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# FRACTAL STRUCTURE IN MULTIPARTICLE PRODUCTION IN $e^+e^-$ ANNIHILATIONS AT 29 GeV

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## ABSTRACT

The fractal structure in multiparticle production in $e^+e^-$ annihilations at 29 GeV has been studied using the HRS detector at PEP. Very high fractal moments have been measured and their implications are discussed in terms of an α-model and a thermodynamical model. Two-dimensional fractal moments in the rapidity and azimuthal angle space have also been measured and the results are compared to the Lund string model predictions.



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The fractal structure in multiparticle production in $e^+e^-$ annihilations at 29 GeV has been studied using the HRS detector at PEP. Very high fractal moments have been measured and their implications are discussed in terms of an $\alpha$-model and a thermodynamical model. Two-dimensional fractal moments in the rapidity and azimuthal angle space have also been measured and the results are compared to the Lund string model predictions.

The theory of fractal geometry[1] has revealed the structure of self-similarity in nature and has been applied successfully in various fields such as physics, astronomy, chemistry, computer graphics, and so on. The application to the multiparticle production in high energy physics has just begun. If quarks and gluons become jets of particles through a cascading process, the particles inside a jet should show a self-similar behavior which leads to a fractal/multifractal structure. Therefore, the analysis techniques developed in fractals are powerful tools to study the underlying physics of the hadronization process.

In this talk we report a study of fractal behavior of the charged particle multiplicity in $e^+e^-$ annihilations, using a high statistics data sample corresponding to a total integrated luminosity of 300 $pb^{-1}$ obtained with the High Resolution Spectrometer (HRS) at PEP. The HRS detector was a solenoidal spectrometer that measured charged particles and electromagnetic energy over 90% of the solid angle. The details of the detector and the event selection criteria can be found elsewhere.[2] A rapidity of each particle is calculated with respect to a thrust axis of all charged particles. The resolution of the rapidity is estimated to be about 0.1 including an ambiguity in determining the true jet axis which is the initial quark direction. The rapidity distribution, after detector corrections for the inclusive data sample is relatively flat in the range between $y = -2$ and $y = +2$, but the uncorrected distribution has a valley near $y =$

0 mainly because of the low tracking efficiency of slow particles and tracks near the beam line. In the following analysis, we use uncorrected variables.

A rapidity interval of total length $Y_0$ between y = -2 and y = +2 is divided into $M_0$ bins of width $\delta = Y_0/M_0$ and let $k_i$ be the number of particles in the $i^{th}$ bin. Since there may be bins that have no particles, we define M to be the number of non-empty bins, which constitute a fractal set. A multifractal moment is defined as follows:[3]

$$G_q = \sum_{j=1}^{M} p_j^q \quad ,$$

where $p_j = k_j/n$ with $n = k_1 + k_2 + \ldots + k_M$, q is a real number, and the summation is carried over non-empty bins only. If the particle production process exhibits self-similar behavior, the moments show a power law relation of $G_q \propto \delta^{\tau(q)}$. This relation does not necessarily occur in the limit of $\delta \to 0$. Therefore, in calculating the power $\tau(q)$, we use the first two points in $G_q$($M_0$ = 2 and 3) and an average of $\ln G_q$ over all events. Once $\tau(q)$ is determined from $G_q$, we can apply the theory of multifractals to calculate the singularity spectrum function $f(\alpha)$ by Legendre transform:

$$\alpha_q = \frac{d\tau(q)}{dq}$$

$$f(\alpha) = q\alpha_q - \tau(q) \quad .$$

The spectrum function $f(\alpha)$ for the inclusive data sample is shown in Fig. 1. The left-hand side of the curve corresponds to multifractal moments with $q > 0$ and the edge points are from high moments with q up to 20. The right-hand side corresponds to the negative moments with q down to -20. The statistical error bars are less than the mark size except near the endpoints. The $f(\alpha)$ stays above zero up to very large moments of $q = \pm 20$, which has thermodynamical implications.

In fluid-dynamics of fully developed turbulence, a geometrical model called "α-model" has been used successfully. It is based on a random cascading process with a two-level probability distribution at each stage. The fractal structure of this model has been studied theoretically and the spectrum functions $f(\alpha)$ are sketched in Fig. 2 according to four different regions in the parameter space.[4] The regions I, II, III, and IV are respectively named a peak transition region, a hole transition region, a full transition region, and a no phase transition region. This is because $f(\alpha)$ can be regarded as a local entropy in a thermodynamical interpretation. Therefore, if

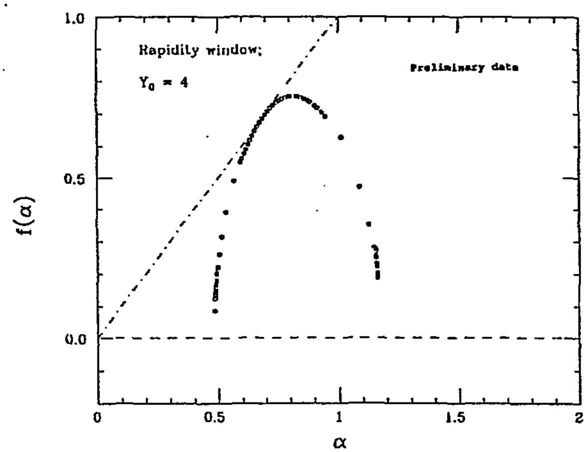


Fig. 1

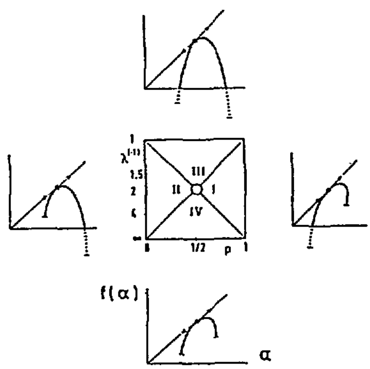


Fig. 2

it reaches zero, the free energy is frozen at the maximum and the system undergoes a phase transition of the spin-glass type. The regions I, II, and III have each corresponding models in nature such as growing crystals and a percolating cluster in a random resistor network. The region IV, however, has few corresponding models so far and it seems that the rapidity distribution of multiparticle production shown in Fig. 1 is in this category. The underlying physics of this process needs to be studied further. The Lund string model[4] reproduces this limiting behavior of f(α) well, although there are some discrepancies for the negative moments where coherence effects are important in the valley regions among jets. More detailed comparisons between data and the Lund model for two-jet and inclusive samples can be found elsewhere.[6]

The multifractal moments of azimuthal angles around the beam axis ($\phi_{beam}$) have been measured. The distribution of $\phi_{beam}$ averaged over many events is flat, although the azimuthal angles in each event are clustered in narrow phase space regions due to the jet structure of events. The resulting $f(\alpha)$ is shown in Fig. 3. The curve seems to reach zero for large q values near ±20, the interpretation of which is still unclear. The shape of $f(\alpha)$ is much wider than that in the rapidity space reflecting the nature of narrowly collimated jets. The fractal dimension given at the peak of the curve is larger than the one in the rapidity space.

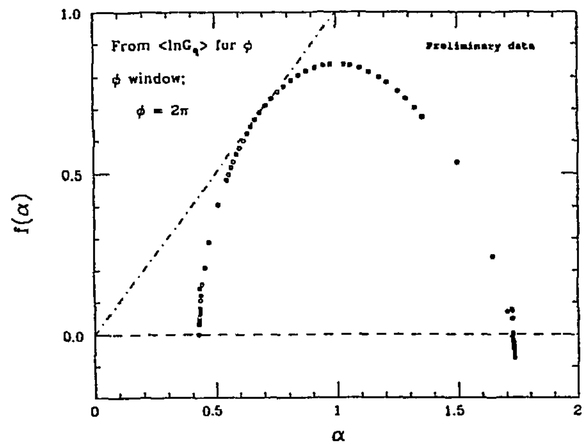


Fig. 3

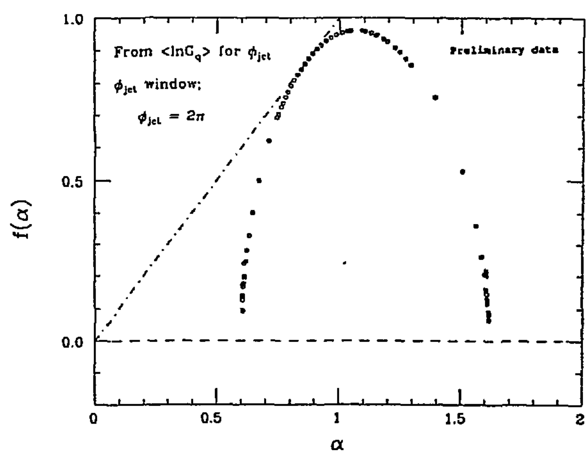


Fig. 4

On the other hand, the $f(\alpha)$ from the multifractal moments of azimuthal angle around the jet axis ($\phi_{jet}$) has a different behavior as shown in Fig. 4. The curve stays positive for moments up to $q = \pm 20$ as in the case of rapidity. The shape is narrower than that in the case of $\phi_{beam}$, and the fractal dimension is close to 1. This is consistent with an observation that particles inside a jet are almost uniformly distributed around the jet axis.

In order to study the fragmentation function of jets, the multifractal moments in the two-dimensional space of y and $\phi_{jet}$ have been measured. The total lengths of $-2 < y < +2$ and $0 < \phi_{jet} < 2\pi$ are divided into the equal number of internals, n, resulting in the total number of grids $M = n^2$ with $n = 1,2,3,\ldots 40$. The moments with various q values are shown in Fig. 5, where the abscissa is basically the number of grids with $\nu_2 = \ln M/\ln 4$. The factor $\ln 4$, instead of $\ln 2$ was used because the size of each rectangular grid is proportional to $\sqrt{M}$ in the two-dimensional case. All the moments saturate faster than in the one-dimensional case due to a faster rise in moments. The singularity spectrum $f(\alpha)$ is calculated using the two points of $M = 1$ and $M = 4$ in $G_q$ and shown in Fig. 6. As expected, the fractal dimension is about 1.7 reflecting the nature of two-dimensional analysis. The Lund string model after detector simulations, shown in Fig. 7, predicts a remarkable agreement with the data.

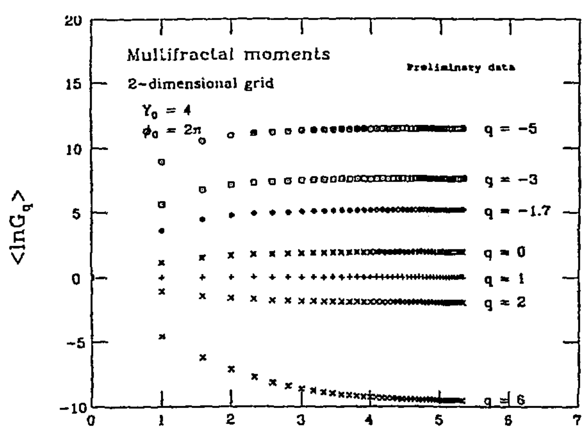


Fig. 5

In conclusion, very high multifractal moments up to $q = \pm 20$ have been measured. The singularity spectrum functions $f(\alpha)$ in rapidity and in azimuthal angle around the jet axis stay positive up to high moments, which is interpreted as a no phase transition in the $\alpha$-model. On the other hand, the $f(\alpha)$ in azimuthal angle around the beam axis reaches zero for very high moments, which is interpreted as

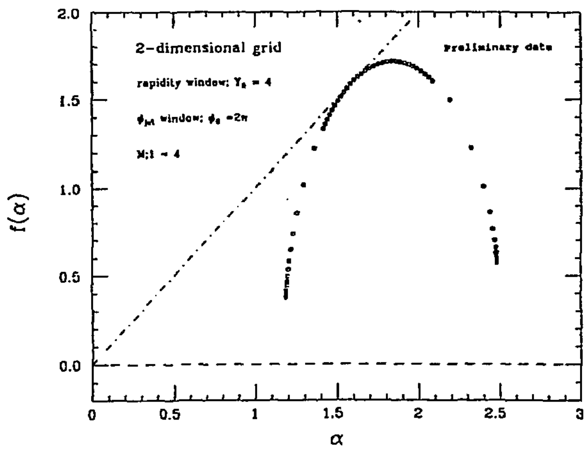


Fig. 6

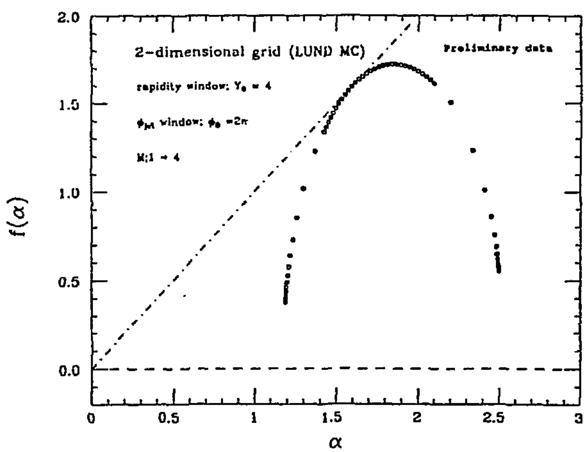


Fig. 7

full transition. More theoretical studies are needed to get a coherent physical picture of these data. The fractal dimensions and the widths of f(α) in two different kinds of azimuthal angles clearly show the difference in the event-by-event distributions of particles in jets. As expected, the particles are narrowly collimated in azimuthal angle around the beam axis, but they are almost uniformly distributed in azimuthal angle around the jet axis.

The two-dimensional multifractal moments in rapidity and azimuthal angle around the jet axis have been measured. All the moments saturate faster than in the one-dimensional case because of a faster rise. The fractal dimension is abut 1.7 reflecting the two-dimensional nature. The Lund string model after detector simulations agrees remarkably well with the data.

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