

69
1-14-80
944

Dr. 2005

LBL-11215
GREMP-10
UC-66d

MASTER

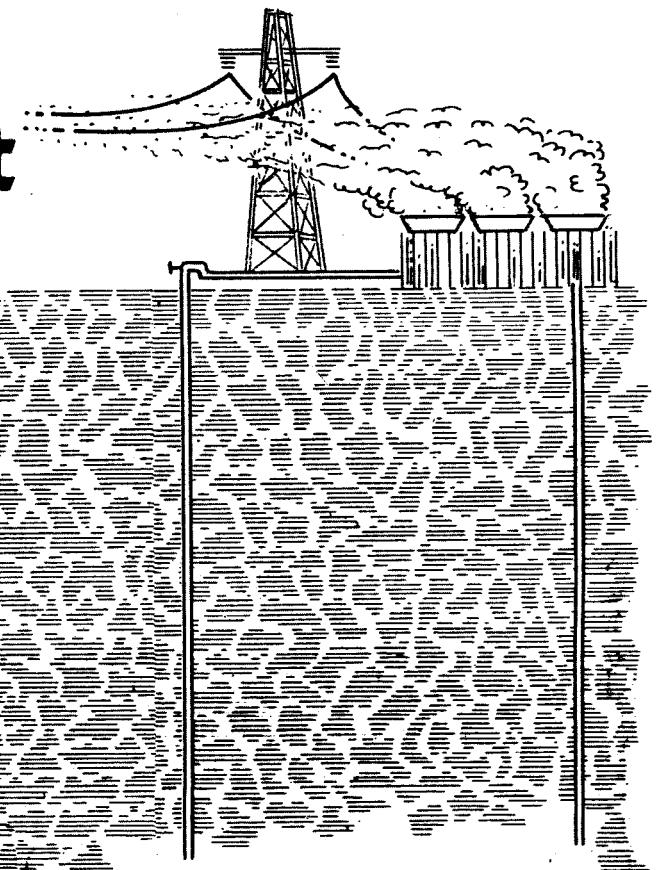
Analysis of Production Decline in Geothermal Reservoirs

Elliot J. Zais
Elliot Zais & Associates, Inc.

Gunnar Bodvarsson
Oregon State University

September 1980

Geothermal Reservoir Engineering Management Program



Earth Sciences Division
Lawrence Berkeley Laboratory
University of California, Berkeley

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price Code: A05

ANALYSIS OF PRODUCTION

DECLINE IN GEOTHERMAL RESERVOIRS

Prepared by

Elliot J. Zais
Elliot Zais & Associates, Inc.
7915 NW Siskin Drive
Corvallis, Oregon 97330

951 2223
cont.

Gunnar Bodvarsson
School of Oceanography
Oregon State University
Corvallis, Oregon 97331

950 0119

Submitted to

LAWRENCE BERKELEY LABORATORY

as an

ACCOUNTING OF RESEARCH

under the

GEOTHERMAL RESERVOIR ENGINEERING MANAGEMENT

Order # 4503010

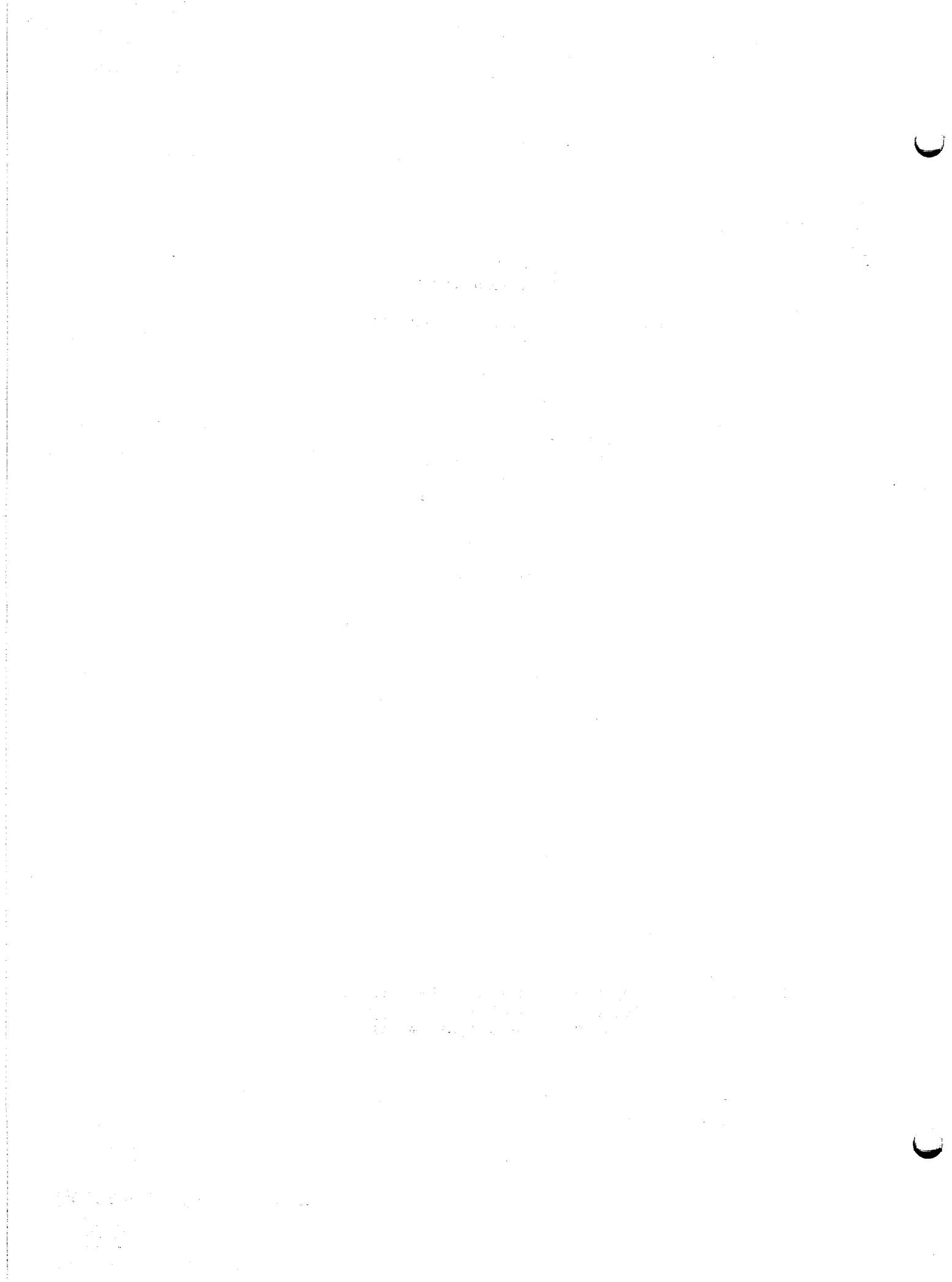
This work was supported by the U.S. Department of Energy.
Division of Geothermal Energy
Under Contract W-7405-ENG-48

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Buy



CONTENTS

	<u>Page</u>
ACKNOWLEDGMENTS	iv
I. INTRODUCTION AND CONCLUSIONS	1
Objectives and Rationale	1
Data and Analysis Methods	1
Conclusions	1
II. THEORY OF RESERVOIR DECLINE MODELS	2
Decline Mechanism	2
Pressure-Flow Fields in Slightly Compressible Formations with Darcy Type Flow	3
Nonstationary Boundaries: Effects of a Free Liquid Surface	4
Reservoir Simulation by Lumping	7
III. REVIEW OF METHODS.	9
Petroleum Reservoirs	9
Geothermal Reservoirs	9
IV. DATA PROCESSING.	21
Data Sets	21
Graphical Treatment of Data	21
Statistical Treatment of Data	23
Discussion of Data Scatter	26
V. RESULTS.	27
Arp's Equations	27
Type-curve Methods	27
Coats' Influence Function Method	27
Bodvarsson's Linearized Free Surface Method	29
VI. STANDARD OPERATING PROCEDURE FOR DATA GATHERING AND ANALYSIS	31
Data Gathering	31
Data Analysis	31
REFERENCES.	32
APPENDIX.	34
Computer Listings and Examples	36
SPSS Programs	36
Example Calculations Using SPSS	39
Estimating Field Influence Functions - Program Usage	47
Influence Function Programs	49
Example Calculations Using INFUNC	57
Linearized Free Surface Programs	61
Example Calculations Using LFS	69

ACKNOWLEDGMENTS

Support for these studies comes from the Department of Energy, Division of Geothermal Energy, through Lawrence Berkeley Laboratory under the Geothermal Reservoir Engineering Management Program, Order #4503010. The authors wish to express their gratitude to the program's technical monitors, Sally

Benson, Alex Graf, Jack Howard, Marcelo Lippmann, Werner Schwarz, and the contract administrator, Paul Marshall of the Lawrence Berkeley Laboratory (LBL) for their advice, consideration and review of this study. The support and encouragement of William Avera, Richard Couch, Nancy Kneisel, Tom Lindstrom, Suzi Maresh, Bill Millison and Jeff Stander of Oregon State University (OSU) are gratefully acknowledged.

I. INTRODUCTION AND CONCLUSIONS

Objectives and Rationale

The major objectives of the Decline Curve project were to

1. Test the decline analysis methods used in the petroleum industry on geothermal production data,
2. Examine and/or develop new analysis methods,
3. Develop a standard operating procedure for analyzing geothermal production data.

Various analysis methods have long been available but they have not been tested on geothermal data because of the lack of publicly available data. The recent release to publication of substantial data sets from Wairakei, New Zealand, Cerro Prieto, Mexico and The Geysers, U.S.A. has made this study possible. Geothermal reservoirs are quite different from petroleum reservoirs in many ways so the analysis methods must be tested using geothermal data.

Data and Analysis Methods

Data and analysis methods were gathered from the petroleum, geothermal, and hydrological literature. The data sets examined include

1. Wairakei, New Zealand - 141 wells
2. Cerro Prieto, Mexico - 18 wells

3. The Geysers, U.S.A. - 27 wells
4. Larderello, Italy - 9 wells and groups
5. Matsukawa and Otake, Japan - 8 wells
6. Olkaria, Kenya - 1 well

The analysis methods tested were

1. Arps's equations
2. Fetkovich type curves
3. Slider's method for Arps
4. Gentry's method for Arps
5. Gentry's & McCray's method
6. Other type curves
7. P/z vs. Q method
8. Coats' influence function method
9. Bodvarsson's Linearized Free Surface
- Green's Function method

Conclusions

The conclusions are

1. The exponential equation fit is satisfactory for geothermal data.
2. The hyperbolic equation should be used only if the data fit well on a hyperbolic type curve.
3. The type curve methods are useful if the data are not too scattered. They work well for vapor dominated systems and poorly for liquid dominated systems.
4. Coats' influence function method can be used even with very scattered data.
5. Bodvarsson's method is still experimental but it shows much promise as a useful tool.

II. THEORY OF RESERVOIR DECLINE MODELS*

(1) Decline mechanism

A geothermal reservoir has essentially three capacitances, (1) fluid/rock compressibility, (2) free liquid surface mobility and (3) reservoir liquid vaporization. In essence, item (3) is also a compressibility effect similar to (1). In this section, we will very briefly review in a semi-quantitative manner, the relative magnitudes of the effects listed above.

Consider a reservoir consisting of a slab of thickness H and of a large horizontal extent. The porosity/permeability can be of the fracture or intergranular type but is assumed to be sufficiently homogeneous that an average porosity ϕ and capacitance s (storage coefficient) can be defined.

On these premises, we find that lowering the pressure by Δp in a vertical column of unit area, releases because of compressibility a total liquid mass of

$$\Delta q_c = \rho s H \Delta p \quad (1)$$

where ρ is the density of the liquid. We can then define a specific release per unit area of

$$dq_e/dp = \rho s H. \quad (2)$$

Let g be the acceleration of gravity. Lowering the pressure by Δp corresponds to a lowering of the free liquid surface by $\Delta p/\rho g$. Hence, for the same Δp , the free surface releases a total of

$$\Delta q_f = \rho \phi \Delta p / \rho g = \phi \Delta p / g, \quad (3)$$

and then the specific release

$$dq_f/dp = \phi / g. \quad (4)$$

Finally, we consider the effect of intergranular vaporization. Let ρ_v be the density of the vapor, L the latent heat of vaporization of the liquid and T the temperature in kelvins. The Clausius-Clapeyron equation for the liquid is then approximately

$$(dp_s/dT)_v = \rho_s L / T \quad (5)$$

where p_s is the vapor pressure along the saturation line that is denoted by the subscript v . Hence, assuming saturation conditions, the lowering of the pressure by Δp lowers T by

$$\Delta T = T \Delta p / \rho_s L \quad (6)$$

and the release of heat per unit volume of the wet formation is

$$\Delta h = \rho_r C T \Delta p / \rho_s L \quad (7)$$

*Chapter II was written by Gunnar Bodvarsson, Geophysics Group, School of Oceanography, Oregon State University, Corvallis, Oregon.

where ρ_r is the density and C is the heat capacity of the wet formation. The release of vapor is then

$$\Delta q_v = \Delta h / L = \rho_r C T \Delta p / \rho_s L^2, \quad (8)$$

and we can thus define a specific rate per unit area of a slab of thickness H

$$dq_f/dp = \rho_r C T H / \rho_s L^2. \quad (9)$$

The ratio of free surface to compressibility effect follows from (2) and (4)

$$(dq_r/dp) / (dq_c/dp) = \phi / g s H \quad (10)$$

Considering porosities in the range $\phi = 0.01$ to 0.2, a thickness of $H = 10^3$ m and taking that $s = 2 \times 10^{-11}$ Pa⁻¹, we find that the ratio given in (10) varies from 50 to 10³. Thus, at normal reservoir conditions the free surface lowering releases a much larger amount of reservoir liquid mass per unit pressure decline than the compressibility.

Along similar lines we obtain the ratio of the vaporization to the compressibility effect on the basis of (2) and (9)

$$(dq_v/dp) / (dq_c/dp) = \rho_r C T / \rho_s L^2. \quad (11)$$

Considering the case of $T = 200^\circ\text{C} = 473$ K and using standard values $\rho_r = 2500$ kg/m³, $C = 10^3$ J/kg·K, $s = 2 \times 10^{-11}$, $\rho_s = 7$ kg/m³ and $L = 2 \times 10^6$ J/kg, we find a ratio of about 2×10^3 . Since ρ_s is the main variable in (11) this ratio will decrease with increasing temperature.

Summing up the results of the present section, we conclude that in the case of liquid dominated reservoirs with common porosities and where no vaporization takes place, the free surface effect is larger than the compressibility effect by a factor of 10^2 - 10^3 . In such cases, the reservoir response to long-term production will be dominated by the free surface effect.

The situation is more complex when vaporization takes place. Theoretically, this effect can release approximately as much fluid mass as the free surface effect. However, in most practical cases where production is initiated at liquid dominated conditions, the vaporization is more or less confined to the local volumes around the boreholes and the ratio in (11) has then to be reduced by a volume factor that may very roughly be of the order of 0.1 or less. The free surface effect would also then dominate the global reservoir response to long term production.

Vapor dominated reservoirs have, as a matter of course, different characteristics. There is no near-surface free liquid surface and ρ_s in equation (2) has then to be replaced by the product $\phi \gamma$ where γ is the steam compressibility. Usually, there is a vaporization at a deep liquid surface and this effect dominates the long term reservoir behavior.

(2) Pressure-flow fields in slightly compressible formations with Darcy type flow

(2.1) Diffusion equation. Let $p(t, P)$ be the pressure field at time t and at the point P in a Darcy type domain B with the stationary boundary surface Σ . Consider a general setting where the permeability k is a linear matrix operator and the kinematic viscosity of the fluid v is also taken to be variable. It is convenient to introduce the fluid conductivity operator $c = k/v$ and express Darcy's law

$$\vec{q} = -c\nu p \quad (12)$$

where \vec{q} is the mass flow density. Moreover, let ρ be the fluid density, s the capacitativity or storage coefficient of the formation and f be a source density. Combining (12) with the equation for the conservation of mass,

$$\nabla \cdot \vec{q} = -\rho s \partial_t p + f \quad (13)$$

we obtain the diffusion equation for the pressure field

$$\rho s \partial_t p + \Pi(c)p = f \quad (14)$$

where $\Pi(c) = -\nabla(c\nabla)$ is the generalized Laplacian operator. Appropriate boundary conditions that may be of the Dirichlet, Neumann, mixed or more complex convolution type, have to be joined to equation (14). The case of a homogeneous/isotropic/isothermal formation results in the simplification $\Pi(c) = c\Pi = -c\nabla^2$ where c is a constant. Moreover, stationary pressure fields satisfy the potential equation

$$\Pi(c)p = f. \quad (15)$$

(2.1i) Eigenfunctions of the Laplacian.

The eigenfunctions $u_n(P)$ of $\Pi(c)$ in B associated with (14) satisfy the equations

$$\Pi(c)u_n = \lambda_n u_n, n = 1, 2, \dots \quad (16)$$

where the constants λ are the eigenvalues and the boundary conditions on Σ are homogeneous of the same type as those satisfied by $p(t, P)$ in (14) and (15).

(2.1ii) Types of solutions. The key to solving equation (14) is the causal impulse response or Green's function $G(P, Q, t)$ which represents the pressure response of the causal system to an instantaneous injection of an unit mass of fluid at $t = 0+$ at the source point Q . This function satisfies the same boundary conditions as the eigenfunctions $u_n(P)$. Solutions to (14) in the case of a general source density $f(t, P)$, non-causal initial values and general boundary conditions can then be expressed in terms of integrals over the Green's function (Duff and Naylor, 1966).

Two fundamental types of expressions for the Green's function are available. First, in the case of simple layered domains B with a boundary Σ composed of a few plane faces, $G(P, Q, t)$ can be

expressed as a sum (or integral) over the fundamental whole space source function

$$G_o(P, Q, t) = (8\rho s)^{-1}(\pi a t)^{-3/2} \exp(-r_{PQ}^2/4at) U_+(t) \quad (17)$$

and its images. The symbol $U_+(t)$ is the causal unit step function, $a = c/\rho s$ the diffusivity, and r_{PQ} is the distance from Q to P . Whenever applicable, sums of this type represent the most elementary local and/or global expressions for $G(P, Q, t)$.

Second, the Green's function can be expanded in a series or integral over the eigenfunctions of $\Pi(c)$. If ρ and s are constants, then

$$G(P, Q, t) = (1/\rho s) \sum_n u_n(P) u_n(Q) \exp(-\lambda_n t/\rho s). \quad (18)$$

The series expansion (18) is of a more general applicability than solutions of the type based on the fundamental source function (17).

The formal link between the two types (17) and (18) is provided by the Poisson summation formula (Stakgold, 1967). It is important to underline that all solutions of the type (17) can be expressed in the form (18).

From the numerical point of view, the form given by (17) is more convenient for the computation of relatively short term field responses, in particular, in the case of layered half-spaces. However, long-term responses in bounded domains are more effectively computed on the basis of (18). This expression is a sum over exponentials where the convergence improves with time.

A different type of solution of (14) that is of interest in the present context can be obtained by operational methods. Limiting ourselves to the pure initial value problem with $p(0, P) = p_0(P)$ in the case of an infinite domain, we can, since ρ , s and $\Pi(c)$ are independent of t , formally express the solution of the homogeneous form of (14) as

$$p = \exp[-t\Pi(c)/\rho s] p_0 \quad (19)$$

where the exponential operator is to be interpreted as a Taylor series in the operator $\Pi(c)$

$$\exp[-t\Pi(c)/\rho s] = 1 - [t\Pi(c)/\rho s] + (\frac{1}{2}) [t\Pi(c)/\rho s]^2 \dots \quad (20)$$

The series represents an iteration process where the convergence is limited to (properly defined) small values of t . The practical applicability is therefore fundamentally different from (18). Moreover, it is of considerable interest that rather general situations with regard to $\Pi(c)$ can be admitted in (19) and (20).

A number of other analytical and/or numerical techniques are available for solving (14). These include the path-integral technique of the Feynman-Kac type (Simon, 1979), compartmentalization or lumping and, as a matter of course, a series of numerical techniques.

(3) Nonstationary boundaries: effects of a free liquid surface

The presence of a free liquid surface in a reservoir requires the introduction of a rather complex non-stationary surface boundary condition. Let Σ now represent the free liquid surface at equilibrium and Ω be the free surface in a perturbed state. The boundary Ω is a surface of constant pressure which without loss of generality can be taken to vanish. The free surface condition (Lamb, 1932) is then expressed

$$\frac{Dp}{Dt} \Big|_{p=0} = 0 \quad (21)$$

where D/Dt is the material derivative. This is an essentially non-linear condition which leads to a much more complex problem setting. Losing the principle of superposition the construction of solutions to the forward problem becomes a difficult task.

Bodvarsson (1977) has shown that when Ω deviates only little from Σ (21) can be simplified and linearized. For this purpose, we place a rectangular coordinate system with the z -axis vertically down such that the (x,y) plane coincides with Σ . Moreover, let the amplitude of Ω relative to Σ be u and the scale of the undulation of Ω be L . Then provided $|u/L| \ll 1$, the condition (21) can be replaced by the approximation

$$(1/w) \partial_t p - \partial_z p = 0 \quad (22)$$

where $w = cg/\phi$ is a new parameter, namely, the free sinking velocity of the pore liquid under gravity (g = acceleration of gravity). Under these circumstances, the solution of the forward problem is obtained by constructing a solution to (14) which satisfies (22) at the free surface and appropriate conditions at other sections of the reservoir boundary.

The presence of a first order derivative with respect to time in the free-surface condition (21) obviously leads to an additional relaxation process analog to the purely diffusive phenomena associated with the first order time derivative in the basic equation (14). As we shall conclude below, the individual time scales of the two phenomena are, however, different.

For the sake of brevity, we shall limit the present discussion to the simplest but practically quite relevant case of the semi-infinite liquid saturated homogeneous, isotropic and isothermal half-space. To consider the pure free-surface related phenomena, we eliminate pressure field diffusion by neglecting the compressibility of the liquid/rock system. As shown in section (I) above, the long term dynamics of liquid reservoirs is dominated by the free surface phenomena. In this setting we can combine the potential equation (15) and the surface condition (22) in one single equation confined to the Σ plane (Bodvarsson, 1978a), which expressed in terms of the fluid surface amplitude $u(t, x, y) = p/\rho g$ takes the form

$$(1/w) \partial_t u + \frac{1}{2} u = f/\rho g c \quad (23)$$

where $\frac{1}{2} = (-\partial_{xx} - \partial_{yy})^{\frac{1}{2}}$ is the square root of the

two-dimensional Laplacian and f is an appropriately defined surface source density. To obtain the pressure field in the space $z > 0$, the boundary values derived from (23) have to be continued into the lower half-space on the basis of standard potential theoretical methods. The fractional order of the Laplacian in (23) is quite unusual, but the operator is well defined and poses no mathematical problems.

Some solutions of equations (23) of practical interest have been obtained by Bodvarsson (1977). Confining ourselves first to the simple semi-infinite half-space, some important results are given below.

(3.i) The source-free case. In a source-free case where $f = 0$, the homogeneous equation (23) is most easily solved by solving

$$-\nabla^2 p = 0, \quad z \geq 0 \quad (24)$$

with the boundary condition (22) combined with a given initial condition which takes the form

$$p = \rho g h_0, \quad t = 0, \quad z = 0 \quad (25)$$

where $h_0(S)$ is a given initial free-surface amplitude.

This solution is obtained immediately by observing that a pressure function of the form

$$p = p(x, y, z + wt) \quad (26)$$

satisfies the boundary condition (22) at all times. Consequently, introducing the Dirichlet type Green's function for the half-space $z \geq 0$ (Duff and Naylor, 1966, page 276) which gives the pressure $p(P)$ in $z > 0$ for a pressure $p_0(S)$ on Σ

$$p(P) = (z/2\pi) \int_{\Sigma} (1/r_{PU}^3) p_0(U) da_U, \quad z \geq 0 \quad (27)$$

where $U = (x', y')$, $da_U = dx' dy'$ and

$$r_{PU} = [(x-x')^2 + (y-y')^2 + z^2]^{\frac{1}{2}} \quad (28)$$

the solution to the present problem is

$$p(P, t) = [\rho g(z+wt)/2\pi] \int_{\Sigma} (1/r_{PUT}^3) h_0(U) da_U, \quad t > 0, \quad z > 0, \quad (29)$$

where

$$r_{PUT} = [(x-x')^2 + (y-y')^2 + (z+wt)^2]^{\frac{1}{2}} \quad (30)$$

The motion of the fluid surface is obtained by letting $z = 0$ in (13) and hence,

$$h(S, t) = (wt/2\pi) \int_{\Sigma} (1/r_{SUT}^3) h_0(U) da_U, \quad t > 0 \quad (31)$$

where now

$$r_{SUT} = [(x-x')^2 + (y-y')^2 + (wt)^2]^{\frac{1}{2}} \quad (32)$$

(3.ii) Flow fields with sources. To select a relevant and important case of flow fields with

sources, we will consider the following situation. Let the fluid at $t = 0$ be in static equilibrium and the fluid surface at $t = 0$ therefore coincide with Σ . Consider a concentrated sink of strength unity at the point $Q = (0, 0, d)$ which at $t = 0+$ starts withdrawing fluid mass at a constant rate equal to unity. In this case we have to solve

$$\Pi p = -\nabla^2 p = (-1/c)\delta(P-Q)U_+(t) \quad (33)$$

where $U_+(t)$ is the causal unit step function for which $U_+(0) = 0$. The boundary condition on Σ is again given by (22) and the initial condition is $p = 0$ at $t = 0$.

A simple method of solving this problem has been given by Bodvarsson (1977). In the present context, it is of some interest to present a different approach via the combined Laplace-Hankel transform method.

Let $\hat{p}(P, s)$ be the Laplace-transform of $p(P, t)$. The transform of (33) and (22) are then

$$\Pi \hat{p} = (-1/(cs))\delta(P-Q) \quad (34)$$

and

$$s\hat{p} - w\frac{\partial \hat{p}}{\partial z} = 0, \quad z = 0 \quad (35)$$

Moreover, let $\tilde{p}(k, z, s)$ be the (two-dimensional) Hankel-transform of $\hat{p}(P, s)$ and $D = d/dz$. The transform of (34) is then

$$k^2\tilde{p} - D^2\tilde{p} = (-1/2\pi cs)\delta(z-d) \quad (36)$$

and (35) takes the form

$$s\tilde{p} - w\frac{\partial \tilde{p}}{\partial z} = 0, \quad z = 0 \quad (37)$$

The solutions of (36) for $z \geq z'$ are of the form $\exp(\pm kz)$ and we thus obtain

$$\tilde{p} = A\exp(kz) + B\exp(-kz), \quad 0 < z < z', \quad (38)$$

and

$$\tilde{p} = C\exp(-kz), \quad z > z', \quad (39)$$

where A , B and C are integration constants (with respect to z). From (37) we obtain the relation

$$A(s - wk) + B(s + wk) = 0, \quad (40)$$

and our solution has to be continuous at $z = d$ such that

$$A\exp(kd) + B\exp(-kd) = C\exp(-kd), \quad (41)$$

Finally, integrating (36) with respect to z from $d-$ to $d+$, we obtain the necessary third condition

$$\left. \frac{d\tilde{p}}{dz} \right|_{d-} = -(1/2\pi cs) \quad (42)$$

which yields the relation

$$-kC\exp(-kd) - k[A\exp(kd) - B\exp(-kd)] = -(1/2\pi cs) \quad (43)$$

Solving (40), (41) and (43) for A , B and C and inserting in (38) leads to

$$p = (-1/4\pi cks)\exp[-k(d-z)] - [(s-wk)/(s+wk)]$$

$$\exp[-k(z+d)] \quad (44)$$

which holds for $z > 0$. Using the identity

$$(s-wk)/(s+wk) = [2/(s+wk)] - (1/s), \quad (45)$$

(44) is easily Hankel-Laplace inverted into (P, t) space (tables in Duff and Naylor, 1966) and the result is

$$p(P, t) = (-1/4\pi c)[1/r_{PQ} + (1/r_{PQ'}) - (2/r_{PQ'}t)] \quad (46)$$

where

$$r_{PQ} = [(x-x')^2 + (y-y')^2 + (z-d)^2]^{1/2}, \quad (47)$$

$$r_{PQ'} = [(x-x')^2 + (y-y')^2 + (z+d)^2]^{1/2} \quad (48)$$

$$r_{PQ't} = [(x-x')^2 + (y-y')^2 + (z+wt+d)^2]^{1/2} \quad (49)$$

The surface elevation $h = p/\rho g$ is

$$h(S, t) = (-1/2\pi \rho g c)[(1/r_{SQ}) - (1/r_{SQ't})] \quad (50)$$

where $S = (x, y)$ and

$$r_{SQ} = [(x-x')^2 + (y-y')^2 + d^2]^{1/2} \quad (51)$$

$$r_{SQ't} = [(x-x')^2 + (y-y')^2 + (wt+d)^2]^{1/2} \quad (52)$$

It is of a particular interest to note that the Hankel-inversion leading to the last term in (46) follows upon a Laplace-inversion on the basis of the Sommerfeld integral

$$\int_0^\infty \exp[-(z+wt+d)k]J_0(rk)dk = 1/r_{PQ't}, \quad (53)$$

which we rewrite

$$\int_0^\infty \exp(-wkt)E(r, z, k)dk = 1/r_{PQ't}, \quad (54)$$

where

$$E(r, z, k) = \exp[-(z+d)k]J_0(r, k), \quad (55)$$

Equation (46) reveals that the effect of the free fluid surface on the pressure drawdown due to the concentrated sink of strength unity starting at time $t = 0$ can be represented by the pressure field due to a stationary image sink of strength unity located at $Q' = (x', y', -d)$ and a moving image source of strength 2 located at $Q't = [x', y', -(wt+d)]$. At time $t = 0+$ the image sink and 1/2 of the image source cancel resulting in an initial pressure field of

$$p(P, 0+) = -[1/(4\pi c)][1/r_{PQ} - (1/r_{PQ'})]. \quad (56)$$

At very large times, that is at $t \gg d/w$, when the image source has retreated far into the negative half space, the third term in (46) becomes negligible and the pressure field reaches its stationary value p_s given by

$$p(P)|_{t \rightarrow \infty} = -[1/(4\pi c)][1/r_{PQ}] + (1/r_{PQ'}), \quad (57)$$

The source-sink situation is illustrated in Figure 1.

It is appropriate to reiterate that the above free surface results have been obtained by neglecting the rock/liquid compressibility.

(3.iii) Flow in slab with a free surface.

The results for the half-space set forth in the previous section are easily generalized to the model of a slab of thickness H and of infinite horizontal extent. As given in equation (46) and shown in Figure 1, the free surface dynamics reduces at any fixed time to a source-sink situation. Applying well known results of elementary potential theory, we can extend equation (46) to the case of the slab by adding an infinite sequence of source-sink images that is obtained by reflecting the source and the two sinks in Fig. 1 at the bottom and the equilibrium surface boundaries. Appropriate reflection coefficients have to be applied in this process. We will refrain from entering into details of the procedure. The practically most important case is obtained when the basement is impermeable and the reflection coefficient at the boundary is equal to unity. As shown above, the equilibrium surface has also a reflection coefficient of unity but on any reflection, we have to observe the splitting of an image source into a stationary image and a double moving image with an opposite sign. The picture is therefore a little more complex than in the usual cases involving single images.

(3.iv) Discussion. Equations (18) and (53) above show that both compressibility and free surface effects lead to decline functions that are sums or integrals over exponentials of negative time. In essence, therefore, the decline processes are governed by very simple functional relationships. Moreover, the analysis in section (I) indicates that from the quantitative point of view, the free surface effect dominates in all liquid reservoirs.

The decline or relaxation time is another parameter of major interest. By definition this is the time t_r during which the amplitude of a stationary wave of wavelength L decreases to $(1/e)$ of its initial value. Inserting a waveform $\exp[-(t/t_r) + ikx]$ where $k = (2\pi/L)$ is the wave-number, into equation (14) gives for compressibility the time $t_r = (1/ak^2)$. Similarly, we find on the basis of (23) for the free surface a value $t_r = (1/wk)$. At the same L , the ratio of the free surface to compressibility time is $(ak/w) = (\phi k/\rho sg)$. Inserting values of interest for long-term reservoir behavior such as, for example, $\phi = 0.1$, $L = 6 \text{ km}$, $s = 3 \times 10^{-11} \text{ Pa}^{-1}$ we find values of this ratio of about 300. This indicates quite clearly that the compressibility phenomena are on a much shorter time scale and smaller magnitude than the free surface phenomena. Our approach of neglecting compressibility in the above analysis is, therefore, well justified.

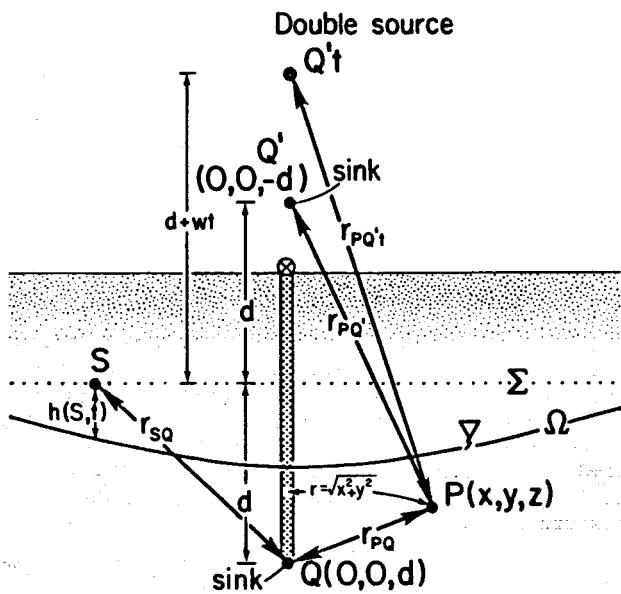


FIG. 1a. Infinite half space of linearized free surface method.

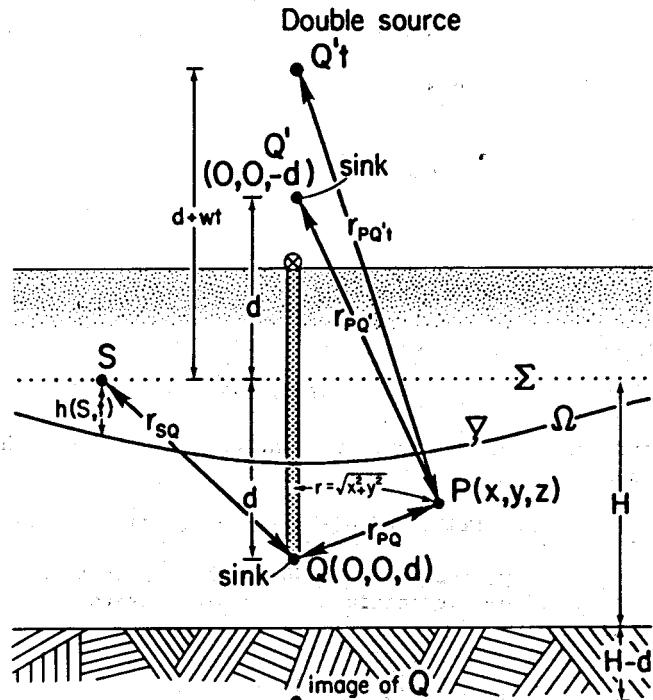


FIG. 1b. Reservoir half space for linearized free surface method with bottom layer.

(4) Reservoir simulation by lumping

The fact that the principal decline functions for liquid reservoirs are of the negative time exponential type suggests the use of lumping as a method of reservoir simulation. Below, we will briefly look onto this possibility.

Consider a liquid geothermal reservoir that is producing a constant mass flow q from a number of wells. We assume that the reservoir pressure is being monitored at a fixed point where a decreasing reference pressure function $p(t)$ is being observed. Moreover, it is being assumed that production, started at time $t = 0$ from equilibrium conditions where we can take that the reference pressure $p(0) = 0$. The producing holes have a bottom-hole pressure of $p_w(t)$ that is also taken from an appropriate reference point as $p(t)$ and therefore $p_w(0) = 0$.

The simplest lumped model to simulate this system is shown in Fig. 2 below.

The model consists of a liquid capacitor or container (I) with vertical walls having an area A . The production q is being extracted from this capacitor over a conductor that has a conductance c_1 . This element represents the contact resistance of the producing holes. Recharge to the container is obtained from a capacitor (II) of an infinite area over another conductor that has a conductance c_2 . Reference pressure in the large capacitor is taken to constant and equal to zero. The liquid level in (I) is measured by the pressure $p_1(t)$. In accordance with Darcy type flow conditions we assume that both conductors are linear and hence that the following system equations hold

$$q = c_1(p_1 - p_w) \quad (58)$$

$$(A/g)Dp_1 = c_2(0 - p_1) - q \quad (59)$$

where g is the acceleration of gravity and $D = d/dt$. Since we don't observe p_w , equation (58) is irrelevant and does not enter into the discussion below. The principal parameters of the simulation system are thus the capacitor area A and the conductance c_2 . Given $q(t)$ and $p_1(t)$ for some fixed time interval starting at $t = 0$, we are now interested in deriving values of A and c_2 such that the model simulates the given reservoir in the optimal way during at least a part of the production time. A

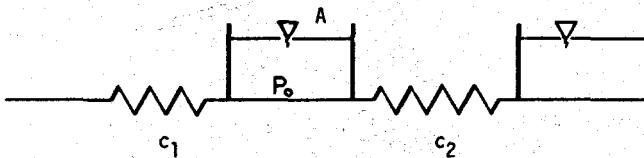


FIG. 2. Lumped parameter model of simulated reservoir.

convenient way of obtaining these values in the following.

Since we have assumed that q is constant, the present decline function $p_1(t)$ is characterized by a smooth negative time exponential behavior. We can then expand the known $p_1(t)$ into a Taylor series in t starting at $t = 0$ and that is truncated at the second order term,

$$p_1(t) = p_1(0) + tDp_1(0) + (t^2/2)D^2p_1(0), \quad (60)$$

where we have abbreviated $Dp_1|_{t=0} = Dp_1(0)$ and $D^2p_1|_{t=0} = D^2p_1(0)$.

Since $p_1(0) = 0$ this series reduces to

$$p_1(t) = tDp_1(0) + (t^2/2)D^2p_1(0), \quad (61)$$

Inserting this expression in equation (59) results in

$$(A/g)[Dp_1(0) + tD^2p_1(0)] = c_2[-tDp_1(0) - (t^2/2)D^2p_1(0)] - q, \quad (62)$$

On a second order approximation, we obtain from the terms in t^0 and t the parameter relations

$$A = -g[q/Dp_1(0)] \quad (63)$$

and

$$c_2 = -(A/g)D^2p_1(0)/Dp_1(0) = (A^2/g^2)D^2p_1(0). \quad (64)$$

Since $p_1(t)$ is a known function, the derivatives at $t = 0$ are also known and we can thus derive A and c_2 from (63) and (64) above.

On the basis of the known parameters, we can then solve equation (59) for a given variable input $q(t)$ and obtain

$$p_1(t) = (g/A) \int_0^t \exp[-gc_2(t-\tau)/A]q(\tau)d\tau \quad (65)$$

as a procedure to predict or extend $p_1(t)$ in time.

An analysis of the above type can be carried out on any field decline functions that have been obtained with sufficient accuracy to derive the derivatives. In most practical cases the mass production function will be a variable $q(t)$ and the input function $p_1(t)$ for the above analysis will then have to be obtained by a deconvolution, that is by solving an equation

$$p_f(t) = \int_0^t p_1(t-\tau)Dq(\tau)d\tau, \quad (66)$$

where $p_f(t)$ is the reference pressure that is the output to $q(t)$. There are no problems in solving (66).

To illustrate the above procedure we will carry out the lumping of the free surface dynamics model leading to equation (46). We obtain then the time derivatives $Dp_1(0)$ and $D^2p_1(0)$ from equation (46) and use (63) and (64) to derive the lumped system parameters. To simplify the procedure,

we consider only the case $q = \text{constant} = \text{unity}$. Omitting elementary details and irrelevant factors, the method, in essence, consists in approximating the function

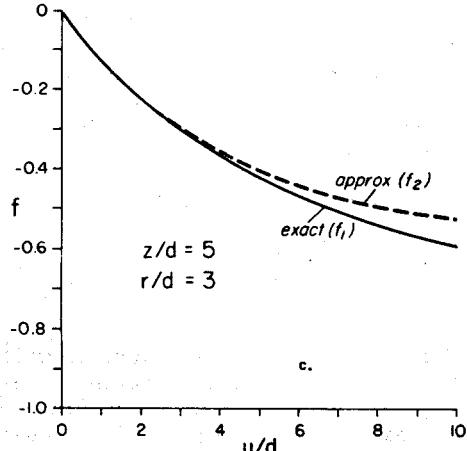
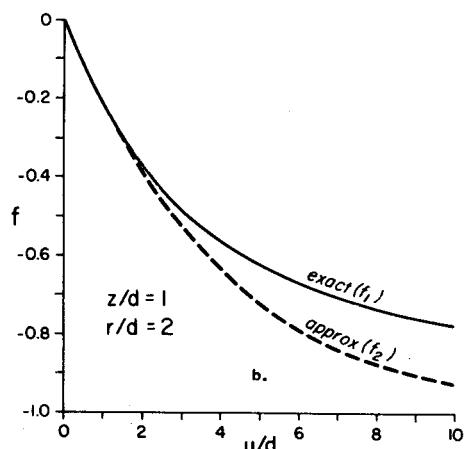
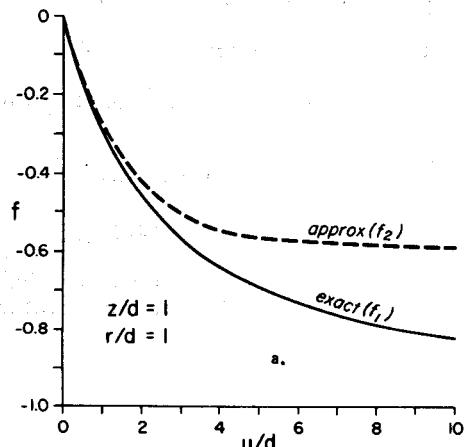
$$f_1(t) = -[1 - (R_0/R_1)] \quad (67)$$

by the function

$$f_2(t) = -[(z+d)^2/R_2^2][1 - \exp[-wtR_2^2/R_0^2(z+d)]], \quad (68)$$

where

$$R_0 = [r^2 + (z+d)^2]^{\frac{1}{2}} \quad (69)$$

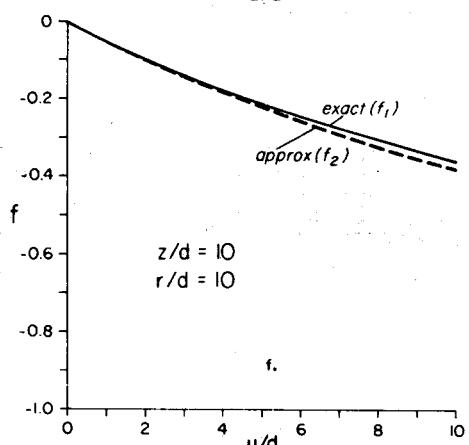
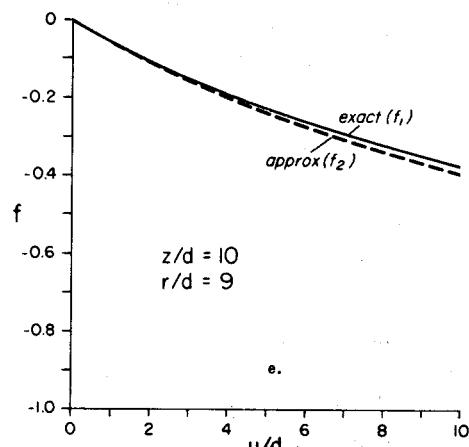
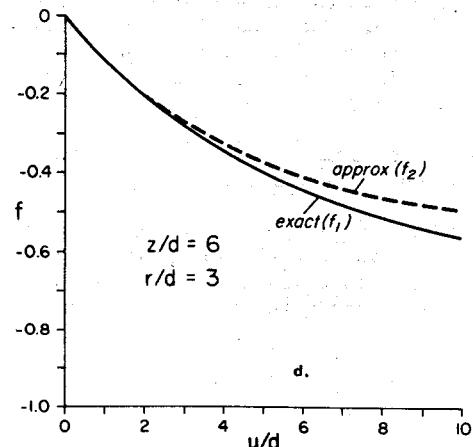


$$R_1 = [r^2 + (z+wt+d)^2]^{\frac{1}{2}} \quad (70)$$

$$R_2 = [2(z+d)^2 - r^2]^{\frac{1}{2}} \quad (71)$$

$$\text{with } r^2 < 2(z+d)^2$$

The results of a numerical evaluation are illustrated in Fig. 3. It is evident that the lumped approximation holds quite well until the factor wt is of the order of a few depths d . Quite often w is of the order of 10^{-5} m/s and d about 10^3 m . In this case the lumped approximation will give good results for a period of a few years.



FIGS. 3a-3f. Comparisons between lumped parameter approximation and exact solution.

III. REVIEW OF METHODS

Petroleum Reservoirs

Production decline methods are probably the most commonly used tool of the reservoir engineer because production data are always recorded and filed whereas temperature and pressure records are far less common. The uses of these methods are at least two fold. First, they are used to predict future production and second, they can provide insight into reservoir mechanisms and geology.

Production data for fields and individual wells are usually plotted on a monthly basis so a year's worth of data might be enough to use with the standard methods. When fields have been produced for a number of years, e.g., 10, production data are plotted on an annual basis and fitted. In the petroleum industry great care must be taken in trying to extrapolate past trends because conditions can change. For example, the reservoir pressure might pass through a bubble point causing dissolved gas to outgas thereby drastically changing flow conditions. The best discussion of and warning about decline methods is in Brons (1963).

Reserve estimates are calculated from predicted future production. If the predictions are bad, the estimates are bad. Brons shows an example using production from two wells, each with constant but different percentage decline rates. When their productions are added together and fitted with a hyperbolic eqn (the best fit) we get a very different reserve estimate from the one obtained by looking at each well separately. As always, the reservoir analyst must supply a great deal of insight.

Decline methods are not directly applicable to new fields except that if the new field appears to be similar to a previously studied field we might make some intelligent guesses about its production characteristics.

Decline methods are used to determine when additional wells should be drilled and when wells should be worked over. Production in individual wells can decrease in a steady regular manner from sand plugging the formation. This can be seen on a production vs. time graph.

Geothermal Reservoirs

The decline methods developed for analyzing oil and gas wells can be used for geothermal wells but we must recognize that petroleum and geothermal reservoirs are very different from each other. These differences can cause production mechanisms to be drastically different in the two cases. Some of the more important differences and their consequences are as follows:

1. Petroleum reservoirs are usually sedimentary formations. Geothermal reservoirs are usually fractured igneous or metamorphic formations. Darcy flow holds in the first case and fracture flow in the second.

- b. Temperature is relatively unimportant in petroleum production. It is critical in geothermal production. High temperatures stress tubing and cement in the wellbore.
- c. Geothermal well flow volumes are often 1 to 2 orders of magnitude greater than petroleum volumes.
- d. Precipitation is much more serious in geothermal wells than in petroleum wells.
- e. Petroleum is a complex mixture with volatile components. Geothermal water is essentially one species.

Fracture size, quantity and distribution are drastically affected by precipitation, changes in temperature and seismic activity. Geothermal reservoirs seem to be much more complex than petroleum reservoirs so methods taken into geothermal work must be examined carefully. We have done this with the data and methods available but more work must be done as we produce more geothermal fields over time.

1. Arps

Arps's (1945, 1956) work forms the basis for all the decline curve methods currently in use. He brought together and codified work on oil reserve estimation that had been done as early as 1908. The commonest methods were graphical in which production q or cumulative production Q was plotted vs. time t . See Fig. 4 from Arps (1956). Examinations of production data showed that data with constant first differences fit an exponential equation while data with constant second differences fit a hyperbolic or harmonic equation. All three equations can be expressed as

$$a = Kq^b = \frac{-dq/dt}{q} \quad (72)$$

where

a = fractional decline

some authors use D = fractional decline

q = production rate of time t

K = constant

b = constant

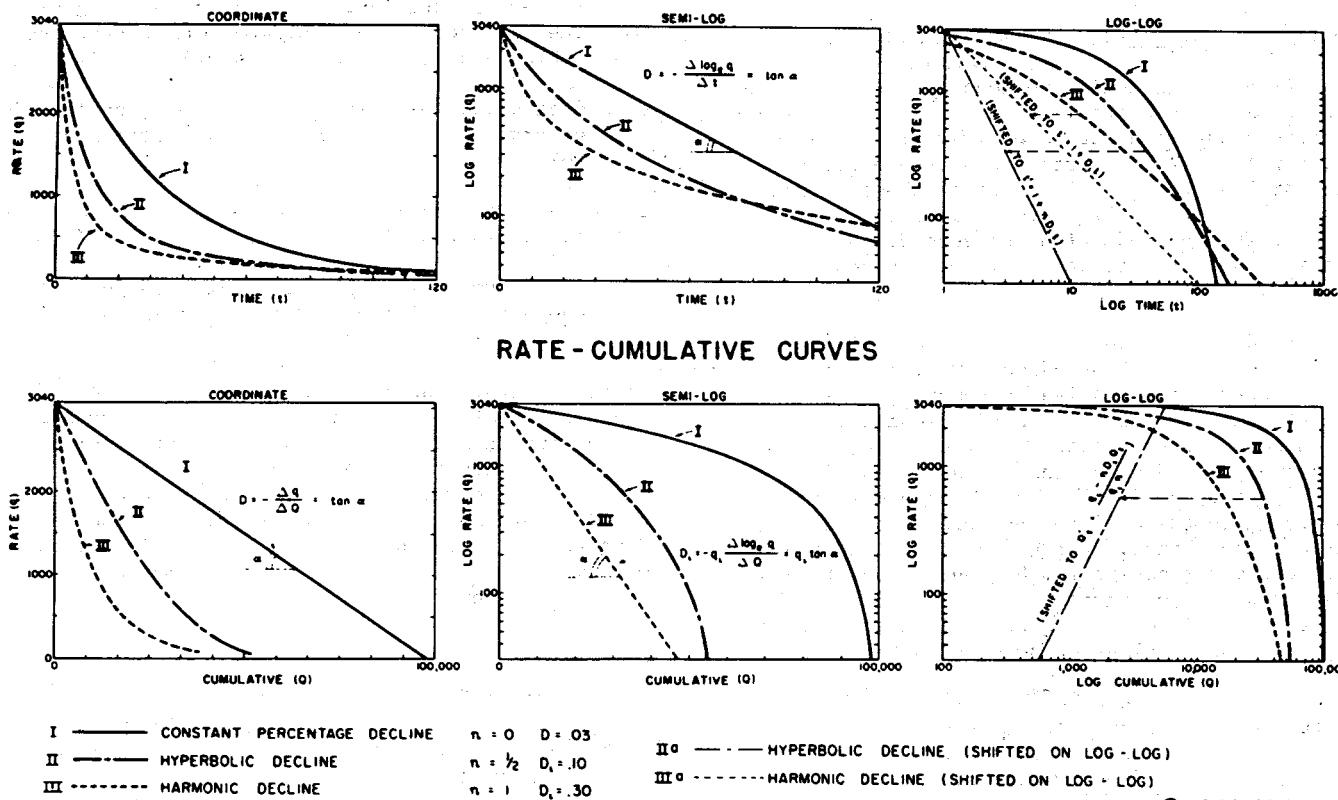
The solutions to equation (72) are shown in Table 1.

Guerrero (1961) gives a good "cookbook" approach to analyzing data using these methods. See Table 2 for problems worked out by Guerrero. Arps's equations were considered to be strictly empirical until 1973 when Fetkovich proposed some theoretical basis for the exponential equation (see below). The hyperbolic equation is still considered to be empirical.

2. Fetkovich

Fetkovich (1973) showed that log-log type curves can be used to analyze production data in an analogous manner to analyzing pressure data. He presented log-log plots of dimensionless flow rate, $q_{Dd} = q(t)/q_i$, vs. dimensionless time, $t_{Dd} = D_i t$, for $0 < b < 1$ and $D_i = 1$ (see Fig. 5). $b = 0$ is the exponential solution while $b = 1$ is the harmonic solution.

RATE-TIME CURVES



© 1956, SPE-AIME

FIG. 4. Three types of production decline curves on coordinate, semi-log, and log-log graph paper (from Arps, 1956).

The exponential curve is given by

$$q_{Dd} = \exp(-D_i t), \quad D_i = 1 \quad (73)$$

while the hyperbolic curves are given by

$$q_{Dd} = (1+bD_i t)^{-1/b} \quad \text{for } 0 < b < 1. \quad (74)$$

Using an overlay technique as shown very clearly in Earlougher (1977), (see Fig. 6), production data can be plotted over the curves and a decline exponent can be picked. For $t_{Dd} < 0.3$ all the curves are coincident.

Fetkovich showed that the exponential decline has a fundamental base by deriving it as a solution to the constant well pressure case. The equation for dimensionless flow rate is

$$q_{Dd} = q(t)/q_i = \exp[-(q_i)_{\max} t/N_{pi}] \quad (75)$$

This equation can be related to (73) by setting

$$D_i = (q_i)_{\max} / N_{pi} \quad (76)$$

then

$$t_{Dd} = \frac{(q_i)_{\max} t}{N_{pi}} \quad (77)$$

We define

$$N_{pi} = \frac{\pi (r_e^2 - r_w^2) \phi c_t h p_i}{5.615 B} \quad (78)$$

$$(q_i)_{\max} = \frac{k h p_i}{141.3 \mu B [\ln(r_e/r_w) - \frac{1}{4}]} \quad (79)$$

Finally we get

$$q_{Dd} = \frac{q(t)}{\frac{kh(p_i - p_{wf})}{141.3 \mu B [\ln(r_e - \frac{1}{4})]}} \quad (80)$$

Fetkovich showed that production decline curve data could be used to derive values for permeability thickness kh which is usually obtained from pressure data. (see Fig. 7a and 7b). Compare kh calculations from rate-time data and pressure time data.

3. Slider's Method

Slider (1968) proposed a simple method of curve matching to obtain the hyperbolic exponent

TABLE 1. CLASSIFICATION OF PRODUCTION DECLINE CURVES

DECLINE TYPE	I. CONSTANT-PERCENTAGE DECLINE	II. HYPERBOLIC DECLINE	III. HARMONIC DECLINE
BASIC CHARACTERISTIC	DECLINE IS CONSTANT $n = 0$	DECLINE IS PROPORTIONAL TO A FRACTIONAL POWER (n) OF THE PRODUCTION RATE $0 < n < 1$	DECLINE IS PROPORTIONAL TO PRODUCTION RATE $n = 1$
	$D = K \cdot q^0 = -\frac{dq/dt}{q}$ $\int_0^t D dt = - \int_{q_i}^{q_t} \frac{dq}{q}$ $-Dt = \log_e \frac{q_t}{q_i}$	$D = K \cdot q^n = -\frac{dq/dt}{q}$ $\text{FOR INITIAL CONDITIONS: } K = \frac{D_i}{q_i^n}$ $\int_0^t \frac{D_i}{q_i^n} \cdot dt = - \int_{q_i}^{q_t} \frac{dq}{q^{n+1}}$ $\frac{n D_i t}{q_i} + q_i^{-n} - q_t^{-n}$	$D = K \cdot q^1 = -\frac{dq/dt}{q}$ $\text{FOR INITIAL CONDITIONS: } K = \frac{D_i}{q_i}$ $\int_0^t \frac{D_i}{q_i} \cdot dt = - \int_{q_i}^{q_t} \frac{dq}{q}$ $\frac{D_i t}{q_i} = \frac{1}{q_i} - \frac{1}{q_t}$
RATE - TIME RELATIONSHIP	$q_t = q_i \cdot e^{-Dt}$ $q_t = q_i \left(1 + n D_i t\right)^{-\frac{1}{n}}$ $q_t = q_i \left(1 + D_i t\right)^{-1}$ $q_t = \int_0^t q_i \cdot dt = \int_0^t q_i \cdot e^{-Dt} \cdot dt$ $q_t = \frac{q_i - q_i \cdot e^{-Dt}}{D}$ <p>Substitute From Rate-time Equation:</p> $q_i \cdot e^{-Dt} = q_t$ <p>To Find:</p>	$q_t = q_i \left(1 + n D_i t\right)^{-\frac{1}{n}}$ $q_t = \int_0^t q_i \cdot dt = \int_0^t \left(1 + n D_i t\right)^{-\frac{1}{n}} \cdot dt$ $q_t = \frac{q_i}{(n-1) D_i} \left[\left(1 + n D_i t\right)^{\frac{n-1}{n}} - 1 \right]$ <p>Substitute From Rate-time Equation:</p> $\left(1 + n D_i t\right) = \left(\frac{q_t}{q_i}\right)^n$ <p>To Find:</p>	$q_t = q_i \left(1 + D_i t\right)^{-1}$ $q_t = \int_0^t q_i \cdot dt = \int_0^t q_i \left(1 + D_i t\right)^{-1} \cdot dt$ $q_t = \frac{q_i}{D_i} \left[\log_e \left(1 + D_i t\right) \right]$ <p>Substitute From Rate-time Equation:</p> $\left(1 + D_i t\right) = \frac{q_t}{q_i}$ <p>To Find:</p>
RATE - CUMULATIVE RELATIONSHIP	$Q_t = \frac{q_t - q_i}{D}$	$Q_t = \frac{q_i^n}{(1-n) D_i} \left(q_i^{1-n} - q_t^{1-n} \right)$	$Q_t = \frac{q_i}{D_i} \log_e \frac{q_i}{q_t}$

TABLE 2. EXAMPLE OF USE OF EXPONENTIAL EQUATION

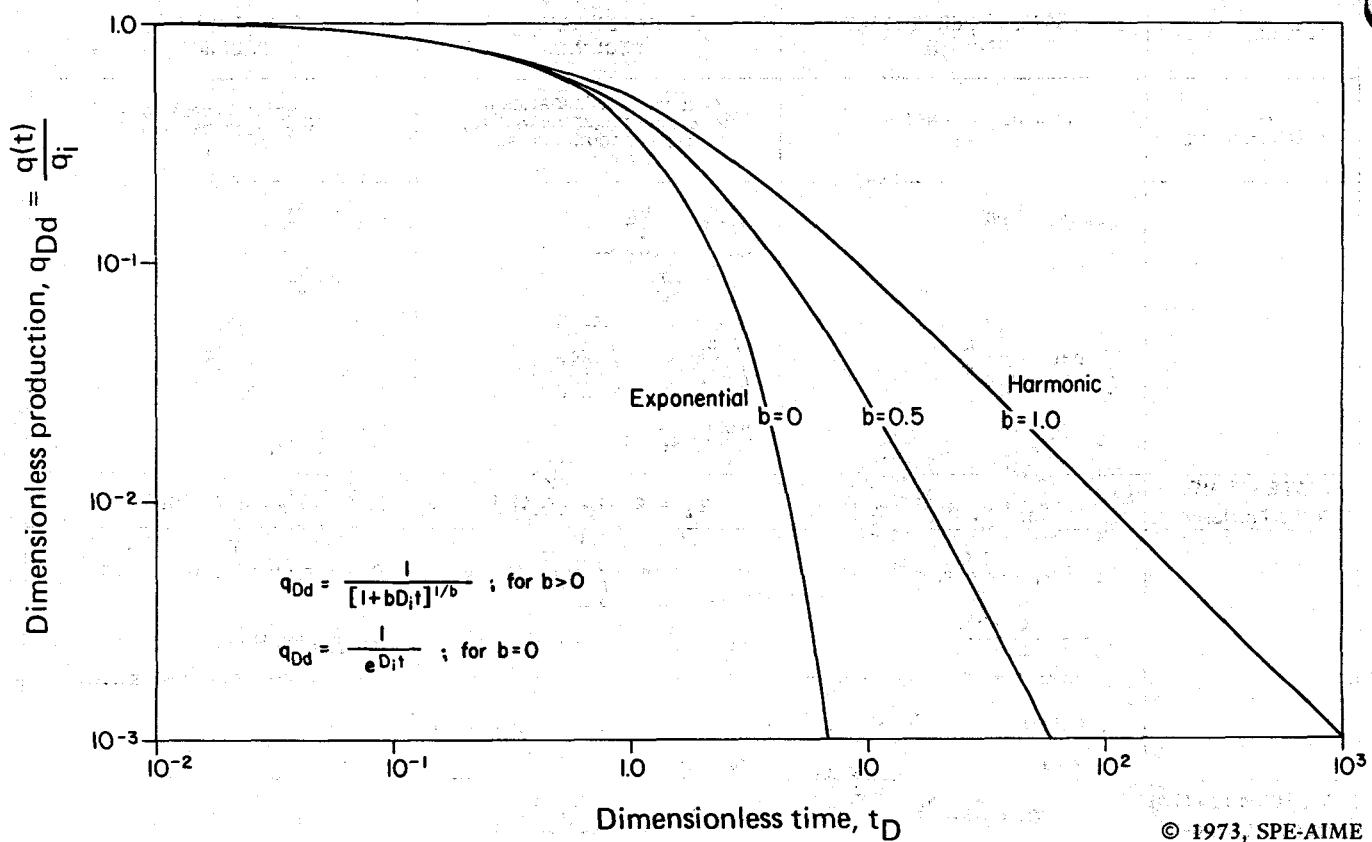


FIG. 5. Log-log type-curve of dimensionless flow rate vs dimensionless time (after Fetkovich, 1973).

b and the initial decline rate q_i . To use the method one needs to construct a set of curves of q/q_i vs. log time for various values of a_1 and b using Arps's hyperbolic equation. Production data can then be plotted on the curves by using a transparent overlay. The overlay can be moved around until the best fit is found thus giving b and a_1 . From equations or from a second set of curves, future production rates q and future cumulative production Q can easily be estimated. This method is easy to apply but it requires a separate set of curves for each possible value of b . Later methods eliminate this shortcoming.

4. Gentry's Method

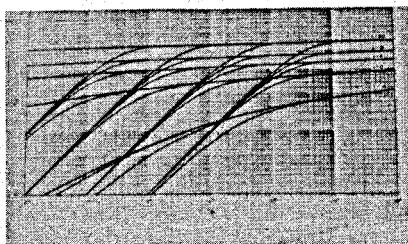
Gentry (1972) developed curves which are much easier to use than Slider's because only one set is needed for all values of b between 0 and 1. (see Figs. 8a and 8b) We can find b from a plot of Q/tq_i vs. $\log q_i/q$. With this b we go to a plot of q_{it} vs. $\log q_i/q$ and find a_1 . This gives us all the factors we need for a reserve analysis.

5. Gentry and McCray

Reservoir analysts have usually assumed that $0 < b < 1$ in the solution of Arps's equations. See

Higgins and Lechtenberg (1970) for exceptions. There is no mathematical basis for this restriction. $b = 0$ and $b = 1$ are special cases, the exponential and harmonic, respectively, but this does not restrict b from being larger than 1. Gentry and McCray (1978) investigated decline curve methods using semi-log plots of q_i/q vs. Q/q_{it} , cartesian plots of q/q_i vs. Q , and semi-log plots of q_i/q vs. $a_1 t$. See Figures 9a, 9b, and 9c. Some of their conclusions are ($N_p = Q$)

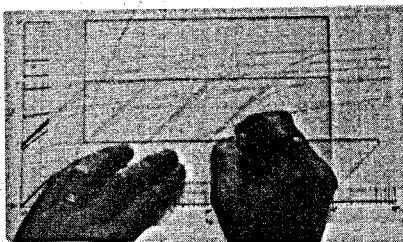
1. The dimensionless curves N_p/q_{it} vs. q_i/q $a_1 t$ vs. q_i/q for a particular fluid-permeability system are not affected by the absolute permeability or size of the reservoir. The behavior of these plots is determined by (1) the characteristics of the contained fluid, (2) the relative permeability characteristics of the reservoir rock, (3) the reservoir drive mechanism, (4) reservoir heterogeneity, and (5) manual manipulation of production.
2. Reservoir heterogeneity tends to increase the magnitude of b as the degree of heterogeneity is increased. It is also apparent that b for a heterogeneous system



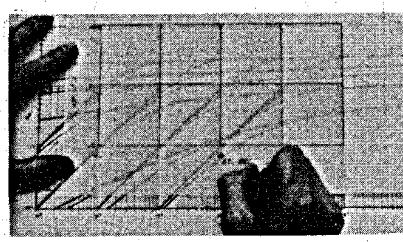
(a) Choose a type curve.



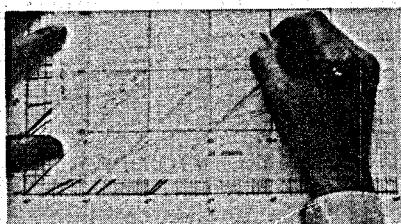
(b) Overlay with tracing paper.



(c) Trace major grid lines.



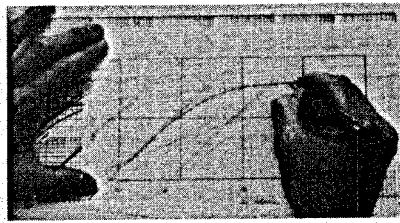
(d) Label axes.



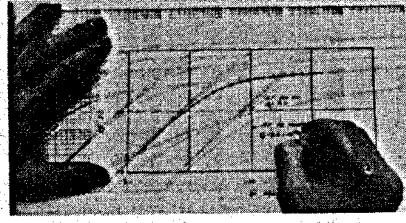
(e) Plot observed data using type-curve grid.



(f) Slide tracing paper to match a type curve.



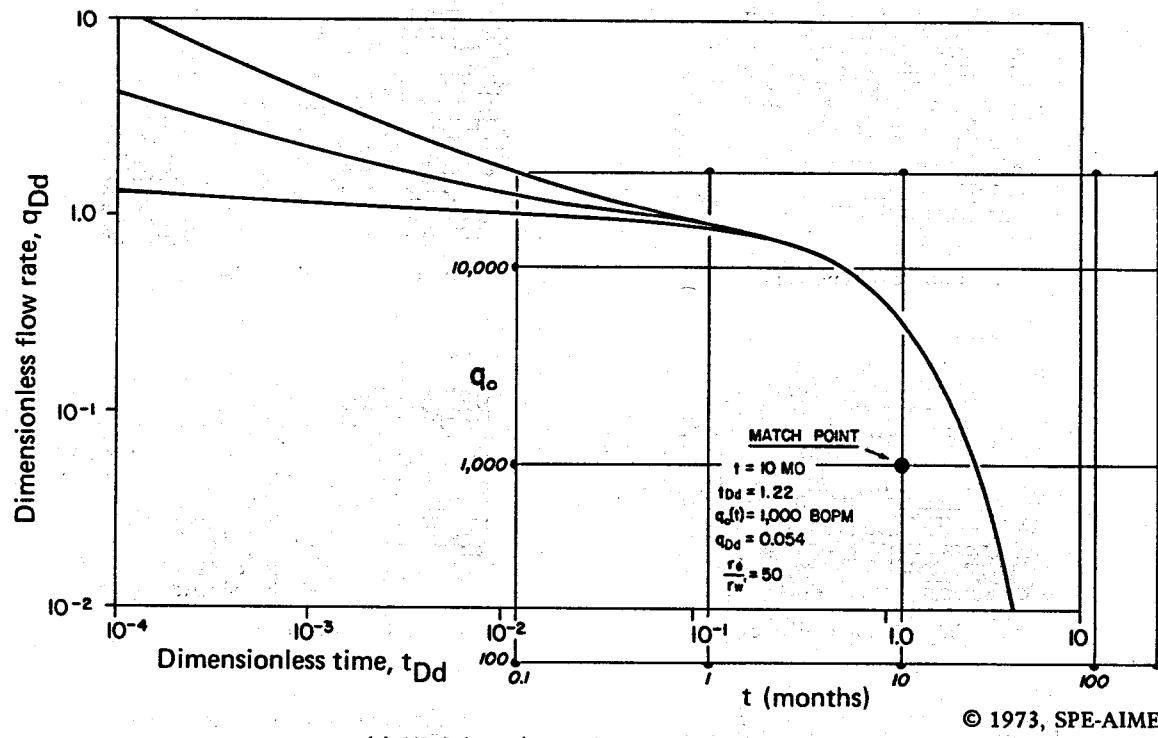
(g) Trace the matched curve.



(h) Pick a match point.

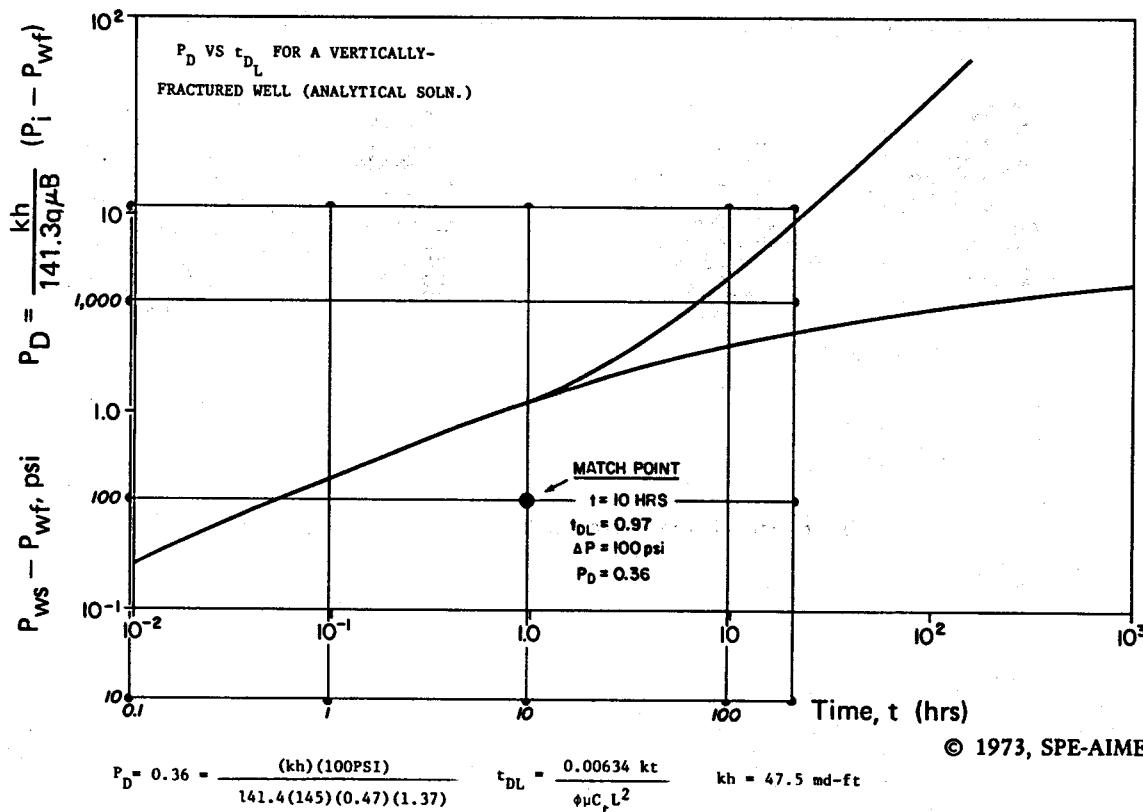
© 1977, SPE-AIME

FIG. 6. Steps in type-curve matching (from Earlougher, 1977).



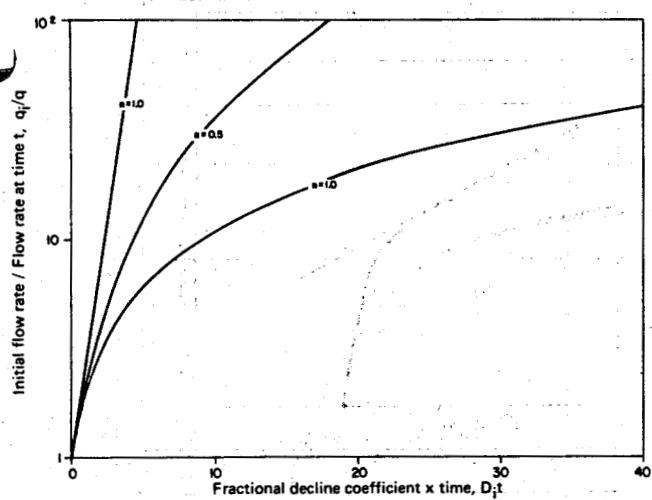
$$kh = \frac{q(t) 141.3 (\ln r_e/r_w - .5) \mu B}{q_{Dd} (P_i - P_w)} = \frac{1000 \text{ BOPM} 141.3 (\ln 50 - .5)}{30.4 \text{ D/M} (.054) (7259)} = 40.5$$

FIG. 7a. Type-curve matching example for calculating Kh using decline curve data (after Fetkovich, 1973).



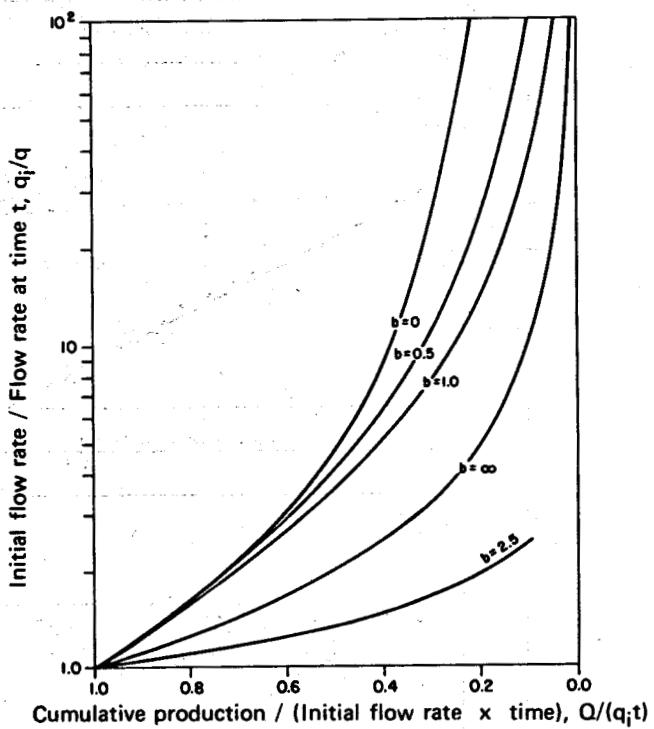
$$P_D = 0.36 = \frac{(kh)(100\text{PSI})}{141.4(145)(0.47)(1.37)} \quad t_{DL} = \frac{0.00634 kt}{\phi \mu C_t L^2} \quad kh = 47.5 \text{ md-ft}$$

FIG. 7b. Type-curve matching example for calculating Kh from pressure buildup (after Fetkovich, 1973).



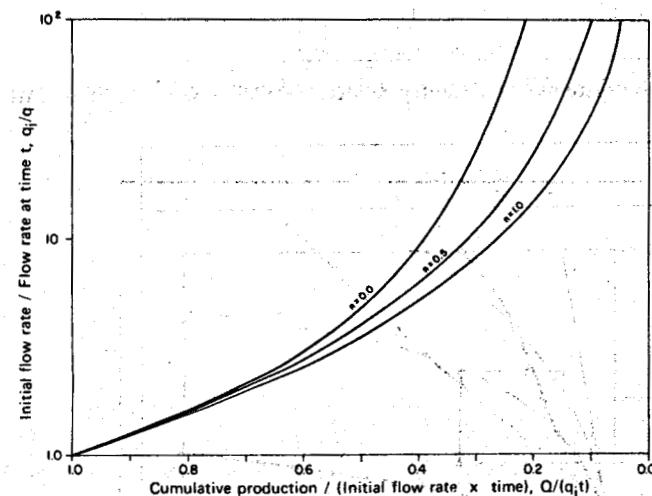
© 1972, SPE-AIME

FIG. 8a. Decline curve analysis chart relating production rate to time (after Gentry, 1972).



© 1978 APE-AIME

FIG. 9a. $Q/(q_i t)$ vs q_i/q (after Gentry & McCray, 1978).



© 1972, SPE-AIME

FIG. 8b. Decline curve analysis chart relating production rate to cumulative production (after Gentry, 1972).

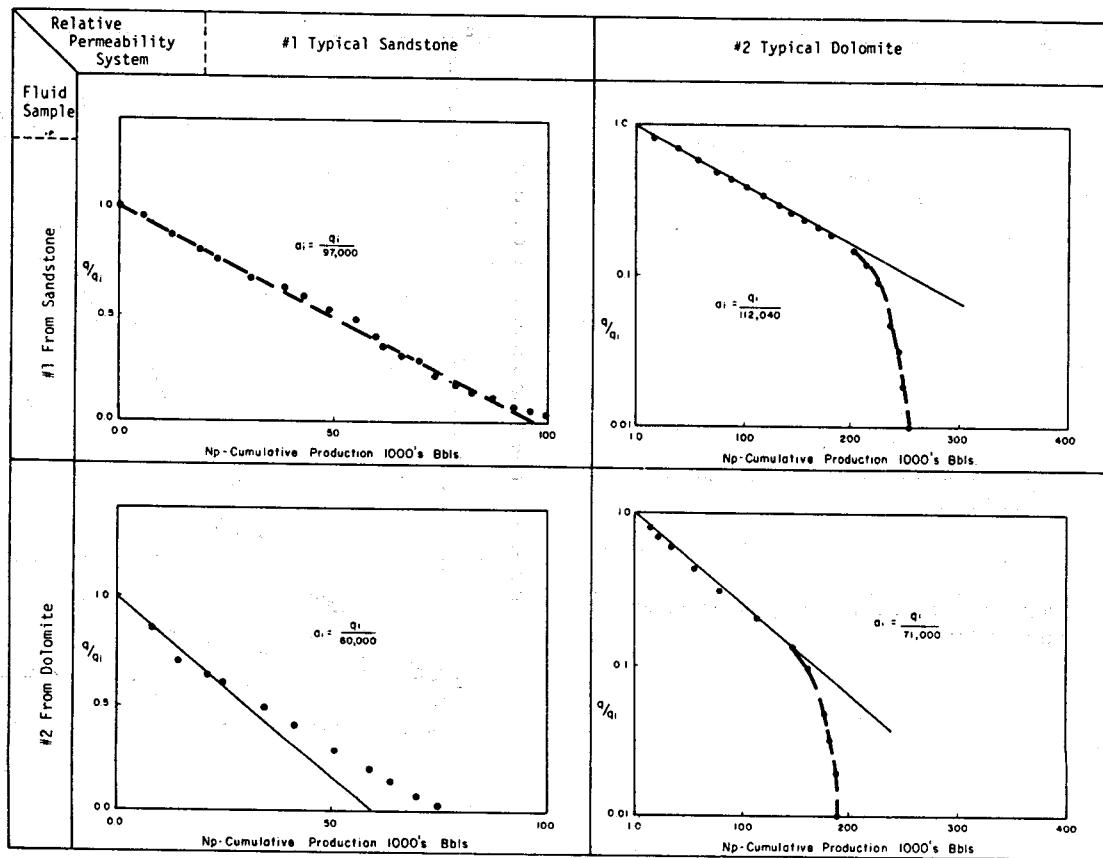


FIG. 9b. Plots of cumulative production vs q/q_i for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME

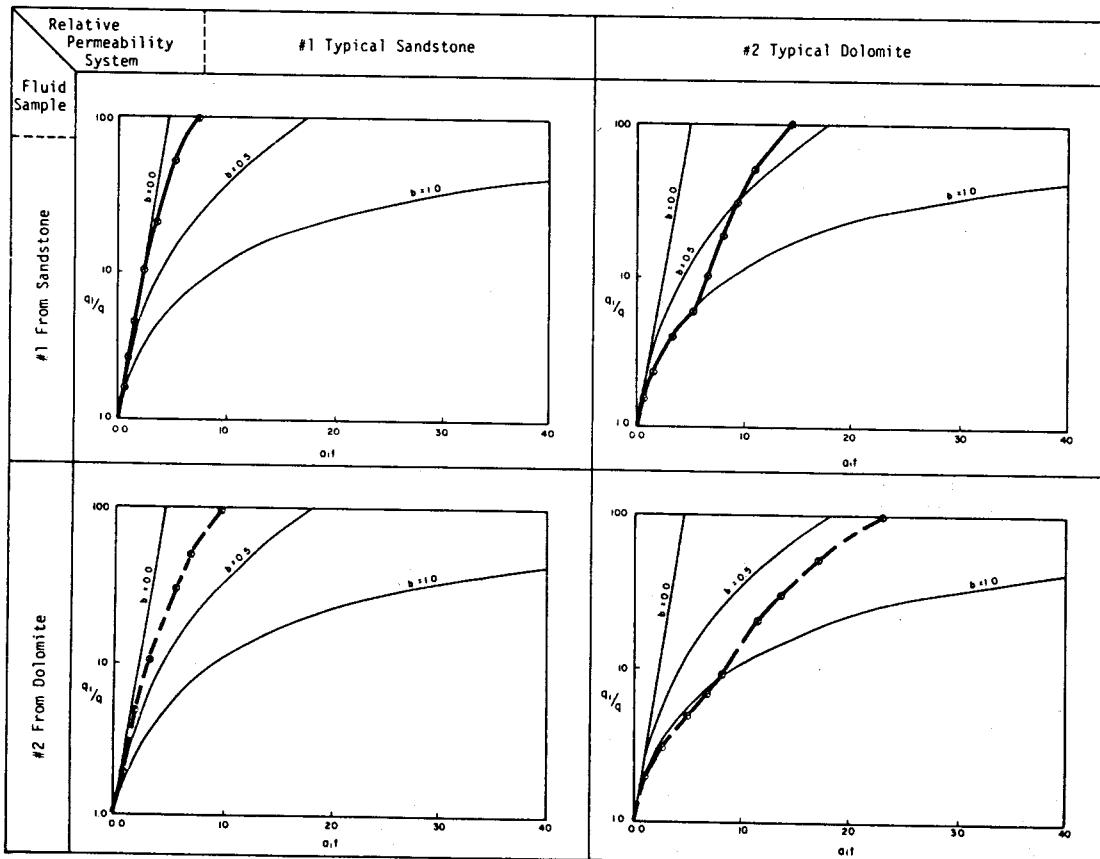


FIG. 9c. Dimensionless $a_i t$ curve histories for four fluid-permeability systems (after Gentry & McCray, 1978). © 1978, SPE-AIME

will increase to a maximum value and then as the ratio q_1/q becomes large, b will decrease and approach its homogeneous value.

3. Reservoir heterogeneity can and does cause b values to be greater than 1.0.

4. Manual manipulation of production can and does cause b values to be greater than 1.0.

5. The dimensionless plots for heterogeneous systems of 1 and 3 md, 3 and 9 md, and 5 and 15 md all plotted the same curve. This indicates that heterogeneous systems in the ratio of 1:3 will plot congruous dimensionless curves.

6. It appears that the relative-permeability characteristics of the reservoir have the greater effect on the decline exponent b , while the fluid characteristics have a greater influence on the constants a_1 and q_1 .

7. The equation

$$N_p/(q_{it}) = (q/q_1)^a$$

may better define certain decline curves than do the Arps equations.

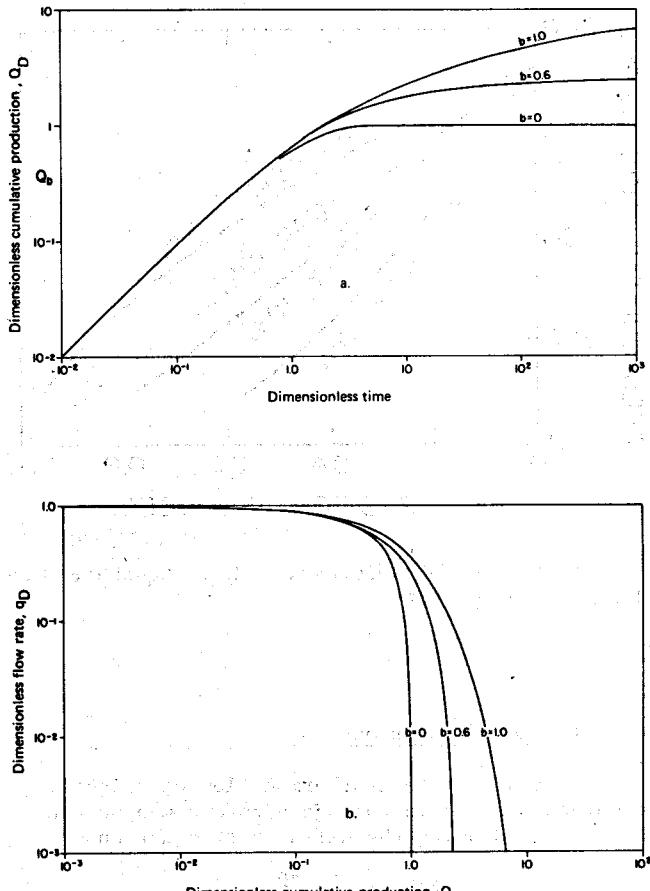
8. The plotting of production data on the $N_p/(q_{it})$ vs. q_1/q curve can be a helpful diagnostic tool for evaluating the production history of a well or lease."

6. Other Type Curves

Fetkovitch developed log-log type curves using dimensionless production vs. dimensionless time, Q_p vs. t_p , but other variables can be used. We tried plots of dimensionless cumulative production vs. dimensionless time and dimensionless production vs. dimensionless cumulative production, q_D vs. Q_p . The plots were made by using the exponential equation and the hyperbolic equation for several values of b . See Figs. 10a and 10b. We had the same data scatter problem with these type curves as we did with Fetkovitch's. A few data sets plotted very nicely on a particular curve, but most sets plotted very ambiguously.

7. P/z vs. Q

The natural gas industry has long used decline curves in which pressure divided by gas deviation factor, p/z , is plotted against cumulative production, Q (Katz, 1959). The straight line can be extrapolated to the economic limit of producing pressure quite easily. Brigham and Morrow (1974) have proposed adapting this method to steam fields. In plotting computer generated data they found that curve shape was strongly influence by porosity. Also, the presence of a boiling interface is critical. "If the wells are completed in the vapor zone it would be natural to graph p/z vs. production, as though this were a gas reservoir, and use an extrapolation of the best straight line as a predictive method to calculate

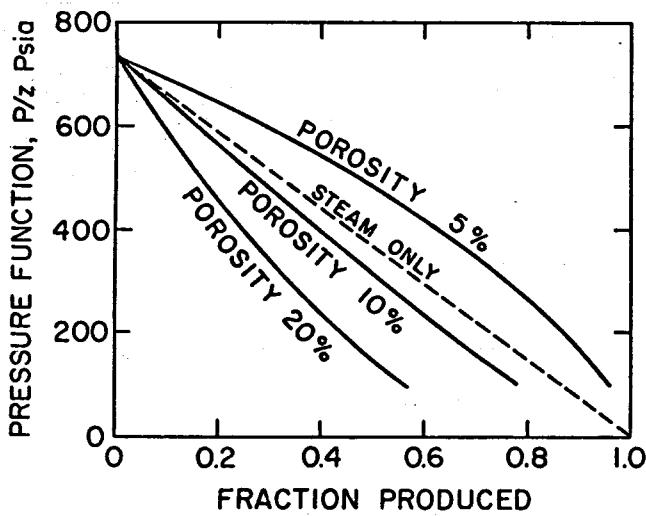


FIGS. 10a-b. Log-log type-curve.

reserves. The efficiency of this technique will be strongly dependent on the porosity if the actual reservoir contains boiling liquid." (see Fig. 11.)

Pruess et al. (1979a, 1979b) have used the simulator SHAFT78 to test the use of p/z vs. Q plots for geothermal reservoirs. They conclude that "... the standard technique of estimating reserves by extrapolating a plot of p/z vs. cumulative production is not applicable to two-phase geothermal reservoirs." and "... in many cases pressure will be a linear function of cumulative production, with the slope allowing an estimate of reservoir volume. Reserve assessment requires knowledge of average porosity and vapor saturation, which cannot be obtained from pressure decline curves."

Brigham (1979) applied p/z techniques to a study of depletion in the Gabbro zone at Larderel-1c, but he stated that the linearity of p/z with cumulative production doesn't hold for the entire life of a reservoir with a boiling interface. He claims that linearity is a good approximation for the first one-third to one-half of the reservoir's life.



© 1974, SPE-AIME

FIG. 11. Pressure depletion vs recovery, falling liquid level (from Brigham & Morrow, 1974).

8. Influence Functions

Unsteady state isothermal flow of slightly compressible liquid through a porous medium can be described using the diffusivity equation

$$\nabla \left(\frac{k \nabla p}{\mu} \right) = \phi \frac{\partial p}{\partial t} \quad (81)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial p}{\partial r} \right) = \phi \frac{\partial p}{\partial t}$$

The equation can be solved using a Green's function approach to derive a "response", "resistance", "memory", or "influence" function. Katz and Coats (1969) in describing water movement in aquifers defined two influence functions: 1) $P(t)$ = the "rate case" influence function which is defined as the pressure drop at the reservoir boundary (a function of time) corresponding to a unit rate (e.g. 1 cu. ft./day) of water influx." For a constant flow rate q we get $p_0 - p(t) = qP(t)$, the constant terminal rate case equation. 2) $Q(t)$ = the "pressure case" influence functions since a constant pressure p_b is specified at the outer boundary. The constant terminal pressure equation is $q(t) = (p_0 - p_b)Q(t)$.

$P(t)$ and $Q(t)$ can be calculated either for idealized models or from field data. Let $F(t) = Q(t)$ or $P(t)$. For an idealized $F(t)$ we must specify "1) model geometry, 2) exterior boundary conditions (e.g. infinite, closed or constant pressure), and 3) model parameters." The specification of reservoir parameters and geometry is particularly difficult in geothermal reservoirs so the calculation of $F(t)$ from field data is more attractive and easier than trying to devise a thoroughly specified model. The advantages of the field method are "1) none of the above choices are required, and 2) an influence function which reflects unknown (and practically speaking, indeterminate) aquifer properties, as reflected

by actual field performance, is determined. Disadvantages are 1) the resemblance of the backed out $F(t)$ to the true function is proportional to the accuracy of field data, and 2) the influence function is obtained only up to the time of last available field data; extrapolation is required for purposes of predicting future water movement."

Coats *et al.* (1964) recommended the use of field influence functions for oil fields with adjacent aquifers. The method is directly applicable to geothermal fields. The influence function F can easily be generated as a function of pressure p and flowrate q using the following equations:

Integral form

$$\Delta p = \int_0^t \frac{dq(t-\tau)}{dt} F(\tau) d\tau = \int_0^t q(t) \frac{dF(t-\tau)}{d\tau} d\tau \quad (82)$$

Discrete form

$$\Delta p_i = p_0 - p_i = \sum_{j=1}^i (q_i - q_{i-1}) F_{i-j+1}$$

Bodvarsson (1980, personal communication) has shown how the influence function problem can be formulated in a slightly different manner. The function F defined by Coats is a unit step response function. Instead of the unit step response, we can use the impulse response h , where $h = dF/dt$. The equation to be solved is then

$$\Delta p = \int_0^t \frac{dq(\tau)}{d\tau} F(t-\tau) d\tau = \int_0^t q(\tau) \frac{dF(t-\tau)}{d\tau} d\tau \quad (83)$$

Integral form $\Delta p = \int q(\tau) h(t-\tau) d\tau$

$$\text{Discrete form } p_0 - p_i = \sum_{j=1}^i q_j h_{i-j+1}$$

The first derivative of the curve from the F formulation should be identical to the curve derived from the h formulation. F can be calculated by hand (Jargon and van Poolen, 1965) and Hutchinson and Sikora, 1959) but we recommend against it. An F can be calculated which fits the data well, but which has no physical meaning.

Katz and Coats cite an example in Katz *et al.* (1963) in which an influence function is calculated by direct methods which exactly reproduces past performance but which cannot be extrapolated. The smoothness constraints below assure a physically meaningful solution and they can be arrived at both intuitively and analytically. From Katz and Coats "if water is injected into an aquifer at a constant rate through some fixed inner aquifer boundary (surface), then intuitively the pressure change at that boundary must always be positive. In addition, the pressure should always increase and the rate of increase should continually decrease with time." The analytical proof for the constraints is given in Coats *et al.*

Linear programming methods such as the package MPOS should be used with the smoothness constraints on F or h :

$$F > 0, t > 0 \quad h > 0, t > 0$$

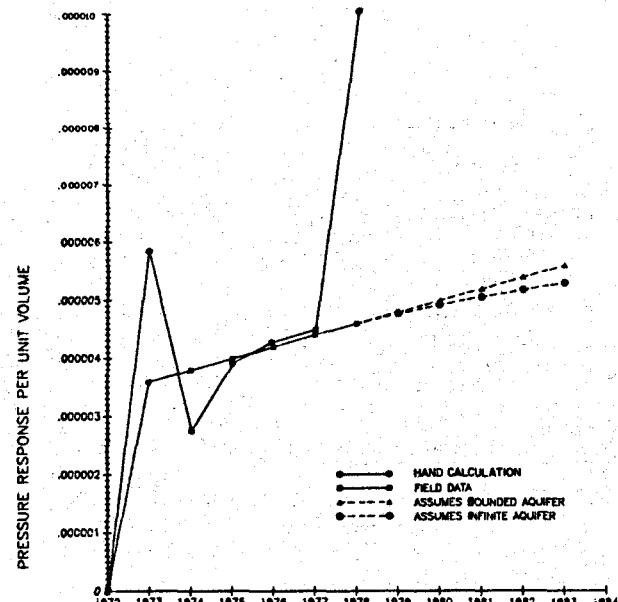
$$\frac{dF}{dt} \geq 0 \quad \frac{dh}{dt} \leq 0$$

$$\frac{d^2F}{dt^2} \leq 0 \quad \frac{d^2h}{dt^2} \geq 0$$

If the data are not "smooth and regular enough" the use of simple hand calculations can lead to the results shown in Fig. 12. The F function will reproduce the pressure very well, but the function cannot be extrapolated and is physically meaningless. The F calculated by the linear program is shown on the same figure for comparison.

Hutchinson and Sikora and Coats et al. discuss the effects of field geometry on the behavior of the pressure drop and the influence function. As production time increases, the rate of pressure change decreases. If the reservoir outcrops, both the pressure drop and the influence function become constant. This is an effect we will look for in geothermal areas which we know have fluid recharge. If the reservoir is infinite-acting or bounded the influence function and the pressure drop will increase monotonically for all time greater than 0.

Hutchinson and Sikora show how to extrapolate calculated influence functions. If a definite straight line has developed from the field data



LIQUID INFLUENCE FUNCTION -- CERRO PRIETO TOTAL FIELD -- 1972-1978
(Fitness measure $\rho = 100058$)

FIG. 12. Liquid influence function - Cerro Prieto total field.

it may be extrapolated and the field assumed to be bounded. If no definite straight line has developed, the last 3 or 4 values of F should be examined. If the average ΔF for these times gives a good match to past performance the curve may be extrapolated using the slope of the average F . The extreme extrapolation assumes an infinite aquifer. In this case

$$\Delta F_{n+1} = \Delta F_n \frac{\log[(n+1)/n]}{\log[n/(n-1)]}$$

All these extrapolations are included in our MPOS program.

9. Linearized Free Surface-Green's Function

One of the main virtues of the influence function method is described above is that it can be used to predict reservoir behavior without specifying a physical model for the reservoir. Long-time behavior of the influence function can tell something about the boundaries. If the reservoir has a free liquid surface and is assumed to be a porous half-space, Fig. 1a, a simple, distributed parameter model can be posited. Bodvarsson (1977) linearized the free surface condition and derived the following equations for pressure

$$p(P, t) = \frac{-q}{4c} \left(\frac{1}{r_{PQ}} + \frac{1}{r_{PQ'}} - \frac{2}{r_{PQ'}t} \right) \quad (84)$$

p = pressure, meters of head

q = flow rate, kg/s (constant)

$$r_{PQ} = ((x-x')^2 + (y-y')^2 + (z-d)^2)^{\frac{1}{2}}$$

$$r_{PQ'} = ((x-x')^2 + (y-y')^2 + (z+d)^2)^{\frac{1}{2}}$$

$$r_{PQ'}t = ((x-x')^2 + (y-y')^2 + (z+wt+d)^2)^{\frac{1}{2}}$$

Bodvarsson (1978) also showed that the impulse response of a linearized free surface can be expressed as

$$G(t, S, Q) = \frac{-1}{2\pi\phi} (wt+d) (x^2 + y^2 + (wt+d)^2)^{-3/2} U_+(t) \quad (85)$$

where

d = depth from free surface to sink

G = Green's function

$S = (x, y)$ a point on the free surface

ϕ = porosity, fraction

ρ = density, kg/m³

$w = kg/(v\phi)$ = sinking velocity, m/s

v = kinematic viscosity, m²/s

$g = 9.8$ m/s²

k = permeability, m²

$u_+(t)$ = unit impulse function

$$r = (x^2 + y^2)^{\frac{1}{2}}, \text{ radial distance from sink.}$$

He obtained the following expression for drawdown in meters for P at a distance r from the sink (wellbore)

$$h(t) = \int_0^t G(t-\tau)q(\tau)d\tau \quad (86)$$

$$\approx \sum_{\tau=0.5}^{t-0.5} G(t-\tau)q(\tau)\Delta\tau$$

See Chpt. II for the derivation of these equations. The impulse response, h , is the drawdown in meters at the point, P, caused by the instantaneous withdrawal of one unit fluid mass at point Q. At a continuous withdrawal the total drawdown at P would be a summation over all fluid sinks. The equation is

$$h_{\text{total}}(P) = \sum_{n=1}^N G_n(t-\tau)q_n(\tau)d\tau \quad (87)$$

This equation for drawdown can be compared directly with the h function formulation of the influence function as described above.

If the reservoir has a relatively impermeable zone below the producing zone the half space assumption can be modified. An image source term or terms as necessary can be added to the equation for G . See Fig. 1b. With one image term the expression is

$$G = \frac{-1}{2\pi\phi\rho} \left[\frac{wt+d}{[x^2 + y^2 + (wt+d)^2]^{3/2}} + \frac{2H-d+wt}{[x^2 + y^2 + (2H-d+wt)^2]^{3/2}} \right] \quad (88)$$

More terms can be added as necessary.

The distributed model described above can be approximated by a lumped parameter model as described more fully in Chpt. II.

IV. DATA PROCESSING

Data Sets

The most complete data set is from Wairakei, New Zealand by Pritchett et al. (1978) published by Systems, Science and Software for Lawrence Berkeley Laboratory. Individual well monthly heat and mass flow rates are given from 1953 through 1976 for 141 wells. Furthermore, a fair amount of pressure and temperature data are presented.

The authors presented a substantial amount of data on the geology and subsidence problems at Wairakei. In addition to this report we received from Malcolm Grant, DSIR, a set of annotated individual well production graphs which indicated when wells were shut in and which steam lines the wells were connected to. See Fig. 13a for map.

The data set for Cerro Prieto by Bermejo et al. (1979) published by Lawrence Berkeley Laboratory included graphical production histories of most of the wells from 1973 to 1978. These graphs were digitized for analysis. The production was broken down into liquid and vapor production. In addition, we received data from Marcelo Lippman, LBL, which showed individual well total mass flow rates. The two data sets were treated separately and then compared. A theoretical pressure drawdown curve was taken from Sanchez and de la Palma (1979). See Fig. 13b for map.

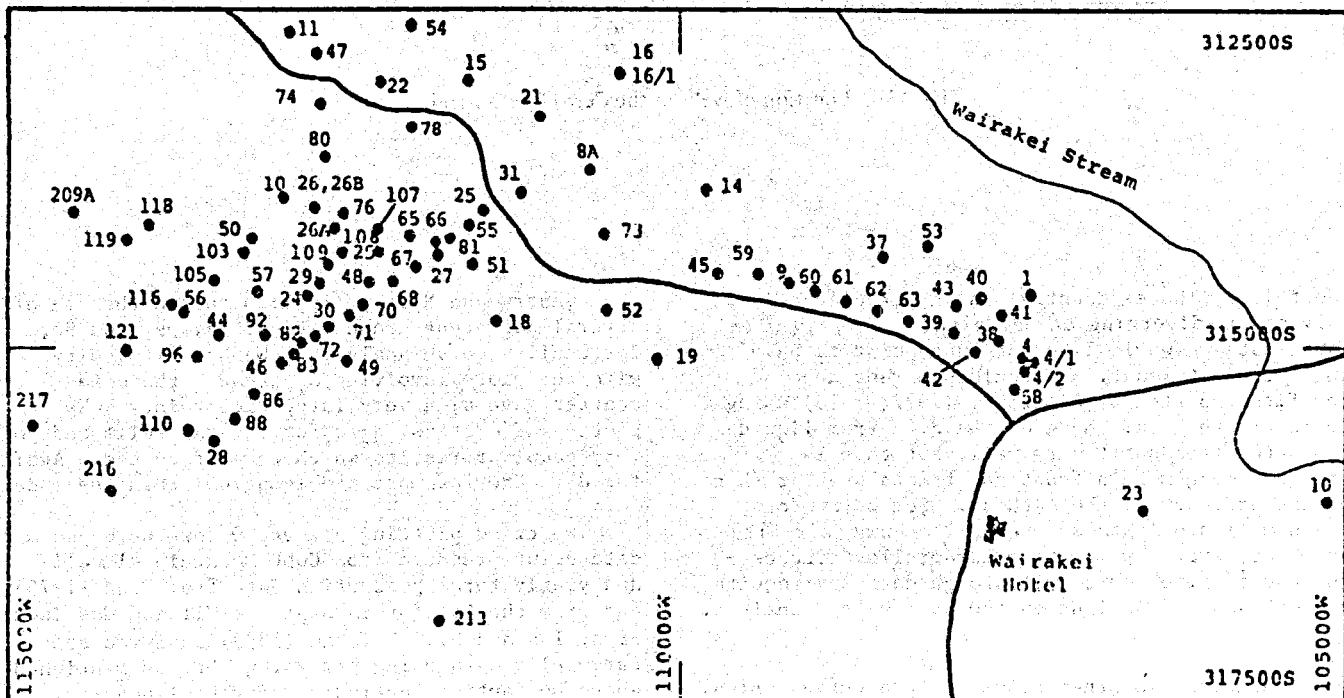
The last large data set is from The Geysers, California, courtesy of the California Dept. of Conservation, Div. of Oil and Gas. The data include production injection and pressure data from 27 wells from March 1971 through December 1979. Additional pressure data are from Lipman, Strobel, and Gulati (1977). See Fig. 13c for map.

The Larderello data were taken from Sestini (1970). The sparse data for other fields were from various sources. See Fig. 13d for map.

Graphical Treatment of Data

The first step in the analysis was graphing all the available production data on cartesian paper using SPSS. These graphs allowed us to eliminate from further consideration wells with severely irregular production such as Bore 11.

Arps (1945, 1956) pointed out that the exponential equation would graph as a straight line on semilog paper. We tried plotting the data for several wells at Wairakei but found that production decline was insufficient to make the semilog plots look very different from the cartesian plots. The log-log plots, however, were significantly different from the cartesian plots so most of the data were plotted on log-log plots. We tried matching the log-log plots against Fetkovich's type curves. For the most part the data scatter rendered the method useless. We were also hindered by the fact that dimensionless time for almost all our wells was fairly short,



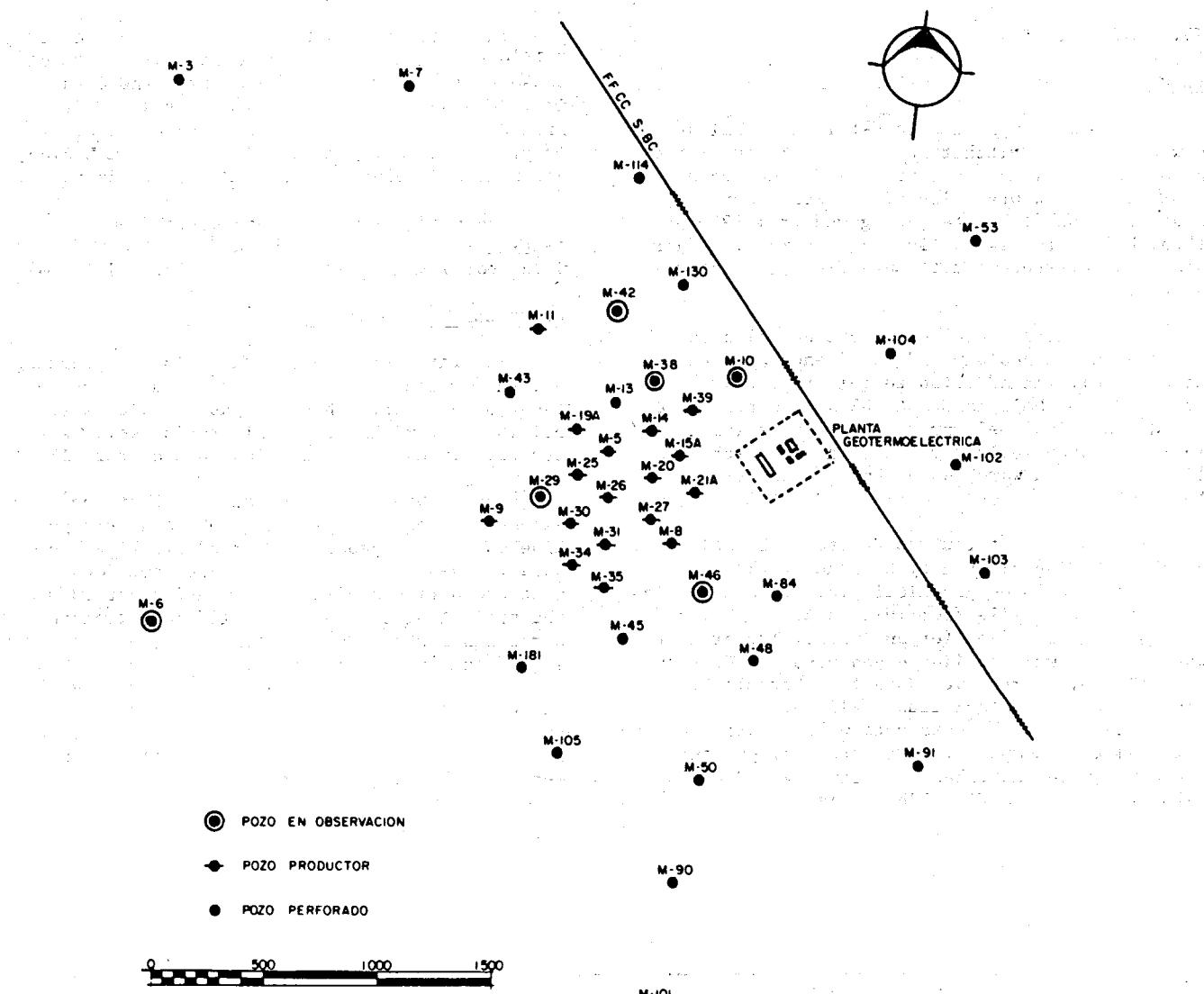


FIG. 13b. Location of wells in the Cerro Prieto Field.

about 1.0. The exponential and hyperbolic curves only start diverging at about $t_{Dd} = 0.2$, so with rough data we would like the last point to have $t_{Dd} = 2.0$, at least. We could not reproduce the fits reported by Rivera-R. (1977, 1978) using Cerro Prieto data. None of the data from liquid-dominated fields fit very well, but this is probably much more a function of data scatter than of the efficacy of the methods. See below for a discussion of data scatter. The only fair fits were for several wells from Larderello. Successful use of type curves with rough data may require a great deal of insight on the part of the analyst.

We tried two other kinds of type curves, Figs. 10a and 10b, with no more success than with Fetkovich's curves. Scatter and small dimensionless time caused problems again.

Gentry and McCray (1978) proposed the use of several different graphs, Figs. 9a, 9b, and 9c, for decline curve analysis. We had difficulty with the plots involving a_1 because the data scatter gave a_1 a very large uncertainty. We plotted $N_p/q_i t$ vs. q_i/q for several wells and got very peculiar results which were of no use. Again, the data are far more problematical than the models.

We tried plotting p/z vs. Q for The Geysers data using pressure from Cobb Mountain #1 well and yearly total production data from Finn (1975) and from the California Dept. of Oil and Gas (see Figs. 14a and b). Brigham (1979) analyzed some Larderello data using p/z vs. Q , but as mentioned above he cautions against expecting linearity after one-third to one-half of the fluid has been produced (see Figs. 15a and 15b for plots of the data).

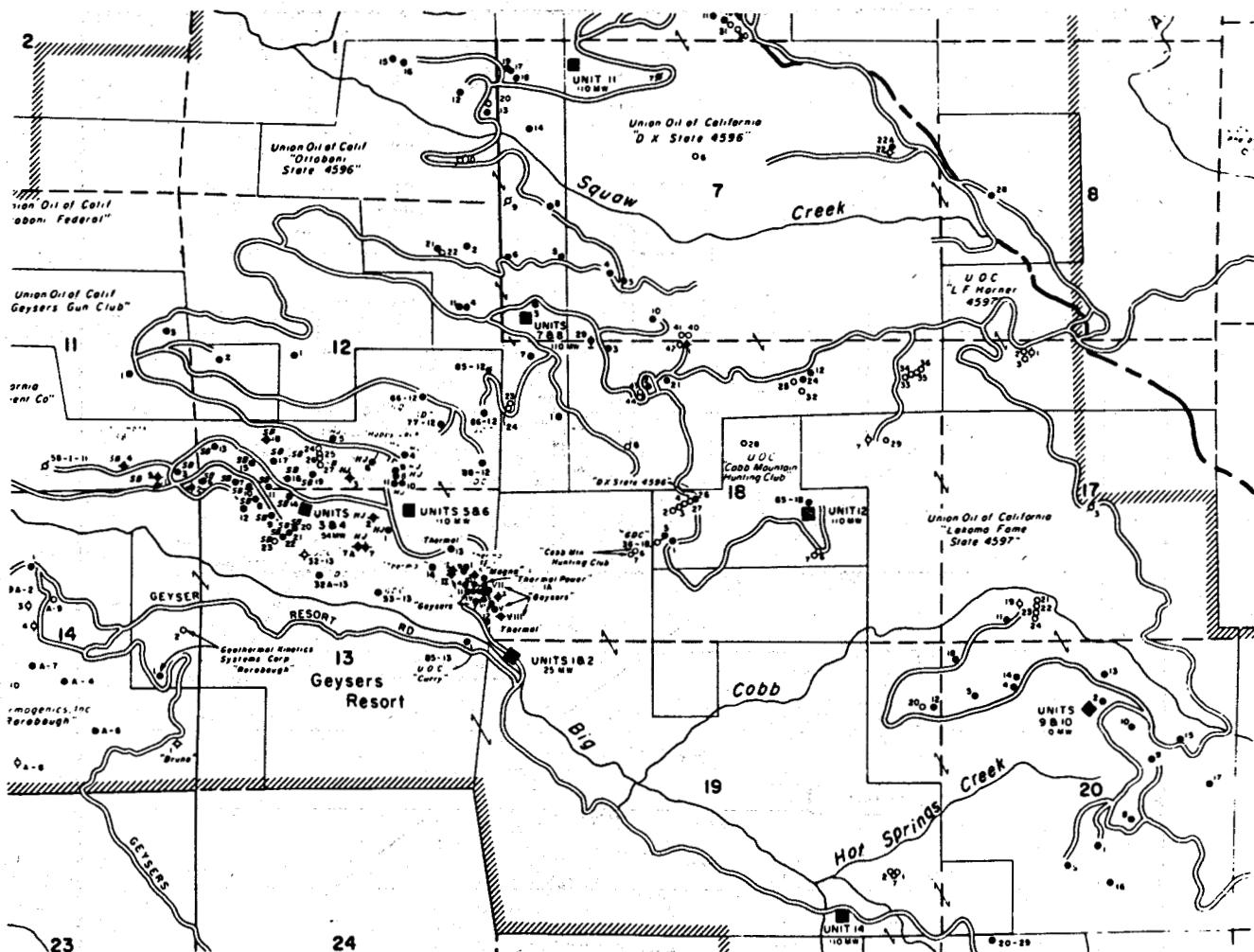


FIG. 13c. The Geysers, California (from California Dept. of Conservation, Division of Oil & Gas, 1978).

Statistical Treatment of Data

Most of the data sets had so much scatter that statistical treatment was the only reasonable approach. We used SPSS (Statistical Package for the Social Sciences) to reduce the data. See Appendix for discussion of SPSS and for the programs we used. SPSS is available at many computer centers and it requires a minimum of data handling.

We used SPSSPLOT to generate cartesian plots of the q vs. t data for all the wells. From the plots we chose wells to analyze further. Some of the wells had drastic rate changes in their histories so only selected parts of their histories were analyzed. We used a non-linear least squares regression subroutine to analyze the exponential equation

$$q(t) = q_i e^{-at}$$

The program requires initial estimates of q_i and a , and it returns $\hat{q}(t)$, the predicted value of q , and best estimates \hat{a} and \hat{q}_i for the fractional decline and the initial flow rate. A fit to the linear equation

$$q(t) = q_f + kt$$

is also generated. The primary statistic generated is R^2 defined as

$$R^2 = \frac{SSR}{SSTO} = \frac{\text{regression sum of squares}}{\text{total sum of squares}}$$

$$1 - \frac{SSE}{SSTO} = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$

$$SSTO = \sum_{i=1}^n (q_i - \bar{q})^2 \text{ where } \bar{q} = \frac{\sum q_i}{n}$$

$$SSR = \sum_{i=1}^n (\bar{q} - \hat{q}_i)^2$$

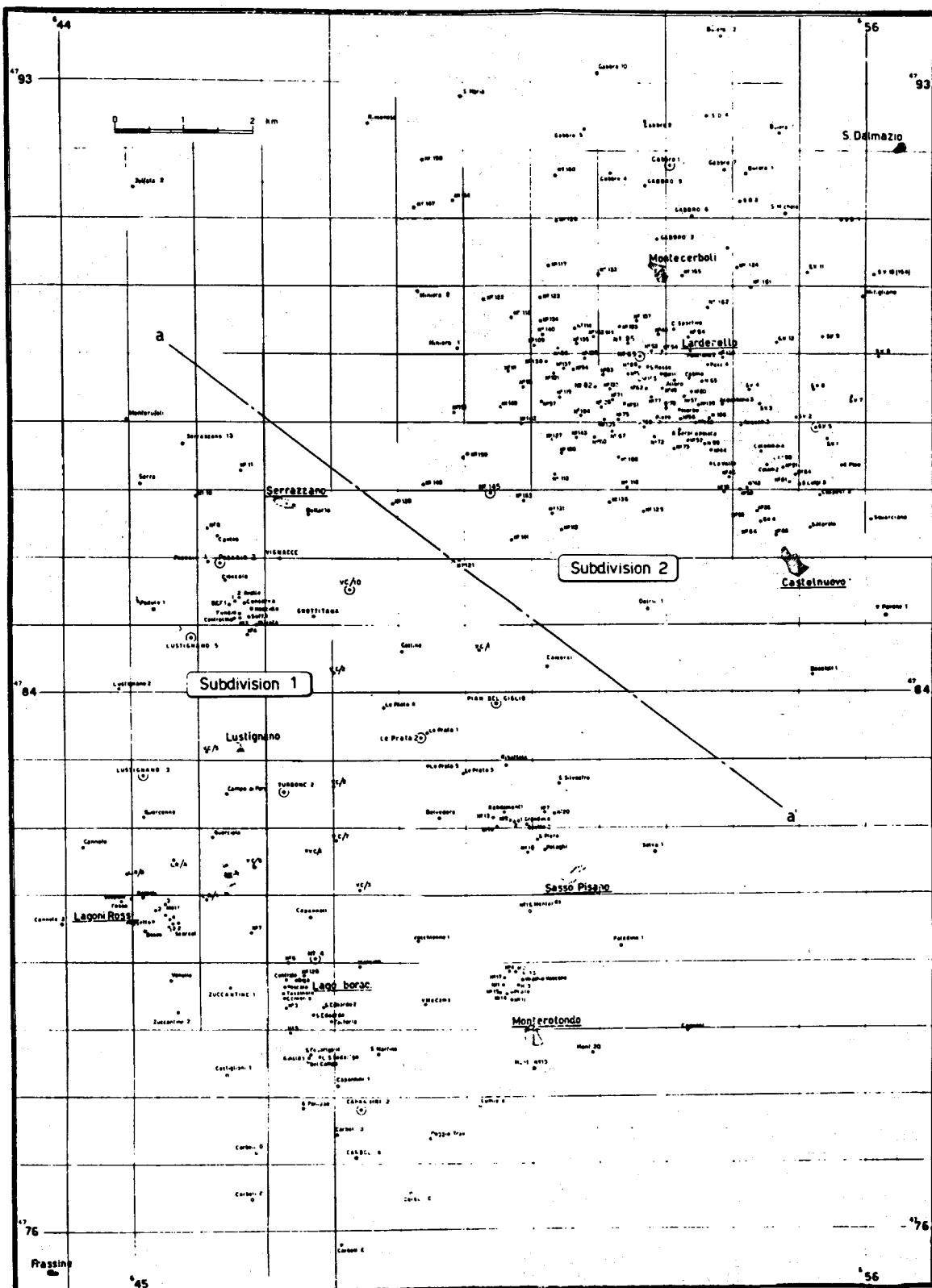


FIG. 13d. Larderello geothermal area (from Sestini, 1970).

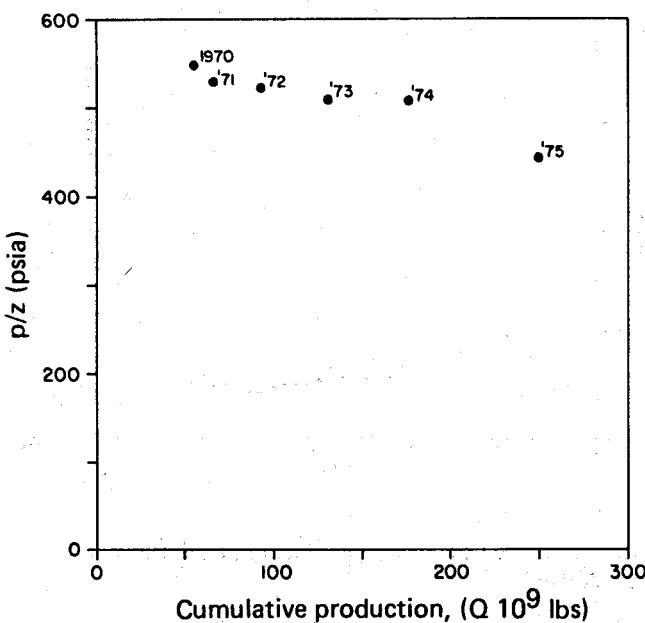


FIG. 14a. The Geysers p/z vs cumulative production, total field (pressure from Cobb Mt. No. 1 and Curry 85, ENEL Proceedings 1977, Strobel et al; production data from Finn, 1975).

the influence function method, we developed a fitness measure, ρ , which is the average fractional deviation of computed pressure differences, Δp , from observed pressure difference, $\Delta p = p_i - p(t)$. For example, if $\rho = 0.1$ and $\Delta p = 100$, the true value is between 90.91 and 111.11 because $\Delta p = \Delta p / (1 + \rho)$.

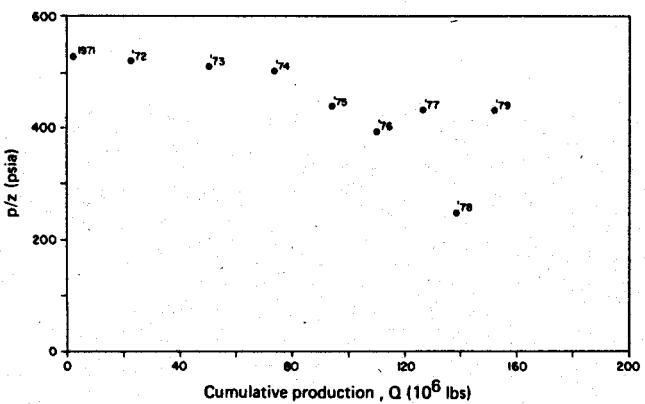


FIG. 14b. The Geysers p/z vs cumulative production, 20-30 wells. (pressure data from ENEL Proceedings 1977; production data from California Division of Oil & Gas, 1971-1979 data on 20-30 selected wells).

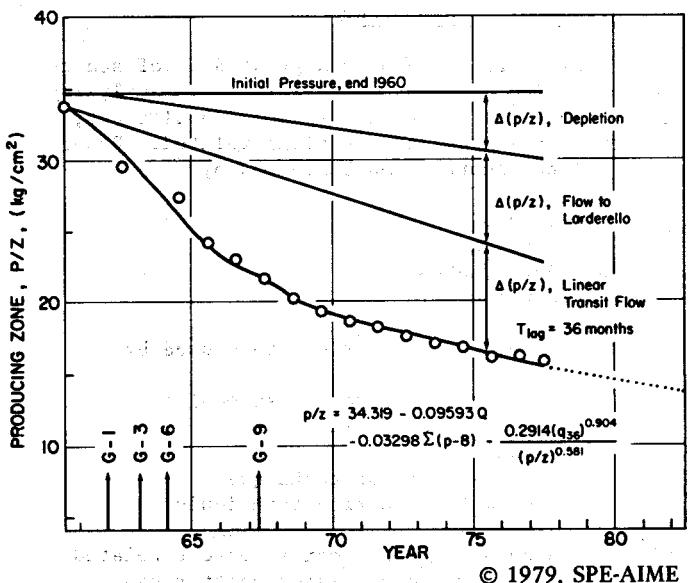


FIG. 15a. Gabbro Zone pressure-production history match, lag time=36 months (from Brigham, 1979).

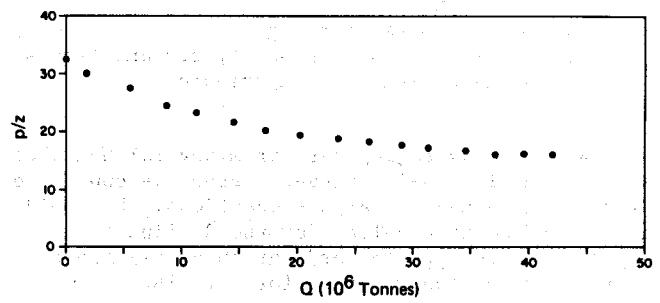


FIG. 15b. p/z vs Q—Larderello data (from Brigham & Neri, 1979).

$$SSE = SSTO - SSR$$

Values of R^2 greater than 0.65 indicate a good fit which can be extrapolated with some confidence. The value 0.65 is arbitrary, but is generally considered to be a good fit for raw data.

For the influence function method, we developed a fitness measure, ρ , which is the average fractional deviation of computed pressure differences, Δp , from observed pressure difference, $\Delta p = p_i - p(t)$. For example, if $\rho = 0.1$ and $\Delta p = 100$, the true value is between 90.91 and 111.11 because $\Delta p = \Delta p / (1 + \rho)$.

Discussion of Data Scatter

Field data often have a great deal of scatter in them which can cause difficulties in analyzing them. The scatter can be of two general types, reservoir related and operations related. Reservoir related scatter can be caused by

- 1) rainfall
- 2) recharge
- 3) earthquakes
- 4) subsidence.

Production related scatter can be caused by

- 1) changes in production schedules
- 2) bad well completions
- 3) workovers
- 4) poor calibration techniques
- 5) poor data gathering techniques

Little can be done to prevent reservoir related scatter, but operations related scatter can always be reduced. Methods for reducing the chance of scatter are discussed in the Standard Operating Procedure section. Scattered data can be analyzed with the following techniques:

- 1) averaging the data
- 2) least squares fitting
- 3) subtracting our known effects and trends
- 4) using insight and experience.

We tried averaging data from several Wairakei and Cerro Prieto wells to see whether we could use Guerrero's method for Arps's equations. We could not get reasonable values for the decline exponent. See Fig. 16 for a graph of six month average production vs. time for Bore 18.

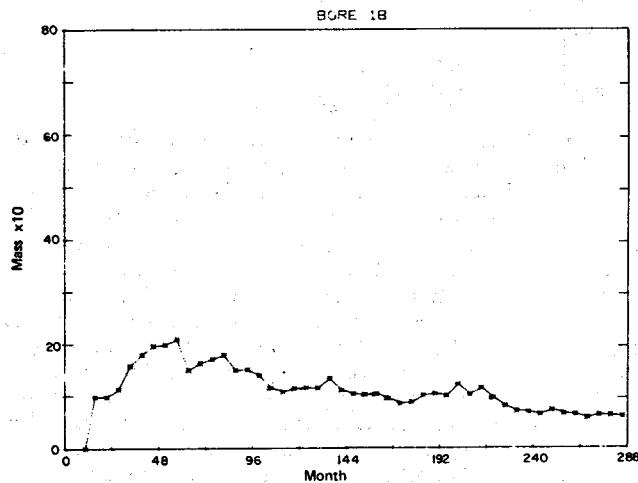


FIG. 16. Six month average production, Bore 18, Wairakei, New Zealand.

The easiest known effects to take into account are periodic shut downs. The annotated production graphs from Wairakei showed that many of the wells were shut in for periods of about 1 month every 1-2 years. The monthly production during these shut-in months is obviously much lower than the preceding and following month's production. If the other data are on a smooth trend, the low values can effectively be ignored in fitting an equation to the trend line. Since these points represent production, however, they should be included in any calculation involving the cumulative production, Q .

V. RESULTS

Arps's Equations

We tested Arps's exponential equation (73) on all individual well data, total field data and on several groups of wells. The results are summarized in Table 3 with complete results in the Appendix. \bar{D} is the average calculated monthly fractional decline. \bar{D} based on total field production from Wairakei, The Geysers, and Cerro Prieto ranges from 0.003 for Wairakei to 0.0115 for The Geysers. This converts to yearly declines of 3.6% and 13.8%, respectively. R^2 for individual wells ranges from 0.0004 for a well at Otake to 0.9712 for a well at Larderello. Eight of the ten wells and groups at Larderello had R^2 's greater than 0.87, indicating a very good fit to the equation. Also, all three wells at Matsukawa had R^2 's greater than 0.76. The wells from The Geysers did not fit as well as the wells from Larderello and Matsukawa, so we cannot draw definite conclusions about vapor-dominated fields and the exponential equation.

Cerro Prieto and Wairakei are both liquid-dominated fields, and their data did not fit the exponential equation quite as well as the vapor-dominated fields. However, for all the fields the equation fit at least several of the well's data quite well. See Figures 17a-g for a fit of the exponential equation to total Wairakei production and to several individual wells.

We only tested a few wells using the hyperbolic equation and got R^2 's greater than 0.989 in all cases. This indicates that the equation is either nearly perfect or that it is a very poor model and will fit virtually any data set. We hold the latter view. Because the equation has no physical basis, we recommend against using it. However, if a particular data set fits a hyperbolic type curve well over a long stretch of dimensionless time, the curve can be used to extrapolate production.

Type-Curve Methods

None of the data sets fit any of the type curves well. The scatter is so high that no value of b can be picked with confidence. Some of the Larderello data fit Fetkovich's exponential curve for up to 80 months but then develop constant production which takes them off the curve. See Figure 18a. Figure 18b shows typical Cerro Prieto data plotted on the same curve. No value of b can reasonably chosen.

Coats' Influence Function Method

Coats' method can be used with any but the most bizarre data because the derivative constraints imposed on the solution method guarantee that either a meaningful solution or no solution is generated. The fitness measure tells how useful

TABLE 3. SUMMARY RESULTS FOR FITS TO EXPONENTIAL EQUATION

Field	D	R ²	# of Wells	Range on Individual R ² 's	# of Wells at R ² >0.65
Wairakei, NZ total prod.	0.0030	0.7847	36	0.0016-0.9049	11
Cerro Prieto B.C., Mexico					
Liquid prod.			17	0.0066-0.8524	6
Total prod.			19	0.0405-0.9409	8
The Geysers, CA., USA	0.01151	0.8103	26	0.0126-0.8127	6
Larderello, Italy			10	0.0416-0.9712	8
Matsukawa, Japan			3	0.7609-0.8633	3
Otake, Japan					
Liquid			4	0.1528-0.7043	1
Vapor			4	0.0004-0.7981	1

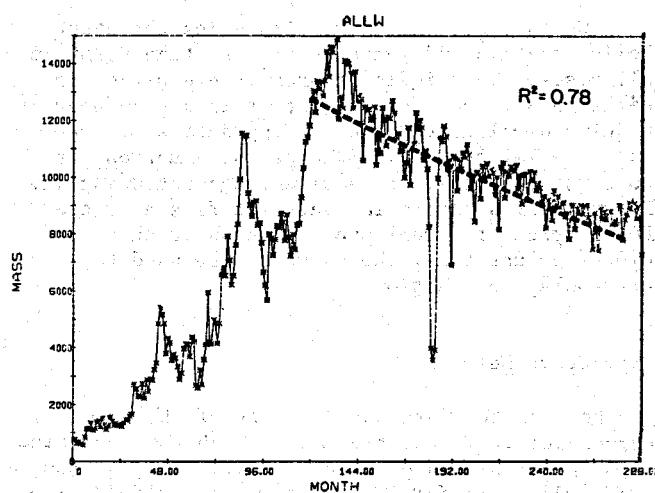


FIG. 17a. Wairakei total production, 1953–1976, with exponential fit 1964–1976.

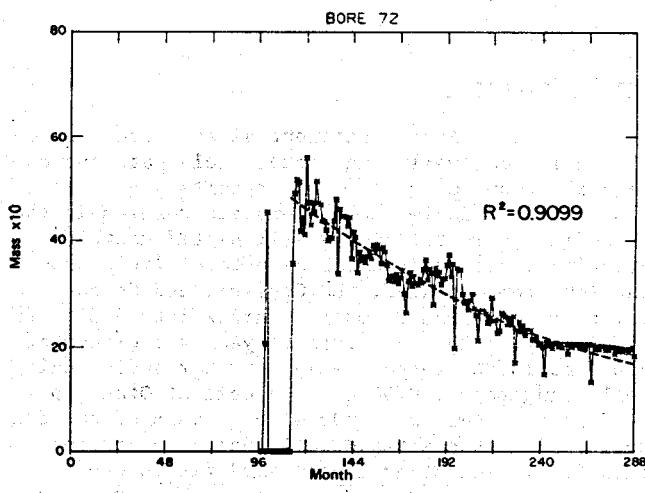


FIG. 17d. Bore 72, Wairakei, New Zealand.

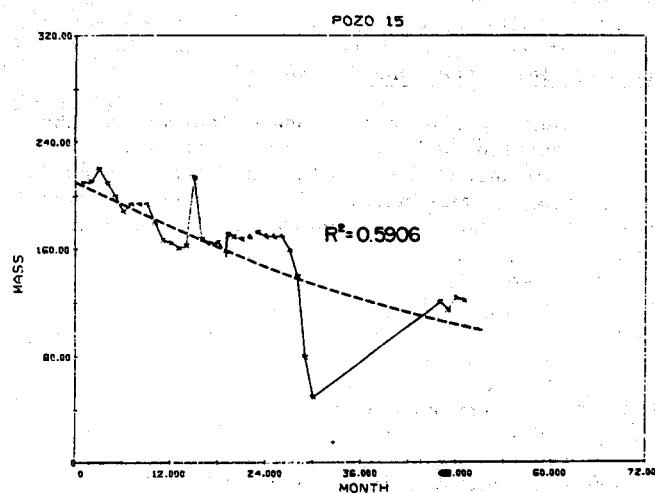


FIG. 17b. Pozo 15, Cerro Prieto, Mexico.

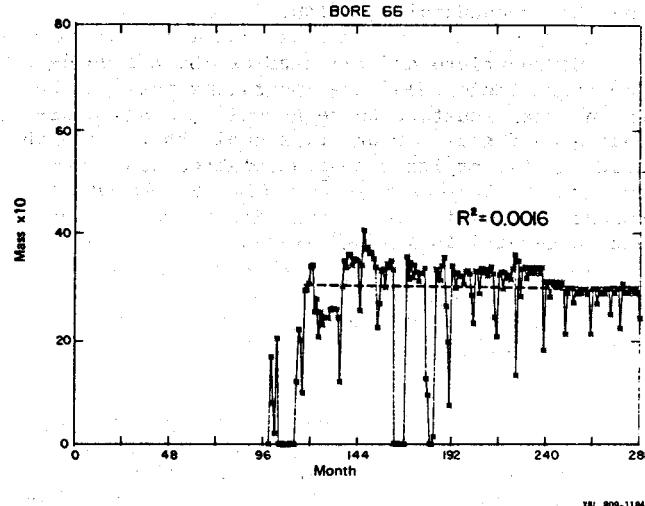


FIG. 17e. Bore 66, Wairakei, New Zealand.

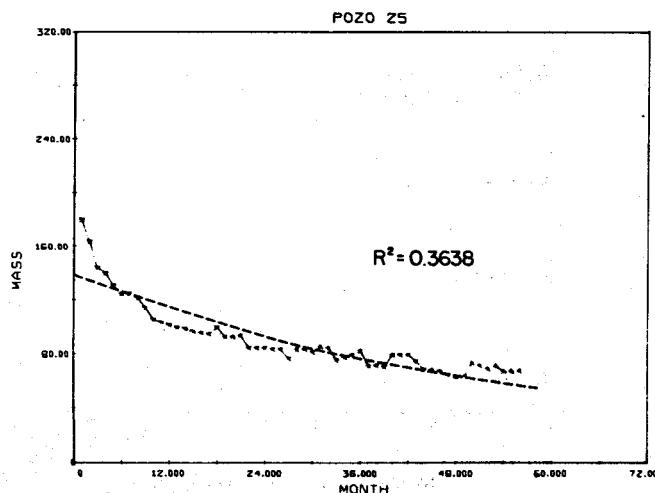


FIG. 17c. Pozo 25, Cerro Prieto, Mexico.

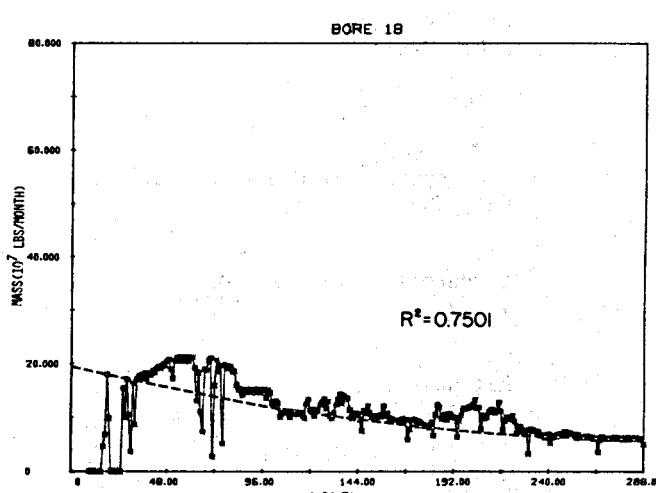


FIG. 17f. Bore 18, Wairakei, New Zealand.

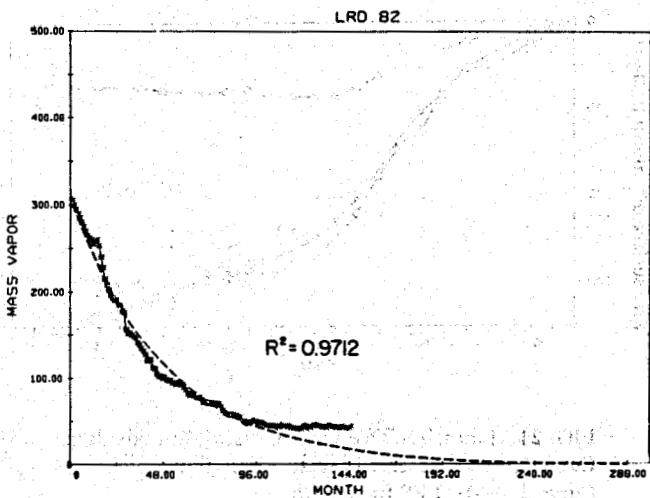


FIG. 17g. Larderello, Italy—Nella Sasso Rosso Nr. 82.

the solution is. We tried the method on Wairakei and Cerro Prieto total production and on Travale 22 from Larderello. The fitness measure, ρ for Travale 22 was 0.038 indicating a very good fit (see Fig. 19). ρ was 0.1001 and 0.3366 for Cerro Prieto liquid production and Wairakei total production, respectively. We tested the predictive value of the method by fitting Wairakei data from 1955-62 and then extrapolating. The pressures obtained using both the infinite and bounded aquifer approximations are shown in Table 4 along with the actual pressures. The pressure drop is calculated as

$$\Delta p = \frac{QF}{t}$$

Where Q is cumulative production, F is the influence function and t is time. Figure 20 shows the calculated influence functions for Wairakei 1955-1962. The high fitness measure indicates very rough data. The observed pressures fall within the fitness measure for the infinite aquifer case.

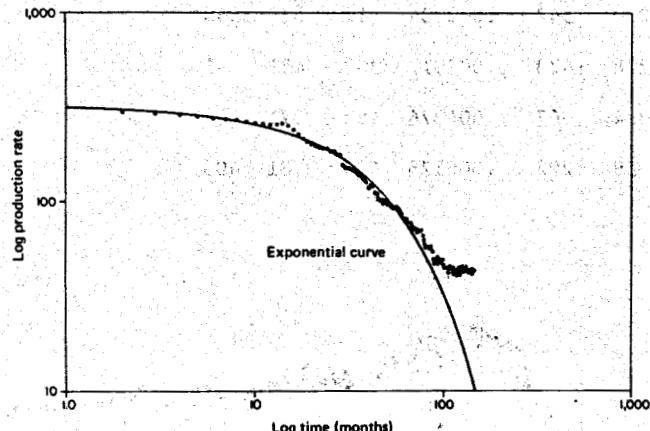


FIG. 18a. Larderello 82 (Nella Sasso Rosso Nr. 82) fit to Fetkovich type-curve.

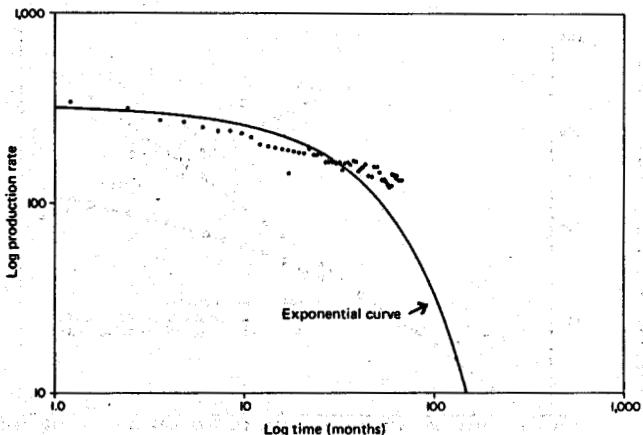
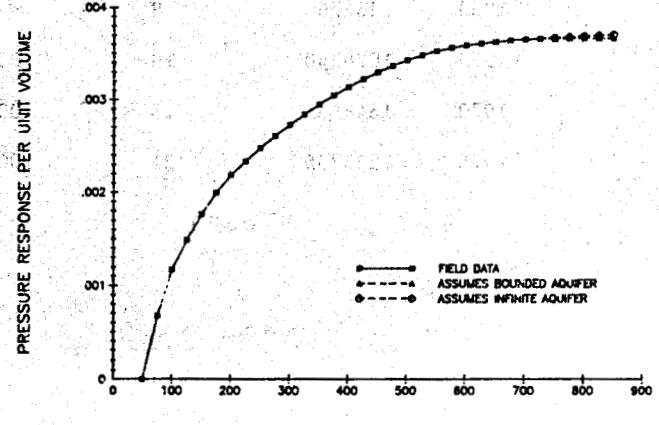


FIG. 18b. Pozo 25, Cerro Prieto, fit to Fetkovich type-curve.

In an infinite aquifer the rate of pressure decline decreases with time as is the case for Wairakei.

Bodvarsson's Linearized Free Surface Method

We had enough data to try the linearized free surface method only with Wairakei. We divided the field into six regions and then assumed that the total production from each region was coming from a virtual well in the "center" of the region. The production depth for each virtual well was the production weighted average center of open zone for the wells in the region. A centroid was chosen for the entire field and then the pressure drawdown at the centroid was calculated for each virtual well and summed to get total drawdown. The drawdown curve obtained is shown in Figure 21 as Curve #2 with the actual drawdown as Curve #1 for comparison. By adding the term for a bottom as described in Chapter III and by adjusting the porosity, ϕ , permeability, k , and depth, H , we obtained Curve #3,



INFLUENCE FUNCTION - TRAVALE 22 -
(Fitness measure $\rho = 0.038239$)

FIG. 19. Influence function—Travale 22.

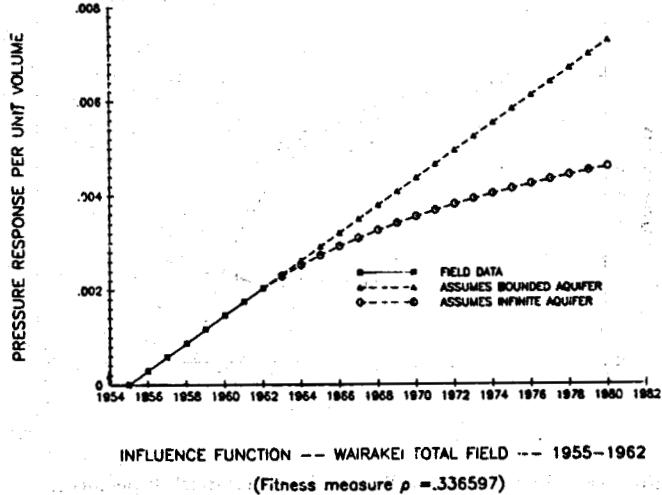


FIG. 20. Influence function—Wairakei total field 1955-1962.

a plausible fit. This method is difficult to use because the necessary geologic and production data are usually lacking or sparse at best.

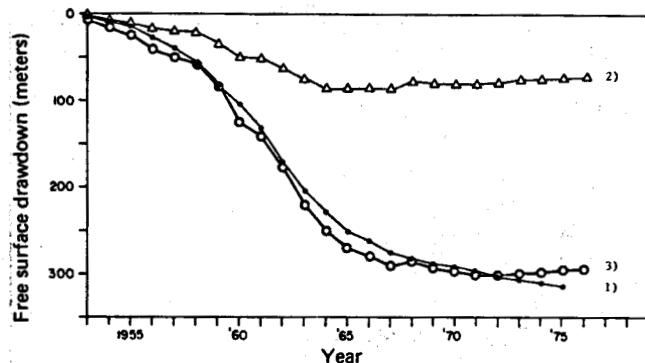


FIG. 21. Linearized free surface fits to Wairakei data.
Curve 1, ●—●: observed pressure drawdown
Curve 2, △—△: LFS fit, 1 term
Curve 3, ○—○: LFS fit, 2 terms

TABLE 4. CHECK OF EXTRAPOLABILITY OF COATS' METHOD USING WAIRAKEI DATA

(Values of the influence function are from Fig. 20)

Year	Cumulative Prod., Q	Producing Time, t, yrs.	F_∞	Δp	p	F_{bound}	Δp	p	p_{obs}
1963	726276	8	.0023	209	542	.002334	212	539	543
1965	10283401	10	.0027	278	473	.002917	300	451	491
1970	1644585	15	.0035	384	367	.004376	480	271	427
1976	2295755	21	.0042	459	292	.006126	670	81	405

VI. STANDARD OPERATING PROCEDURE FOR DATA GATHERING AND ANALYSIS

Data Gathering

The most important step for the analysis of data is the proper collection of it. The data must be as complete and clear as possible so that "bad data" can be eliminated as a possible cause of unusual results in the analysis. Some steps for ensuring good data gathering are

- 1) Set up regular testing schedules and stick to them.
- 2) Set up calibration schedules for all instruments used such as pressure gauges and temperature bombs.
- 3) Keep an updated calibration log for each instrument.
- 4) Use clear standard forms for recording data.

A data chart for routine measurements should include at least the following information

- 1) Well name and location
- 2) Date and time
- 3) Pressure-well head, tubing, bottom hole, meter run, etc. gauge or absolute
- 4) Temperature
- 5) Flow rate
- 6) Location of test points
- 7) Units for all measured quantities
- 8) Well status
- 9) Type of test being conducted - buildup, interference, etc.
- 10) Zone being tested
- 11) Instrument numbers
- 12) Name of tester

Data Analysis

The data can be analyzed by wells, by groups of wells and by fields.

Graphs

Graphing the data provides an easy way of examining the data for unusual behavior such as occasional high, low or erratic production. Such data sets can be flagged for special attention. The data should be plotted and analyzed according to Arps (1945, 1956) using cartesian semi-log, and log-log plots of production vs. time. However, this provides only a "quick look" and further analysis should be done. If the data are smooth enough, log-log type curves and Gentry's and McCray's curves can be used to fit current data and to extrapolate to future behavior. If the field is vapor dominated, p/z vs. Q plots can be used but only with great caution.

Least Squares Fits to Arps's Equations

Production data (q vs. t) should be fit to Arps's exponential equation using a non-linear least squares program. The program should calculate R^2 to indicate goodness of fit. A reasonably high value of R^2 , e.g., greater than 0.65, allows extrapolation with some degree of confidence.

We recommend against using the computer to fit data to Arps's hyperbolic equation for the reasons described in Chapter V. However, if the data fall very well on a particular type curve then one may reasonably predict future production using that curve.

COMPLETE RESULTS FOR FITS TO EXPONENTIAL EQUATION

Field Well #	Start Date	End Date	Calculated Fractional Decline, $Dx10^3$	R^2
<u>Wairakei - Total Production</u>				
18	1-56	12-76	4.86	0.75
20	4-59	12-76	6.00	0.32
22	12-59	12-76	12.47	0.79
24	1-60	12-66	7.52	0.60
24	5-58	12-76	3.26	0.46
26A	1-63	12-76	5.38	0.52
26B	10-62	12-76	5.29	0.68
27	8-58	12-76	2.05	0.20
28	1-64	12-76	1.74	0.19
30	3-57	12-76	4.90	0.72
39	1-64	12-76	3.69	0.53
41	1-59	12-66	7.02	0.43
42	1-60	12-66	17.48	0.80
43	12-58	12-66	5.04	0.40
44	7-62	12-76	5.10	0.78
46	12-58	12-76	4.86	0.74
47	3-59	12-76	1.34	0.14
47	1-63	11-67	7.87	0.40
47	6-58	12-76	0.47	0.01
48	5-62, excl 5-68-3-69	12-76	11.22	0.80
55	5-62	12-76	3.05	0.52
56	8-62	12-76	9.81	0.86
57	9-62	12-76	6.01	0.69
58	8-61	12-76	2.84	0.45
66	5-64	12-76	1.03	0.06
67	8-60	12-76	4.67	0.61
70	1-65	12-76	1.94	0.38
71	5-63	12-76	1.77	0.20
72	7-62	12-76	5.97	0.90
74	12-63	4-66	2.50	0.27
76	12-62	12-76	5.72	0.59
80	1-63	12-76	6.81	0.72
81	12-60	5-62	3.99	0.59
82	1-56	12-76	3.40	0.33
83	9-63	12-76	2.54	0.38
88	1-64	12-76	0.45	0.01
108	8-64	12-76	2.87	0.51
ALL WELLS	1-64	12-76	3.00	0.78

COMPLETE RESULTS FOR FITS TO EXPONENTIAL EQUATION
(Continued)

Field	Well #	Start Date	End Date	Calculated Fractional Decline, $D \times 10^3$	R^2
<u>Cerro Prieto - Liquid Production</u>					
5	3-73	7-78	4.72	0.55	
8	6-73	7-78	14.36	0.64	
9	3-73	12-77	10.26	0.13	
11	3-73	7-78	24.22	0.73	
14	8-76	8-78	4.65	0.09	
15	8-74	8-78	14.55	0.59	
19	2-75	7-78	-13.79	0.54	
20	8-73	7-78	24.71	0.79	
21	9-74	7-78	-15.02	0.62	
25	12-73	7-78	16.08	0.36	
26	8-73	7-78	-20.87	0.76	
27	8-76	7-78	-2.06	0.01	
30	12-73	7-78	7.80	0.47	
31	8-73	7-78	4.08	0.58	
34	7-73	9-75	22.67	0.84	
35	3-74	7-78	6.82	0.84	

Cerro Prieto - Total Production

Field	Well #	Start Date	End Date	Calculated Fractional Decline, $D \times 10^3$	R^2
5	1-73	12-79	8.01	0.81	
8	1-73	12-79	18.12	0.81	
9	1-73	12-79	99.09	0.15	
11	1-73	12-79	19.87	0.72	
14	1-73	12-79	9.74	0.52	
15	1-73	12-79	17.31	0.73	
19	1-73	12-79	-1.40	0.06	
20	1-73	12-79	25.31	0.83	
21	1-73	12-79	4.60	0.19	
25	1-73	12-79	9.52	0.58	
26	1-73	12-79	-8.36	0.31	
27	1-73	12-79	2.76	0.04	
29	1-73	12-79	7.55	0.22	
30	1-73	12-79	6.93	0.68	
31	1-73	12-79	5.84	0.67	
34	1-73	12-79	27.20	0.58	
35	1-73	12-79	8.07	0.94	
39	1-73	12-79	33.26	0.63	
42	1-73	12-79	7.18	0.32	

Geyser - Total Production

Field	Well #	Start Date	End Date	Calculated Fractional Decline, $D \times 10^3$	R^2
<u>Geyser Total Field</u>					
DX-2	3-74	11-79	7.98	0.44	
DX-3	11-72	11-79	8.10	0.54	
DX-4	8-72	11-79	10.86	0.69	
DX-5	8-71	11-79	1.16	0.01	
DX-10	8-71	11-79	5.00	0.16	
DX-10	3-74	2-78	9.80	0.35	
SUFBK3	8-71	11-79	11.59	0.49	
GDC-32A	12-72	11-79	9.94	0.54	
GDC-53	3-72	11-79	19.01	0.32	
GDC-66	4-73	11-79	9.56	0.54	
GDC-77	5-72	11-79	9.85	0.31	
GDC-85	5-72	11-79	7.17	0.38	
GDC-85	5-73	12-74	37.98	0.82	
GDC-85	2-75	11-76	24.34	0.89	
GDC-86	5-73	11-79	7.69	0.42	
GDC-88	3-72	11-79	11.76	0.61	
HAPYJK1	11-71	11-79	16.10	0.61	
HAPYJK2	11-71	11-79	13.17	0.60	
HAPYJK7	11-71	11-79	2.45	0.15	
HAPYJK8	11-71	11-79	10.13	0.48	
HAPYJK9	11-72	11-79	6.18	0.22	
OS-1	6-73	11-79	9.66	0.59	
OS-2	9-72	11-79	6.03	0.23	
OS-3	9-72	11-79	13.70	0.46	
OS-3	6-75	9-77	25.36	0.89	
OS-4	11-72	11-79	11.64	0.54	
OS-5	7-74	11-79	23.99	0.63	
OS-6	8-72	11-79	9.58	0.59	
OS-7	11-72	11-79	11.58	0.69	
OS-8	8-72	11-79	10.80	0.81	

Geyser Total Field

Field	Well #	Start Date	End Date	Calculated Fractional Decline, $D \times 10^3$	R^2
Olkaria	--	--	--	10.84	0.85

Influence Functions

If adequate production and pressure data are available, they should be analyzed using Coats's influence function method and a computer program with the constraints described in Chapter III. Data preparation is straightforward and data handling is minimal. The first half of a data set can be modeled and extrapolated in several different ways. Comparing the extrapolation with the second half of the data can give insight to the placement of reservoir boundaries such as faults or outcrops.

Bodvarsson's linearized free surface method should be tried if the reservoir has a free liquid surface, and if enough data are available to estimate a sinking velocity.

REFERENCES

Alonso E., H., B. Dominguez A., et al., 1978, Recent Activities at the Cerro Prieto Field, LBL-8538, Dec., Lawrence Berkeley Laboratory.

Arps, J., 1945, Analysis of Decline Curves, Trans. AIME, 160

Arps, J., 1956, Estimation of Primary Oil Reserves, Trans. AIME, 207

Arps, J., F. Brons, A. van Everdingen, R. Buchwald, and A. Smith, 1967, A Statistical Study of Recovery Efficiency, Am. Pet. Inst. API Bull. D14, October

Bermejo M., F., C. Cortéz A., and A. Aragón A., 1979, Cambios Fisicos Y Termodinamicos Observados en el Yacimiento Geotermico de Cerro Prieto, in Proceedings First Symposium on the Cerro Prieto Geothermal Field, Baja California, Mexico, Sept. 20-22, 1978, LBL-7098, Lawrence Berkeley Laboratory

Bodvarsson, G., 1977, Unconfined Aquifer Flow with a Linearized Free Surface Condition, Jokull, 27

Bodvarsson, G., 1978, Mechanisms of Reservoir Testing, in Proceedings of the Fourth Workshop on Geothermal Reservoir Engineering, Dec. 13-15, 1978, Stanford University, SGP-TR-30.

Bodvarsson, G., 1980, Personal Communication

Brigham, W. and G. Neri, 1979, Preliminary Results on a Depletion Model for the Gabbro Zone (Northern Part of Larderello Field) in Proceedings Fifth Workshop Geothermal Reservoir Engineering, H.J. Ramey, Jr. and P. Kruger, eds., Stanford, Dec. 12-14, SGP-TR-40.

Brigham, W. and W. Morrow, 1974, P/Z Behavior for Geothermal Steam Reservoirs, Soc. Pet. Eng. SPE 4899

Brons, F., 1963, On the Use and Misuse of Production Decline Curves, Prod. Monthly

California Dept. of Conservation, Div. of Oil & Gas, 1978, The Geysers, (geothermal map), G3-1

California Division of Oil and Gas, Public Production Data on the Geysers Geothermal Field, Released 1979

Cerullo, R. and M. Romeo, 1970, A Review of the Curve-Fitting Method of Least Squares as Applied to Petroleum Engineering, USDI Bu. Mines Info. Circ. 8449

Coats, K., 1973, Elements of Reservoir Simulation, Intercomp/Human Resources Development Corp.

Coats, K., L. Rapoport, J. McCord, and W. Drews, 1964, Determination of Aquifer Influence Functions from Field Data, J. Pet. Tech., Dec.

Cohen, C., and J. Stein, 1977, Multipurpose Optimization System User's Guide, Version 4, Vogelback Computing Center, Northwestern University, Evanston, Il.

Duff, G. and D. Naylor, 1966, Differential Equations of Applied Mathematics, John Wiley & Sons, Inc., New York

Earlougher, Jr., R. C., 1977, Advances in Well Test Analysis, Monograph 5, Society of Petroleum Engineers of AIME

Economides, M., W. Brigham, et al., 1979, Influence Functions and their Application to Geothermal Well Testing, in Geothermal Resources Council, Trans., 3, Sept.

Fetkovich, M., 1973, Decline Curve Analysis Using Type Curves, Soc. Pet. Eng. SPE 4629

Finn, D.F.X., 1975, Price of Steam at The Geysers, in Proceedings, 2nd UN Symposium on the Development and Use of Geothermal Resources, p. 2298

Garrison, L.E., 1972, Geothermal Steam in The Geysers - Clear Lake Region, California, GSA Bull., v. 83, p. 1449, May

Gennai, N. and G. Sestini, 1964, Studio sulle Cause del Decline delle Portate di Vapore Endogeno nel Tempo, Richerche di Termotecnica, Nr. 14

Gentry, R., 1972, Decline-Curve Analysis, J. Pet. Tech, Jan.

Gentry, R. and A. McCray, 1978, The Effect of Reservoir and Fluid Properties on Production Decline Curves, J. Pet. Tech., Sept.

Grant, Malcolm, 1979, Wairakei Production Graphs

Guerrero, E., 1961a, How to Determine Performance and Ultimate Recovery by Exponential Decline-Curve Analysis, Oil and Gas J., May 22

Guerrero, E., 1961b, How to Determine Performance and Ultimate Recovery of a Reservoir Declining Hyperbolically, Oil and Gas J., July 17

Gurley, J., 1963, A Productivity and Economic Projection Method - Ohio Clinton Sand Gas Wells, J. Pet. Tech., Nov.

Higgins, R. and H. Lechtenberg, 1970, How to Predict Oil-Field Performance, Oil and Gas J., Sept. 14

Higgins, R. and H. Lechtenberg, 1969, Merits of Decline Equations Based on Production History of 90 Reservoirs, Soc. Pet. Eng. SPE 2450

Hutchinson, T. and V. Sikora, 1959, A Generalized Water-Drive Analysis, Trans. AIME, 216

Jargon, J. and H. van Poolen, 1965, Unit Response Function from Varying-Rate Data, J. Pet. Tech., August

Katz, D. and K. Coats, 1968, Underground Storage of Fluids, Ulrich's Books, The U. of Michigan

Katz, D., M. Tek, et al., 1963, Movement of Underground Water in Contact with Natural Gas, American Gas Assoc., New York

Lamb, H., 1932, Hydrodynamics, 6th ed., Dover, New York

Lefkovits, H., C. Matthews, 1958, Application of Decline Curves to Gravity-Drainage Reservoirs in the Stripper Stage, Trans. AIME, 213

Lindstrom, F., 1980, FUNMIN: An Online-Interactive Mixed Simplex-Marquardt Least Squares or Likelihood Minimization Algorithm, Tech. Rep. 73, Dept. of Stat., OSU, Corvallis

Locke, C., L. Schrider, and M. Romeo, 1968, A Unique Approach to Oil-Production Decline Curve Analysis with Applications, Soc. Pet. Eng. SPE 2224

Mannon, R., 1965, Oil Production Forecasting by Decline Curve Analysis Soc. Pet. Eng. SPE 1254

Matthews, C. and H. Lefkovits, 1956, Gravity Drainage Performance of Depletion-Type Reservoirs in the Stripper Stage, Trans. AIME, 207

Muskat, M., 1949, Physical Principles of Oil Production, McGraw-Hill

Nie, N., C. Hull, J. Jenkins, K. Steinbrenner and D. Bent, 1975, Statistical Package for the Social Sciences, 2nd Edition, McGraw-Hill

Onodera, S., T. Masukawa, M. Fukuda, and K. Aosaki, 1977, The Inclination of Flow Rates of Steam and Hot Water from Otake Production Wells, and the Effect of Reinjection, Nihon Koogyo Kaishi (Japan Mining Journal), 93 1069 (177-3)

Palama, A., 1976, Funzioni di Influenza in Campi Geotermici, ENEL Provisorio 511-Z 18, Ottobre

Pirson, S., 1946, Mathematical Methods of Decline Curve Extrapolation and Reserve Calculation, The Oil Weekly, Sept. 9

Pritchett, J., L. Rice, and S. Garg, 1978, Reservoir Engineering Data: Wairakei Geothermal Field, New Zealand, SS-R-78-3597-1, Systems, Science and Software

Pruess, K., G. Bodvarsson, R. Schroeder et al., 1979, Simulation of the Depletion of Two-Phase Geothermal Reservoirs, Lawrence Berkeley Laboratory LBL-9606, August

Pruess, K. and R. Schroeder, 1979, Basic Theory and Equations Used in the Two-Phase Multi-dimensionsl Geothermal Reservoir Simulator, SHAFT79, Lawrence Berkeley Laboratory LBL-9464, July

Pruess, K., J. Zerzan, R. Schroeder, and P. Witherspoon, 1979, Description of the Three-Dimensional Two-Phase Simulator SHAFT78 for Use in Geothermal Reservoir Studies, Lawrence Berkeley Laboratory LBL-8802, January

Rivera R., J., 1977, Decline Curve Analysis - A Useful Reservoir Engineering Tool for Predicting the Performance of Geothermal Wells, Geothermal Res. Council, Trans., 1

Rivera-R., J., 1978, Application of Type Curve Procedures for the Analysis of Production Data from Geothermal Wells, Geothermal Res. Council, Trans., 2

Robinson, B., 1977, SPSS Subprogram NONLINEAR - Nonlinear Regression, Manual No. 433, Vogelback Computing Center, Northwestern University, Evanston, IL, August

Sánchez, R., J. and A. de la Peña, 1979, Preliminary Geohydrological Study of the Cerro Prieto Geothermal Field, in Abstracts of Second Symposium on the Cerro Prieto Geothermal Field, Baja California, Mexico, Lawrence Berkeley Laboratory

Sestini, G., 1970, Superheating of Geothermal Steam, Proc. of the U.N. Symposium on the Development & Utilization of Geothermal Resources, Pisa.

Shea, G., R. Higgins, and H. Lechtenberg, 1964, Decline and Forecast Studies Based on Performances of Selected California Oilfields, J. Pet. Tech., Sept.

Slider, H.C. 1968, A Simplified Method of Hyperbolic Decline Curve Analysis, J. Pet. Tech., January

Simon, B., 1974, Functional Integration and Quantum Physics, Academic Press, New York

Stakgold, I., 1967, Boundary Value Problems of Mathematical Physics, vol. 1, The MacMillan Co., New York

Sunde, E., 1968, Earth Conduction Effects in Transmission Systems, Dover, New York

Tsarevich, K. A. and I.F. Kuranov, 1956, Calculation of the Flow Rates for the Center Well in a Circular Reservoir Under Elastic Conditions, extract from Part I of Problems of Reservoir Hydrodynamics and Thermodynamics (Voprosy Gidrodinamiki i Termodinamiki Plasta), ed., E. M. Minskii, Ministerstvo Neftyanoi Promyshlennosti SSSR, Vsesoyuzni Neftegazovyi Nauchno-Issledovatelskii Institut (VNII), Leningrad Gostoptekhizdat

Tuccy, J., 1977, SPSS Subprogram PLOT: Digital (Calcomp) Plotting, Document No. 409 (Rev. A), Vogelback Computing Center, Northwestern University, Evanston, IL, October

APPENDIX

Statistical Package for the Social Sciences SPSS

SPSS is a set of programs developed for general statistical analysis. We have used the two subprograms PLOT (Tuccy, 1977) and NON-LINEAR (Robinson, 1977) quite extensively. The listings for our SPSS main programs which used these subprograms are given below. SPSS2 will plot a set of data. SPSS4 will do a nonlinear least squares fit using the exponential equation. B(1) and B(2) are initial guesses for initial production, q_0 , and monthly fractional decline, D. The other program names are self-explanatory. A complete description of SPSS is given in Nie, 1975.

Multiple Purpose Optimization System MPOS

MPOS is a linear programming package designed to solve a wide variety of linear programming problems. Coats' influence method can be formulated as a linear programming problem as follows.

$$\sum_{j=1}^1 q_{i-j} x_j + u_i - v_i = b \quad (1) \quad i = 1, 2, \dots, n$$

$$x_j = F_j - F_{j-1}$$

$$x_i \geq 0 \quad (2a) \quad i = 1, 2, \dots, n$$

$$x_{n-1} - x_n \geq 0 \quad (2b) \quad i = 1, 2, \dots, n$$

$$x_{i+1} - 2x_i + x_{i-1} \geq 0 \quad (2c) \quad i = 1, 2, \dots, n$$

Objective function $\sum (u_i + v_i) = \text{minimum}$ where
 u_i and v_i are slack variables, b_i is the observed pressure change Δp , and $q_{i-j} x_j$ is the calculated pressure change. MPOS generates a tableau for the calculation which looks like the following for $n = 4$.

$$\begin{array}{ccccccccccccc}
 & x_1 & x_2 & x_3 & x_4 & u_1 & u_2 & u_3 & u_4 & v_1 & v_2 & v_3 & v_4 & b \\
 \text{Objective function} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
 \text{2 a)} & \left\{ \begin{array}{cccccc} q_1 & & & & 1 & & & & -1 & & & & & = b_1 \\ q_2 & q_1 & & & & 1 & & & & & -1 & & & = b_2 \\ q_3 & q_2 & q_1 & & & & 1 & & & & & -1 & & = b_3 \\ q_4 & q_3 & q_2 & q_1 & & & & 1 & & & & & & = b_4 \end{array} \right. \\
 \text{2 c)} & & & & 1 & -1 & & & & & & & & \geq 0 \\
 \text{2 d)} & & & & \left\{ \begin{array}{cccccc} 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \end{array} \right. & & & & & & & & & \geq 0 \\
 & & & & & & & & & & & & & \geq 0
 \end{array}$$

Values for u_i and v_i are given in the output so that the fitness measures, ρ , can be calculated directly as

$$\rho = \frac{1}{n} \sum_{i=1}^n \frac{1}{b_i} (u_i + v_i)$$

The listing for program INFUNC is given below.

COMPUTER LISTINGS AND EXAMPLES

SPSS PROGRAMS

```
===== SPSS2 =====
111
PAGESIZE      NOEJECT
RUN NAME      PLOT ONLY -- POZO 5
VARIABLE LIST  MONTH53, MASS
INPUT FORMAT   FIXED (24X,F3.0,27X,F7.0)
N OF CASES    UNKNOWN
READ INPUT DATA
PLOT          PLOTS=MASS(0,320) WITH MONTH53(0,72) /
              TITLE=POZO 5/
              TITLEX=MONTH/
              TITLEY=MASS/
              SIZE=10.5,8.0/
              XDIV=12/
              YDIV=8/
              SYMBOLS=-7/
              1,10
```

OPTIONS
FINISH

```
===== SPSS4 =====
111
PAGESIZE      NOEJECT
RUN NAME      NONLINEAR REGRESSION -- BORE 108
VARIABLE LIST  MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT   FIXED (9X,F5.0,5X,2F5.0,F20.2)
N OF CASES    159
COMPUTE        PMASS=MASS
MISSING VALUES MASS(0)
IF             (YEAR LT 1964)MASS=0
IF             (YEAR EQ 1964 AND AMONTH LT 8)MASS=0
COMPUTE        MONTHR=MONTH53-140
READ INPUT DATA
*REJECT IF
NONLINEAR
MODEL
DERIVATIVES
PARAMETERS
REGRESSION
PLOT          (MASS EQ 0)
              VARTABLES=MASS WITH MONTHR,NB=2
              YHAT=1000000*B(1)*EXP(B(2)*.00001*MONTHR)
              G(1)=EXP(B(2)*.00001*MONTHR)*1000000
              G(2)=10*MONTHR*B(1)*EXP(B(2)*.00001*MONTHR)
              B(1)=435.299562
              B(2)=-392.55
              VARTABLES=MASS,MONTHR/
              REGRESSION=MASS WITH MONTHR/
              PLOTS=PMASS(0,800000000) WITH MONTH53(0,288) /
              TITLE=BORE 108/
              TITLEX=MONTH/
              TITLEY=MASS/
              SIZE=10.5,8.0/
              XDIV=12/
              YDIV=8/
              SYMBOLS=-7/
              1,10
```

OPTIONS
FINISH

==== SPSS5 =====

```
***  

PAGESIZE      NOEJFCT  

RUN NAME      SEMILOG PLOT ONLY -- BORE 22  

VARIABLE LIST MONTH53,AMONTH,YEAR,MASS  

INPUT FORMAT  FIXED (9X,F5.0,5X,2F5.0,F20.21)  

N OF CASES    UNKNOWN  

COMPUTE       LOGMASS=LG10(MASS+1)  

READ INPUT DATA  

PLOT          PLOTS=LOGMASS(0,9) WITH MONTH53(0,2681)/  

              TITLE=BORE 22/  

              TITLEX=MONTH/  

              TITLEY=LOG MASS/  

              SIZE=10.5,8.0/  

              XDIV=12/  

              YDIV=9/  

              SYMBOLS=-7/  

              1.10  

OPTIONS  

FINISH
```

===== SPSS6 =====

```
***  

PAGESIZE      NOEJCT  

RUN NAME      LOG-LOG PLOT ONLY -- BORE 72  

VARIABLE LIST MONTH53,AMONTH,YEAR,MASS  

INPUT FORMAT  FIXED (9X,F5.0,5X,2F5.0,F20.21)  

N OF CASES    UNKNOWN  

COMPUTE       LOGMASS=LG10(MASS+1)  

COMPUTE       LOGT=LG10(MONTH53)  

READ INPUT DATA  

PLOT          PLOTS=LOGMASS(4,9) WITH LOGT(0,3)/  

              TITLE=BORE 72/  

              TITLEX=LOG TIME/  

              TITLEY=LOG MASS/  

              SIZE=-9.0,-15.0/  

              XDIV=12/  

              YDIV=20/  

              SYMBOLS=-7/  

              1.10  

OPTIONS  

FINISH
```

===== SPSSGEN =====

```
***  

PAGESIZE      NOEJCT  

RUN NAME      GENTRY SEMILOG PLOT -- BORE 22  

VARIABLE LIST XTM,YPRD,XCM  

INPUT FORMAT  FIXED(10X,F3.0,5X,F20.0,5X,F20.0)  

N OF CASES    UNKNOWN  

COMPUTE       QINIT=354000000  

MISSING VALUES YPRO(0)  

COMPUTE       QIDQ=LG10(QINIT/YPRD)  

ASSIGN MISSING QIDQ(0.1)  

COMPUTE       RQCDQIT=1-XCM/(QINIT*XTM)  

READ INPUT DATA  

CODESCRIPTIVE QIDQ,RQCDQIT  

STATISTICS    ALL  

PLOT          PLOTS=QIDQ(0,2) WITH RQCDQIT(0,1)/  

              TITLE=BORE 22/  

              TITLEX=BORE22 CUM. PROD. OVER INITIAL PROD.*TIME/  

              TITLEY=LOG(INITIAL PROD. OVER CURRENT PROD.)/  

              SIZE=-15,-10/  

              XDIV=20/  

              YDIV=16/  

              SYMBOLS=-7/  

              10  

OPTIONS  

FINISH
```

===== SPGEN2 =====

111
 PAGESIZE NOEJECT
 RUN NAME GENTRY CARTESIAN FOR A-INITIAL -BORE 20
 VARIABLE LIST XTM, YPRD, XCM
 INPUT FORMAT FIXED(10X, F3.0, 5X, F20.0, 5X, F20.0)
 N OF CASES UNKNOWN
 COMPUTE QINIT=489000000
 MISSING VALUES YPRD(0)
 COMPUTE QDQI=YPRD/QINIT
 ASSIGN MISSING QDQI(0.01)
 READ INPUT DATA
 CONDESCRIPTIVE STATISTICS
 PLOT QDQI
 ALL
 PLOTS=QDQI(0, 1.5) WITH XCM(0, 100000000000)
 TITLE=BORE 20/
 TITLEX=BORE 20 PROD RATIO VS CUM PROD/
 SIZE=15, 10/
 XDIV=20/
 YDIV=12/
 SYMBOLS=-9/
 OPTIONS 10
 FINISH

===== SPSSCUM =====

111
 PAGESIZE NOEJECT
 RUN NAME CUMULATIVE VS CURRENT PRODUCTION, BORE 24
 VARIABLE LIST XTM, YPRD
 INPUT FORMAT FIXED(14X, F5.0, 10X, F20.2)
 N OF CASES UNKNOWN
 COMPUTE XCM=ACCUM(YPRD)
 WRITE CASES (10X, F3.0, 5X, F20.0, 5X, F20.0)
 XTM, YPRD, XCM
 READ INPUT DATA
 PLOT PLOTS=YPRD(0, 800000000) WITH XCM(0, 100000000001)
 TITLE=BORE 24/
 TITLEX=CUMULATIVE PRODUCTION/
 TITLEY=CURRENT PRODUCTION/
 SIZE=10.5, 8.0/
 XDIV=10/
 YDIV=8/
 SYMBOLS=-7/
 PLOT PLOTS=XCM(0, 10000000000) WITH XTM(0, 288)
 TITLE=BORE 24/
 TITLEX=TIME IN MONTHS/
 TITLEY=CUMULATIVE PRODUCTION/
 XDIV=12/
 YDIV=10/
 SYMBOLS=-7/
 FINISH

===== SPSSAVG =====

111
 PAGESIZE NOEJECT
 RUN NAME OUTPUT SEMI-ANNUAL AVERAGES -- BORE 72
 VARIABLE LIST MONTH53, AMONTH, YEAR, MASS
 INPUT FORMAT FIXED(9X, F5.0, 5X, 2F5.0, F20.2)
 N OF CASES UNKNOWN
 MISSING VALUES MASS(0)
 IF (AMONTH LE 6) HAFYEAR=1
 IF (AMONTH GE 7) HAFYEAR=2
 READ INPUT DATA
 AGGREGATE GROUPVARS=YEAR, HAFYEAR/
 VARIABLES=MONTH53, MASS/
 AGGSTATS=MEAN/
 OPTIONS 3
 STATISTICS 3
 FINISH

VOGELBACK COMPUTING CENTER
NORTHWESTERN UNIVERSITY

79/07/25. 08.28.54. PAGE 1

SPSS - STATISTICAL PACKAGE FOR THE SOCIAL SCIENCES

VERSION 7.0 -- JUNE 27 1977

PAGESIZE 110EJECT
RUN NAME NONLINEAR REGRESSION -- BORE 18
VARIABLE LIST MONTH53,AMONTH,YEAR,MASS
INPUT FORMAT F14.0 (9A,F5.0,5X,2F5.0,F20.2)

ACCORDING TO YOUR INPUT FORMAT, VARIABLES ARE TO BE READ AS FOLLOWS

VARIABLE FORMAT RECORD COLUMNS

MONTH53	F 5. 0	1	10- 14
AMONTH	F 5. 0	1	20- 24
YEAR	F 5. 0	1	25- 29
MASS	F20. 2	1	30- 49

THE INPUT FORMAT PROVIDES FOR 4 VARIABLES. 4 WILL BE READ
IT PROVIDES FOR 1 RECORDS ("CARDS") PER CASE. A MAXIMUM OF 49 COLUMNS ARE USED ON A RECORD.

WARNING - A NUMERIC VARIABLE HAS A WIDTH GREATER THAN 14. SMALL ROUNDING/TRUNCATION ERRORS MAY OCCUR.

N OF CASES 283
IF (YEAR LT 1956)MASS=0
COMPUTE MUNTHR=MONTH53-37
ASSIGN MISSING MUNTHR(-1)
REJECT IF (MASS EQ 0)
READ INPUT DATA

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.28.54. PAGE 2

NONLINEAR VARIABLES=MASS WITH MUNTHR,NB=2
MODEL YHAT=1000000+8.444*EXP(3(2)*.00001*MUNTHR)
DERIVATI G(1)=EXP(3(2)*.00001*MUNTHR)*1000000
G(2)=10*MUNTHR*3(1)*EXP(3(2)*.00001*MUNTHR)
PARAMETERS 3(1)=185.615892
3(2)=-467.18
STATISTICS 9
FINISH

SPSS RELOADED

00061400 CM NEEDED FOR NONLINEAR

OPTION - 1
IGNORE MISSING VALUE INDICATORS

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.28.54. PAGE 3

FILE NONAME (CREATION DATE = 79/07/25.)

NONLINEAR PROBLEM SUMMARY

252 CASES
1 DEPENDENT VARIABLE(S)
2 PARAMETERS
50 ITERATION LIMIT
METHOD = MARQUARDT
TOL1 = 1.580000E-08 REL. CHANGE IN A PARAMETER
TOL2 = 0 REL. CHANGE IN SUM OF SQUARES
TOL3 = 0 RATIO TO INITIAL SUM OF SQUARES
TOL4 = 1.000000E-08 PIVOT TOLERANCE

PARAMETERS

NO.	NAME	INITIAL VALUE
1	B1	1.8561509E+02
2	B2	-4.6718000E+02

0 DERIVATIVE ERRORS WERE DETECTED

SETUP TIME = .475 SECONDS

THE LAST ITERATION

ITERATION NO.	5	BASE POINT	TEST POINT
SUM OF SQUARES		1.2714952E+17	1.2714952E+17
L		0	0
LAMBDA		0	0
GAMMA		1.0000000E+00	1.0000000E+00
ANGLE IN DEGREES		8.7176	8.7257
MAX. PIVOT REDUCTION		3.9411999E-01	3.9411999E-01
PAR.	1	B1	S 1.9510662E+02
	2	B2	-4.6583609E+02

CUMULATIVE NO. OF FUNCTION CALLS = 6
ITERATION TIME = .262 SECONDS
CUMULATIVE TIME = 1.0 SECONDS

ITERATION TERMINATES

MAX. RELATIVE CHANGE IN A PARAMETER .LT. TOL(1) = 1.500000E-00

NONLINEAR REGRESSION -- BORE 10

79/07/25. 08.28.54.

PAGE 4

FILE: NONAME (CREATION DATE = 79/07/25.)

FINAL PARAMETER VALUES

FINAL VALUE SUM OF SQUARES = 1.2714952E+17

R² = .7601

PAR.	1	31	1.9510662E+02
2	82	-4.8583609E+02	

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.28.54.

PAGE 5

FILE: NONAME (CREATION DATE = 79/07/25.)

FINAL FUNCTION VALUES AND RESIDUALS

ROOT MEAN SQUARE RESIDUAL = 2.2552119E+07

THIS IS THE SCALE UNIT IN THE GRAPH OF THE RESIDUALS.

D.F. = 250

CASE NO.	VAR NG.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL
1	1	1.9510662E+08	1.8200000E+08	-1.3106622E+07	*****
2	1	1.9416192E+08	1.7100000E+08	-2.3161623E+07	*****
3	1	1.9322001E+08	1.8200000E+08	-1.1220006E+07	*****
4	1	1.9228355E+08	1.7900000E+08	-1.3283550E+07	*****
5	1	1.9135163E+08	1.6700000E+08	-4.3516331E+06	***
6	1	1.9042423E+08	1.6200000E+08	-7.4242324E+06	****
7	1	1.8950133E+08	1.9200000E+08	2.4986735E+06	**
8	1	1.8856289E+08	1.9500000E+08	6.4171065E+06	****
9	1	1.8766891E+08	1.9100000E+08	3.3310882E+06	**
10	1	1.8675936E+08	2.0000000E+08	1.3240640E+07	*****
11	1	1.8585422E+08	1.9600000E+08	1.0145784E+07	*****
12	1	1.8495346E+08	2.0500000E+08	2.0046541E+07	*****
13	1	1.8405707E+08	2.0800000E+08	2.3942933E+07	*****
14	1	1.8316502E+08	1.9000000E+08	6.8344795E+06	****
15	1	1.8227730E+08	1.7200000E+08	-1.0277297E+07	*****
16	1	1.8139388E+08	2.0500000E+08	2.3606124E+07	*****
17	1	1.8051474E+08	2.1200000E+08	3.1485264E+07	*****
18	1	1.7963986E+08	2.0500000E+08	2.53604436E+07	*****
19	1	1.7876922E+08	2.1200000E+08	3.3230781E+07	*****
20	1	1.7790288E+08	2.1200000E+08	3.4097200E+07	*****
21	1	1.7704058E+08	2.0500000E+08	2.7959420E+07	*****
22	1	1.7618254E+08	2.1200000E+08	3.5817461E+07	*****
23	1	1.7532866E+08	2.0500000E+08	2.9671344E+07	*****

24	1	1.7447691E+08	2.1200000E+08	3.7521088E+07	*****
25	1	1.7363329E+08	2.1200000E+08	3.8366713E+07	*****
26	1	1.7279176E+08	1.9200000E+08	1.9208241E+07	*****
27	1	1.7295431E+08	1.3000000E+08	4.1954311E+07	*****
28	1	1.7112092E+08	1.8400000E+08	1.2879079E+07	*****
29	1	1.7029157E+08	1.1000000E+08	-6.0291570E+07	*****
30	1	1.6946624E+08	7.3000000E+07	-9.6466238E+07	*****
31	1	1.6864491E+08	1.9000000E+08	2.1355093E+07	*****
32	1	1.6782756E+08	1.8800000E+08	2.0172444E+07	*****
33	1	1.6701417E+08	2.0700000E+08	3.9985033E+07	*****
34	1	1.6620472E+08	2.1100000E+08	4.4795200E+07	*****
35	1	1.6539920E+08	2.7000000E+07	-1.3839920E+08	*****
36	1	1.6459753E+08	1.6000000E+08	-4.5975754E+06	***
37	1	1.6379964E+08	2.0700000E+08	4.3200160E+07	*****
38	1	1.6300597E+08	1.8600000E+08	2.2994028E+07	*****
39	1	1.6221595E+08	1.9700000E+08	3.4784050E+07	*****
40	1	1.6142976E+08	5.1000000E+07	-1.1042976E+08	*****
41	1	1.6064738E+08	1.9900000E+08	3.8352624E+07	*****
42	1	1.5986679E+08	1.9100000E+08	3.1131214E+07	*****
43	1	1.5909397E+08	1.9600000E+08	3.6906030E+07	*****
44	1	1.5832291E+08	1.9500000E+08	3.6077092E+07	*****
45	1	1.5755558E+08	1.8500000E+08	2.7444416E+07	*****
46	1	1.5679198E+08	1.8700000E+08	3.0208022E+07	*****

NONLINEAR REGRESSION -- BORE 13

79/07/25. 08.23.54. PAGE 0

FILE NONAME (CREATION DATE = 79/07/25.)

42

FINAL FUNCTION VALUES AND RESIDUALS

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL				
					-2	-1	0	+1	+2
47	1	1.5603207E+08	1.6100000E+08	4.3673262E+06	***	**	***	***	***
48	1	1.5527535E+08	1.5300000E+08	-2.2758522E+06	**	**	***	***	***
49	1	1.5452338E+08	1.5300000E+08	-1.5232956E+06	**	**	***	***	***
50	1	1.5377439E+08	1.4300000E+08	-1.0774386E+07	*****	*****	*****	*****	*****
51	1	1.5302911E+08	1.5300000E+08	-2.9106781E+04	*	*	*	*	*
52	1	1.5226744E+08	1.4600000E+08	-4.2674393E+06	***	***	***	***	***
53	1	1.5154937E+08	1.5300000E+08	1.4506337E+06	**	**	***	***	***
54	1	1.5081437E+08	1.4800000E+08	-2.8148704E+06	**	**	***	***	***
55	1	1.5006393E+08	1.4300000E+08	-2.9160656E+06	**	**	***	***	***
56	1	1.4935654E+08	1.5300000E+08	3.6434592E+06	***	***	***	***	***
57	1	1.4863267E+08	1.4800000E+08	-6.3267269E+05	*	*	*	*	*
58	1	1.4791231E+08	1.5300000E+08	5.0876872E+06	***	***	***	***	***
59	1	1.4719944E+08	1.4800000E+08	8.0455577E+05	*	*	*	*	*
60	1	1.4646205E+08	1.5300000E+08	6.5179500E+06	***	***	***	***	***
61	1	1.4577211E+08	1.5200000E+08	-6.22780675E+06	***	***	***	***	***
62	1	1.4506562E+08	1.3700000E+08	-8.0656174E+06	*****	*****	*****	*****	*****
63	1	1.4436255E+08	1.5200000E+08	7.6374545E+06	*****	*****	*****	*****	*****
64	1	1.4366288E+08	1.4700000E+08	3.3371189E+06	*****	*****	*****	*****	*****
65	1	1.4296661E+08	1.2800000E+08	-1.4966604E+07	*****	*****	*****	*****	*****
66	1	1.4227371E+08	1.2700000E+08	-1.5273709E+07	*****	*****	*****	*****	*****
67	1	1.4158447E+08	1.3200000E+08	-9.5841683E+06	*****	*****	*****	*****	*****
68	1	1.4089797E+08	1.2700000E+08	-1.3897970E+07	*****	*****	*****	*****	*****
69	1	1.4021518E+08	1.0100000E+08	-3.9215036E+07	*****	*****	*****	*****	*****
70	1	1.395533E+08	1.1300000E+08	-2.653518E+07	*****	*****	*****	*****	*****
71	1	1.3885926E+08	1.0900000E+08	-2.98592e3E+07	*****	*****	*****	*****	*****
72	1	1.3818627E+08	1.1200000E+08	-2.6186271E+07	*****	*****	*****	*****	*****

73	1	1.37.54E+08	1.1200000E+08	-2.55163E+07
74	1	1.368506E+08	1.0180000E+08	-3.5650056E+07
75	1	1.3618680E+08	1.1100000E+08	-2.5186801E+07
76	1	1.3552676E+08	1.0700000E+08	-2.4526761E+07
77	1	1.3486992E+08	1.1000000E+08	-2.4865920E+07
78	1	1.3421626E+08	1.0600000E+08	-2.6216263E+07
79	1	1.3356577E+08	1.0900000E+08	-2.4565773E+07
80	1	1.3291844E+08	1.0900000E+08	-2.3918436E+07
81	1	1.3227424E+08	9.9000000E+07	-3.3274236E+07
82	1	1.3163316E+08	1.2600000E+08	-5.6831598E+06
83	1	1.3099519E+08	1.3400000E+08	3.0048114E+06
84	1	1.3036031E+08	1.1200000E+08	-1.8360318E+07
85	1	1.2972851E+08	1.1600000E+08	-1.3728509E+07
86	1	1.2909977E+08	1.0300000E+08	-2.6099769E+07
87	1	1.2847408E+08	1.1400000E+08	-1.4474077E+07
88	1	1.2785142E+08	1.1200000E+08	-1.5051410E+07
89	1	1.2723170E+08	1.2400000E+08	-3.2317757E+06
90	1	1.2661519E+08	1.2800000E+08	1.3846631E+06
91	1	1.2600149E+08	1.3500000E+08	8.9985132E+06
92	1	1.2539081E+08	1.1600000E+08	-9.3908107E+06
93	1	1.2478309E+08	1.3000000E+08	5.2169057E+06
94	1	1.2417832E+08	1.0300000E+08	-2.1178323E+07
95	1	1.2357648E+08	9.9000000E+07	-2.4576483E+07
96	1	1.2297756E+08	1.0900000E+08	-1.3977560E+07

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.23.54. PAGE 7

FILE NUNAME (CREATION DATE = 79/07/25.)

43

F I N A L F U N C T I O N V A L U E S A N D R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL				
					-2	-1	0	+1	+2
97	1	1.2238154E+08	1.2400000E+08	1.6184601E+06	***	***	***	***	***
98	1	1.2178841E+08	1.3400000E+08	-1.2211532E+07	*****	*****	*****	*****	*****
99	1	1.2119815E+08	1.4400000E+08	2.2801849E+07	*****	*****	*****	*****	*****
100	1	1.2061075E+08	1.2600000E+08	5.3892451E+06	***	***	***	***	***
101	1	1.2002621E+08	1.4100000E+08	2.0973795E+07	*****	*****	*****	*****	*****
102	1	1.1944449E+08	1.3500000E+08	1.5555511E+07	*****	*****	*****	*****	*****
103	1	1.1886559E+08	1.3700000E+08	1.8134408E+07	*****	*****	*****	*****	*****
104	1	1.18249505E+08	1.1400000E+08	-4.2845086E+06	***	***	***	***	***
105	1	1.1771620E+08	9.9000000E+07	-1.8716281E+07	*****	*****	*****	*****	*****
106	1	1.1714568E+08	1.0200000E+08	-1.5145681E+07	*****	*****	*****	*****	*****
107	1	1.1657752E+08	1.0600000E+08	-6.5779248E+06	****	****	****	****	****
108	1	1.1681292E+08	1.0500000E+08	-1.0012921E+07	****	****	****	****	****
109	1	1.1545066E+08	9.6000000E+07	-1.9450655E+07	*****	*****	*****	*****	*****
110	1	1.1449111E+08	7.6000000E+07	-3.4891114E+07	*****	*****	*****	*****	*****
111	1	1.1334298E+08	1.1200000E+08	-2.3342857E+06	**	**	**	**	**
112	1	1.1378016E+08	1.0700000E+08	-6.7801556E+06	****	****	****	****	****
113	1	1.1322871E+08	1.2100000E+08	7.7712880E+06	****	****	****	****	****
114	1	1.1267944E+08	1.0900000E+08	-3.6799394E+06	***	***	***	***	***
115	1	1.1213383E+08	1.0100000E+08	-1.1133827E+07	*****	*****	*****	*****	*****
116	1	1.1159036E+08	1.0130000E+08	-1.0590362E+07	*****	*****	*****	*****	*****
117	1	1.1104953E+08	1.0100000E+08	-1.0049531E+07	****	****	****	****	****
118	1	1.1051132E+08	1.0300000E+08	-7.5113203E+06	****	****	****	****	****
119	1	1.097572E+08	1.0100000E+08	-8.9757185E+06	****	****	****	****	****
120	1	1.0944271E+08	1.0500000E+08	-4.427126E+06	***	***	***	***	***
121	1	1.0891229E+08	1.2100000E+08	1.2087710E+07	*****	*****	*****	*****	*****

122	1	1.0838444E+08	1.0200000E+08	-6.3844380E+06	****
123	1	1.0785914E+08	1.0800000E+08	1.4085564E+05	*
124	1	1.0733640E+08	9.7000000E+07	-1.0336397E+07	*****
125	1	1.0661615E+08	9.8000000E+07	0.0161023E+06	*****
126	1	1.0629849E+08	9.4000000E+07	-1.2298489E+07	*****
127	1	1.0578331E+08	9.4000000E+07	-1.1783305E+07	*****
128	1	1.0527062E+08	9.3000000E+07	-1.2270618E+07	*****
129	1	1.0476042E+08	9.3000000E+07	-1.1760416E+07	*****
130	1	1.0425269E+08	9.7000000E+07	-7.2526865E+06	****
131	1	1.0374742E+08	9.2000000E+07	-1.1747410E+07	*****
132	1	1.0324460E+08	9.7000000E+07	-6.2445978E+06	****
133	1	1.0274421E+08	5.9444265E+07	-6.3299930E+07	*****
134	1	1.0224626E+08	7.8424571E+07	-2.3817686E+07	*****
135	1	1.0175071E+08	9.5534265E+07	-6.2164274E+06	****
136	1	1.0125757E+08	9.6081427E+07	-5.1761426E+06	***
137	1	1.0076682E+08	9.5632855E+07	-4.9339619E+06	***
138	1	1.0027844E+08	8.5059993E+07	-1.5218445E+07	*****
139	1	9.9792435E+07	9.3390569E+07	-6.3939663E+06	****
140	1	9.9308783E+07	9.0221427E+07	-9.0873565E+06	****
141	1	9.8827476E+07	8.6810570E+07	-1.2008906E+07	*****
142	1	9.8348501E+07	8.5194294E+07	-1.3154217E+07	*****
143	1	9.7871847E+07	8.4359990E+07	-1.3511049E+07	*****
144	1	9.7397503E+07	8.1637141E+07	-1.5760362E+07	*****
145	1	9.6925459E+07	9.1932055E+07	-4.9926038E+06	***
146	1	9.6455702E+07	6.6801426E+07	-2.9654274E+07	*****

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.28.54. PAGE 8

FILE NONAME (CREATION DATE = 79/07/25.)

F I N A L F U N C T I O N V A L U E S A N D R E S I D U A L S

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL				
					-2	-1	0	+1	+2
147	1	9.5968222E+07	1.0540143E+08	9.4132042E+06	*****	*****	*****	*****	*****
148	1	9.5523007E+07	1.2439143E+08	2.0668421E+07	*****	*****	*****	*****	*****
149	1	9.5060048E+07	1.1844143E+08	2.3801378E+07	*****	*****	*****	*****	*****
150	1	9.4599332E+07	9.7515712E+07	2.9162803E+06	**	**	**	**	**
151	1	9.4140849E+07	1.0432143E+08	1.0100578E+07	*****	*****	*****	*****	*****
152	1	9.3684588E+07	1.0849286E+08	1.2008268E+07	*****	*****	*****	*****	*****
153	1	9.3230530E+07	9.5679990E+07	2.6494600E+06	**	**	**	**	**
154	1	9.2778689E+07	1.0763143E+08	1.4912730E+07	*****	*****	*****	*****	*****
155	1	9.2329030E+07	1.0516857E+08	1.2839540E+07	*****	*****	*****	*****	*****
156	1	9.1881550E+07	1.0072714E+08	8.8455911E+06	*****	*****	*****	*****	*****
157	1	9.1436239E+07	9.9811427E+07	7.5751882E+06	*****	*****	*****	*****	*****
158	1	9.0993086E+07	6.4111428E+07	-2.6681658E+07	*****	*****	*****	*****	*****
159	1	9.0552081E+07	9.4319990E+07	3.7679172E+06	**	**	**	**	**
160	1	9.0113213E+07	1.0449428E+08	1.4361071E+07	*****	*****	*****	*****	*****
161	1	8.9676472E+07	1.1667571E+08	2.7199240E+07	*****	*****	*****	*****	*****
162	1	8.9241848E+07	1.1589000E+08	2.6648149E+07	*****	*****	*****	*****	*****
163	1	8.8809331E+07	1.1663420E+08	2.7824953E+07	*****	*****	*****	*****	*****
164	1	8.8378909E+07	1.1997000E+08	3.1591089E+07	*****	*****	*****	*****	*****
165	1	8.7950574E+07	1.1884714E+08	3.0046668E+07	*****	*****	*****	*****	*****
166	1	8.7524315E+07	1.2435000E+08	3.6825684E+07	*****	*****	*****	*****	*****
167	1	8.7108121E+07	1.3389571E+08	4.5995591E+07	*****	*****	*****	*****	*****
168	1	8.66784E+07	1.1828000E+08	3.16027E+07	*****	*****	*****	*****	*****
169	1	8.625892E+07	1.1866245E+08	3.2404902E+07	*****	*****	*****	*****	*****
170	1	8.5839836E+07	7.63339999E+07	-7.4998375E+06	*****	*****	*****	*****	*****

171	1	8.54107E+07	1.3365423E+08	1.82305E+07	*****
172	1	8.5009794E+07	9.9784285E+07	1.4774491E+07	*****
173	1	8.4597737E+07	9.9935712E+07	1.5387925E+07	*****
174	1	8.4107777E+07	1.1079714E+08	2.6669363E+07	*****
175	1	8.3779755E+07	1.1359714E+08	3.0117387E+07	*****
176	1	8.3373710E+07	1.1357429E+08	3.0200575E+07	*****
177	1	8.2969632E+07	1.0843714E+08	2.5467508E+07	*****
178	1	8.2567514E+07	1.1157039E+08	2.9002485E+07	*****
179	1	8.2167344E+07	1.2813000E+08	4.5962653E+07	*****
180	1	8.1769113E+07	1.1160236E+08	2.3913743E+07	*****
181	1	8.1372813E+07	7.7159949E+07	-4.2128139E+06	**
182	1	8.0978433E+07	9.4088570E+07	1.3110137E+07	*****
183	1	8.0585965E+07	1.0074571E+08	2.0159747E+07	*****
184	1	8.0195399E+07	9.0005712E+07	1.6450313E+07	*****
185	1	7.9806725E+07	1.0002424E+08	2.0217558E+07	*****
186	1	7.9419936E+07	1.0315574E+08	2.3735770E+07	*****
187	1	7.9035021E+07	9.333712E+07	1.4302121E+07	*****
188	1	7.8651972E+07	7.1094999E+07	-7.5519726E+06	***
189	1	7.8270779E+07	6.44385713E+07	6.1149344E+06	***
190	1	7.7891433E+07	6.3351427E+07	5.4599938E+06	***
191	1	7.7513926E+07	7.5327142E+07	-2.1867843E+06	**
192	1	7.7138249E+07	7.1921420E+07	-5.2168218E+06	**
193	1	7.6764392E+07	7.7012856E+07	1.0484636E+06	*
194	1	7.6392348E+07	3.3182657E+07	-4.3209491E+07	*****
195	1	7.6022106E+07	7.8944284E+07	2.9221777E+06	**
196	1	7.5653659E+07	7.0382055E+07	2.7291958E+06	**

NONLINEAR REGRESSION -- BORE 18

79/07/25. 03.28.54. PAGE 9

FILE NONAME (CREATION-GATE = 79/07/25.)

FINAL FUNCTION VALUES AND RESIDUALS

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL				
					-2	-1	0	+1	+2
197	1	7.5286998E+07	7.5297142E+07	1.0144161E+04	*	*	*	*	*
198	1	7.4922114E+07	7.5715712E+07	7.9359848E+05	*	*	*	*	*
199	1	7.4558998E+07	7.4444284E+07	-1.1471364E+05	*	*	*	*	*
200	1	7.4197642E+07	6.7602855E+07	-6.5947866E+06	***	***	***	***	***
201	1	7.3838037E+07	6.8151427E+07	-5.6866100E+06	***	***	***	***	***
202	1	7.3480175E+07	6.3952056E+07	-3.5273191E+06	***	***	***	***	***
203	1	7.3124048E+07	6.3441428E+07	-9.6022197E+06	***	***	***	***	***
204	1	7.2769646E+07	6.99448570E+07	-2.4210763E+06	**	**	**	**	**
205	1	7.2416963E+07	5.2595714E+07	-1.9021249E+07	*****	*****	*****	*****	*****
206	1	7.2065988E+07	6.2952856E+07	-9.1131321E+06	*****	*****	*****	*****	*****
207	1	7.1716715E+07	6.7544570E+07	-4.1681446E+06	***	***	***	***	***
208	1	7.1369134E+07	6.4332457E+07	-7.0362774E+06	***	***	***	***	***
209	1	7.1023238E+07	6.9946570E+07	-1.07466679E+06	*	*	*	*	*
210	1	7.0679018E+07	6.7148570E+07	-3.5304482E+06	***	***	***	***	***
211	1	7.0336467E+07	6.9945713E+07	-3.9075380E+05	*	*	*	*	*
212	1	6.9995576E+07	7.4034284E+07	4.0307884E+06	***	***	***	***	***
213	1	6.9656337E+07	7.1830571E+07	2.1622344E+06	**	**	**	**	**
214	1	6.9318742E+07	7.2631428E+07	3.2126663E+06	***	***	***	***	***
215	1	6.8982783E+07	7.2859999E+07	3.0772160E+06	***	***	***	***	***
216	1	6.8648453E+07	6.9465713E+07	8.1726045E+05	*	*	*	*	*
217	1	6.8315742E+07	6.89848570E+07	6.7282754E+05	*	*	*	*	*
218	1	6.7984645E+07	6.2394571E+07	-5.5060744E+06	***	***	***	***	***
219	1	6.7655152E+07	6.9005714E+07	1.4303620E+06	**	**	**	**	**

220	1	6.7327256E+07	6.3991427E+07	-3.3356290E+06	**
221	1	6.7080949E+07	6.4130571E+07	-2.6623707E+06	**
222	1	6.6676224E+07	6.4272857E+07	-2.4033660E+06	**
223	1	6.6353072E+07	6.6754285E+07	-4.8121219E+05	**
224	1	6.6031487E+07	6.4804285E+07	-1.2272025E+06	**
225	1	6.5711460E+07	6.2657143E+07	-3.8543177E+06	**
226	1	6.5392984E+07	6.4488571E+07	-9.0441347E+05	**
227	1	6.5076052E+07	6.8964286E+07	-4.1117660E+06	***
228	1	6.4760656E+07	6.2565713E+07	-2.1949431E+06	**
229	1	6.4446700E+07	3.5972657E+07	-2.8473932E+07	*****
230	1	6.4134442E+07	5.7982857E+07	-6.1515851E+06	****
231	1	6.3823618E+07	6.4855713E+07	1.0321035E+06	*
232	1	6.3514283E+07	6.1041428E+07	-2.4728559E+06	**
233	1	6.3206456E+07	6.2651428E+07	-5.5582846E+05	*
234	1	6.2900121E+07	6.0900000E+07	-2.0001219E+06	**
235	1	6.2595271E+07	6.2775714E+07	1.8644247E+05	*
236	1	6.2291898E+07	6.3324285E+07	1.0323860E+06	*
237	1	6.1989996E+07	6.2302856E+07	3.1286036E+05	*
238	1	6.1689556E+07	6.4124285E+07	2.4347207E+06	**
239	1	6.1390573E+07	6.1105714E+07	-2.8465902E+05	*
240	1	6.1093039E+07	6.3297142E+07	2.2041027E+06	**
241	1	6.0796947E+07	6.3243999E+07	-2.4530519E+06	**
242	1	6.0502289E+07	5.9364285E+07	-1.1380045E+06	**
243	1	6.0209068E+07	6.3874285E+07	2.8652241E+06	**
244	1	5.9917252E+07	6.1322856E+07	1.4056030E+06	**
245	1	5.9626859E+07	6.3275714E+07	3.6488547E+06	***
246	1	5.9337873E+07	6.0945714E+07	1.6078409E+06	**

NONLINEAR REGRESSION -- BORE 18

79/07/25. 08.28.54. PAGE 10

94

FILE NONAME (CREATION DATE = 79/07/25.)

FINAL FUNCTION VALUES AND RESIDUALS

CASE NO.	VAR NO.	PREDICTION	OBSERVATION	RESIDUAL	GRAPH OF RESIDUAL			
					-2	-1	0	+1
247	1	5.9050287E+07	6.3087143E+07	4.0368555E+06	***			
248	1	5.8764095E+07	6.3228570E+07	4.4644749E+06	***			
249	1	5.8479290E+07	6.0767142E+07	2.2878517E+06	***			
250	1	5.8195866E+07	6.1889571E+07	3.6927051E+06	***			
251	1	5.7913815E+07	6.0577142E+07	2.6633269E+06	***			
252	1	5.7633131E+07	5.0198572E+07	-7.4345597E+06	****			

TIME SINCE END OF THE LAST ITERATION = 1.533 SEGUNDOS
TOTAL TIME = 3.368 SECONDS

NONLINEAR REGRESSION -- BORE 18

79/07/25. 09.25.54. PAGE 11

RUN COMPLETED

NUMBER OF CONTROL CARDS READ 13

ESTIMATING FIELD INFLUENCE FUNCTIONS -- PROGRAM USAGE

The linear programming technique of Coats, Rapoport, McCord and Drews (JPT, Dec., 1964), is used to determine the aquifer influence functions from field data. Several programs have been written to take raw data and convert it into a tableau used by the M.P.O.S linear programming package, then extract the results and reformat it. The procedure is as follows:

1. INPUT DATA

The input data must follow the following arrangement:

Card No.	Information
1	Header Card
2	Input Format. Fields are integer-real-real for time-pressure-volume
3	N = number of data cards to follow
4	First Data Card (contains data for time, pressure, and volume)
N+4	Last Data Card
repeat for more data sets	

2. GENERATING THE INFLUENCE FUNCTION

The procedure to do this is simply:

GET,INFUNC. or GET,INFUNC/UN=BK005C.
INFUNC,IN.

where IN is the file containing the input data as described above.

The results will be found in file OUT and are arranged as follows:

- 1) header card
- 2) card containing number of data points and fitness value RHO in (I5,5X,F17.0) field
- 3) N data cards in the format (I5,5X,4F17.7) in order of time, F, delta-F, P, Q.
- 4) E-O-F mark
- 5) repetition of 1) to 3) for each data set

These results should be saved for future plotting. There is also a file named RESULTS generated which contains output from the M.P.O.S. package which should be sent to the line printer with carriage control.

3. PLOTTING THE INFLUENCE FUNCTION

To plot the influence functions generated above, it is necessary to use the INFPLT program. The necessary libraries and object decks will have been gotten by INFUNC. All that is necessary is to enter the command:

INFPLT,OUT

where OUT is the file generated in step 2) above. The program should be run from a graphics terminal. Output can be sent to the GERBER plotter or plotted on a 4662 plotter. The program will prompt the user for the necessary information.

1980

March 13,

Influence Function Programs

```

PROC,INFUNC,IN,OUT,RESULTS.
* -----
* THIS PROCEDURE WILL TAKE RAW TIME/PRESSURE/VOLUME DATA AND
* CALL THE PROPER PROGRAMS TO PRODUCE AN INFLUENCE FUNCTION USING
* THE LINEAR PROGRAMMING ALGORITHM OF GATS, ET AL (1964).
* -----
* CALLING SEQUENCE: BEGIN,INFUNC,JPRCC,IN,OUT,RESULTS.
* -----
* WHERE:
* -----
*   IN    FILE WITH DATA SETS IN PROPER FORMAT.
*   OUT   FILE WHICH WILL CONTAIN TIME, INFLUENCE FUNCTION, DELTA-F,
*          PRESSURE, AND VOLUME IN (15.5X17.7) FORMAT.
*   RESULT FILE WHICH WILL CONTAIN OUTPUT FROM M.P.O.S. LINEAR PROGRAMMING
*          PACKAGE.
* -----
* *CGPY,INF4,OUTPUT.
* *REVERT.
IF(FILE(JLIB,NOT,AS))GET,JLIB/UN=AET150.
LIBRARY=OSULIB/JLIB/COMPLOT.
BEGIN,GET,PROG,P,JPROGS,TABGEN,INFPLUT.
IF(FILE(MPOS,NOT,AS))ATTACH,MPOS/UN=LIBRARY.
REWIND,IN.
SETL,100.
RETURN,ZTAB.
TABGEN(IN,ZTAB)
COPY,INF1.
MPOS(ZTAB,RESULTS)
COPY,INF2.
BEGIN,REFUM,JPF0C,RESULTS,IN,OUT.
IF(FILE(Z123Z,AS))REVERT. ERROR ENCOUNTERED.
COPY,INF3.
RETURN,INF1,INF2,INF3.
REVERT,PJRC/INFUNC.
*DATA,INF1
L.P. TABLEAU GENERATED EIN LOCAL FILE <ZTAB>
ENTERING M.P.O.S.
*DATA,INF2
M.P.O.S. RUN COMPLETE. ERESULTS ARE EIN LOCAL FILE <RESULTS>
BEGIN,DATA,REFORMATTING.
*DATA,INF3
REFORMAT COMPLETE. INFLUENCE FUNCTION PLUS ORIGINAL DATA ARE NOW
EIN LOCAL FILE <OUT>. DATA SETS ARE SEPARATED BY E-O-F MARKS.
USE <APPEND,JINFUNC,OUT> TO ADD TO PREVIOUS ERESULTS,
OTHERWISE <SAVE,OUT=NEWNAME> TO SAVE THE OUTPUT.
*DATA,INF4
PROGRAMS ARE NOT WORKING TEMPORARILY. CALL JEFF AT 754-4515 OR -4599

```

```

*TABGEN
PROGRAM TABGEN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
DIMENSION P(300), Q(300), BUFF(8), FORMI(8)
DIMENSTON HDR(8)
DATA ZERO/0/, ONE/1./, ONEG/-1./, THOM/-2./, X/1HX/, V/1HV/
DATA U/1HU/, EQ/1HO/, DASH/1H-/
C
T HIS PROGRAM READS IN PRESSURE AND PRODUCTION DATA AND
CREATES INPUT FOR USE IN LINEAR PROGRAMMING ROUTINE M.P.O.S.
C
READ HEADER AND INPUT AND OUTPUT FORMATS
C
1 READ(5,101) HDR
IF.EOF(5))9922
2 READ(5,101) FORMI
101 FORMAT(8A10)
C
READ NUMBER OF DATA POINTS
READ DUMMY VARIABLE IN TIME FIELD, READ PRESSURE AND PRODUCTION DATA
C
READ(5,*) N
DO 5 I=1,N
READ(5,FORMI) IDUM, P(I), Q(I)
5 CONTINUE
C
SETUP INDEXES
C
N1 = N-1
N2 = N1+2
N3 = N1+2
N4 = 2*N1-1
C
COMPUTE DELTA-P
C
DO 10 I=2,N
10 P(I) = ABS(P(1) - P(I))
C
PRINT M.P.O.S. CONTROL CARDS
C
WRITE(6,201)
201 FORMAT(2TITLE*)
WRITE(6,101) HDR
WRITE(6,202)
202 FORMAT(2REVISED//2VARIABLES*)
PRINT (6,*) X1 TO X2,N1
PRINT (6,*) U1 TO U2,N1
PRINT (6,*) V1 TO V2,N1
WRITE(6,203) 2*N1-1
203 FORMAT(1PACKED//1MINIMIZE//1CONSTRAINTS1,I5)
WRITE(6,103) (EQ,I=1,N1), (DASH,I=1,N3)
103 FORMAT(8A1)
WRITE(6,204) FORMO
204 FORMAT(1FORMAT//(2I5,F20.5)//1READ*)
C
REWRITE DATA AS INPUT TO M.P.O.S. IN TABLEAJ FORMAT
C
WRITE OBJECTIVE FUNCTION
C
111 FORMAT(2I5,F20.5)
WRITE(6,111) (ZERO,I+N1,ONE,I=1,N2)
C
WRITE SECOND ORDER CONSTRAINTS
C
DO 20 J=1,N1
WRITE(6,111) (J,K,Q(J-K+2),K=1,J)
WRITE(6,111) J,N1+J,ONE
WRITE(6,111) J,N2+J,ONEG
WRITE(6,111) J,ZERO,P(J+1)
20 CONTINUE
C
WRITE THIRD ORDER CONSTRAINTS
C
WRITE(6,111) N1+1, N1-1, ONE
WRITE(6,111) N1+1, N1, ONEG
DO 22 J=N3,N4
K = J-N3
WRITE(6,111) J, K+1, ONE
WRITE(6,111) J, K+2, THOM
WRITE(6,111) J, K+3, ONE
22 CONTINUE

```

C
WRITE(6,111) ZERO,ZERO,ZERO
WRITE(6,265)
205 FORMAT(%CPTIMIZE%)
GO TO 1
99 REWIND 6
STOP
END

*INFPLO
 * PROGRAM INFPLOT(OUT,EXTRAP,INPUT,OUTPUT,TAPE1C=0,TAPES=INPUT,
 * TAPE8=OUT,TAPE7=EXTRAP,TAPE6=OUTPUT)

 PLOTS THE INFLUENCE FUNCTION FOR GEOTHERMAL FIELD

TIME AND VALUE OF THE FUNCTION AS GENERATED BY PROGRAM <LPM> ARE READ
 AND PLOTTED ON TEKTRONIX 4662 PLOTTER

LOGICAL UNITS FOR INPUT/OUTPUT:

LUN 8 READS INFLUENCE FUNCTION AS GENERATED BY PROCEDURE <REFORM>
 FORMAT IS ONE HEADER CARD FOLLOWED BY TIME AND FUNCTION VALUE CARDS
 IN (F5.0,5X,F17.6) FORMAT. SEPARATE DECKS SEPARATED BY --EOF--.
 DEFAULT DATA FILE NAME IS <OUT>.
 LUN 6 WRITES MESSAGES TO OPERATOR ON FILE <OUTPUT>
 LUN 7 WRITES COMPUTED EXTRAPOLATIONS ON DEFAULT FILE <EXTRAP>
 LUN 5 ASSIGNED TO FILE <INPUT> FOR RECEIVING OPERATOR INSTRUCTIONS

 INTEGER EXLAB1,EXLAB2(3),EXLAB3(3),EXLAB4(4)
 DIMENSION HDR(8), T(301), F(301), YLAB(8), BUFF(2), FOOT(8)
 DATA YES /YES/
 DATA MARK/0/
 DATA YLAB/PRESSURE RESPONSE PER UNIT VOLUME /
 DATA EXLAB1 /FIELD DATA/
 DATA EXLAB2 /ASSUMES BOUNDED AQUIFER/
 DATA EXLAB3 /ASSUMES INFINITE AQUIFER/
 DATA EXLAB4 /ASSUMES IMMEDIATELY BOUNDED AQUIFER/
 DATA FOOT /F ITNESS MEASURE AQ<v = /

 INITIALIZATION -- DETERMINE WHICH, IF ANY, EXTRAPOLATIONS ARE REQUIRED

 CALL ALPHAS
 PRINT *,FOO YOU WISH TO PLOT ON THE GERBER#
 LTYP=4
 IF(NOTES(5).EQ.0) LTYP=1
 PRINT *,ENTER NUMBER OF TIME PERIODS FOR EXTRAPOLATION#
 READ (5,*) NEXT
 IF(EOF(5).EQ.1) NEXT=0
 IF(NEXT.EQ.0) GO TO 1
 WRITE(6,104) EXLAB2
 104 FORMAT(ENTER ##YES## IF YOU DESIRE EXTRAPOLATION WHICH #,5A10)
 L2 = NOYES(5)
 WRITE(6,104) EXLAB3
 L3 = NOYES(5)
 WRITE(6,104) EXLAB4
 L4 = NOYES(5)

 READ HEADER FOR X-LABEL

 1 READ (6,101) HDR
 IF(EOF(6)) 9,10
 10 N=0
 READ(6,107) RHO
 107 FORMAT(10X,F17.7)
 IF(MARK.EQ.0) GO TO 15
 CALL TEKPAUS
 101 FORMAT(8A10)

 READ DATA

 15 N = N + 1
 READ (6,102) T(N), F(N)
 102 FORMAT(F5.0,5X,F17.6)
 IF(N.GE.301) GO TO 20
 IF(EOF(6)) 20,15
 20 N=N-1

 PLOTTING SECTION

 C PHYSICAL PROPERTIES SETUP

```

IF(MARK.EQ.1) GO TO 30
MARK=1
WIDTH=7.
HEIGHT=5.
CALL PLUTYPE(LTYP)
CALL TKTYPE(4662)
CALL BAUD(1200)
CALL SIZE(WIDTH+1.0,HEIGHT+1.0)

```

 RANGING

```

30 NINTS=4
FINCR=F(N)-F(N-1)
TINCR = T(2) - T(1)
CALL RANGE(0.,F(N)+NEXT*FINCR,NINTS,YMIN,YMAX,YTIC)
CALL RANGE(T(1), T(N)+NEXT*TINCR, 8, XMIN, XMAX, XTIC)
IF(XTIC.LT.1.0) XTIC=1.0

```

 SCALING (INCHES/USER UNIT)

```

KBIAS=2.
YBIAS=1.5
KDELT = XMAX - XMIN
XFACT=(WIDTH-XBIAS)/XDELT
YFACT=(HEIGHT-YBIAS)/(YMAX-YMIN)
CALL SCALE(XFACT,YFACT,XBIAS,YBIAS,XMIN,YMIN)

```

 GRID GENERATION AND DIMENSIONING

```

CDEB CALL ERASE
CDEB CALL ALPHAS
CDEB LSIZE=.67
CDEB PRINT(7,*)XMIN=!,XMIN,Z XMAX=!,XMAX,Z XTIC=!,XTIC,Z XFACT=!,XFACT
CDEB PRINT(7,*)YMIN=!,YMIN,Z YMAX=!,YMAX,Z YTIC=!,YTIC,Z YFACT=!,YFACT
CDEB CALL AXIS(XMIN,XMAX,XMIN,YMIN,YMAX,YTIC,0,9)
CDEB CALL AXLABL
* (1,XMAX,XMIN,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1.,YDISP)
* CALL AXLABL
* (2,XMAX,XMIN,XTIC,YMAX,YMIN,YTIC,XFACT,YFACT,CSIZE,1,1.,XDISP)

```

 WRITE AXIS LABELS

```

CSIZE = .1
NCHARS = IGNORC8(1H,HDR,1,80)
X = XDELT - SYMHID(CSIZE,NCHARS,HDR)/XFACT
K = XMIN + X/2.
CALL SYMBOL(X,-.7/YFACT,0.,CSIZE,NCHARS,HDR)
CALL FNUM(RHO,0.000001,BUFF,NCHAR)
CALL MOVEST(BUFF,1,FOOT,25,NCHAR)
CALL MOVEST(2H):,1,FOOT,25+NCHAR,2)
X = XDELT - SYMHID(CSIZE,80,FOOT)/XFACT
K = XMIN + X/2.
CALL SYMBOL(X,-.98/YFACT,0.,CSIZE,80,FOOT)

```

```

NCHARS=33
Y = YMAX - SYMHID(CSIZE,NCHARS,YLAB)/YFACT
Y = Y/2.
K = -.8/XFACT + XMIN
CALL SYMBOL(X,Y,90.,CSIZE,NCHARS,YLAB)

```

 PLOT THE VALUES

```

CALL LINE(T,F,17,N)

```

 CHECK IF EXTRAPOLATION WAS REQUESTED BY USER

```

IF(NEXT.LE.0) GO TO 1
IF(N.LT.5) GO TO 1
CALL DASHSZ(.05,.03,.07,.05)

```

C PERFORM FIRST EXTRAPOLATION -- ASSUME BOUNDED AQUIFER.
 C ALGORITHM: AVERAGE LAST THREE INTERVALS.

C
 C IF(L2.EQ.0) GO TO 42
 FDELT = (F(N)-F(N-3)) / (T(N)-T(N-3))
 C CALL DASHES
 CALL PLOT(T(N),F(N),0,0)
 WRITE(7,101) H02
 WRITE(7,103) NEXT,EXLAB2
 103 FORMAT(13,2 TIME PERIODS EXTRAPOLATION #,4A10)

C
 DO 48 I=1,NEXT
 Y = F(N) + FDELT*I
 K = T(N) + I * TINCR
 WRITE(7,102) X,Y
 CALL PLOT(X,Y,1,19)
 48 CONTINUE

C PERFORM SECOND EXTRAPOLATION -- ASSUME INFINITE AQUIFER
 C ALGORITHM: HUTCHINSON & SIKORA, 1959, P.172, EQN. 16

C
 42 IF(L3.EQ.0) GO TO 52
 CALL DASHES
 CALL PLOT(T(N),F(N),0,0)
 WRITE(7,103) NEXT,EXLAB3
 KB=1(I)
 Y = F(N)
 DELTY = TINCR
 DO 50 I=1,NEXT
 K = T(N) + I * TINCR
 XX = X - X0
 D = (XX+1) / XX
 F = XX / (XX-1)
 DELTY = DELTY + ALOG(D) / ALOG(E)
 Y = Y + DELTY
 WRITE(7,102) X,Y
 CALL PLOT(X,Y,1,27)
 50 CONTINUE

C PERFORM THIRD EXTRAPOLATION -- ASSUME IMMEDIATELY BOUNDED AQUIFER
 C ALGORITHM: USE LAST INTERVAL

C
 52 IF(L4.EQ.0) GO TO 62
 FDELT = (F(N)-F(N-1)) / (T(N)-T(N-1))
 C CALL DASHES
 CALL PLOT(T(N),F(N),0,0)
 WRITE(7,103) NEXT,EXLAB3
 C
 DO 60 I=1,NEXT
 Y = F(N) + FDELT * I
 K = T(N) + I * TINCR
 WRITE(7,102) X,Y
 CALL PLOT(X,Y,1,21)
 60 CONTINUE

C 62 CONTINUE

C
 C WRITE LABELS

C
 CALL SCALE(1.,1.,XBIAS,YBIAS,0.,0.)
 X = WIDTH/3.
 Y=HEIGHT/5.
 CALL VECTORS
 CSIZE=.87
 MM=2.
 CALL WMARK(X,Y,CSIZE,17)
 CALL SYMBOL(X,Y,0,CSIZE,10,EXLAB31)
 CALL DASHES
 IF(L2.EQ.0) GO TO 71
 X = WIDTH/3.
 Y=HEIGHT/5 - CSIZE * MM
 MM = MM+2
 CALL WMARK(X,Y,CSIZE,19)
 CALL SYMBOL(X,Y,0,CSIZE,23,EXLAB21)
 CALL DASHES

```
71 IF(L3.EQ.0) GO TO 72
  X = WIDTH/3.
  Y = HEIGHT/5. - CSIZE * MM
  MM = MM + 2
  CALL WMARK(X,Y,CSIZE,27)
  CALL SYMBOL(X,Y,0,CSIZE,24,EXLAB3)
  CALL DASHES
72 IF(L4.EQ.0) GO TO 1
  X = WIDTH/3
  Y = HEIGHT/5 - CSIZE*MM
  CALL WMARK(X,Y,CSIZE,21)
  CALL SYMBOL(X,Y,0,CSIZE,35,EXLAB4)
  GO TO 1
```

C
C END

```
99 CALL PLOTEND
999 STOP #END INFPLOT#
END
```

• PROC,REFRM,IN,DATA,CUT.

• THIS PROCEDURE WILL REFORMAT THE OUTPLT FROM M.P.O.S. LINEAR PROGRAMMING
• AND MERGE IT WITH THE TIME VALUES FROM THE ORIGINAL DATA. THE FINAL
• FORM IS DESIGNED TO BE USED WITH THE PLOTTING PROGRAM <INFPLOT>.

• FILES:

• IN CONTAINS OUTPUT FROM M.P.O.S.
• DATA CONTAINS ORIGINAL DATA USED TO GENERATE M.P.O.S. TABLEAU.
• OUT FILE WHICH WILL CONTAIN THE OUTPUT USED TO CALL <INFPLOT>

• RETURN,OUT,EDUMP,NUL,Z123Z.

• NOEXTS
• XEDIT,IN,I=EDIT1,L=EDUMP,NH.

• IFE,FILE(Z123Z,.NOT.AS),B.

• ONEEXTS

• XEDIT,NUL,I=EDIT2,L=EDUMP,NH.

• IFE(FILE(LPM,.NOT.AS))BEGIN,GETPROG,P,,LPM.

• LPM(OUT,OUT,DATA)

• RETURN,EDIT1,EDIT2,BADAT.

• REVERT,OUT.

• REVERT,PROC/REFRM. <OUT> CONVERTED.

• ENDIF(B)

• COPY,BADAT.

• RETURN,BADAT,EDIT1,EDIT2.

• REVERT,PROC/REFRM. ERROR FOUND.

• EDATA,EDIT1

• NOBELLS

• MM 81 84

• YSL/NO FEAS/;L/NO FEAS/0:COPY Z123Z 1;STOP

• YSL/Y OF R/;L/SUM/0 IN-3:CSW//INF//COPY NUL 1:N7:COPY NUL /SLACK/;L/0BJ...N/-

• STOP

• EDATA,EDIT2

• NOBELLS

• D/SLACK/

• N2

• YSL-K1 /;L/ K1 /0:N=1:HEOF:N2:-

• MM 78 84

• YSLW/---INF/;LW/---INF/0;0:-C

• Q

• EDATA,BADAT

• RUN ABORTED.

• FILE <DATA> CONTAINS A <DATA> SET FOR WHICH NO FEASIBLE SOLUTION
• CAN BE FOUND BY THE LINEAR PROGRAMMING PACKAGE.
• PLEASE EXAMINE FILE <IN> TO DETERMINE WHICH SET IS NOT FEASIBLE.

ENTERING PHASE II

ITNO. 33 IN VA-- 38 VAR- 33 TO LOWER BOUND -ZMIN= -216.307277
 1 MPO VERSION 3.2 NORTHWESTERN UNIVERSITY

60/03/13. 10.42.13.

PAGE 4

 * PROBLEM NUMBER 1 *

USING REVISED
 INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	LOWER BOUND	UPPER BOUND
1	X1	1		0.015534	0.0000000	0.1000	INF
2	X2	2		0.014569	0.0000000	0.1000	INF
3	X3	3		0.0113546	0.0000000	0.1000	INF
4	X4	4		0.0112526	0.0000000	0.1000	INF
5	X5	5		0.0111509	0.0000000	0.1000	INF
6	X6	6		0.0110470	0.0000000	0.1000	INF
7	X7	7		0.0110471	0.0000000	0.1000	INF
8	X8	8		0.008453	0.0000000	0.1000	INF
9	X9	9		0.007434	0.0000000	0.1000	INF
10	X10	10		0.006615	0.0000000	0.1000	INF
11	X11	11		0.0053396	0.0000000	0.1000	INF
12	X12	12		0.004378	0.0000000	0.1000	INF
13	X13	13		0.0033359	0.0000000	0.1000	INF
14	X14	14		0.002236	0.0000000	0.1000	INF
15	X15	15		0.0011321	0.0000000	0.1000	INF
16	X16	16		0.0011321	0.0000000	0.1000	INF
17	X17	17		0.0011321	0.0000000	0.1000	INF
18	X18	18		0.0011321	0.0000000	0.1000	INF
19	X19	19		0.0011321	0.0000000	0.1000	INF
20	U1	20		100.0000	-1.0000000	0.1000	INF
21	U2	21		0.000000	-1.0000000	0.1000	INF
22	U3	22		0.000000	-1.0000000	0.1000	INF
23	U4	23		0.000000	-1.0000000	0.1000	INF
24	U5	24		0.000000	-1.0000000	0.1000	INF
25	U6	25		0.000000	-1.0000000	0.1000	INF
26	U7	26		0.000000	-1.0000000	0.1000	INF
27	U8	27		0.000000	-1.0000000	0.1000	INF
28	U9	28		0.000000	-1.0000000	0.1000	INF
29	U10	29		0.000000	-1.0000000	0.1000	INF
30	U11	30		0.000000	-1.0000000	0.1000	INF
31	U12	31		0.000000	-1.0000000	0.1000	INF
32	U13	32		0.000000	-1.0000000	0.1000	INF
33	U14	33		0.000000	-1.0000000	0.1000	INF
34	U15	34		0.000000	-1.0000000	0.1000	INF
35	U16	35		0.000000	-1.0000000	0.1000	INF
36	U17	36		0.000000	-1.0000000	0.1000	INF
37	U18	37		0.000000	-1.0000000	0.1000	INF
38	U19	38		0.000000	-1.0000000	0.1000	INF
39	V1	39		15.982000	0.0000000	0.1000	INF
40	V2	40		25.8280040	0.0000000	0.1000	INF
41	V3	41		37.650044	0.0000000	0.1000	INF
42	V4	42		41.4257540	0.0000000	0.1000	INF
43	V5	43		53.9923934	0.0000000	0.1000	INF
44	V6	44		6.0000000	-1.0000000	0.1000	INF
45	V7	45		6.0000000	-1.0000000	0.1000	INF
46	V8	46		6.0000000	-1.0000000	0.1000	INF
47	V9	47		6.0000000	-1.0000000	0.1000	INF
48	V10	48		2.7927569	-2.0000000	0.1000	INF
49	V11	49		1.9074233	-2.0000000	0.1000	INF
50	V12	50		0.0000000	-2.0000000	0.1000	INF
51	V13	51		0.0000000	-2.0000000	0.1000	INF
52	V14	52		0.0000000	-1.3166268	0.1000	INF
53	V15	53		1.2754649	0.0000000	0.1000	INF
54	V16	54		2.0046635	0.0000000	0.1000	INF
55	V17	55		1.6040838	0.0000000	0.1000	INF

1 MPO VERSION 3.2 NORTHWESTERN UNIVERSITY

60/03/13. 10.42.13.

PAGE 5

 * PROBLEM NUMBER 1 *

USING REVISED
 INFLUENCE FUNCTION -- BORE28 -- 1956 TO 1975

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	LOWER BOUND	UPPER BOUND
47	V9	1	L3	0.0000000	-2.0000000	0.1000	INF
48	V10	2		2.7927569	0.0000000	0.1000	INF
49	V11	3		1.9074233	0.0000000	0.1000	INF
50	V12	4		0.0000000	-2.0000000	0.1000	INF
51	V13	5		0.0000000	-2.0000000	0.1000	INF
52	V14	6		0.0000000	-1.3166268	0.1000	INF
53	V15	7		1.2754649	0.0000000	0.1000	INF
54	V16	8		2.0046635	0.0000000	0.1000	INF
55	V17	9		1.6040838	0.0000000	0.1000	INF

6
 7 V16
 8 V19
 9 --SLACK
 10 --SLACK
 11 --SLACK
 12 --SLACK
 13 --SLACK
 14 --SLACK
 15 --SLACK
 16 --SLACK
 17 --SLACK
 18 --SLACK
 19 --SLACK
 20 --SLACK
 21 20
 22 23
 23 24
 24 25
 25 26
 26 27
 27 28
 28 29
 29 30
 30 31
 31 32
 32 33
 33 34
 34 35
 35 36
 36 37
 37 38
 38 39
 39 40
 40 41
 41 42
 42 43
 43 44
 44 45
 45 46
 46 47
 47 48
 48 49
 49 50
 50 51
 51 52
 52 53
 53 54
 54 55
 55 56
 56 57
 57 58
 58 59
 59 60
 60 61
 61 62
 62 63
 63 64
 64 65
 65 66
 66 67
 67 68
 68 69
 69 70
 70 71
 71 72
 72 73
 73 74
 74 75
 75 76
 76 77
 77 78
 78 79
 79 80
 80 81
 81 82
 82 83
 83 84
 84 85
 85 86
 86 87
 87 88
 88 89
 89 90
 90 91
 91 92
 92 1
 1 MFUS VERSION 3.2

NORTHWESTERN UNIVERSITY

SUMMARY OF RESULTS

VAR	VAR	POW	STATUS	ACTIVITY	OPPORTUNITY	LOWER	UPPER
NO	NAME	NO		LEVEL	CJST	BOUND	BOUND
93	--ARTIF	0	18	0000000000000000	0.0000000000000000	0.0000000000000000	INF
94	--ARTIF	1	19	0000000000000000	0.0000000000000000	0.0000000000000000	INF
95	--ARTIF	2	20	0000000000000000	0.0000000000000000	0.0000000000000000	INF
96	--ARTIF	3	21	0000000000000000	0.0000000000000000	0.0000000000000000	INF
97	--ARTIF	4	22	0000000000000000	0.0000000000000000	0.0000000000000000	INF
98	--ARTIF	5	23	0000000000000000	0.0000000000000000	0.0000000000000000	INF
99	--ARTIF	6	24	0000000000000000	0.0000000000000000	0.0000000000000000	INF
100	--ARTIF	7	25	0000000000000000	0.0000000000000000	0.0000000000000000	INF
101	--ARTIF	8	26	0000000000000000	0.0000000000000000	0.0000000000000000	INF
102	--ARTIF	9	27	0000000000000000	0.0000000000000000	0.0000000000000000	INF
103	--ARTIF	10	28	0000000000000000	0.0000000000000000	0.0000000000000000	INF
104	--ARTIF	11	29	0000000000000000	0.0000000000000000	0.0000000000000000	INF
105	--ARTIF	12	30	0000000000000000	0.0000000000000000	0.0000000000000000	INF
106	--ARTIF	13	31	0000000000000000	0.0000000000000000	0.0000000000000000	INF
107	--ARTIF	14	32	0000000000000000	0.0000000000000000	0.0000000000000000	INF
108	--ARTIF	15	33	0000000000000000	0.0000000000000000	0.0000000000000000	INF
109	--ARTIF	16	34	0000000000000000	0.0000000000000000	0.0000000000000000	INF
110	--ARTIF	17	35	0000000000000000	0.0000000000000000	0.0000000000000000	INF
111	--ARTIF	18	36	0000000000000000	0.0000000000000000	0.0000000000000000	INF
112	--ARTIF	19	37	0000000000000000	0.0000000000000000	0.0000000000000000	INF

MINIMUM VALUE OF THE OBJECTIVE FUNCTION = 216.067277

CALCULATION TIME WAS 4.1520 SECONDS FOR 93 ITERATIONS.

DATA STORAGE MEMORY = 004564(OCTAL) TOTAL MEMORY = 045003(OCTAL)

2 TOTAL TIME FOR THIS PROBL 1 WAS 5.007 SECONDS

80/03/13. 10.42.13.

PAGE 6

Linearized Free Surface Programs

*LFS

• PROGRAM LFS(INPUT=65/150,OUTPUT=65/150,LFSUM,TAPES=INPUT,
 TAPE6=OUT FUT,TAPE7,TAPE8=LFSUM)

FIELD DRAWDOWN AND PRESSURE WITH A LINEARIZED FREE SURFACE CONDITION

PROGRAM BY OREGON SYSTEMS ANALYSTS
 6660 RESERVOIR ROAD
 CORVALLIS, OREGON 97330
 PHONE: 503/929-5622

COPYRIGHT ELLIOT ZAIS AND ASSOCIATES, 1980.

THIS PROGRAM CALCULATES PRESSURE AT A FIELD POINT USING A DISCRETIZED ESTIMATE OF THE GREENS FUNCTION. THE FIELD POINT LIES ON THE STATIC EQUILIBRIUM FREE SURFACE AT SOME RADIAL DISTANCE FROM THE BOREHOLE.

THE PROGRAM READS IN ONE DATA SET AT A TIME AND WILL CALCULATE THE FIELD PRESSURE FOR UP TO SIX RADIAL DISTANCES FROM THE BORE. DATA SETS MAY BE SEPARATED BY AN E-O-R BUT NOT BY AN E-O-F. A SINGLE DATA SET CONSISTS OF CONTROL CARDS, PARAMETERS, AND TIME SERIES DATA FOR FLOW OF THE REAL OR VIRTUAL WELL BEING ANALYZED. THIS TIME-FLOW DATA MAY BE READ EITHER IN FREE-FORMAT FROM THE SAME INPUT STREAM AS THE CONTROL CARDS, OR IT MAY BE READ FROM A DESIGNATED PERMANENT INDIRECT ACCESS FILE.

A SINGLE DATA SET CONSISTS OF CONTROL CARDS AND DATA AS DESCRIBED BELOW. ORDER OF INPUT IS UNIMPORTANT EXCEPT THAT THE <BEGIN> CARD MUST COME FIRST, THE <END> CARD MUST COME LAST.

- 1) A CARD WITH THE WORD <BEGIN> IN COLS 1-5, FOLLOWED BY A TITLE CARD OF UP TO 80 CHARACTERS
- 2) A CARD WITH THE WORD <PARAMETER> IN COLS 1-10 WITH THE FOLLOWING PARAMETERS ENTERED IN FREE FORMAT (BLANK SEPARATED) AND IN ORDER. ONE OR MORE LINES (CARDS) MAY BE USED.
 - A) THE TIME INCREMENT IN SECONDS (DTEE)
 - B) THE WELL DEPTH IN METERS (D)
 - C) THE POROSITY (PHI) WHICH IS DIMENSIONLESS
 - D) THE DENSITY (RHO) IN KILOGRAMS/CUBIC METER
 - E) THE PERMEABILITY (K) IN SQUARE METERS
 - F) THE DYNAMIC VISCOSITY (MU) IN KILOGRAMS/METER-SECONDS
 - G) THE P.Z. THICKNESS (H) IN METERS
 - H) THE NUMBER OF RADII TO BE USED IN THE CALCULATIONS (MAXIMUM OF EIGHT)
 - I) THE VALUES OF THE RADII (IN METERS)

**** NOTE ****

THE P.Z. THICKNESS, ITEM G, IS USED IN THE SECOND TERM OF THE EXPANSION OF THE GREENS FUNCTIONS. IF <H>=0, THE SECOND TERM IS NOT INCLUDED IN THE GREENS FUNCTION CALCULATION.

- 3) A <DATA> CARD WHICH SPECIFIES LOCATION OF THE TIME AND FLOW DATA. THERE ARE TWO FORMS OF THE DATA CARD DEPENDING ON HOW THE INPUT IS TO BE READ.
 - A) FREE FORMAT -- FIRST READ IN A <DATA N> CARD WITH <N> EQUAL TO THE NUMBER OF DATA POINTS, E.G. <DATA 27>. ON FOLLOWING LINE OR LINES ENTER <N> DATA POINTS FOR TIME, FOLLOWED BY <N> DATA POINTS FOR FLOW.
 - B) STORED ON FILE -- READ IN A <DATA N FILE FORM (S)> CARD WHERE <FILE> IS THE FILE NAME, <FORM> IS THE INPUT FORMAT, <S> IS AN OPTIONAL INTEGER FROM 1 TO 9 WHICH TELLS THE PROGRAM TO SKIP THAT MANY CARDS BEFORE READING THE DATA. THE FILE MUST BE AN INDIRECT ACCESS PERMANENT FILE. ANY LOCAL FILE WITH THE SAME NAME WILL BE IRRETRIEVABLY LOST. DATA IN THIS CASE IS ORGANIZED IN COLUMNS AS TIME-FLOW AND MUST BE READ IN INTEGER-REAL FORMAT. AN EXAMPLE IS:

<DATA 27 LFSDAT (110,15X,F15.5) 3 >

WHICH MEANS READ 27 OBSERVATIONS OFF THE PERMANENT FILE NAMED <LFSDAT> USING THE SPECIFIED FORMAT AND SKIPPING THE FIRST 3 LINES OF THE FILE./

4) AN OPTIONAL CARD WHICH HAS THE WORD <GREEN> IN COLUMNS 1-5. THIS WILL PRINT THE VALUES OF THE GREENS FUNCTION FOR VALUES OF REAL TIME AND ALL RADII.

5) AN OPTIONAL <LINES N> CARD WHERE N IS THE MAXIMUM NUMBER OF OUTPUT LINES ON A PAGE. THE DEFAULT IS 66 LINES. IF <N> IS NOT SPECIFIED, THE DEFAULT FOR <LINES> IS N=51. N MUST HAVE THE RANGE 10-99. THIS CARD NEED ONLY BE ENTERED ONCE PER RUN.

6) AN OPTIONAL <MULT> CARD WHICH INDICATES THE PROGRAM IS TO START SUMMING PRESSURE DROPS FOR A CENTRAL POINT WHICH IS AT A DISTANCE EQUAL TO THE FIRST SPECIFIED RADIUS FOR ALL WELLS AT EACH TIME PERIOD. THIS SUMMATION WILL CONTINUE UNTIL A <MULTEND> CARD IS FOUND IN A DATA SET OR UNTIL THE END-OF-FILE IS REACHED. ADDITIONAL <MULT> CARDS ENCOUNTERED BETWEEN THE FIRST ONE AND THE <MULTEND> OR E-O-F ARE EXTRANEOUS AND ARE IGNORED.

7) AN OPTIONAL <CONVERT CODE> CARD WHICH TELLS THE PROGRAM TO CONVERT THE FLOW DATA FROM THE UNITS GIVEN TO KG/SEC. VALID ARGUMENTS FOR <CODE> ARE:

CODE:

CONVERSION FROM:

A
B
C
D
E
F
G
Z

TON/HOUR

10E6 LB S/YR

TONNE/HOUR

LB/S/HR

LBS/HOUR

LBS/DAY

1000 LBS/MONTH

EXTERNALLY SUPPLIED MULTIPLICATIVE FACTOR GIVEN AS REAL NUMBER FOLLOWING CODE LETTER <Z>.

8) AN OPTIONAL <FIELD P> CARD WHICH SPECIFIES AN INITIAL PRESSURE P IN POUNDS-PER-SQUARE-INCH (PSI). IF THIS CARD IS SUPPLIED THEN A TABLE GIVING FIELD PRESSURE OVER TIME IS PRODUCED FROM THE CALCULATED DRAWDOWNS. IF <P> IS LE. ZERO AN ERROR MESSAGE IS GIVEN AND THE OPTION IS SKIPPED.

9) AN END CARD WHICH HAS THE WORD <END> IN COLUMNS 1-3.

INPUT AND OUTPUT LOGICAL UNITS:

UNIT 5: DATA SET AND CONTROL CARD INPUT

UNIT 6: OUTPUT

UNIT 7: OPTIONAL UNIT ASSIGNED TO FILE SPECIFIED IN <FILE> CARD

UNIT 6: TEMPORARY FILE <LFSUH> ASSIGNED TO STORE HEADERS

```

REAL K, MU, CENTER(100), FIELD(8,100), FPHEAD(7), CONV(10)
REAL FP(100), Q(100), GREENS(8,100), R(8), RSQ(8), PARAMS(1E)
INTEGER NAMES(2,19), ITEST(8), JCONV(2)
INTEGER BUFF(8), IFORM(8), HDR(8), NTIME(100), FPOHED(7), GHEAD(7)
INTEGER DASHT(14), CHEAD(7), HBUF(13)
COMMON /FPAR/ DTEE, PHI, RHO, K, MU, H, M, R
EQUIVALENCE(DTEE,PARAMS)
DATA ICONV/10HZABCDEF/1, IACTIV/0/
DATA CARDS/10HBEPOGLMCF/, IACTIV/0/
DATA NAMES/#EXTERNAL#/ , #TONS/HOURS#, # #, #10E6 POUND#, #S/YEAR#,
#TONNES/HOL#, #R #, #POUNDS/YEAR#, #R #,
#POUNDS/HOU#, #R #, #POUNDS/DAY#, #, #, #1000 POJNDF, #S/MONTH#/
DATA CONV/0.0, .2520669, .014377925, .277777, 1.4377928E-8,
.12683347E-4, 5.2513947E-6, 1.7274324E-04/
DATA GRAVITY/9.801018/, THOPI/6.283183/ DAS4/14*10H-----/
DATA LINES/66/, BLANK/10H /, DOT/1H/, CF1/1.15E0122/
DATA FPHEAD
#/ FIELD PRESSURE AT SPECIFIED DISTANCE FROM BOREHOLE #/
DATA FPOHED
#/ FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE #/
DATA GHEAD
#/ VALUES OF GREENS FUNCTION AT SPECIFIED DISTANCE FROM BOREHOLE#/
DATA CHEAD
#/ FIELD DRAWDOWN AT CENTRAL POINT IN METERS OF HEAD #/
DATA FLO/#FLON#/, NOLD/0/, IBAD/0/
DATA TPH/#TPRIME#/ , IMULT/0/, CENTER/100*0.0/, NMULT/0/
GFUNC(A,B,ASQ) = (A/(B+ASQ))**1.5 11

```

```

REWIND 8
CPU = SECOND(DUM) - .025

READ A CONTROL CARD, DETERMINE WHICH TYPE IT IS AND GO TO PROPER SECTION
1 READ(5,101) ITEST
IF(EOF(5).EQ.1) GO TO 990
CALL FINDC(CARDS,9,ITEST,1,1,IFIND,ICF,IRET)
PRINT *,#IACTIV,IRET,IFIND,ICF:#,IACTIV,IRET,IFIND,ICF
CD WRITE(6,101) ITEST
IF(IRET.NE.1) GO TO 1
IF(IACTIV.EQ.0.AND.ICF.GT.2) GO TO 1
IBEG = LOC(CIBLANK,ITEST,1,80)
GO TO (801,802,803,804,805,806,807,808,809), ICF
B E P D G L M C F

CHECK FOR INACTIVE STATUS, THEN
READ THE HEADER CARD WHICH CONTAINS AN IDENTIFIER FOR THIS DATA SET
801 IF(IACTIV.EQ.1) GO TO 997
IACTIV = 1
READ(5,101) HDR
101 FORMAT(1A10)
IF(EOF(5).EQ.1) GO TO 995
WRITE(6,138)
138 FORMAT(1H1)
CALL TIME(TIM)
CALL DATE(DAT)
PZERO = 0.0
FACTOR = 0.6
IGREEN=0
ISIG = 0
GO TO 1

READ PARAMETERS IN FREE FORMAT
803 IEND=16
KK=0
3 KKK=KK+1
IF(KK.GT.IEND) GO TO 1
4 PARAMS(KK) = XPAR(1,ITEST,IBEG,80,IRET)
CD PRINT *,#IBEG,IRET,KK,PARAMS(KK):#,IBEG,IRET,KK,PARAMS(KK)
IBEG = IRET
IF(IRET.GT.0) GO TO 5
C READ ANOTHER CARD
IBEG = 1
READ(5,101) ITEST
IF(EOF(5).EQ.1) GO TO 983
GO TO 4
5 IF(KK.NE.8) GO TO 3
C ADJUST LOOP TERMINUS IF LESS THAN 8 RADIUS SPECIFICATIONS
M = PARAMS(8)
TX = M
IF(M.GT.8) M=8
IEND = 8+M
GO TO 3

READ TIME AND FLOW DATA
IF <FILE> OPTION, READ FROM FILE, ELSE READ FROM <INPUT>
804 N = XPAR(2,ITEST,1,80,IRET)
NN=N
IF(N.LE.0.OR.N.GT.100) NN=0
IF(IRET.LE.0) GO TO 996
CALL FNDWRD(3,ITEST,1,80,NAME,LEN,IRET)
IF(IRET.EQ.0) GO TO 10

DATA IS LOCATED ON A SEPARATE FILE.
THE FORMAT IS INTEGER-REAL FOR TIME-FLOW COLUMNS
GET FORMAT FROM THE <DATA> CARD
CLOSE UNIT 7, ASSIGN FILE TO UNIT 7 AND READ DATA.
CALL FNDWRD(4,ITEST,1,80,IFORM,LEN,IRET)
LENF = (LEN-1)/10 + 1
IF(IRET.LE.0) GO TO 996
NSKIP = XPAR(5,ITEST,1,80,IRET)

C ISIG=1
CALL FCLOSE(7)
CALL UGET(7,NAME)
IF(NSKIP.LE.0) GO TO 16
DO 15 I=1,NSKIP

```

```

      READ(7,101) DUMMY
15  CONTINUE
16  CONTINUE
C   READ 17,IFORM1 (INTIME(I), Q(I), I=1,N)
      IF(EOF(7).EQ.1) GO TO 992
      GO TO 1
C   ----- DATA IS LOCATED ON THE INPUT UNIT.
C   READ, IN FREE FORMAT, ALL DATA FOR TIME, FOLLOWED BY ALL DATA
C   FOR FLOW.
C   ----- 18 READ(15,*) (INTIME(I), I=1,N), (Q(I), I=1,N)
      IF(EOF(15).EQ.1) GO TO 995
      GO TO 1
C   ----- CHECK FOR FOLLOWING OPTIONAL CARDS:
C   ----- <GREEN> CARD
      885 IGREEN=1
      GO TO 1
C   ----- <LINES> CARD
      886 LINES = XPAR(2,ITEST,1,80,IRET)
      IF(IRET.LE.0) LINES=51
      GO TO 1
C   ----- <MULT> AND <MULTEND> CARD
      887 IMULT=1
      IF(ICOMPAR(7HMULTEND,1,ITEST,1,7).EQ.0) IMULT=2
      GO TO 1
C   ----- <CONVERT> CARD
      888 CALL FNDWRD(2,ITEST,1,20,JCONV,LEN,IBEG)
      IF(IBEG.EQ.0.OR.LEN.GT.1) GO TO 998
      CALL FINDC(ICONV,10,JCONV,1,1,IFIND,NAMEF,IRET)
      IF(IRET.GT.1) GO TO 998
      FACTOR = CONV(NAMEF)
      IF(NAMEF.GT.1) GO TO 37
      FACTOR = XPAR(3,ITEST,1,80,IRET)
      IF(IRET.LE.0) GO TO 998
      37 DO 36 I=1,N
      36 Q(I) = Q(I) * FACTOR
      GO TO 1
C   ----- <FIELD> CARD
      889 PZERO = XPAR(2,ITEST,1,80,IRET)
      IF(IRET.LE.0) GO TO 980
      IF(PZERO.LE.0) GO TO 980
      GO TO 1
C   ----- MANDATORY <END> CARD. START ANALYSIS.
      892 IF(IACTIV.EQ.0) GO TO 997
      IF(IACTIV.LT.0) GO TO 1
      IACTIV = 0
C   ----- CALCULATE SQUARE OF RADIUS
      IF(NN.EQ.0) GO TO 390
      DO 22 IRAD=1,M
      22 RSQ(IRAD) = R(IRAD) * R(IRAD)
C   ----- CALCULATE SINKING VELOCITY, W.
      IF(MU.EQ.0.OR.PHI.EQ.0) GO TO 994
      W = 1K * GRAVITY * RHO / (MU * PHI)
C   ----- CALCULATE GREENS FUNCTION FOR VALUES OF REAL TIME T AND ALL RADII.
      IF(RHO.EQ.0) GO TO 994
      G1 = -1. / (THOPI * PHI * RHO)
C   *****LOOP OVER ALL TIME PERIODS*****

```


WRITE(6,122) DAT, TIM

 PRINT RESULTS OF CENTRAL POINT DRAWDOWN IF IMULT > 1.
 IMULT = 0 IMPLIES NO SUMMING GOING ON
 IMULT = 1 IMPLIES SUMMATION OCCURRING
 IMULT = 2 IMPLIES FINISH SUMMATION, PRINT RESULTS, AND RESET COUNTERS
 IMULT = 3 IMPLIES E-O-F ENCOUNTERED. PRINT RESULTS AND END.

```

46 IF(IMULT.EQ.0) GO TO 50
NMULT = NMULT + 1
WRITE(6,119) HDR, R11
119 FORMAT(1X,0/10,1X,EDISTANCE TO CENTRAL POINT =#,F13.2)
IF(NOLD.NE.N.AND.NMULT.GT.1) IBAD=1
NOLD=N
IF(IMULT.EQ.1) GO TO 50
51 REWIND 8
WRITE(6,116)
NEED = NEED + N + NMULT + 8
CALL PEJECT(NEED,LINE,6)
IF(IBAD.EQ.1) PRINT *
***** ERROR: DATA SETS HAVE DIFFERENT NUMBER OF TIME STEPS
IBAD=6
DO 48 I=1,NMULT
READ(6,120) HBUF
WRITE(6,121) HBUF
120 FORMAT(13A10)
121 FORMAT(4X,13A10)
48 CONTINUE
WRITE(6,112) DASH
WRITE(6,139) CHEAD
139 FORMAT(7,7TIME#,T21,7A10)
WRITE(6,112) DASH
WRITE(6,123) (INTIME(I),CENTER(I),I=1,N)
123 FORMAT(10,10X,F13.3)
WRITE(6,112) DASH
WRITE(6,122) DAT, TIM
122 FORMAT(2REFERENCE DATE AND TIME#,A10,3X,A10)
IF(IMULT.EQ.3) GO TO 991
DO 49 KK=1,N
49 CENTER(KK) = 0.0
IMULT=0
NMULT=0
REWIND 8

```

 RETURN TO BEGINNING AND SEE IF ANOTHER DATA SET IS AVAILABLE

50 GO TO 1

 NORMAL EXIT

```

998 IF(IMULT.EQ.0) GO TO 991
IMULT=3
GO TO 51
991 CPU = -CPU+SECOND(DUM)
WRITE(6,109) CPU
109 FORMAT(1//# L.F.S. PROGRAM UTILIZED#,F6.3,# SECONDS CPU TIME#)
STOP FEND LFS

```

 ERROR EXITS

```

992 PRINT *,"***** PREMATURE END-OF-FILE ENCOUNTERED ON DATA FILE"
GO TO 1000
993 PRINT *,"***** ##END## CARD NOT FOUND. RUN ABORTED"
GO TO 991
994 PRINT *,"***** VALUE FOR RHO, MU, OR PHI IS EQUAL TO ZERO."
GO TO 1000
995 PRINT *,"***** PREMATURE END-OF-FILE ENCOUNTERED ON INPUT UNIT"
GO TO 999
996 PRINT *,"***** ERROR IN ##DATA## CARD. "
GO TO 1000
997 PRINT *,"***** ERROR. ##BEGIN## AND ##END## CARDS OUT OF ORDER"
GO TO 999
998 PRINT *,"***** ERROR IN ##CONVERT## CARD. "
GO TO 1000
980 PRINT *,"***** INITIAL PRESSURE SPECIFICATION IS ABSENT, ZERO OR"
PRINT *,"AN INTEGER. FIELD PRESSURES CANNOT BE CALCULATED."
PZERO = 0.0
GO TO 1
981 PRINT *,"***** ERROR IN ##CONVERT## CARD. NO DECIMAL POINT IN"
PRINT *,"FACTOR SPECIFICATION. "
GO TO 1000

```

```
982 PRINT *,***** NO ##BEGIN## CARD, OR OUT OF PLACE.#
  GO TO 999
983 PRINT *,***** ERROR IN PARAMETER LIST.#
  GO TO 999
984 PRINT *,***** ERROR IN CONTROL CARDS. #
  GO TO 1000
985 PRINT *,***** NUMBER OF DATA POINTS EXCEEDS MAXIMUM OF 100#
  GO TO 1000
999 PRINT *,***** RUN ABORTED#
  GO TO 991
1000 PRINT *,***** ANALYSIS ON THIS DATA SET ABORTED.#
  IACTIVE=-1
  GO TO 1
END
```

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----

L.F.S. FOR WAIRAKEI GROUP 1, 1953 TO 1976
REFERENCE DATE AND TIME: 08/05/11 17.16.36

TIME AND FLOW DATA READ FROM FILE #HLFSG1 # WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 3 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR

1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24						
DELTA-TAU:	3.156E+07	SECONDS					
FLUID LEVEL (D):	1.610E+02	METERS					
POROSITY (PHI):	4.000E-01						
DENSITY (RHO):	8.140E+02	KG/CU. METERS					
PERMEABILITY (K):	3.004E-14	SO. METERS					
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND					
P.Z. THICKNESS (H):	9.200E+02	METERS					
SINKING VELOCITY (W):	3.445E-06	METER/SECND					
7 RADIUS SPECIFICATIONS:	990.00	0.01	941.00	1692.00	1540.00	1588.00	1312.00

L.F.S. FOR WAIRAKEI GROUP 1, 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
		990.00	0.00	941.00	1692.00	1540.00	1588.00	
1953	36.779	253	9.488	.277	.097	.115	.109	.153
1954	57.382	692	15.189	.760	.260	.310	.293	.416
1955	74.396	1303	25.744	1.420	.490	.585	.552	.784
1956	28.159	3360	83.902	3.676	1.280	1.525	1.441	2.038
1957	201.320	5.092	84.089	5.567	1.938	1.310	2.182	3.098
1958	269.126	7.233	105.430	7.884	2.799	2.331	3.140	4.442
1959	461.877	10.655	164.705	11.588	4.197	4.284	4.715	6.614
1960	41.149	13.884	176.049	15.068	5.555	6.583	6.231	8.701
1961	374.660	16.680	175.002	18.042	6.822	8.861	7.639	10.595
1962	328.536	18.870	165.807	20.337	7.945	9.352	8.873	12.199
1963	373.984	21.028	176.248	22.582	9.129	10.698	10.165	13.838
1964	356.932	22.845	176.494	24.457	10.216	11.917	11.341	15.285
1965	311.598	24.132	166.341	25.755	11.119	12.909	12.304	16.414
1966	311.268	25.177	163.665	25.195	11.940	13.795	13.170	17.393
1967	280.095	25.833	154.889	27.419	12.603	14.491	13.856	16.117
1968	248.952	26.116	143.944	27.645	13.105	14.995	14.361	16.590
1969	246.311	26.243	139.157	27.714	13.521	15.399	14.770	16.938
1970	238.904	26.246	136.974	27.668	13.852	15.706	15.887	19.173
1971	258.148	26.345	138.007	27.725	14.170	16.814	16.402	19.428
1972	235.243	26.306	132.792	27.650	14.410	16.217	15.615	19.562
1973	209.163	26.081	124.225	27.376	14.538	16.304	15.716	19.559
1974	211.675	25.847	121.548	27.096	14.636	16.362	15.788	19.531
1975	206.465	25.597	119.179	26.806	14.700	16.386	15.826	19.472
1976	252.694	25.671	129.325	26.877	14.861	16.532	15.977	19.589

REFERENCE DATE AND TIME: 08/05/11. 17.16.36.

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----

L.F.S. FOR MAIRAKEI GROUP 2 , 1953 TO 1976
REFERENCE DATE AND TIME: 00/05/11 17.16.38.

TIME AND FLOW DATA READ FROM FILE #MLFSG2 * WITH FORMAT \$I10,15X,F15.2

FLOW DATA WERE CONVERTED FROM TYPE B (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.4377929E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (NIT)	29
DELTA-TAU:	3.156E+07
FLUID LEVEL (D):	2.670E+02
POROSITY (PHI):	4.000E-02
DENSITY (RHO):	6.140E+02
PERMEABILITY (K):	3.000E-14
DYNAMIC VISCOSITY (MU):	1.099E-04
PZ. THICKNESS (H):	6.200E+02
SINKING VELOCITY (W):	5.445E-02
7. RADIUS SPECIFICATIONS:	211.00 0.00 941.00 1017.00 734.00 709.00 398.00

L.F.S. FOR MAIRAKEI GROUP 2 , 1953 TO 1976

TIME	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)						
		211.00	0.00	941.00	1017.00	734.00	709.00	398.00
METERS:								
1953	63.637	5.332	8.238	.576	.499	.914	.975	2.639
1954	98.683	10.749	15.623	1.441	1.261	2.261	2.388	5.832
1955	144.864	15.613	21.473	2.476	3.162	3.761	3.980	9.020
1956	177.436	24.479	34.280	4.095	3.591	6.133	6.475	14.269
1957	102.438	23.388	31.159	5.011	4.427	7.286	7.654	15.275
1958	93.530	22.392	29.345	5.675	5.055	6.083	6.369	15.496
1959	325.356	62.437	59.212	8.325	7.404	11.897	12.679	25.309
1960	351.569	56.271	76.816	11.272	10.824	16.141	16.936	34.378
1961	313.209	61.974	61.964	13.785	12.293	19.511	20.431	39.640
1962	445.515	97.398	103.491	17.267	15.426	20.305	25.474	49.101
1963	448.913	88.614	107.804	20.731	14.965	20.941	38.244	52.218
1964	361.504	87.975	112.595	23.051	20.736	31.647	32.991	59.423
1965	315.826	84.906	106.646	24.596	22.246	33.134	34.644	59.255
1966	212.693	75.248	91.529	24.890	22.664	32.751	33.928	55.862
1967	207.113	76.269	94.532	25.574	23.397	33.200	34.338	55.893
1968	224.627	71.560	87.219	25.599	23.531	32.759	33.818	52.819
1969	258.010	71.583	86.498	25.654	23.840	32.858	33.850	52.637
1970	253.196	71.572	88.220	26.064	24.091	32.890	33.983	52.409
1971	240.284	71.493	87.494	26.230	24.292	32.940	33.934	52.268
1972	241.900	70.446	86.387	26.339	24.435	32.931	33.911	51.907
1973	233.597	69.375	84.835	26.371	24.504	32.827	33.786	51.351
1974	231.000	68.572	83.779	26.379	24.545	32.712	33.662	50.875
1975	231.724	68.097	83.218	26.390	24.584	32.627	33.553	50.556
1976	233.634	68.072	83.279	26.440	24.652	32.621	33.561	50.484

REFERENCE DATE AND TIME: 00/05/11 17.16.38.

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----

L.F.S. FOR HAIRAKEI GROUP 3 , 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11 17.16.41.

TIME AND FLOW DATA READ FROM FILE #WLFSG3 # WITH FORMAT:(I10,15X,F15.2)

FLOW DATA HERE CONVERTED FROM TYPE 6 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.63779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24						
DELTA-TAU:	3.15E+07	SECONDS					
FLUID LEVEL (D):	5.270E+02	METERS					
POROSITY (PHI):	4.00E-01						
DENSITY (RHO):	8.14E+02	KG/CU. METERS					
PERMEABILITY (K):	3.08E-14	SQ. METERS					
DYNAMIC VISCOSITY (MU):	1.099E-04	KG/METER-SECOND					
PZ. THICKNESS (H):	1.200E+03	METERS					
SINKING VELOCITY (W):	5.445E-06	METER/SEC.CND					
7 RADIUS SPECIFICATIONS:	827.00	0.00	1692.00	1017.00	360.00	533.00	727.00

L.F.S. FOR HAIRAKEI GROUP 3 , 1953 TO 1976

TIME METERS:	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE					
		(METERS OF HEAD)	1692.00	1017.00	360.00	533.00	727.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1957	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1958	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1959	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1961	4.426	0.52	0.200	0.115	0.037	0.134	0.094
1962	0.000	0.48	0.126	0.015	0.036	0.038	0.077
1963	16.463	2.38	0.820	0.071	0.173	0.570	0.412
1964	12.365	3.64	1.091	0.115	0.271	0.793	0.597
1965	12.634	4.70	1.270	0.157	0.357	0.955	0.740
1966	13.515	5.80	1.477	0.203	0.447	1.129	0.888
1967	11.613	6.54	1.546	0.241	0.512	1.209	0.970
1968	11.212	7.12	1.596	0.276	0.566	1.267	1.031
1969	10.838	7.59	1.629	0.307	0.610	1.308	1.076
1970	9.753	7.86	1.616	0.332	0.640	1.314	1.094
1971	9.084	8.03	1.590	0.353	0.660	1.346	1.097
1972	8.884	815	1.574	0.371	0.676	1.381	1.099
1973	8.478	821	1.551	0.385	0.686	1.289	1.096
1974	8.277	825	1.533	0.397	0.694	1.279	1.092
1975	7.927	826	1.509	0.407	0.698	1.264	1.083
1976	7.233	819	1.465	0.414	0.696	1.235	1.064

REFERENCE DATE AND TIME: 80/05/11. 17.16.41.

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----

L.F.S. FOR WAIRAKEI GROUP 4, 1953 TO 1976
REFERENCE DATE AND TIME: 80/05/11. 17.16.47.

TIME AND FLOW DATA READ FROM FILE #NLFSG4 # WITH FORMAT: II18,15X,F19.2

FLOW DATA HERE CONVERTED FROM TYPE B (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.4377929E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	26
DELTA-TAU:	3.15E+07 SECONDS
FLUID LEVEL (D):	3.10E+02 METERS
POROSITY (PHI):	4.00E-01
DENSITY (RHO):	8.10E-03 KG/CU. METERS
PERMEABILITY (K):	3.00E-14 SQ. METERS
DYNAMIC VISCOSITY (MU):	1.00E-04 KG/METER-SECOND
P.L. THICKNESS (H):	9.20E+02 METERS
SINKING VELOCITY (W):	2.44E-16 METER/SECOND
7 RADIUS SPECIFICATIONS:	576.00 0.00 154.00 734.00 360.00 194.00 388.00

L.F.S. FOR WAIRAKEI GROUP 4, 1953 TO 1976

TIME METERS	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)					
		576.00	0.00	154.00	734.00	360.00	194.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	30.999	7.703	3.231	1.17	0.465	1.405	2.386
1957	54.392	1.874	7.307	0.331	1.273	3.501	5.595
1958	69.891	3.249	11.192	0.616	2.266	5.739	8.762
1959	118.408	5.331	17.633	1.063	3.777	9.227	13.918
1960	102.342	6.948	20.437	1.493	5.044	11.451	16.551
1961	73.859	7.631	19.644	1.613	5.718	11.831	16.238
1962	101.034	8.755	32.248	2.213	6.643	13.412	18.459
1963	156.217	10.986	22.560	2.808	8.105	12.173	12.674
1964	232.333	14.681	41.674	3.687	10.987	23.457	33.683
1965	219.709	17.625	47.834	4.532	13.294	27.569	38.645
1966	217.918	20.853	50.664	5.361	15.308	30.631	42.098
1967	189.461	21.394	50.292	6.052	16.604	31.680	42.431
1968	142.395	21.397	45.929	6.518	16.958	30.476	39.512
1969	211.203	22.894	51.366	7.181	18.198	32.881	43.465
1970	198.020	23.989	52.844	7.762	19.159	34.214	44.937
1971	204.707	25.138	54.388	8.338	20.193	35.631	46.764
1972	196.194	25.907	55.270	8.840	20.890	36.411	47.299
1973	186.059	26.335	54.755	9.268	21.381	36.589	47.102
1974	189.273	26.700	55.111	9.670	21.848	36.974	47.455
1975	186.611	27.130	55.198	10.030	22.238	37.240	47.620
1976	182.956	27.375	55.038	10.347	22.522	37.357	47.583

REFERENCE DATE AND TIME: 80/05/11. 17.16.47.

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONCITION -----

L.F.S. FOR HAIRAKEI GROUP 5 , 1953 TO 1976
REFERENCE DATE AND TIME: 00/05/11. 17.16.49.

TIME AND FLOW DATA READ FROM FILE #NLFSG5 * WITH FORMAT:(I10,15X,F15.2)

FLOW DATA WERE CONVERTED FROM TYPE 3 (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24
DELTA-TAU:	3.156E+07 SECONDS
FLUID LEVEL (D):	3.303E+02 METERS
POROSITY (PHI):	4.000E-01
DENSITY (RHO):	8.000E+02 KG/CU. METERS
PERMEABILITY (H):	3.000E-14 SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04 KG/METER-SECOND
P.Z. THICKNESS (H):	9.000E+02 METERS
SINKING VELOCITY (W):	5.445E-06 METER/SECOND
7 RADIUS SPECIFICATIONS:	602.00 0.00 1588.00 709.00 533.00 194.00 311.00

L.F.S. FOR HAIRAKEI GROUP 5 , 1953 TO 1976

TIME (METERS)	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)					
		602.00	0.00	1588.00	709.00	533.00	194.00
1953	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1954	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1955	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1956	37.771	6.816	3.597	1.136	0.611	0.979	2.731
1957	68.065	2.186	8.355	0.393	1.668	2.614	6.544
1958	55.139	3.119	9.791	0.617	2.473	6.653	7.990
1959	67.418	4.569	13.960	0.960	3.663	5.313	6.211
1960	156.978	7.304	23.023	1.558	5.855	8.506	11.365
1961	98.000	8.445	22.503	1.961	6.987	9.676	10.616
1962	100.775	9.378	23.199	2.356	7.789	10.628	10.982
1963	220.385	12.694	35.322	3.163	10.456	14.507	19.697
1964	252.649	16.350	44.397	4.097	13.459	18.690	27.260
1965	267.156	19.807	52.259	5.086	16.386	22.550	37.656
1966	295.035	23.326	59.688	6.164	19.404	26.451	43.681
1967	297.996	26.378	64.811	7.232	22.094	29.763	50.157
1968	200.481	26.870	59.167	7.912	22.845	29.972	54.883
1969	243.564	28.467	61.258	8.672	24.029	30.274	44.885
1970	255.676	29.390	63.725	9.418	24.265	32.570	46.327
1971	248.775	30.443	64.799	10.091	26.280	33.646	46.474
1972	237.479	31.130	66.709	10.675	26.997	34.297	56.357
1973	206.392	31.071	62.066	11.698	27.121	34.673	54.496
1974	190.889	30.688	59.481	11.412	26.958	34.511	52.493
1975	166.862	29.672	55.925	11.591	26.405	32.473	49.675
1976	195.237	29.806	56.750	11.829	26.300	32.396	50.083

REFERENCE DATE AND TIME: 00/05/11. 17.16.49.

----- FIELD POINT PRESSURES WITH LINEARIZED FREE SURFACE CONDITION -----

L.F.S. FOR HAIRAKEI GROUP 6 , 1953 TO 1976
REFERENCE DATE AND TIME: 00/05/11 17.16.51.

TIME AND FLOW DATA READ FROM FILE #WLFSG6 * WITH FORMAT #I10.15X,F15.2

FLOW DATA WERE CONVERTED FROM TYPE B (10E6 POUNDS/YEAR) TO KG/SEC BY THE MULTIPLICATIVE FACTOR 1.43779290E-02

INPUT PARAMETERS:

NUMBER OF DATA POINTS (N):	24
DELTA-TAU:	3.156E+07 SECONDS
FLUID LEVEL (D):	3.380E+02 METERS
POROSITY (PHI):	4.000E-01
DENSITY (RHO):	6.14E+02 KG/CU. METERS
PERMEABILITY (K):	3.00E-14 SQ. METERS
DYNAMIC VISCOSITY (MU):	1.099E-04 KG/METER-SECOND
P.Z. THICKNESS (H):	9.200E+02 METERS
SINKING VELOCITY (W):	5.445E-06 METER/SECOND
7 RADIUS SPECIFICATIONS:	318.00 0.08 1312.00 398.00 727.00 388.00 311.00

L.F.S. FOR HAIRAKEI GROUP 6 , 1953 TO 1976

TIME METERS:	FLOW (KG/SEC)	FIELD DRAWDOWN AT SPECIFIED DISTANCE FROM BOREHOLE (METERS OF HEAD)					
		318.00	0.08	1312.00	398.00	727.00	388.00
1953	12.192	.606	1.121	.065	1.474	.190	.466
1954	40.690	2.444	4.332	.284	1.942	.816	1.899
1955	110.739	7.312	12.524	.886	5.772	2.494	5.040
1956	102.874	10.168	16.314	1.470	8.385	3.950	8.592
1957	191.134	16.648	26.867	2.512	13.742	6.581	14.077
1958	181.255	20.891	32.186	3.520	17.541	6.858	17.931
1959	194.634	24.644	36.964	4.565	20.931	11.034	21.367
1960	438.642	39.611	62.675	6.857	33.059	16.720	24.992
1961	375.163	47.110	71.094	8.837	39.926	22.013	40.766
1962	565.225	62.146	94.880	11.752	52.561	27.686	53.977
1963	643.978	86.873	134.281	16.098	73.125	38.040	74.721
1964	916.162	108.279	163.951	20.816	91.772	48.618	93.698
1965	680.202	122.183	179.607	25.235	104.708	57.427	106.762
1966	904.380	134.052	193.558	29.588	119.772	65.376	117.927
1967	820.991	139.218	196.181	33.176	121.394	70.933	123.506
1968	691.723	137.412	188.308	35.790	121.137	73.579	123.976
1969	790.872	142.005	195.150	38.583	125.326	77.181	127.306
1970	745.861	143.699	195.569	40.688	127.318	79.723	129.265
1971	754.146	145.917	197.752	43.600	129.574	82.889	131.516
1972	720.564	146.347	196.738	44.737	130.381	83.701	132.281
1973	688.694	145.555	194.128	46.125	130.105	84.642	131.986
1974	661.901	144.151	191.805	47.216	129.219	85.085	130.999
1975	666.569	143.713	190.097	48.192	128.984	85.564	130.738
1976	615.195	141.337	185.496	48.818	127.226	85.329	128.909

REFERENCE DATE AND TIME: 00/05/11 17.16.51.

L.F.S. FOR HAIRAKEI GROUP 1 , 1953 TO 1976
L.F.S. FOR HAIRAKEI GROUP 2 , 1953 TO 1976
L.F.S. FOR HAIRAKEI GROUP 3 , 1953 TO 1976
L.F.S. FOR HAIRAKEI GROUP 4 , 1953 TO 1976
L.F.S. FOR HAIRAKEI GROUP 5 , 1953 TO 1976
L.F.S. FOR HAIRAKEI GROUP 6 , 1953 TO 1976

DISTANCE TO CENTRAL POINT = 990.00
DISTANCE TO CENTRAL POINT = 211.00
DISTANCE TO CENTRAL POINT = 821.00
DISTANCE TO CENTRAL POINT = 578.00
DISTANCE TO CENTRAL POINT = 602.00
DISTANCE TO CENTRAL POINT = 318.00

TIME FIELD DRAWDOWN AT CENTRAL POINT IN METERS OF HEAD

1953 6.190
1954 13.885
1955 24.128
1956 39.516
1957 49.604
1958 57.481
1959 87.635
1960 124.219
1961 141.892
1962 176.596
1963 226.432
1964 250.494
1965 269.123
1966 275.427
1967 289.748
1968 214.867
1969 291.851
1970 295.782
1971 299.838
1972 300.953
1973 299.238
1974 296.963
1975 295.235
1976 293.079

REFERENCE DATE AND TIME: 30/05/11. 17.16.51.

L.F.S. PROGRAM UTILIZED 3.415 SECONDS CFU TIME