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CHARACTERIZATION FOR SENSOR FAILURE DETECTION**

**Authors:** Z. Frei and B. R. Upadhyaya

**Affiliation:** The University of Tennessee  
Knoxville, Tennessee

**Mailing Address:** Dr. B. R. Upadhyaya  
Department of Nuclear Engineering  
The University of Tennessee  
Knoxville, TN 37996-2300

**Phone:** (615) 974-5048

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Z. Frei\* and B. R. Upadhyaya  
The University of Tennessee, Knoxville, Tennessee

ABSTRACT

Independent estimation of critical signals is required for use in consistency checking of instrument outputs, and for isolating common-mode failures. When measurement redundancy is not available, this method can be used for monitoring sensor degradation or sensor drifts by on-line prediction of the sensor output. A methodology is developed to characterize a given critical signal as a function of other related variables in a process by a nonlinear minimum-term model. This steady-state characterization is fully automated and implemented on an IBM PC, and is applied for sensor failure monitoring in a power plant and a process control industrial system.

1. INTRODUCTION

Instrumentation channel outputs in large dynamic systems such as power plants, process industry and others are used in plant control systems, protection systems and monitoring systems. In order to reduce the burden of decision-making on the part of the plant operators, and to improve the reliability of operation, on-line systems are being developed for signal validation<sup>[1]</sup>. These methods often require the prediction of one or more instrument outputs as an independent source of information for sensor

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\* Roland Eotvos University, Budapest, Hungary.

verification. The objective of this paper is to present a systematic methodology for modeling a critical signal in a process as a function of other signals related to a subsystem. To achieve this goal we have developed a steady-state nonlinear modeling technique based on the measurement during a learning phase of the system.

The general form of this predictive model is given by

$$y_1 = C_0 + \sum_{i=1}^N C_i \phi_i(\underline{x}) \quad (1)$$

where  $y_1$  is the sensor output to be monitored,  $\underline{x} = (x_1, x_2, \dots, x_n)$  is the vector of subsystem variables that influence  $y_1$ , and  $\{\phi_i, i=1, 2, \dots, N\}$  are nonlinear terms which along with  $\{c_i\}$  minimize the prediction error.

Desrochers and Mohseni<sup>[2]</sup> developed a method of nonlinear polynomial fitting. We have adopted the general approach given in this paper and developed a new algorithm. The following aspects of the development are our contribution to this approach.

- (1) A complete software package to handle data, optimize model selection, and predict sensor output.
- (2) Application to different industrial processes to determine the validity of the method.
- (3) Choice of operating regions to determine best minimum-term models.
- (4) A more accurate model by including a constant term in Eq. (1) and the coefficient estimation at the end of the overall optimization procedure.

Our study showed that some of the conclusions in Ref.[2] were not fully valid. For example, the optimal (N+1) term model may not include all the terms of the optimal N-term model. Computing the  $\{C_i\}$  sequence as the model terms are selected will not result in the same sequence if it is computed at the end of optimization. The problem statement is given in Sect. 2. The minimum-term, optimal nonlinear modeling algorithm is described in Sect. 3. Sect. 4 contains a discussion of the implementational aspects of this algorithm with application to an operating power plant. Concluding remarks are given in Sect. 5.

## 2. PROBLEM STATEMENT

The analytical measurement or prediction of a critical signal  $y_1$  as a function of related variables in a subsystem, during steady-state or quasi steady-state operating condition is given by

$$y_1 = f_1(\underline{x}) = f_1(x_1, x_2, \dots, x_n) \quad (2)$$

This is shown schematically in Fig. 1. No assumption is made that the  $\underline{x}$  variables are independent of  $y_1$ .

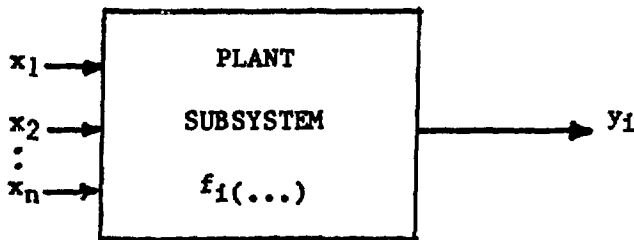


Figure 1. A plant subsystem showing the relationship between a critical signal  $y_1$  and the signal set  $(x_1, x_2, \dots, x_n)$  that influences the behavior of  $y_1$ .

In particular, we want to generate a functional relationship of the form

$$y_1 = C_0 + \sum_{i=1}^N C_i \phi_i(\underline{x}) \quad (3)$$

where  $(C_0, C_1, \dots, C_N)$  is a constant vector, and each  $\phi_i(\cdot)$  is a nonlinear term (cross product) with a specified maximum polynomial order. Our objective is to generate a minimum-term, optimal model from measurements  $(y_1, y_2, \dots, y_L, \underline{x}^1, \underline{x}^2, \dots, \underline{x}^L)$ , where  $L$  is the number of data points. This is referred to as the data-driven predictive model (DPM).

We will present a detailed algorithm to determine an optimal model and consider the following aspects.

- (1) Selection of the polynomial order and the number of terms  $N$  in the empirical model.
- (2) Scaling of variables in order to avoid numerical instability.
- (3) Error propagation due to the fitting procedure and measurement uncertainties.

### 3. NONLINEAR SYSTEM MODELING ALGORITHM

The general nonlinear steady-state system predictor has the form

$$y = \sum_{i=1}^N C_i \phi_i(\underline{x}) + C_0 \quad (4)$$

After choosing the functionals  $\phi_i(\underline{x})$ , the coefficients  $(C_0, C_1, \dots, C_N)$  are calculated using the least-squares procedure. Each  $\phi_i$  is a nonlinear term or cross product of the components of  $\underline{x}$ . The number of possible cross

product terms is a function of the polynomial order and the number of components in  $\underline{x}$ . In order to handle the nonlinear models, and to reduce the number of terms, or even the order of the model, several theories have been developed<sup>[3]</sup>. Among them the algorithm proposed by Desrochers and Mohseni<sup>[2]</sup> is found to be useful for our applications. Their algorithm is not generally optimal as stated in Ref.<sup>[2]</sup>.

We want to determine a minimum-term fit of the form of Eq.(4), without evaluating all possible cross product terms. The basic principle of the algorithm is given below. Let L be the number of data vectors, and M be the number of possible cross products. Using each cross product we can calculate M vectors, each of dimension L.

$$\underline{v}(i) = \{\phi_i(\underline{x}(k))\} \quad (5)$$

$i=1, 2, \dots, M; k=1, 2, \dots, L$ .  $\phi_i$  is the  $i$ -th cross product. Thus, we have an L-dimensional Euclidean space, with M vectors in this space. In this space the scalar signal  $y$  is represented as an L-dimensional vector. Our goal is to select N vectors,  $N < M$ , to give an optimal fit in the least-squares sense.

The computer algorithm follows the basic mathematical steps similar to those described in [2].

STEP 1. Determine the cross-product term closest to the vector  $\underline{y}^L$ . This will provide us with one-term model. Calculate the projection matrices

$$P(i) = \frac{[\underline{v}^j(i)][\underline{v}^j(i)]^T}{[\underline{v}^j(i)]^T[\underline{v}^j(i)]}, \quad i=1,2,\dots,M. \quad (6)$$

where  $j$  is the loop counter.

STEP 2. Project the actual output vector  $\underline{y}^L$  to different directions and calculate the scalar length.

$$\underline{y}^{Lj}(i) = P(i)\underline{y}^{Lj} \quad (7)$$

and 
$$R(i) = [\underline{y}^{Lj}(i)]^T[\underline{y}^{Lj}(i)] \quad (8)$$

STEP 3. Find the largest scalar length from among  $R(i)$ , and identify the cross product term based on the largest scalar length as the next term in the model.

STEP 4. If we need more terms, project all the remaining vectors onto the subspace orthogonal to the vector we have selected.  $\underline{y}^L$  is also projected onto this subspace. The cross product term that best approximates the error in  $\underline{y}^L$  is selected as above. Calculate the projection matrix corresponding to the selected term, and project the remaining vector onto the space orthogonal to the vector selected previously.

$$P(i_k) = \frac{[\underline{v}^j(i_k)][\underline{v}^j(i_k)]^T}{[\underline{v}^j(i_k)]^T[\underline{v}^j(i_k)]} \quad (9)$$

and

$$\underline{v}^{j+1}(i) = M^j \underline{v}^j(i), \quad i=1,2,\dots,M \quad (10)$$

$$i \neq i_1, l=1,2,\dots,j.$$

where

$$M^j = I - \sum_{l=1}^j P(i_l) \quad (11)$$

STEP 5. Project the output vector onto this subspace, check its length and select the next best term.

$$\underline{y}^{L_{j+1}}(i) = M_j \underline{y}^{L_j}(i) \quad (12)$$

Determine the error. Stop, or go to STEP 1.

Once the polynomial terms are selected the coefficients ( $C_0, C_1, \dots, C_N$ ) are determined from the least squares solution

$$\min_{(C_0, C_1, \dots, C_N)} \sum_{k=1}^M [y(k) - C_0 - \sum_{i=1}^N C_i \phi_i(\underline{x}(k))]^2 \quad (13)$$

#### 4. IMPLEMENTATION AND APPLICATION

##### 4.1 Implementation

We have developed a complete software package for implementation on an IBM PC. The software system consists of general routines to manage large data files, scaling to provide mathematically tractable data sets, a routine to perform the nonlinear modeling steps described in Sect. 3, a more general program to find the best polynomial order and an optimal number of terms in the model. Both input and output information are displayed. Once a model is created, it can be used to predict test data points and to evaluate the accuracy of measurements.

The following observations concerning the present work are made as compared with earlier work[2].

1. Selection of the best nonlinear model is based on evaluating the model of Eq. (4), rather than comparing only the residual errors after each step.
2. Prediction error is calculated explicitly.
3. As stated in Ref.[2], the best N-term model is not generally obtained after N steps.
4. Data scaling used in our algorithm is very important to derive an optimal fit.
5. It is necessary to generate models for different operating regimes.

The detailed description of the software system is given in Ref.[4].

#### 4.2 Application to Sensor Fault Monitoring in a Power Plant

The method described in Sect. 3 and the automated signal characterization (nonlinear modeling) algorithm are applied to data from an operating nuclear power plant. First, application to a reactor simulation data is presented. Figure 2 is a schematic of a U-tube type steam generator showing the variables influencing its dynamics. An optimal model is developed to predict the steam flow rate ( $y$ , lbm/sec) as a function of reactor power ( $x_1$ , %), hot leg temperature ( $x_2$ , F), cold leg temperature ( $x_3$ , F) and steam pressure ( $x_4$ , psi). The model has the form

$$y = 5.864x_1 - 0.04838 x_1 x_4 - 0.00118x_4^2 + 0.6428 \quad (14)$$

Figure 3 shows the comparison of the simulated measurement and the model prediction. It is clear that when there is no measurement error, very accurate models can be generated for prediction purposes.

The second example utilizes actual operating data from a pressurized water reactor (PWR) plant, similar to a Westinghouse, 1150 MWE, four-loop plant. Certain points must be made in implementation to operating systems. It is not always possible to determine a single model to characterize all the operating conditions. Thus several models may be required. If the database contains bad measurements, then these points must be removed from the set in order not to force the model fit through such observations. This aspect was illustrated<sup>[4]</sup> with application to a rolling mill system. In the current case study, our goal is to predict the hot leg temperature ( $y$ , F) as a function of steam generator pressure ( $x_1$ , psi), primary coolant flow rate ( $x_2$ , lbm/sec), feedwater flow rate ( $x_3$ , lbm/sec), and reactor power ( $x_4$ , %). The following model is generated.

$$y = 6.915x_2 + 5.541x_3 - 0.519x_2x_3 - 0.009x_3^2 - 47.34 \quad (15)$$

Note that some of the variables may not appear in the model; for example  $x_1$  and  $x_4$  in Eq. (15) are absent. Figure 4 shows the model performance comparing the measurement with prediction for the data points used in model fitting. The prediction error is also shown in the comparison. Figure 5 shows a comparison of the measurement (hot leg temperature) and prediction for test data points not used in developing the model. The performance is very good except for two measurements. We have to declare these points as suspicious.

For cases where a large database is used to characterize one or more critical signals it is necessary to cluster the database and develop models for each cluster. We have applied pattern clustering algorithms in application to a rolling mill system<sup>[5]</sup>. In general, database clustering must be considered in characterizing different operating conditions of a system.

## 5. CONCLUDING REMARKS

We have developed an automated algorithm for fitting an optimal nonlinear model to predict critical signals for purposes of sensor failure monitoring and process diagnostics in process control systems. Numerical studies using operational data from a nuclear power plant and a rolling mill[5] have established the feasibility and applicability of this analytical predictive method. The improvement in the previous methods has produced a robust algorithm useful for signal validation problems. The linear predictive model is a special case of the modeling discussed here. The modular architecture of the software allows easy interface with a general failure diagnostics system.

## ACKNOWLEDGMENT

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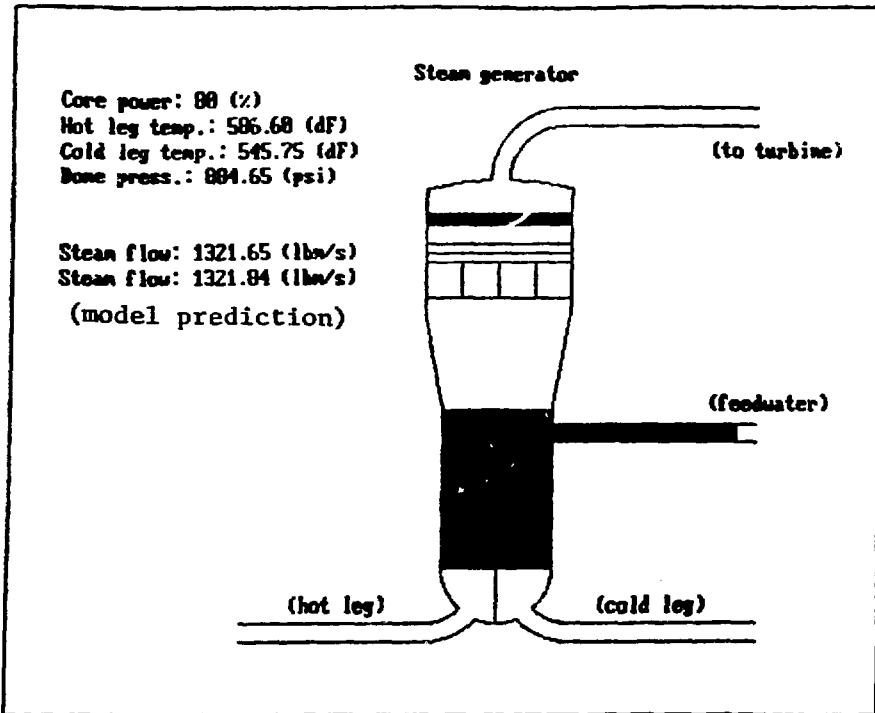


Figure 2. Schematic showing the PWR U-tube steam generator and process variables for one measurement.

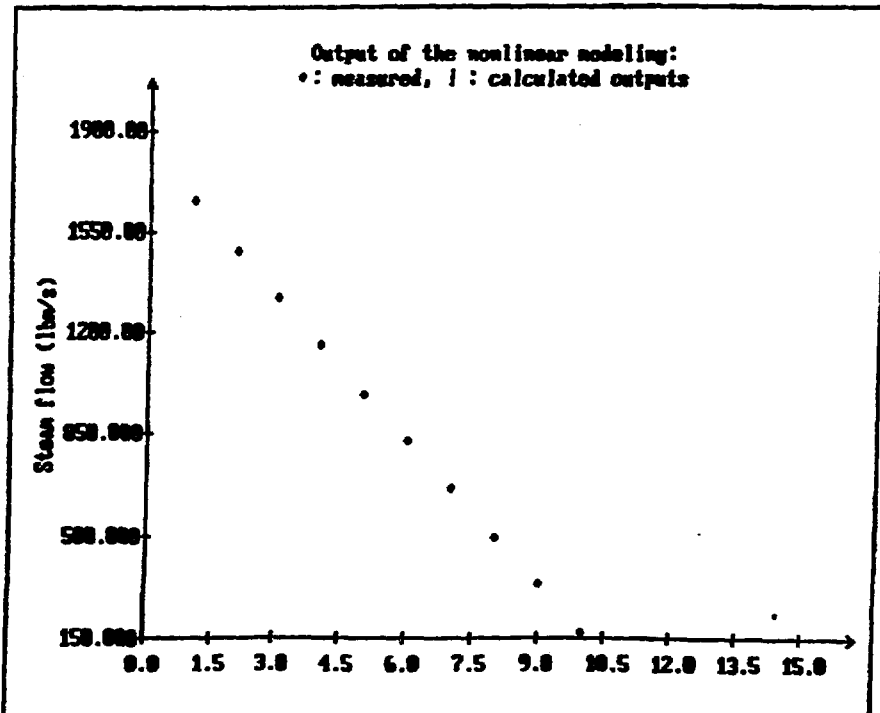


Figure 3. Measured (from simulation) and predicted values of steam flow rate signal, as a function of measurement points.

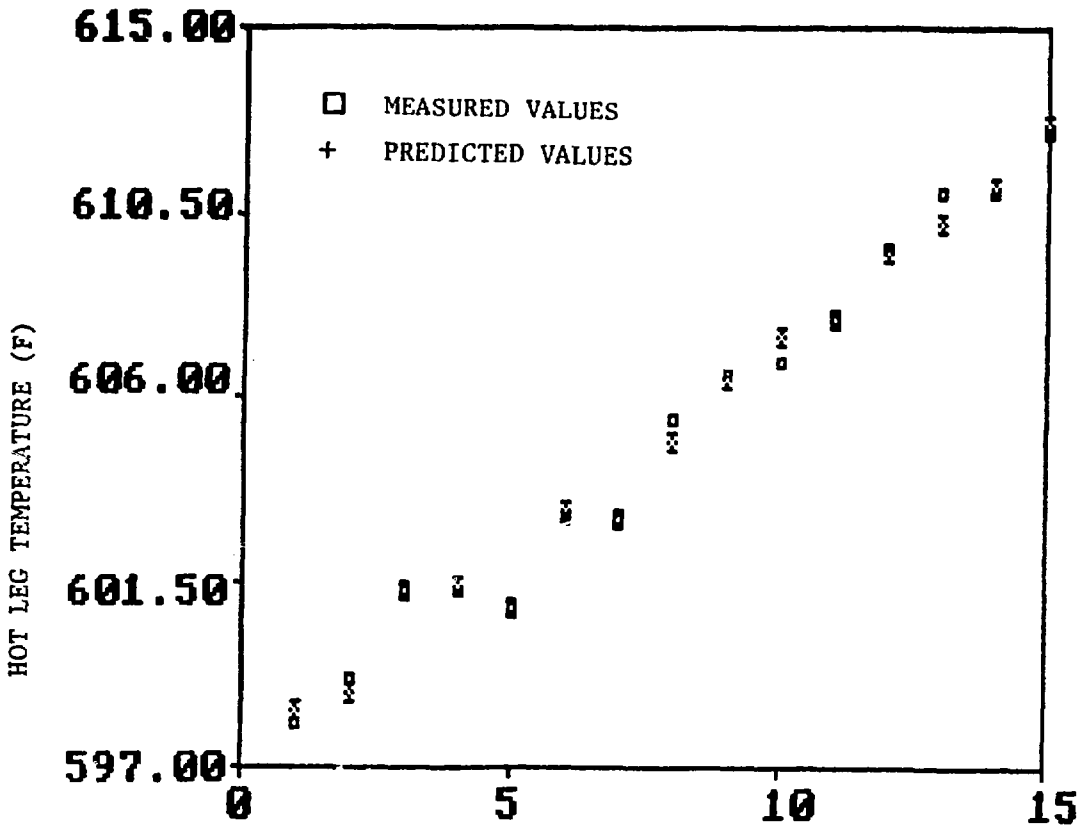


Figure 4. Comparison of measured and predicted values of hot leg temperature in an operating PWR (measurements used in model generation).

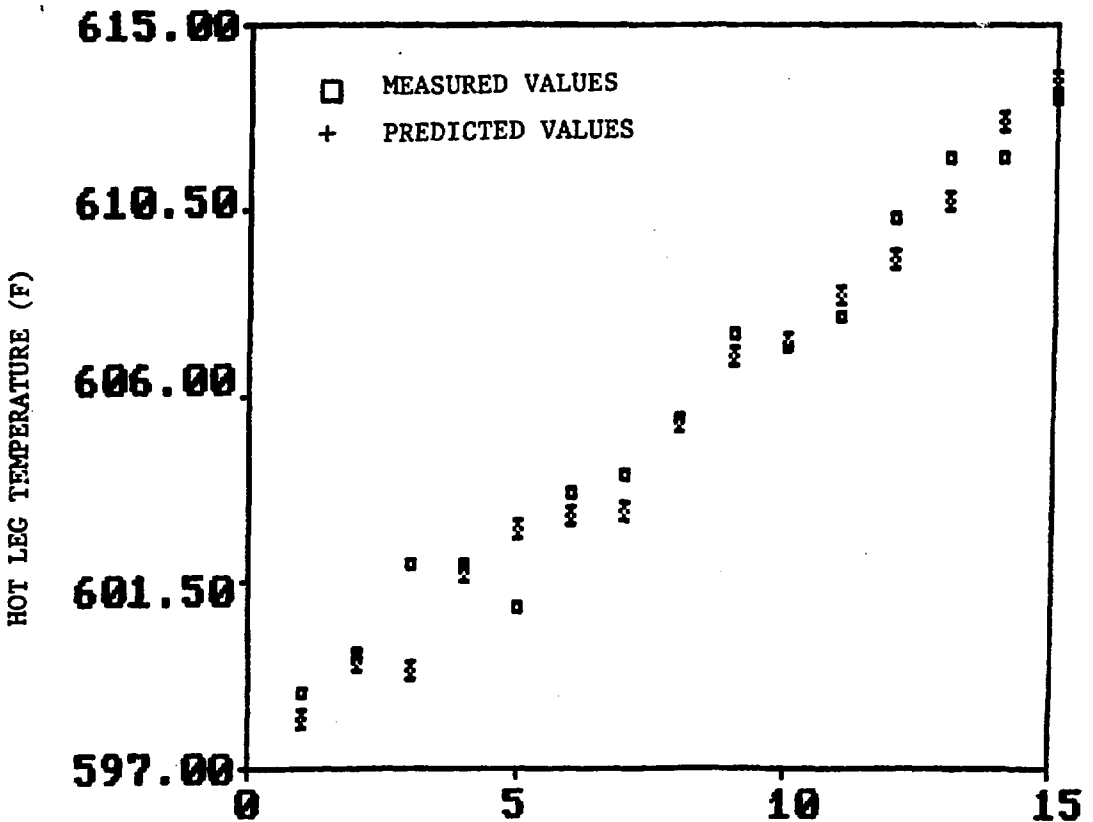


Figure 5. Comparison of measured and predicted values of hot leg temperature in an operating PWR (measurements not used for model generation).