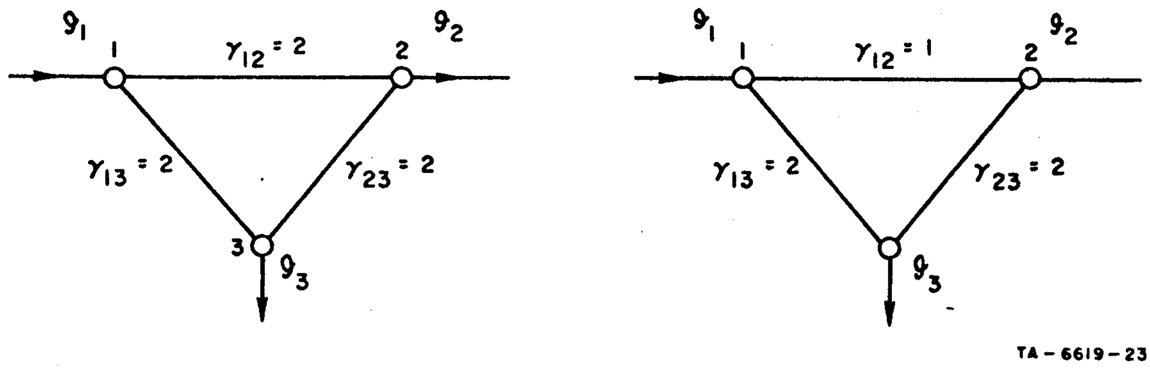

FIG. V-11 A 3-NODE MODEL

These results indicate that according to approximate method the transient stability limit at node 1 is between 1.2 and 1.3, whereas, according to swing curve computations, the limit lies between 1.15 and 1.2. Thus, the results of approximate method and swing curve computation differ from each other by only 5 to 10 percent.

C. Concluding Remarks

The results of the 8- and 3-node model examples presented above (and other examples not reported here) indicate that the approximate method can be very useful in providing a quick, yet fairly accurate estimate of the transient stability power limits at various nodes of a power system. Alternatively, if the transient stability of a system is to be tested for a given fault, the approximate method can be used to test the stability in a much shorter time, and with much fewer computations, than the conventional method of computing swing curves. The saving in time and the simplicity of calculations in the approximate method can thus be very valuable in the transmission system planning, at least in the early stages of the design. Figures V-6 and V-7 are very useful for a quick reference to test the stability and the measure of stability for a given condition.

The approximate method requires only the results of the pre-fault and post-fault steady-state load-flow studies. After this, only a few algebraic computations are required to test the stability or to calculate the transient stability limits. Even with the knowledge of only the pre-fault operating angles and pre-fault and post-fault admittances (i.e., the knowledge of post-fault steady-state operating angles is not absolutely essential, although accuracy is improved if these are known), it is possible to test the transient stability or to calculate the transient stability limits at various nodes approximately, since in the calculation of y_i^* (post-fault amplitude of equivalent power-angle curve) the <u>pre-fault</u> values of angles $\left(\theta_j - \theta_k\right)$ $j,k \neq i$ can also be used without introducing serious errors. Thus, further time and computation can be spared with only a small loss in accuracy.

The results of 8-node model also confirm the suspicion that the uniform limit of 36° on all voltage angles is too conservative. With the input power of 16,513 at node 2, the operating angle $\left(\theta_2 - \theta_1\right)$ and $\left(\theta_2 - \theta_4\right)$ are 46° and 48°, respectively, and the system is still stable when 2 × 500 kV lines drop from branch 1. Thus, it appears that the use of a uniform 36° angle criterion will probably result in an overdesign of the system.

It is possible to extend the ideas of the approximate method and to modify it suitably to take into account the effects of governor and voltage regulators.

Furthermore, it is possible to explore the potential of improving the accuracy of the approximate method by not neglecting the variations in the angles $\left(\theta_j - \theta_k\right)$; $j,k \neq i$ (which in this section were assumed to adopt the post-fault values immediately after tripping of the line) during the transient period. Assuming however that these variations are small, the terms involving $\left(\theta_j - \theta_k\right)$ could be linearized. It is expected that the resulting system of dynamic equations, which will contain mostly linear terms, would be much easier to analyze and would be relatively more accurate than the approximate method presented in this section.

VI THE DISCRETE NOMINAL

A. Introduction

1. Reasons for a Nominal Expansion Schedule

In the problem of the optimal expansion schedule of a transmission system, it is assumed that the initial state of the system and the injection functions $\mathcal{J}_i(t)$ for the entire planning period are given. A "feasible" expansion schedule is a set of line additions (or deletions) that ensure that all the existing constraints are permanently satisfied during the planning period. Let S be the set of all feasible expansion schedules. If investment costs and operating costs (power losses) are calculated, then an optimal expansion schedule, which minimizes its total cost over all feasible schedules in S, can be defined.

To determine this optimal schedule, a two-phase method can be considered: first find a "nominal" expansion schedule, which is an element of set S. Then, in the next phase, gradually modify the previous schedule in a way that minimizes total cost, without departing from the set S of feasible schedules.

The determination of a feasible nominal schedule is the subject of this section. This schedule must not be regarded as a final result by itself, but merely as a starting solution for phase 2, where it will be gradually improved with respect to total cost. The nominal solution must be both feasible and easy to obtain.

2. Continuous vs. Discrete Nominal

In principle, any feasible schedule could be regarded as a possible starting solution for a successive approximation method. However, the closer this nominal is to the optimal solution ultimately sought, the faster convergence can be expected. Therefore, some cost considerations must be included in the determination of the nominal, as far as they do not overcomplicate the search for the first approximate solution.

The dimensionality of the cost optimization problem is greatly reduced by breaking the total cost optimization into a sequence of separate optimizations, one for each year, assuming the results just obtained for the preceding year. Given the state of the system in year (t - 1), the nominal procedure will find the most economical state of the system that will yet be able to meet the requirements of year t. Thus, long-term effects, such as economics of scale, are temporarily left aside in this determination of a nominal. However, emphasis is to be put on the feasibility of the resulting nominal, rather than on its overall economy.

For the above annual optimization, it may seem convenient to temporarily release the constraint that the lines have to be chosen from a discrete catalog, and to look instead for a "continuous" annual optimization of the branch capacities γ_k. Once this continuous expansion schedule is obtained, it will represent the "ideal" growth of the system; it will then be possible to approximate those γ's with actual combinations of lines and to obtain a fairly good starting solution for the successive approximations to come.

An alternate way of obtaining a nominal solution is explained in this section. In this method, continuous variation of the γ's never has to be assumed, and only discrete line additions are to be performed. Feasibility is always guaranteed, and this is the principal requirement for the nominal solution.

B. The Discrete Nominal Optimization Method

1. Reliability Constraints

Reliability has been defined as not exceeding a maximal angular difference in any branch of the network. This formulation is well suited to a purely mathematical model, as applied to the continuous version of the problem. We could keep it in our discrete nominal optimization, but a more sophisticated (and realistic) formulation of the reliability constraints can be substituted for it without leading to overwhelming difficulties.

Each type of line is characterized by a maximal flow capacity T_{max}, beyond which the line is automatically cut by protection devices. Since each type of line is also characterized by a coefficient $\gamma = V^2/X$ (per unit of length), the upper bound actually applies to the ratio T/γ, and this ratio is precisely the angular difference ψ (per unit of line length). Thus, directly introducing capacity constraints on the line flows T is equivalent to imposing a maximal angular difference $\alpha = T_{max}/\gamma$ for each single line of the network.

Given a certain layout of the system, the angles θ_i are computed at each node, and then the line flows T_k are readily available, using the formula:

$$T_k = \gamma_k \left(\theta_i - \theta_j\right) \quad . \qquad \text{(VI-1)}$$

[Here k refers to a line of the network.] The system is acceptable if none of its lines shows any overload.

We shall say that the system is reliable if the loss of any line, possibly the strongest, does not cause any overload. Thus, in order to be declared reliable, a configuration must pass a series of r tests, each one proving that it resists the loss of the strongest line in one of its r branches.

2. General Survey of the Method

The present study is made with the assumption that the demands of flow are monotonically growing during the planning period. This is usually true in the power industry; however, some additional refinements ought to be added if we wanted the model to apply to any growth pattern, possibly non-monotonical.

Let us assume a Q-year planning period. The algorithm performs Q iterations, one for each year, starting in year 1. The state of the system in year t is obtained from the state found in year (t - 1), by performing the least costly set of line additions that enables the system to keep up with the new injections in year t.

The procedure works as follows: the year t injections are applied to the final system of year (t - 1). The strongest line of each branch is identified and, in turn, each of these r lines is supposed to be cut. Power flows are computed, and the line that has been cut is reinserted into the network prior to the cutting of the next line. If overloads arise anywhere when a line has been cut, then the program will compute the most economical capacity addition to a single branch that makes all these overloads disappear. An extensive use is made of the first order sensitivity equations relating $\Delta\psi$ to $\Delta\gamma$.[5] The capacity additions are performed in terms of actual lines taken from the catalog.

3. Flow Chart

The flow chart (Fig. VI-1) illustrates the explanations of the preceding paragraph. Each of its elements will be explained with more detail.

a. Initiate

The state of the system at the end of year 0 is given, and it is assumed to meet all the reliability requirements.

b. Read Injections $\mathcal{J}_i(t)$

In the present state of the algorithm, the injections are assumed not to decrease from one year to the next.

c. Ordering Procedure for the Branches

The results of the nominal algorithm are sensitive to the order in which the branches are considered for cutting their respective strongest lines. The following ordering rule seems appropriate: a preliminary power-flow computation is performed without cutting any line. For each branch k we compute $\Delta\psi_k$, the additional angular difference that would bring the first overload in some line of branch k (all $\Delta\psi_k$ are positive if no overload exists yet in the network). The r branches are then ranked by increasing $\Delta\psi_k$'s, i.e., the closer to saturation are to be considered first in the scanning that follows. These branches are likely to be the most critical, and thus it seems

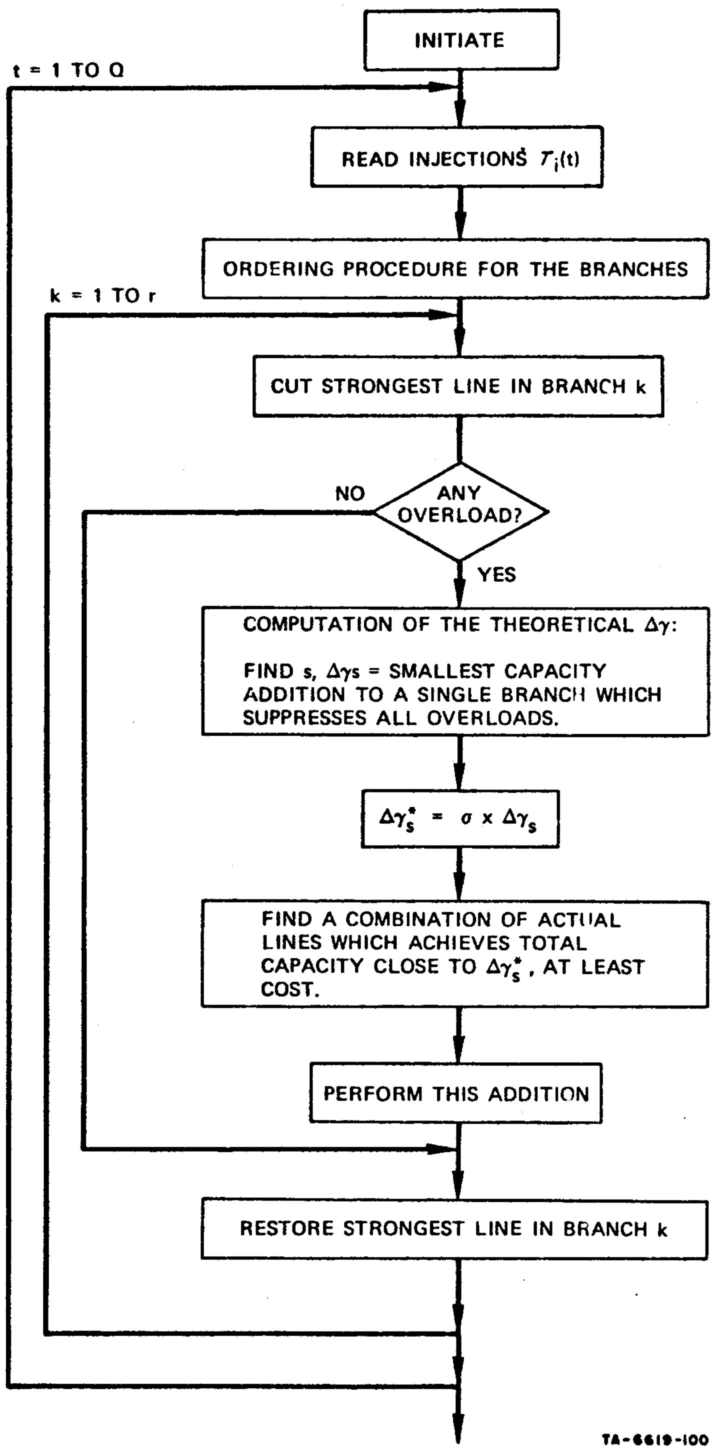

FIG. VI-1 FLOW CHART OF NOMINAL EXPANSION SCHEDULE reasonable to look at them first. However, alternate ranking rules could also be imagined.

d. Power Flows

After the flows T have been obtained, an overload coefficient can be computed for each line of the network:

$$\rho = \frac{T}{T_{max}} \quad . \tag{VI-2}$$

If $\rho < 1$, then the line will carry the load; otherwise it is overloaded.

e. Computation of the Theoretical $\Delta\gamma$

For each branch k where overloads exist, let $\Delta\psi_k$ be the smallest variation in ψ_k that would cause all the overloads of branch k to disappear. Once $\Delta\psi_k$'s have been computed for all the overloaded branches, we search for the smallest $\Delta\gamma_s$ that, if applied to a single branch s, would yield at least a $\Delta\psi_k$ angular variation in each of the saturated branches k, and thus desaturate all of them.

To compute $\Delta\gamma_s$, we use the first-order sensitivity matrix B*

$$\Delta\psi = G_\theta^{-1} G_\gamma \Delta\gamma = B \Delta\gamma \quad . \tag{VI-3}$$

Since we have decided that only one branch k' will be modified to cure the overloads,

$$\Delta\psi_k = B_{kk'} \Delta\gamma_{k'} \tag{VI-4}$$

and then

$$\Delta\gamma_s = \min_{k'} \max_{k} \frac{\Delta\psi_k}{B_{kk'}} \quad . \tag{VI-5}$$

*Developed in Sec. V and in Ref. 5.

The computer printouts presented with this report show that branch s is usually the one that has just been deprived of its strongest line (which means that s = k on the flow chart). This fact seems to indicate that the best way to cure overloads by a single-branch addition is to add capacity to the very branch that has generated the overloads by a drop in its capacity. However, the procedure will pick another branch, if the required addition to it is less costly.

The error brought by the use of a first-order formula can be very much reduced now by performing a second iteration of the above computation, as in a Newton-Raphson computation. γ_s is replaced by $\gamma_s + \Delta\gamma_s$, and the power flows are computed again. The correction to add to $\Delta\gamma_s$ is obtained by the same first-order approximation as above. In the computational examples that so far have been tested, the latter correction is usually much smaller than the first approximation $\Delta\gamma_s$. This fact is a check of the validity of our first-order sensitivity formula.

Finally, the addition of a capacity $\Delta\gamma_s$ to the single branch s is guaranteed to make all overload disappear throughout the network. Furthermore, this is the smallest single branch capacity addition that has this effect. The resulting $\Delta\gamma_s$ value is called "Theoretical $\Delta\gamma$" in the computer printouts.

f. $\underline{\Delta\gamma_s^* = \sigma \times \Delta\gamma_s}$

σ is a factor (2 in the given example) by which $\Delta\gamma_s$ is multiplied before the actual capacity addition is performed. This is an attempt to take into account economies of size in a very simple manner.

$\Delta\gamma_s$ is a minimal capacity addition, which allows the system to be reliable under year t's injections, but not under larger injections. If, as we have assumed, demands are monotonically increasing, then separate capacity additions will have to be performed every year. By multiplying the required capacity addition by a factor $\sigma > 1$, we force this addition to be large enough to eliminate requiring any new one during the next few years.

g. Find a Combination of Actual Lines Achieving a Total Capacity Close to $\Delta\gamma_s^*$, at Least Cost

An optimal algorithm for this subproblem can be established. However, in the current procedure, a simpler policy is used: a series of permissible line combinations has been established, each of them achieving a specific $\Delta\gamma$ per unit of length. All branches have to be treated separately, for the actual γ's are obtained by multiplying these figures by the respective branch lengths; the resulting numbers are entered into a tableau.

Once $\Delta\gamma_s^*$ has been obtained, now s of this tableau is considered, and the procedure selects the combination that realizes a $\Delta\gamma$ just higher than $\Delta\gamma_s^*$ (unless there exists a combination with a $\Delta\gamma$ within the interval $[90/100\ \Delta\gamma_s^*,\ \Delta\gamma_s^*]$, which would be preferred).

The procedure described just above is but one among several selection procedures that can be imagined.

C. Computational Example

The nominal expansion schedule of a 5-node, 7-branch system is shown here. Three types of lines are available: 230 kV, 345 kV, 500 kV. The planning period extends over 5 years. Figure VI-2 shows the initial state of the system. In this example, the node injections have been assumed to increase at an 8 percent rate every year.

The required line additions are shown in Table VI-1.

Table VI-1

LINE ADDITIONS REQUIRED BY THE NOMINAL EXPANSION PROGRAM

Year	Branch	Number and Type to be Added
1	1	2 × 500 kV
2	3	1 × 500 kV
2	2	1 × 500 kV
3	6	1 × 230 kV
4	3	1 × 500 kV
4	2	1 × 345 kV
4	6	1 × 230 kV
5	2	1 × 500 kV

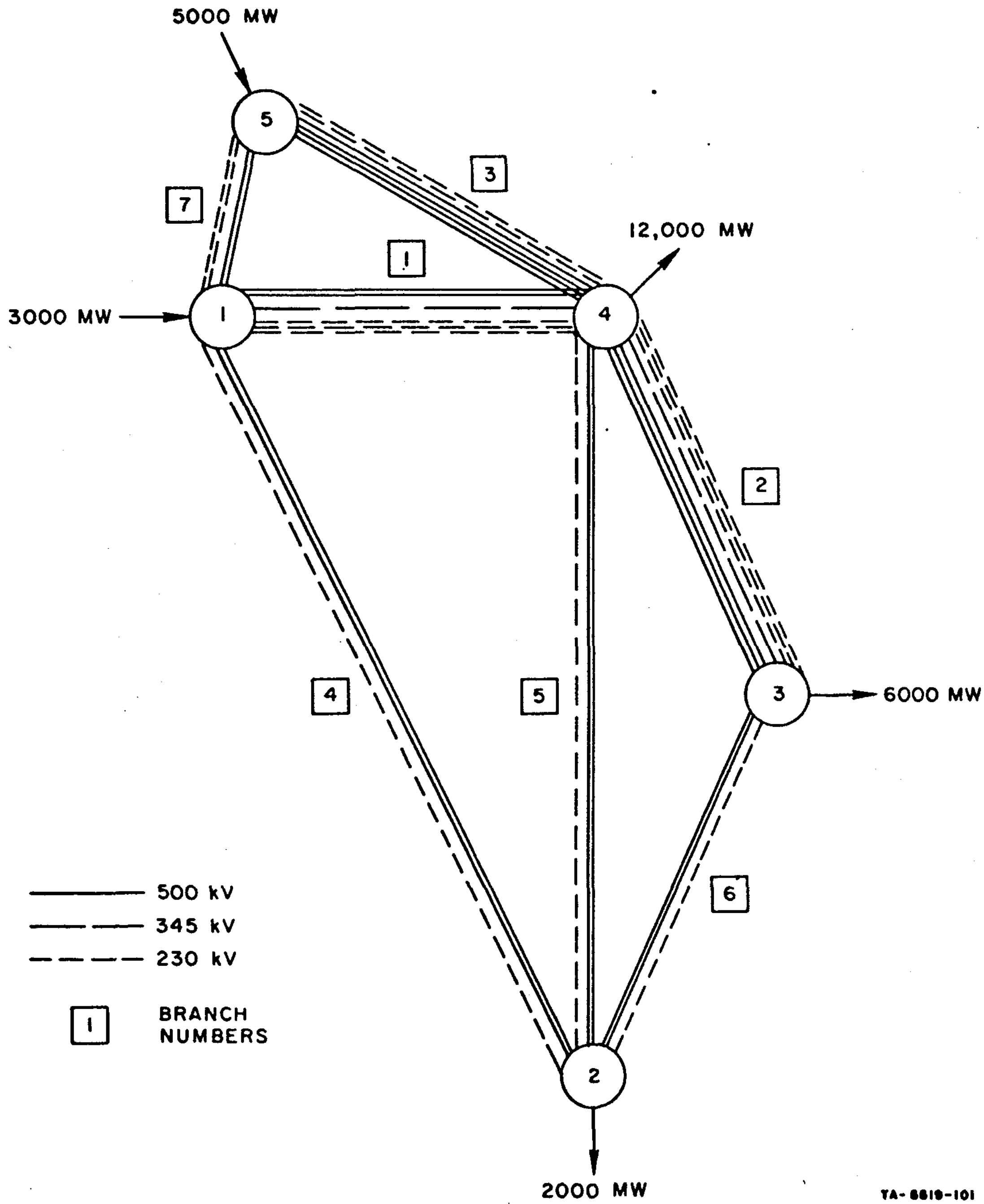

FIG. VI-2 STATE OF THE SYSTEM AT THE END OF YEAR 0 (The dimensions on this figure are approximately proportional to the actual line lengths.)

VII OPTIMAL DISCRETE APPROACH TO OVERALL TRANSMISSION-SYSTEM PLANNING

A. General Survey of the Proposed Method

Section VI described a way to obtain a "nominal" expansion schedule for a transmission network. The prime goal of this procedure was to guarantee feasibility of the resulting schedule with respect to the reliability constraints. Minimization of total cost was an important, but nevertheless secondary, objective.

Our purpose is now to analyze the ways of improving the above schedule until possibly reaching the optimal expansion schedule of the system. The principal difficulty is the high dimension of the problem, which discourages methods like straightforward dynamic programming. An operational method of successive approximations in dynamic programming has been suggested by R. E. Bellman,[44] and successfully applied to some high dimensional practical cases.[36,37] It is this method that we shall use here to improve our nominal expansion schedule. The need to satisfy the reliability constraints on line flows will require some special techniques, which will be explained below.

A series of iterations are performed, each yielding a schedule that is still feasible, but less costly than the preceding one. At each iteration, we shall assume that (r - 1) of the r branch schedules are temporarily fixed, and we shall try to improve the r^{th} branch schedule above. Once it is done, we turn to another branch schedule, and repeat the operation until no branch schedule can be modified any longer in a profitable way.

It is very important that at no step of the procedure the current expansion schedule become infeasible with respect to the reliability constraints. The nominal expansion schedule from which we shall start has itself been constructed to be, above all, feasible.

A condition for such a procedure to lead to a true optimum is that there exists some degree of coupling between the different branch schedules, which are separately modified in the successive approximation procedure. This is a question whose theoretical nature will have to be deeply analyzed.

B. Improvement of the Expansion Schedule of a Branch k

1. Introduction

We shall now describe one iteration of the successive approximation method. Let us assume that a feasible expansion schedule for all the network has been obtained. We shall try to reduce the total cost of this schedule by modifying branch k's expansion schedule above. The reliability constraints are to be satisfied every year. As explained in Sec. VI, the system must be able to resist the loss of the strongest line of any of the branches.

2. Determination of the Feasible Trajectory Domain

We call a trajectory the succession of the values taken by branch k's capacity through its expansion schedule (Fig. VII-1). Let $\gamma_{k,t}$ be the value of γ_k in year t. Our objective is to modify the current trajectory in an optimal way, without violating any reliability constraint.

We use the first-order sensitivity relation

$$\Delta\psi = B\ \Delta\gamma \qquad \text{(VI-3)}$$

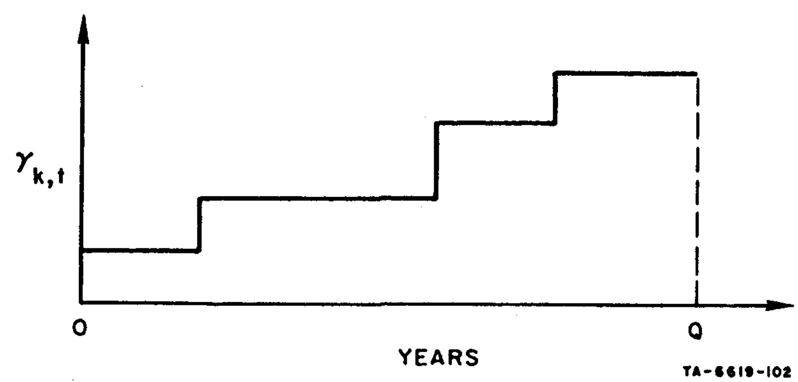

FIG. VII-1 EXAMPLE OF A TRAJECTORY: BRANCH k EXPANSION SCHEDULE to find the set of possible variations $\Delta\gamma_{k,t}$ around the current feasible trajectory. This computation is almost identical to the determination of the "theoretical $\Delta\gamma$" explained in Sec. VI. The procedure successively cuts (and restores afterwards) the strongest line of each branch, and computes the matrix B at each time. Since the system we start from is assumed to meet the reliability constraints, no overloads should appear after any of these cuts. During each of the cuts, we use the elements of the corresponding matrix B to find the smallest $\Delta\gamma_{k,t}$ (in absolute value) that will bring the first overload at any place in the network. $\Delta\gamma_{k,t}$ is usually negative (overload brought by reduction of capacity of branch k), but sometimes it is positive, which means that there is an upper bound to the potential increase of branch k's capacity.

A second (Newton-Raphson) iteration can be performed (like in the nominal procedure) to obtain more accuracy for the bound.

Once the r bounds on $\Delta\gamma_{k,t}$ have been obtained, they are compared with another, and a "most restrictive" interval is found, i.e.,

$$\underline{\gamma}_{k,t} \le \gamma_{k,t} \le \overline{\gamma}_{k,t} \quad , \qquad \text{(VII-1)}$$

which guarantees reliability for all values of $\gamma_{k,t}$ within it. The flow chart of the above operations is represented in Fig. VII-2.

Thus a "permissible trajectory domain" is defined by an upper-limiting and a lower-limiting trajectory (Fig. VII-3). We shall now restrict our search for improved branch k's expansion schedules to within this domain, and thus all the reliability constraints will be automatically taken care of.

Remark: If $\underline{\gamma}_{k,t} < 0$, the actual lower bound on $\gamma_{k,t}$ is taken to be 0 (or a small positive value). This simply means that the system is not very sensitive to what happens to branch k in year t.

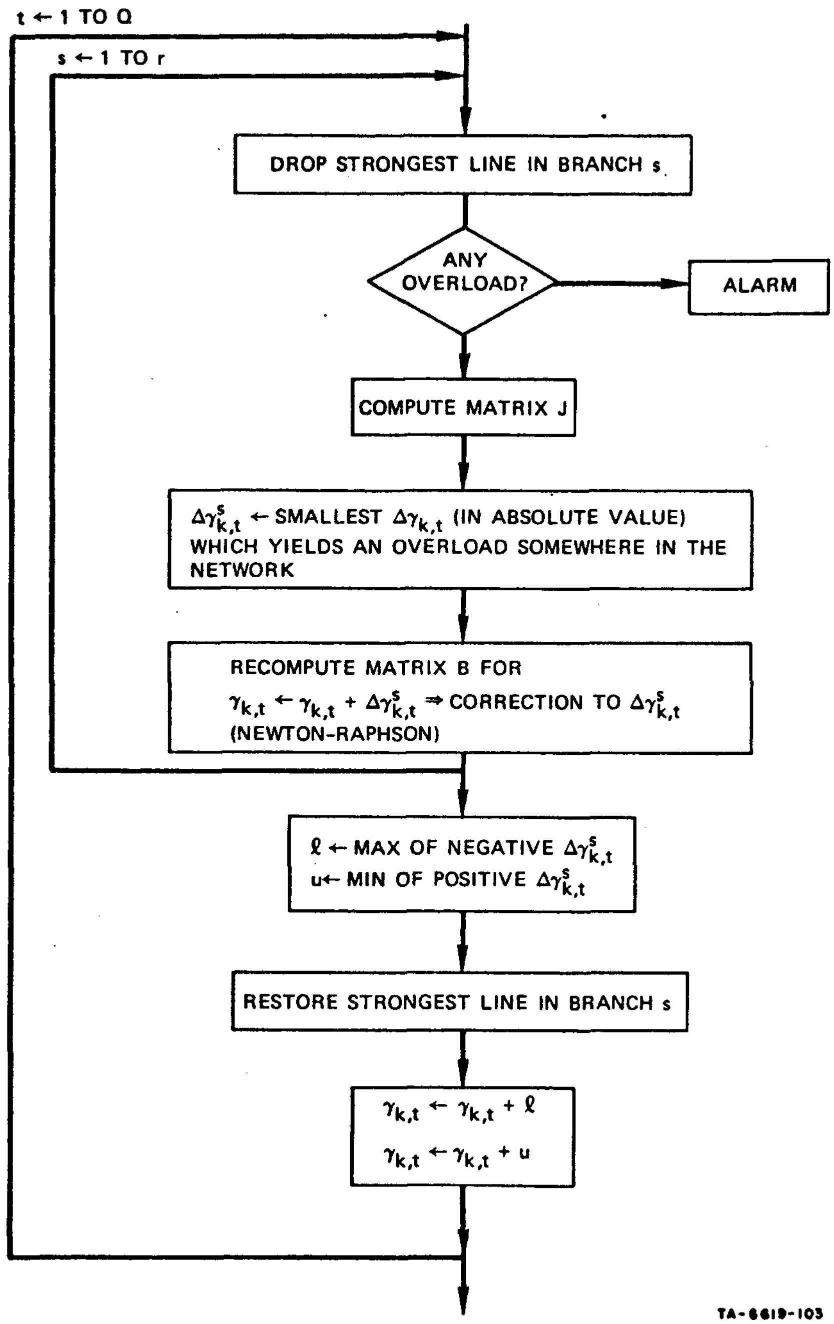

FIG. VII-2 FLOW CHART OF THE DETERMINATION OF THE PERMISSIBLE DOMAIN OF VARIATION OF $\gamma_{k,t}$, FOR A GIVEN BRANCH k AT TIME t

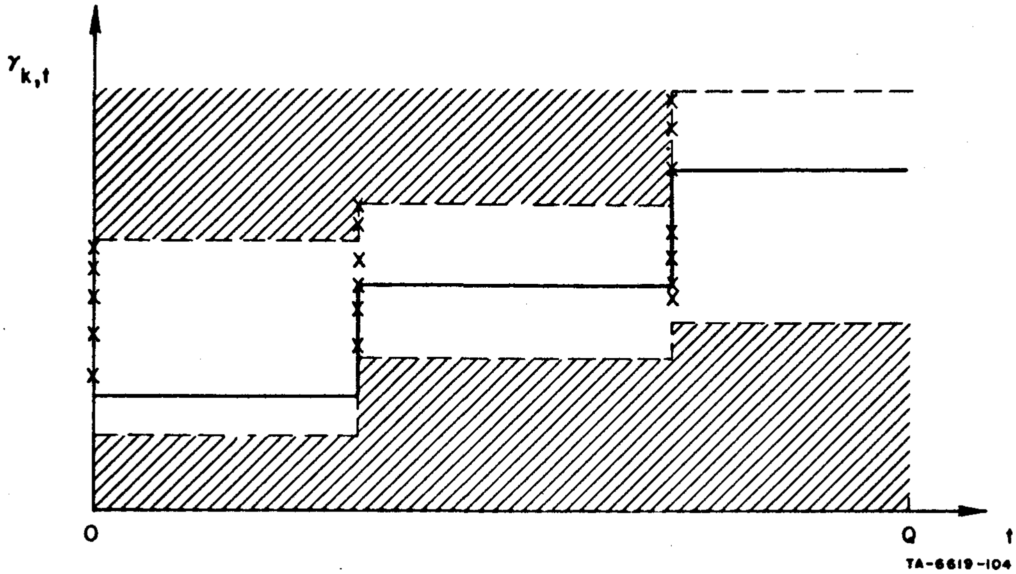

FIG. VII-3 SET OF THE ACCEPTABLE TRAJECTORIES

3. Finding Acceptable Line Combinations

The capacity γ_k of branch k cannot actually vary in a continuous fashion, since branch k must be composed of a combination of actual lines taken from a catalog. The number of possible combinations between several types of lines is unlimited, but we shall accept only those combinations that will achieve a total branch capacity $\gamma_{k,t}$ within the permissible interval defined in the preceding paragraph.

Since we naturally prefer the low cost combinations, this is really a kind of a "knapsack problem"; the set of the possible trajectories is thus reduced to those that are sequences of acceptable line combinations. In Fig. VII-3, these combinations appear as points (with a definite $\gamma_{k,t}$), and the possible trajectories are the lines joining these points from year to year throughout the planning period. Their feasibility with respect to all the reliability constraints is guaranteed, since they all belong to the permissible trajectory domain.

4. Dynamic Programming Optimization of Branch k's Expansion Schedule

The problem of determining the least costly trajectory among all those passing through the acceptable line combinations can be easily solved by a single-dimensional dynamic programming procedure. The state of branch k at time zero is known, although we do not know in what state we want it to be at the end of the planning period. Therefore, we choose forward dynamic programming, instead of backward.

The cost function to be minimized is the sum of the investment costs and the operating expenses. They are analyzed in the two paragraphs below.

a. Capital Investment Costs

The accounting model used here has already been explained in Sec. II-D-2-b. In the computational examples presented below, no geographical constraint has been taken into account. Therefore, no line is ever torn down and there are no penalty costs.

b. Operating Expenses

They represent the cost of the losses of power throughout the network, which are studied with more detail in Appendices A and B and in Ref. 2. It is very convenient in the dynamic programming algorithm to use the second-order approximation formula that is developed in Appendix B. It is then sufficient to compute the losses exactly for only one trajectory and to use the second-order approximation for the neighboring trajectories. This saves a large amount of computer time.

C. Computational Examples

1. Preliminary Remarks

The following examples are intended to illustrate the sequential optimization method explained above. In each case, a nominal expansion schedule has been determined by the procedure explained in Sec. VI. The resulting schedules are shown below with the corresponding expansion costs.

The expansion cost of a schedule is defined as the compounded sum of all the expenses incurred during the planning period that are affected by the decision variables. It includes both the cost of losses and the cost of new capital investment, which are computed as explained in Sec. II. But it does not include the installments corresponding to the lines built before the beginning of the planning period, even if many of these installments are actually paid during the planning period.

This explains why in the results shown below the cost of losses seems surprisingly to be much larger than the capital expense: the cost of losses reflects the power losses in the whole system, while the capital expense shown here corresponds only to the line additions that take place during the planning period. The capital expense for the previously built lines, though a major cost item during the planning period, does not appear here, since no decision variable can affect it.

2. Expansion Schedule of a 3-Node System

The system is shown in its initial state in Fig. VII-4. We assume a 4-year planning period, from the end of year 0 to the end of year 4. Every year the power injections increase by 8 percent.

We see that after one dynamic programming optimization of branch 2, the two expansion schedules become identical. In fact, although the nominal schedules are different, the domains of feasible trajectories around the nominal are quite similar. Figure VII-5 illustrates this fact: it shows, in both cases, the nominal trajectories of branch 2 together with the respective domains of feasibility and the final common trajectory after the dynamic programming optimization.

Tables VII-1 and VII-2 show the line additions to the initial system. The nominal is obtained by the method of Sec. VI; the dynamic programming algorithm is then performed to reduce the total expansion cost of each branch in sequence. The tables show the first two iterations.*

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*The branches are scanned by order of decreasing investment cost in the nominal schedule. Therefore, branch 2 comes first, followed by branch 3.

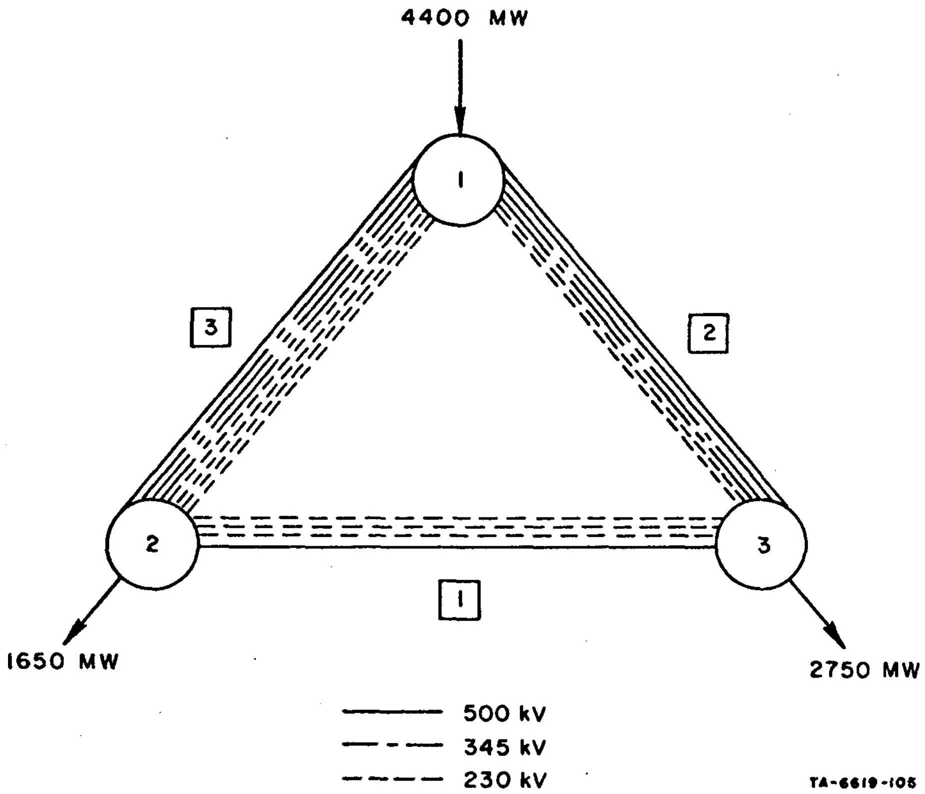

FIG. VII-4 INITIAL STATE OF THE SYSTEM

The two tables differ by their nominals. The nominal of Table VII-1 has been obtained by a strict application of the procedure described in Sec. VI. But Table VII-2 shows a voluntarily overinvested nominal, where the "theoretical $\Delta\gamma$" of Sec. VI is multiplied by 4 instead of 2 before choosing the actual line additions.

3. Expansion Schedule of a Single Branch

The complete procedure is now applied to the expansion of a 2-node, 1-branch system over a 5-year period. The initial state of the system is shown in Fig. VII-6.

The injections are assumed to increase by 7 percent every year. Figure VII-7 shows the nominal trajectory and the domain of feasibility. The permissible configurations are represented by the corresponding points on the figure.

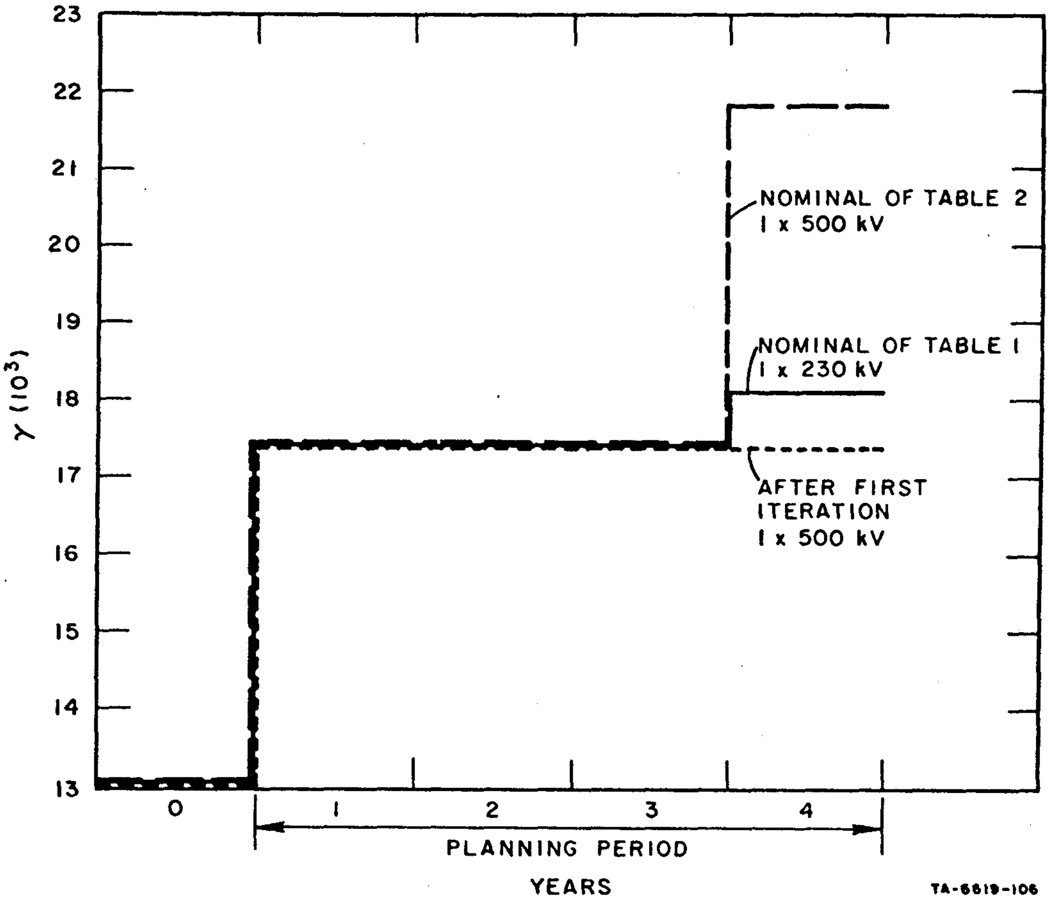

FIG. VII-5 EXPANSION SCHEDULE OF BRANCH 2 (3–Node System)

Table VII-1

DISCRETE OPTIMIZATION OF THE EXPANSION OF A 3-NODE SYSTEM

"Small" additions in the nominal.

	Nominal			After Dynamic Programming Optimization of Branch 2			After Dynamic Programming Optimization of Branch 3		
	Year	Branch	Additions	Year	Branch	Additions	Year	Branch	Additions
(In Thousands of Dollars)	1	2	1 X 500 kV	1	2	1 X 500 kV	1	2	1 X 500 kV
	4	2	1 X 230 kV			*			
	4	3	1 X 345 kV	4	3	1 X 345 kV	4	3	1 X 345 kV
Capital Expense	2402			2205			2205		
Cost of Losses	3083			3103			3103		
Total Cost of Expansion	5485			5308			5308		

*The addition of the 230 kV line is found to be unnecessary after the 345 kV addition in branch 4 has been performed.

Table VII-2

DISCRETE OPTIMIZATION OF THE EXPANSION OF A 3-NODE SYSTEM

"Large" additions in the nominal.

	Nominal			After Dynamic Programming Optimization of Branch 3			After Dynamic Programming Optimization of Branch 3		
	Year	Branch	Additions	Year	Branch	Additions	Year	Branch	Additions
(In Thousands of Dollars)	1	2	1 X 500 kV	1	2	1 X 500 kV	1	2	1 X 500 kV
	4	2	1 X 500 kV						
	4	3	1 X 345 kV	4	3	1 X 345 kV	4	3	1 X 345 kV
Capital Expense	2662			2205			2205		
Cost of Losses	2986			3103			3103		
Total Cost of Expansion	5648			5308			5308		

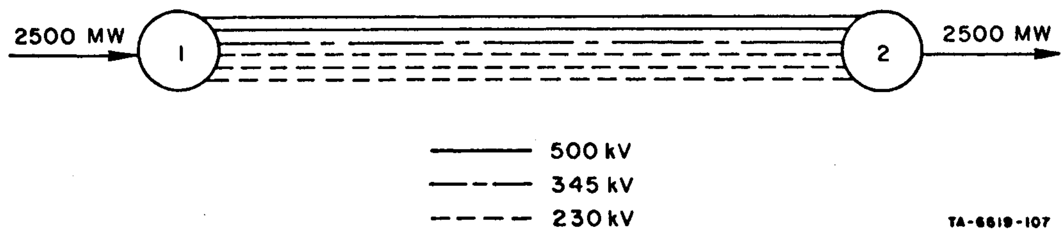

FIG. VII-6 STATE OF THE SYSTEM AT THE END OF YEAR 0

The dynamic programming shows that in this case no permissible trajectory can be found that is less costly than the nominal. However, this is not yet an absolute proof that we have reached the optimum: the discrete combinations of lines that are investigated in the dynamic programming do not scan all the possibilities; the procedure to obtain them (Sec. VI-B-2-f) has just been designed to give a simple illustration of

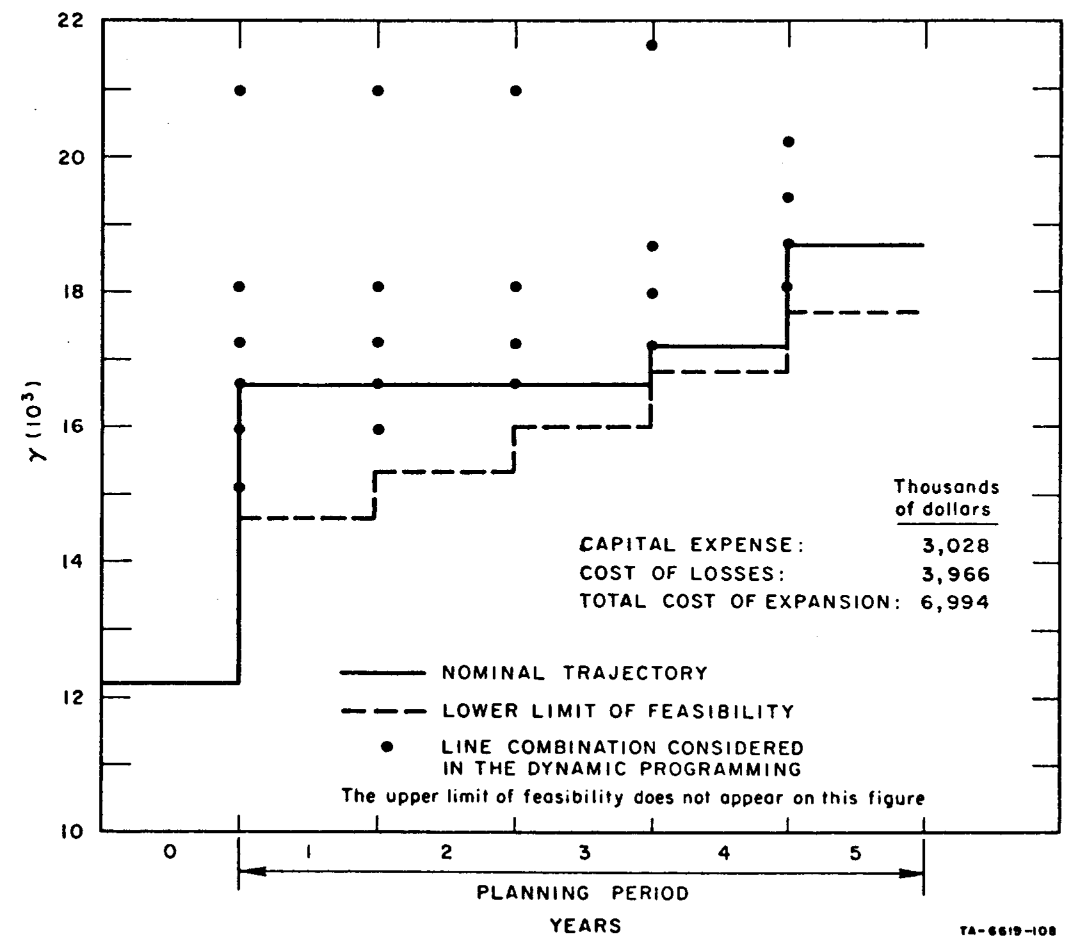

FIG. VII-7 EXPANSION SCHEDULE OF A SINGLE BRANCH the optimization method. It will be necessary--and probably not too difficult--to refine it further.

4. Capability of Existing Program

In the course of the project, more complicated examples than the 3-node and 2-node system of Figs. VII-4 and VII-6 were treated. The major amount of computer experimentation was performed on the 5-node 7-branch system of Fig. VI-2. The existing program accepts systems of up to 15 nodes and 20 branches. To save on computation cost, it was never run on a system of that size in the course of this study project.

5. Remark on Couplings[36]

The treatment of more complicated examples shows the great importance of "couplings" between the separate branch schedules, i.e., the fact that modifying the schedule of one branch will change the cost function for the study of the next branch. Without such interaction, one could not expect to find a true optimum for the whole system's expansion schedule.

The couplings are introduced here by the operating cost function (losses). Indeed, the program will tend to reduce capital investment, by delaying construction of new lines. But this tendency is counterweighted by two needs:

(1) The reliability constraints, which prevent reducing the strength of the system too much.

(2) The cost of losses, which increases when a branch is underequipped.

The cost of losses is expected to require reinforcement of the main branches when the less important ones are being reduced, i.e., it should reorient the layout of the entire system in a better fashion.

The unit cost of the MWH lost is thus a very important variable of the program. If it is too low, then the couplings introduced by the losses are not sensitive. If it is too high, then the program tends to build up the system with the prime incentive of reducing the cost of losses. A suitable intermediate value seems to be \$2.2 per MWH. By a mere coincidence, this value seems a reasonable estimate of the economical cost of the loss of a MWH.

It is still necessary to better understand the role of couplings and to develop theoretical criteria for convergence. At this stage, it seems likely that the results of the continuous nominal considered in Sec. III could be of great help here.

VIII OUTAGE ANALYSIS, OPTIMUM LOAD CURTAILMENT, AND EXPECTED UNSUPPLIED ENERGY

A. Problem Formulation

The massive power failure of 9 November 1965, and 17 of the 20 major failures that have occurred since that time have been essentially cascading failures, namely the loss of one line (e.g., due to a faulty operation of a relay or due to short circuit) resulting in the overloading of other lines, or causing excessive swings in the synchronous machines, which were tripped in turn by their protective relays and breakers. With these considerations in view, the Federal Power Commission[31] has correctly stressed the importance of analyzing, designing, and coordinating various interconnected power systems to make them as invulnerable as possible to such cascading phenomena.

The sudden loss of a heavily loaded line from a branch of a power system can result in the instability of the system due to either or both of the following reasons.

(1) Loss of Synchronism--One or more generators may undergo excessive swings and may be disconnected from the system. This, in turn, may produce further shock to the system, and more machines or transmission lines may be tripped so that cascading takes place.

(2) Overloading of Transmission Lines--The line loadings, particularly in the branch losing the line and in the other branches in the vicinity, may become excessive, and the respective overload protective relays may disconnect further lines, thereby starting a cascading operation, causing more and more lines of the system to trip, one after the other; consequently, the entire system may either disintegrate completely or may degenerate into subsystems (sometimes called islands) operating independently of each other.

The loss of stability due to excessive transient swing of the machines was discussed in Sec. V and in Ref. 5. In the present section, we consider outages due to overloading of transmission lines.

When a critical line is overloaded, two types of situations may occur:

(1) There is no planned curtailment of load or generation. Then, the power unbalances in the post-fault system may cause the overloading and subsequent tripping of transmission lines in a cascading manner, following the unscheduled outage of a critical line.

(2) There exists a policy of load- or generation-curtailment, or both, during an emergency to prevent the disintegration of the transmission system and reduce the amount of energy lost while the fault condition prevails.

Therefore, two types of programs were developed in the study:

(1) A line-outage-analysis program,[7] which simulates the transmission system disintegration so as to determine the energy lost because of the fault condition. This program is to be used when there is no planned curtailment of load or generation.

(2) An optimum-load-curtailment program,[10,35] which provides an emergency curtailment policy to minimize the amount of energy lost as a result of an initial fault, while maintaining the integrity of the power grid and preventing cascading failures.

B. Line-Outage-Analysis Program--No Planned Load Curtailment

1. Program Description

The program is written for an n-node, r-branch power system network assuming the following information is available as input data:

(1) Topography of the system, i.e.,

(a) Specification of the nodes connected to each other.

(b) Specification of the number of lines, their respective reactance X in ohms per mile, voltage rating V in kilovolt of each line, and the length L_{ij} in miles of all branches.

(2) The allowable load-carrying capacity $\bar{T}^{\ell}_{ij}$ in MW of each line (indicating the desired setting of the protective relays) in branch ij.

(3) The net injections $\mathcal{J}_i$ at all nodes.

(4) The initial perturbation (i.e., specification of the type of the line and the branch from which the line is tripped).

With the above input data available, the program goes through the following steps. It computes:

(1) The quantity γ per mile, defined by

$$\frac{V^2}{X}$$

for each line, where X is the reactance per mile.

(2) The line capacity γ^{ℓ}_{ij}, defined by

$$\gamma^{\ell}_{ij} = \frac{V^2}{L_{ij} X_{\ell}}$$

for each line ℓ in branch ij for the unperturbed condition, L_{ij} being the length in miles of the respective branch.

(3) The branch capacity γ_{ij}, defined by

$$\gamma_{ij} = \sum_{\ell} \gamma_{ij}^{\ell}$$

for all branches in the unperturbed condition.

(4) It then solves the linearized power-flow equations

$$\mathcal{J}_i = \sum_j \gamma_{ij} \left(\theta_i - \theta_j\right) \qquad \text{(VIII-1)}$$

for the angles θ_2, θ_3, ..., θ_n (assuming $\theta_1 = 0$ is taken as reference). The linearized power-flow equation (VIII-1) and assumption $r \ll X$ (implying $Z \simeq X$) have been used for simplicity. The program can, however, be easily modified if exact power-flow equations are to be used and line resistance is not to be neglected.

(5) Knowing θ_i and θ_j, the power-flow T_{ij}^{ℓ} in each line of the branch ij is calculated using the relation

$$T_{ij}^{\ell} = \gamma_{ij}^{\ell} \left(\theta_i - \theta_j\right) \qquad \text{(unperturbed condition)} \quad .$$

(6) Using the values of T_{ij}^{ℓ}, a quantity α_{ij}^{ℓ}, defined by

$$\alpha_{ij}^{\ell} = 1 + \frac{T_{ij}^{\ell} - \bar{T}_{ij}^{\ell}}{\bar{T}_{ij}^{\ell}} \quad ,$$

is computed. α_{ij}^{ℓ} could be called the <u>line overload factor</u>. It indicates the loading of line ℓ in branch ij as a fraction of the maximum allowable load for this line. For example, $\alpha_{ij}^{\ell} = 0.8$ would mean that the line is carrying 80 percent of the maximum allowable load and $\alpha_{ij}^{\ell} = 1.2$ would mean that the line is carrying an overload of 20 percent. Thus, whenever α_{ij}^{ℓ} is greater than 1, it means the line is overloaded.

(7) The program first computes the factors α_{ij}^{ℓ} for the unperturbed conditions and prints them out. In a properly designed system, the factors α_{ij}^{ℓ} in the unperturbed conditions are all ≤ 1. It is now assumed that branch rs loses its biggest line.

(8) The program then computes again the new γ_{ij}^{ℓ}, solves the power-flow equations, calculates new values of T_{ij}^{ℓ} and $\alpha_{ij,rs}^{\ell}$, where the second index refers to the perturbation in branch rs.

(9) If $\alpha_{ij,rs}^{\ell}$ are all ≤ 1 again, the program stops, indicating that there is no overload in any line due to the presumed perturbation in branch rs. The new values of T_{ij}^{ℓ} and $\alpha_{ij,rs}^{\ell}$ are printed out.

(10) If any one or more α_{ij}^{ℓ} become >1 due to perturbation, then the line for which α_{ij}^{ℓ} has the highest value is removed and the new γ_{ij}, T_{ij}^{ℓ}, and α_{ij}^{ℓ} are recalculated for the remaining lines. The same logic as is mentioned above is applied, and the process is continued.

(11) If, as a result of the required continued disconnection of various lines, a node becomes isolated from the rest of the network, this node is suppressed. The injections at one or more of the remaining production nodes are

readjusted (assuming that at least one of the remaining production nodes is equipped with load frequency control and that stability and synchronism of the rest of the system is not lost due to the suppression of the node) so that the net injection is equal to the net consumption. The calculation of α_{ij}^{ℓ} for the lines in the remaining network is repeated as before, and the process of removing the most overloaded line at each stage is continued till no line is found to be overloaded or till the system disintegrates completely.

In its present form, the program is not completely automatic and does not include any synthesis aspects. The suppression of the isolated node and the readjustment of net injections is done manually, but could easily be made automatic. Some possible refinements to the program are discussed in Ref. 7.

2. Results

The program was applied to the 8-node model shown in Fig. VIII-1. The connection of various nodes and net injections and consumptions at the nodes are indicated. The topography of the grid and the characteristics of the lines used are shown in Fig. VIII-1. The maximum allowed loading is indicated in Table VIII-1(a) and Table VIII-1(b). The initial perturbation was assumed to be the loss of the strongest line in branch 1. The following stages of disintegration were predicted by the computer.

Stage 1: All of the lines from branch 1 trip, one after the other, as a result of the initial perturbation (see Fig. VIII-2).

Stage 2: All of the lines from branch 2 trip, one after the other, cutting off node 1 from the rest of the system (see Fig. VIII-3).

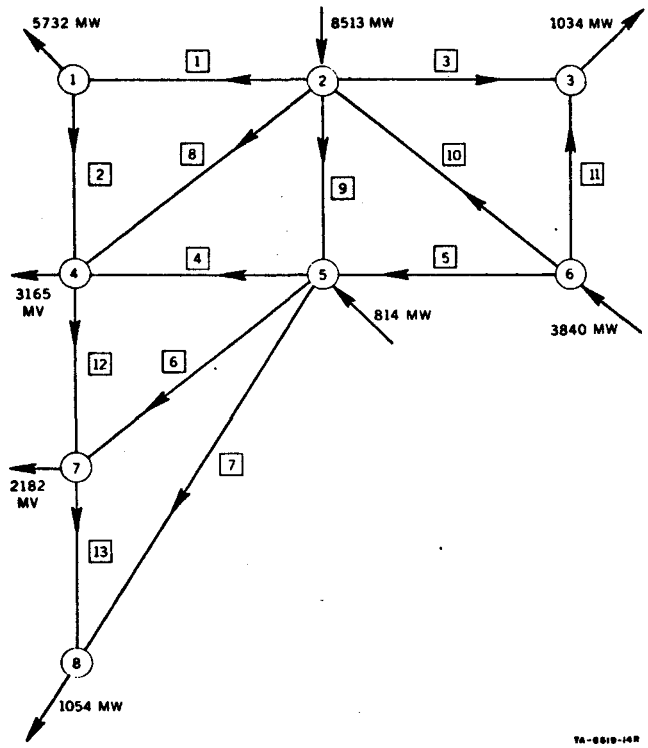

FIG. VIII-1 8-NODE MODEL, SHOWING INITIAL NET INJECTIONS AND NET CONSUMPTIONS AT VARIOUS NODES

<u>Stage 3</u>: Node 1 is suppressed manually, and the injections at nodes 2, 5, and 6 are reduced proportionally to account for the lost consumption at node 1. The network is solved as a 7-node problem. A line in branch 13 becomes overloaded and is tripped (see Fig. VIII-4).

<u>Stage 4</u>: The line in branch 7 trips due to overload, thus cutting off node 8 from the system (see Fig. VIII-5).

<u>Stage 5</u>: Node 8 is suppressed manually, and the injections at nodes 2, 5, and 6 are readjusted again to account for the loss of consumption at node 8. The network is

Table VIII-1(a)

TOPOGRAPHY OF THE GRID FOR 8-NODE MODEL

Branch	Origin	Extremity	Length	Type of Line	Number
1	1	2	180		
				4	4
				3	3
				2	1
				1	4
2	1	4	200		
				4	2
				1	1
3	2	3	240		
				1	4
4	4	5	120		
				4	2
				3	1
				1	3
5	5	6	240		
				4	4
				3	1
				1	3
6	5	7	160		
				4	4
				1	2
7	5	8	280		
				2	1
8	2	4	280		
				4	2
				1	1
9	2	5	200		
				4	2
				1	1
10	2	6	240		
				4	2
				1	1
11	3	6	160		
				4	2
				1	1
12	4	7	100		
				4	2
				1	2
13	7	8	80		
				4	2
				1	2

Table VIII-1(b)

CATALOG OF LINES FOR 8-NODE MODEL

Type	kV	X/mile	Maximum Allowable Load (MW)
1	230	0.812	160
2	230	0.799	200
3	345	0.803	440
4	500	0.5706	1200

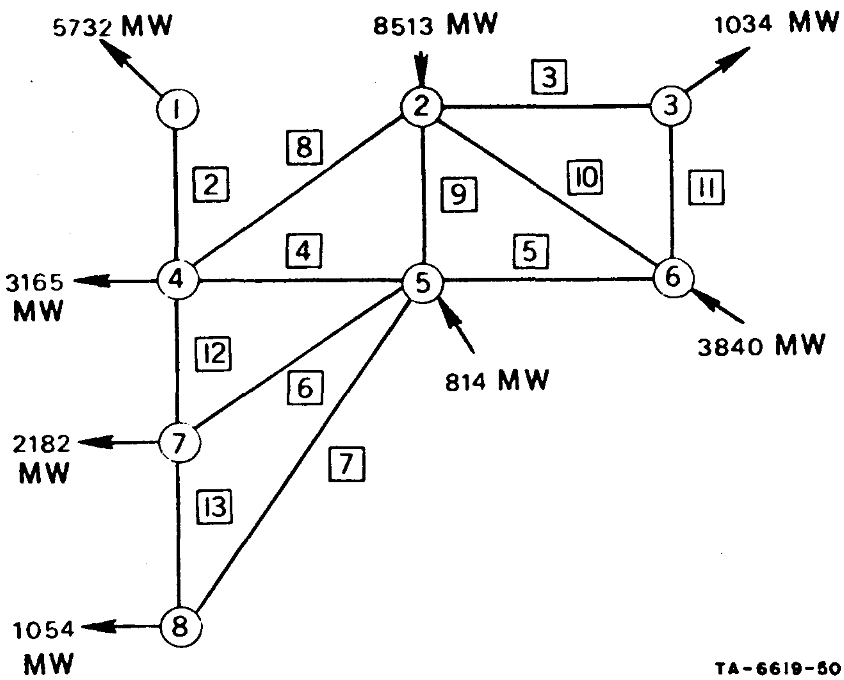

FIG. VIII-2 STAGE 1 — ALL LINES IN BRANCH 1 TRIPPED

now solved as a 6-node problem (see Fig. VIII-5).

Stage 6: All of the lines in branches 3 and 11 trip successively due to overloading, and node 3 is also isolated from the network (see Fig. VIII-6).

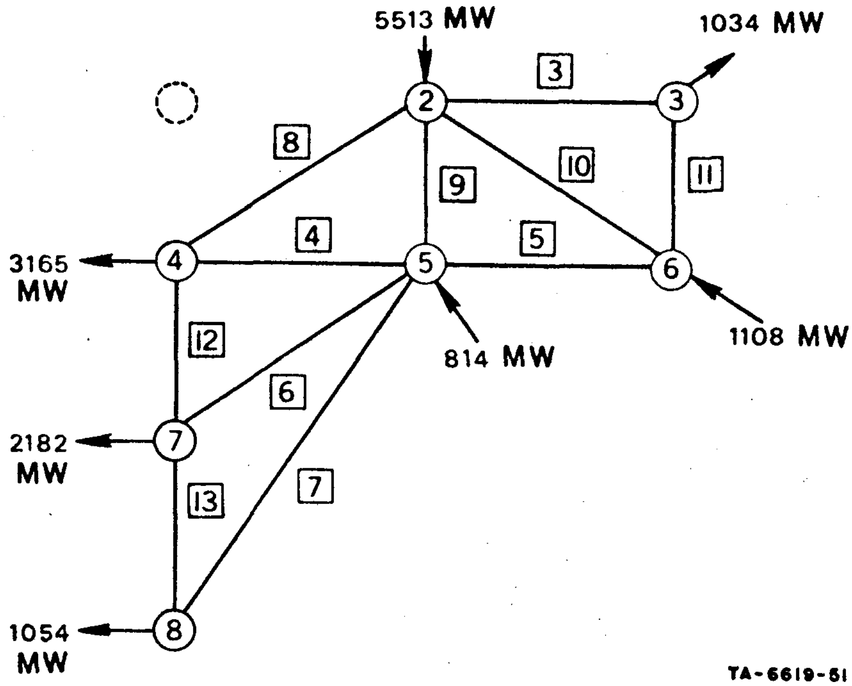

FIG. VIII-3 STAGE 2 — ALL LINES IN BRANCH 2 TRIPPED

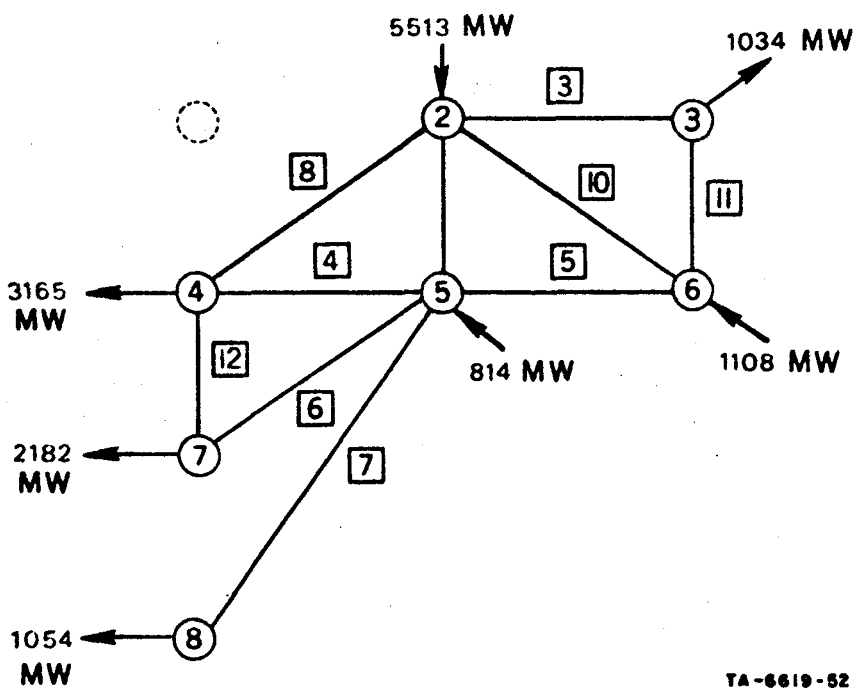

FIG. VIII-4 STAGE 3 — ALL LINES IN BRANCH 13 TRIPPED

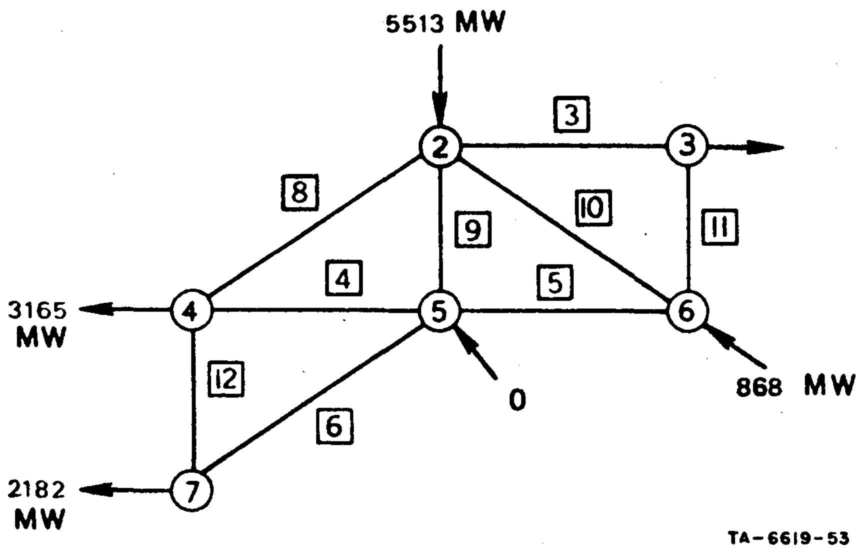

FIG. VIII-5 STAGES 4 AND 5 — ALL LINES IN BRANCH 7 TRIPPED

<u>Stage 7</u>: Node 3 is removed manually, and injections at nodes 2, 5, and 6 are readjusted. One of the 230 kV lines in branch 8 is overloaded and is tripped. After this, no other line is found overloaded, and the program is stopped in this stage (see Fig. VIII-7).

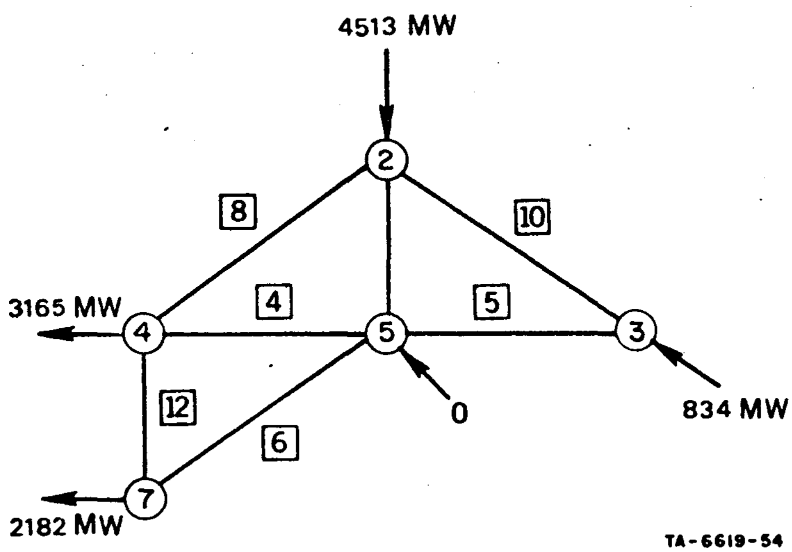

FIG. VIII-6 STAGE 6 — ALL LINES IN BRANCHES 3 AND 11 TRIPPED

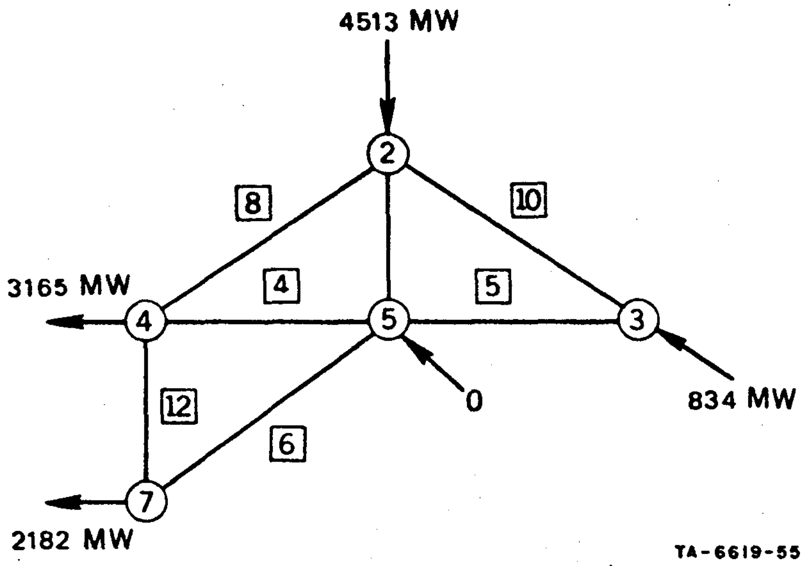

FIG. VIII-7 STAGE 7 — ONE LINE OF 230 kV TRIPS FROM BRANCH 9; NO OTHER LINE IS OVERLOADED AND THE PROGRAM STOPS

At the end of the process it is seen that 3 nodes are isolated--nodes 1, 3, and 8. The total load curtailed is 7820 MW--i.e., 59 percent of the load supplied by the pre-fault system.

C. Optimum-Load-Curtailment Program--Planned Load Curtailment[10,35]

1. Program Description

Consider an n-node, r-branch power system, defined by

(1) The topography of the network,

(2) The parameters of the individual transmission lines ($\gamma = V^2/X$),

(3) The maximum load-carrying capacity of each transmission line,

(4) The net injections $P_i - C_i$ at each node $i = 1, 2, \ldots, n$,

(5) A set of n linear equations modeling the power flow in the system, e.g.,

$$P_i - C_i = \sum_j \gamma_{ij} \left(\theta_i - \theta_j\right) \quad , \qquad \text{(VIII-2)}$$

where γ_{ij} is the parameter of the branch connecting nodes i and j, θ_i, and θ_j are the voltage phase angles at nodes i and j, respectively, and the summation is performed over all nodes j connected to node i = 1, 2, ..., n, and

(6) The initial perturbation, i.e., the specification of the type of the faulted line and the branch from which it was removed.

With the above input data, find the minimum amount of load that must be dropped in the system to prevent cascading fault due to subsequent overloading of transmission lines.

The minimization of curtailment in the post-fault system can be formulated assuming certain realistic approximations discussed in Ref. 10 as a linear programming problem. The objective is to minimize a linear function of the unsupplied demand,

$$\min_{P_i, C_i} \sum_i \xi_i \left(C_i^* - C_i\right) \quad ; \quad i = 1, \ldots, n \quad , \qquad \text{(VIII-3)}$$

where the C_i^* are the connected loads in the pre-fault system, and the C_i are the loads that can be supplied in the post-fault system. An identical objective is to maximize a linear function of the satisfied demand,

$$\max_{P_i, C_i} \sum_i \xi_i C_i \quad ; \quad i = 1, \ldots, n \quad , \qquad \text{(VIII-4)}$$

where the ξ_i represent the priorities assigned to the various connected demands C_i^*. Both the minimization, (VIII-3), and the maximization, (VIII-4) are subject to the (linearized) network equations

$$P_i - C_i = \sum_j \gamma_{ij} \left(\theta_i - \theta_j\right) \quad ; \quad i = 2, \ldots, n \quad \text{(VIII-5)}$$

$$\sum_{i=1}^{n} \left(P_i - C_i\right) = 0 \quad , \quad \text{(VIII-6)}$$

where the set of n dependent equations, Eq. (VIII-2), has been replaced by the set of n - 1 independent equations, Eq. (VIII-5), and the relation Eq. (VIII-6), and also subject to the inequality constraints:

$$\left|\gamma_{ij}^{\ell} \left(\theta_i - \theta_j\right)\right| \leq \bar{T}_{ij}^{\ell} \quad \text{(VIII-7)}$$

for all branches ij and all lines ℓ in the branch ij, and

$$C_i^m \leq C_i \leq C_i^* \quad ; \quad (i = 1, \ldots, n) \quad \text{(VIII-8)}$$

$$P_i^m \leq P_i \leq P_i^M \quad ; \quad (i = 1, \ldots, n) \quad . \quad \text{(VIII-9)}$$

Here γ_{ij}^{ℓ} is the capacity of line ℓ in branch ij and $\bar{T}_{ij}^{\ell}$ is the maximum transmission capability of line ℓ in branch ij (in MW). The superscripts m and M denote minimum and maximum, respectively. Node 1 is considered the reference node, with $\theta_1 = 0$.

If a branch ij is composed of L parallel transmission lines, then

$$\gamma_{ij} = \sum_{\ell=1}^{L} \gamma_{ij}^{\ell} \quad ; \quad (\ell = 1, 2, \ldots, L) \quad . \quad \text{(VIII-10)}$$

Therefore, the minimum load curtailment problem can be formulated as a linear programming problem, the solution of which is well known. Practically, however, the method would rapidly become computationally infeasible, because the number of constraints, directly proportional to the number of transmission lines, becomes very large even for a small network, when the individual branches are composed of many parallel lines. Fortunately, the dimension of the problem can be greatly reduced, as it has been shown in Ref. 10, by replacing the set of constraints (VIII-7) by the equivalent set:

$$\left|\gamma_{ij}\left(\theta_i - \theta_j\right)\right| \leq \bar{T}_{ij} \qquad \text{for all branches } ij \quad ,$$

where $\bar{T}_{ij}$ is defined to be:

$$\bar{T}_{ij} = \min_{\ell} \frac{\gamma_{ij}\,\bar{T}^{\ell}_{ij}}{\gamma^{\ell}_{ij}} \qquad \ell = 1, 2, \ldots, L$$

and represents the maximum allowed flow in branch ij such that no line in the branch is overloaded. Hence, it suffices to consider the individual branches in the program instead of the individual lines.

The linear programming problem is then readily solved with a simplex method.

It is recalled that a simplified linear "DC model" has been used to describe the network. The more accurate AC model would require nonlinear techniques, such as gradient procedures.[35] Such techniques were not considered here, since the improvement in accuracy that they permit may not justify the complications that they entail.

D. Results

The program was applied to the same 8-node model discussed above (Fig. VIII-1). Again the initial perturbation was the loss of the strongest line in branch 1, but subsequent overloadings were prevented by intentional curtailment such that the total satisfied demand is maximized in the post-fault system. No priorities were assigned in this example (ξ_i = 1 for all i).

The results are given in Table VII-2. The total load curtailed was 681.48 MW, or 5.18 percent of the total load supplied by the pre-fault system. Thus, a drastic saving can be obtained if, instead of the network being allowed to disintegrate, it is controlled to curtail load optimally.

Table VIII-2

OPTIMUM LOAD CURTAILMENT PROGRAM RESULTS*

Values at the Nodes				
Node	Voltage Angle (Radians)	Production (MW)	Consumption (MW) Post-Fault	Consumption (MW) Pre-Fault
1	0.000000	0.0000	5050.5248	5732.0000
2	0.442099	7831.5248	0.0000	0.0000
3	0.334708	0.0000	1034.0000	1034.0000
4	-0.015397	0.0000	3165.0000	3165.0000
5	0.177802	814.0000	0.0000	0.0000
6	0.490625	3840.0000	0.0000	0.0000
7	-0.056106	0.0000	2182.0000	2182.0000
8	-0.134021	0.0000	1054.0000	1054.0000

Flows in the Branches		
Branch	Flow (MW) Post-Fault	Flow (MW) Pre-Fault
1	-5123.0000	-5874.76
2	72.4752	142.770
3	116.6050	113.380
4	-1964.0924	-1936.31
5	-2732.2586	-2715.241
6	2752.5678	2737.908
7	73.6680	73.379
8	1538.1966	1510.64
9	1244.0696	1218.352
10	-190.3464	-204.139
11	-917.3950	-920.608
12	409.7642	424.718
13	980.3320	980.518
Total Load Curtailed: 681.4752		

*Angles are in radians; productions, consumptions, and flows are in MW.

Load was curtailed by the program at node 1, the origin of branch 1, in which the line fault occurred; this result could intuitively be expected here. This is not, however, a general result. A different network structure, the introduction of priorities in the load curtailment policy, the particular location of the fault, and the allocation of generation reserves could all contribute to a different result. It is possible, for instance, that a fault in branch 1 connecting nodes 1 and 2 may require load curtailment at node 8 in order to minimize the cost of the power failure resulting from that initial fault.

In theory, the program described above must be applied to the whole network for each critical contingency. This is not necessarily impossible, the simplex method being very efficient and subject to further drastic improvement for sparse networks. The example described here, involving 50 equations and 80 variables (slacks included), was run in 9.2 seconds on the B-5500 computer with a very crude available program.

If the time required to obtain a solution in real time is excessive in relation to the time available for curtailment, or if it is uneconomical to telemeter all of the required data to the dispatching center, this program can still be used to set the protection logic (e.g., under-frequency protections[45]) to prevent cascading faults. The setting of this logic can be adjusted with time of day to accommodate load variations.

E. Extensions

In addition to an optimal load curtailment policy (which gives the locations and magnitude of the loads that should be curtailed in order to minimize the total loss of energy while keeping the integrity of the network), the program provides very useful sensitivity information by computing the dual variables associated with the constraints.

For the example network considered, these dual variables are shown in Table VIII-3 and were shown in Ref. 10 to yield to the following results:

(1) Branch 1 is the only branch saturated. Thus, it can be expected that additional faults could occur in the

Table VIII-3

DUAL VARIABLES

Constraint	Dual	
1	1.0000	$\Sigma P_i - \Sigma C_i = 0$, Eq. (VIII-6)
2	-1.0000	Power Flows, Eq. (VIII-5)
3	-0.8717	
4	-0.6456	
5	-0.7638	
6	-0.8480	
7	-0.7099	
8	-0.7109	
9	0.0000	$\overline{T}_{max}$ for Branches, Eq. (VIII-7)
10	0.0000	
11	0.0000	
12	0.0000	
13	0.0000	
14	0.0000	
15	0.0000	
16	0.0000	
17	0.0000	
18	0.0000	
19	0.0000	
20	0.0000	
21	0.0000	
22	1.2623	$\overline{T}_{max}$ for Branches, Eq. (VIII-7)
23	0.0000	
24	0.0000	
25	0.0000	
26	0.0000	
27	0.0000	
28	0.0000	
29	0.0000	
30	0.0000	
31	0.0000	
32	0.0000	
33	0.0000	
34	0.0000	
35	1.0000	P^M at Nodes, Eq. (VIII-9)
36	0.0000	
37	0.1283	
38	0.3544	
39	0.2362	
40	0.1520	
41	0.2901	
42	0.2891	
43	0.0000	C^* at Nodes, Eq. (VIII-8)
44	0.0000	
45	0.8717	
46	0.6456	
47	0.0000	
48	0.0000	
49	0.7099	
50	0.7109	

system without causing overloads. This is because the dual variables associated with the maximum flow constraints in all the branches other than branch 1 are zero.

(2) An increase in the capacity of branch 1 by 1 MW would permit an additional load of 1.2626 MW to be supplied by the example network. Thus it would be profitable to invest in branch 1.

(3) The dual variables associated with the constraints on P_i and C_i being positive, it can be concluded that the optimum solution would require less curtailment of load if more power were supplied at nodes 5 and 6 and more load were allowed at nodes 3, 4, 7, and 8.

F. Probability of Lost Energy

A useful aspect of the above programs is that they can be employed to compute predicted loss of energy resulting from some faulty condition in a given system. This prediction, in turn, can be used to assess the reliability of the system. Furthermore, by introducing appropriate penalty factors for lost energy, these calculations can be helpful in the analysis, modification, or extension of subsystems, where it may be admissible to have occasional power disconnections, as long as such disconnections do not result in cascading failures.

In the 8-node example, it was found that with the assumed loading limits for the various lines, the loss of one stronger line from branch 1 resulted in ultimately losing 7820 MW if no optimal load curtailment policy existed, 681 MW if such a policy was used. If the total average time for which this loss is incurred is denoted by τ, then the total loss of energy per occurrence of the fault under consideration is $C\tau$, where C is the sum of all unsupplied powers as a result of a particular fault and τ is the average time elapsed between disconnection of the loads and their re-connection.

Assuming, for simplicity that the topography of the system does not change appreciably during a test period (say one year) but considering the daily and seasonal variations of injections and consumptions during the test period at various nodes, one can calculate the total amount of expected unsupplied energy (or lost energy) during the test period as follows:

We assume that during the test period, the system can be represented by N discrete "states," e.g., a possible "state" is the set of consumptions and injections at different nodes during the winter peak load conditions. Let index j represent a particular state. Furthermore, let

C_{ij} = average power in MW disconnected as a result of a fault F_i (e.g., loss of a certain line in a certain branch) for the system in the state j.

τ_i = average restoration time for the lost power due to fault F_i, expressed as a fraction of the test period.

μ_i = number of times the fault F_i is expected to occur during the test period (e.g., if on the average 1 × 500 kV line trips from a branch once in three months, then $\mu = 4$ for a test period of one year).

τ_j = fraction of test period for which the system operated in state j.

The total expected unsupplied energy during the test period is then given by

$$\bar{E} = \sum_{j=1}^{N} \sum_{i=1}^{m} C_{ij} \, \tau_i \, \mu_i \, \tau_j \quad ,$$

where m is the number of various types of fault being considered for the system during the test period. The choice of N and m will depend upon the desired accuracy and the extent of labor required in calculating the probable amount of lost energy. Note that the product $\tau_i \, \mu_i$ expresses the fraction of the test period for which the fault F_i continues to exist. Denoting the product $\tau_i \, \mu_i$ by η_i, we can properly call the factor η_i the probability of fault condition F_i. If the total energy that was desired to be supplied during the test period is E, then we can define a "system reliability" factor as:

$$D = \frac{E - \bar{E}}{E} \quad .$$

Ideally, the factor D should be 1. The more near one the value of D, the more reliable the overall system.

The factor D, however, is not very meaningful from the point of view of an individual customer. A system might have a very high overall reliability factor, yet a certain customer (a certain consumption node) might be suffering disproportionately. Therefore, to indicate the reliability of supply at a node r, we can introduce a customer reliability factor, which can be easily obtained from the results of the line-outage-analysis program as described below.

Considering a node r, we locate all those faults F_k that cause a disconnection of supply at node r. The total probable amount of unsupplied energy during the test period at node r is then given by

$$\sum_j \sum_k C_{kj,r} \, \mu_k \, \tau_k \, \tau_j \quad ,$$

where $C_{kj,r}$ is the lost consumption at node r due to fault F_k in system state j, μ_k is the number of times fault F_k occurs during test period, and τ_k is the restoration time for the supply after fault F_k. If E_r is the total energy that was desired to be supplied at node r during the test period, a customer reliability factor at node r can be defined as

$$D_r = \frac{E_r - \sum_j \sum_k C_{kj,r} \, \mu_k \, \tau_k}{E_r} \quad .$$

As in the case of D, the value of D_r should ideally be 1. Both the factors D and D_r, as well as the expected lost energy $\bar{E}$, can be used along with a suitable penalty factor to design and modify a system.

This approach is primarily suggested for analysis, modification, or expansion of subsystems where it may be admissible to have occasional temporary power failures, i.e., where it may not be economically justifiable to make the system "failure free" to a very high degree.

These calculations can easily be extended to include multiple faults. If two faults F_i and F_i' are statistically independent and their probabilities as defined above are η_i and η_i', then the probable amount of lost energy due to faults F_i and F_i' occurring at the same time is

$$\sum_j \sum_{ii'} C_{ii',j}\, \eta_i\, \eta_i'\, \tau_j \quad ,$$

where $C_{ii'}$ is the amount of power disconnected as a result of simultaneous faults F_i and F_i' in system state j. If F_i and F_i' are correlated, then the combined probability $\eta_{i,i'}$ can be obtained by making proper assumptions concerning the effect of failure F_i on η_i'.

G. Conclusions

The work summarized in this section (and described in greater detail in Refs. 7 and 10) is not a central part of this study, since it is not concerned directly with the synthesis of a transmission system.

There is, however, an indirect relation, since the system's reliability determines the outcome of an outage and since load curtailment, if properly implemented, increases reliability in an economical manner.

The reader will have noticed that the discrete nominal schedule discussed in Sec. VI is the synthesis counterpart of the outage analysis described here. In fact, the idea of the discrete nominal schedule was generated in the course of our work on outage analysis.

Our concern with outage analysis methods and emergency load curtailment optimization was stimulated to a large extent by the discussions on power system reliability[27,31,32,38] that have appeared in the recent literature. Indeed, power system planners are faced with the need to synthesize systems that represent an acceptable trade-off between reliability--measured for example in expected MWH's lost per year--and capital cost. It may well be that the criterion which best satisfies this trade-off reads as follows:

> "In the event of a single, unscheduled outage, no load shall be lost and no additional outages shall be provoked."
>
> "In the event of a second, third, etc., ..., outage load curtailment designed to minimize the unsupplied energy shall be permissible."

With these (or similar) reliability criteria, the techniques described in this section permit an efficient computation of the unsupplied energy for a specified fault. More generally, their repetitive application permits the computation of the <u>expected</u> energy lost as a function of equipment outage probabilities, network structures, and loads, as outlined above.

IX SUBSYSTEM OPTIMIZATION IN THE PRESENCE OF UNCERTAINTY

A. Statement of Problem

This study is concerned with the optimal planning of parts of the complete transmission system (referred to as subsystems). Relatively detailed models for the network, the demand statistics, and the investment costs are used.

The objective is to obtain a procedure allowing the decision maker to plan the future characteristics of each individual piece of equipment, such as a line, a transformer, a switchgear.

The problem is particular in the sense that the uncertainty of the future should be taken into account. The possible variations of future demands may be large, and consideration of a single-value forecast may be questionable. Probability distributions are therefore introduced to weight the penalty incurred as the result of a bad forecast.

The nature of the uncertainty is twofold:

(1) There is a fairly constant deviation about a predictable trend. For instance, in planning the expansion of an electrical system, an averaging effect is observed in the variations of load due to deviations in the regional economies.

(2) There is a possibility of sudden, discrete, local variations, occurring at sparse, discrete intervals of time so that no averaging effect is observed. For instance, in an electrical system a large load may be encountered suddenly because of an industrial plant being moved in.

The introduction of uncertainty in the planning problems means that exogenous data, which would be considered as constants in a deterministic case, are now random variables, a model of which will be described in Sec. X.

B. Formulation of Problem

The uncertainty of future exogenous requirements and the discrete nature of the planning variables rules out most of the classical optimization techniques, such as the gradient procedures, integer programming, or branch and bound algorithms. Dynamic programming, however, can theoretically be used without difficulty. The practical use of this technique is made possible through the particular formulation of the problem given below.

1. Planning Period

The planning period is divided into T + 1 subperiods (numbered 0 through T) in which planning decisions are made. The index t will represent the t^{th} subperiod. For illustration purposes, the Olympia/Port Angeles subsystem [shown in Fig. IX-1(a)] will be considered throughout this section. Pertinent numerical data were made available by the Bonneville Power Administration. The subperiod is a year, and the planning horizon is 15 years. This subsystem is discussed extensively in Ref. 9.

2. Variables

To consider the locations and ratings of each individual piece of equipment and the magnitudes of each stochastic demand would make the dynamic programming approach computationally infeasible. The dimensionality of the problem is reduced by considering only two state variables and one decision variable.

a. State Variables

(1) Equipment Variable

The equipment variable x_t identifies the configuration of the system at time t. Its value $x_t^{(i)}$ is the identification number i of a complete design that contains all the information required to:

- Build the system
- Determine its performance for given loads.

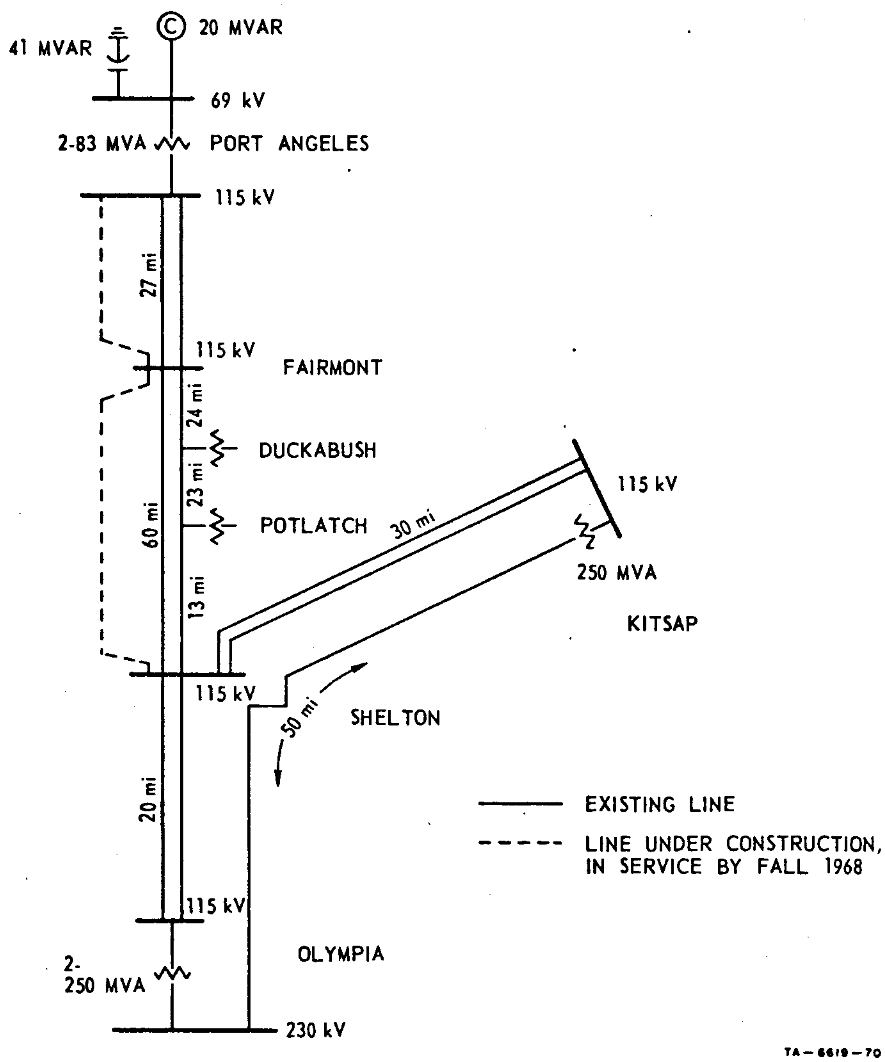

FIG. IX-1(a) EXISTING CONFIGURATION OF THE OLYMPIA/PORT ANGELES TRANSMISSION SUBSYSTEM IN 1968

There is a finite set of designs, which are reasonable from the point of view of electrical engineering and among which the planner can choose at each stage.

In the sample network, 8 possible configurations numbered 0 through 7 were selected. Thus:

$x_0^{(0)}$ states that configuration 0 is used at time 0.

$x_{10}^{(4)}$ states that configuration 4 is used at time 10.

... etc.

These configurations are given in Fig. IX-1(b). Configuration 0 is the existing configuration of the Olympia/Port Angeles transmission subsystem, which is given in Fig. IX-1(a).

(2) Demand Variables

The location of the future loads is considered to be known. Moreover, only one demand is assumed to be stochastic. The latter assumption was based upon the fact that most loads are fairly predictable, though one of them (Port Angeles in the sample subsystem) is allowed to show wide variations about a predictable trend. The stochastic demand will be denoted by d_t at time t.

Figure IX-2 gives the future deterministic loads and the minimum, average, and maximum predicted values of the stochastic load in the Olympia/Port Angeles subsystem.

b. Decision Variable

Assume that a transition from configuration $x_t^{(i)}$ to configuration $x_{t+1}^{(j)}$ is decided at time t. Symbolically, this transition will be denoted by the "transition" equation:

$$x_{t+1}^{(j)} = x_t^{(i)} + u_t^{(i,j)} \quad , \qquad \text{(IX-1)}$$

where $u_t^{(i,j)}$ is the value at time t of the decision variable u_t causing i at t to become configuration j at t + 1.

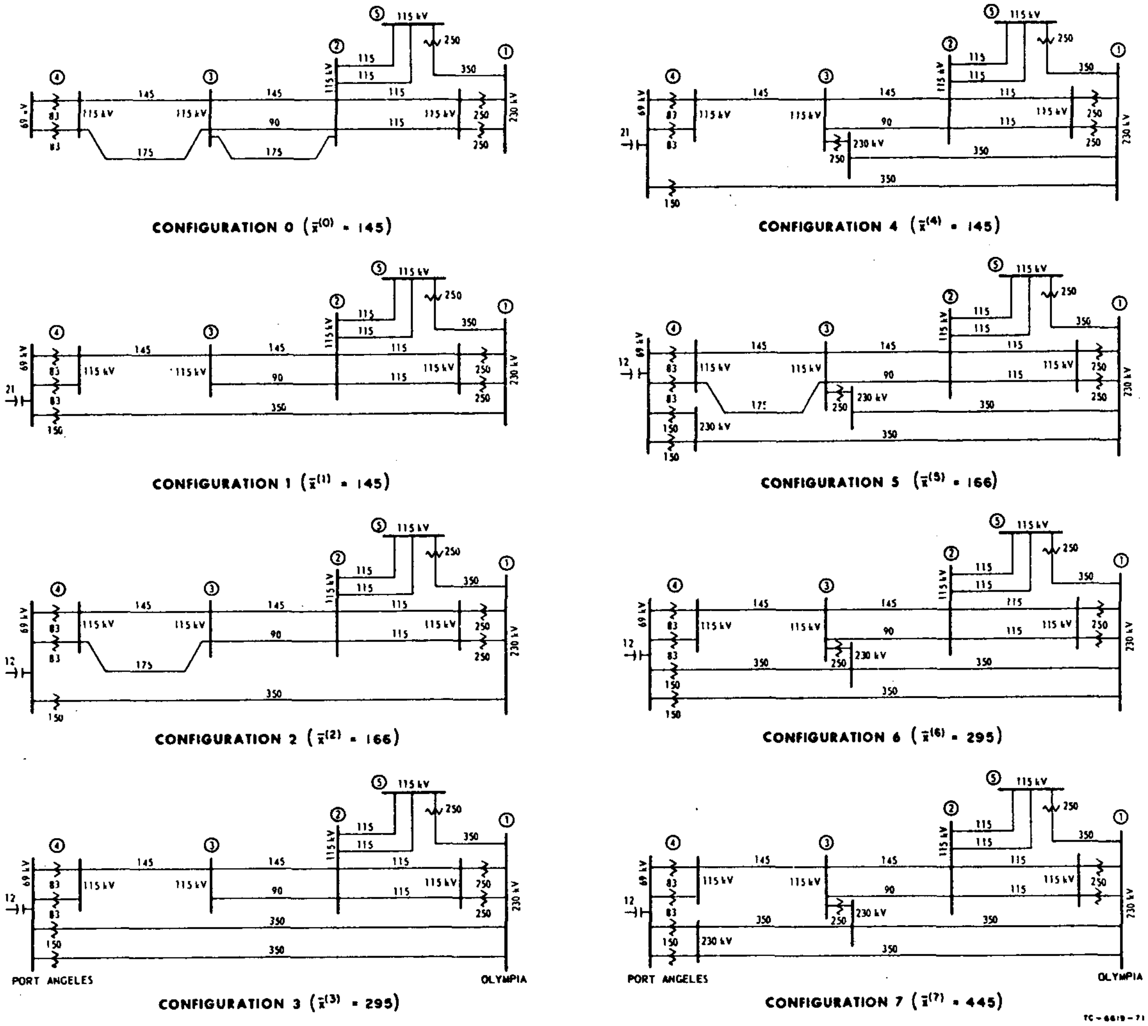

FIG. IX-1(b) SUBSYSTEM CONFIGURATIONS i = 0, 1, . . ., 7 FOR THE OLYMPIA/PORT ANGELES EXPANSION PLAN. Transmission-line thermal limits and transformer ratings are shown in MVA units. The value $\bar{x}^{(i)}$ denotes the maximum power that can be delivered to node 4 by configuration i, with the largest-capacity line out of service. The node numbers are shown by circles.

YEAR	NODE 2	NODE 3	MIN.DEM	AVE.DEM	MAX.DEM	NODE 5	NODE 6
1968	35.00	50.00	106.00	106.00	106.00	155.00	0.00
1969	36.00	53.00	110.00	110.00	110.00	162.00	0.00
1970	38.00	55.00	108.82	113.50	118.19	172.00	0.00
1971	40.00	57.00	107.65	117.12	126.98	180.00	0.00
1972	42.00	61.00	106.49	120.85	136.44	190.00	0.00
1973	44.00	64.00	105.35	124.69	146.59	201.00	0.00
1974	46.00	67.00	104.21	128.66	157.50	213.00	0.00
1975	49.00	70.00	103.09	132.76	169.22	226.00	0.00
1976	51.00	73.00	101.98	136.99	181.82	241.00	0.00
1977	53.00	76.00	100.89	141.35	195.35	258.00	0.00
1978	56.00	80.00	99.80	145.85	209.89	278.00	0.00
1979	58.00	83.00	98.73	150.49	225.52	298.00	0.00
1980	60.00	87.00	97.67	155.28	242.30	310.00	0.00
1981	63.00	91.00	96.62	160.23	260.34	335.00	0.00
1982	66.00	94.00	95.58	165.33	279.71	360.00	0.00
1983	69.00	98.00	94.55	170.59	300.53	380.00	0.00

FIG. IX-2 FORECAST OF DEMAND (The demand is perfectly known at nodes 2, 3, 5, 6 and stochastic at node 4. The minimum, average, and maximum values of demand predicted at node 4 are given. All quantities are given in MW.

In the example, $u_t = 1$ signifies that configuration 1 should be adopted at time t + 1, $u_t = 2$ that configuration 2 should be adopted, etc.

c. Constraints

(1) Reliability Constraint

A properly formulated reliability constraint will also enter the planning procedure. This reliability constraint must be defined in terms of either the tolerable loss in equipment (e.g., transmission lines) without load curtailment, or the tolerable degree of curtailment for given contingencies.

The approach taken in the study was:

The subsystem configuration i at time t must be able to supply the expected demands at each node at time t + 1, with any one of the transmission lines out of service and without any curtailment.

For the Olympia/Port Angeles example, the reliability constraint is simplified to consider only the expected demand at Port Angeles (node 4), since the demand growth at other consumer nodes is assumed to be known. The constraint becomes, in this example:

$$\bar{x}_t^{(i)} \geq x_t' \qquad \text{(IX-2)}$$

where $\bar{x}_t^{(i)}$ is the maximum power that can be delivered to Port Angeles by configuration i with the largest-capacity transmission line out of service, and x_t' denotes the expected demand at Port Angeles at time t + 1.

The values of $\bar{x}^{(i)}$ are readily determined from the transmission line configurations of Fig. IX-1(b) and are listed with each state i = 0, 1, ..., 7.

(2) Constraints on the Equipment and the Decision Variables

It was assumed that only transitions toward a higher investment are allowed. This assumption could be easily relaxed.

d. Cost Function

The cost function comprises two, and possibly three, components:

(1) The investment cost ℓ', which depends on the investment decision u_t. It is most convenient to assume that the full cash price [discounted by $1/(1 + \xi)^t$] is paid at t when the equipment is purchased. The value of the equipment at terminal time T is then subtracted with the proper discounting from the terminal cost. In other words, the investment component of the terminal cost $I(x_t, T)$ is minus the resale value of this equipment in a fair market, divided by the discounting factor $1/(1 + \xi)^t$.

If, in the interest of accuracy, the fair market value of the equipment is taken into account, then its age must be kept track of in the computer. This can be done readily with forward dynamic programming, but is not possible, at least not rigorously, with backward dynamic programming limited to the state variables defined above. In Ref. 9 approximate procedures for keeping track of equipment age with backward dynamic programming are discussed. Knowledge of equipment age is desirable not only to evaluate the system's terminal value, but also to consider the following two eventualities:

(a) Equipment (e.g., switchgear) is removed at time t but can be used elsewhere in the system. The proper procedure is to add the cost of removal and to subtract the fair market value at the time of removal t, both properly discounted, to the transition cost.

(b) Equipment (e.g., transmission lines) is retired at time t and cannot be used elsewhere in the system. The proper procedure is to subtract from the transition cost at t the salvage value (which may depend on age) and to add the cost of demolition.

In the Olympia/Port Angeles example, the investment cost incurred in a transition from design i to design j at time t was taken to be:

$$\ell'\left[x_t^{(i)},\ x_{t+1}^{(j)},\ t\right] = \frac{C^{(i,j)}}{(1+\xi)^t} \quad . \qquad \text{(IX-3)}$$

The values of $C^{(i,j)}$ are shown in Table IX-1. They were computed considering that the salvage value of the equipment removed is 2/3 of the original value of the equipment. The discounting factor ξ was taken to be 3.125 percent.

(2) <u>The cost of losses ℓ''</u>, which is a function of the state x_t. For given technical equipments and a description of the load, the yearly losses can be readily computed as discussed in Ref. 9. It was assumed that the cost of losses is constant and equal to $2/MWH.

(3) <u>The penalty cost ℓ''', associated with constraint violations</u>. A function of the form ℓ''' may be introduced to penalize the planner for temporary lack of "reliability," the latter being defined in terms of expected yearly loss of load due to equipment failure. Such a function was not included in the example considered.

3. <u>Notion of Lead Time</u>

It is assumed in the example that <u>the load at Port Angeles is known one year in advance</u>. Thus d_{t+1} is known at time t. The demand state is therefore defined as

Table IX-1

TRANSITION COSTS $C^{(i,j)}$ ASSOCIATED WITH AN INVESTMENT DECISION $u_t^{(i,j)}$ TO GO FROM STATE $x_t^{(i)}$ TO STATE $x_{t+1}^{(j)}$, FOR THE OLYMPIA/PORT ANGELES EXTENSION EXAMPLE*

$x_t^{(i)}$ \ $x_{t+1}^{(j)}$	Transition Costs (millions of dollars)							
	j = 0	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7
i = 0	0	2.440	2.868	5.423	5.103	6.686	6.886	7.486
i = 1		0	0.623	2.998	2.663	4.471	4.491	5.091
i = 2			0	2.555	2.708	3.818	4.018	4.618
i = 3				0	0.408	1.443	1.463	2.063
i = 4					0	1.808	1.828	2.428
i = 5						0	0.200	0.800
i = 6							0	0.600
i = 7								0

*States i, j = 0, 1, ..., 7 are shown in Fig. IX-1(b). Transitions from higher to lower investment state are not considered here.

$$x'_t \triangleq d_{t+1} \tag{IX-4}$$

at time t. Since at time t, the decision u_t is made that will materialize in equipment state x_{t+1} at time t + 1, the definition (IX-4) means that:

(1) At time t + 1 the demand d_{t+1} can certainly be satisfied by equipment state x_{t+1}.

(2) The lead time to implement a new configuration is the same as the time the demand is known in advance (e.g., one year).

This last assumption may be questioned, as one year appears too short to implement a new configuration. In fact, this assumption could be easily relaxed, and different lead times could be used for implementation and demand knowledge. This was not done in the example, in order to reduce programming effort and computer time.

C. Objective

The objective is to minimize the sum of the investment and operating costs during the planning period--i.e., to minimize the functional:

$$J = E \sum_{t=0}^{T} \ell'\left(x_t,\ x_{t+1},\ t\right) + \ell''\left(x_{t+1},\ d_{t+1},\ t\right) + \ell'''\left(x_t,\ x_{t+1},\ u_t,\ d_{t+1},\ t\right) \quad \text{(IX-5)}$$

or, if ℓ''' is neglected:

$$J = E \sum_{t=0}^{T} \ell'\left(x_t,\ u_t,\ t\right) + \ell''\left(x_t,\ u_t,\ x_t',\ t\right) \quad , \quad \text{(IX-6)}$$

where the symbol E denotes the expectation taken over the random variables $d_2,\ d_3,\ \ldots,\ d_T$.

If $I\left(x_t,\ x_t',\ t\right)$ denotes the minimum expected cost incurred between time t and time T when optimal decisions are made starting in equipment state x_t at time t while the demand is known to be x_t' at time t + 1, then by Bellman's principle of optimality,[44] the minimization of Eq. (IX-6) is equivalent to the sequence of minimizations:

$$I\left(x_t,x_t',t\right) = \min_{u_t} E\left\{\ell'\left(x_t,u_t,t\right) + \ell''\left(x_t,u_t,x_t',t\right) + I\left(x_{t+1},x_{t+1}',\ t+1\right)\right\} \quad , \quad \text{(IX-7)}$$

where the expectation is taken over the effect of the random variable x_{t+1}' or equivalently d_{t+2}. It is important to note that <u>the only statistics needed for the optimization are the conditional probabilities</u>:

$$p\left[d_{t+2} \middle| d_{t+1}\right] = p\left[x_{t+1}' \middle| x_t'\right] \quad . \quad \text{(IX-8)}$$

These conditional probabilities are computed for each t and are printed out by the program. The major problem in computing these probabilities is interpolation, due to the necessity of quantizing the possible levels of the demands $d_{t+1},\ d_{t+2}$.

At time T, the function $I\left(x_T, x'_T, T\right)$ becomes $I\left(x_T, T\right)$ and equals minus the salvage value of the equipment in use at time T.

The result of the stochastic optimization is not one sequence of optimal investments u_t, but a sequence of decision tables $u\left(x_t, d_{t+1}, t\right)$. Every year the planner, who knows the actual configuration of his subsystem and the demand that will materialize the next year, finds in the decision table the optimum investment he should make in order to minimize the expected cost he will incur between that time and the end of the planning period.

D. The Program

A program was developed and run on a B-5500 computer. The inputs and outputs are summarized below.

1. Inputs

Option I--Compact form:

- Inverses of the M matrix for each possible configuration. The M matrix is defined by the equation:

$$\mathfrak{J} = M \, \psi \quad , \qquad \text{(IX-9)}$$

where $\mathfrak{J}$ is the vector of the injections at the nodes, and ψ is the vector of the angular differences between two connected nodes.

- Estimates $\hat{a}$ and $\hat{b}$ of the parameters a and b of the demand model.[4,9]

Option II--Detailed form:

- Topography of the possible network configurations.
- Quantitative values of the resistances, reactances, and voltages.

- Past values of the stochastic demand for N years.

In Both Cases:

- Coefficients of the loss formula
- Length of the planning period
- Desired quantization of demand
- Future deterministic demands
- Investment costs
- Maximum loads allowed by the prescribed reliability constraint, for each configuration.

2. Outputs

Option I--Limited output:

- Sequence of decision tables $u(x_t, d_{t+1}, t)$ indicating in every year t the optimum configuration be adopted if the actual configuration is x_t and if the demand next year will be d_{t+1}.

Option II--Extended output:

- Topography of the possible network configurations.
- Matrices M and M^{-1} for each possible configuration.
- Future load forecasts giving the deterministic demands and the minimum, average, and maximum value of the stochastic demand.
- Computed estimates $\hat{a}$ and $\hat{b}$ of the parameters a and b of the demand model.[9]
- Sequence of decision tables $u(x_t, d_{t+1}, t)$ as in Option I.

- Conditional probabilities $p\left[d_{t+1} \middle| d_t\right]$ for every year t.

A sample of the decision and conditional probability tables is given in Fig. IX-3.

E. Results

1. Decision Tables

This program was tested on the Olympia/Port Angeles subsystem.[9] In the solution obtained:

- The demand d_t is quantized in 15 levels. The stochastic model used is described in Sec. X and in Ref. 9.
- The discounting factor ξ used is 3.125 percent.
- The salvage value of the equipment removed during the 15-year planning period is 2/3 of its original value.
- The value of the equipment at the end of the planning period is 2/3 of its original value.

Typical computer printouts of the decision tables are given in Fig. IX-3. From these decision tables, one can summarize an "optimal decision chart" as shown in Fig. IX-4. The best policy is to go to configuration 1 in the first year and stay there for at least three more years. Thereafter, transitions to configuration 3, depending on the demand that actually materializes at Port Angeles.

It is instructive to compare the expansion policy of Fig. IX-4 with the plans recommended at present by the BPA staff. The "recommended plan" and the "alternate plan" are shown in Fig. IX-5. It appears that the recommended plan is based on the assumption that the new industrial load will be connected to Port Angeles, while the alternate plan considers the possibility that part of that load will be supplied from Fairmont (node 3).

I (DEMAND AT TIME 11 , STATE AT TIME 10)

d_{11} \ x_{10}	0	1	2	3	4	5	6	7
98.730	2452.7097 1	659.0081 1	466.3293 2	-931.7781 3	-477.5472 4	-1330.9204 6	-1477.9451 6	-1731.3218 7
113.030	2631.5277 1	837.8260 1	630.3377 2	-880.3160 3	-334.8563 4	-1271.3902 6	-1418.4150 6	-1671.7916 7
127.331	2930.8867 1	1137.1850 1	882.9199 2	-818.8399 3	-124.4189 4	-1200.2783 6	-1347.3030 6	-1600.6797 7
141.631	3253.1307 1	1453.5008 3	1127.8410 3	-750.3999 3	75.6683 6	-1121.1130 6	-1268.1377 6	-1521.5144 7
155.932	3334.0005 2	1523.5300 3	1197.8703 3	-680.3706 3	156.6706 6	-1040.1107 6	-1187.1354 6	-1440.5121 7
170.232	3410.3423 3	1598.2625 3	1272.6027 3	-605.6382 3	243.1114 6	-953.6699 6	-1100.6946 6	-1354.0713 7
184.533	3485.0385 3	1672.9588 3	1347.2990 3	-530.9419 3	329.5091 6	-867.2722 6	-1014.2969 6	-1267.6736 7
198.833	3567.0863 3	1755.0065 3	1429.3467 3	-448.8942 3	424.4087 6	-772.3725 6	-919.3973 6	-1172.773[illegible] 7
213.134	3658.2601 3	1846.1804 3	1520.5206 3	-357.7203 3	527.1734 6	-669.6079 6	-816.6326 6	-1070.7801 7
227.434	3781.7663 3	1969.6866 3	1644.0268 3	-234.2141 3	637.7527 6	-559.0286 6	-706.0533 6	-969.4508 7
241.735	4023.4993 3	2211.4196 3	1885.7598 3	7.5189 3	789.6647 6	-407.1165 6	-554.1413 6	-852.4452 7
256.035	4309.3910 3	2497.3112 3	2171.6514 3	293.4106 3	945.6040 6	-251.1772 6	-398.2020 6	-740.9458 7
270.336	4662.5613 3	2850.4815 3	2524.8217 3	646.5809 3	1126.4539 6	-70.3274 6	-217.3521 6	-610.1006 7
284.636	4922.2969 3	3110.2171 3	2784.5573 3	906.3165 3	1280.1222 6	83.3409 6	-63.6838 6	-483.2136 7
298.937	5143.1016 7	3382.4805 7	3034.7670 7	1156.5261 7	1424.8463 7	228.0650 7	81.0403 7	-360.0339 7
313.237	5274.0931 7	3513.4720 7	3165.7586 7	1287.5177 7	1555.8378 7	359.0565 7	212.0318 7	-229.0424 7

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(a) Decision Table at Time 10.

Example: If demand at Time 11 (1979) is 141.631 MW and if state at Time 10 (1978) is State 2, then the optimal decision at Time 10 is to adopt Configuration 3. The expected cost incurred to the end is $1.127,841.

FIG. IX-3 TYPICAL COMPUTER PRINTOUTS FOR THE OLYMPIA/PORT ANGELES EXAMPLE, WITH THE NEW INDUSTRIAL LOAD AND DISCOUNTING FACTOR 3.125 PERCENT

I (DEMAND AT TIME 12 , STATE AT TIME 11)

d_{12} \ x_{11}	0	1	2	3	4	5	6	7
97.668	2043.7330 0	304.4701 1	115.3953 2	-1191.8066 3	-822.3842 4	-1608.9179 6	-1751.4874 6	-2004.8640 7
114.261	2166.6445 1	427.2975 1	234.7721 2	-1145.7446 3	-715.0784 4	-1555.6349 6	-1698.2044 6	-1951.5810 7
130.853	2417.0126 1	677.6656 1	453.5696 2	-1092.8051 3	-530.8335 4	-1494.3982 6	-1636.9676 6	-1890.3443 7
147.445	2861.9011 2	1110.5548 3	794.7635 3	-1026.5610 3	-257.2590 6	-1417.7742 6	-1560.3436 6	-1813.7203 7
164.037	2928.9886 3	1171.8203 3	856.0290 3	-965.2954 3	-186.3947 6	-1346.9099 6	-1489.4793 6	-1742.8560 7
180.629	2999.6219 3	1242.4536 3	926.6623 3	-894.6622 3	-104.6968 6	-1265.2120 6	-1407.7814 6	-1661.1581 7
197.222	3072.8527 3	1315.6844 3	999.8931 3	-821.4314 3	-19.9957 6	-1180.5109 6	-1323.0804 6	-1576.4570 7
213.814	3145.8569 3	1388.6886 3	1072.8973 3	-748.4272 3	64.4423 6	-1096.0729 6	-1238.6423 6	-1492.0190 7
230.406	3228.1999 3	1471.0317 3	1155.2404 3	-666.0841 3	159.6804 6	-1000.8348 6	-1143.4042 6	-1396.7809 7
246.998	3380.8961 3	1623.7278 3	1307.9365 3	-513.3880 3	271.7498 6	-888.7654 6	-1031.3349 6	-1303.2115 7
263.591	3690.2430 3	1933.0747 3	1617.2834 3	-204.0411 3	433.3460 6	-727.1692 6	-869.7386 6	-1194.4698 7
280.183	4026.8815 3	2269.7133 3	1953.9220 3	132.5975 3	596.5647 6	-563.9505 6	-706.5199 6	-1085.1575 7
296.775	4374.1052 7	2666.8363 7	2329.6596 7	508.3351 7	768.5243 7	-391.9909 7	-534.5603 7	-962.2686 7
313.367	4489.2554 7	2781.9865 7	2444.8098 7	623.4853 7	883.6745 7	-276.8407 7	-419.4101 7	-847.1184 7
329.959	4619.4571 7	2912.1881 7	2575.0114 7	753.6869 7	1013.8761 7	-146.6391 7	-289.2085 7	-716.9168 7
346.552	4753.6028 7	3046.3339 7	2709.1572 7	887.8327 7	1148.0219 7	-12.4933 7	-155.0627 7	-582.7710 7

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(b) Decision Table at Time 11.

Same interpretation as decision table at Time 10.

FIG. IX-3 (Continued)

TIME 10

CONDITIONAL PROBABILITIES DK+1/DK AT TIME 11

0.38	0.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.54	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.67	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.79	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.04	0.79	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.08	0.83	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.21	0.75	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.67	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.58	0.04	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.54	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.50	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.58	0.37	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.63	0.33	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.50	0.33	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.67	0.17	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.50	0.17

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(c) Conditional Probabilities at Time 10.

Demands are quantized in 16 levels. Each of the sixteen lines correspond to a demand level at Time 11. Each of the sixteen columns corresponds to a demand level at Time 12.

Example: If at Time 11 the level of the demand is 4, then the probability that at Time 12 the demand level will be 5 is 0.21 (corresponding to Line 4 and Column 5 in the table).

FIG. IX-3 (Concluded)

The expansion schedule obtained by dynamic programming (Fig. IX-4) is quite similar to the "recommended plan" of the BPA staff (Fig. IX-5), with the notable exception that the computer results favor configuration 1 instead of configuration 2.

Certain equipment states (e.g., 3 and 7) appear to be "stable" states both without and with discounting, while other states (notably 4 and 5) appear "unstable," in the sense that if the system is initially in these states, the optimal decision is usually to go to a higher investment state.

The decision tables thus identify the preferred configurations and allow the planner to eliminate certain consistently "unstable" configurations.

2. Interpretation of Results

The decision tables, obtained as the output of the computer program, are subject to the following interpretations:

(1) For every state $\{x_t, d_{t+1}\}$ at time t the planner is given the optimum decision u_t. For instance, if at time 9 the actual configuration is configuration 1 and if demand at node 4 is going to be next year 132.83 MW (known at time 9), then configuration 3 should be adopted (see Fig. IX-3). A set of decision tables constitutes the decision policy.

(2) For every state $\{x_t, d_{t+1}\}$ at time t <u>the planner is given the minimum expected cost to the end</u>. For instance, if at time 9, $x_9 = 1$ and $d_{10} = 132.83$, this minimum cost is $1,828,130 (see Fig. IX-3).

(3) In deterministic dynamic programming, the tables $I(x_t, t)$ are used at the end of the

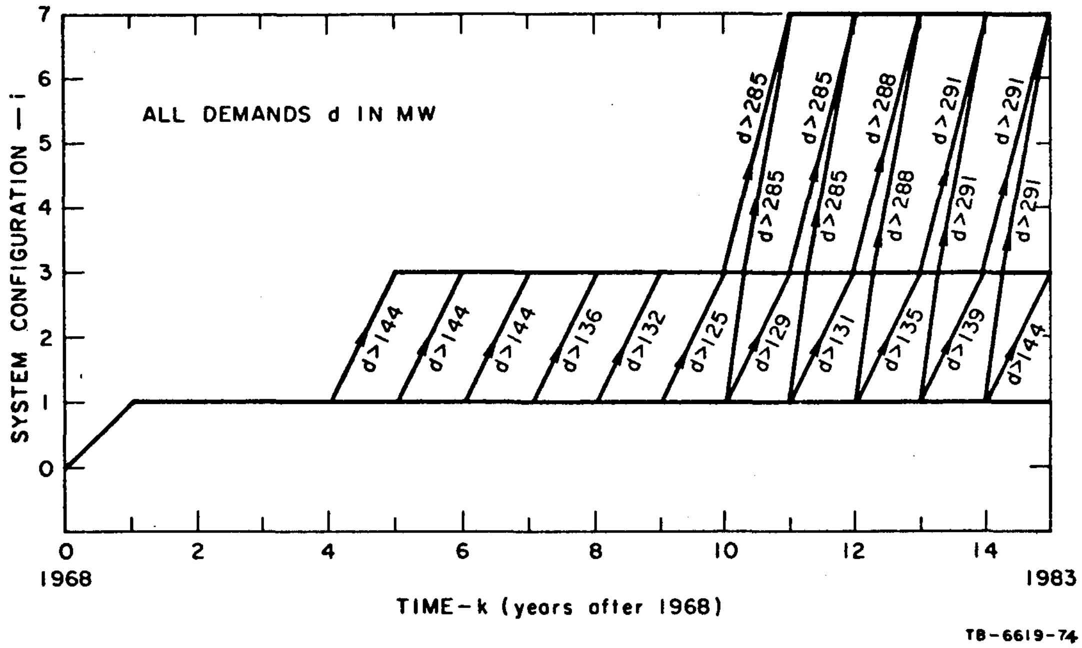

FIG. IX-4 OLYMPIA/PORT ANGELES EXPANSION SCHEDULE, BASED ON THE DECISION TABLES OBTAINED FOR THE EXAMPLE PROGRAM OF SEC. IX-C. For system configurations $i = 0, 1, \ldots, 7$ see Fig. IX-1(b). Arrows indicate transitions to a higher-cost equipment state; the decision at time t is made if the demand at time t + 1 exceeds the values indicated. New equipment is installed at time t + 1.

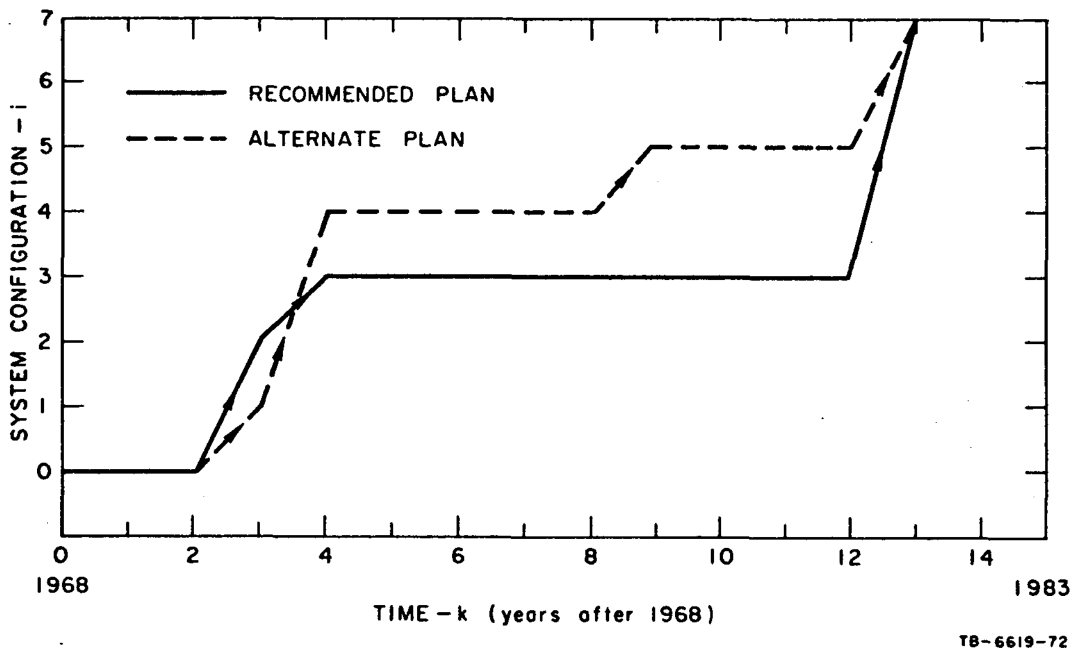

FIG. IX-5 OLYMPIA/PORT ANGELES EXPANSION SCHEDULE, BASED ON BPA STUDIES (Source: Ref. 9). For system configurations $i = 0, 1, \ldots, 7$ see Fig. IX-1(b). Arrows indicate transitions to a higher-cost equipment state; the decision is made at time t and new equipment is installed at t + 1.

optimization to determine a schedule that, starting from the actual state x_o, gives the sequence of optimal decisions u_o, u_1, ..., u_{T-1}. This sequence can thus be known at time 0. Here this schedule is not very meaningful, since the decisions should be adjusted to next year's demand. In other words, the schedule u_o, u_1, ..., u_{T-1} cannot be known at time 0; only u_o can be determined then. At time 1, when d_2 is known, u_1 can be determined, and so on.

(4) If, for some reason, the planner is forced to disobey the recommendation contained in the decision table, the expected additional cost he incurs is readily computable. This was done in Ref. 9, where it was shown that if configuration 3 is adopted instead of the optimum configuration 2 in 1978, given that the demand would be 156 MW in 1979, then the expected waste of money is $161,065.

F. Conclusions and Recommendations

This sample problem led to the following conclusions:

(1) Optimum subsystem planning with complete technical detail and with uncertainty about the future can be solved practically by dynamic programming. Thus, the idea outlined in Ref. 4 is feasible.

(2) In order to obtain meaningful results, the planner is forced at the outset of the program to make certain important assumptions, based on his present knowledge about the future, such as the following:

- Divide the total planning period into an appropriate number of planning stages $t = 0, 1, \ldots, T$. At each planning stage t, a decision will be made to change or keep the system configuration; this decision will be based on the knowledge of the true state x_t and the imperfect information about the future states.

- Define an appropriate demand growth model, and assign probability distribution for the random demand variables.
- Narrow down the large number of possible system configurations to a manageable number of generic configurations (say, 10). Each of these generic configurations may have several variations that can be investigated at a later stage when the preferred configurations have already been identified by the program.
- Define rigorously the reliability criteria for the subsystem.
- Assign realistic discounting factors and terminal-equipment values.
- Organize the equipment cost and lead-time input data in a systematic and consistent manner, so as to form a basis for a computerized data bank.

(3) In return for the systematic effort spent in preparation of the input data, the computer printout provides a great deal of information. For example, the expected total costs are obtained, loss due to inability of the planner to follow an optimal policy is easily calculated, sensitivity analysis with respect to certain variables (e.g., demands) or assumptions (e.g., terminal-equipment value, interest rate) can be readily performed.[9]

(4) Even if some of the assumptions are quite crude, the program can be meaningfully applied to a practical planning problem, and the results give a good insight into the problem. The preparation of input data helps the planner to approach a "fuzzy" problem in a well-organized manner, and evaluation of the output data provides important feedback information in the engineering design phase.

The results obtained so far are promising. Dynamic programming appears to be especially suited for the optimal planning of electrical subsystems, where the uncertainty of the future plays an important part. It provides a policy solution, so that the planner can make every year the best decision that can be made with his up-to-date knowledge of the future. Moreover, this solution is obtained efficiently. A typical subsystem such as the Olympia/Port Angeles network can be run in less than a minute and a half on a B-5500 computer.

X LOAD FORECASTING

A. Statement of the Problem

When planning the expansion of a network, the result obviously depends on the accuracy with which the location and the magnitude of future productions--assumed to be known accurately here--and demands are taken into account. These demands are random variables. The problem is to determine their statistics as a function of time as required by the optimization discussed in Sec. IX.

The demand in a particular year can be characterized in terms of its peak, average, RMS, or, more accurately, as a function of time. The latter description is necessary to compute the losses in the network. Peak loads are a determining factor in reliability studies.

As a network expansion schedule is determined primarily by investment costs and reliability considerations, only the determination of peak loads was considered in this study, and approximate values were used for the losses. The accurate loss computation discussed in Ref. 2 could be introduced in the program as an easy future refinement.

The load-forecasting problem is now stated as follows:

Find for every year t the statistics of the demands d_t where d_t refers to the peak attained in year t.

B. Problem Analysis

The analysis of past records and experience shows the following characteristics:

(1) There is a general trend in the overall consumption of electrical power in a specific area. The larger the area, the smaller are the deviations from this trend.

(2) There are local deviations from this trend. Therefore, though in a given year the total load may

increase by an accurately predictable amount of, say, 5 percent, it may increase by only 3 percent in Los Angeles and by 7 percent in Seattle. These deviations are generally correlated in time. They have three components:

- A fairly predictable element that, added to the overall trend, materializes in the predictable local trend, which we shall denote by a.
- A purely random element with a zero mean.
- A purely random element (represented by new industrial load) with a nonzero mean.

1. <u>Markov Model for Load Forecasting</u>[4]

If d_{t+1} is the demand at time t + 1, then

$$d_{t+1} = d_t \left(1 + a + bw_t\right) + r_t \quad , \qquad \text{(X-1)}$$

where

d_t = demand at time t

a = predictable mean increase

w_t = white random deviation about the predictable mean increase (with zero mean)

b = constant used to normalize the white noise w_t to unit variance

r_t = additional white random variable added to include the possibility of a new large industrial demand.

Figure X-1 gives a possible future demand curve if a = 7 percent, b = 1.15 percent, and r_t = 0.

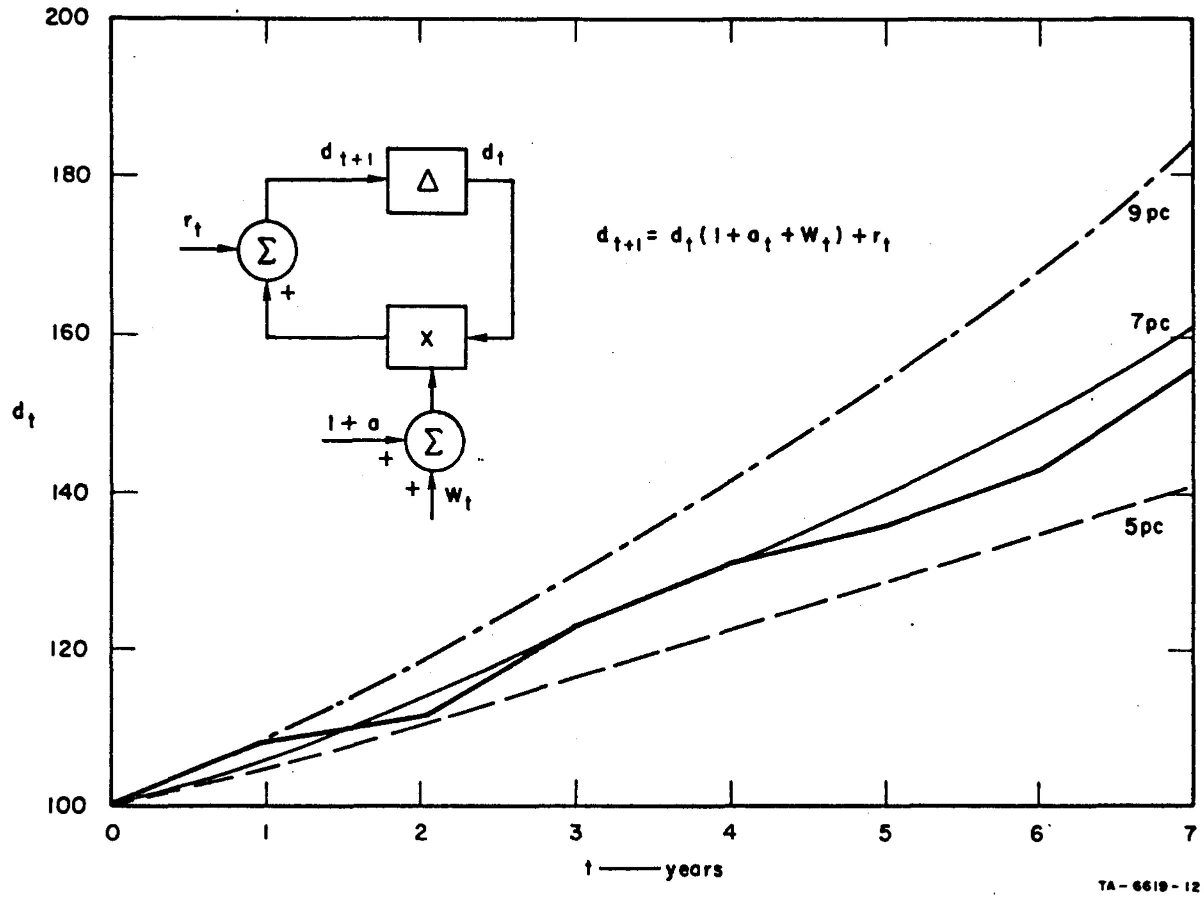

FIG. X-1 POSSIBLE FUTURE DEMAND CURVE FOR 7-PERCENT YEARLY MEAN LOAD INCREASE AND WHITE DEVIATIONS OF ±2 PERCENT MAXIMUM ABOUT THE 7-PERCENT DEMAND INCREASE PROJECTED FROM THE POINT REACHED IN THE PREVIOUS YEAR. The random variable r_t is assumed to be zero.

2. Estimation of the Parameters

The parameters a and b were estimated by the method of moments,[9] with the model:

$$d_{t+1} = d_t \left(1 + a + bw_t\right) \quad , \tag{X-2}$$

where for all t the mean $w_t = 0$ and the variance $\sigma^2_{w_t} = 1$, and where the w_t are uncorrelated in time and uniformly distributed. Denote

$$z_t = \frac{d_{t+1}}{d_t} - 1 \quad , \tag{X-3}$$

then

$$a + bw_t - z_t = 0 \quad . \tag{X-4}$$

Suppose that n values z^*_t can be observed from past data. Then the estimates of a and b--i.e., $\hat{a}$ and $\hat{b}$--are:

$$\hat{a} = \bar{z}^*$$

$$\hat{b}^2 = \sum_{t=1}^{n} \frac{z^{*2}_t}{n} - \hat{a}^2 = \frac{1}{n} \sum_{t=1}^{n} \left(z^{*2}_t - n\bar{z}^*\right) \quad . \tag{X-5}$$

From these results, and with these equations and future projections given by BPA for the Olympia/Port Angeles expansion, the values of $\hat{a}$ and $\hat{b}$ were estimated to be 3.2 percent and 2.46 percent, respectively. These estimates were used in the optimization of Sec. IX.

3. Variable r_t

The statistics of this random variable should be obtained from past experience and economic predictions.

For the Olympia/Port Angeles example, we rather arbitrarily assumed:

$$r_t = \begin{cases} 0 & \text{if } t < 5 \\ \left.\begin{array}{ll} 0 & \text{with probability } 0.25 \\ 5 & \text{with probability } 0.50 \\ 10 & \text{with probability } 0.25 \end{array}\right\} & \text{if } t \geq 5 \end{cases} .$$

C. Results

The load forecasting program is an independent subroutine, which can be used as such or incorporated into the subsystem planning program of Sec. IX.

Inputs

Option 1:

- Estimates of parameters a and b.

Option 2:

- Past values of demand d_t.

In both cases:

- Statistics of r_t for the next T years are given as:

$$r_t = \rho_t \text{ with the probability } \pi_t$$

  (if ρ_t and π_t are the same for all t's, they need only to be given once).

- Desired number ND of quantization levels for future d_t's.

Outputs

- Estimates of parameters a and b.
- For each future year t with t = 0, 1, ..., T:
  - Minimum value of d_t: $d_{t\ min}$
  - Maximum value of d_t: $d_{t\ max}$

- Average value of d_t: $d_{t\ ave}$
- Absolute probability $p\left[d_t/0\right]$ that d_t will be δ_t, where δ_t is one of the ND quantized level of demand, between $d_{t\ min}$ and $d_{t\ max}$, that could materialize in year t.
- Conditional probabilities $p\left[d_{t+1}/d_t\right]$ that the demand d_t, occupying level d_t at time t will occupy the level d_{t+1} at time t + 1.

These conditional probabilities are the only load data required in the subsystem optimal planning program of Sec. IX. A sample of the tables giving $p\left[d_t/0\right]$ and $p\left[d_{t+1}/d_t\right]$ is shown in Fig. X-2.

A graphical representation of these tables, obtained from the load analysis at Port Angeles for the next eighteen years, omitting the possibility of a new industrial customer (r_t = 0 with probability 1) is given in Fig. X-3.

It seems that the exceptionally high values of $d_{t\ max}$ obtained by the model are somewhat unrealistic. This does not mean that the model is inadequate. These values should not be considered alone, but together with their probability of occurrence, which is very small.

D. Conclusion

It is not claimed at this time that the model used here for illustrative purposes should be retained for actual subsystem expansion studies. Past records should be analyzed and future technological and economic changes should be correctly interpreted to obtain an improved statistical model. However, these improved models will enter into the stochastic optimization of Sec. IX in the same general manner that the present model did.

ABSOLUTE PROBABILITIES DT/DO AT TIME 1977

132.9	134.7	136.4	138.1	139.8	141.6	143.3	145.0	146.8	148.5	150.2
0.000	0.001	0.017	0.127	0.275	0.316	0.205	0.054	0.005	0.000	0.000

CONDITIONAL PROBABILITIES DK+1/DK AT TIME 1977

d_t \ d_{t+1}	136.1	138.1	140.1	142.1	144.1	146.2	148.2	150.2	152.2	154.2	156.2
132.9	0.455	0.545	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
134.7	0.000	0.545	0.455	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
136.4	0.000	0.000	0.636	0.364	0.000	0.000	0.000	0.000	0.000	0.000	0.000
138.1	0.000	0.000	0.000	0.727	0.273	0.000	0.000	0.000	0.000	0.000	0.000
139.8	0.000	0.000	0.000	0.000	0.818	0.182	0.000	0.000	0.000	0.000	0.000
141.6	0.000	0.000	0.000	0.000	0.091	0.818	0.091	0.000	0.000	0.000	0.000
143.3	0.000	0.000	0.000	0.000	0.000	0.182	0.818	0.000	0.000	0.000	0.000
145.0	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.727	0.000	0.000	0.000
146.8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.364	0.636	0.000	0.000
148.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.455	0.545	0.000
150.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.545	0.455

ABSOLUTE PROBABILITIES DT/DO AT TIME 1978

136.1	138.1	140.1	142.1	144.1	146.2	148.2	150.2	152.2	154.2	156.2
0.000	0.001	0.011	0.099	0.288	0.345	0.211	0.041	0.003	0.000	0.000

CONDITIONAL PROBABILITIES DK+1/D AT TIME 1978

139.4	141.7	144.0	146.3	148.6	150.9	153.2	155.5	157.8	160.1	162.4

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Examples: Probability that the demand at node 4 is 136.4 MW in 1977 given that it is 106 MW in 1968 : 0.017.

Probability that the demand at node 4 is 142.1 MW in 1978 given that it is 136.4 MW in 1977 : 0.364.

FIG. X-2 PROBABILITIES WITHOUT THE NEW INDUSTRIAL LOAD (Olympia/Port Angeles subsystem)

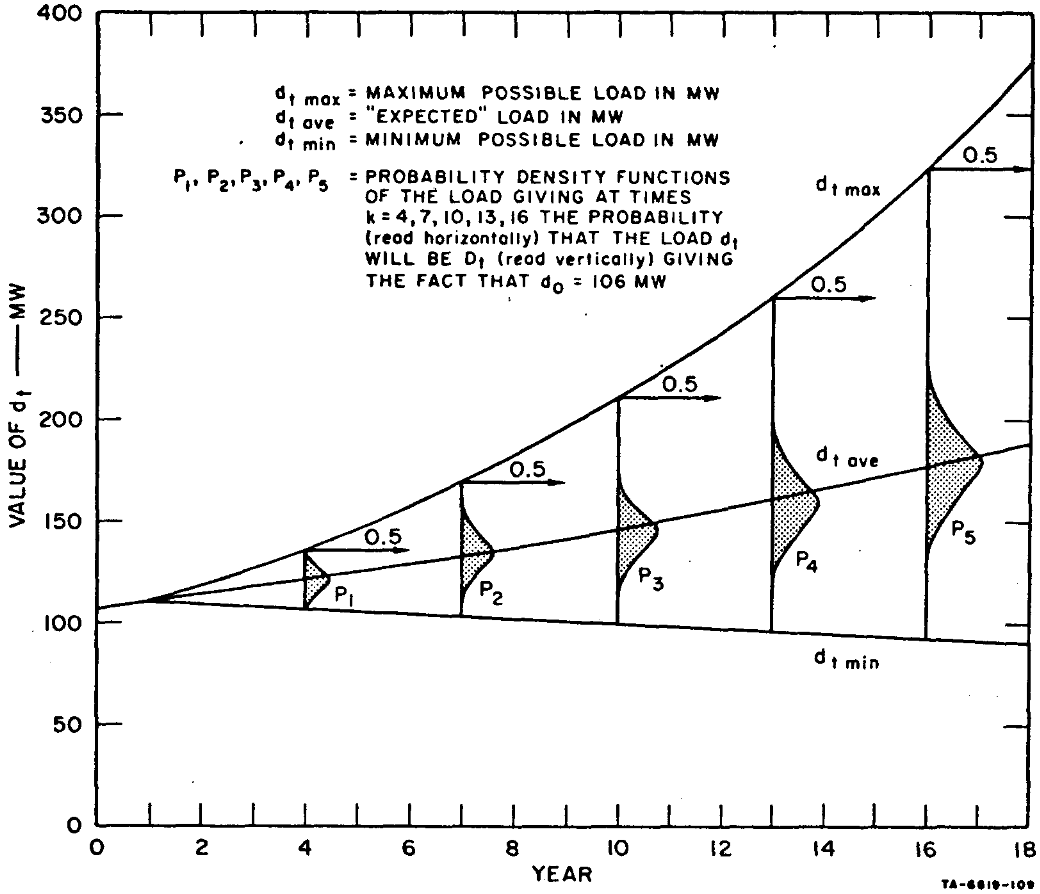

FIG. X-3 FUTURE LOAD AT PORT ANGELES

XI CONCLUSIONS

Since the technical memoranda[2-10] and some of the previous sections already contain conclusions pertaining to the detailed technical material treated, the aim of this section will be to provide an overall view of the highlights of the project and to give recommendations on how to proceed from here.

Perhaps the most important contribution of the project has been to break a very complex system problem down into an orderly sequence of sub-problems, each of which is amenable to solution by known techniques of mathematical analysis and numerical computation. Thus, the problem of long-term system planning was partitioned into overall system planning, where the discrete character of line additions, the reliability constraint, the system losses, and the effects of series compensation were considered; and into subsystem planning, where the uncertainty of future loads was incorporated.

In addition to an orderly breakdown of this complex problem, the feasibility and practicality of the analysis and computational methods developed in the course of the project were proven. Dynamic programming was used extensively after the computational difficulties inherent in this otherwise extremely powerful procedure had been eliminated by a proper problem formulation and by the recently developed successive-approximation technique. In connection with planning problems, expert use of dynamic programming far surpasses the cost/benefit analysis procedures of current fame and the linear programming approaches that have become well established for simpler economic optimization problems.

The project has not only been an application of dynamic programming to network expansion problems. We feel that the efficient computational procedures developed for analyzing system reliability and transient stability will be of great value to the power engineer in operation as well as in planning. We feel that the discrete nominal will soon permit the efficient approximate study of complex projected future systems. We finally feel that the stochastic optimization approach will add much to the effectiveness with which parts of power systems are presently evaluated and expanded, and with which future load requirements are estimated.

The feasibility of these techniques having been proven by experimental programs, it remains necessary to convert these programs into reliable routine engineering tools for the Bonneville Power Administration. This will require a sizeable effort, which appears economically highly attractive, in view of the tremendous dollar savings that even a one-year investment delay of a major facility brings about. As authors, we feel committed to the R&D work we performed and are prepared to cooperate with the Bonneville Power Administration to develop these routine programs. As a first step, we propose to work out, in Portland, with BPA system engineers example problems of present concern to the Administration. This will familiarize the BPA staff with the techniques developed much better than could be done with a report. Thereafter, a plan for performing the service programming required will need to be evolved; being familiar with the intricacies of these programs, we are prepared to advise the Bonneville Power Administration in evolving this plan in accordance with a logical step-by-step schedule.

A major fiction maintained throughout the project is that future generation is known to the transmission system designer. In actual fact, the problems of transmission and generation planning are coupled and should be treated as such, even though there may exist administrative boundaries between the transmission and generation planning departments. From a mathematical point of view, generation planning (including nuclear and hydro) can be handled by the same fundamental techniques discussed in this report for transmission planning, although there are numerous differences of detail. Thereafter, the two planning procedures should be joined in a single program, the general structure of which is known.

Finally, a conscientious effort should be made to apply some of the analysis techniques developed to problems that may appear to be very different, but in fact are not. In the course of our discussions with power engineers working in the area of automatic dispatching centers (BPA, TVA, AEP, and EDF), we became aware of the fact the reliability and transient stability tests worked out for planning problems could be used readily, with minor modifications, as real-time monitors of network security. We feel that similar applications could be found for the loss computation, outage analysis, and emergency load curtailment procedures discussed in this report.

Appendix A

OPTIMAL CONTROL OF NETWORK WITH QUADRATIC COST

1. Introduction

This appendix is a comprehensive derivation of the network model equations of Sec. II-D-1. It starts with some general considerations on systems, and then derives particular results for networks with a quadratic cost function. By writing that the system is optimally operated, one obtains a set of system equations that are equivalent to the ones directly derived from the Kirchoff laws of electrical networks. In particular, the voltage phase angles θ are shown to be also the Lagrange multipliers of this optimization.

2. General Considerations on Systems

In the study of a system we shall distinguish:

(1) The external world

(2) The design of the system

(3) The operation of the system.

Accordingly, the behavior of the system is influenced by several types of data:

(1) The external data w, on which neither the designer nor the operator of the system can act,

(2) The design parameters p, which are chosen by the designer of the system.

(3) The operating variables u, which are determined by the operator at each instant of the operation of the system.

(4) The dependent variables x, which result, at each instant of time t, from the current (w, p, u) values.

These relations among these variables constitute a set of algebraic equations:

$$G(w, p, u, x) = 0 \quad . \qquad \text{(A-1)}$$

All the above variables may be subject to inequality constraints.[42]

System Performance

Usually the ownership of a system entails two costs:

(1) The capital cost C(p)

(2) The operating cost L(w, p, u, x).

Our aim is to find an optimal policy for the design of the system, which implies that we wish to minimize, over the planning period of the system, the sum total of the capital and operating costs.

It is convenient and realistic for the system designer to assume that the operator will always operate the system optimally, i.e., solve the following problem:

For a system with state equations

$$G(w, p, u, x) = 0 \quad :$$

Given w at an instant t, find p that minimizes the capital cost C(p), while p, u, x satisfy certain inequality constraints, the control variables u being set to minimize the operating cost L(w, p, u, x).

These are the same hypotheses as for the continuous nominal of Sec. III above.

3. Example of Network Flow Optimization with Quadratic Cost

a. System Equations

Let $G = \{N, \mathcal{G}\}$ be a directed graph with n nodes and r arcs. The subscripts i or j will refer to nodes, while k will refer to the arc joining the nodes (ij). As an illustration, we shall consider the following 5-node, 7-branch network (Fig. A-1).

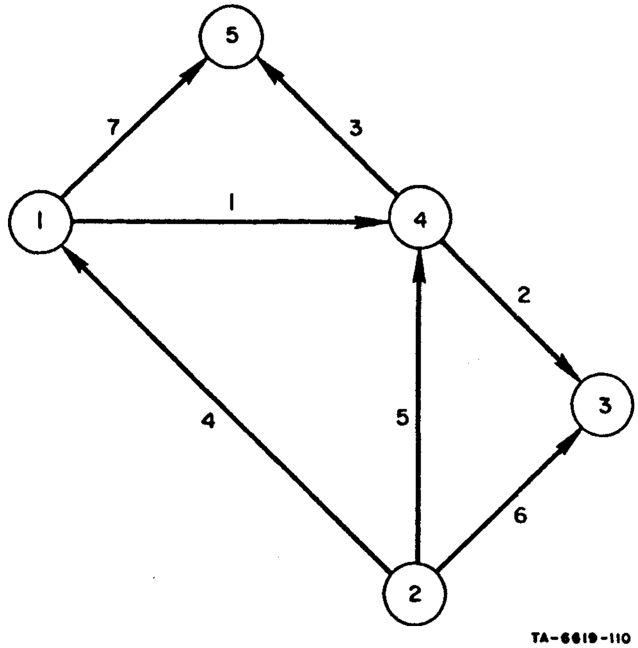

FIG. A-1 EXAMPLE NETWORK

In each arc k there exists a flow T_k, which has a sign with respect to the (arbitrarily given) orientation of the arc. The flow T_k is not limited at this time by any "capacity" constraint in either direction. At every node i an equation of the following type holds:

$$\mathcal{I}_i = \sum_{k \in K_i} T_k - \sum_{k' \in K'_i} T_{k'} \qquad i = 1, \ldots, n \qquad \text{(A-2)}$$

K_i is the set of arcs originating at node i

K'_i is the set of arcs ending at node i.

$\mathcal{I}_i$ is the injection of flow at node i, from the external world. It has a given value for each node, and the conservation of flow implies:

$$\sum_{i=1}^{n} \mathcal{I}_i = 0 \quad , \qquad \text{(A-3)}$$

which allows the elimination of one equation in system (A-2).

b. System Performance

We define an operating cost L attached to the flow through the network, and we assume this function to be quadratic:

$$L = \frac{1}{2} \sum_{k=1}^{r} \sum_{k'=1}^{r} q_{kk'} T_k T_{k'} \quad , \tag{A-4}$$

where $q_{kk'}$ are the elements of a positive definite quadratic matrix Q. A particular case arises when the matrix Q is diagonal, i.e.,

$$q_{kk'} = \begin{cases} s_k & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases} .$$

Then

$$L = \frac{1}{2} \sum_{k=1}^{r} s_k T_k^2 \quad .$$

The operating cost of a branch is then proportional to the square of flow through it.

The matrix Q is determined by the design of the system. Hence, we shall consider its elements to be design parameters.

Since the system (A-2) has (n - 1) independent equations and r variables, there exist (r - n + 1) degrees of freedom in the choice of the T_k's.

With reference to the general discussion of Sec. 2, we now identify the variables w, p, u, and x.

- The ϑ_i are the external data w.
- The $q_{kk'}$ are the design parameters p.
- (r - n + 1) of the T_k are the operating variables u.
- (n - 1) of the T_k are the dependent variables x.

The operator's role is to choose (r - n + 1) of the T_k's in such a way that the quadratic function L is minimized, given the Q matrix and the injections $\mathcal{I}_i$.

The designer's role is to determine the elements of the Q matrix such that the sum of the capital and operating costs is minimized.

Our immediate concern is to solve the operator's problem, assuming a design Q has been provided.

c. Optimization Using Lagrange Multipliers

Let A be the incidence matrix of the system shown in Fig. A-1

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array} .$$

Equations (A-2) can then be written as:

$$AT = \mathcal{I} \tag{A-5}$$

where

$\mathcal{I}$ is the vector of the injections $\mathcal{I}_i$

T is the vector of the flows T_k.

The Lagrangian function associated with Eqs. (A-4) and (A-5) is

$$\mathcal{L} = \frac{1}{2} T' QT - \lambda'(AT - I) \tag{A-6}$$

$$= \frac{1}{2} T' QT - \left(T' A' - I'\right)\lambda \quad , \tag{A-7}$$

where λ is the column-vector of the Lagrange multipliers λ_i assigned to each equation (A-2). The optimality conditions become:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{\partial \mathcal{L}}{\partial T} = 0 \quad . \tag{A-8}$$

The optimality condition (A-8) is written explicitly as:

$$- AT + \mathcal{J} = 0 \tag{A-9}$$

$$QT - A'\lambda = 0 \quad . \tag{A-10}$$

Assuming that the matrix Q has an increase:

$$T = Q^{-1} A'\lambda \tag{A-11}$$

$$A Q^{-1} A'\lambda = \mathcal{J} \quad . \tag{A-12}$$

Equations (A-11) and (A-12) provide the solution to the problem of optimal dispatching of flow. Given A, Q, and $\mathcal{J}$, we compute the dual variables λ by solving the system of Eqs. (A-12). Thereafter, we compute the flows T from Eq. (A-11).

Using a commonly accepted approximate model for AC transmission systems, we define the matrix Q as

$$Q = \begin{bmatrix} s_1 & & (0) \\ & \ddots & \\ (0) & & s_2 \end{bmatrix}$$

with

$$s_k = \frac{2 R_k}{V_k^2} \quad ; \quad k = 1, \ldots, r \tag{A-13}$$

$$\begin{cases} R_k = \text{resistance of line } k \\ V_k = \text{voltage of line } k. \end{cases}$$

We assume that the ratio reactance/resistance is the same for all the branches of the system, which is approximately true when we limit ourselves to EHV lines.

Since

$$s_k > 0 \quad \forall \; k = 1, \ldots, r$$

$$Q^{-1} = \begin{bmatrix} \frac{1}{s_1} & & (0) \\ & \ddots & \\ (0) & & \frac{1}{s_r} \end{bmatrix}$$

As in Sec. II-D-1, we define the variables

$$\mu = \frac{X_k}{R_k}$$

$$\gamma_k = \frac{V_k^2}{X_k} = \frac{2}{\mu}\frac{1}{s_k} \quad .$$

Then

$$Q^{-1} = \frac{\mu}{2}\begin{bmatrix} \gamma_1 & & (0) \\ & \ddots & \\ (0) & & \gamma_r \end{bmatrix} \quad . \tag{A-14}$$

With this notation, Eqs. (A-11) and (A-12) become:

$$T_k = \frac{\mu}{2}\gamma_k\left(\lambda_i - \lambda_j\right) \tag{A-15}$$

$$\begin{cases} k = 1, \ldots, r \\ \text{i and j are the origin and the extremity of branch k.} \end{cases}$$

We define:

$$\theta_i = \frac{\mu}{2} \lambda_i \qquad i = 1, \ldots, n \quad ,$$

then

$$\sum_{k \epsilon K(i)} \gamma_k \left[\theta_i - \theta_j(k)\right] = \mathcal{J}_i \qquad i = 1, 2, \ldots, n \quad . \qquad \text{(A-16)}$$

K(i) is the set of the indices of the branches incident with node i.

j(k) is the other end of branch k incident with node i.

Equations (A-15) and (A-16) can be directly derived from the laws of AC networks. θ_i is then the voltage phase angle at node i.[33] However, the angular differences $\left(\theta_i - \theta_j\right)$ are assumed to remain small for this model to be accurate--sin $\left(\theta_i - \theta_j\right)$ is replaced by $\left(\theta_i - \theta_j\right)$--the θ's have thus a physical and directly measurable value.

We have previously shown that the θ's are proportional to the Lagrange multipliers λ_i in the flow optimization problem. This suggests that the laws of electrical networks play the role of an operator who dispatches the flows T_k optimally, so as to minimize the total loss of power.

Finally, we see that the network model of Sec. II-D-1 is not specific to electrical power transmission, but that it applies to a broader class of network flow problems with a quadratic performance criterion.

Appendix B

SYSTEM LOSSES AS A FUNCTION OF BRANCH CAPACITY AND INJECTIONS

1. Introduction

In this appendix a second-order approximation of the power losses in terms of the branch capacities γ is derived. This approximation is needed in the dynamic programming part of Sec. VII.

The system and the loss expressions are the same as given in Sec. II-D-1 and in Appendix A:[33]

$$\sum_{j \in K(i)} \gamma_{ij} \left(\theta_i - \theta_j\right) = \vartheta_i \tag{B-1}$$

$i = 1, \ldots, n - 1$

$K(i)$ = set of all nodes connected with node i

$$L = \frac{1}{2\mu} \sum_{i,j} \gamma_{ij} \left(\theta_i - \theta_j\right)^2 \tag{B-2}$$

where

$$\mu = X_{ij}/R_{ij}$$
$$\gamma_{ij} = V_{ij}^2/X_{ij}.$$

In the interest of minimizing the programming effort, we assumed a constant $\mu = X/R$ ratio throughout the network. This is not a necessary assumption, and individual X/R ratios could be easily added to the program. We can write Eqs. (B-1) and (B-2) in a more compact form:

$$G(\theta, \gamma) = 0 \tag{B-3}$$

$$L = L(\theta, \gamma) \tag{B-4}$$

with

$$\gamma = [\gamma_1, \ldots, \gamma_r]$$

$$\theta = [\theta_1, \ldots, \theta_{n-1}] \quad \theta_n = 0 \quad .$$

For given γ the system (B-3) can be solved to yield $\theta = \theta(\gamma)$. We then substitute θ into (B-4) and obtain:

$$L = L[\theta(\gamma), \gamma] = L^*(\gamma) \quad . \tag{B-5}$$

However, solution of the system (B-3) involves the inversion of a (n - 1) by (n - 1) matrix, and the result is far from being a simple function of γ. We prefer to derive a second-order sensitivity equation of the form:

$$L^*(\gamma + \Delta\gamma) = L(\gamma) + A\ \Delta\gamma + \frac{1}{2}\Delta\gamma'\ B\ \Delta\gamma + \xi(\Delta\gamma) \quad , \tag{B-6}$$

where the prime denotes transposition

A is a vector with r elements;

B is a r × r symmetric positive definite matrix, and

$$\lim_{\Delta\gamma \to 0} \frac{\xi(\Delta\gamma)}{\Delta\gamma'\Delta\gamma} = 0 \quad . \tag{B-7}$$

2. Derivation of the Terms of Equation (B-6)

This derivation is patterned after the second-order sensitivity methods developed in Refs. 2, 30, and 42. In what follows, we shall omit the $\xi(\Delta\gamma)$ terms in the Taylor Series expansions.

$$\Delta L = L_\theta\ \Delta\theta + L_\gamma\ \Delta\gamma + \frac{1}{2}\Delta\theta'\ L_{\theta\theta}\ \Delta\theta + \frac{1}{2}\Delta\gamma'\ L_{\gamma\gamma}\ \Delta\gamma + \Delta\theta'\ L_{\theta\gamma}\ \Delta\gamma \tag{B-8}$$

$$0 = G_\theta\ \Delta\theta + G_\gamma\ \Delta\gamma + \frac{1}{2}\Delta\theta'\ G_{\theta\theta}\ \Delta\theta + \frac{1}{2}\Delta\gamma'\ G_{\gamma\gamma}\ \Delta\gamma + \Delta\theta'\ G_{\theta\gamma}\ \Delta\gamma \quad . \tag{B-9}$$

The Lagrangian $\mathcal{L}(\theta, \gamma)$ is defined as:

$$\mathcal{L}(\theta, \gamma) = L(\theta, \gamma) + \lambda' G(\theta, \gamma) \quad , \tag{B-10}$$

where the dual vector λ is defined by:

$$L_\theta + \lambda' G_\theta = 0 \quad . \tag{B-11}$$

It can be shown readily that the incremental loss ΔL of Eq. (B-8) can be expressed in terms of the Lagrangian $\mathcal{L}$ by:

$$\Delta L = \mathcal{L}_\gamma \Delta\gamma + \frac{1}{2} \Delta\theta' \mathcal{L}_{\theta\theta} \Delta\theta + \frac{1}{2} \Delta\gamma' \mathcal{L}_{\gamma\gamma} \Delta\gamma + \Delta\theta' \mathcal{L}_{\theta\gamma} \Delta\gamma \quad . \tag{B-12}$$

$\Delta\theta$ must now be replaced by its expression in terms of $\Delta\gamma$. In Eq. (B-12) $\Delta\theta$ appears only in the second-order terms, and this equation is limited to the second order. Therefore, a first-order expansion of relation comes from Eq. (B-9):

$$\Delta\theta = - G_\theta^{-1} G_\gamma \Delta\gamma \quad , \tag{B-13}$$

assuming that G_θ^{-1} exists. Let E denote the matrix:

$$E = - G_\theta^{-1} G_\gamma \quad . \tag{B-14}$$

Then

$$\Delta L = \mathcal{L}_\gamma \Delta\gamma + \frac{1}{2} \Delta\gamma' \left[E' \mathcal{L}_{\theta\theta} E + \mathcal{L}_{\gamma\gamma} + 2E' \mathcal{L}_{\theta\gamma} \right] \Delta\gamma \tag{B-15}$$

and

$$A = \mathcal{L}_\gamma$$

$$B = E' \mathcal{L}_{\theta\theta} E + \mathcal{L}_{\gamma\gamma} + 2E' \mathcal{L}_{\theta\gamma} \quad . \tag{B-16}$$

The partial derivatives involved in the above equations are readily computed from the γ and the $\mathcal{J}$ data. Once A and B have been obtained for a configuration γ, the sensitivity of the losses to variations in γ is then given, to the second order, by formula (B-15). The accuracy of this approximation is illustrated by a numerical example in Sec. 4 of this appendix.

3. Time-Varying Loads

It is possible to make the node injections $\mathcal{J}_i$ appear explicitly in Eq. (B-15). A and B are both bilinear in $\mathcal{J}_i$:

$$A = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \bar{a}_{ij}\, \mathcal{J}_i\, \mathcal{J}_j$$

$$B = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \bar{\bar{b}}_{ij}\, \mathcal{J}_i\, \mathcal{J}_j \quad , \qquad \text{(B-17)}$$

where $\bar{a}_{ij}$ is a tensor of the third order and $\bar{\bar{b}}_{ij}$ a tensor of the fourth order.

If the loads $\mathcal{J}_i$ are varying with time t, then we can define the total loss of energy L_E during a given period $\mathfrak{T}$

$$L_E = \int_0^{\mathfrak{T}} L(t)dt \quad . \qquad \text{(B-18)}$$

And formula (B-15) integrated over time yields:

$$\Delta L = \left[\int_0^{\mathfrak{T}} A(t)dt\right]\Delta\gamma + \frac{1}{2}\,\Delta\gamma'\left[\int_0^{\mathfrak{T}} B(t)dt\right]\Delta\gamma \quad , \qquad \text{(B-19)}$$

where

$$\int_0^{\mathfrak{T}} A(t)dt = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \bar{a}_{ij} \int_0^{\mathfrak{T}} \mathcal{J}_i(t)\, \mathcal{J}_j(t)dt$$

$$\int_0^{\mathfrak{T}} B(t)dt = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \bar{\bar{b}}_{ij} \int_0^{\mathfrak{T}} \mathcal{J}_i(t)\, \mathcal{J}_j(t)dt \quad . \qquad \text{(B-20)}$$

Finally, the only necessary information about time-varying loads is that contained in the matrix Φ, defined as:

$$\Phi_{ij} = \int_0^{\mathcal{T}} \mathcal{J}_i(t)\ \mathcal{J}_j(t)dt \qquad \text{(B-21)}$$

and easily computable from past records. Detailed knowledge of each of the injection functions $\mathcal{J}_i(t)$ is not required.

4. Numerical Example

We apply the above results to the 8-node, 13-branch model of the BPA grid (Fig. II-2). The branch capacities γ are shown on Table B-1 below. We compute the losses $L(\gamma)$, and we use formula (B-15) to approximate the new losses $L(\gamma + \Delta\gamma)$ when the capacities have been modified by $\Delta\gamma$.

Table B-1

DESIGN PARAMETERS AND VARIATIONS

Branch	γ	$\Delta\gamma$
1	14,022	-2000
2	4,707	0
3	1,086	0
4	10,166	-1000
5	8,734	-1000
6	11,768	1500
7	236	0
8	3,362	0
9	4,707	0
10	3,923	0
11	5,884	1000
12	10,066	0
13	12,582	0

In Table B-2, the second-order approximation of the new losses is compared with their exact value, which has been separately computed.

Table B-2

COMPARISON OF THE RESULTS

$L(\gamma)$	559.3 MW
$L(\gamma) + A\ \Delta\gamma$	597.2 MW
$L(\gamma) + A\ \Delta\gamma + \frac{1}{2}\ \Delta\gamma'\ B\ \Delta\gamma$	602.6 MW
$L(\gamma + \Delta\gamma)$	603.9 MW

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