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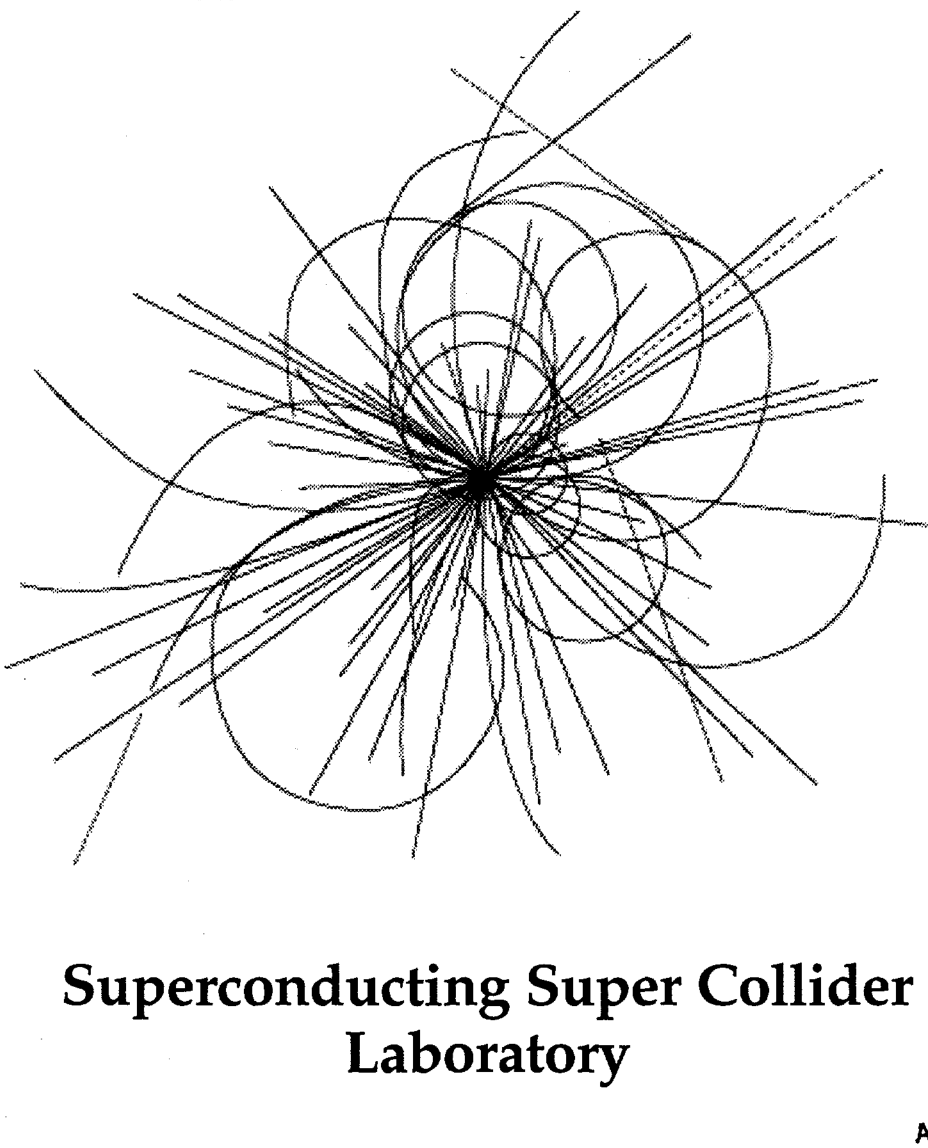

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Emittance Growth Due to Power Ripple in a Hadron Collider*

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MASTER

EMITTANCE GROWTH DUE TO POWER RIPPLE IN A HADRON COLLIDER

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Abstract. Emittance growth of the beam caused by power supply ripple in the Superconducting Super Collider (SSC) is studied. We find that due to nonlinear terms in the equation of the betatron oscillations, the emittance grows only during an initial transient period, after which its increase saturates at some level. An analytical formula is derived that gives the fractional increase of the emittance as a function of the amplitude of the ripple. Numerical estimates for the SSC are presented.

1 INTRODUCTION

In this paper, we consider the effect of magnetic field oscillations in an accelerator on the emittance growth of the beam. Small fluctuations of the field are usually produced by the ripple in the power supplies. For the Superconducting Super Collider (SSC), these oscillations are expected to occur at frequencies equal to an integer times 60 Hz, with the maximum amplitude of the ripple at 720 Hz.[1] Since the revolution frequency in the SSC is as low as 3.4 kHz, it might happen that one of the ripple frequencies resonates with the betatron oscillation of the beam. In this case, the betatron oscillations being amplified would cause a blowup of the beam. In the linear theory, such a resonant growth proceeds without limitations and results in a constant emittance growth rate. However, due to a small nonlinearity of the betatron oscillations, the tune for a given particle is a function of its amplitude, and the increase of the latter would produce detuning from the resonant frequency and cessation of amplitude growth. From this qualitative picture we expect a saturation of the emittance growth due to nonlinearity of the machine. A quantitative theory of such an effect is developed, allowing one to find the tolerances for the ripple in power supplies based on the allowable increase of the beam emittance.

Typically, the expected amplitude of the fluctuation of the magnetic field at the SSC is very small, $\delta B/B = 10^{-8}$ – 10^{-9} (Reference 2). This makes it extremely difficult to simulate the nonlinear saturation of the emittance growth using existing computer codes. On the other hand, the smallness of the oscillating component of the magnetic field

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allows us to develop an analytic perturbation theory in parameter $\delta B/B$. Mathematically, this problem is very similar to that of nonlinear Landau damping in plasma[3] and nonlinear damping of a sound wave in a liquid with gas bubbles.[4] We will use below the approach developed in Reference 4.

In Section 2, we derive a Hamiltonian governing the particle betatron oscillations near the resonance, with the ripple accounting for the nonlinearity of the motion. In Section 3, the equilibrium distribution function of the beam is found, corresponding to the regime in which the oscillations have saturated. Using this distribution function we calculate the total increase of the emittance of the beam. Results of computer simulations presented in this section confirm our analytic formulas and also show the transient process leading to the equilibrium. In Section 4, we formulate the applicability conditions of our theory and present estimates for the allowable amplitude of the ripple for the SSC.

2 SINGLE-PARTICLE DYNAMICS IN THE PRESENCE OF NONLINEARITY AND MAGNETIC-FIELD PERTURBATION

We start from a linear equation that describes particle motion in the presence of a time-dependent perturbation of the magnetic field in one of the dipole magnets. This equation is given by

$$\frac{d^2\eta}{d\zeta^2}+\eta=\lambda(t)\sum_{m=-\infty}^{\infty}\delta(\zeta-m\mu-\zeta_0) , \tag{1}$$

where $\eta = y/\sqrt{\beta\beta_0}$, y is the transverse offset of the particle, β_0 is the beta function at the position of the magnet, $\mu = 2\pi\nu$, and the function $\lambda(t)$ is proportional to the strength of the perturbed magnetic field δB,

$$\lambda(t)=\frac{\delta B(t)l}{B\rho}, \tag{2}$$

where B is the magnetic field on the orbit and ρ is the bending radius. In Eq. (2), we have assumed that the length l of the dipole magnet is much shorter than the betatron wavelength. This allows us to model the additional force acting on the particle from the perturbed magnetic field by a series of kicks given by the periodic delta function on the right-hand side (rhs) of Eq. (1). The betatron phase ζ in Eq. (1) plays the role of a time variable and changes from $-\infty$ to $+\infty$, each μ interval corresponding to a complete turn. Let us also assume that

$$\lambda(t) = \lambda_0 \cos(\omega t), \tag{3}$$

where ω is the frequency of the ripple.

In what follows, we will utilize the linear dependence t versus ζ, $t = \zeta / \nu\Omega$, where Ω is the revolution frequency. This assumption is justified for accelerators with a large tune, $\nu \gg 1$, because nonlinear terms in the relation between t and ζ are small in the parameter ν^{-1}. With such a relation, $\lambda(t)$ in Eq. (1) can be considered as a function of ζ,

$$\lambda(\zeta) = \lambda_0 \cos\left(\frac{\omega}{\nu\Omega}\zeta\right). \tag{4}$$

Our consideration will be based on the Hamiltonian formulation of the problem. Defining a conjugate momentum $p = d\eta/d\zeta$, Eq. (1) can be written as a pair of Hamiltonian equations:

$$\dot{p} = -\frac{\partial H}{\partial \eta}, \quad \dot{\eta} = \frac{\partial H}{\partial p}, \tag{5}$$

where the dot denotes the derivative with respect to ζ, and the Hamiltonian H is

$$H(p,\eta) = \frac{1}{2}p^2 + \frac{1}{2}\eta^2 - \eta\lambda_0 \cos\left(\frac{\omega}{\nu\Omega}\zeta\right)\sum_{m=-\infty}^{\infty}\delta(\zeta - m\mu - \zeta_0). \tag{6}$$

The next step is to transform to the action-angle variables J and ϕ of the unperturbed Hamiltonian according to

$$\eta = \sqrt{2J}\cos(\phi + \zeta), \quad p = -\sqrt{2J}\sin(\phi + \zeta), \tag{7}$$

which gives

$$H(J,\phi) = -\sqrt{2J}\lambda_0 \cos(\phi + \zeta)\cos\left(\frac{\omega}{\nu\Omega}\zeta\right)\sum_{m=-\infty}^{\infty}\delta(\zeta - m\mu - \zeta_0). \tag{8}$$

Now we assume that the ripple frequency ω is close to one of the sideband betatron harmonics; that is, for some n, the parameter $\Delta = \left(\nu \pm n \pm \frac{\omega}{\Omega} \right)$ is much smaller than the tune, $\Delta/\nu \ll 1$. This allows us to use the method of averaging for the Hamiltonian (8) (see, for example, Reference 5), keeping in it the only harmonic of the perturbation that resonates with the beam,

$$H(J,\phi) \to -\sqrt{\frac{J}{2}} \frac{\lambda_0}{\mu} \cos\left(\phi + \frac{\Delta}{\nu} \zeta + \phi_0 \right), \tag{9}$$

where $\phi_0 = \pm n \zeta_0 / \nu$ is the initial phase. It is convenient now to choose, instead of the phase coordinate ϕ, a new variable $\varphi = \phi + \frac{\Delta}{\nu} \zeta + \phi_0$, which gives rise to a time-independent Hamiltonian,

$$H(J,\varphi) = \frac{\Delta}{\nu} J - \sqrt{\frac{J}{2}} \frac{\lambda_0}{\mu} \cos \varphi. \tag{10}$$

At this point we will introduce into the problem a small nonlinearity that generates a weak dependence of the tune versus the action. Far from machine resonances, the only effect of the nonlinearity is that it makes ν a function of J. For the sake of simplicity, consider here a linear dependence, $\delta \nu = \alpha \nu J$, usually produced by the sextupole and octupole components of the magnetic field. It is easy to see that this kind of nonlinearity is accounted for by adding a quadratic term to our Hamiltonian:

$$H(J,\varphi) = \frac{\Delta}{\nu} J + \frac{1}{2} \alpha J^2 - \sqrt{\frac{J}{2}} \frac{\lambda_0}{\mu} \cos \varphi. \tag{11}$$

The coefficient α may have either a positive or negative sign on a particular machine.

Due to the effect of nonlinearity, different particles in the beam will acquire different tunes. In the limit when the amplitude of the ripple is small enough, only those particles that have the tune equal to the resonant one will effectively interact with the perturbation. The value of the action, $J = J_r$, which corresponds to these resonant particles, can be found from the condition that total tune of the particle is equal to a sideband harmonic of the ripple, $\nu + \delta \nu = \pm n \pm \omega/\Omega$, which gives

$$J_r = -\frac{\Delta}{\nu\alpha}. \tag{12}$$

By definition, J_r must be positive, so that Δ has to be negative for positive α and *vice versa*; otherwise, the ripple does not resonate with any particle in the beam. For particles having J in the vicinity of J_r (we will see below that those particles make the dominant contribution to the emittance growth), one can expand the Hamiltonian in the difference $y = J - J_r$ with the result

$$H(y,\varphi) = \frac{1}{2}\alpha y^2 - \sqrt{\frac{J_r}{2}}\frac{\lambda_0}{\mu}\cos\varphi. \tag{13}$$

As follows from Eq. (13), we have reduced our problem of the nonlinear particle motion in the vicinity of the resonance to the Hamiltonian of the pendulum.[5] This Hamiltonian has the familiar phase portrait shown in Figure 1, with the separatrix dividing the closed trajectories from the periodic ones. In the next section, we will apply this Hamiltonian to the evolution of the distribution function of particles in the beam.

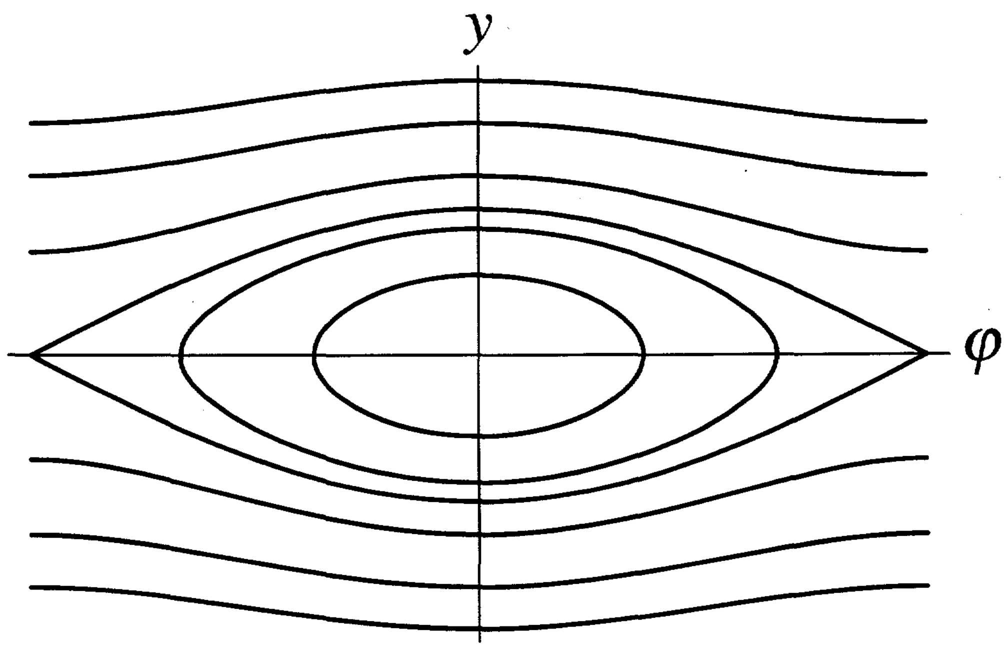

FIGURE 1. Phase portrait of the Hamiltonian (13) for positive α.

3 DISTRIBUTION FUNCTION OF THE BEAM

We start from the initial Gaussian distribution function ψ_i of the beam particles, which in terms of the action J has the form,

$$\psi_i = \frac{1}{2\pi J_0} e^{-J/J_0}, \tag{14}$$

where J_0 gives the width of the distribution in J. The parameter J_0 can be expressed as $J_0 = \varepsilon_0/\beta_0$, where ε_0 is the initial emittance of the beam.

Due to the lasting perturbation of the magnetic field $\delta B(t)$, this function will be changing with time. To find $\psi(t)$ one has to solve the appropriate Vlasov equation with the initial condition given by Eq. (14). A distinctive feature of this solution is that after a transient period the distribution function arrives at an equilibrium state.[3,4] In this paper, we restrict our attention to a simpler problem of determining the final distribution function ψ_f corresponding to this equilibrium, and we also give an estimate of the transient time. Knowledge of ψ_f allows us to calculate the ultimate increase of emittance due to the ripple.

Because of the assumed smallness of the perturbation, the distribution function is perturbed only in the vicinity of the resonant value J_r. Furthermore, the amount by which ψ is perturbed in this region is also relatively small. Taking this into account, we consider the difference

$$\delta\psi = \psi_f - \psi_i, \tag{15}$$

and since we are interested in the region of small y, expand ψ_i around the resonant value J_r:

$$\psi_i = \psi_i(J_r) + y\psi_i', \tag{16}$$

where $\psi_i' = (d\psi_i/dJ)\big|_{J=J_r}$.

The final equilibrium distribution function ψ_f can be easily found without invoking the Vlasov equation if one notices that according to the ergodicity principle, in the equilibrium the distribution function is a function of the integrals of motion. In our case, that means that ψ_f must be a function of the Hamiltonian (13), $\psi_f = \psi_f(\mathsf{H})$. (To avoid confusion, we use the symbol H in the argument of ψ_f and denote by H the function given by Eq. (13).) The dependence $\psi_f(\mathsf{H})$ is given by the following formula:[4]

$$\psi_f(\mathsf{H}) = \frac{\int \psi_i \delta(\mathsf{H} - H(y,\varphi))dyd\varphi}{\int \delta(\mathsf{H} - H(y,\varphi))dyd\varphi}. \tag{17}$$

Instead of integrating over y, we first perform integration over H, using the relation $dy = dH(\partial H/\partial y)^{-1} = dH(\dot{\varphi})^{-1}$. This eliminates δ-functions in Eq. (17) and gives

$$\psi_f(\mathsf{H}) = \frac{\int \psi_i \frac{d\varphi}{\dot{\varphi}}}{\int \frac{d\varphi}{\dot{\varphi}}} = \psi_i(J_r) + \psi_i' \frac{\int y \frac{d\varphi}{\dot{\varphi}}}{\int \frac{d\varphi}{\dot{\varphi}}}, \tag{18}$$

where Eq. (16) has been used. In Eq. (18), the integration is performed along the trajectories of the Hamiltonian (13) over one period of motion in the plane y, φ. Noting that $\dot{\varphi} = \partial H/\partial y = \alpha y$, for closed trajectories inside the separatrix we obtain $\int y d\varphi/\dot{\varphi} = \alpha^{-1}\oint d\varphi = 0$, and the second term on the rhs of Eq. (18) vanishes. That means that for $\mathsf{H} < a$, where

$$a = \lambda_0 \sqrt{J_r/2}/\mu, \tag{19}$$

one obtains

$$\psi_f(\mathsf{H}) = \psi_i(J_r). \tag{20}$$

Outside of the separatrix, $\mathsf{H} > a$, we have from Eq. (13),

$$y = \pm\left(\frac{2}{\alpha}\right)^{1/2} (\mathsf{H} + a\cos\varphi)^{1/2}, \tag{21}$$

which gives

$$\frac{\int y \frac{d\varphi}{\dot{\varphi}}}{\int \frac{d\varphi}{\dot{\varphi}}} = 2\pi \left(\frac{2}{\alpha}\right)^{1/2} \left[\int_{-\pi}^{\pi} d\varphi (\mathsf{H} + a\cos\varphi)^{-1/2} \right]^{-1}. \tag{22}$$

Combining Eqs. (15), (18), and (19) one finds the following expression for the perturbation of the distribution function:

$$\delta\psi = \psi_i' \left\{ 2\pi \left(\frac{2}{\alpha}\right)^{1/2} \left[\int_{-\pi}^{\pi} d\varphi (H + a\cos\varphi)^{-1/2} \right]^{-1} - y \right\}, \quad H > a, \tag{23}$$

$$\delta\psi = -\psi_i' y, \quad H < a.$$

The graph of the function $\delta\psi$ is shown in Figure 2. As a simple analysis shows, $\delta\psi$ tends to zero in the limit $\mathsf{H} \gg a$, which means that, indeed, the perturbation of the distribution function is localized in the region $|y| \sim (a/\alpha)^{1/2}$ in accordance with our original assumption.

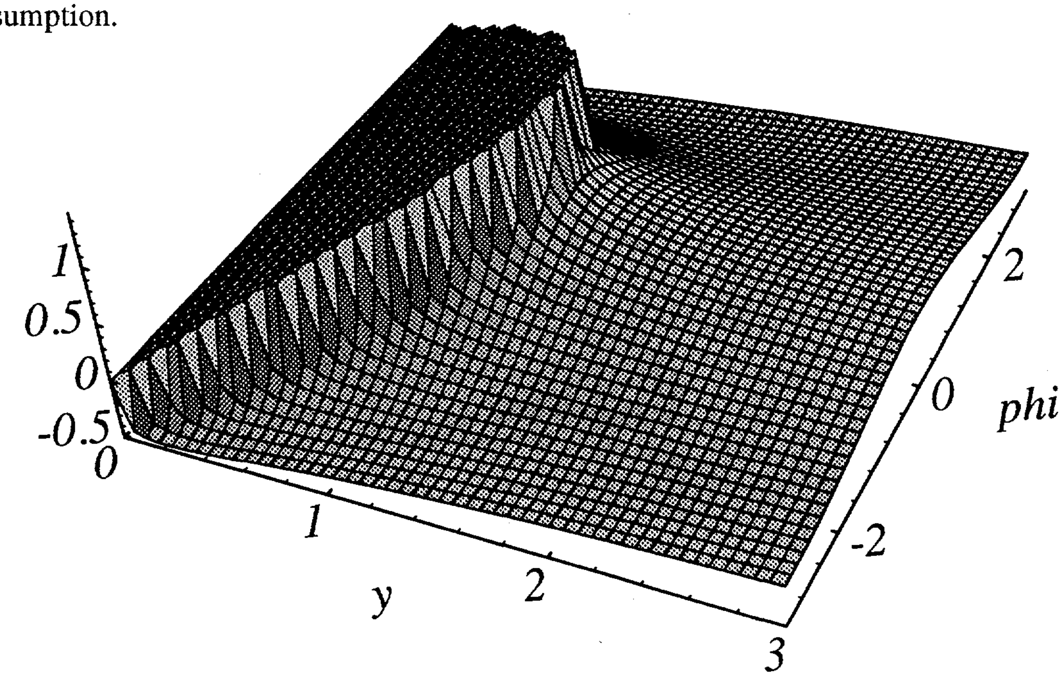

FIGURE 2. Perturbation of the distribution function $\delta\psi$ as a function of y and φ. For negative y, $\delta\psi$ is given by the antisymmetry condition, $\delta\psi(-y,\varphi) = -\delta\psi(y,\varphi)$.

Having found the perturbation of the distribution function, we are now in position to compute an increase $\Delta\varepsilon = \varepsilon_f - \varepsilon_i$ of the emittance of the beam. Applying the definition

$$\varepsilon = \beta_0 \iint J\psi dJ d\varphi \tag{24}$$

to Eq. (23), one finds

$$\Delta\varepsilon = -2^{5/2} \pi\gamma\beta_0 \left(\frac{a}{\alpha}\right)^{3/2} \psi_i', \tag{25}$$

where the numerical factor γ is

$$\gamma = \int_{-\pi}^{\pi} d\varphi \int_{1}^{\infty} d\xi \left[\frac{1}{2\pi}\sqrt{\xi + \cos\varphi} - \left(\int_{-\pi}^{\pi} \frac{d\varphi}{\sqrt{\xi + \cos\varphi}} \right)^{-1} \right] + \frac{1}{3\pi} \int_{-\pi}^{\pi} d\varphi (1 + \cos\varphi)^{3/2} = 1.1. \tag{26}$$

Note that $\Delta\varepsilon > 0$ because ψ_i' is negative.

The result (25) can be rewritten in a more practical form if one introduces the tune spread in the beam $\delta\nu_{NL}$ as the tune difference between the particles with zero amplitude of the betatron oscillations and that equal to σ,

$$\delta\nu_{NL} = |\alpha| J_0 \nu. \tag{27}$$

Then the relative increase of emittance $\Delta\varepsilon/\varepsilon_0$ (where ε_0 is the initial emittance of the beam, which, according to Eq. (24), is $\varepsilon_0 = \beta_0 J_0 = \sigma^2/\beta$) takes the following form:

$$\frac{\Delta\varepsilon}{\varepsilon_0} = 2^{-3/4} \gamma \pi^{-3/2} \left| \frac{\Delta}{\delta\nu_{NL}} \right|^{3/4} \exp\left(-\left| \frac{\Delta}{\delta\nu_{NL}} \right| \right) \left(\frac{\lambda_0}{\delta\nu_{NL}} \right)^{3/2} \left(\frac{\beta\beta_0}{\sigma^2} \right)^{3/4}. \tag{28}$$

Maximum value of $\Delta\varepsilon$ is reached for $|\Delta| = 3\delta\nu_{NL}/4$ and is equal to

$$\frac{\Delta\varepsilon}{\varepsilon_0} = 0.045 \left(\frac{\lambda_0}{\delta\nu_{NL}} \right)^{3/2} \left(\frac{\beta\beta_0}{\sigma^2} \right)^{3/4}. \tag{29}$$

This equation expresses the increase of the emittance of the beam in terms of the amplitude of the ripple, the tune spread in the beam, and the initial emittance of the beam.

As a check of our analytic results we performed computer simulations in which an ensemble of 10^5 particles was tracked, governed by the Hamiltonian (6) with the nonlinear term $\alpha J^2/2$ added to it. A result of such a simulation for $\lambda_0 = 3 \cdot 10^{-3}$, $\alpha = 0.1$, $\nu = 0.28$, and $\omega/\Omega = 0.3$ is shown in Figure 3. As is seen from this figure, an initial linear growth of emittance anticipated from a linear theory turns out to be a transient process which, after reaching a local maximum, comes to oscillations around an asymptotic value of $\Delta\varepsilon/\varepsilon_0$. Note the good agreement between simulation and the analytical prediction for asymptotic $\Delta\varepsilon$. The duration of the transient process can be characterized by the number of turns N of the first maximum in Figure 3. From the theory, it follows that this time is associated with nonlinear oscillations described by the Hamiltonian (13). A simple scaling analysis predicts that N behaves in accordance with the scaling law, $N \propto \delta\nu_{NL}^{-1}\left(\varepsilon_0 / \Delta\varepsilon\right)^{1/3}$. Comparison with the curve in Figure 3 allows one to find approximately the unknown numerical factor in this scaling:

$$N = \frac{\left(\varepsilon_0/\Delta\varepsilon\right)^{1/3}}{\delta\nu_{NL}} \quad . \tag{30}$$

This equation complements Eq. (29) and can be used for estimates of the characteristic time at which the final equilibrium state of the beam is established.

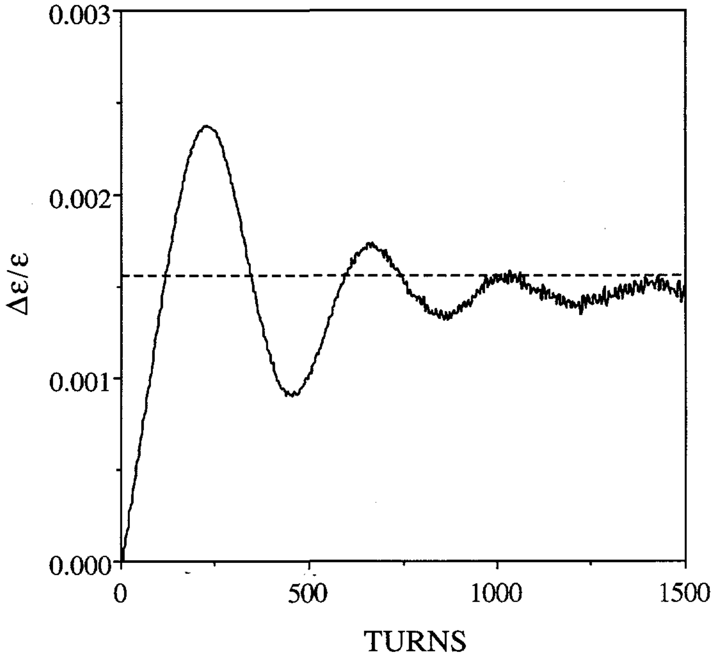

FIGURE 3. Plot of the relative emittance increase as a function of number of turns. The horizontal dashed line shows the asymptotic value of $\Delta\varepsilon/\varepsilon_0$ calculated with the use of Eq. (28).

4 DISCUSSION

The main result of this paper consists in the prediction of the saturation level of the emittance growth caused by the resonance with the ripple. Our approach is based on the use of a small parameter associated with the amplitude of the ripple. To find out the condition under which our approach is valid, we should require the width of the resonance to be much smaller than the width of the distribution function. The width of the resonance δJ_{res} can be estimated from the Hamiltonian (13) as the amplitude of the oscillations on the separatrix, $\delta J_{res} \sim (a/\alpha)^{1/2}$, giving a condition of applicability $\delta J_{res} \ll J_0$. Comparing this inequality with Eq. (25) one concludes that within a numerical factor of the order of unity the condition of applicability of the theory can be also formulated as $\Delta\varepsilon/\varepsilon_0 \ll 1$; that is, the theory is valid for the ripple, which causes a relatively small increase of the emittance.

The theory can be easily generalized for nonlinearity $\delta\nu(J)$ of the kind other than quadratic dependence—for example, for those caused by the beam-beam interaction. For rough estimates one can use Eq. (29) for any type of nonlinearity by simply putting in it a proper $\delta\nu_{NL}$ (defined as a tune difference between particles having a zero amplitude and that equal to σ).

As an application of results obtained, let us estimate a tolerable level of the ripple for the SSC parameters at injection energy 2 TeV, assuming that the allowable increase of the emittance is $\Delta\varepsilon/\varepsilon_0 = 10^{-2}$. From Eq. (29) one finds

$$\frac{\delta B}{B} = 7.9\delta\nu_{NL}\frac{\rho}{l}\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)^{2/3}\left(\frac{\sigma^2}{\beta\beta_0}\right)^{1/2}. \tag{31}$$

For the collider,[6] the magnet length l = 15 m, bending radius ρ = 9.8 km, and nominal normalized emittance $\gamma\sigma^2/\beta = 10^{-6}$m. Taking the average beta function β_0 = 100 m as the value of β at the position of the magnet, and using $\delta\nu_{NL} = 10^{-5}$ (Reference 7), for the natural nonlinearity of the ring, one finds $\delta B/B = 5.4\cdot 10^{-9}$. However, this result refers to the case of only one magnet experiencing ripple oscillations of the magnetic field. The SSC collider will have 10 power supplies that produce the ripple that attenuates with the distance propagating from the drive points.[1] Effectively, the result of all the power supplies can be estimated as equal to 20–30 times of the perturbation of the magnetic field in the magnet closest to the driving point. With this amplification factor due to the large number of magnets involved in the ripple oscillations the tolerable level of fluctuations per magnet becomes $\delta B/B = (2\div 3)\cdot 10^{-10}$, which lies below the expected value of fluctuations.

At the full energy of 20 TeV, the nonlinearity will be dominated by beam-beam interaction with the nominal tune shift $\delta\nu_{NL}$ = 0.004. Although beam–beam interaction is characterized by a more complex dependence of the tune shift versus the amplitude of the betatron oscillation than that assumed in Eq. (27), for an estimate we can still use Eq. (31). Using the same values for other parameters as above, one finds the tolerable level of the ripple $\delta B/B = (2\div 3)\cdot 10^{-8}$.

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