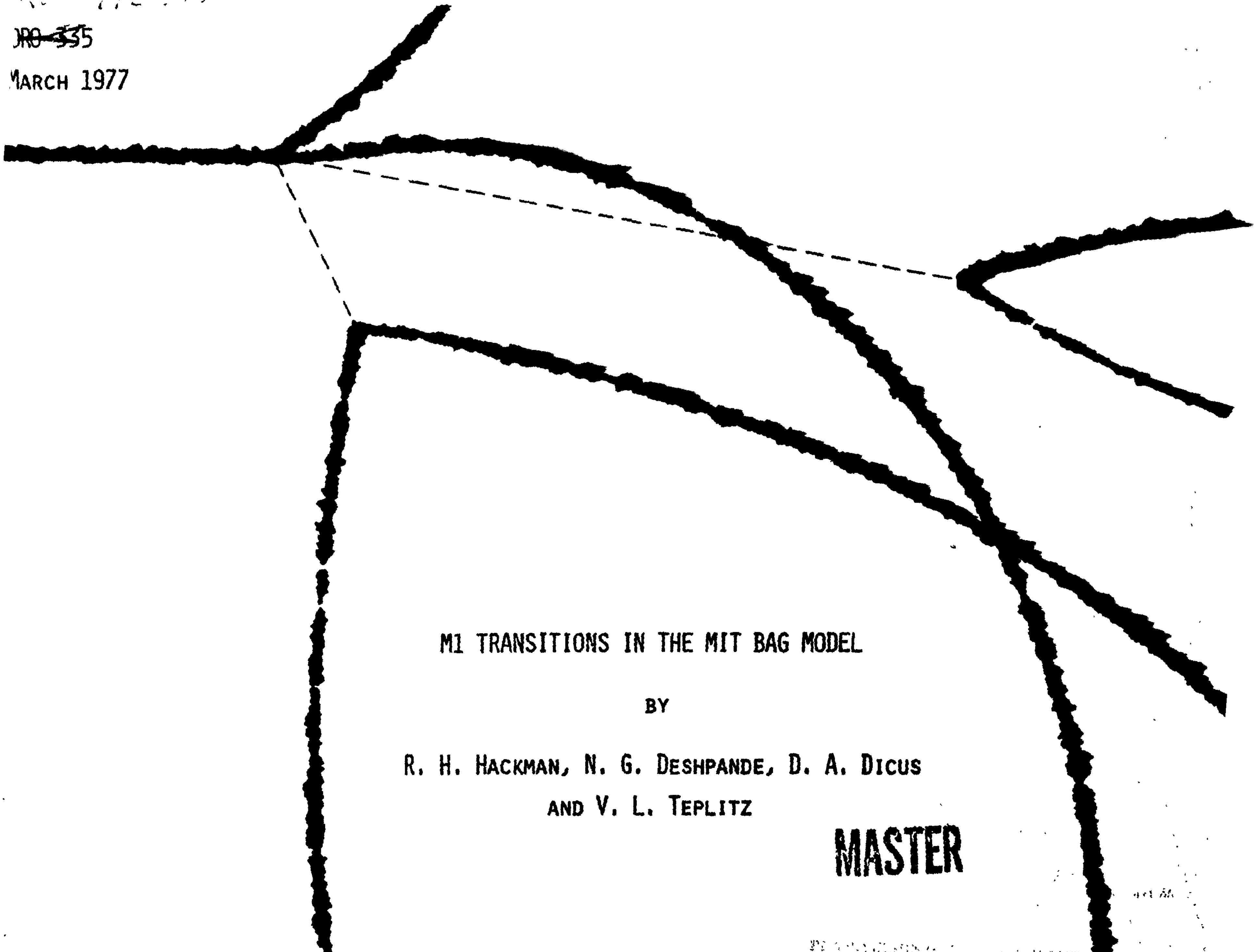
ORO-335
MARCH 1977



M1 TRANSITIONS IN THE MIT BAG MODEL

BY

R. H. HACKMAN, N. G. DESHPANDE, D. A. DICUS
AND V. L. TEPLITZ

Abstract

We investigate, in the MIT bag model, the M1 transitions of low lying hadrons. We perform the following calculations: (1) We recompute 32 hadron masses with a choice of bag parameters designed to give the correct values for the proton magnetic moment, μ_p, and several masses, $M_{[illegible]}$, M [illegible], M, [illegible] and M_p; (2) We [illegible] ... [illegible] mixing in an untrustworthy approximation; and (3) We compute the widths for 38 M1-transitions.

I. Introduction

In a series of papers[1-7] the properties of the simple, plausible, well-determined, confined-quark, M.I.T. bag model have been explored. With only a few adjustable parameters the model has a variety of successful qualitative and quantitative predictions. The purpose of the present paper is to apply the model to M1 transitions ($\Delta \rightarrow N + \gamma$, $\rho \rightarrow \pi + \gamma$, etc.).

Within the model, the most extensively studied systems are static bags, those with s-wave quarks, without radial excitation, and without excitation of the bag surface. Most of the work to date has been within the framework of the spherical cavity approximation in which the bag shape is not properly treated as a dynamical variable and the quark wave functions are parameterized by a (static) bag radius. The present work is also within this approximation, and, to a great extent, its chief result is to illuminate the limitations of the approximation.

The usual bag parameterization gives a proton magnetic moment too small by about 30%: In Section II we introduce a new parameter into the bag energy, adjust it so that the proton magnetic moment is predicted correctly, and then recalculate the masses, radii, magnetic moments, charge radii, and axial vector coupling constants of static bags. In Section III we calculate baryon and boson, static bag, M1-transitions. Calculations involving isoscalar, pseudoscalar mesons suffer from

additional uncertainty arising from mixing induced by transitions to pure gluon states. We segregate these calculations into an appendix. In Section IV we conclude briefly. We include in our calculations the full panoply of static bags within SU_4 symmetry[8].

Static bag M1 transitions for a few key cases have been studied by Hays and Ulehla[9] and by Katz and Tatur[10]. These authors used the usual bag parameterization with the bad prediction for the proton magnetic moment and thus could not be expected to get good results for transitions like $\Delta \rightarrow N + \gamma$. In addition, in Reference 10, a simplifying assumption was made in evaluating the integrals. Related work on non-M1 transitions to non-static bag states has been performed by Hey, Holstein, and Sidhu[11]. Many other authors have discussed M1 transitions within other models[12].

II. Recalculation of Bag Particle Properties

A. The Bag Model

The calculations of quark wave functions, bag masses, and bag radii has been reviewed in several places[3,7]. Its essential features are: (1) The free Dirac equation is solved for each quark subject to a linear boundary condition that ensures the vanishing at the surface of the bag of all (vector) currents carrying quantum numbers. (2) Color gluon fields interacting with quark fields are included; their electric and magnetic energies are calculated; (3) The total energy is found from adding to the energies found in (1) and (2) a volume bag energy and a zero point energy; (4) A quadratic boundary condition is imposed that the sum of the quark and gluon pressures balance the external pressure, B, locally on the bag's surface. For the $\ell = 0$ states with no radial or surface excitations (static bags) this last step reduces to performing the above steps for varying bag radius R and then minimizing the total energy as a function of R. This calculation was performed in Reference 3 for non-charmed quarks and in Reference 4 for charmed quarks. The free parameters were fit to the N, Δ, ω, Ω and ψ masses. Good results were obtained for ratios of magnetic moments to the proton magnetic moment. The proton magnetic moment itself, however, came out too small by about one-third. In calculating M1 transitions it seems clearly desirable to add an extra term phenomenologically, preferably one that has some theoretical justification, so as the ensure a good value of other magnetic quantities including μ_p. Kuti[13] has pointed out that a confinement scheme based on a surface tension rather than a volume pressure provides an alternative basis for a phenomenology. Including both a volume energy $\frac{4}{3}\pi BR^3$ and a surface energy $4\pi SR^2$ and refitting all bag masses and radii while adjusting S to produce the correct value of μ_p is not, however, possible. The bag model makes a strong correlation between boson and fermion masses and radii. Within the parameterization of Reference 3, the only differences

between the forms of boson and fermion masses are an extra quark kinetic energy and different, but fixed, coefficients for the magnetic gluon energy. Both energies vary as 1/R for bags of non-strange quarks; μ_p, on the other hand, varies as R. Thus it is not possible to add any extra term to the energy that has the same form for mesons and baryons and fit μ_p, M_p, M_ω and M_Δ.

A simple term that one can add to the energy, which does differ between baryons and mesons is $C|N_q - N_{\bar{q}}|$ where N_q and $N_{\bar{q}}$ are the numbers of quarks and antiquarks, with C a constant. Such a term is a crude approximation to effects arising from differences between mesons and baryons in valence quark, higher order, gluon exchange diagrams.

With this addition the expression for the energy is

$$M(R) = E_V + E_0 + E_Q + E_M + E_E + \Delta E \tag{1}$$

where the six terms are:

(i) the bag volume energy

$$E_V = \frac{4}{3}\pi BR^3 \tag{2}$$

(ii) the finite part of the zero point energy

$$E_0 = -\frac{Z_0}{R} \tag{3}$$

(iii) the sum of single particle quark energies

$$E_Q = \sum_i \omega_i \tag{4}$$

(iv) the color magnetic energy

$$E_M = -4\pi\alpha_c \sum_a \sum_{i>j} \int_{Bag} d^3x \vec{B}_i^a \cdot \vec{B}_j^a \tag{5}$$

where α_c is the square of the rationalized quark-gluon coupling constant, a is the color index and i and j are quark labels;
(v) the color electric energy

$$E_E = 2\pi\alpha_c \sum_a \sum_{i,j} \int_{Bag} d^3x \vec{E}_i^a \cdot \vec{E}_j^a \tag{6}$$

(vi) the "bag type" energy, not present in previous papers,

$$\Delta E = C|N_q - N_{\bar{q}}| \tag{7}$$

E_M and E_E are evaluated in detail in Reference 3. The quark energy ω is given by

$$\omega(m,R) = \frac{1}{R}[x^2 + \lambda^2]^{1/2} \tag{8}$$

where $\lambda = mR$ and x obeys the eigenvalue equation

$$\tan x = x/[1 - \lambda - (x^2 + \lambda^2)^{1/2}] \tag{9}$$

Magnetic moments are given by

$$\mu = \frac{R}{6} \frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha - 1) + \lambda} \tag{10}$$

where $\alpha = R\omega$. We recall that for quark mass $m = 0$, the solution of (9) is $x(mR) = x(0) = 2.0428$. We now determine the parameters B, Z_o, α_c, m, m_c, and C by fitting (1) to the masses M_ω, M_P, M_Δ, M_Ω, and M_D and (10) to the value of the proton magnetic moment, μ_P. In Table I we compare the present values of the parameters with the results of References 3 and 4. In Tables II and III we compare the values of the masses and radii for baryons and mesons. In general, the fit to experimental particle masses[14] of the new parameterization is as good as that of References 3 and 4. It is significantly worse in only one case, that of the K-meson mass which is now too high by about 50 MeV. The π meson is similarly too high as it was in Reference 3; following this paper's discussion of the effects of raising the light quark mass from zero, one infers that raising it in this case would give the K mass correctly and move the π mass toward 140 MeV. We therefore give a second solution of the parameters with $m_\prime$ not equal zero and adjusted such as to give proper value for the kaon mass. The parameters of this solution (B) are also given in Table I and the corresponding mass and radius values are in Tables II and III. One sees that the pion mass moves, in going from solution A to solution B, by about 40 MeV toward its experimental value. Also the proton radius increases in B to compensate for the decrease in the light quark magnetic moment.

In Table IV we give values, deduced from the quark wave function according to the prescription of Table I of Reference 3, for static magnetic moments and electric charge radii for some particles, comparing with experiment where possible[14,15]. The axial vector coupling constant can also be deduced from Table I of Reference 3. Table V gives values for the correction factor to the SU(6) values. For example, for $n \to pe\bar{\nu}$, the ratio g_A/g_V as given by the parameters of Reference 3 would be 0.653 times the SU(6) value, 5/3, namely 1.09. Our case B gives a better value, 0.717, so we find g_A/g_V = 1.20 which is closer to the experimental value of 1.25. Thus one sees that the new $C \neq 0$ parameterization gives at least as good a fit to these static quantities as the C = 0 parameterization of References 3 and 4, while having the important advantage of giving quark wave functions that give the proper μ_p. For some quantities, such as g_A, the fit is much better. We will use these solutions in Section III in calculating M1 transition rates but two important reservations as to their creditability are necessary: (1) Both solutions give a large quark gluon coupling constant, $\alpha_c \approx 1$, while the expressions of (5) and (6) are explicitly first order in α_c. Inclusion of higher order contributions could make important modifications in predicted bag

wave functions so that the present calculations of M1 transition rates, while suggestive, cannot be considered definitive bag tests. (2) For $c\bar{c}$ and 3c states there is little quark kinetic energy and inclusion of higher order terms in α_c is considered to be still more important. This is because, in the absence of outward quark pressure to balance the inward bag pressure, one expects gluon pressure to provide the chief balancing force. A treatment using, instead of the approximation of Reference 3, a Born-Oppenheimer approach is in progress[16]. M1 transitions involving $c\bar{c}$ and 3c systems in the present paper must therefore be considered a merely suggestive basis for comparison with the results expected from the Born-Oppenheimer approach.

We note that both solution A ($m_{\ell} = 0$) and solution B ($m_{\ell} \neq 0$) are surprisingly accurate for the masses of Σ_c and Λ_c when compared to the tentative experimental values given in Reference 14. The "old" bag results of Reference 4 are not so close. Because Reference 4 fixed m_c from ψ while we fix m_c from D, this result tends to give credence to both the general bag picture and to assertion (2) above that the techniques of References 3 and 4 and the present paper are less reliable for the $c\bar{c}$ and 3c states.

III. The M1 Transition Rates

We now calculate the M1-transition rates. Following Reference 6, electromagnetic transition rates are given by the imaginary part of the magnetic contribution to the mass difference,

$$\Gamma/2 = \mathrm{Im}\left\{\frac{e^2}{(2\pi)^2}\sum_{\eta,\eta'}\int_{\mathrm{Bag}} d^3x\,d^3y\;\frac{\sin k|\vec{x}-\vec{y}|}{|\vec{x}-\vec{y}|}\right.$$

$$\left.\langle i,\eta|J_\mu(\vec{x},0)|f,\eta\rangle\langle f,\eta'|J^\mu(y,0)|i,\eta'\rangle\right\} \tag{11}$$

In (11) the sum on (η,η') is a sum over the emitting quark; both $\eta = \eta'$ and $\eta \neq \eta'$ are to be kept. J_μ is given by

$$J_\mu(x) = \sum_\alpha Q_\alpha \bar{q}_\alpha(x)\gamma_\mu q_\alpha(x) \tag{12}$$

where Q_α is the charge of the α-th quark . Expanding (12) yields

$$\vec{J}(\vec{x},0) = -\frac{1}{4\pi}\sum_\alpha\sum_{m_1,m_2} N_{\alpha_1}N_{\alpha_2}$$

$$\times\, b^{+}_{\alpha_2}(m_2)Q_\alpha b_{\alpha_1}(m_1)U^{+}_{m_2}(\hat{x}\cdot\vec{\sigma})U_{m_1}$$

$$\times\left[j_0\left(\frac{x_{\alpha_1}x}{R_1}\right)j_1\left(\frac{x_{\alpha_2}x}{R_2}\right)\left(\frac{\omega_\alpha + m}{\omega_{\alpha_1}}\right)^{1/2}\left(\frac{\omega_{\alpha_2} - m}{\omega_{\alpha_2}}\right)^{1/2}\right.$$

$$\left.+\, j_0\left(\frac{x_{\alpha_2}x}{R_2}\right)j_1\left(\frac{x_{\alpha_1}x}{R_1}\right)\left(\frac{\omega_{\alpha_1} - m}{\omega_{\alpha_1}}\right)^{1/2}\left(\frac{\omega_{\alpha_2} + m}{\omega_{\alpha_2}}\right)^{1/2}\right] \tag{13}$$

Here N_α is the quark normalization

$$N_\alpha^{-2} = R^3[j_0(x_\alpha)]^2\,\frac{2\omega_\alpha(\omega_\alpha - \frac{1}{R}) + \frac{m_\alpha}{R}}{\omega_\alpha(\omega_\alpha - m_\alpha)} \tag{14}$$

and we have used the expression for the quark wave function

$$q(r,t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} \left(\frac{\omega + m}{\omega}\right)^{1/2} i j_0\left(\frac{xr}{R}\right) U \\ -\left(\frac{\omega - m}{\omega}\right)^{1/2} j_1\left(\frac{xr}{R}\right) \vec{\sigma}\cdot\hat{r} U \end{pmatrix} \tag{15}$$

The expression in brackets in (13) takes into account the fact that the general wave function in the initial (i) and final (f) bags are different. In (11) we use the expression

$$\frac{e^{ik|x - y|}}{|x - y|} = 4\pi i k \sum_{\ell=0}^{\infty} j_\ell(kx) h_\ell^{(1)}(ky)$$

$$\times \sum_{m=-\ell}^{\ell} Y_{\ell m}(\Omega_x) Y_{\ell m}(\Omega_y) \tag{16}$$

We then obtain for the full M1 width

$$\Gamma = \frac{e^2}{4\pi} k^3 \left(\frac{16}{3}\right) \sum_{\substack{\alpha,\beta \\ \alpha\geq\beta}} \mu_\alpha \mu_\beta C_{\alpha\beta}^{PQ} \tag{17}$$

where the transition moment, μ_α, is given by

$$\mu_\alpha = \frac{1}{2k} N_{\alpha_1} N_{\alpha_2} \int_0^R dx x^2 j_1(ky)$$

$$\times \left[j_0\left(\frac{x_{\alpha_1} x}{R_1}\right) j_1\left(\frac{x_{\alpha_2} x}{R_2}\right) \left(\frac{\omega_{\alpha_1} + m_\alpha}{\omega_{\alpha_1}}\right)^{1/2} \left(\frac{\omega_{\alpha_1} - m_\alpha}{\omega_{\alpha_2}}\right)^{1/2} \right.$$

$$\left. + j_0\left(\frac{x_{\alpha_2} x}{R_2}\right) j_1\left(\frac{x_{\alpha_1} x}{R_1}\right) \left(\frac{\omega_{\alpha_2} + m_\alpha}{\omega_{\alpha_2}}\right)^{1/2} \left(\frac{\omega_{\alpha_1} - m_\alpha}{\omega_{\alpha_1}}\right)^{1/2} \right] \tag{18}$$

In the limit of $k \to 0$ and $R_1 = R_2$, this reproduces the expression for the static magnetic moment of Reference 3. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ are, as in Reference 6, given by

$$C_{\alpha\beta}^{PQ} = \sum_{m_1,m_2} \sum_{k_1,k_2} \langle P|b_\alpha^+(m_2) Q_\alpha b_\alpha(m_1)|Q\rangle$$

$$\times \langle Q|b_\beta^+(k_1) Q_\beta b_\beta(k_2)|P\rangle U_{m_2}^+ \sigma^i U_{m_1} U_{k_1}^+ \sigma_i U_{k_2} \tag{19}$$

The $C_{\alpha\beta}^{PQ}$ are listed in Tables VI and VII for the cases of interest to the present work. In Tables VIII and IX we give the values of the transition moments, from (18), for fermions and bosons for the two cases considered in Section II.

In evaluating the integrals in (11) or (18) it is necessary to choose the value of the upper limit of the integral and its relation to the radii R_1 and R_2 in the integrands. We have chosen the following ansatz: given R_1 and R_2, for a particular transition, from the results of Section II we take $R = \frac{1}{2}(R_1 + R_2)$; with this value of R we recalculate the eigenvalue x(R) in (9) and replace R_1 and R_2 by R everywhere in (18). Then $\alpha_1 = \alpha_2 = \alpha$. The motivation for this ansatz is the realization that the static cavity model of Reference 3 is an approximation to the "real" physical situation in which the bag shape is a *dynamical* variable with its own wave function. It should be recognized that our procedure gives a slightly larger transition moment than choosing $R = \min(R_1,R_2)$.

An important point to notice is that the r-dependence of the current operator in the transition moment is

$$\frac{1}{k}j_1(kr) = O_{Tr}$$

in comparison to the simple static dependence

$$O_s = r$$

This difference tends to cut down the size of the transition moment for $kr \gtrsim 1$. For transitions in this region it means that experimental static and transition moments provide

independent information; it is not true that changing the bag parameterization to ensure a large R and a correct μ_p, as was done in Section II, will give a large moment for the corresponding transition, $\Delta \to P + \gamma$. For a given transition, once $kR \gtrsim 1$, increasing R becomes progressively less effective in increasing the transition moment, μ_{Tr}. Roughly, μ_{Tr} measures the product of one large and one small component of the two different wave functions at $r \approx \frac{1}{k}$ while the static moment, μ_s, measures the product of large and small components of the same wave function at $r = R$.

In Tables X and XI, we give the results of the two (A for $m_\ell = 0$, B for $m_\ell \neq 0$) bag parameterizations for M1 radiative widths.

The comparison between the predicted and experimental values for the measured cases (ΔN, $\rho\pi$, $\omega\pi$, and K^*K) is not spectacular. The 9:1 ratio between $\omega\pi$ and $\rho\pi$ is a result of $\omega - \rho$ degeneracy in any quark model and no amount of adjusting the model will ever change it. The ΔN value is too small, despite our parameterization that gives a good value for μ_p, for the reasons mentioned above. In this regard it is interesting to notice that all the cases correspond to $kR \gtrsim 1$.

IV. Discussion

Consider first our reparameterization of the static bag model. This gives, as discussed, reasonably good results for magnetic moments and masses (except for K and, of course, π), satisfactory results for axial vector coupling constants, and not unreasonable results for charge radii. It cannot, however, be claimed that there is any evidence for the form of our reparameterization -- the addition of $C|N_q-N_{\bar{q}}|$ to the energy. In the first place it might be that the solution of the problem posed by the low value of μ_p lies in an anomolous quark magnetic moment rather than in a larger proton radius although this seems somewhat outside the spirit of a quark model. But even if modification of the static bag wave functions is called for, our addition to the Hamiltonian is not unique; other changes, such as making the constant Z_o in the zero point energy different for baryons than for bosons, are possible. Our value for C does, however, indicate the size of the necessary change.

The universal slope, α', of Regge trajectories is another semi-measurable quantity. Johnson and Thorn[2] have shown that for massless light quarks, α' is proportional to $(B\alpha_c)^{\frac{1}{2}}$ and

that the old bag parameters give a good numerical value when compared to slopes derived from particle spectroscopy. Our reparameterization gives a larger α_c but a smaller B so its corrections are appropriately directed. Numerically, however, α' falls for Model A(B) to 63% (55%) of its "old bag" value; we consider this comparison uncomfortable but not necessarily compelling.

Consider now the M1-transitions obtained with models A and B. One of our most important results is probably the realization that increasing the bag radius for Δ and P in order to fit μ_p does not give the M1-transition rate correctly. The rate for $\Delta \to N\gamma$ is suppressed by two other factors: (1) the maximum value of the matrix element of the current in Eq. (13) falls with increasing $\Delta R = R_\Delta - R_N$; this tends to suppress transition moments relative to static moments. (2) The ratio $j_1(kR)/kR$ is approximately $1 - k^2R^2/10$; for Δ - N this effect suppresses the transition moment relative to the static moment by about 40% for our parameterization and 20% in the "old bag". The result of these two effects is that the rate for $\Delta \to N\gamma$ in the "old bag" of Ref. 3, 200 keV, is not easily brought into agreement with experiment. If the rate is scaled upward by $\left[(\mu_p)_{exp}/(\mu_p)_{Bag}\right]^2 \simeq 2$, in a crude attempt to take into account an anomolous quark magnetic moment, it is still 14 standard deviations short of experiment. It should be noted that introdcution of anomolous quark moments in the bag model raises calculational difficulties in evaluating the contribution of hadron center of mass motion to momentum dependent terms; the q in $\bar{\psi}_j \sigma_{\mu\nu} q^\nu \psi_i$ is $p_j - p_i$ where $p_i = -i(\partial_R + \partial_r)$.

Our value for $K^* \to K\gamma$ is well within experimental error with either the predicted or experimental masses but, since our K mass is too heavy and the error is large this is, at best, a modest success. The ratio of $\eta' \to \rho\gamma$ to $\eta' \to \omega\gamma$ has recently been measured accurately.[7] Our solutions for the mixing, as explained in the appendix, make the η essentially pure $s\bar{s}$ so that we can only predict zero for both these decays. Finally the radiative decays into pions are not, because of the large momentum transfer involved, expected to be reliable in the static bag approximation.

In general the available data is much too limited to make a meaningful assessment of bag predictions or to serve as a guide for modifications of the model. It is highly desirable for this model and others[12] that further careful M1 measurements be undertaken.

Acknowledgement

It is a pleasure to acknowledge stimulating conversations with and helpful advice from Professor Kenneth Johnson.

Appendix

We treat here M1-transitions involving the isoscalar pseudoscalar mesons (η, η', η_c). It is generally thought that within both the bag model and other confinement schemes, these particle states are dynamically mixed through the SU_4-singlet, two colored gluon, intermediate state. Each is a linear combination of $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ where $L\bar{L}$ is the isosinglet combination of $u\bar{u}$ and $d\bar{d}$ quarks. Our approach, following that of Reference 3, is to make a three parameter expression for the mixing and to determine the three parameters by requiring the eigenvalues of the mass matrix, when minimized with respect to the radius R, to be the experimental masses. This approach suffers from two difficulties: (1) The $c\bar{c}$ system, as discussed in Section II, should not be treated to lowest order in α_c; (2) One result of the static cavity approximation in which the radius, R, is treated as a fixed parameter rather than a quantum mechanical variable is that the wave functions we find will not be orthogonal if they have different radii. The orthogonality that they must enjoy by virtue of the self-adjointness of the bag Hamiltonian with the two bag boundary conditions requires the R wave functions to check numerically. In the present approximation bag states having different R's are considered orthogonal by virtue of the unknown R wave function.

We write the mass matrix as

$$\mathcal{M} - MI = \begin{pmatrix} E_0 - M + 2\beta & \sqrt{2}\beta & \sqrt{2}\beta \\ \sqrt{2}\beta & E_s - M + \beta & \beta \\ \sqrt{2}\beta & \beta & E_c - M + \beta \end{pmatrix} \quad (A.1)$$

Here E_0, E_s, and E_c are the $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ system energies as determined by (1) in Section II. M is the eigenvalue to be determined. β is the singlet state two gluon mixing. The two gluon state is assumed, by its singlet nature, to couple equally to $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ states; hence the couplings to $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ are in the ratio $\sqrt{2}$, 1, 1. We parameterize β as follows:

$$\beta = a + bM + \frac{c}{R} \quad (A\cdot 2)$$

We must solve for the six quantities a, b, c, R_η, $R_{\eta'}$, and R_{η_c}. We define $f(R,\beta(R,M),M)$ by

$$f(R,\beta(R,M),M) = \det(\mathcal{M} - MI) \quad (A\cdot 3)$$

We have three mass equations

$$f(R_i,\beta(R_i,M_i),M_i) = 0 \quad i = \eta, \eta', \eta_c \quad (A\cdot 4)$$

and three radius equations

$$0 = \frac{dM}{dR} = [\frac{\partial f}{\partial M} + \frac{\partial f}{\partial \beta}\frac{\partial \beta}{\partial M}]^{-1}[\frac{\partial f}{\partial R} + \frac{\partial f}{\partial \beta}\frac{\partial \beta}{\partial R}] \quad (A.5)$$

We have solved (A.4) and (A.5) numerically for the $m_{\ell} = 0$ bag parameter cases of Table I. The results for R_i, a, b, and c, and the $\ell\bar{\ell}$, $s\bar{s}$, and $c\bar{c}$ mixing are shown in Table XII. The mixing parameters are defined as

$$\alpha_i|\ell\bar{\ell}\rangle + \beta_i|s\bar{s}\rangle + \gamma_i|c\bar{c}\rangle \quad (A.6)$$

No solution to (A.4) and (A.5) was found using the $m_{\ell} \neq 0$ case from Table I. Using these results we may calculate the M1 widths involving the η's by proceeding as in Section III and then multiplying by α_i^2, β_i^2, or γ_i^2 as appropriate for the quark structure of the meson. It is important to remember to multiply the pseudoscalar into vector plus photon rates by two to take into account the sum over final helicity states. The results of these calculations are given in Table XIII.

References

1. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).
2. A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974).
3. T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); for an expanded discussion of the axial vector constant see E. Golowich, Phys. Rev. D 12, 2108 (1975) and J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rev. D 12, 2875 (1975).
4. R. L. Jaffe and J. Kiskis, Phys. Rev. D 13, 1355 (1976).
5. F. E. Low, Phys. Rev. D 12, 163 (1975).
6. N. G. Deshpande, D. A. Dicus, K. Johnson, and V. L. Teplitz, Phys. Rev. D 15, 1885 (1977).
7. The bag equations and their solutions are reviewed by K. Johnson, Acta Phys. Polon. B6, 865 (1975), and in a more general context by P. Hasenfratz and J. Kuti, Physics Reports (to be published).
8. P. Tarjanne and V. L. Teplitz, Phys. Rev. Lett. 11, 447 (1963); D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Phys. Lett. 11, 190 (1964); J. D. Bjorken and S. L. Glashow, ibid. 11, 255 (1964); Y. Hara, Phys. Rev. 134, B701 (1964); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
9. P. Hays and M. V. Ulehla, Phys. Rev. D 13, 1339 (1976).
10. J. Katz and S. Tatur, Institute fur Theoretische Physik der Freien Universitat Berlin, Preprint (Jan. 1977). These authors make the approximation $j_1(kr)/k \approx \frac{1}{3}r$ derived in Sec. III.

11. A. Hey, B. R. Holstein, and D. P. Sidhu, Radiative Transitions in the MIT Bag, University of Massachusetts Preprint (1977).
12. R. Von Royen and V. F. Weisskopf, Nuovo Cimento L A, 617 (1967); H. Fritzsch and J. D. Jackson, CERN preprint Th 2264 (December 1976); J. D. Jackson, 1976 SLAC Summer Institute on Particle Physics, Lecture No. 4; D. H. Boal, Phys. Rev. Letters 37, 1333 (1976); J. Sucher, Relativistic Magnetic Dipole Transitions in Atoms, Ions, and Psions, Invited Talk presented at the Fifth International Conference on Atomic Physics held at the University of California, Berkeley, Calif., July, 1976 (Maryland Technical Report No. 77-029); M. T. Vaughn, Phys. Rev. D 13, 2621 (1976); N. Chase and M. T. Vaughn, Radiative Decays in a Quark Loop Model, Northeastern University preprint (Oct. 1976) and Physics Letters 61B, 175 (1976); A. Bohm and R. Teese, Phys. Rev. Letters 38, 629 (1977).
13. P. Hasenfratz, J. Kuti, and A. S. Szalay, Central Research Institute for Physics, Budapest Preprint KFKI-75-30 (1975); presented by J. Kuti at the Xth Rencontre de Moriond, 1975 (Meribel les Allues).
14. Particle Data Group, Rev. Mod. Phys. 48, No. 2, Part II (1976). For charmed particles see the supplement: Phys. Letters 68B, 1 (1977). Recent results on D and D* are given by I. Peruzzi et al., Phys. Rev. Letters 39, 1301 (1977).

15. A new measurement of the pion charge radius is given by E. B. Dally et al., Phys. Rev. Letters 39, 1176 (1977). This group has a measurement of the K charge radius in progress. Hyperon charge radius measurements at FNAL are possible in the future. We are grateful to T. Toohig for a discussion of the experimental situation.
16. K. Johnson, in preparation.
17. C. J. Zanfino, et al., Phys. Rev. Letters 38, 930 (1977). We thank Professor Arno Bohm for informing us of this reference.

Table Captions

Table I. Values of the bag parameters. Energies are in GeV and distances are in GeV^{-1}.

Table II. Baryon masses (GeV) and radii (GeV^{-1}) for the three sets of parameters of Table I.

Table III. Boson masses (GeV) and radii (GeV^{-1}) for the three sets of parameters of Table I.

Table IV. Magnetic moments and charge radii for cases of interest for the three sets of bag parameters of Table I.

Table V. Correction factors for the SU(6) values of g_A/g_V.

Table VI. The magnetic transition coefficients $C^{PQ}_{\alpha\beta}$ as defined in (19) for fermions.

Table VII. The magnetic transition coefficients $C^{PQ}_{\alpha\beta}$ of (19) for bosons. The parameters α_i, β_i, and γ_i are given in Table XII.

Table VIII. Baryon quark transition moments, μ_α, from (18). They are calculated by using the bag masses of Table II, not the experimental masses.

Table IX. Boson quark transition moments, μ_α, from (18). They are calculated using the bag masses from Table II.

Table X. Baryon M1 transition rates for the two cases of Table I (Solution A is $m_{/}$ equal to zero and solution B is $m_{/}$ equal to 79 MeV). The rates in parenthesis use experimental masses for each particle with the radius of Table II.

Table XI. Boson M1 transition rates for the two cases of Table I. The values in parenthesis use the experimental particle masses.

Table XII. (a) Values of the η, η', and η_c radii and of the mixing parameters in the parameterization of Equation (A.2). (b) Values of the η, η', and η_c direction cosines along $|L\bar{L}\rangle$, $|s\bar{s}\rangle$, and $|c\bar{c}\rangle$ for the $m_{/} = 0$ solution.

Table XIII. M1 transition rates involving isoscalar, pseudoscalar mesons for the $m_{/} = 0$ solutions of Tables III and XII.

Table I

	B	Z_0	α_c	m_l	m_s	m_c	C
Ref. 3 and 4	4.54×10^{4}	1.84	0.55	0	0.279	1.551	-
<u>This paper</u>							
Solution A	1.21×10^{4}	0.567	0.826	0	0.266	1.506	0.139
Solution B	6.65×10^{-5}	0.497	1.16	0.079	0.317	1.534	0.139

Table II

			Ref. 3 or 4		This paper Soln. A		This paper Soln. B	
Particle	Quarks	M_{exp}	M	R	M_A	R_A	M_R	R_B
P	$\ell\ell\ell$	0.938	0.938	5.00	0.937	7.34	0.938	8.27
Λ	$\ell\ell s$	1.116	1.105	4.95	1.109	7.24	1.110	8.10
Σ	$\ell\ell s$	1.189	1.144	4.95	1.165	7.25	1.170	8 19
Ξ	ℓss	1.321	1.289	4.91	1.304	7.13	1.308	7.97
N*(Δ)	$\ell\ell\ell$	1.236	1.233	5.48	1.236	8.17	1.236	9.61
Σ*	$\ell\ell s$	1.385	1.382	5.43	1.386	8.07	1.385	9.45
Ξ*	ℓss	1.533	1.529	5.39	1.528	7.98	1.530	9.27
Ω^-	sss	1.672	1.672	5.35	1.666	7.88	1.672	9.10
$C_1(\Sigma_c)$	$\ell\ell c$	2.430	2.357	4.78	2.495	7.65	2.428	8.41
$C_0(\Lambda_c)$	$c(\ell\ell)_{anti}$	2.260.	2.214	4.63	2.253	6.84	2.257	7.68
S	$c(s\ell)_{sym}$		2.507	4.75	2.564	7.14	2.576	8.22
A	$c(s\ell)_{anti}$		2.396	4.58	2.450	6.75	2.459	7.54

Table II (continued)

Particle	Quarks	M_{exp}	Ref. 3 or 4 M	Ref. 3 or 4 R	This paper Soln. A M_A	This paper Soln. A R_A	This paper Soln. B M_B	This paper Soln. B R_B
T	ccs		2.653	4.71	2.705	7.04	2.720	8.03
X_U	$cc\ell$		3.538	4.27	3.636	6.43	3.654	7.33
X_s	ccs		3.690	4.25	3.781	6.36	3.803	7.15
C_1^*	$c\ell\ell$		2.461	5.12	2.495	7.65	2.499	8.97
S*	$c(s\ell)_{sym}$		2.603	5.02	2.633	7.54	2.640	8.76
T*	css		2.742	5.02	2.766	7.43	2.778	8.55
X_U^*	$cc\ell$		3.661	4.69	3.727	7.01	3.743	8.15
X_s^*	ccs		3.795	4.64	3.854	6.88	3.875	7.90
Ω_c	ccc		4.827	4.21				

Table III

Particle	Quarks	M_{exp}	Ref. 3 or 4 M	Ref. 3 or 4 R	This paper Soln. A M_A	This paper Soln. A R_A	This paper Soln. B M_B	This paper Soln. B R_B
ρ	$\ell\bar{\ell}$	0.77	0.783	4.71	0.785	7.30	0.783	8.62
K*	$\ell\bar{s}$	0.892	0.928	4.65	0.928	7.18	0.926	8.42
ω	$\ell\bar{\ell}$	0.783	0.783	4.71	0.785	7.30	0.783	8.62
ϕ	$\ell\bar{\ell}$	0.019	1.068	4.61	1.063	7.09	1.063	8.21
K	$\ell\bar{s}$	0.495	0.497	3.26	0.545	5.15	0.495	4.01
π	$\ell\bar{\ell}$	0.139	0.280	3.34	0.298	5.28	0.243	4.33
D	$c\bar{\ell}$	1.865	1.726	2.80	1.864	5.31	1.870	5.71
F	$c\bar{s}$		1.885	2.84	2.015	5.27	2.022	5.55
D*	$c\bar{\ell}$	2.007	1.969	4.18	2.015	6.59	2.020	7.72
F*	$c\bar{s}$		2.099	4.12	2.139	6.14	2.149	7.44
ψ	$c\bar{c}$	3.095	3.095	3.53	3.194	5.60	3.216	6.39

Table IV

Particle	$(\mu/\mu_p)_{exp}$	Ref. 3 μ/μ_p	Soln. A μ/μ_p	Soln. B μ/μ_p	$(R_c)_{exp}$	Ref. 3 R_c	Soln. A R_c	Soln. B R_c
P	1.0	1.0	1.0	1.0	0.88±0.03	0.73	1.06	1.14
N	-0.685	-0.67	-0.67	-0.67	-0.12±0.01	0	0	0
Σ^+	0.938±0.15	0.97	0.95	0.96			1.08	1.16
Σ^0		0.31	0.30	0.30			0.27	0.27
Σ^-	-0.530±0.13	-0.36	-0.36	-0.36			1.01	1.10
Ξ^0		-0.56	-0.52	-0.51			0.37	0.37
Ξ^-	-0.663±0.27	-0.23	-0.19	-0.19			0.96	1.04
Λ	-0.240±0.021	-0.255	-0.23	-0.23			0.27	0.27
π^+	-	-	-	-	0.56±0.04	0.49	0.76	0.60
π^0	-	-	-	-			0	0
K^+	-	-	-	-			.72	.55
K^0	-	-	-	-			.17	.11

Table V

	Ref. 3	Soln. A	Soln. B
ΔS = 0	0.653	0.653	0.717
ΔS = 1	0.707	0.723	0.776

Table VI

Transition	q_1q_1	q_1q_3	q_1q_3	q_2q_2	q_2q_3	q_3q_3
$N^* \to N\gamma$	4/27	8/27	8/27	4/27	8/27	4/27
$\Sigma^{*+} \to \Sigma^+\gamma$	4/27	8/27	8/27	4/27	8/27	4/27
$\Sigma^{*0} \to \Sigma^0\gamma$	4/27	8/27	-4/27	4/27	-4/27	1/27
$\Sigma^{*-} \to \Sigma^-\gamma$	4/27	-4/27	-4/27	1/27	2/27	1/27
$\Sigma^{*0} \to \Lambda\gamma$	0	0	0	12/27	12/27	3/27
$\Xi^{*0} \to \Xi^0\gamma$	1/27	2/27	8/27	1/27	8/27	16/27
$\Xi^{*-} \to \Xi^-\gamma$	1/27	2/27	-4/27	1/27	-4/27	4/27
$\Sigma^0 \to \Lambda\gamma$	0	0	0	4/27	4/27	1/27
$\Sigma_c^{*++} \to \Sigma_c^{++}\gamma$	16/27	-16/27	-16/27	4/27	8/27	4/27
$\Sigma_c^{*+} \to \Sigma_c^+\gamma$	16/27	8/27	-32/27	1/27	-8/27	16/27
$\Sigma_c^{*0} \to \Sigma_c^0\gamma$	16/27	8/27	8/27	1/27	2/27	1/27
$\Sigma_c^{*+} \to \Lambda_c\gamma$	0	0	0	3/27	12/27	12/27
$\Sigma c \to \Lambda c\gamma$	0	0	0	4/27	4/27	1/27
$S^{*+} \to S^+\gamma$	16/27	8/27	-32/27	1/27	-8/27	16/27
$S^{*+} \to A^+\gamma$	0	0	0	3/27	12/27	12/27
$X_U^{*++} \to X_U^{++}\gamma$	4/27	8/27	-16/27	4/27	-16/27	16/27
$X_U^{*+} \to X_U^+\gamma$	4/27	8/27	8/27	4/27	8/27	4/27
$X_s^{*+} \to X_s^+\gamma$	4/27	8/27	8/27	4/27	8/27	4/27
$T_0^* \to T_0^*\gamma$	16/27	8/27	8/27	1/27	2/27	1/27

Table VII

Transition	q_1q_1	q_1q_2	q_2q_2
$\rho \to \pi\gamma$	12/27	-12/27	3/27
$\rho \to \eta\gamma$	12/27 α_1^2	12/27 α_1^2	3/27 α_1^2
$\omega \to \pi\gamma$	12/27	12/27	3/27
$\omega \to \eta\gamma$	12/27 α_1^2	-12/27 α_1^2	3/27 α_1^2
$K^{*+} \to K^+\gamma$	12/27	-12/27	3/27
$K^{*0} \to K^0\gamma$	3/27	6/27	3/27
$\phi \to \eta\gamma$	3/27 β_1^2	6/27 β_1^2	3/27 β_1^2
$\phi \to \eta'\gamma$	3/27 β_2^2	6/27 β_2^2	3/27 β_2^2
$\eta' \to \rho\gamma$	12/27 α_2^2	12/27 α_2^2	3/27 α_2^2
$\eta' \to \omega\gamma$	12/27 α_2^2	-12/27 α_2^2	3/27 α_2^2
$\eta_c \to \phi\gamma$	3/27 β_3^2	6/27 β_3^2	3/27 β_3^2
$\eta_c \to \omega\gamma$	12/27 α_3^2	-12/27 α_3^2	3/27 α_3^2
$\eta_c \to \rho\gamma$	12/27 α_3^2	12/27 α_3^2	3/27 α_3^2
$\Psi_c \to \eta\gamma$	12/27 γ_1^2	24/27 γ_1^2	12/27 γ_1^2
$\Psi_c \to \eta'\gamma$	12/27 γ_2^2	24/27 γ_2^2	12/27 γ_2^2
$\Psi_c \to \eta_c\gamma$	12/27 γ_3^2	24/27 γ_3^2	12/27 γ_3^2
$D^{*+} \to D^+\gamma$	12/27	-12/27	3/27
$D^{*0} \to D^0\gamma$	12/27	24/27	12/27
$F^{*+} \to F^+\gamma$	12/27	-12/27	3/27

Table VIII

	Soln. A			Soln. B		
	μ_{q_1}	μ_{q_2}	μ_{q_3}	μ_{q_1}	μ_{q_2}	μ_{q_3}
$N^*\to N\gamma$	.5913	.5913	.5913	.5586	.5586	.5586
$\Sigma^*\to\Sigma\gamma$	.4573	.6600	.6600	.4427	.6476	.6476
$\Sigma^*\to\Lambda\gamma$	.4248	.6076	.6076	.4000	.5781	.5781
$\Xi^*\to\Xi\gamma$	.4527	.4527	.6498	.4352	.4352	.6311
$\Sigma\to\Lambda\gamma$	.5056	.7252	.7252	.5009	.7235	.7235
$\Sigma_c^*\to\Sigma_c\gamma$	.1563	.7374	.7374	.1549	.7586	.7586
$\Sigma_c^*\to\Lambda_c\gamma$	.1373	.6098	.6098	.1305	.5912	.5912
$S^*\to S\gamma$	.1566	.5070	.7304	.1552	.5102	.7474
$S^*\to A\gamma$	.1459	.4602	.6509	.1471	.4705	.6755
$X_U^*\to X_U\gamma$	.1543	.1543	.6641	.1522	.1522	.6771
$X_S^*\to X_S\gamma$	.1553	.1553	.4750	.1541	.1541	.4782
$T_0^*\to T_0\gamma$	.1568	.5043	.5043	.1556	.5056	.5056

Table IX

	Soln. A		Soln. B	
	μ_{q_1}	μ_{q_2}	μ_{q_1}	μ_{q_2}
$\rho\to\pi\gamma$	.4720	.4720	.4240	.4240
$\omega\to\pi\gamma$	.4730	.4730	.4243	.4243
$K^*\to K\gamma$	.3707	.4938	.3369	.4381
$D^*\to D\gamma$	.1494	.5731	.1475	.5821
$F^*\to F\gamma$	.1511	.4299	.1492	.4298

Table X

Transition	Exp. (keV)	Transition Rate (keV) Soln. A	Transition Rate (keV) Soln. B
$N^*\to N\gamma$	700±70	338.51 (338.51)	291.50 (291.50)
$\Sigma^{*+}\to\Sigma^+\gamma$		153.27 (113.06)	135.86 (108.10)
$\Sigma^{*0}\to\Sigma^0\gamma$	< 1800	30.08 (22.13)	26.43 (20.99)
$\Sigma^{*-}\to\Sigma^-\gamma$		1.99 (1.50)	1.89 (1.53)
$\Sigma^{*0}\to\Lambda\gamma$	< 2200	222.79 (211.23	197.69 (190.91)
$\Xi^{*0}\to\Xi^0\gamma$		158.39 (146.33)	145.03 (137.15)
$\Xi^{*-}\to\Xi^-\gamma$	< 360	2.00 (1.86)	1.93 (1.84)
$\Sigma^0\to\Lambda\gamma$		1.11 (2.67)	1.36 (2.66)
$\Sigma_c^{*++}\to\Sigma_c^{++}\gamma$		3.27	2.88
$\Sigma_c^{*+}\to\Sigma_c^+\gamma$		1.52	1.36
$\Sigma_c^{*0}\to\Sigma_c^0\gamma$		2.67	2.26
$\Sigma_c^{*+}\to\Lambda_c\gamma$		176.73	166.17
$\Sigma_c^+\to\Lambda_c\gamma$		22.91	24.57
$S^{*+}\to S^+\gamma$		1.46	1.26
$S^{*+}\to A^+\gamma$		74.01	40.24
$X_U^{*++}\to X_U^{++}\gamma$		4.35	5.75
$X_U^{*+}\to X_U^+\gamma$		3.96	5.03
$X_S^{*+}\to X_S^+\gamma$		1.35	1.29
$T_0^*\to T_0\gamma$		.85	.73

Table XI

Transition	Exp. (keV)	Soln. A (keV)	Soln. B (keV)
$\phi\to\pi\gamma$	5.7± 2.1	0	0
$\rho\to\pi\gamma$	35 ±10	36.54 (43.45)	34.43 (37.47)
$\omega\to\pi\gamma$	880 ±60	326.85 (398.72)	310.30 (344.04)
$K^{*+}\to K^+\gamma$	< 80	7.45 (7.71)	8.09 (7.48)
$K^{*0}\to K^0\gamma$	75 ±35	90.80 (93.72)	93.94 (81.90)
$D^{*+}\to D^+\gamma$		1.00 (.82)	1.07 (.89)
$D^{*0}\to D^0\gamma$		27.73 (22.57)	27.75 (23.05)
$F^{*+}\to F^+\gamma$		.12	.14

Table XII

(a)

R_η	$R_{\eta'}$	R_{η_c}	a	b	c
5.00	2.32	3.62	0.696	-0.237	-1.09

(b)

i	α_i	β_i	γ_i
η	-0.765	0.642	0.053
η'	0.002	1.0	0.0
η_c	0.117	0.101	-0.988

Table XIII

Transition	Exp. (keV)	Bag (A) (keV)
$\phi\to\eta\gamma$	64±10	43.72
$\phi\to\eta'\gamma$		2.39
$\Psi_c\to\eta_c\gamma$	< 3.5	21.00
$\Psi_c\to\eta\gamma$	0.55±0.01	0
$\Psi_c\to\eta'\gamma$	0.152±0.117	0
$\eta'\to\rho\gamma$	< 300	0
$\eta'\to\omega\gamma$	< 50	0
$\rho\to\eta\gamma$	50±13 76±15	58.33
$\omega\to\eta\gamma$	3 +2.5, -1.8 29±7	6.36
$\eta_c\to\phi\gamma$		0
$\eta_c\to\omega\gamma$		0.06
$\eta_c\to\rho\gamma$		0.54