

LA-UR- 98-1178

Approved for public release;  
distribution is unlimited.

Title:

FINITE CAVITY EXPANSION METHOD FOR  
NEAR-SURFACE EFFECTS AND LAYERING DURING  
EARTH PENETRATION (U)

CONF-981030--

Author(s):

R. W. MACEK, ESA-EA  
T. A. DUFFEY, ESA-EA

Submitted to:

INTERNATIONAL SYMPOSIUM ON IMPACT AND  
PENETRATION PROBLEMS (ICES)  
ATLANTA, GA  
OCTOBER 5-9, 1998

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED 

MASTER

**Los Alamos**  
NATIONAL LABORATORY

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. The Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

### **DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

## **DISCLAIMER**

**Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.**

# Finite Cavity Expansion Method for Near-Surface Effects and Layering During Earth Penetration

Richard W. Macek, Thomas Duffey  
Group ESA-EA MS P946  
Los Alamos National Laboratory  
Los Alamos, NM 87544, USA

## Summary

A finite spherical cavity expansion technique is developed to simulate the loading on projectiles penetrating geologic media. Damaged Mohr-Coulomb plasticity models and general pressure-dependent damaged plasticity models are used with incompressible kinematics to approximate a wide range of targets. The finite cavity expansion approximation together with directional sampling reasonably captures near surface and layering effects without resort to ad hoc or empirical correction factors. The Mohr-Coulomb models are integrated exactly to provide a very efficient loading algorithm for use with conventional implicit or explicit finite element structural analysis. The more general constitutive model requires numerical integration and leads to a more computationally intensive procedure. However, subcycling is easily implemented with the numerical integration and thus an efficient loading method is readily achieved even for large complex simulations using explicit finite element analysis. The utility of the finite cavity expansion approach is demonstrated by comparison of simulations to measured test data from projectiles penetrating rock and soil targets.

## Introduction

The general subject of the penetration of various media by projectiles has recently been surveyed by Backman and Goldsmith [1], Corbett, Reid, and Johnson [2], and Heuzé [3]. Typically penetration problems are solved by either empirical, analytical, or numerical methods. In general empirical methods are most often employed when the depth of penetration and the trajectory of the projectile in the target are desired. Furthermore, the empirical methods are the most successful when the penetrator can be approximated as a rigid body. If the structural response of the projectile during penetration is the main concern, then analytical or numerical approaches have shown more success.

Full numerical approaches to penetration have the most firm theoretical basis. With these both the target and the projectile are discretized and integrated numerically in both space and time. Lagrangian, Eulerian, and coupled Lagrangian-Eulerian methods have been successfully used. Any of these methods provide generality in both the geometrical and constitutive representation of the target. However, they are very computationally intensive and the strictly Lagrangian approaches often suffer from mesh entanglement in the target which can prematurely terminate the analysis.

One of the more popular analytical techniques is cavity expansion coupled to a flexible body projectile model [3]. With this approach the pressure on the projectile is typically computed analytically by assuming that the penetrator is uniformly (one dimensionally) expanding either a spherical or cylindrical cavity in an infinite medium at a constant velocity (the velocity normal to the projectile's outer surface). The projectile, itself, is usually modeled with conventional finite elements. While this method is very computationally efficient and accurately predicts the axial response of the penetrator, it often errs significantly in predicting the lateral response. This deficiency is presumed to be due to inadequate geometrical representation of the target

near free surfaces or interfaces which dominate the lateral loading. Currently ad hoc or empirical near surface pressure corrections are often used to improve the lateral response but they tend to lack generality. The finite cavity expansion approach described in this paper to first order overcomes this inadequacy by solving for the pressure based on expanding a spherical cavity in a layered finite radius sphere. Further improvement is achieved by directional sampling.

The simulation method used for a given problem clearly depends on the objective of the analysis, the desired accuracy, and the cost of the solution. For geologic targets the variability of the medium and the accuracy of the constitutive data often eclipse the integration accuracy achieved with the full numerical approaches. For projectile structural response cavity expansion may be a relatively accurate and economical approach especially when considering the cost and complexity of the full numerical techniques.

### Finite Cavity Expansion Technique

With the finite cavity expansion technique any point A on the penetrator surface is assumed to be uniformly expanding a layered sphere with finite radii as shown in Figure 1. The boundary condition on the

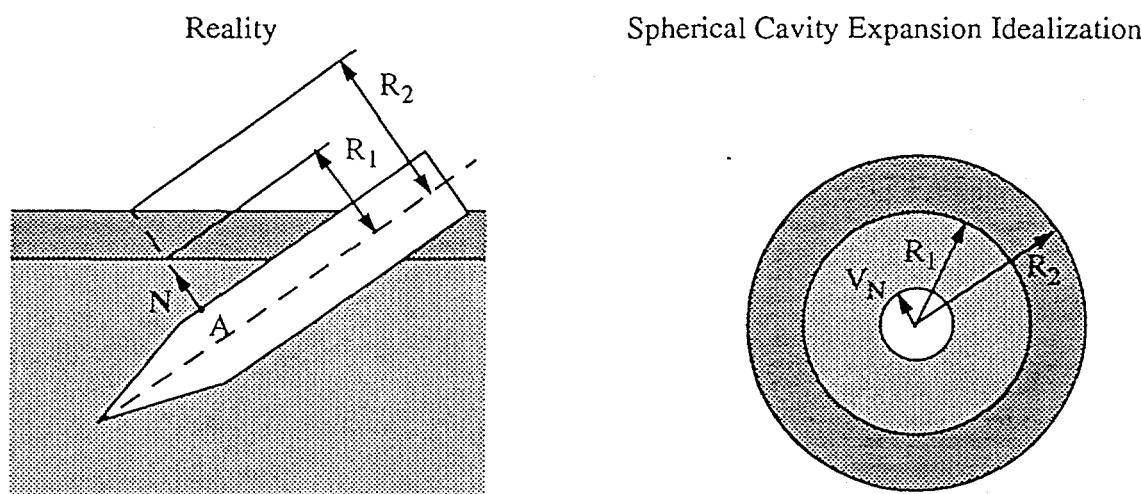


Figure 1. Geometry idealization.

exterior free surface is zero pressure while the interior velocity is the local velocity normal to the penetrator surface  $V_N$ . The layer radii are measured along the local normal  $N$  to the penetrator surface. In practice we have found that a better approximation is often obtained when the pressure is averaged (using Gauss quadrature) from radii measured along several directions symmetrically distributed about the local normal. To date we have only sampled within  $180^\circ$  in the plane containing the local penetrator normal and the layer normals.

The equations governing the solution for spherically symmetric cavity expansion consist of incompressible kinematics, the material constitutive relations, and the radial equilibrium equation. For the finite cavity expansion solution incompressible kinematics are essential because without this assumption time dependent wave propagation and reflection would need to be tracked relative to the moving penetrator. Obviously this would greatly complicate the solution procedure. The role of compressibility effects are isolated in [4] and are believed to be of second order for earth penetration.

The incompressible kinematic equations are as follows:

$$r^{*3} - r_i^{*3} = r^3 - r_i^3 \quad (1)$$

$$\varepsilon_\theta - \varepsilon_r = \frac{r_i^{*3} - r_i^3}{r^{*3}} = \varepsilon_e \quad (2)$$

$$V_r = \frac{r_i^{*2}}{r^{*2}} V_i \quad (3)$$

$$a_r = \frac{r_i^{*2}}{r^{*2}} \dot{V}_i + 2r_i^{*} V_i^2 \left( \frac{1}{r^{*2}} - \frac{r_i^{*3}}{r^{*5}} \right) \quad (4)$$

In the above equations radii  $r$  with the \* superscript refer to the deformed configuration while radii without a superscript refer to the undeformed configuration. The  $i$  subscript refers to quantities measured at the inner surface of the layer  $i$ . Also  $V$ ,  $a_r$ ,  $\varepsilon_\theta$ ,  $\varepsilon_r$  are radial velocity, radial acceleration, hoop strain, and radial strain, respectively.

A variety of material models have been implemented with the finite cavity expansion solution. The simplest elastic-perfectly-plastic models with damaged Mohr-Coulomb yield criteria permit a closed form solution of the governing equations. Two damage criteria (reduction of yield strength  $Y$ ) have been investigated; one assumes that damage occurs at critical value of tensile hoop strain  $\varepsilon_\theta = \varepsilon_u$  and the other assumes damage occurs immediately after the initial yield surface is reached. With the former approach damage always occurs next to the penetrator and the size of the damaged zone is only dependent on the radius of the penetrator and the critical strain. With the latter approach damage is present throughout the plastic zone. Figure 2. illustrates the damaged Mohr-Coulomb plasticity models. A more general pressure and strain

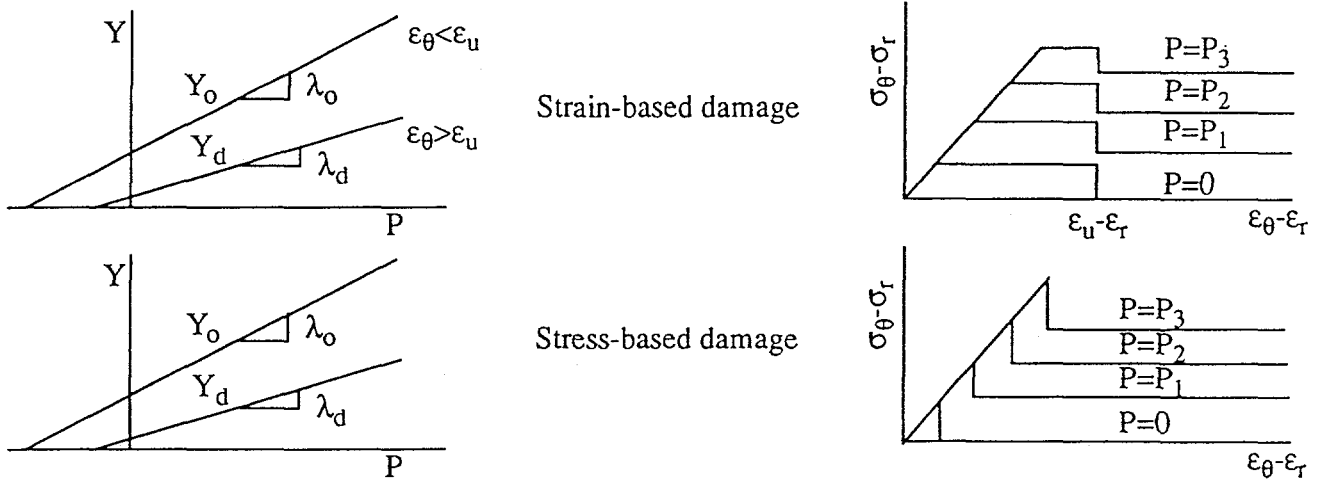


Figure 2. Damaged Mohr-Coulomb constitutive models.

rate dependent damaged yield model has also been implemented. With this model a piecewise linear representation is used for the yield surface and the transition between the undamaged and damaged yield surfaces occurs linearly over some amount of plastic strain  $\bar{\epsilon}_p$ . The strain rate dependence for both the undamaged and damaged yield surfaces follows a capped power law form. Mathematically the general plasticity model is defined by the following equations:

$$Y(P, \dot{\epsilon}_{min}, \epsilon_p) = \frac{(\bar{\epsilon}_p - \epsilon_p)}{\bar{\epsilon}_p} Y_o(P, \dot{\epsilon}_{min}) + \frac{\epsilon_p}{\bar{\epsilon}_p} Y_d(P, \dot{\epsilon}_{min}, \bar{\epsilon}_p) \quad 0 < \epsilon_p < \bar{\epsilon}_p \quad (5)$$

$$Y(P, \dot{\epsilon}_{min}, \epsilon_p) = Y_d(P, \dot{\epsilon}_{min}, \bar{\epsilon}_p) \quad \epsilon_p > \bar{\epsilon}_p \quad (6)$$

$$Y(P, \dot{\epsilon}, \epsilon_p) = Y(P, \dot{\epsilon}_{min}, \epsilon_p) \quad \dot{\epsilon} < \dot{\epsilon}_{min} \quad (7)$$

$$Y(P, \dot{\epsilon}, \epsilon_p) = \left( \frac{\dot{\epsilon}}{\dot{\epsilon}_{min}} \right)^\alpha Y(P, \dot{\epsilon}_{min}, \epsilon_p) \quad \dot{\epsilon}_{min} < \dot{\epsilon} < \dot{\epsilon}_{max} \quad (8)$$

$$Y(P, \dot{\epsilon}, \epsilon_p) = \left( \frac{\dot{\epsilon}_{max}}{\dot{\epsilon}_{min}} \right)^\alpha Y(P, \dot{\epsilon}_{min}, \epsilon_p) \quad \dot{\epsilon} > \dot{\epsilon}_{max} \quad (9)$$

Figure 3. graphically illustrates the model and shows its capability to capture brittle-to-ductile transition behavior commonly seen in rock.

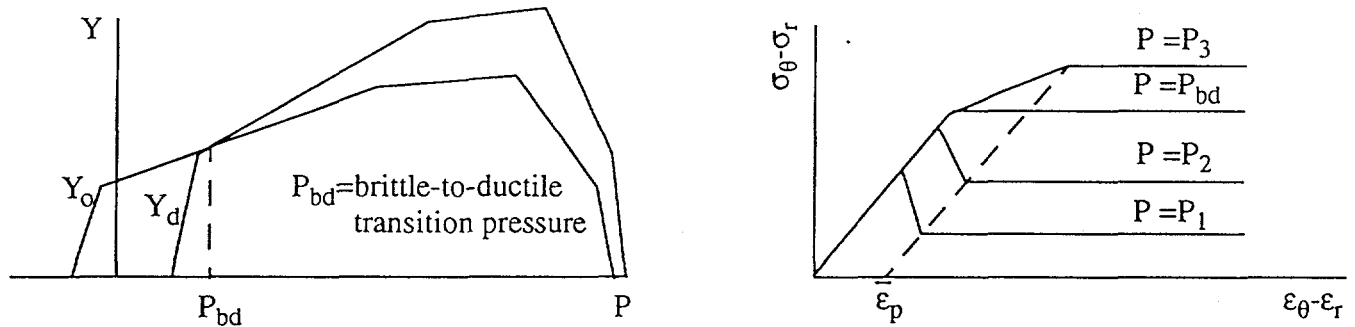


Figure 3. General plasticity model.

The radial equilibrium equations for a spherically symmetric elastic-plastic system are

$$\frac{d\sigma_r}{dr^*} + \frac{2}{r^*}(\sigma_r - \sigma_\theta) = \frac{d\sigma_r}{dr^*} + \frac{4}{3}E \left( \frac{r_i^{*3} - r_i^3}{r^{*3}} \right) = \rho a_r \quad \text{elastic} \quad (10)$$

$$\frac{d\sigma_r}{dr^*} = \rho a_r + \frac{2Y}{r^*} \quad \text{plastic} \quad (11)$$

The equilibrium equations together with the kinematic and constitutive equations can be integrated exactly for the damaged Mohr-Coulomb models. With the more general material model numerical integration using a fifth order Runge-Kutta method with adaptive stepsize control is employed. The cavity expansion integration is coupled to a general purpose Lagrangian finite element (FE) code via a user-supplied loading routine to compute the structural response of the penetrator. The FE code supplies the normal direction and the velocity boundary condition to the cavity expansion integration and the user-supplied loading routine returns the pressure on the penetrator. With an explicit FE code subcycling may be economically advantageous if the more general constitutive model is employed. This can be easily accomplished by simply updating the pressure less frequently than the structural response. In many applications the pressure can be updated as infrequently as once in every ten or one hundred structural time steps because the low frequency modes dominate the velocity history.

### Simulation of Earth Penetration Experiments

The first experiment simulated was normal penetration into a rock half-space as reported in [5]. In this analysis the stress-based damaged Mohr-Coulomb constitutive model was used. The measured and computed axial response are compared in Figure 4. As is evident the axial deceleration responses are very close.

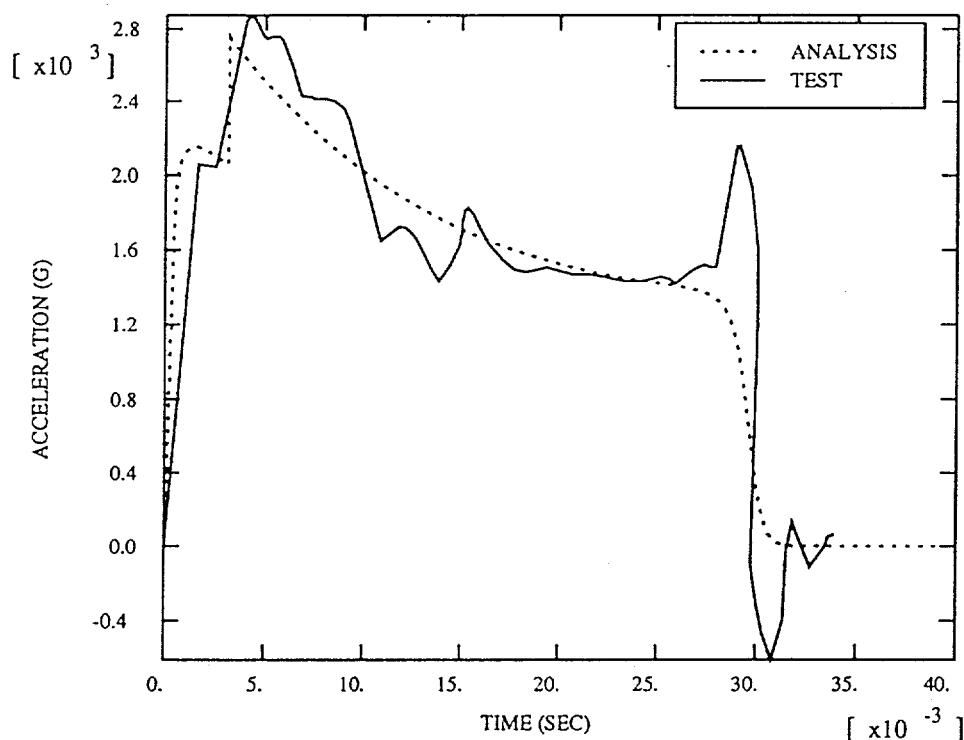


Figure 4. Measured and computed axial deceleration for penetration into rock.

The second experiment was oblique impact into a single layer of frozen soil. This is an especially challenging test for the cavity expansion approach since the projectile completely perforated and exited the target, i.e. there were two free surfaces in the simulation. In this analysis the general constitutive model was utilized with a strong strain rate dependence [6]. Comparison of the axial and lateral accelerations are shown in Figures 5. Again the axial decelerations compare very well. The general shape of the lateral acceleration pulse is captured with the analysis; however the peak magnitude is about 27% lower than the test. Considering the scatter in the material data from which the constitutive properties were derived, this is a very reasonable comparison.



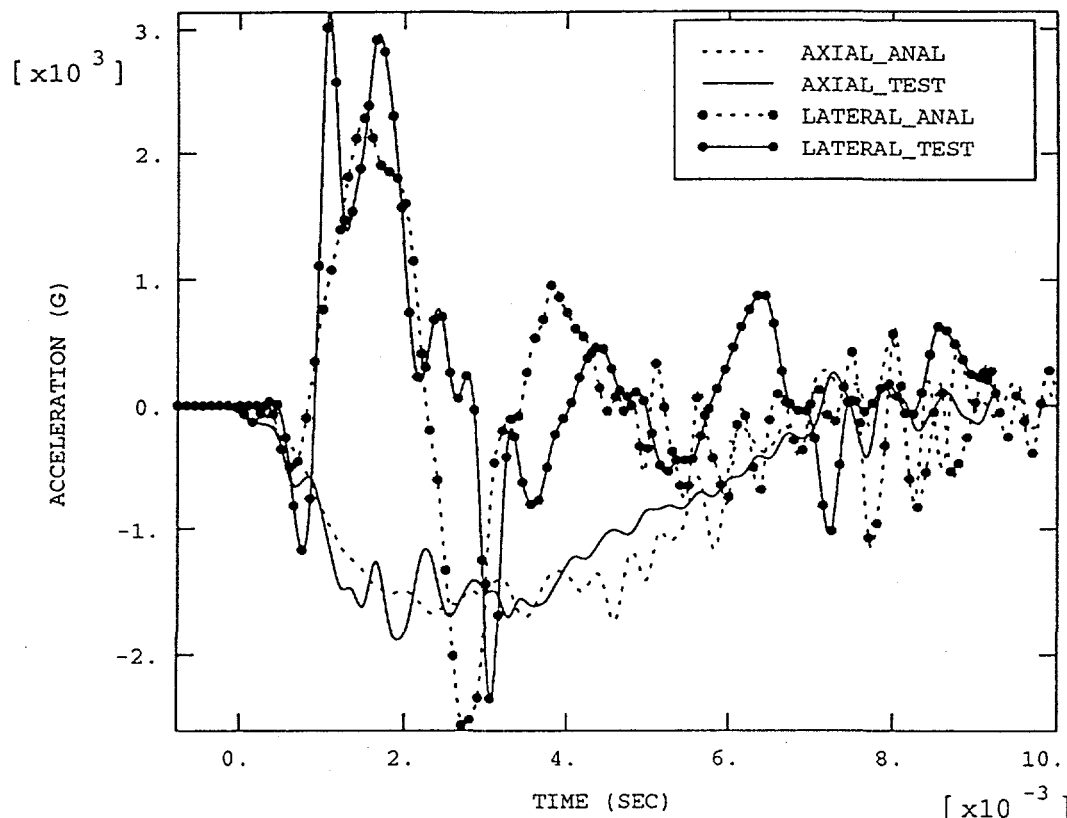


Figure 5. Measured and computed axial and lateral decelerations for penetration into frozen soil.

### Conclusions

Based on the analysis of a limited number of earth penetration experiments the cavity expansion technique developed in this paper appears to reasonably capture near surface and layering effects in structural response simulations. In the future comparison to a broader experimental data base may extend the applicability of this method.

### References

1. Backman, M. E. and Goldsmith, W. (1978), "The Mechanics of Penetration of Projectiles into Targets", International Journal of Engineering Sciences, Vol. 16, pp. 1-99.
2. Corbett, G. G., Reid, S. R., and Johnson, W. (1996), "Impact Loading of Plates and Shells by Free-Flying Projectiles", International Journal of Impact Engineering, Vol. 18, No. 2, pp 144-230.
3. Heuzé, F. E., (1989), "An Overview of Projectile Penetration into Geological Material, with Emphasis on Rocks", Computational Techniques for Contact, Impact, Penetration and Perforation of Solids, Schwer, L. E., Salamon, N. J., and Liu, W. K. (Eds.), American Society of Mechanical Engineers, pp. 275-308.
4. Forrestal, M. J. and Luk, V. K. (1988), "Dynamic Spherical Cavity-Expansion in a Compressible Elastic-Plastic Solid", Journal of Applied Mechanics, Vol. 55, p. 275-279.
5. Longcope, D. B. and Forrestal, M. J. (1983), "Penetration of Targets Described by a Mohr-Coulomb Failure Criterion with Tension Cutoff", Journal of Applied Mechanics, Vol. 50, pp. 327-333.
6. Chamberlain, E., Groves, C., and Perham, R. (1972), "The Mechanical Behavior of Frozen Earth Materials Under High Pressure Triaxial Test Conditions", Geotechnique, Vol. 22, No. 3, pp. 469-483.