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Abstract

This is the final report of a three-year, Laboratory Directed Research and Development (LDRD) project conducted at the Los Alamos National Laboratory (LANL). This project has focused on developing and experimentally verifying a suite of analytical tools for identifying the onset of damage in structural and mechanical systems from changes in their vibration characteristics. A MATLAB-based computer code referred to as Damage Identification And MOdal aNalysis of Data (DIAMOND) was developed. The code was then extensively exercised on data obtained from a variety of test structures. The most notable structure was an in situ bridge located ten mile north of Truth or Consequences, New Mexico. The suite of tools contained in DIAMOND is now being applied to the nuclear weapons enhanced surveillance program and an industrial partner has asked to enter into a partnership so that they can implement routines from DIAMOND into their commercial damage assessment hardware for large civil engineering structures. Because of the large volume of requests from around the world for DIAMOND, it can now be downloaded from the web site: http://esaea-www.esa.lanl.gov/damage_id.

Background and Research Objectives

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical, and aerospace engineering communities. Current damage detection methods are either visual or localized experimental methods such as acoustic or resonant ultrasonic (RUS) methods, magnetic field methods,

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*Principal Investigator, e-mail: farrar@lanl.gov radiography, eddy-current methods and thermal field methods. All these experimental methods require that the vicinity of the damage is known *a priori* and that the portion of the structure being inspected is readily accessible. Subjected to these limitations, these experimental methods can detect damage on or near the surface of the structure. The need for more global damage detection methods that can be applied to complex structures has led to the development of methods that examine changes in the vibration characteristics of the structure.

Global damage or fault detection, as determined by changes in the dynamic properties or response of structures, is a subject that has received considerable attention in the technical literature beginning approximately 30 years ago. In general, the concept of *damage* implies a comparison between the current state of the structure and some previous baseline state that often is considered to be the undamaged condition. Based on the amount of information provided regarding the damage state, vibration-based damage identification methods can be classified as providing four levels of damage detection. The four levels are: 1)identify that damage has occurred; 2) identify that damage has occurred and determine the location of damage; 3) identify that damage has occurred, locate the damage, and estimate its severity; and 4) identify that damage has occurred, locate the damage, estimate its severity, and determine the remaining useful life of the structure.

Vibration-based damage detection methods can be further classified as model- or nonmodel-based methods. This study investigates both methods. Model-based methods assume that the system will behave according to some physical model such as that described by the standard linear, second-order differential equation for a vibrating system. Nonmodel-based methods attempt to identify changes in the system by simply looking for changes in patterns of the vibration signatures. Typically nonmodel-based methods require some training data from an undamaged and damaged system. Model-based approaches can be classified as linear and nonlinear methods. The distinction here is in the type of model that will be used to characterize the response of the structure. Damage may cause nonlinear response in a structure that previously could be accurately simulated with a linear model.

Finally, methods can also be classified based on the time scale over which the monitoring is to take place. In some cases there is a need to monitor the structure only after some extreme event such as an earthquake. Other situations such as monitoring for fatigue crack growth require the data to be acquired continuously or at relatively short time intervals.

The basic premise of global vibration-based damage detection is that the dynamic response measures such as acceleration time histories and properties derived from them, most notably resonant frequencies, mode shapes, and modal damping, are a function of the physical properties of the structure (mass, damping, stiffness, and boundary conditions). Therefore, changes in physical properties of the structure, such as its stiffness or flexibility, will cause changes in the measured response and/or derived modal properties.

Therefore, the research goal of this project was to develop a robust suite of tools that analyze measured vibration data to determine the location of damage in a structure. This goal could not be accomplished unless these tools were verified on data obtained from "real-world" structures. To this end, numerous laboratory specimens and *in situ* structures would have to be tested in varying configurations and with varying amounts of damage. An important feature that needs to be incorporated into a vibration-based damage-identification method is the ability to discriminate between the changes in the identified vibration characteristics caused by damage and those caused by changing test conditions. To this end, statistical modal analysis procedures would have to be developed (under a complimentary LDRD project as it turns out). The coupling of statistical analysis procedures with experimental modal analysis procedures and vibration-based damage detection algorithms is the unique contribution that this project has made to the engineering community.

Importance to LANL's Science and Technology Base and National R&D Needs

The ability to detect damage at an early stage in any mechanical or structural system will enhance the safety and reliability of that system. Also, repairs of damage at an early stage are typically much more cost-effective than those undertaken when the damage has progressed to a more severe state. Because of the pervasive need for damage detection methods across all engineering disciplines, the potential impact of this technology on society is tremendous. As an example, there are currently no quantifiable methods to determine if a building or bridge is structurally sound after a significant earthquake. Instead, visual inspections are performed by engineers in a very rapid manner. Many of these engineers are pressed into service on short notice and have little experience in performing such assessments. If an array of accelerometers can be located through a building, changes in the vibration characteristics measured during aftershocks could potentially give a quantifiable measure of the damage in the structure. Similarly, the only surveillance tools that assess mechanical defects in nuclear weapons systems on a global basis are visual inspections. If vibration-based methods can be shown to identify certain types of damage in these systems, these methods represent one of the first rapid, nondestructive condition-assessment tools that can be applied to surveillance of the nuclear weapons stockpile. The potential economic impact of such technology, whether it is applied to commercial and municipal systems or to stockpile surveillance, will be immense.

The goals of this program have required the coupling of several different technologies at Los Alamos and extending the state of the art in several of these disciplines. These technologies include dynamic testing and data acquisition applied to vibration testing of large-scale structures, statistical analysis of measured experimental data, and numerical simulation of the dynamic response of structures and mechanical systems.

Scientific Approach and Accomplishments

The approach taken in this study was that there is no single vibration-based damage detection method that can perform equally well on all problems. Therefore, the code that was developed is a suite of graphical interface software algorithms that can numerically simulate vibration tests and apply various modal analysis, damage identification, and finite-element model refinement techniques to measured or simulated modal vibration data. This toolbox is known as DIAMOND (Damage Identification And MOdal aNalysis of Data). DIAMOND is written in MATLAB [1], a numerical matrix math application that is available on all major computer platforms. DIAMOND is unique in three primary ways:

- DIAMOND contains several of the most widely used modal curve-fitting algorithms. Thus the user may analyze the data using more than one technique and compare the results directly. This modal identification capability is coupled with a numerical test-simulation capability that allows the user to directly explore the effects of various test conditions on the identified modal parameters.
- The damage identification and finite-element model-refinement modules are graphically interactive, so the operation is intuitive and the results are displayed visually as well as numerically. This feature allows the user to easily interpret the results in terms of structural damage.
- DIAMOND has statistical analysis capability built into all three major analysis modules: modal analysis, damage identification, and finite-element model refinement. The statistical analysis capability allows the user to determine the magnitude of the uncertainties associated with the results. No other software package for modal analysis or damage identification has this capability.

The development of DIAMOND was motivated primarily by the lack of graphical implementation of modern damage identification and finite-element model-refinement algorithms. Also, the desire to have a variety of modal curve-fitting techniques available and the capability to generate numerical data with which to compare the results of each technique was a motivating factor. The authors are unaware of any commercial software package that integrates all of these features.

Overview of DIAMOND

DIAMOND is divided into four primary modules at the top level: numerical vibration test simulator, experimental modal curve fitting and statistical analysis, damage identification, and finite-element model refinement. In this summary, the three analysis-

Experimental Modal Analysis / Statistical Analysis of Modal Data

The experimental modal analysis module provides a series of tools for plotting the data in various forms, plotting data indicator functions, defining sensor geometry, performing modal curve fits, analyzing the results of modal curve fits, and analyzing the variance of identified modal parameters as a function of the noise in the measurements as defined by the measured coherence function.

The most important feature of the experimental modal analysis module is the variety of modal parameter-identification algorithms that are available. These include:

- Operating shapes, which is simply “peak picking” or “slicing” the frequency response function (FRF) matrix at a particular frequency bin.
- Eigensystem Realization Algorithm (ERA) [2], which is a low-order time-domain modal-parameter-estimation algorithm.
- Complex exponential algorithm, which is a high-order time-domain modal-parameter-estimation algorithm. The specific algorithm implemented is the Polyreference Time Domain [3] approach.
- Rational Polynomial Curve fit [4], which is a high-order frequency-domain technique that uses orthogonal polynomials to estimate the coefficients of a rational polynomial representation of the frequency response function.
- Nonlinear least squares fit, which uses a Levenberg-Marquardt, nonlinear, least-squares, curve-fitting routine [5] to estimate modal frequencies and modal damping ratios from the unfiltered Fourier spectral responses of a base-excited structure.

Any of these modal identification algorithms can be implemented in a statistical Monte Carlo [6] technique. In such an analysis, a series of perturbed data sets, based on the statistics of the measured FRFs as defined by the measured coherence functions, are generated and propagated through the selected algorithm. The statistics on the results are then used as uncertainty bounds on the identified modal parameters.

Damage Identification

The algorithms contained in the damage identification module of DIAMOND can be classified as modal-based, finite-element refinement-based, or nonlinear. The damage identification module presents a number of different algorithms:

Strain energy methods are based on the work of Stubbs [7], Cornwell [8], and others. The basic idea of these methods is the division of the structure into a series of beam or plate-like elements, and then the estimation of the strain energy stored in each element both before and after damage. The curvatures (second-derivatives with respect to space) of the mode shapes are used to approximate the strain energy content.

Flexibility methods all use some measure of the change in the modal flexibility matrix, estimated from the mass-normalized measured mode shapes and squared modal frequencies. The modal flexibility matrix is used to estimate the static displacements that the structure would undergo as a result of a specified loading pattern. The uniform load flexibility method [9] involves specifying a unit load at all measurement degrees of freedom (DOF), then comparing the change in the resulting displacement pattern before and after damage. The point flexibility method [10] specifies the application of a unit load at each measurement DOF one at a time, then looking for a change in the resulting displacements at the same point before and after damage.

The selective flexibility method, which is still under development, uses one of the above two flexibility approaches but filters the modes used to form the flexibility matrix according to their relative statistical uncertainty. The idea of this method is to exclude modes with a high uncertainty from the analysis to avoid biasing the results.

The residual flexibility method [11] also uses one of the above two flexibility approaches but includes the estimate of the residual flexibility, which is the contribution to the flexibility matrix from the modes above the bandwidth of interest. The resulting flexibility matrix is a closer approximation to the true static flexibility matrix than is the modal flexibility matrix.

Finite-element model correlation-based damage-identification techniques are based on the comparison of the finite-element model-correlation results from before damage to those after damage. The correlation techniques are discussed in the next section.

Nonlinear damage identification techniques are based on different theories of nonlinear signal processing. They are a widely varying group of methods including time-frequency methods and nonmodel based pattern recognition methods. These methods are reviewed and discussed in Ref. [12].

Finite-Element Model Refinement

The finite-element model-refinement module consists of four options: pre-processing for update analysis, optimal matrix updating, sensitivity-based model update, and post-processing of update results. The pre-processing phase of the model correlation analysis involves the selection of which modal parameters (i.e. modal frequencies and mode shapes) should be used in the correlation, as well as which finite-element model parameters should be updated.

The optimal matrix update methods are based on the minimization of the error in the structural eigenproblem using a closed-form, direct solution. The minimum rank perturbation technique (MRPT) [13] is one such method that produces a minimum-rank perturbation of the structural stiffness, damping, and/or mass matrices reduced to the measurement degrees of freedom. The minimum rank element update (MREU) [14] is a similar technique that produces perturbations at the elemental, rather than the matrix, level. The Baruch updating technique [15] minimizes an error function of the eigenequation using a closed-form function of the mass and stiffness matrices.

The sensitivity-based model update methods also seek to minimize the error in the structural eigenequation, but do so using a Newton-Raphson-type technique based on solving for the perturbations such that the gradient of the error function is near zero [6]. Thus these methods require the computation of the sensitivity of the structural eigenproblem to the parameters that are to be updated. The Hemez/Alvin algorithm [16],[17] computes the sensitivities at the elemental level, then assembles them to produce the global sensitivity matrices. The Ricles/Kosmatka [18] algorithm computes a "hybrid" sensitivity matrix using both analytical and experimental sensitivities.

Over the course of this three-year project these algorithms were applied to many different structures. Length limitations preclude the discussion of these individual applications. The DIAMOND software package has been requested by people from all over the world studying vibration-based damage detection. The latest version of DIAMOND can now be downloaded from the web site: http://esaea-www.esa.lanl.gov/damage_id.

In addition to the standard measures of success for a LDRD project such as follow-on funding and publications, several other non-traditional measures of success have resulted from this program. First, the Society for Experimental Mechanics requested that we teach a short course on vibration-based damage detection. This course was held in conjunction with the International Modal Analysis Conference at Orlando FL, in February, 1997, and had 30 attendees. Subsequently, this course was scheduled to be held in Tokyo June, 1997, under sponsorship of the Japan Society for the Promotion of Sciences; in Melbourne, Australia September, 1997, in conjunction with International Conference on

Aerospace Structures; and in February, 1999, again with the International Modal Analysis Conference. The researchers on this project have been asked to be keynote speakers on the subject of vibration-based damage detection at three international meetings including a NATO sponsored workshop on vibration testing and analysis. We have also been asked to be on external advisory boards for damage detection studies at universities and for proposed damage detection experiments planned for NASA's space station. Follow-on funding has led to the application of this technology to nuclear weapons stockpile surveillance. In summary, through the work done on this and another LDRD project, Los Alamos is currently recognized as the leader in the rapidly emerging field of vibration-based damage detection.

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