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## HEATING OF FIELD-REVERSED PLASMA RINGS ESTIMATED WITH TWO SCALING MODELS

J. W. Shearer

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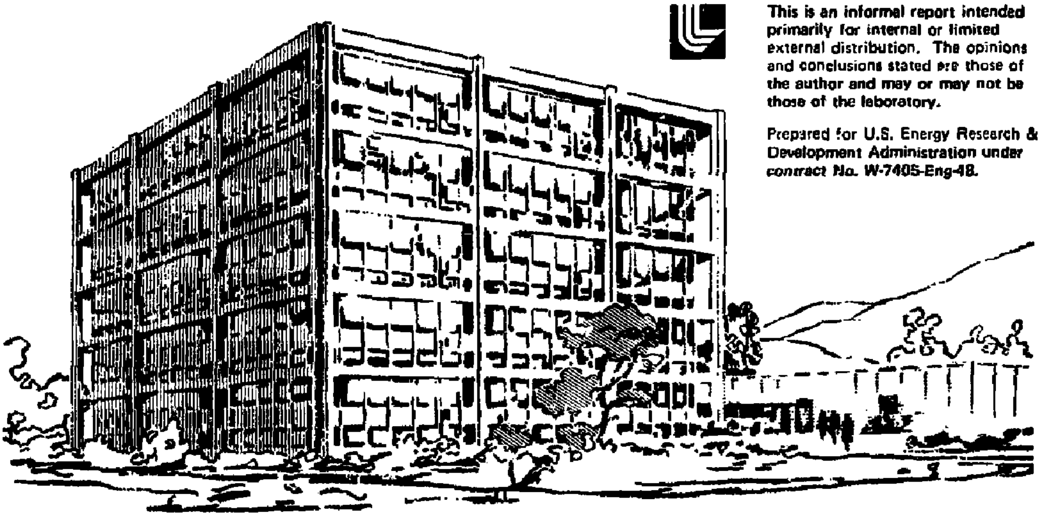



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ABSTRACT

Scaling calculations are presented of the one temperature heating of a field-reversed plasma ring. Two sharp-boundary models of the ring are considered: the long thin approximation and a pinch model. Isobaric, adiabatic, and isovolumetric cases are considered, corresponding to various ways of heating the plasma in a real experiment by using neutral beams, or by raising the magnetic field. It is found that the shape of the plasma changes markedly with heating. The least sensitive shape change (as a function of temperature) is found for the isovolumetric heating case, which can be achieved by combining neutral beam heating with compression. The complications introduced by this heating problem suggest that it is desirable, if possible, to create a field reversed ring which is already quite hot, rather than cold.

# HEATING OF FIELD REVERSED PLASMA RINGS ESTIMATED WITH TWO SCALING MODELS

J. W. Shearer

1. Introduction: The methods under consideration for producing a field reversed plasma ring (in a mirror machine) usually create a plasma whose temperature is lower than the desired fusion reaction temperatures.[1-4] The subsequent heating to elevated temperatures would then procede either via adiabatic heating[5][6] and/or neutral beam heating.[7] This report is an approximate scaling calculation of this subsequent heating process.

Suppose one starts with a stationary field-reversed plasma ring of major radius R, minor radius a, length 2L, average density n, and temperature T contained in a magnetic field B. Then one would like to know what happens to these parameters as the plasma is heated. In addition the FLR stability parameter $s \equiv a/\rho_i$ is of interest, where $\rho_i$ is the ion gyro-radius. Also, changes in the neutral beam adsorption parameter $M \equiv \int \langle\sigma\rangle n dr$ may be important for some of the cases.

Two simplified field-reversed plasma models are used to consider the gross effects of heating a plausible plasma target. The heating time is assumed to be short compared to the lifetime of the plasma ring; thus, diffusion and stability questions are ignored. The heating time is assumed to be long, however, compared to the acoustic ringing time, so that one can use equilibrium models.

2. Long Thin Model (Figure 1a).

This is a two-dimensional model in which end effects are neglected. The plasma has sharp boundaries, and $\beta = 1$. Pressure equilibrium is:

$$P = 2nkT \qquad (1)$$

in the single temperature approximation. In the absence of field diffusion, the reversed field flux $\Phi_R$ is conserved:

$$\Phi_R = \pi\ (R-a)^2\ B = (2\pi)^{3/2}\ (R-a)^2\ P^{1/2} \tag{2}$$

A third relation is needed; we choose conservation of particles N (per unit length L)

$$\frac{N}{L} = \pi\ [(R+a)^2 - (R-a)^2]\ n = 4\pi aRn \tag{3}$$

Particle conservation is a reasonable condition for the case of adiabatic heating; for neutral beam heating it is reasonable for cases where charge exchange is the dominant absorption process. Otherwise, equation (3) is an assumption of balance between particle injection and losses.

For scaling purposes equations (1)-(3) are recast in the following forms:

$$1 = \frac{n}{n_0}\ \frac{T}{T_0}\ \left(\frac{P}{P_0}\right)^{-1} \tag{4}$$

$$1 = \left(\frac{R-a}{R_0-a_0}\right) \left(\frac{P}{P_0}\right)^{1/2} \tag{5}$$



$$1 = \frac{aRn}{a_0R_0n_0} \tag{6}$$



where the initial conditions are indicated by the zero subscripts. These three equations are solved for the three unknowns a, R, and n, with P and T is written in the form:

$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^G \tag{7}$$

Figure 1a

Long Thin Model

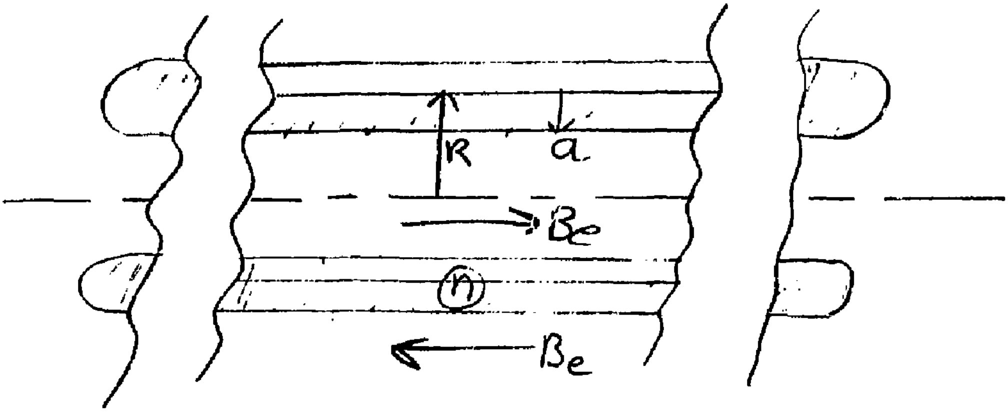
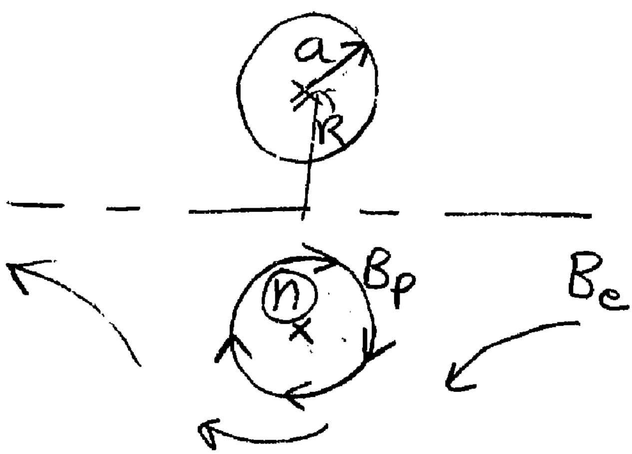



Figure 1b

Pinch Model



For adiabatic compressional heating:

$$G_{adiab.} = \gamma/\gamma-1 \tag{8}$$

On the other hand, if the pressure is held constant, then G = 0, corresponding to the case of neutral beam heating in a constant magnetic field. Intermediate values of G correspond to heating by both neutral beams and adiabatic compression.

Simultaneous solution of equations (4)-(7) leads to the following equations:

$$\frac{a}{a_0} = \frac{R_0 - a_0}{2a_0} \left(\frac{T}{T_0}\right)^{-G/4} \left[-1 + \left(1 + \frac{4a_0R_0}{(R_0-a_0)^2} \left(\frac{T}{T_0}\right)^{1-G/2}\right)^{1/2}\right] \tag{9}$$

$$\frac{n}{n_0} = \left(\frac{T}{T_0}\right)^{G-1} \tag{10}$$

$$\frac{R}{R_0} = \left(\frac{a}{a_0}\frac{n}{n_0}\right)^{-1} \tag{11}$$

from which the auxiliary variables S and M can be found.

$$\frac{S}{S_0} = \frac{a}{a_0} \left(\frac{T}{T_0}\right)^{(G-1)/2} \tag{12}$$

$$\frac{M}{M_0} = \frac{R_0}{R} = \left(\frac{a}{a_0}\right)\left(\frac{n}{n_0}\right) \tag{13}$$

These equations will be used for the examples given later in this report.

3. Pinch Model of the Ring (Figure 1b)

Conceptually this second approach is a "pinched bicycle tire" geometry; its best accuracy should be for large aspect ratio rings ($R \gg a$). Pressure equilibrium in this case corresponds to the Bennett pinch equation:

$$I^2 = 4\pi a^2 \dot{n} kT \tag{14}$$

The flux equation is derived from the expression for the inductance L of a current ring.[8]

$$L \cong 4\pi R \left\{ \left[ 1 + \frac{1}{8}\left(\frac{a}{R}\right)^2 \right] \ln\left(8\,\frac{R}{a}\right) + \frac{1}{24}\left(\frac{a}{R}\right)^2 - \frac{7}{4} \right\} \tag{15}$$

The quadratic terms are small and are neglected in this treatment; thus the flux $\Psi$ is:

$$\Psi = IL \cong 4\pi\ IR \left\{ \ln\left(8\,\frac{R}{a}\right) - \frac{7}{4} \right\} \tag{16}$$

The third equation is the conservation of particles in the volume of the torus:

$$N = 4\pi^2\ nRa^2 \tag{17}$$

This relation is analogous to equation (3) of the previous model; similar comments are applicable here.

In order to derive scaling relations similar to the previous model, one first expresses the current I in terms of the plasma pressure P:

$$I = \frac{1}{2}\, aB_p = a\,\sqrt{2\pi P} \tag{18}$$

where $B_p$ is the poloidal field at the plasma boundary. Then, by invoking the previous condition on P and T (equations (7) and (8)), one derives from equations (14-18) the following scaling equations:

$$1 = \left(\frac{n}{n_0}\right)\left(\frac{T}{T_0}\right)\left(\frac{P}{P_0}\right)^{-1} \tag{19}$$

$$1 = \left(\frac{a}{a_0}\right)\left(\frac{R}{R_0}\right)\left(\frac{P}{P_0}\right)^{1/2} \quad \frac{\ln 8\ R/a - 7/4}{\ln 8\ R_0/a_0 - 7/4} \tag{20}$$

$$1 = \left(\frac{a}{a_0}\right)^2\left(\frac{R}{R_0}\right)\left(\frac{n}{n_0}\right) \tag{21}$$

$$1 = \left(\frac{T}{T_0}\right)^G\left(\frac{P}{P_0}\right)^{-1} \tag{22}$$

The solution of these scaling equations is conveniently written in the following form:

$$\frac{T}{T_0} = \left[\frac{(\ln 8\ R_0/a_0 - 7/4)\ (a/a_0)}{\ln 8\ R_0 a_0^{\,2}/a^3 - 7/4}\right]^{2/(2 - G)} \tag{23}$$

$$\frac{R}{R_0} = \left(\frac{a}{a_0}\right)^{-2}\left(\frac{T}{T_0}\right)^{1-G} \tag{24}$$

$$\frac{n}{n_0} = \left(\frac{T}{T_0}\right)^{G-1} \tag{25}$$

from which the following auxiliary variables can be obtained:

$$\frac{S}{S_o} = \frac{a}{a_o}\left(\frac{T}{T_o}\right)^{\frac{G-1}{T}} \tag{26}$$

$$\frac{M}{M_o} = \frac{a}{a_o}\left(\frac{T}{T_o}\right)^{G-1} \tag{27}$$

In addition, one must know how the external magnetic field $B_e$ must be varied for this scaling. In this model, $B_e$ is given by:

$$B_e = \frac{a}{R} B_p = \frac{a}{R}\sqrt{8\pi P} \tag{28}$$

From which one can obtain the scaling relation:

$$\frac{B_e}{B_{eo}} = \left(\frac{a}{a_o}\right)^3 \left(\frac{T}{T_o}\right)^{\frac{3}{2}G-1} \tag{29}$$

Comparison of equation (23) with equation (29) demonstrates that there is no simple algebraic relationship between the temperature T and the external field $B_e$ in this scaling. This is a consequence of the non-linearity of the flux equation in the pinched geometry. If one defines the plasma beta in terms of the pressure ratio:

$$\beta \equiv \frac{8\pi P}{B_e^{\,2}} = \left(\frac{B_p}{B_e}\right)^2 = \left(\frac{R}{a}\right)^2 \tag{30}$$

then in this scaling model we have:

$$\frac{\beta}{\beta_o} = \left(\frac{a}{a_o}\right)^{-6} \left(\frac{T}{T_o}\right)^{2-2G} \tag{31}$$

These results are useful for adiabatic compression calculations, but they are not helpful for the case of neutral beam heating in a constant external field. For the neutral beam heating case a different set of scaling equations can be obtained. Constancy of $B_e$ implies (from equation (28)) the following scaling relation.

$$1 = \left(\frac{a}{a_0}\right)^2 \left(\frac{R}{R_0}\right)^{-2} \left(\frac{p}{p_0}\right) \qquad (32)$$

Equation (32) then replaces equation (22); the other equations (19-21) remain unchanged. The results are then:

$$\frac{a}{a_0} = 8\frac{R_0}{a_0}\left(\frac{T}{T_0}\right)^{1/3} e^{-\left\{7/4 + (\ln 8 \frac{R_0}{a_0} - 7/4)\,(T/T_0)^{-2/3}\right\}} \qquad (33)$$

$$\frac{R}{R_0} = \left(\frac{T}{T_0}\right)^{1/3} \qquad (34)$$

$$\frac{n}{n_0} = \left(\frac{a}{a_0}\right)^{-2} \left(\frac{T}{T_0}\right)^{-1/3} \qquad (35)$$

$$\frac{p}{p_0} = \left(\frac{a}{a_0}\right)^{-2} \left(\frac{T}{T_0}\right)^{2/3} \qquad (36)$$

The auxiliary variables are also simply expressed:

$$\frac{S}{S_0} = \left(\frac{T}{T_0}\right)^{-1/6} \qquad (37)$$

$$\frac{M}{M_0} = \left(\frac{a}{a_0}\right)^{-1} \left(\frac{T}{T_0}\right)^{-1/3} \tag{38}$$

$$\frac{\beta}{\beta_0} = \left(\frac{a}{a_0}\right)^{-2} \left(\frac{T}{T_0}\right)^{2/3} \tag{39}$$

Note that as the aspect ratio changes, the pinched plasma pressure changes during the heating, even though the external pressure is constant in this case.

## 4. Heating without Compression (Isobaric)

For a first example consider a field-reversed plasma immersed in a constant magnetic field whose temperature is being raised solely by neutral beam heating. Such would be the case, for example, for neutral beam heating of a field-reversed plasma confined by the magnetic field of a constant current cryogenic coil.

For the initial conditions the following plasma parameters were chosen:

$$\left.\begin{array}{l} R_0 = 10\text{cm} \\ a_0 = 2.5\text{cm} \\ s_0 = 2.5 \end{array}\right\} \quad R_0/a_0 = 4 \tag{40}$$

The large aspect ratio, $R_0/a_0$ corresponds to a generous amount of initially field-reversed flux.

In the long thin model, constancy of pressure implies $G = 0$ (equation (7)). Then for this example equations (9-13) can be written:

$$a = 3.75 \left\{ -1 + 1 + (16/9)\ (T/T_0)^{1/2} \right\}$$

$$R = (25/a)\ (T/T_0)$$

$$(n/n_0) = (T_0/T)$$

$$s = a(T_0/T)^{1/2}$$

$$(M/M_0) = (a/2.5)\ (T_0/T) \qquad (41)$$

The pinch model scaling equations for this case were just derived (equations 33-39); for these initial conditions (equation (40)), one finds:

$$a = 80\ (T/T_0)^{\frac{1}{3}}\ e^{-\left\{1.75 + (1.716)\ (T/T_0)^{-2/3}\right\}}$$

$$R = 10\ (T/T_0)^{1/3}$$

$$(n/n_0) = (6\text{[illegible]}.5/a^2R)$$

$$(P/P_0) = (n/n_0)\ (T/T_0)$$

$$S = (a)\ (n/n_0)^{1/2}\ (T/T_0)^{1/2}$$

$$(M/M_0) = (0.4a)\ (n/n_0)$$

$$\beta = (100/a^2)\ (T/T_0)^{2/3} \qquad (42)$$

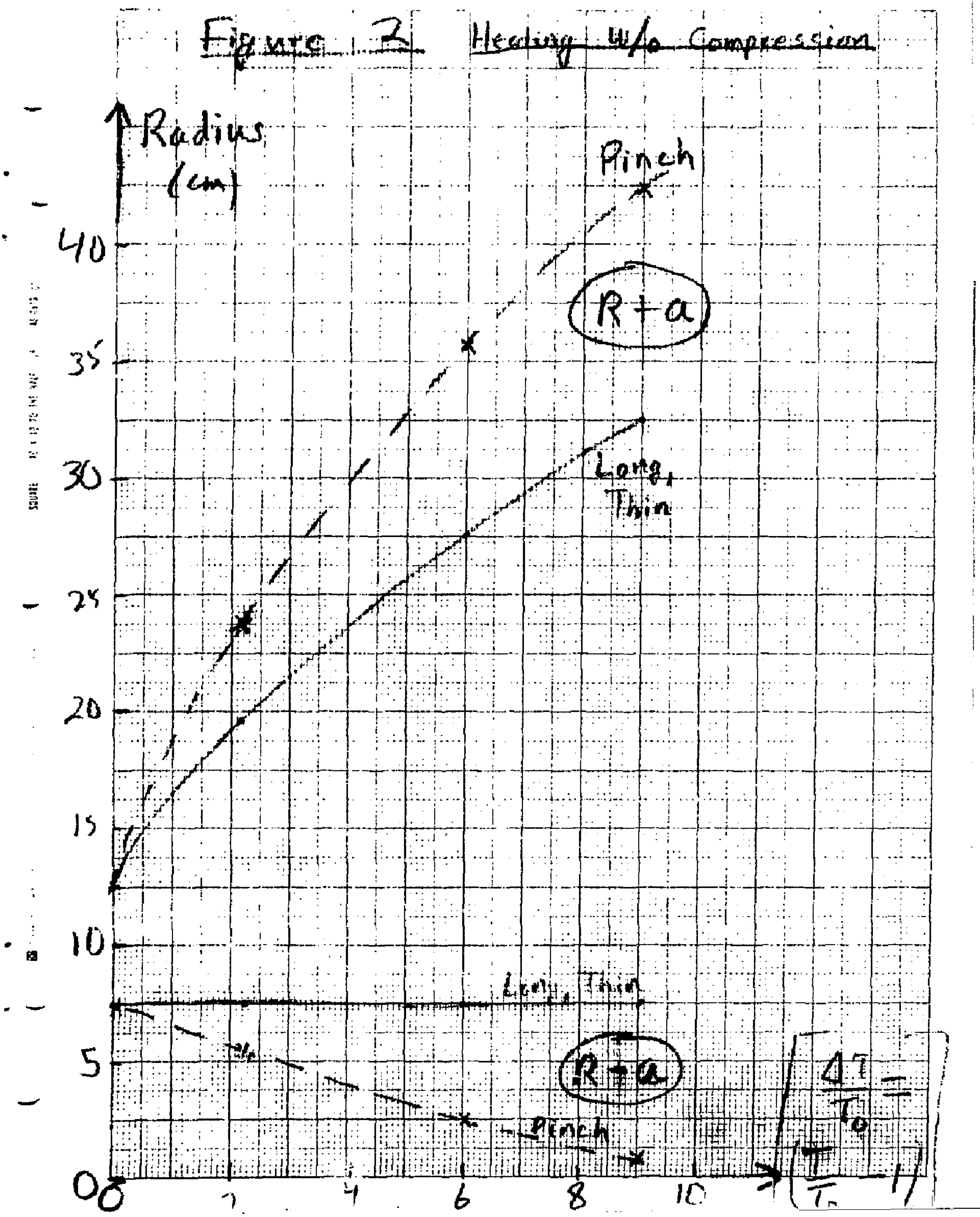
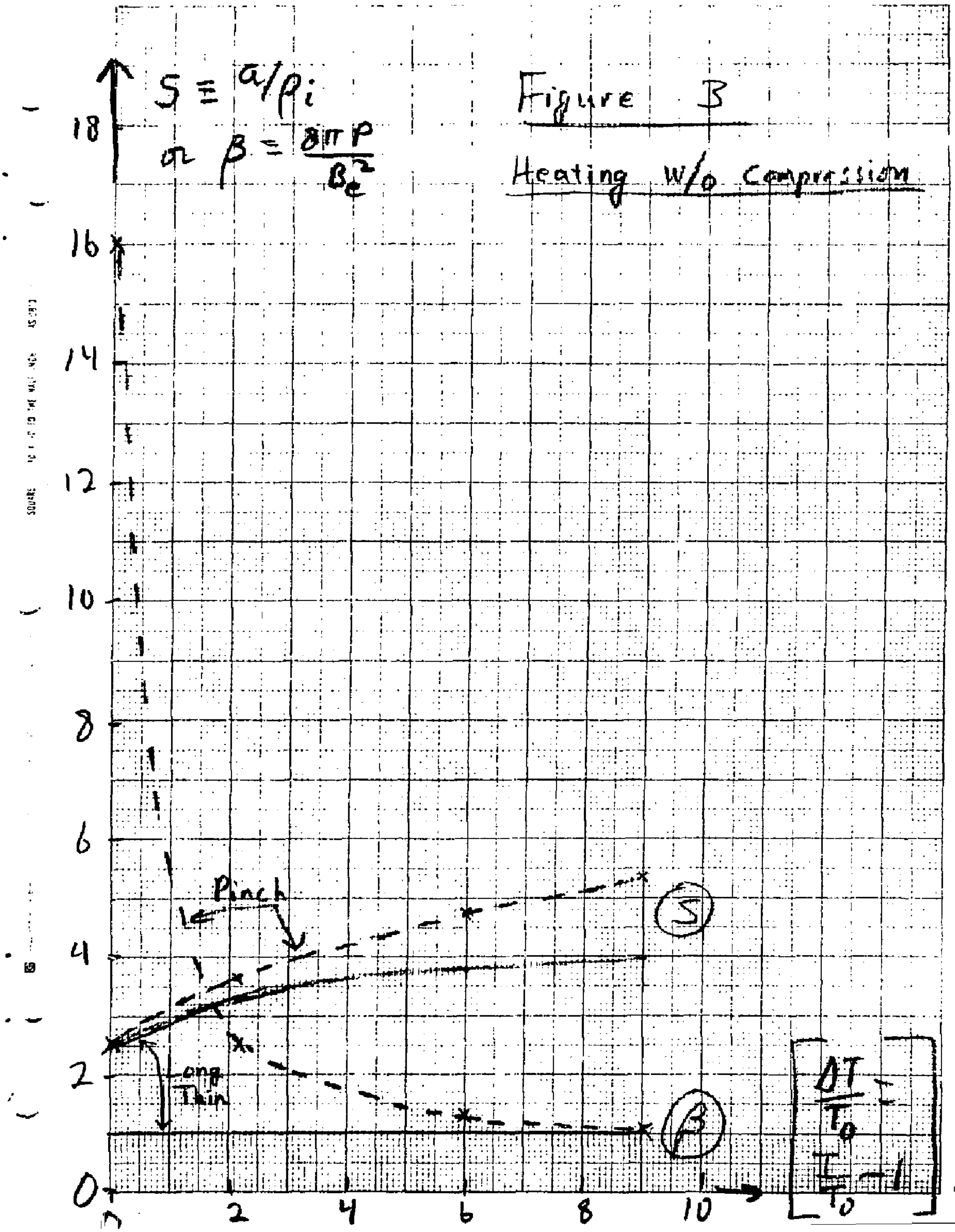


Figure 2 Heating w/o Compression
Radius (cm)
Pinch
R+a
Long, Thin
Long, Thin
R+a
Pinch
0
2
4
6
8
10
5
10
15
20
25
30
35
40
$\frac{\Delta T}{T_0} = \left(\frac{T}{T_0} - 1\right)$


S ≡ a/ρi
or β = 8πP/Be²
Figure 3
Heating w/o Compression
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
Pinch
Long Thin
S
β
ΔT/T₀
T/T₀ − 1

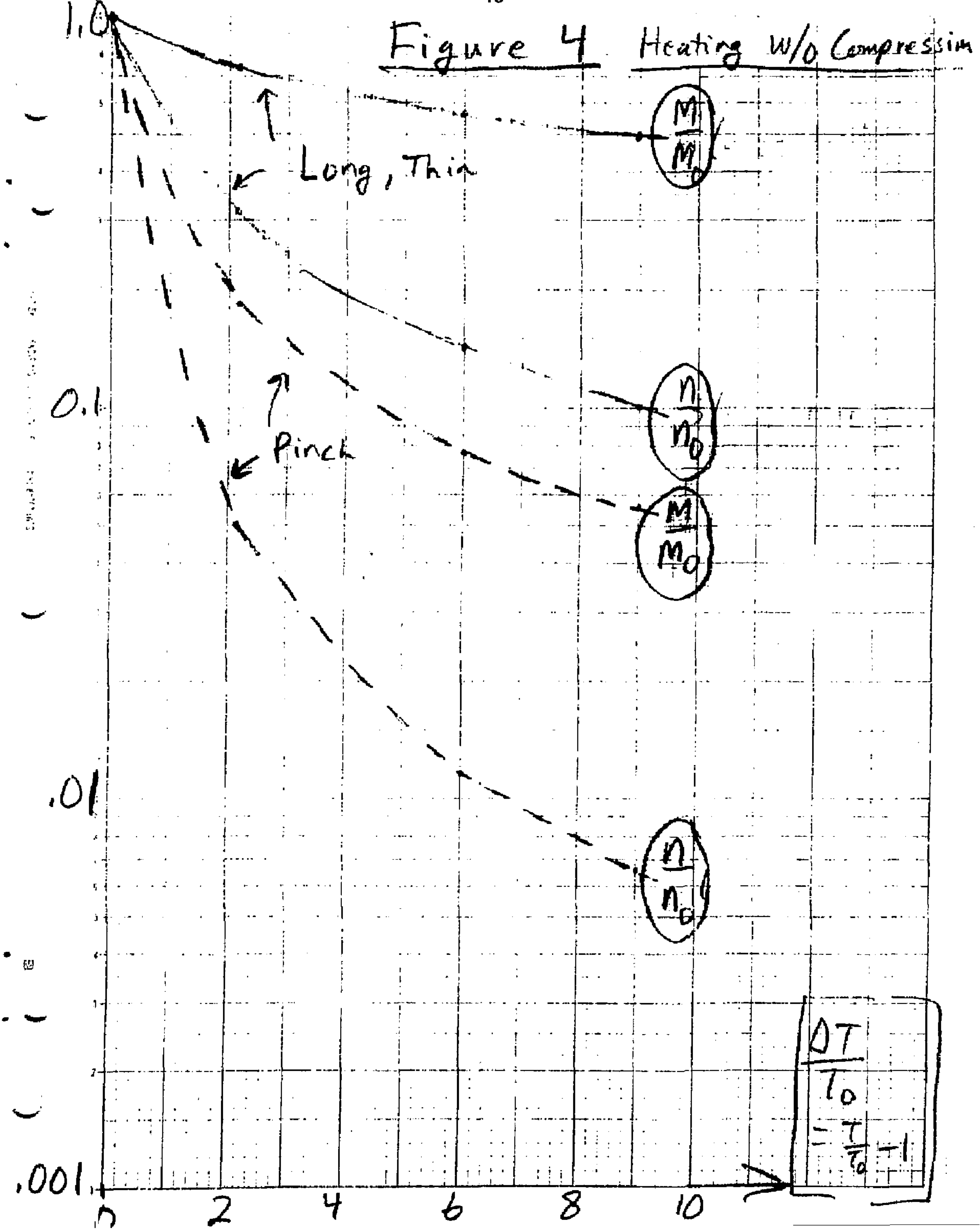


Figure 4 Heating w/o Compression
1.0
0.1
.01
.001
0
2
4
6
8
10
Long, Thin
Pinch
M/M0
n/n0
M/M0
n/n0
ΔT/T0 = T/T0 − 1

Figures 2, 3, and 4 present the results of applying equations (41) and (42) to the initial conditions of equation (40). It is found that heating the plasma lowers its aspect ratio drastically (figure 2). The change in the pinch model is especially dramatic; its plasma $\beta$ drops from $\beta_0 = 16$ to a value close to unity (figure 3). On the other hand, the FLR parameter s is rather insensitive to the heating process (figure 3). The density n and the neutral beam absorption parameter M both decrease with heating, especially for the pinch model (figure 4).

The calculations are not extended to higher temperatures ($T > 10T_0$), because it is believed that the changes in plasma parameters (such as the aspect ratio R/a) would be too great to be physically realistic. The decrease in density makes it difficult to match the absorption efficiency of the neutral beams to the plasma geometry over a wide range of temperatures. The analysis of stability questions becomes more involved.

As long as the rise in temperature with heating is not too great ($T < 10T_0$), figures (2-4) indicate that the shape changes may be tolerable.

5. Heating by Compression (Adiabatic)

As a second example consider a field-reversed plasma ring which is compressed by a slow "adiabatic" increase in the magnetic field. Such an increase might be produced by liner compression,[6] by increasing the current in external coils, or by projecting the plasma ring into a higher magnetic field region.[5] The same initial conditions are used as for the previous example (equation (40)). The specific heat ratio $\alpha$ is set equal to 5/3, corresponding to a collisional fully ionized plasma. Thus, for equation (8) one finds G = 2.5. Substitution of this value in equations (9-13) and equations (23-31) leads to a set of scaling relations similar to equations (41-42). This will be left as an exercise for the reader; the results are plotted in figures 5-7.

26 April 78

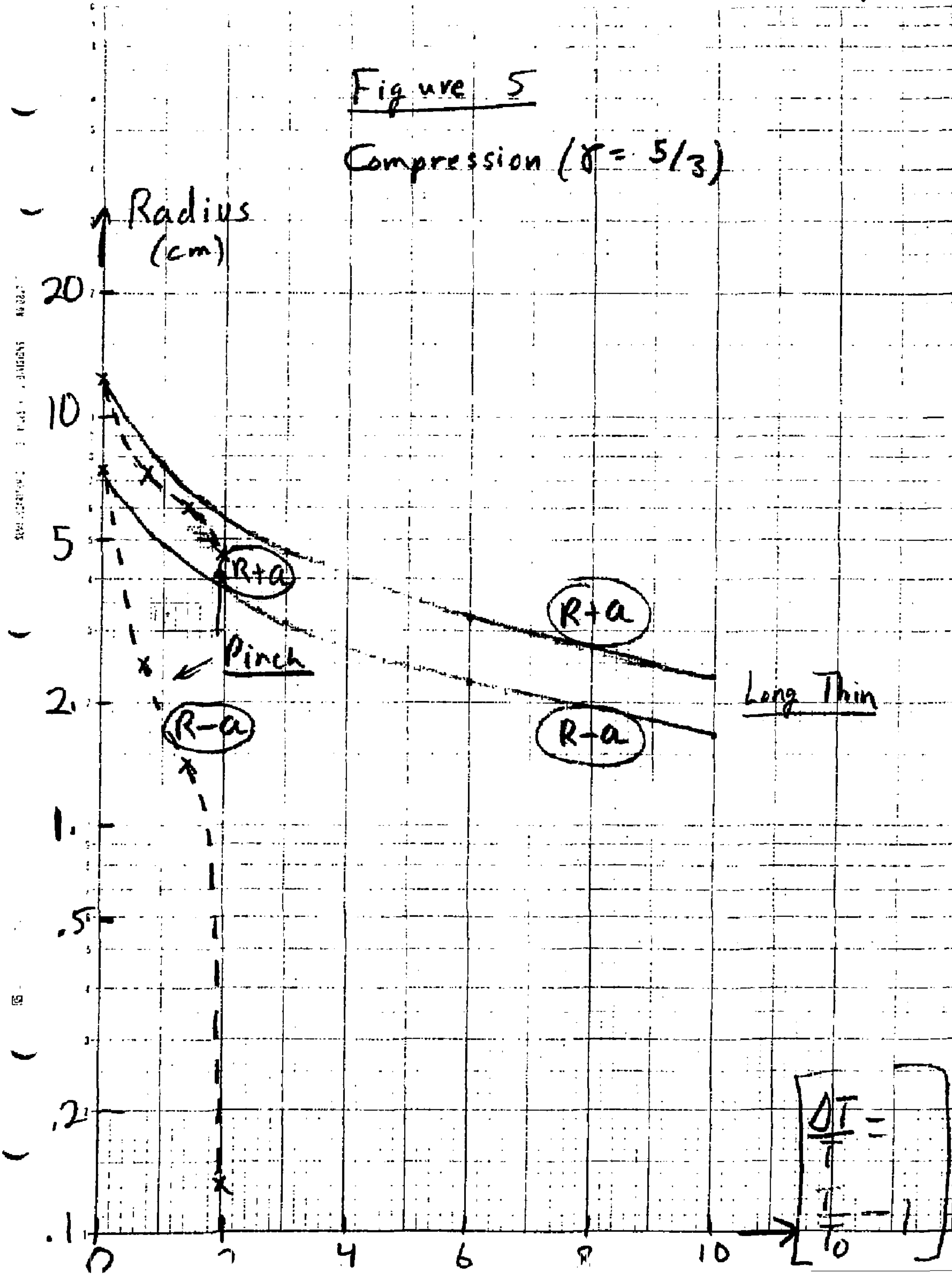



Figure 5

Compression ($\gamma = 5/3$)

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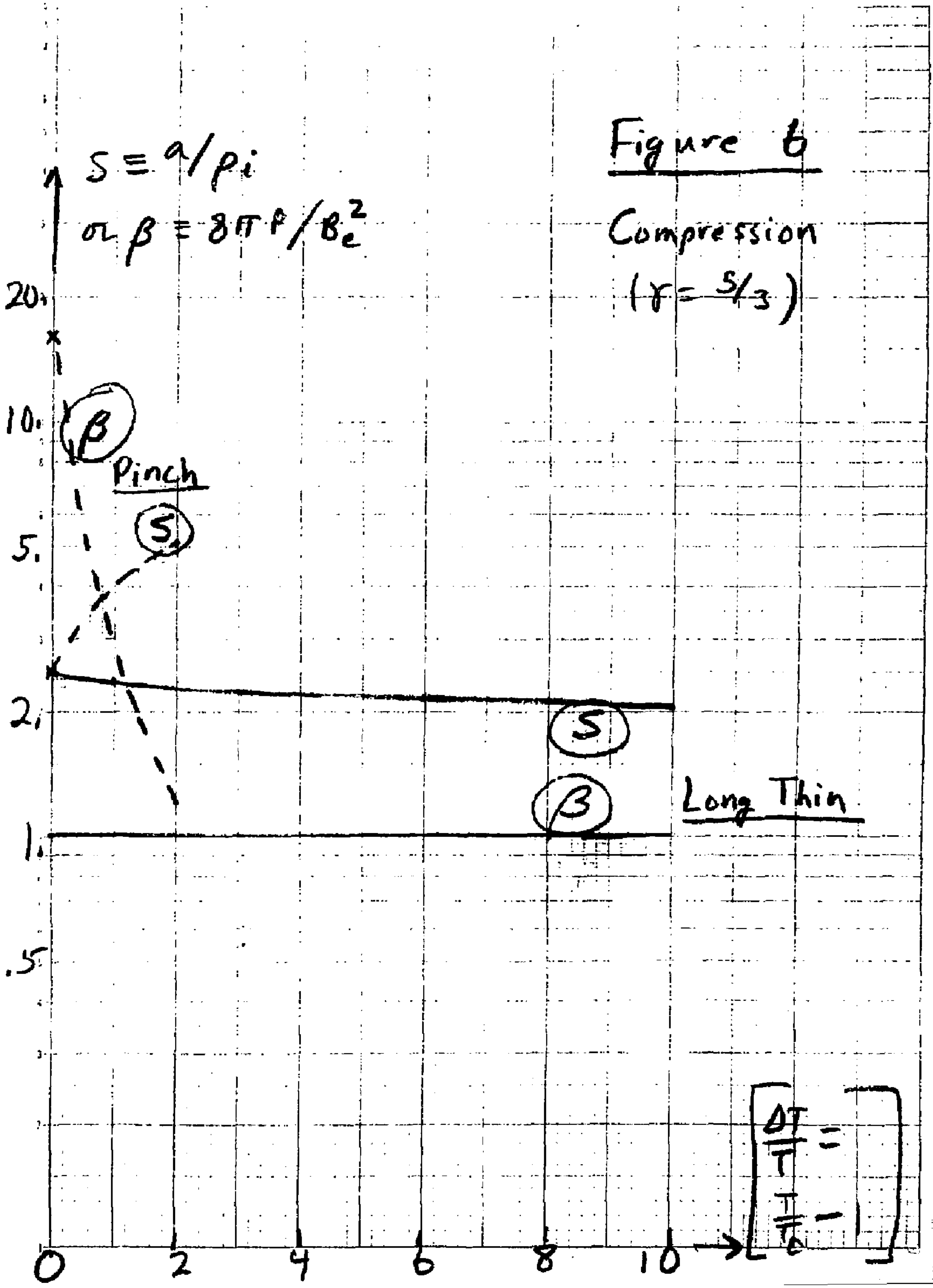


Figure 6
Compression
($\gamma = 5/3$)
$S \equiv a/\rho_i$
or $\beta \equiv 8\pi P/B_e^2$
Pinch
Long Thin
β
S
20
10
5
2
1
.5
0
2
4
6
8
10
$\frac{\Delta T}{T} = \frac{T}{T_0} -$

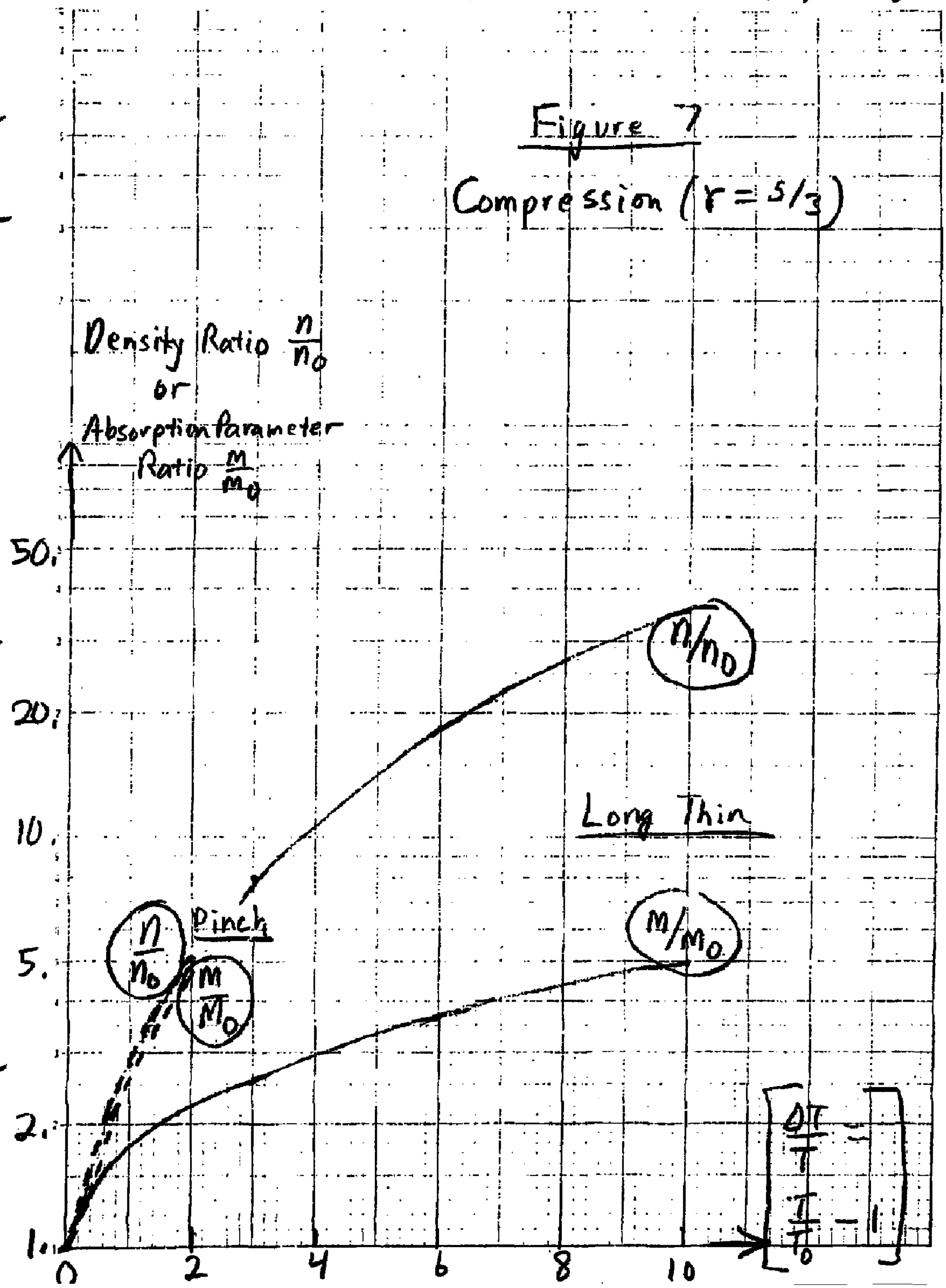


Figure 7
Compression ($\gamma = 5/3$)
Density Ratio $\frac{n}{n_0}$
or
Absorption Parameter
Ratio $\frac{M}{M_0}$
50
20
10
5
2
1
0
2
4
6
8
10
$n/n_0$
Long Thin
$M/M_0$
$\frac{n}{n_0}$
Pinch
$\frac{M}{M_0}$
$\left[\frac{\Delta T}{T} = \frac{T}{T_0} - 1\right]$

The aspect ratio of the pinch model (figure 5) decreases even more drastically in this case than it does for neutral beam heating; in fact, this model is not applicable for a temperature increase greater than a factor of three. Calculations of the external field $B_e$ (equation (29), with $G = 2.5$) show that $B_e/B_{eo} \cong 15$ when $T/T_o = 3$. Thus, this modest increase in plasma temperature requires a more than 200-fold increase in the external magnetic field pressure. As one might expect, the parameter $\beta$ (figure 6) shows a corresponding sharp drop.

The long thin model, on the other hand, is found to be well-behaved over a much wider range of temperatures and pressures, as shown in figures (5-7). The aspect ratio increases slightly, and the whole plasma shrinks. These results are in substantial agreement with recent liner compression calculations done elsewhere.(6)

It seems possible from these estimates that an initial "bicycle tire" geometry might evolve into a long thin plasma layer during compression. But this is still a speculative thought.

6. Heating at Constant Density (Isovolumetric)

In this third example both the temperature and the pressure are linearly increased together. Then $G = 1$ (equation (7)), and the density n is constant (equations (4) and (19)). This is an intermediate case to the two previous examples; experimentally, it would require programming of the neutral beam intensity and the external magnetic field so that they act together.

In this example, as in the last one, the numerical work is omitted; the pertinent results are displayed in figures 8 and 9. The density ratio $n/n_o$ is not plotted because it is unity for this case. The absorption parameter M is likewise omitted, because it changes by less than a factor of 1.5 over this temperature range.

Inspection of the results of this case show that both plasma models are

28 April 78

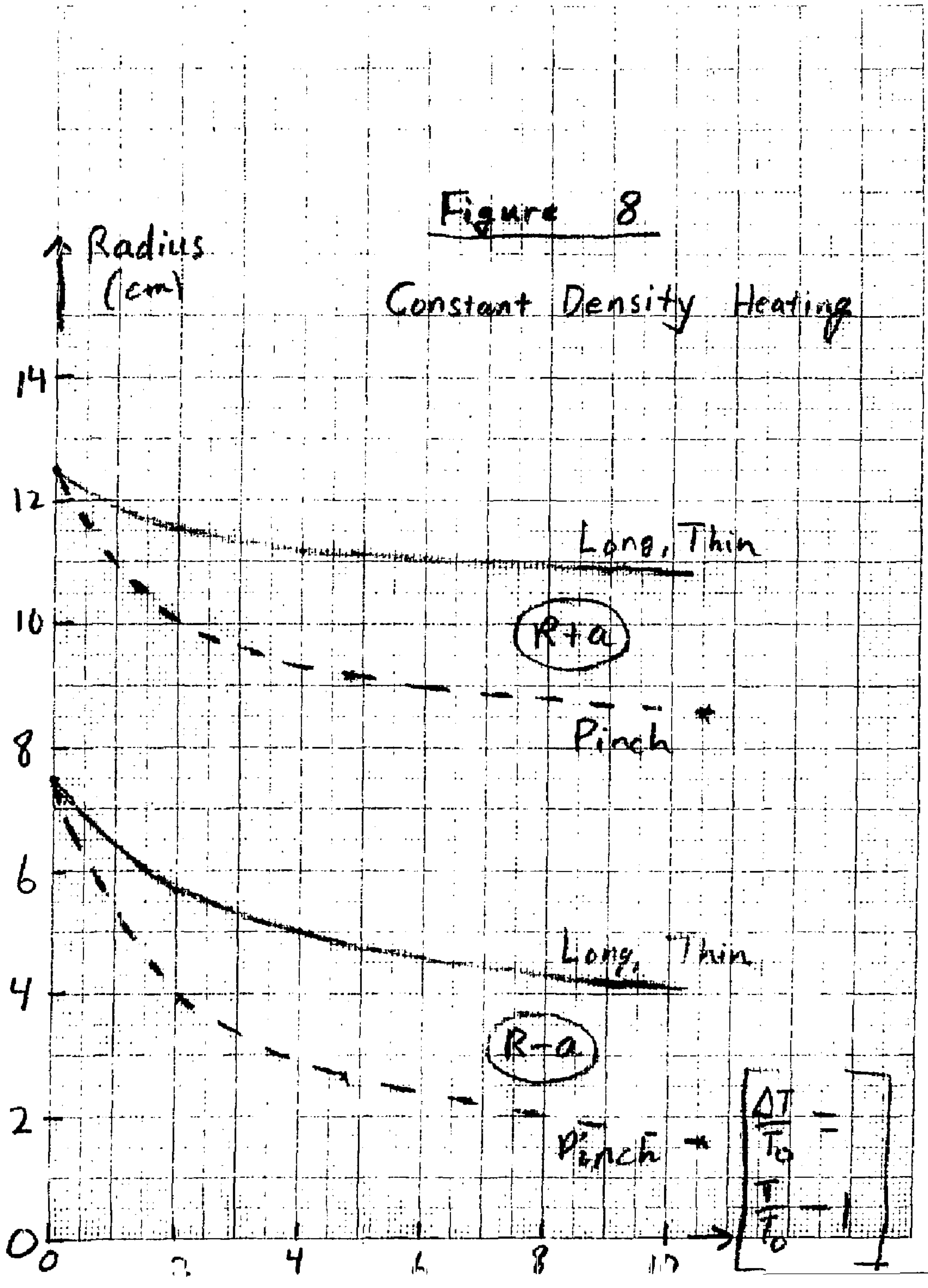
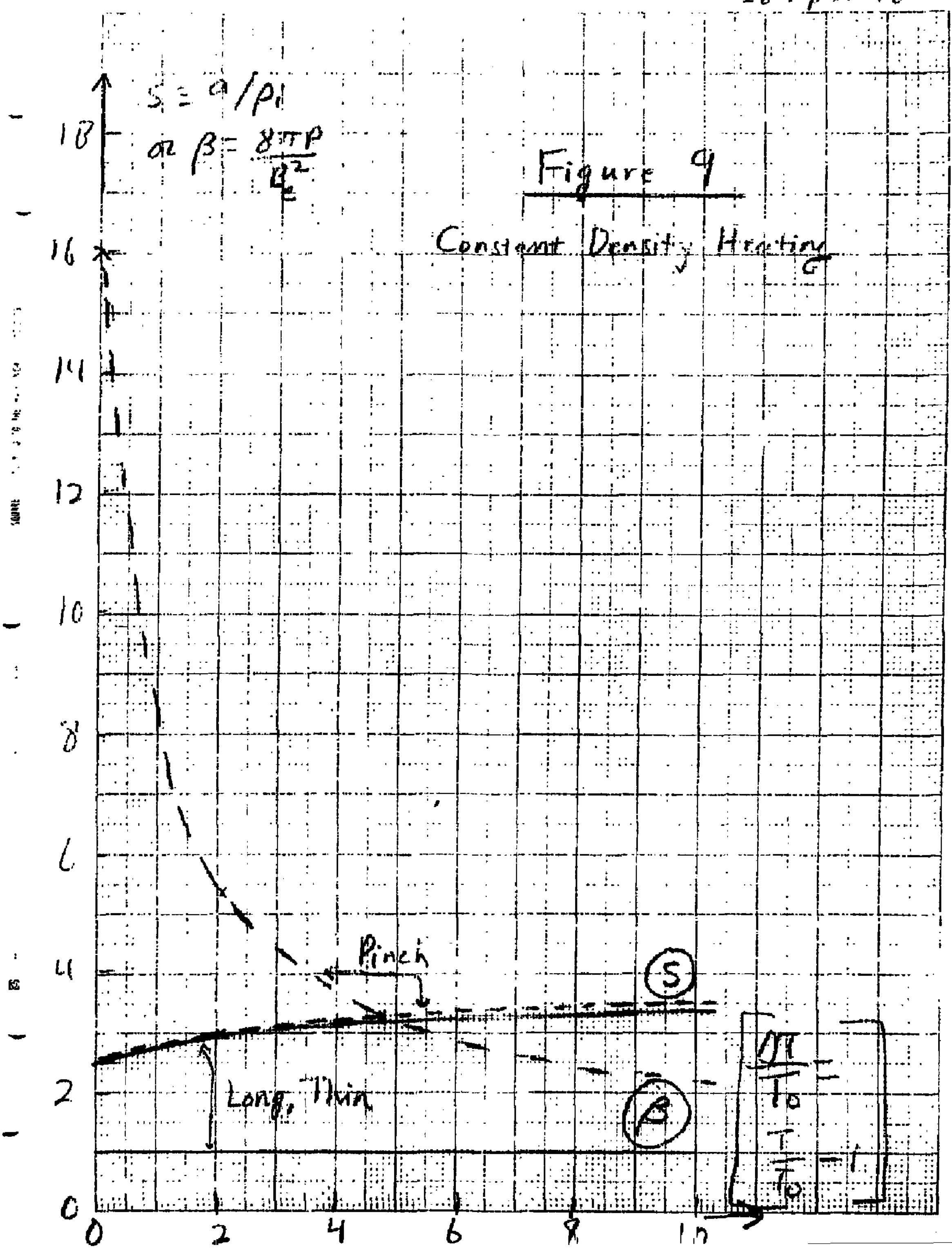


Figure 8
Constant Density Heating
Radius (cm)
14
12
10
8
6
4
2
0
Long, Thin
R+a
Pinch
Long, Thin
R-a
Pinch
0
2
4
6
8
10
[ΔT/T0 = T/T0 − 1]

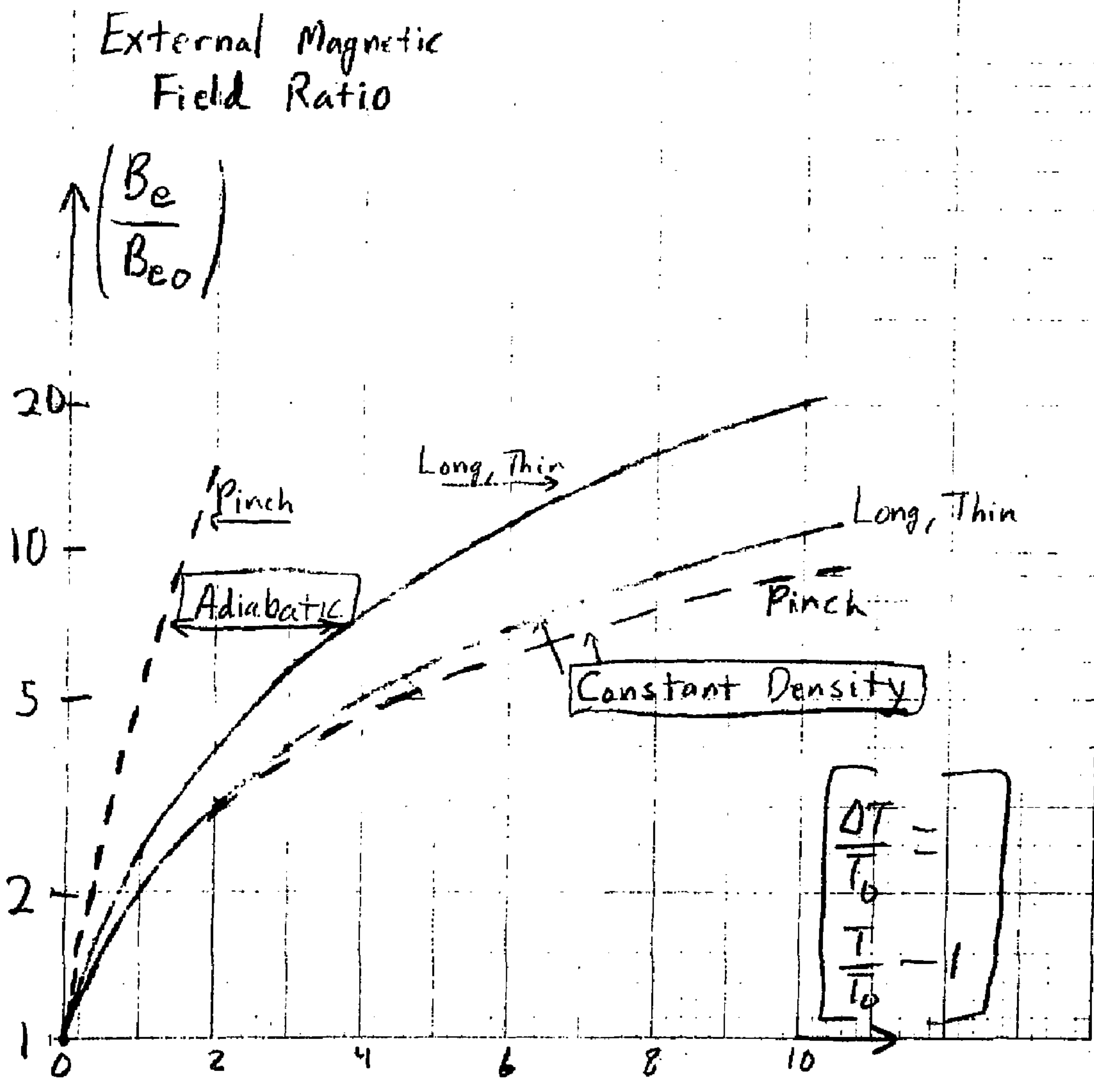
28 April '78


S = a/ρ_i
or β = 8πp/B_z^2
Figure 9
Constant Density Heating
Pinch
S
Long, Thin
β
ΔT/T_0 = T/T_0 - 1
0
2
4
6
8
10
12
14
16
18

# Figure 10

## External Magnetic Field vs. Temperature

$B_e$ vs. $\Delta T$



better behaved for the constant density case than for either of the two previous extremes. It is thus evident that it is necessary to increase both the pressure and the temperature in some programmed fashion, in order to heat the field-reversed plasma ring without drastic distortion of its shape. This example may not be the optimum programming for minimum shape distortion, but it is obviously more stable than the two previous methods of heating.

The programming of the external field that is needed for the adiabatic and constant density examples has been calculated from equations (2) and (29), and is presented in figure 10. In particular, for the constant density case, the external pressure ($\propto B_e^2$) is seen to vary approximately as the square of the temperature for both plasma models. The practical problems of raising the field over such a range would have to be studied for each experimental configuration.

## 7. Conclusions

Although these scaling relations are approximations to the real geometry of a field-reversed ring, it is hoped that they will bracket the true behavior. An additional simplification in the analysis is the neglect of any possible toroidal magnetic field ($B_\theta$).

The results indicate that heating a field-reversed plasma ring by neutral beam injection in a constant external magnetic field is possible over a modest temperature range ($T \lesssim 10T_o$), but that achievement of larger final temperatures ($T \gtrsim 10T_0$) appears problematical due to distortion of the shape of the plasma ring (changes in the aspect ratio R/a). If larger temperature ratios are required, it appears to be necessary to raise the external magnetic field as well as the temperature.

Experimentally, the simplest approach appears to be to try to create the field-reversed plasma ring at a sufficiently high temperature so that it can be heated in a constant external magnetic field. This implies that the temperature of the trapped plasma ring should be greater than one tenth of the final desired temperature.

## 8. Acknowledgements

I have benefited from discussions with James Eddleman, Charles Hartman, Tom Simonen, Adrian Smith, and William Turner.

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