NOVEMBER 1980

PPPL-1720
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Double Layers Without Current

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ABSTRACT

The steady-state solution of the nonlinear Vlasov-Poisson equations is reduced to a nonlinear eigenvalue problem for the case of double-layer (potential drop) boundary conditions. Solutions with no relative electron-ion drifts are found. The kinetic stability is discussed. Suggestions for creating these states in experiments and computer simulations are offered.

Both laboratory experiments[1-3] and space-plasma observations[4-6] have shown that plasmas can develop states which have a narrow, isolated region of rapid potential change surrounded by large regions of effectively uniform plasma potential. Such states are called double-layers because of the dipole-sheet nature of the space change distribution required. Theoretical models[1,7-9] of double layers have generally required a relative electron-ion drift (i.e., a plasma current), but recently computer simulations[10] and studies of thermal barrier cells in tandem mirror devices[11-12] have found states with abrupt potential drops with little or no plasma current. Currentless double layers have a particular significance for two reasons: (1) Their $\vec{E}\cdot\vec{J}$ energy dissipation vanishes so that no external energy source is required to maintain them; and (2), in contrast to collisionless shocks,[13] they involve no mass flow and, hence, no supply of streaming plasma is necessary. A currentless solitary wave solution has recently been found by Hasegawa and Sato.[14]

The goal of this letter is to find solutions to the Vlasov-Poisson equations which exhibit the following properties: (1) An isolated region of abrupt potential change exists surrounded by regions where the plasma is quasineutral and the potential is constant. (2) On the high-density side of the potential change, the plasma has Maxwellian velocity distributions for both ions and electrons although the respective temperatures may be different. (3) On the low-density side, the electron velocity distribution remains Maxwellian while the ion distribution is composed of counterstreaming ion beams. There is no net current. (4) The potential decreases from the high-density to the low-density side.

The key to obtaining these solutions is to recognize that electrostatically trapped ions can exist on the low-density, low-potential side. We will regard the density of these trapped ions to be an adjustable parameter which, together with the magnitude of the potential drop, provides us with two parameters which are just sufficient to satisfy the two criteria for a double-layer solution: that the low-density side be quasineutral and that the total charge in the double layer be zero. Hence, the trapped ion density and the potential drop are the two components of a nonlinear, two-component eigenvalue problem which determines the double-layer solution.

Our model is that of a one-dimensional Vlasov-Poisson plasma, and we shall define a nondimensional potential ψ related to the convential potential by

$$\psi = -e\phi/T_e , \tag{1}$$

and choose the $\psi = 0$ level to be on the high density side. Hence, ψ will be positive and monotonically increasing. The steady-state Vlasov equation is solved by any function of energy. We assume the electron distribution function is everywhere Maxwellian. Our model for the ion distribution functions f is

$$\left(\frac{2\pi T_i}{M}\right)^{1/2} \frac{f}{n_o} = h = \begin{cases} e^{-\varepsilon} & \varepsilon > -\Delta \\ 0 & \varepsilon < -\Delta \end{cases} , \tag{2}$$

where

$$\varepsilon = (Mv^2/2T_i) - \psi\tau , \qquad \tau = T_e/T_i . \tag{3}$$

The positive parameter Δ governs the density of electrostatically trapped ions (those with $\varepsilon < 0$). The electron and ion densities can then be expressed as

$$n_e = n_o e^{-\psi} \quad , \quad n_i = n_o g(\psi, \Delta) \equiv n_o \int_{-\psi\tau}^{\infty} h[\pi(\varepsilon + \psi\tau)]^{-1/2} d\varepsilon \ , \tag{4}$$

and the Poisson equation is

$$\partial^2\psi/\partial\xi^2 = g(\psi, \Delta) - e^{-\psi} \equiv G(\psi, \Delta) \quad , \tag{5}$$

where $\xi = x/\lambda_D$ and $\lambda_D \equiv (T_e/4\pi n_o e^2)^{1/2}$ is the Debye length. Evaluation of the integral in Eq. (4) leads to

$$g(\psi, \Delta) = \begin{cases} e^{\psi\tau} & \psi\tau < \Delta \\ e^{\psi\tau} \int_{(\psi\tau-\Delta)^{1/2}}^{\infty} 2e^{-t^2} dt/\sqrt{\pi} & \psi\tau > \Delta \end{cases} \ . \tag{6}$$

Double layer solutions to Eq. (5) can occur if the net charge density vanishes ·· $\xi \to \pm\infty$. Our assumption that $\psi \to 0$ as $\xi \to -\infty$ is consistent with this condition. This requirement, combined with the asymptotic dependence $\psi \to \psi_o$ as $\xi \to +\infty$, leads to the equation

$$g(\psi_o, \Delta) - e^{-\psi_o} \equiv G(\psi_o, \Delta) = 0 \quad , \tag{7}$$

as one of the two nonlinear equations relating the potential change ψ_o and Δ.

The electric field must also vanish as $\xi \to \pm\infty$. Multiplying Eq. (5) by $\partial\psi/\partial\xi$ and integrating we find

$$\left(\frac{\partial\psi}{\partial\eta}\right)_{\infty}^{2} - \left(\frac{\partial\psi}{\partial\eta}\right)_{-\infty}^{2} = 2 \int_0^{\psi_o} d\psi \left[g(\psi, \Delta) - e^{-\psi}\right]$$

$$= 2 \int_0^{\psi_o} d\psi \ G(\psi) \equiv V(\psi_o, \Delta) = 0 \ . \tag{8}$$

Integration by parts simplifies Eq. (8) to read

$$\frac{1}{\tau}\left[e^{\psi_o \tau}\int_{(\psi_o\tau-\Delta)^{1/2}}^{\infty}(2/\sqrt{\pi})e^{-t^2}\,dt + (2/\sqrt{\pi})(\psi_o\tau-\Delta)^{1/2}\,e^{\Delta}-1\right] - 1 + e^{\psi_o} = 0 \,, \tag{8a}$$

which is the second equation relating ψ_o and Δ. Equations (7) and (8) are the nonlinear equations for the two-component eigenvalue (ψ_o, Δ). Figure 1 presents solutions of these equations for a range of values of the electron-to-ion temperature ratio τ. We note that in addition to the potential change, these double layers have a distinct density change $\Delta n/n_o = 1 - e^{-\psi_o}$.

Equations (7) and (8) coupled with the condition

$$V(\psi, \Delta) = 2\int_0^{\psi} G(\psi', \Delta)\,d\psi' \geq 0 \qquad 0 \leq \psi \leq \psi_o, \tag{9}$$

represent both necessary and sufficient conditions for the existence of a double-layer solution. Necessity follows from the arguments directly preceding Eqs. (7) and (8). Sufficiency will be demonstrated by construction. The integral

$$\int_{\delta\psi_1}^{\psi} d\psi' \left[V(\psi', \Delta)\right]^{-1/2} = \xi - \xi_1 \qquad \delta\psi_1 < \psi < \psi_o - \delta\psi_2 \,, \tag{10}$$

provides the relation between ψ and ξ given that $\psi = \delta\psi_1$ at $\xi = \xi_1$. The end points must be treated specially because the integral formally diverges there. The quantities $\delta\psi_1$, $\delta\psi_2$ can be taken arbitrarily small, so that a Taylor expansion of G is possible. Hence, near $\psi = 0$, $V = G'\psi^2$ and the integral

$$\int_{\psi}^{\delta\psi_1} (G')^{-1/2}\, d\psi/\psi = \xi_1 - \xi , \qquad (11)$$

provides the relation

$$\psi = \delta\psi_1 \exp[(G')^{1/2} (\xi - \xi_1)] , \qquad (12)$$

which shows that the solution exponentially decays to zero as $\xi \to -\infty$. Similar arguments yield an exponentially decaying approach to ψ_o as $\psi \to +\infty$. These arguments coupled with convergent integral Eq. (10) show that, given a V satisfying Eq. (8) and Eq. (9), a solution can be explicitly constructed. Figure 2 shows representative quantities. We note that if there were no trapped ions, then it would be impossible to satisfy Eq. (9).

The two-component eigenvalue is composed of the potential change ψ_o plus an additional component (in our case Δ) which permits a variation of the plasma distribution function. Thus, in general, a double layer cannot occur because the plasma will not have the correct value of Δ. However, a plasma distribution function may vary slowly in space (compared to a Debye length) as a result of changes in mirror ratio, for example. It follows that these slow spatial variations permit a parameter like Δ to assume the correct value at one point in space which is where the double layer will occur. Hence, the physical interpretation of the two-component eigenvalue problem is that one component determines the potential change, the second component determines the point where the double layer occurs.

Double layers must be stable to exist. Clearly, the solution given here is stable to waves in the electron plasma frequency range because the electron velocity distribution is everywhere Maxwellian. On the low-density side, the stability situation is that of counterstreaming ion beams.[15-17] We shall confine our attention to electrostatic stability criteria. When the model of a magnetic field-free plasma is appropriate, zero-frequency modes of the ion-acoustic branch are most unstable.[15] A linear stability analysis[18] yields stability functions for both parallel propagating modes $S_{\parallel}$ and obliquely propagating modes S_o at the maximally unstable angle θ_M determined from $\tan\theta_M = (0.66)(\psi_o\tau - \Delta)^{1/2}$. The stability criteria are

$$\left.\begin{matrix} S_{\parallel} \\ S_o \end{matrix}\right\} = \left\{\begin{matrix} 1.00 \\ 1.28 \end{matrix}\right\} \tau e^{\psi_o+\Delta} \left[\pi(\psi_o\tau - \Delta)\right]^{-1/2} - \tau - 1 < 0 \ . \tag{13}$$

Figure 1c shows that solutions are stable to parallel propagating modes for all τ and to oblique modes for $\tau < 0.8$. We conjecture that there exist other distribution functions without the abrupt energy cutoff which satisfy the eigenvalue problem and which are stable for larger τ values.

If the potential drop occurs along a magnetic field, then we must address the question of stability with respect to electrostatic ion-cyclotron waves. Theory[16,17] experiment[17] and space observations[19] have shown that instabilities occur in this situation. An analysis shows that purely growing modes are unstable for distribution function Eq. (2), but that this conclusion depends on the abrupt energy cutoff. Again, future work must search for distributions which satisfy Eqs. (7) - (9) and remain stable to ion-cyclotron modes.

While the dynamics of the formation of a double layer are outside the scope of this letter, currentless double layers are consistent with the presence of a negatively-biased transparent grid. This can be seen by extending the definition of V to higher values of ψ

$$V(\psi_g, \Delta) = 2 \int_{\psi_0}^{\psi_g} d\psi\, G(\psi) > 0 , \tag{14}$$

$$\left(\frac{\partial\psi}{\partial\xi}\right)_{\psi_g} = \frac{\lambda_D eE}{T} = - \frac{\lambda_D e\sigma_S}{2T} = \left[V(\psi_g, \Delta)\right]^{1/2} , \tag{15}$$

where σ_S is the (negative) surface charge density of a grid at potential $\phi_g = -\psi_g T_e/e$. Hence, the introduction of a negatively charged grid in an otherwise symmetric plasma device such as a triple plasma device[1] or magnetic mirror could lead to the formation of a currentless double layer. The computational simulation analogy is the gradual buildup of a fixed, negative charge sheet.

In conclusion we have shown that solution of a two-component nonlinear eigenvalue problem determines the structure of a double layer and specific results for a currentless double layer are presented in Fig. 1. We argued that the physical information conveyed by the two components is the potential drop and the position of the double layer. Definitive statements regarding the kinetic stability with respect to electrostatic ion-cyclotron modes await future work.

ACKNOWLEDGMENTS

We are happy to acknowledge stimulating discussions with Drs. P. K. Kaw and H. Okuda.

This work was supported jointly by the U. S. Air Force Office of Scientific Research under Contract PR # 80-00656 with Princeton University and by the U. S. Department of Energy Contract No. DE-AC02-76-CH03073.

APPENDIX

Ion-Acoustic Stability of a Double-Layer Model

Ion-acoustic modes in a counterstreaming-ion-beam plasma are accurately described by a theory which ignores the effect of a magnetic field on the ions because the frequencies and growth rates (if unstable) of these modes are well above the ion-gyrofrequency Ω_i provided the inequality $\omega_{pi}^2 \gg \Omega_i^2$ holds. This inequality is strongly satisfied in most fusion and space plasmas. Forslund and Shonk[14] have shown that zero-frequency modes are the most unstable. Therefore, our stability criterion is that for marginally stable zero-frequency modes exist. A quasineutral dispersion relation provides a good description of ion-acoustic modes. Straightforward linear stability analysis yields the stability criterion

$$S = \frac{T_e}{M} \int \frac{dv}{v} \left(\frac{\partial f}{\partial v}\right) - 1 = \frac{\tau}{2} \int \frac{dw}{w} \frac{\partial f}{\partial w} - 1 \leq 0 \ , \qquad (A1)$$

where

$$w = v\left(\frac{M}{2T_i}\right)^{1/2} , \qquad \tau = \frac{T_e}{T_i} , \qquad (A2)$$

f is the one-dimensional velocity distribution along the direction of the wavevector, and the equality is the marginal stability condition. Let $\hat{z}$ be the direction parallel to the double-layer potential drop and $\hat{x}$ an orthogonal direction in which the ion velocity distribution will remain Maxwellian. Let (s, u) denote nondimensional velocities in the $(\hat{x}, \hat{z})$ directions respectively and let the wavevector $\vec{k}$ be in the $\hat{n}$ direction

$$\vec{k} = k\hat{n} = k(n_x\hat{x} + n_z\hat{z}) \ . \tag{A3}$$

Our double-layer model then gives the two-dimensional velocity distribution

$$h(s, u) = \begin{cases} (c/\pi)e^{-(s^2+u^2)} & u^2 > u_o^2 \\ 0 & u^2 < u_o^2 \end{cases} , \tag{A4}$$

where c is a normalizing constant

$$c^{-1} = (2/\sqrt{\pi}) \int_{u_o}^{\infty} e^{-u^2} du \ , \tag{A5}$$

and $u_o = (\psi_o\tau - \Delta)^{1/2}$. The one-dimensional velocity distribution f(w) is

$$f(w) = \int_{-\infty}^{\infty} ds \int_{|u|>u_o}^{\infty} du \ (c/\pi)e^{-(s^2+u^2)} \ \delta(n_x s + n_z u - w) \ , \tag{A6}$$

$$= \int_{|u|>u_o}^{\infty} (\frac{c}{n_x \pi}) du \ \exp[-(\frac{w - n_z u}{n_x})^2 - u^2] \ . \tag{A7}$$

Evaluating $\partial f/\partial w$ and using the resulting expression in Eq. (A1), one finds

$$s + 1 = -\tau \int_{-\infty}^{\infty} (\frac{c}{\pi n_x^3}) dw \int_{|u|>u_o}^{\infty} \exp[-(\frac{w - n_z u}{n_x})^2 - u^2] du$$

$$+ \tau \int_{-\infty}^{\infty} (\frac{c n_z u}{w \pi n_x^3}) dw \int_{|u|>u_o}^{\infty} du \ \exp(-\frac{w^2}{n_x^2} - \frac{u^2}{n_x^2} + \frac{2wn_z u}{n_x^2}) \ . \tag{A8}$$

The next step is to interchange the order of integration and perform the w-integration

$$S + 1 = -\frac{\tau}{n_x^2} + \tau \int_{u_o}^{\infty} du \left(\frac{2n_z cu}{\sqrt{\pi}\, n_x^3}\right) e^{-(u^2/n_x^2)} I\left(\frac{2n_z u}{n_x}\right) , \qquad (A9)$$

where

$$I(\alpha) = \int_{-\infty}^{\infty} \frac{d\eta}{\sqrt{\pi}\,\eta} e^{-\eta^2} \sinh(\alpha\eta) = 2 \int_0^{\alpha/2} e^{t^2} dt . \qquad (A10)$$

The second term in Eq. (A9) can be integrated by parts to give

$$S + 1 = -\tau + \frac{\tau c}{\sqrt{\pi}\, u_o} e^{-u_o^2} F\left(\frac{n_z u_o}{n_x}\right) , \qquad (A11)$$

where the function

$$F(\beta) = 2\beta^2 \int_0^1 e^{\beta^2(t^2-1)} dt = \beta e^{-\beta^2} I(2\beta) , \qquad (A12)$$

depends only on the orientation of the wavevector. Two orientations are of interest: First, parallel propagation where $\beta \to \infty$ and $F = 1$. Secondly, the direction for which $\beta = 1.51$ where F achieves its maximum value $F = 1.28$. Combining the relation $c = e^{\psi_o(\tau+1)}$, derived from Eq. (7), with Eq. (A11), one obtains the stability functions of Eq. (13).

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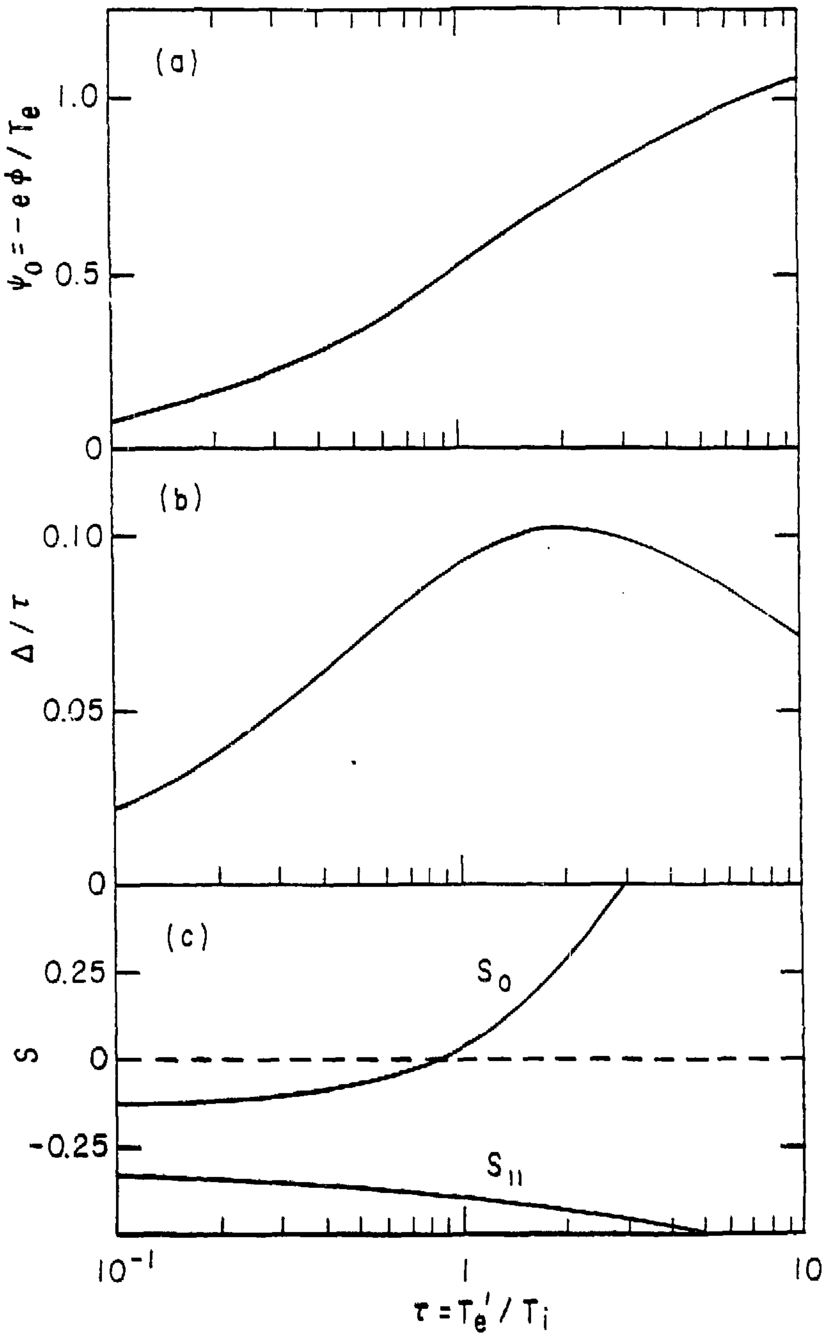

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Fig. 1(a,b). Solutions of the nonlinear eigenvalue problem: Note that Δ/τ is quite constant. (c). The stability functions S and S_0 [Eq. (13)]. S < 0 represents stability.

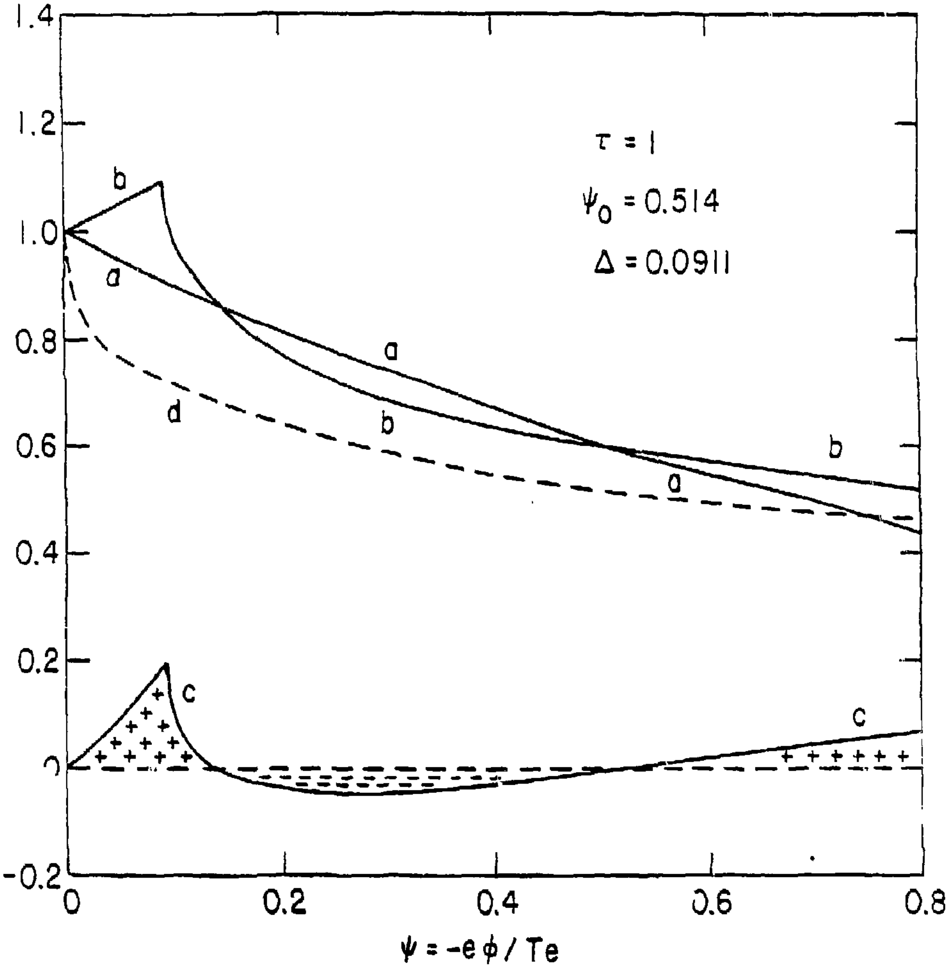

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Fig. 2. Electron and ion densities as a function of potential for $\tau = 1$. Curve a is the electron density $e^{-\psi}$. Curve b is the ion density $g(\psi,\Delta)$ [Eq. (6)]. Curve c is the difference $G(\psi,\Delta)$ and depicts regions of positive and negative charge density. Dashed curved would be the ion density if $\Delta = 0$. It is evident that the required region of positive charge density cannot exist for $\Delta = 0$.