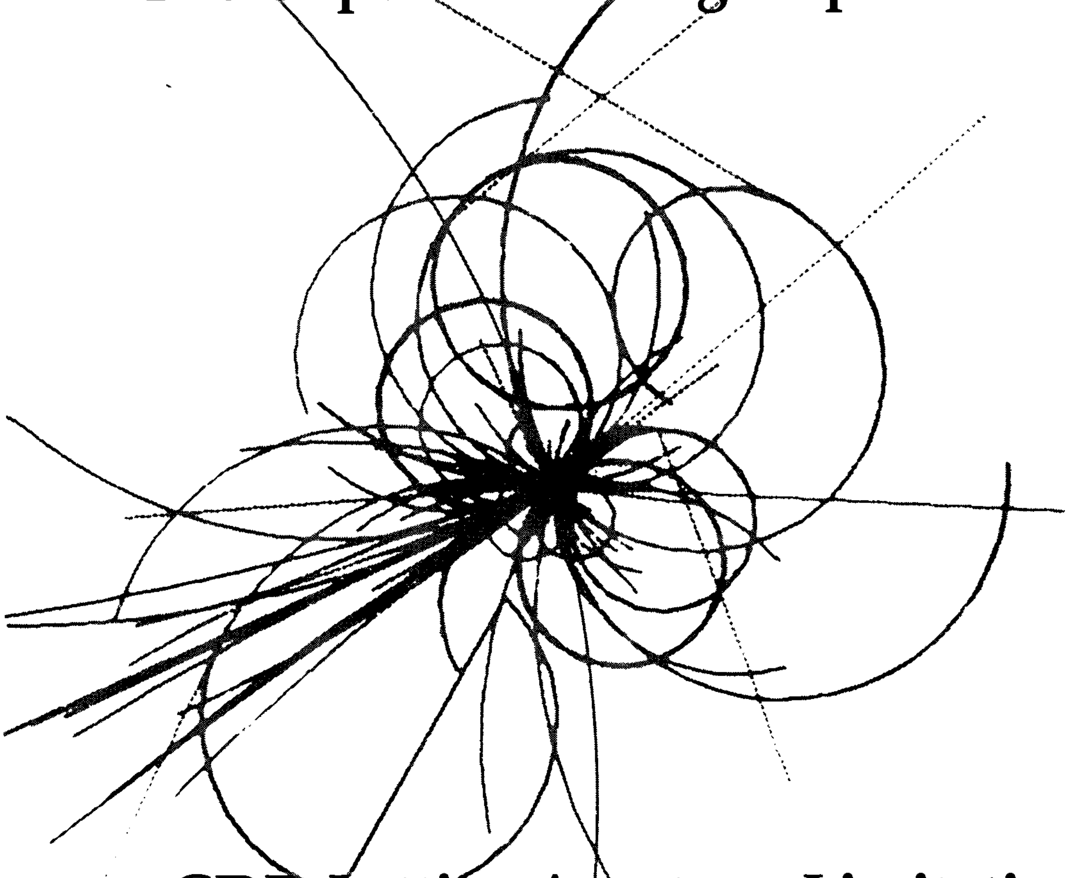
SSC-109

The Superconducting Super Collider



CDR Lattice Aperture Limitations Due to Magnetic Imperfections in the IR Quadrupole Triplets

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1. Introduction

The effect of magnetic imperfections in the arc dipole magnets on the linear and dynamic aperture of a variety of test-lattices has been explored in a number of particle tracking studies.These results have been incorporated in the Conceptual Design Report (CDR) [1] and contributed to establish tolerances for the arc dipole magnet imperfections. Sorting and/or binning of the magnets according to a particular multipole moment (usually b_2 or a_2) proved most effective. The importance of the IR quadrupole imperfections to the dynamic aperture due to the large β - function values at their locations was first documented in tracking studies by Dell [2] and Schachinger [3] . In the present report we explore the dependence of the linear and dynamic apertures on lattice parameters such as the β - function value β^* and the crossing angle α at the interaction points in the low- β IRs for the nominal [3] IR quad triplet multipole errors. Furthermore, the sensitivity of the dynamic aperture to the number of corrected multipole orders, to the residual multipole strengths and to the strengths of the higher, uncorrected multipole moments is examined. It should be understood, that the rms-values of the magnetic multipoles moments are implied, whenever we refer to "multipole errors".

2. Lattice and Tracking Procedure

The lattice that was used for the present tracking study is the CDR lattice [4] in the collision mode. This realistic lattice uses a regular cell with 60° phase advance, has 8 insertions : 2 low-β IRs, 2 medium-β IRs, 2 utility modules and 2 future IRs (presently identical with the utility insertions). The four IRs include vertical dipole magnets to bring the beams to collision at the IP. Throughout this study the parameters of the medium-β IRs are kept at their nominal CDR values. The IR optics used to study the aperture dependence on β^* are based on the IR tuning curves [5,6].

The tracking was carried out with the recently developped code "FASTRACK" [7], which uses the input structure and the main tracking loop of "RACETRACK" [8], but otherwise is quite different from that code. This is particularly true for its tracking speed. In its CRAY version, "tuned" for the huge SSC ring size, the tracking speed has been improved by more than a factor of 100 in case of the realistic lattice. Since the IR quadrupole triplets are in a dispersion free region of the lattice all tracking has been done on momentum only. Each data point represents the average of 5 tracking runs using the same set of 5 different error distributions (i.e.5 different random number seeds).

The IR quad triplet magnets and the adjacent couple of dipole magnets are common to both beams. Thus the closed orbit, depending on the crossing angle α, will significantly deviate from the axes of those elements. To take this into account to first order in the tracking the beam got displaced at the entrance of the first and the exit of the last common elements. If the transfer matrix for (y, p_y) from the first common magnet BV to the interaction point IP is

$$M(BV \rightarrow IP) = \begin{bmatrix} m_{11}, m_{12} \\ m_{21}, m_{22} \end{bmatrix} \tag{1}$$

then the beam displacement required for a given crossing angle α is

$$\Delta y = -\frac{\alpha}{2} m_{12} \tag{2.a}$$

$$\Delta p_y = \frac{\alpha}{2} m_{11} \tag{2.b}$$

In "FASTRACK" the lateral displacement is accomplished by rotating the (y, p_y) phase space through 90 degrees, followed by a vertical dipole kick with strength given by (2.a) and a restoring phase space rotation of - 90 degrees. Leaving the last common magnet the closed orbit displacement resulting from the finite crossing angle is brought back to zero by an analogous procedure.

For each individual tracking run the closed orbit has been determined first. Next, the tunes and chromaticities with respect to this orbit got adjusted to their nominal values:

$$Q_x = 0.265 \qquad Q_y = 0.285 \tag{3.a}$$

$$\xi_h = 0.0 \qquad \xi_v = 0.0 \tag{3.b}$$

The dynamic aperture was determined then by tracking a set of 10 particles with initial conditions equally spaced along $\varepsilon_x = \varepsilon_y$ for 400 turns. Everytime a particle is lost the initial conditions are reduced by an appropriate factor until all 10 particles remain stable for 400 turns. The largest of the ten initial conditions is then the dynamic aperture. To improve the resolution of the aperture scan the 10 initial amplitudes were chosen between $a_o/2$ and a_o . Thus, for a given error distribution the dynamic aperture is determined with an accuracy of ± 2.5 % .

3. Magnet Field Quality

The rms values for the multipole moments of the various magnets used in this study are given in table 1. The errors of the arc dipole magnets (column 1) are the tolerances given in the CDR [1], except for the b_2 - term, which is reduced to 20 % of its nominal value. This reflects the effective reduction of the b_2 - rms value achievable using a grouping of the magnets (according to their b_2 - values) into 7 bins [9]. For the vertical dipole magnets in the IRs (columns 2 & 3) these same values are scaled from 4 cm to 7.5 and 16 cm coil diameter, respectively, and the the skew and normal moments are interchanged. The quadrupole errors derive from scaling the 5 cm coil diameter Fermilab quadrupole data to the appropriate coil dimensions [3]. The errors for the cell quadrupoles (4 cm coil diameter) are given in column 4, while column 5 lists those of the 5 cm coil diameter quadrupoles in the utility insertions. Finally, in column 6 there are the nominal IR quad triplet tolerances. The quadrupole errors a_1 and b_1 of the dipole magnets are set to 0 assuming a complete correction of these terms.

The homogeneity of the magnetic field B

$$B_y + i B_x = B_0 \sum_{n=0}^{N} (b_n + i a_n)(x + i y)^n \tag{4}$$

can be expressed conveniently in terms of the variance of its magnitude

$$< |B|^2 > = B_0 \sum_{n=0}^{N} | b_n^{rms} + i a_n^{rms} |^2 | r |^{2n} \tag{5}$$

For particles with $\varepsilon_x = \varepsilon_y = \varepsilon$ the magnitude of $| r |^{2n}$ is given by

$$| r |^{2n} = \varepsilon^n \, (\beta_x \cos^2\phi + \beta_y \cos^2\psi)^n \tag{6}$$

For an uncoupled system ϕ and ψ are independent variables and averaging over both provides

$$< | r |^{2n} > = (\varepsilon \beta_x)^n \sum_{k=0}^{n} \left(\frac{\beta_y}{\beta_x} \right)^k \binom{n}{k} \frac{(2(n-k)-1)!!}{(2(n-k))!!} \frac{(2k-1)!!}{(2k)!!} \tag{7}$$

Therefore the variance of B is

$$<< |B|^2 >> = \sum_{n=0}^{N} | b_n + i a_n |^2 (\varepsilon \beta_x)^n \sum_{k=0}^{n} \left(\frac{\beta_y}{\beta_x} \right)^k \binom{n}{k} \frac{(2(n-k)-1)!!}{(2(n-k))!!} \frac{(2k-1)!!}{(2k)!!} \tag{8}$$

Equation (8) has been used to generate the field plots in fig. 1 and fig. 2 .The arc dipole field variance is shown in fig.1 . In the log-log plot the r - dependence is given as a straight line of slope of 2.0 over most of the aperture range, demonstrating the dominance of the sextupole components in the dipole magnet. For a displacement of about 9 mm the deviation from a straight line becomes noticeable in qualitative agreement with the linear aperture value quoted in the CDR . The situation changes drastically in case of the IR quad triplet magnets, whose field variances are shown in fig.2, where it is assumed that the multipole moments of order $n = 2, 3, 4$ and 5 are corrected to residual rms-values of 0.05 units. While a large fraction of the aperture is still dominated by the sextupole components, the higher order multipoles become apparent very abruptly, causing a sharp kink in the r - dependence of the field variance leading to a slope of almost 9 (in a log-log plot). This pattern impacts the linear aperture concept severely, as is documented in fig.3 , where the relative, maximum variation of the first order invariant ("smear") for 400 revolutions is plotted versus the initial (arc-) amplitude. The steep rise of the field variance caused by the uncorrected high order multipole moments ($n > 5$) is responsible for moving the linear and dynamic apertures on top of each other ("brick wall effect").

4. Aperture Dependence on β^*

The linear and dynamic apertures have been determined for 5 different β^* - values and two error configurations. The results are summarized in fig.4 . For the case where only the arc dipole errors have been included the linear and dynamic apertures are nearly constant over the range of β^*-values investigated, demonstrating a perfect match between the IR optics and the complementary lattice. If in addition to the arc dipole errors the IR quad errors (col. 6 of table 1) are included, the aperture shows a pronounced dependence on β^* . As stated above the linear and dynamic apertures coincide in this case. If the field variance of the IR quad triplet is the aperture limiting cause due to the "brick wall", then one should expect the dynamic aperture to scale with the maximum β - value in the triplet :

$$a_{ARC} = a_{IRQ} \sqrt{\frac{\beta_{ARC}}{\beta_{IRQ}}} \qquad (9)$$

where a_{IRQ} is the aperture limit in the triplet due to the "brick wall". The aperture prediction according to (9) is compared with the actual tracking data in fig. 5 .The qualitative agreement confirms the validity of the "brick wall model". However, in the region of interest, where β^* is between 0.3 and 1.0 m, the aperture grows almost linearly at a rate of 5 mm per 1 m change in β^* .

This is quite favorable in case the required aperture should be raised. Assuming an increase in required aperture by 1 mm , β^* would have to go to 0.7 m . Thus, an aperture gain of 20 % can be accomplished by a 40 % increase of β^* (with respect to the nominal case). The increase in the number of particles / bunch required to maintain the luminosity would amount to 18 %. Though substantial, this still should be well within the capability of the refrigeration system.

Since we have included only the magnetic errors in the IR quad triplet elements (beside the arc dipole errors), it is important for the general validity of this study to show, that the aperture is indeed dominated by the errors in the IR triplets even in the presence of any other magnetic imperfections. This is documented in fig.6 for seven different error configurations. The errors used here are those listed in table 1. Within the resolution given by the error bars the dynamic aperture does not change anymore once the IR triplet errors are "turned on", regardless what other imperfections are included.

The dynamic aperture for the nominal CDR IR quad triplet tolerances and the CDR collision optics ($\beta^* = 0.5$ m) as determined by averaging the results of 5 different error distributions is

$$a_{ARC} = 3.81 \pm 0.16 \text{ mm} \quad . \tag{10}$$

It is interesting to convert this value to the IR quad Q2 according to eq.(9)

$$a_{IRQ} = a_{ARC} \sqrt{\frac{11{,}000}{662}} \cong 15.6 \text{ mm} \quad . \tag{11}$$

This quantity must be compared to the bore tube radius of 16.13 mm in the IR quad triplet magnets: In radial dimension the dynamic aperture comprises over 95 % of the physical aperture in these elements. Tighter tolerances for their multipole moments therefore would not result in an increased dynamic aperture due to the physical aperture limitation.

5. Aperture Dependence on Crossing angle α

The dynamic aperture has been determined for 4 different crossing angle values. The nominal errors of the arc dipoles and the IR quad triplets have been included (table 1, columns 1 & 6). The dependence is slight enough to allow for quite some flexibility in the crossing angle around the nominal value of $\alpha = 75$ µrad. Again, the results are in agreement with the "brick wall model" : The closed orbit displacements in the IR quad triplets are proportional to the crossing angle α . By reducing for each quadrupole the aperture value obtained for $\alpha = 0$ by the closed orbit displacement, one arrives at a dependence on α as shown by the three lines in fig.7 .

6. Aperture Dependence on Residual Errors

Sofar we have used for the IR quad triplet errors the tolerances specified in the CDR. The correction of all multipole moments up to order 5 to 0.05 units might be hard to achieve for several reasons, the most obvious being the limitation in measuring accuracy. To explore the impact on the dynamic aperture by relaxing the quadrupole triplet multipole tolerances the following tracking runs have been performed : For two IR optics with $\beta^* = 0.5$ m and 1.0 m the IR quad triplet multipoles of order n = 2, 3, 4 and 5 successively increased from 0.05 to 0.5 units, while the higher orders n = 6, 7, 8 and 9 were kept at their nominal values. The results are summarized in fig.8 . For both IR optics studied the reduction in aperture due to looser tolerances is fairly modest and shows within the tracking accuracy the same dependence. The "brick wall model" offers a qualitative understanding of these results : The "brick wall" is primarily determined by the uncorrected high multipole moments (cf. fig.2). Relaxing the low order tolerances results in a reduction of the observed kink in the field variance and in only a very slow closing in of the dynamic aperture limit due to the steepness of the field variance. This conlusion applies equally to the linear aperture within the range of $\sigma_{a,k}$, $\sigma_{b,k}$ ($2<k<5$) that was studied. However, this is not so for the tracking results shown in fig.9 . Here we have explored for the same two IR optics as above the dependence on the number of multipole moments that are corrected to 0.1 units while the values from column 4 in table 1 are given to the uncorrected moments. Once again the two optics hardly differ in the dependence on the number of corrected multipoles and the aperture reduction going from four to only one corrected multipole order is rather moderate. However, in order not to cut into the linear aperture at the sextupole and octupole moments have to be corrected. The effects due to the decapole and dodecapole moments is smaller than the resolution of the tracking results reported here. Therefore these effects, if they exist, must be less than 5 % .

Another aspect to consider is the contribution of the IR quad triplet elements to the variation of the first order invariant ("smear"). The "smear" as plotted in fig.3 for the nominal collision optics and nominal errors grows linearly with amplitude over 75% of the aperture. It turns out, however, that in this case the slope in the linear region is determined by the sextupole moments in the arc dipole magnets. This is demonstrated by the results plotted in fig.10 : For a given error distribution the "smear" as a function of amplitude has been explored for five tolerance values of σ_k ($2\leq k\leq 5$). In fig.10 the cases with $\sigma_k = 0.0$, 0.2, and 0.5 are plottted. The data points of the other cases with $\sigma_k = 0.05$ and 0.1 are coinciding with those for $\sigma_k = 0$ (within the resolution of the plotting symbols). In order to maintain the linear regime over an aperture range as large as possible, the IR quad multipole moments σ_k ($2\leq k\leq 5$) should be corrected to a rms-value of 0.1 . For this value the corrected IR quad triplet is matched to the arc dipole with respect to their field quality. Any further relaxation of the tolerances results in a loss of aperture range with linear "smear".

7. Aperture Dependence on High Order Multipoles

The effect of the high order ($n > 5$) multipoles in the IR quad triplet on the dynamic aperture is illustrated with fig.11 . When the rms values for these multipoles are set to zero the aperture increases by 50 % compared to the nominal case. Because of the steep slope of the field variance (cf. fig.2), however, increasing the strength of the high order multipoles by 50 % has no noticeable effect on either the dynamic or the linear aperture.

8. Conclusions

We reconfirmed that the dynamic aperture limitation for the realistic CDR lattice are clearly dominated by the multipole moments of the IR quadrupole triplet in the collision optics. The high order multipoles ($n > 5$) cause a steep rise of the field variance producing a "brick wall effect". The aperture dependence on the β^*- value in the range of interest ($0.3 < \beta^* < 1.0$) is quite linear with a slope of 5 mm per 1 m change in β^*. Together with the relative weak dependence on crossing angle α, residual multipole errors after correction and on number of multipole orders corrected, this should provide sufficient flexibility to choose a proper configuration of all these parameters. Thus it is possible to relax the multipole moments tolerances of order 2 through 5 and to regain the resulting aperture loss by increasing the β^*- value; however, going to a smaller β^*-value is not possible without either a substantial loss of dynamic aperture or by replacing the present IR quad triplet elements with magnets of a larger bore tube diameter and a correction scheme that includes higher multipoles. The topics of amplitude dependent tune shifts (on- and off-momentum) as well as the off-momentum dynamic aperture are subjects of future studies.

9. References

1. "Conceptual Design of the Superconducting Super Collider", SSC-SR-2020
2. G.F. Dell,"Summary of Tracking for the SSC Storage Lattice", SSC-N-132
3. L. Schachinger,"The Effects of Varying β^* on the SSC CDR Collision Lattice",SSC-N-224
4. J.M. Peterson, "SSC Quadrupole Errors Scaled from Tevatron", SSC-?
5. E.D. Courant, A. A. Garren, D.E. Johnson and K. Steffen, "SSC Conceptual Design Report Lattice", SSC-N-139
6. A.A. Garren , D.E. Johnson, "SSC Medium and Low β IR Tuning Curves", SSC-N-140
7. D.E. Johnson , private communications.
8. E. Forest, B.T. Leemann, "FASTRACK", to be published.
9. A. Wrulich, "RACETRACK", DESY 84-026, March 1984
10. R.E. Meller, "Moments of a Binned Distribution", SSC-N-237

Table 1 : RMS multipole moments in units of 10^{-4} at 1 cm .

RMS VALUE	ARC DIPOLE	7.5 cm IR DIPOLE	16 cm IR DIPOLE	ARC QUAD	UTILITY QUAD	IR TRIPLET QUAD
a_2	0.61	0.07	0.008	3.09	2.45	0.05
a_3	0.69	0.018	0.001	1.23	0.84	0.05
a_4	0.14	0.005	0.000	0.34	0.20	0.05
a_5	0.16	0.002	0.000	0.13	0.065	0.05
a_6	0.034	0.001	0.000	0.058	0.025	0.058
a_7	0.030	0.000	0.000	0.000	0.000	0.000
a_8	0.0064	0.000	0.000	0.021	0.007	0.021
a_9	0.0056	0.000	0.000	0.019	0.005	0.019
b_2	0.40	0.14	0.015	3.35	2.66	0.05
b_3	0.35	0.075	0.004	0.58	0.39	0.05
b_4	0.59	0.013	0.000	0.32	0.19	0.05
b_5	0.059	0.004	0.000	0.29	0.14	0.05
b_6	0.075	0.001	0.000	0.056	0.024	0.056
b_7	0.016	0.000	0.000	0.000	0.000	0.000
b_8	0.021	0.000	0.000	0.025	0.008	0.025
b_9	0.003	0.000	0.000	0.021	0.006	0.021

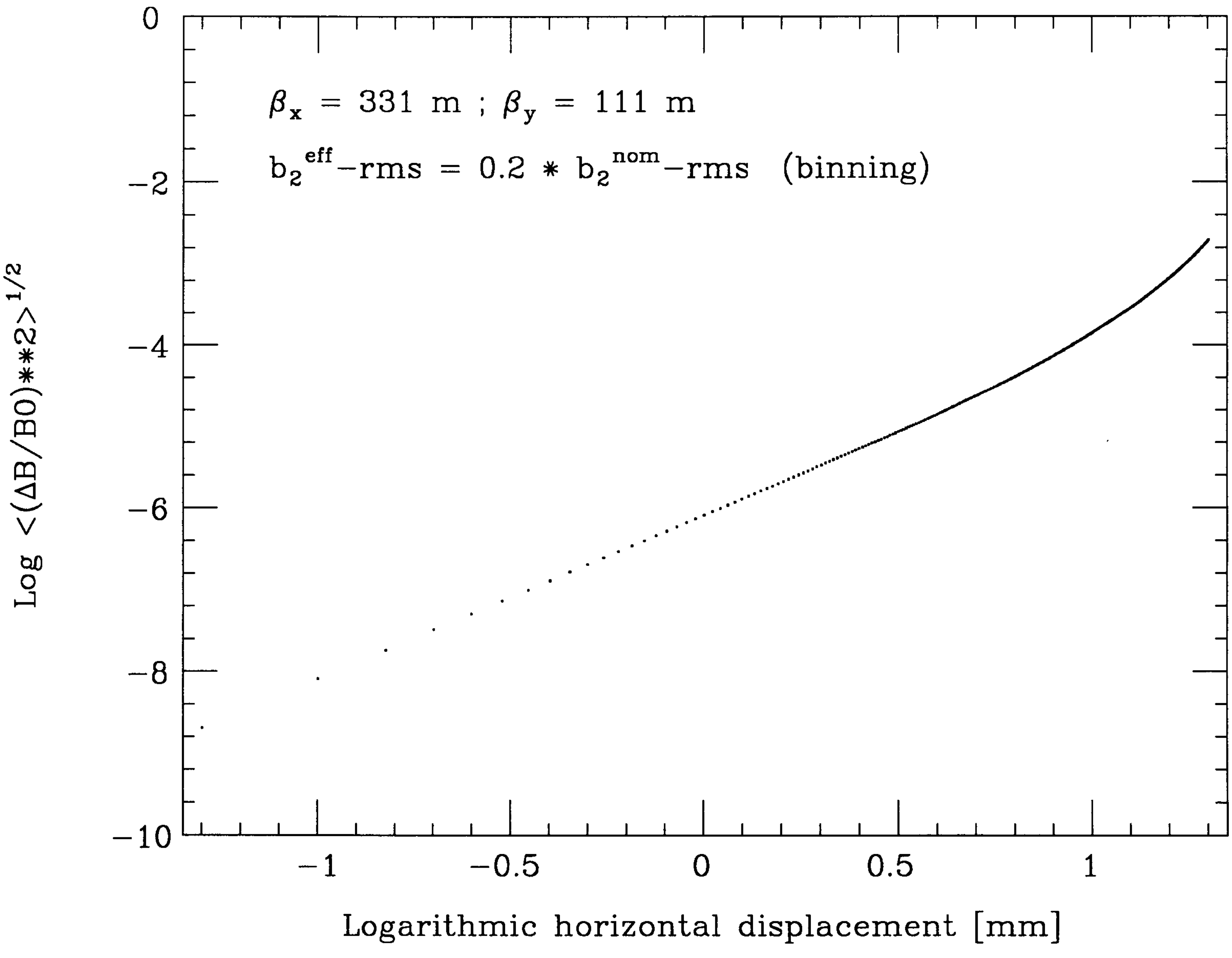

Fig. 1 : Arc Dipole Magnet : Field Quality

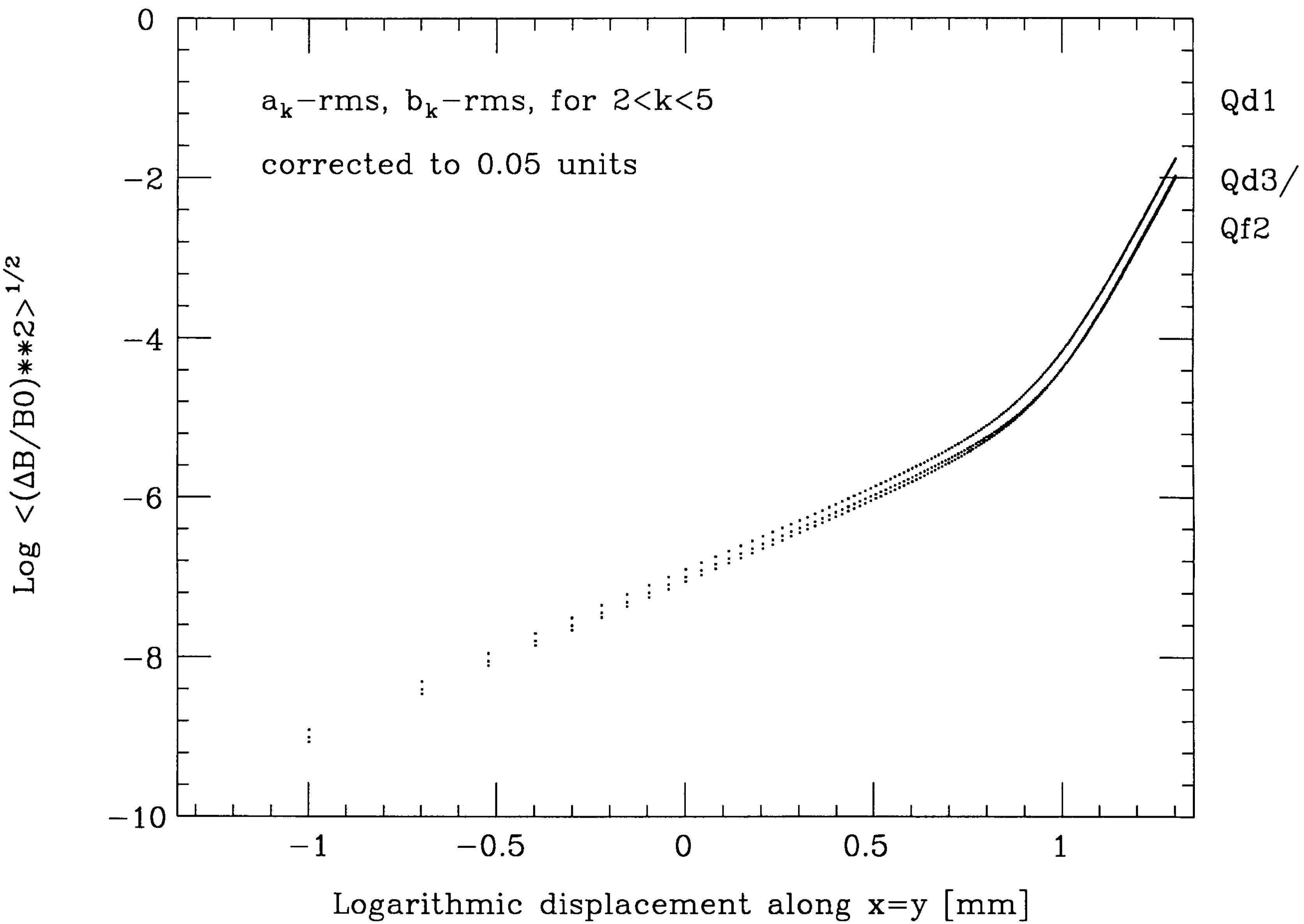

Fig. 2 : IR Quadrupole Triplet Field Quality

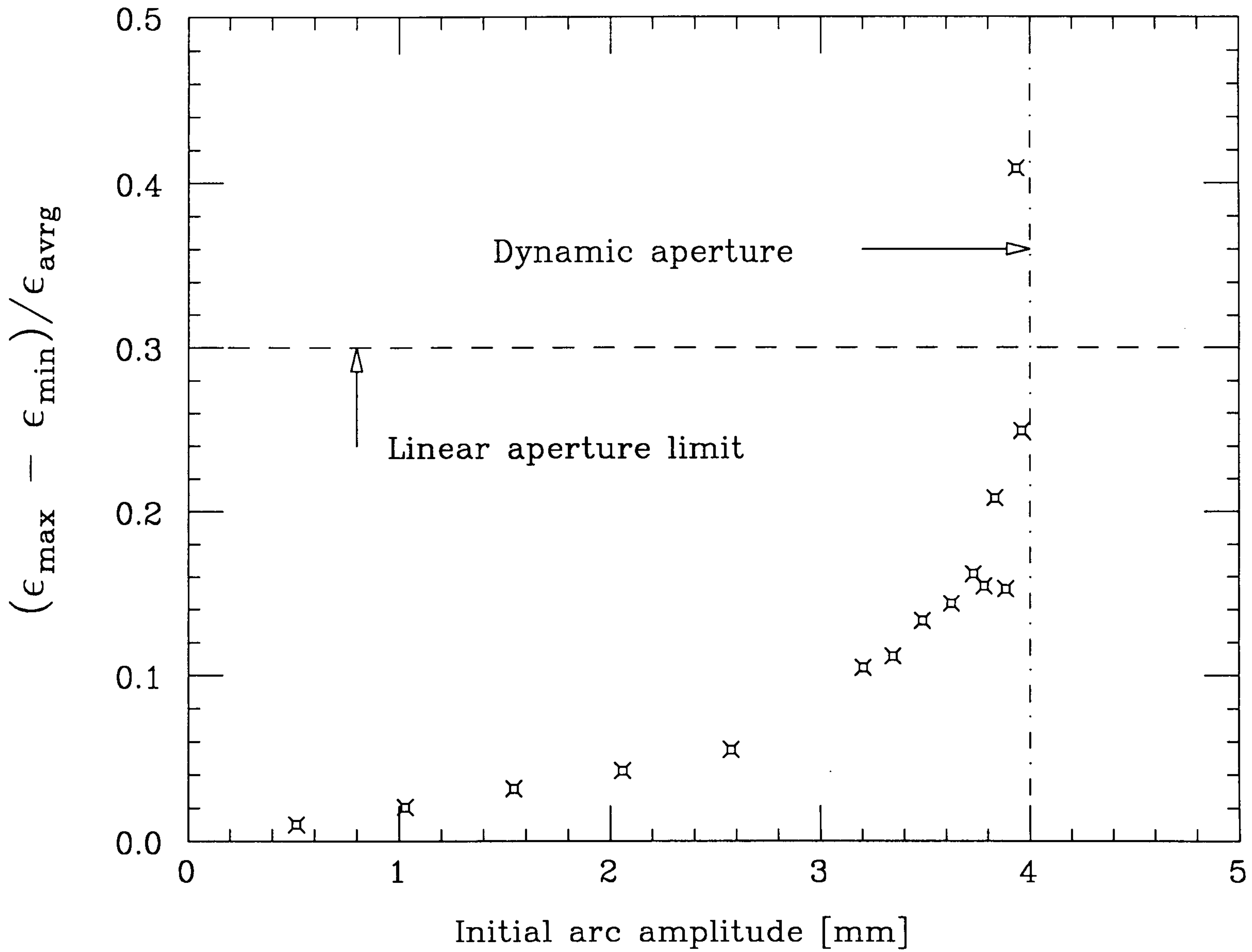

Fig. 3 : Variation of Linear Invariant vs. Amplitude

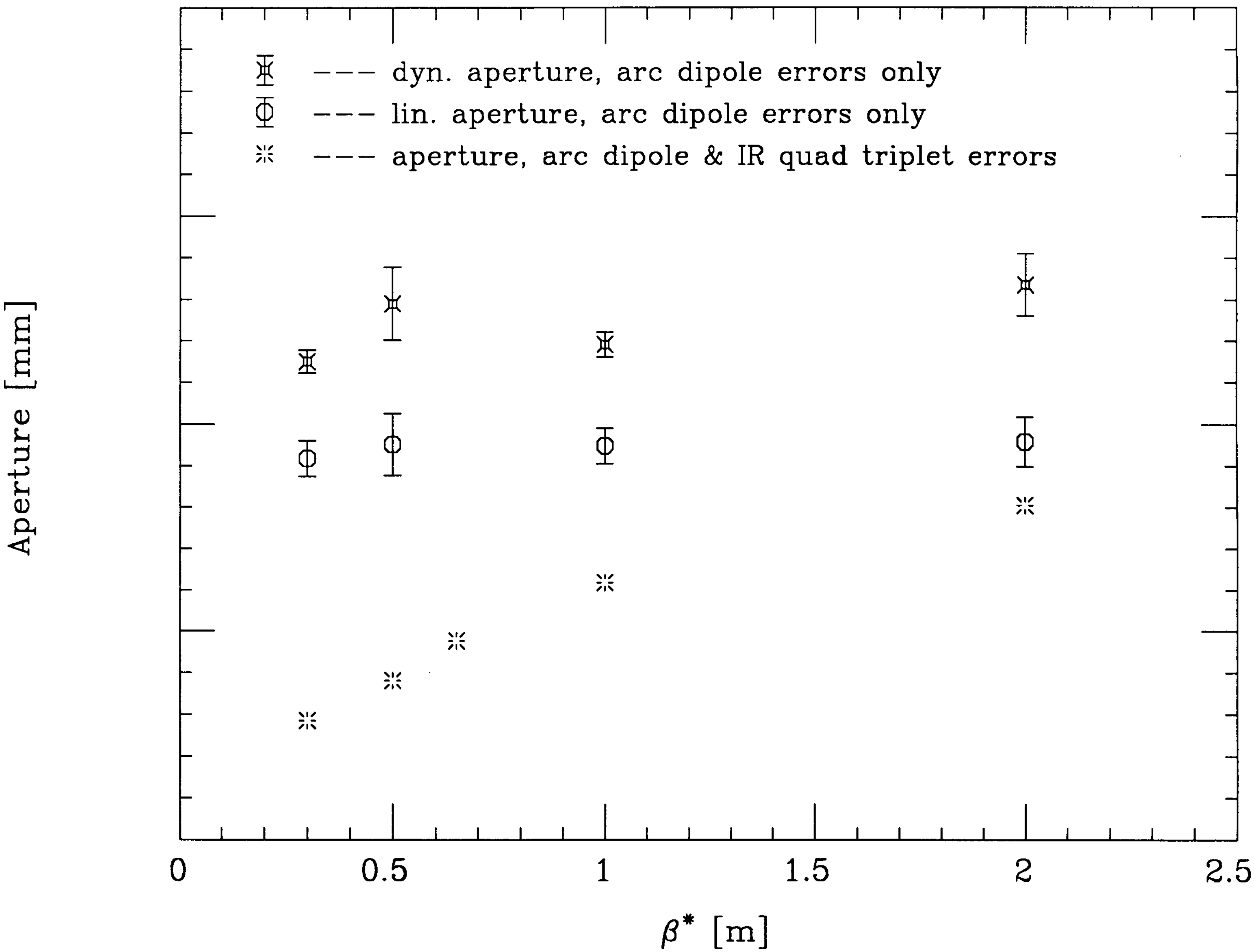

Fig. 4 : Aperture Dependence on β^*

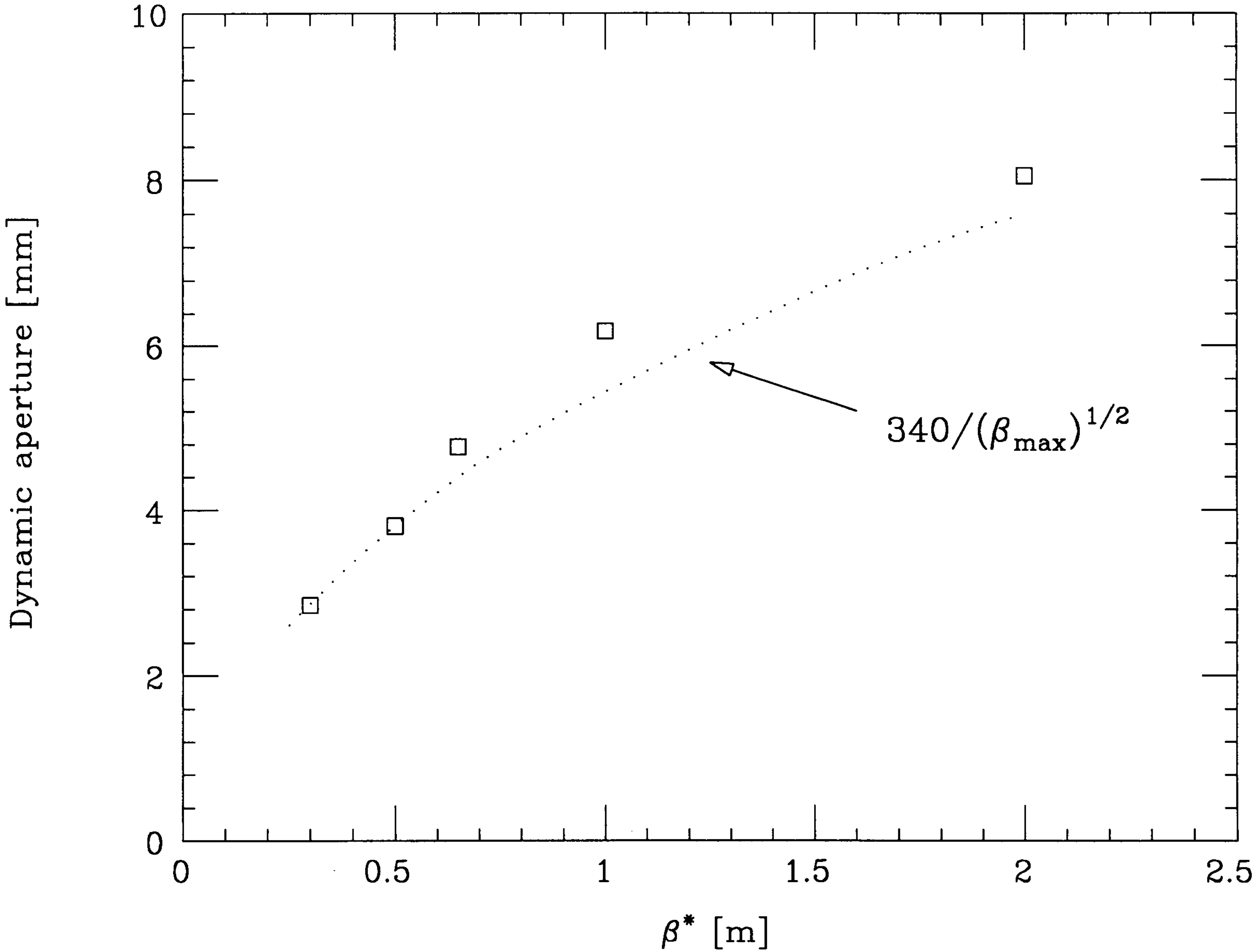

Fig. 5 : Aperture Dependence on β^*

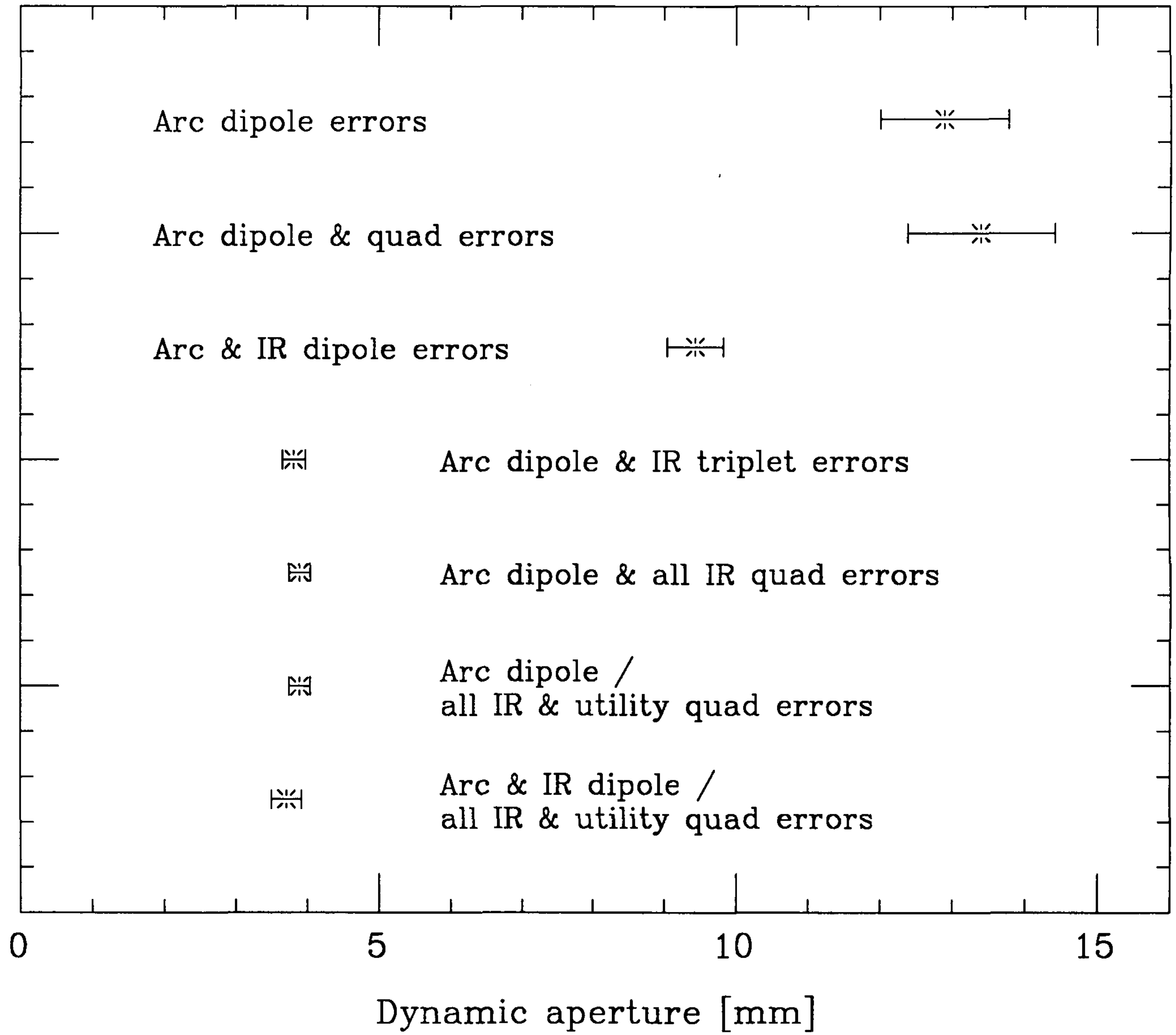

Fig. 6 : Comparison Of Various Error Configurations

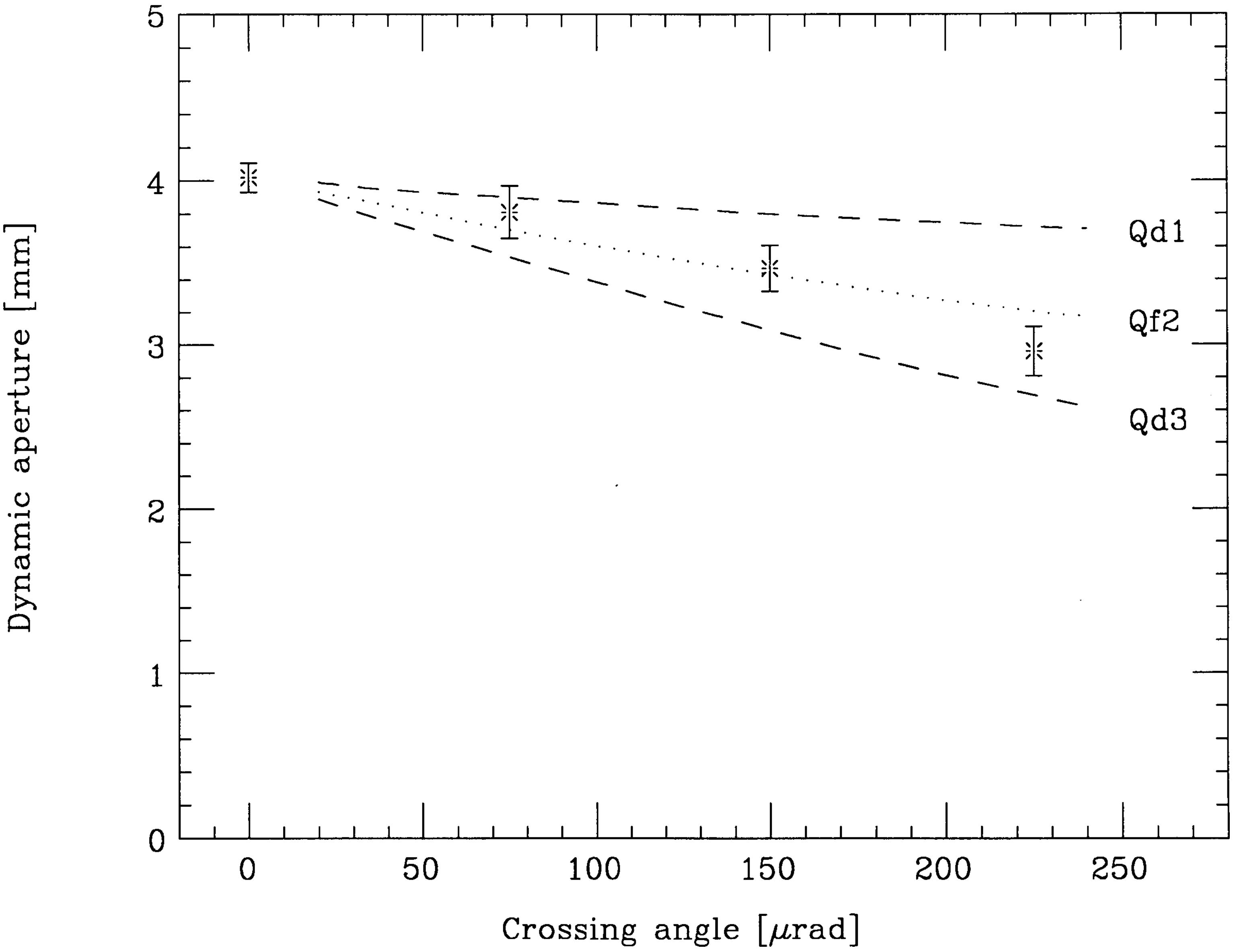

Fig. 7 : Aperture Dependence on Crossing Angle

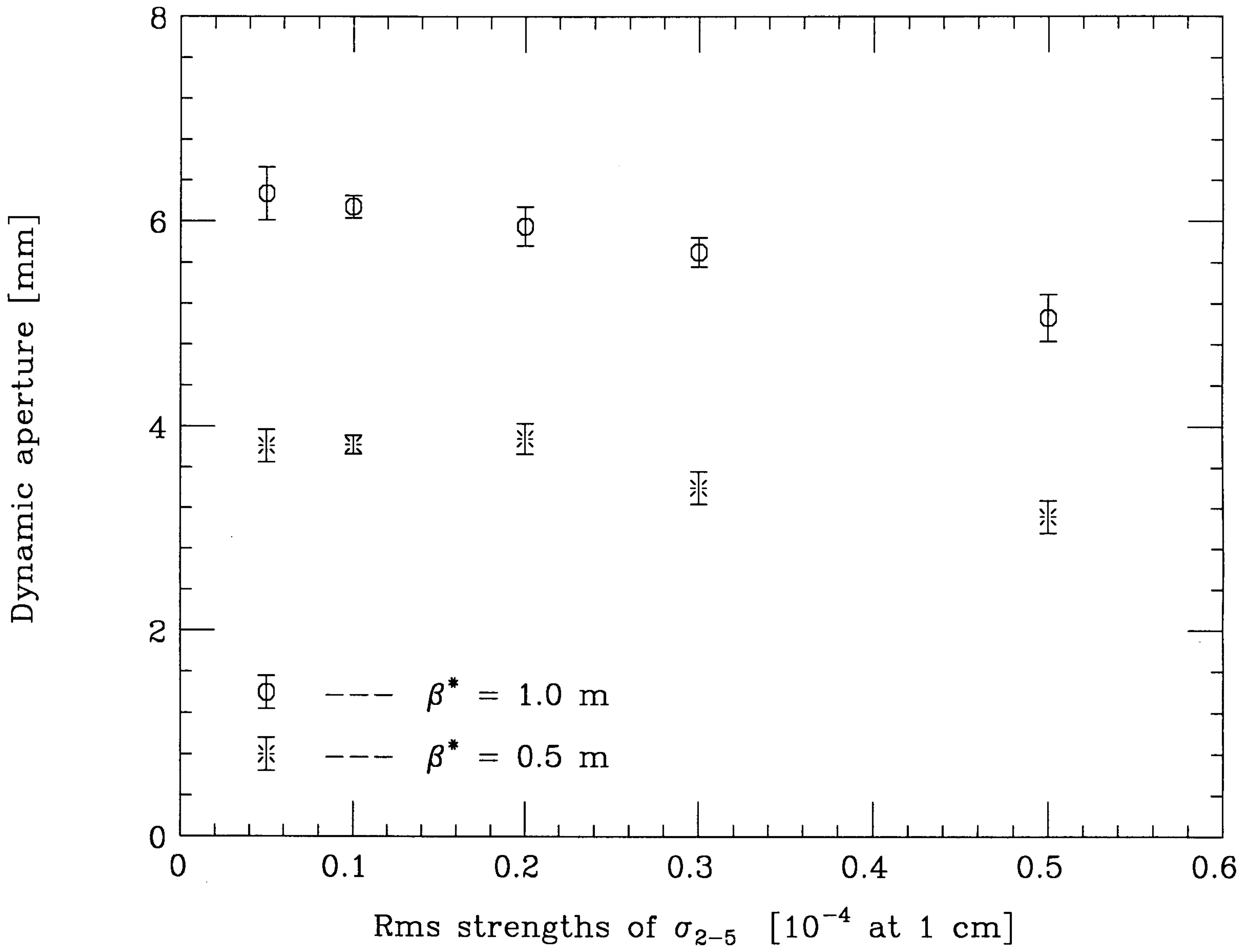

Fig. 8 : Dependence On Residual Errors

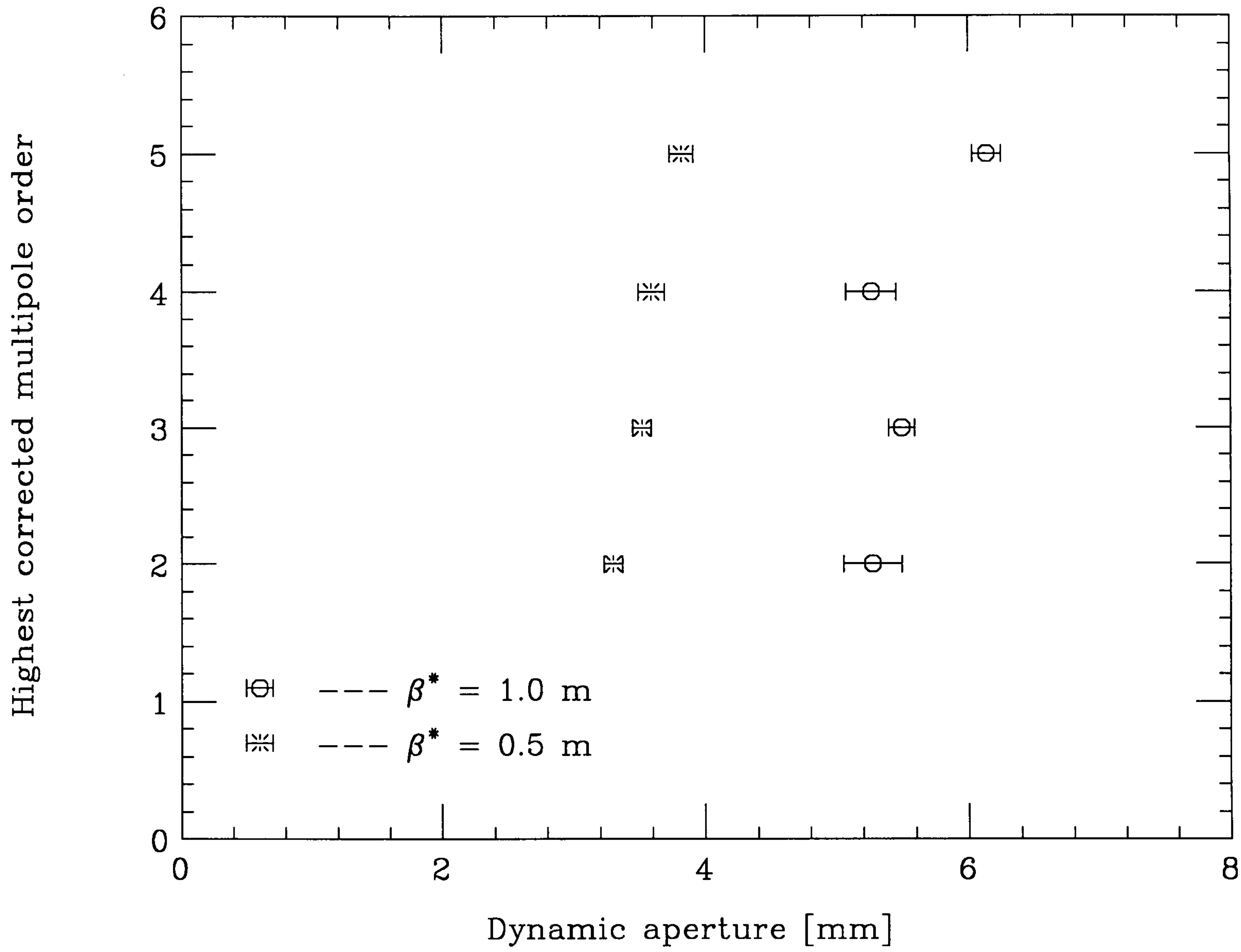

Fig. 9 : Dependence on Number of Corrected Multipoles

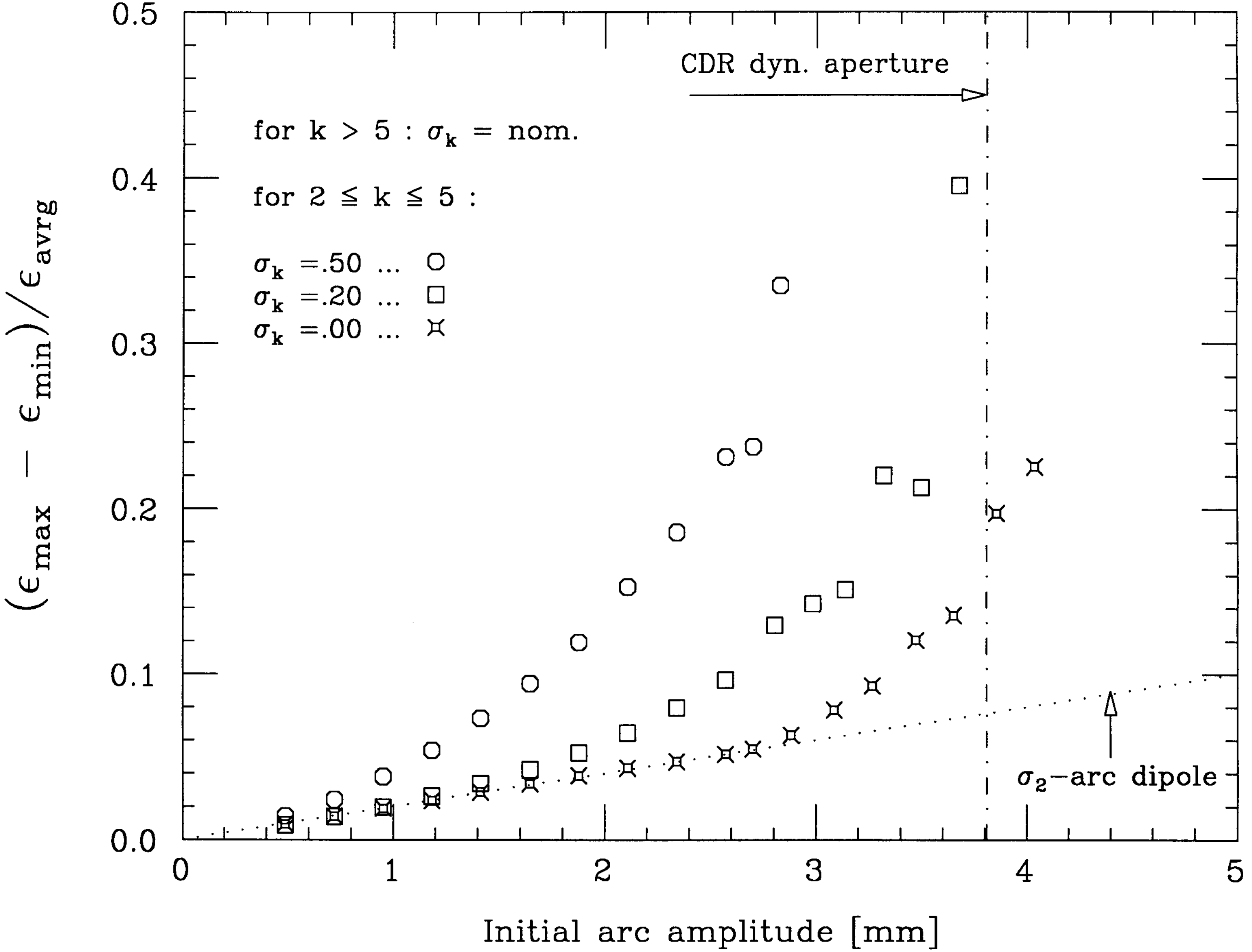

Fig. 10 : Variation of Linear Invariant with σ_k

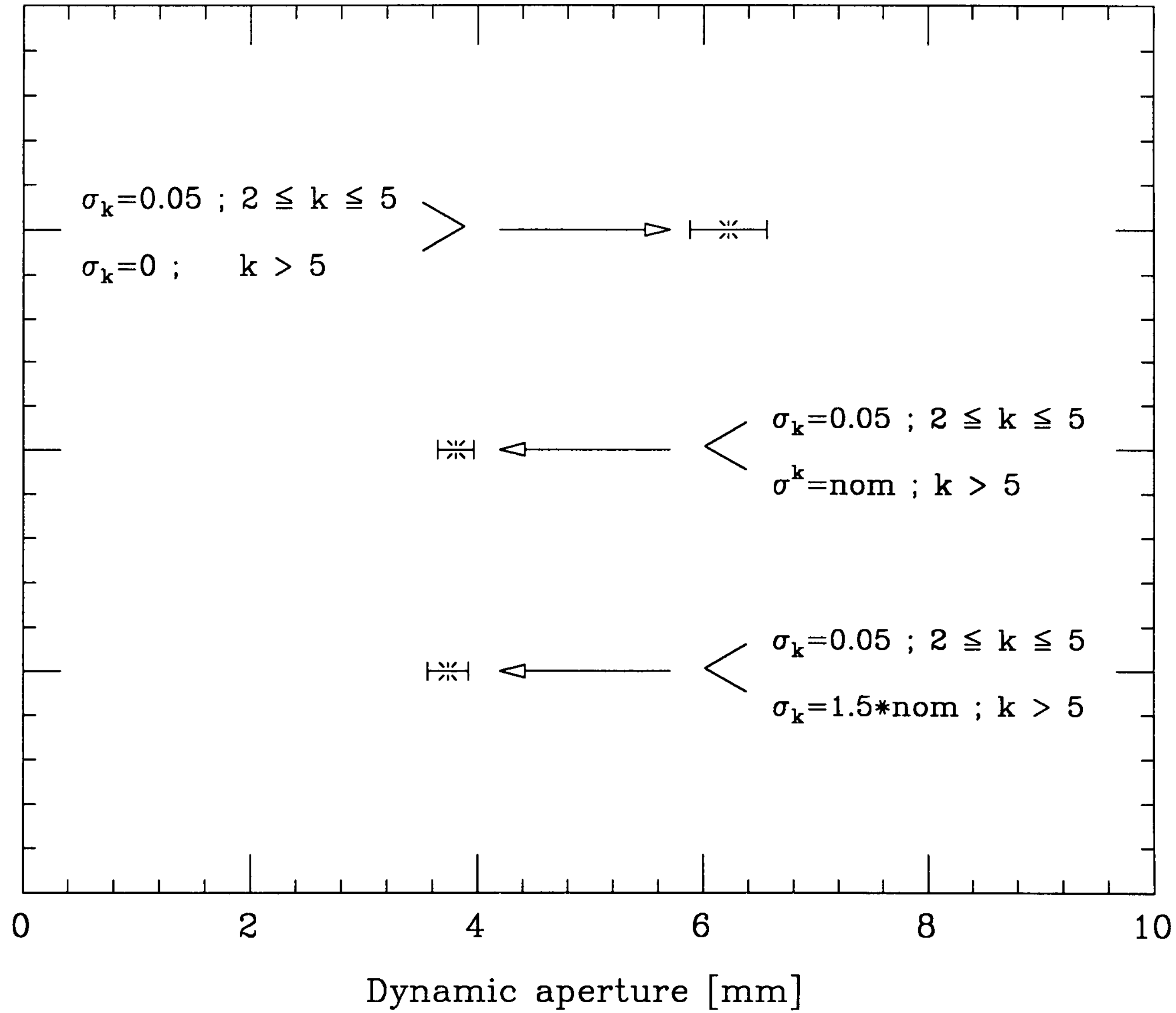

Fig. 11 : Dependence on High Order Multipoles