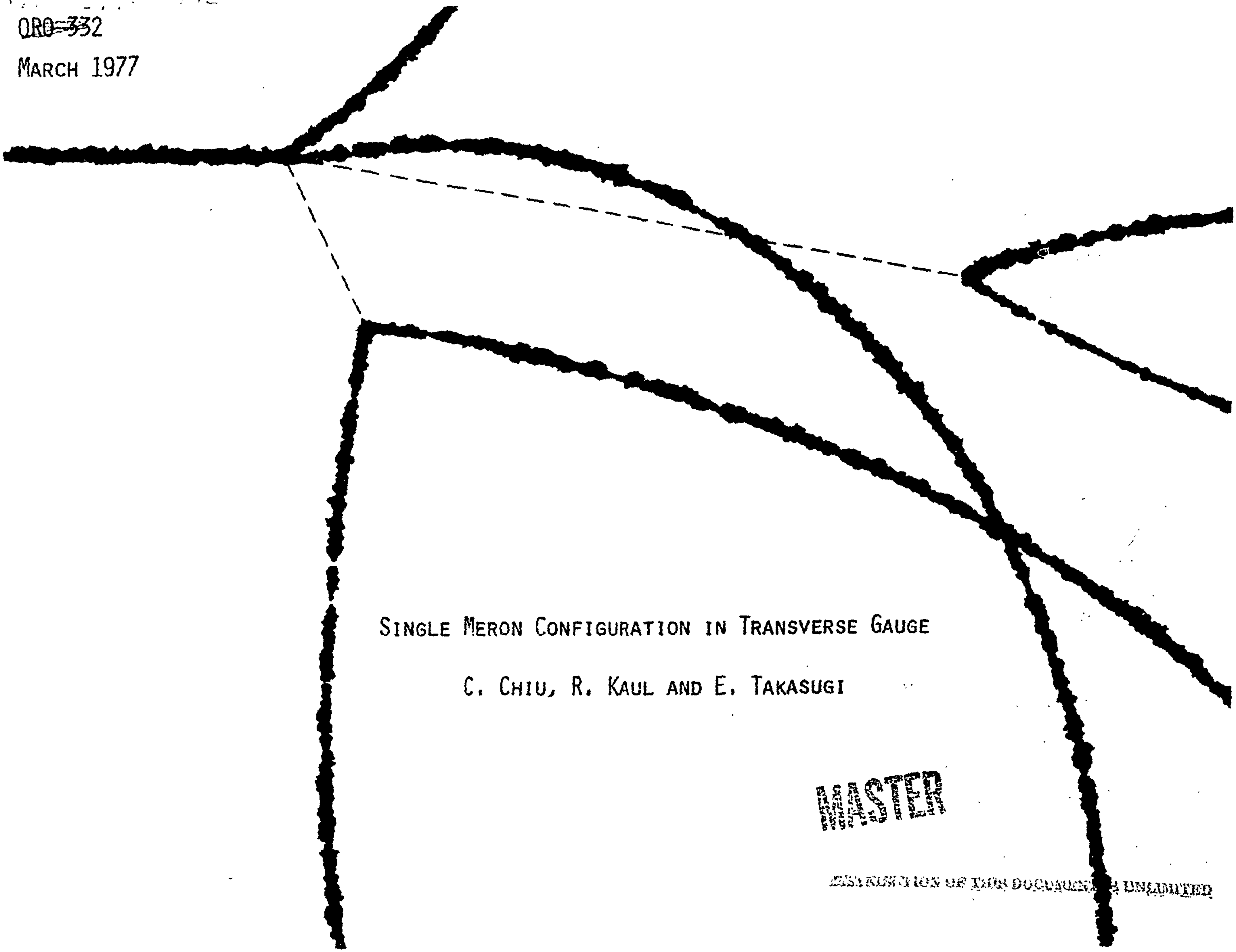
ORO-332

MARCH 1977



SINGLE MERON CONFIGURATION IN TRANSVERSE GAUGE

C. CHIU, R. KAUL AND E. TAKASUGI

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ORO 352

Single Meron Configuration in Transverse Gauge*

[illegible], [illegible], and T. Takasagi

Center for Particle Theory
Department of Physics
University of Texas
Austin, Texas 78712

*Work supported in part by the U.S. Energy Research Development Administration under Contract No. E(4-1)3992.

Abstract

The Alfaro-Fubini-Furlan single meron configuration for SU(2) Yang-Mills theory is studied in the transverse gauge. It is shown that, as the Euclidean time varies from distant past to distant future, this configuration evolves from the supervacuum to the Gribov vacuum which is the nonsingular vacuum with the topological charge 1/2.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Gribov's recent observation that the Coulomb (or Landau) gauge condition does not fix the gauge completely in non-Abelian gauge theories has led to quite a few peculiar physical results.[1] In particular, gauge fixing degeneracy implies non-uniqueness of vacuum in transverse gauge.[1,2] In this gauge, within SU(2) Yang-Mills theory, in addition to the usual perturbative vacuum, $A_\mu(x) = 0$ (supervacuum) with topological charge zero, there exist other space-dependent vacua with "topological" charges 1/2 and -1/2 (Gribov vacua). In contrast to the trivial vacuum $A_\mu(x) = 0$, these vacua, $A_\mu(x) = U^{-1}(x)\partial_\mu U(x)$, do not satisfy the asymptotic conditions $U(x) \to 1$ as $|\vec{x}| \to \infty$. Also the perturbative vacuum does not tunnel into any of these vacua.[2] However, Sciuto,[3] also Abbott and Eguchi[4] have independently argued that the one instanton configuration of Belavin, Polyakov, Schwartz, and Tyupkin (BPST) in the transverse gauge causes tunneling between the Gribov vacuum with topological charge -1/2 and the other Gribov vacuum with topological charge 1/2 as Euclidean time evolves from distant past to distant future.[3,4] It has also been suggested that the multi-instanton configurations cannot be written at all in the transverse form.[2] However, Jackiw, Muzinich and Rebbi have argued that discontinuities in the time evolution of the transverse potentials are essential in order to accommodate various configurations with the arbitrary Pontryagin indices.[6]

In this short note we shall study the transverse forms of the Alfaro-Fubini-Furlan (AFF) single meron configuration.[7] These configurations, it will be argued, connect the supervacuum to the Gribov vacuum, i.e., the non-singular vacuum with the topological charge $\pm 1/2$ as Euclidean time evolves from the distant past to the distant future.

AFF single meron configuration in SU(2) Yang-Mills theory[7] can be expressed as

$$A_\mu(x) = \frac{1}{2} g^{-1}(x)\partial_\mu g(x) , \tag{1}$$

where

$$A_\mu(x) = A_\mu^a(x)\tau^a/2i ,$$

$$g(x) = \frac{-t + i\vec{x}\cdot\vec{\tau}}{\sqrt{t^2 + r^2}} \equiv \exp\left[i\beta(r,t)\,\frac{\vec{x}\cdot\vec{\tau}}{r} \right] , \tag{2}$$

and

$$\beta(r,t) = -\tan^{-1}\left(\frac{r}{t}\right), \qquad (0 \leq \beta \leq \pi) . \tag{3}$$

$\tau^a/2$ are SU(2) generators and t is the Euclidean time which ranges from $-\infty$ to ∞. In order to study the transverse form of this meron solution, we perform a gauge transformation $h(x)$, such that

$$B_\mu(\vec{x},t) = h^{-1}(x)A_\mu(x)h(x) + h^{-1}(x)\partial_\mu h(x) , \tag{4}$$

and

$$\partial_i B_i(\vec{x},t) = 0 \ . \qquad (5)$$

This transverse gauge potential can be written as

$$B_\mu(\vec{x},t) = \frac{1}{2}\{(g^{1/2}G)^{-1}\partial_\mu(g^{1/2}G) + (g^{-1/2}G)^{-1}\partial_\mu(g^{-1/2}G)\} \ , \qquad (6)$$

where

$$G(x) = g^{1/2}(x)h(x) \equiv \exp\left\{i\left[\alpha(r,t) - \frac{\pi}{2}\right]\frac{\vec{x}\cdot\vec{\tau}}{r}\right\} \ . \qquad (7)$$

Since $\beta(r,t\to\infty) = \pi$ and $\beta(r,t\to-\infty) = 0$, the gauge gield B_μ goes to vacua as $t \to \pm\infty$,

$$B_\mu(r,t\to-\infty) = G^{-1}(x)\partial_\mu G(x) \ ,$$

$$B_\mu(r,t\to\infty) = \bar{G}^{-1}(x)\partial_\mu \bar{G}(x) \ , \qquad (8)$$

where $G(x)$ is given by (7) and $\bar{G}(x)$ is given by

$$\bar{G}(x) = \exp\left\{i\alpha(r,t)\frac{\vec{x}\cdot\vec{\tau}}{r}\right\} \ . \qquad (9)$$

Field B_μ is written in terms of α and β as

$$B_i = -\frac{\tau^a}{2i}\left\{\varepsilon_{iaj}\frac{x_j}{r^2}(1 - \psi\cos 2\alpha) + \left(\delta_{ai} - \frac{x_a x_i}{r^2}\right)\frac{\psi\sin 2\alpha}{r} + \frac{x_a x_i}{r^2}2\alpha'\right\} \ , \qquad (10)$$

and

$$B_4 = -\frac{\tau^a}{2i}\frac{x_a}{r}2\partial_t\alpha \ , \qquad (11)$$

where prime denotes differentiation with respect to r and

$$\psi = -\cos\beta = t/\sqrt{t^2 + r^2} \ . \qquad (12)$$

The transversality condition for the meron configuration can be obtained by substituting (6) into (5):

$$\alpha'' + \frac{2}{r}\alpha' - \frac{\psi}{r^2}\sin 2\alpha = 0 \ . \qquad (13)$$

As $t \gg r$ ($-t \gg r$), this condition reduces to the damped pendulum equation for α ($\bar{\alpha} = \alpha \pm \pi/2$) with a periodic potential $V(\alpha) = -\sin^2\alpha$ as discussed by Gribov,[1] and Wadia and Yoneya[8]:

$$\alpha'' + \frac{2}{r}\alpha' - \frac{\sin 2\alpha}{r^2} = 0 \ . \qquad (14)$$

We now turn to the solutions of (13). We shall require that the field B_μ of (10) and (11) be continuous at $r = \infty$ for all t, and it is nonsingular except at the origin[a]. The trivial solutions to Eq. (13) are:

$$\alpha(r,t) = \frac{n\pi}{2} \ , \qquad n = 0,\pm1,\pm2,\ldots \ . \qquad (15)$$

However, they are not compatible with the above requirement. More specifically, for even n, the field is singular at r = 0, along the negative t axis; while for odd n, it is singular at r = 0, along the positive t axis.

Now the nontrivial solutions. We introduce function f with $0 \leq f < \pi$. We shall see later, for α solutions it is possible to write, for t > 0

$$\alpha(r,t) = \pm f + n\pi \ , \tag{16}$$

and the corresponding differential equation is given by

$$f'' + \frac{2}{r}f' - \frac{1}{r^2\sqrt{1 + \left(\frac{r}{t}\right)^2}}\sin 2f = 0 \ . \tag{17}$$

For a general fixed positive t, the solution f is a function of the independent variable r and is specified by two constants of integration, which are chosen to be the value of f and its slope at r = 0,

$$f = 0 \quad \text{and} \quad f' = C \ . \tag{18}$$

The set of all possible solutions at this fixed time t $F(r,t;C)$ corresponds to having "the slope parameter" C varying from 0 to ∞.

Now as t varies, the t-dependence of f is determined

through a specific choice for the slope parameter function $C \equiv C(t)$. We are not able to write down a close form for the solution, although the solution in two specific regions can be easily obtained. In particular, for f near zero and $r \ll t$ we have

$$f(r,t) = C(t)r \ , \tag{19}$$

and for f near $\pi/2$ and $r \gg t$,

$$f(r,t) = \frac{\pi}{2} + \frac{D(t)}{r} \ , \tag{20}$$

where D(t) is completely specified, once C(t) is given.

Now we turn our attention to the general form of C(t), so that as $t \to 0$, $C(t) \to \infty$ (in this limit B_μ develops a singularity at origin); and as $t \to \infty$ C(t) approaches some finite nonzero constant. More specifically for small t, we take f(r,t) to be that obtained through a scale transformation on the initial solution at $t = t_o$, $F(r,t_o;C(t_o))$:

$$f(r,t) \equiv F(r,t;C(t)) = F\left(\frac{rt_o}{t},t_o;C(t_o)\right) \ , \tag{21}$$

for small t. One can easily check that this implies having

$$C(t) = \frac{C(t_o)t_o}{t} \ . \tag{22}$$

As t approaches zero, the scaling factor t_o/t increases indefinitely. In turn the structure of $F(r,t_o;C(t_o))$ at finite domain of r is pushed toward the point r = 0 in $F(r,t;C(t))$, and thus f(r,t) approaches $\pi/2$ for every point of r, except r = 0. In other words, as t approaches zero from the positive direction

$$f(r,0) = \frac{\pi}{2} , \quad \text{for } r > \epsilon , \tag{23}$$

and the structure is cumulated in the region $0 < r < \epsilon$, with ϵ being arbitrarily small.

A numerical example for the small t behavior for the case C = 100/t is shown in Fig. 1a,b,c.

Now the large t limit. As $t \to \infty$, we require C(t) to approach a constant. This is both the necessary and sufficient conditions, which allow f(r,t) to be smoothly joined onto the solution, f(r), of the damped pendulum equation of Gribov (14), where the corresponding r-slope is given by $f'(0) = C(\infty)$. The numerical example for large t, and that for the Gribov vacuum with C = 10 are shown in Figs. 1d and 1e respectively.

So far we have only concentrated on the behavior of f(r,t) near t = 0 and at large t. The general solutions explored here are associated with some general smooth t-dependent form for C(t), with its small t behavior specified by Eq. (22). We observe that the small t behavior can be further generalized to

include all possible forms of C(t), where the function is continuous for positive t, and as t approaches zero, C(t) smoothly approaches infinity. Our reasoning for this is as follows. The infinite r-slope at r = 0 and t = 0, i.e. $C(0) = \infty$ is necessary to arrive at the behavior of given in (23). However, it appears that the specific rate for approaching infinity given in (22) is not a necessary condition.

For a given positive t solution, f, the corresponding general solutions for α for t > 0 are given by (16). For t < 0, the corresponding solutions are given by

$$\alpha(r,t) = \pm f(r,t) + \left(m + \frac{1}{2}\right)\pi , \tag{24}$$

where $m = 0, \pm 1, \pm 2, \ldots$. Solutions $\alpha(r,t)$ both for t > 0 and t < 0 given by (15), (16), and (24) are to be matched at t = 0 by requiring the continuity of $\alpha\ (r = \infty, t)$ and $\partial\alpha\ (r = \infty, t)/\partial t$. From (10) and (11), one sees that this guarantees the continuity of $B_\mu(\vec{x},t)$ at $r = \infty$ with respect to t. Thus we get the following four general classes of solutions, up to modulus π:

$$\begin{aligned}
&\text{(i)} \quad \alpha(r,t) = \pi/2, \quad t < 0, \quad \text{and} \quad \pi - f(r,t), \quad t > 0 , \\
&\text{(ii)} \quad \alpha(r,t) = \pi/2 + f(r,t), \quad t < 0 \quad \text{and} \quad \pi, \quad t > 0 , \\
&\text{(iii)} \quad \alpha(r,t) = \pi/2, \quad t < 0, \quad \text{and} \quad f(r,t), \quad t > 0 , \\
&\text{(iv)} \quad \alpha(r,t) = \pi/2 - f(r,t), \quad t < 0 \quad \text{and} \quad 0, \quad t > 0 .
\end{aligned} \tag{25}$$

We note that these four solutions are continuous everywhere except at the origin which is a cummulation point. From (10) and (11) the field configuration $B_\mu(\vec{x},t)$ is singular at the origin and discontinuous along the line $0 < r < \infty$ at $t = 0$, and continuous everywhere else.

The topological charge is defined by[5]

$$q = \frac{1}{16\pi^2}\int_{S_\infty} dS_\mu \varepsilon_{\mu\nu\rho\sigma} \mathrm{Tr}(B_\nu \partial_\rho B_\sigma + \tfrac{2}{3} B_\nu B_\rho B_\sigma) \ . \qquad (26)$$

The surface of integration S_∞ is taken to be a cylinder with its axis along the t-axis. In this case q is decomposed into three pieces: two pieces are associated with the flux crossing the $t = \pm T$ ($T \gg 0$) surfaces, $\phi_\pm$, and one associated with the flux crossing the lateral surface of the cylinder, ϕ_L,

$$q = \phi_+ - \phi_- + \phi_L \ . \qquad (27)$$

For the potential (6), or equivalently (10) and (11), we obtain[b]

$$\phi_\pm = \lim_{T\to\pm\infty} \frac{-1}{\pi}\left[\alpha - \tfrac{1}{2}\psi\sin 2\alpha\right]_{r=0}^{r=\infty} , \qquad (28)$$

and

$$\phi_L = \lim_{T\to\infty} \frac{1}{\pi}\left[\alpha - \tfrac{1}{2}\psi\sin 2\alpha\right]_{-T}^{T}\Bigg|_{r=\infty} \ . \qquad (29)$$

Let us examine the topological nature of the various solutions. The trivial solution $\alpha = \pi/2$ given in (15) leads to a transverse field B_μ

$$B_i^a(\vec{x},t) = \tfrac{1}{2}(1 + t/\sqrt{r^2 + t^2})\varepsilon_{aij}\frac{x_j}{r^2} , \qquad B_4^a(\vec{x},t) = 0 , \qquad (30)$$

which has a singularity along the line, $r = 0$, and $t > 0$. The topological charge is readily calculated from (28) and (29). It gives $\phi_\pm = 0$ and $\phi_L = 0$, since α is constant for all space-time.

The topological charge[c] of the original meron configuration of (1) is $\phi_+ = 1/4$, $\phi_- = -1/4$, $\phi_L = 0$ and $q = 1/2$. One can see explicitly how this charge is lost in the course of the singular gauge transformation of Eq. (4). In particular, this transformation is $h = \exp\{i\gamma(\vec{x},t)\vec{x}\cdot\vec{\tau}/r\}$ with $\gamma(\vec{x},t) \equiv \alpha - (\beta+\pi)/2 = -\beta/2$. This turns out to carry a topological charge $-1/2$ distributed on the three sides of the cylinder as $\phi_+ = -1/4$, $\phi_- = 1/4$ and $\phi_L = 0$.

For the configuration (i) of Eq. (25), from Eqs. (28) and (29), it gives $\phi_+ = 1/2$, $\phi_- = 0$, $\phi_L = 0$, or a topological charge 1/2. For this case, the gauge transformation h is associated with a zero topological charge, with $\phi_+ = 1/4$, $\phi_- = 1/4$ and $\phi_L = 0$. The corresponding field B_μ is singular at the origin, $r=0$, $t=0_+$ and discontinuous along the axis $0 < r < \infty$ at $t = 0$. The topological charge $q = 1/2$ is also located at the origin. Our demonstration for this is as follows. We observe that the right hand side of (28) and (29) can be evaluated for any shapes of coaxial cylinders. First consider a flat disk-like cylinder with its two bases at $t = 0_{\pm}$ and r extends to ∞. For the case, $\phi_+ = 1/2$, $\phi_- = 0$ and $\phi_L = 0$, or a topological charge 1/2. On the other hand, consider a long thin cylinder with its bases at $t = \pm\infty$ and its radius $r = 0_+$. Here one finds $\phi_{\pm} = 0$ and $\phi_L = 1/2$ or again $q = 1/2$. The topological charge 1/2 must be located in the region common to both cylinders, which is the origin. Thus the configuration (i) being associated with $q = 1/2$ is a meron in the transverse gauge, which connects the supervacuum to the Gribov vacuum with again $q = 1/2$ as the Euclidean time evolves from distant past to distant future.

Similarly solution (ii) is a single meron configuration in the transverse gauge which connects the vacuum with the topological charge -1/2 to the supervacuum. Solution (iii) connects the supervacuum to the Gribov vacuum with the topological charge -1/2 and solution (iv) connects the Gribov vacuum with the topological charge 1/2 to the supervacuum. More detailed informations are summarized in Table I.

Finally we remark that other configurations which have the magnitude of the topological charge $|q|$ greater than 1/2 can also be constructed if we allow the discontinuity for $\alpha(r,t)$ at $r = \infty$ for some t. The appearance of a cummulation point in $\alpha(r,t)$ at $r = t = 0$ for our transverse configuration (i) - (iv) is not unusual. The BPST instanton configuration

in the transverse gauge obtained by Sciuto[3] also has a similar cummulation point at $r = t = 0$. As expected, our transverse configurations(i)-(iv) have zero Euclidean energy and the action $S = 3\pi^2 \ln R$, where R is the size of space and time.

We would like to thank Professor E.C.G. Sudarshan for several useful discussions and critical comments and Professor S. A. Zaidi for kindly showing us his program for the non-linear differential equation.

References

1. V. N. Gribov, Lecture at the 12th Winter School of the Leningrad Nuclear Physics Institute (1977), translated by H.D.I. Abarbanel, SLAC preprint, SLAC-TRANS-176.
2. M. Ademollo, E. Napolitano, and S. Sciuto, CERN preprint TH-2412 (1977).
3. S. Sciuto, Phys. Lett. 71B, 129 (1977).
4. L. F. Abbott and T. Eguchi, Phys. Lett. 72B, 215 (1977).
5. A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Yu. S. Tyupkin, Phys. Lett. 59B, 85 (1975).
6. R. Jackiw, I. Muzinich, and C. Rebbi, Brookhaven preprint BNL-23360 (1977).
7. V. de Alfaro, S. Fubini, and G. Furlan, Phys. Lett. 65B, 163 (1976).
8. S. Wadia and T. Yoneya, Phys. Lett. 66B, 341 (1977).

Footnotes

a. Let us contrast our criteria with that considered in Ref. 6, where a discontinuity in the transverse field at $r = \infty$ for some t is necessary in order to accommodate arbitrary Pontryagin indices. We consider here the field configuration with half-integer topological charge.

b. The limit $r \to \infty$ is always taken before the $t \to \pm\infty$ limit.

c. Topological charge defined by (26) for meron $A_\mu = (1/2)g^{-1}\partial_\mu g$ and vacuum $A_\mu = g^{-1}\partial_\mu g$, $g = \exp\{i\gamma(\vec{x},t)\vec{x}\cdot\vec{\tau}/r\}$ is given by $\phi_\pm = -\lim_{t\to\pm\infty} \frac{(1+\delta)}{2\pi} \left[\gamma - \frac{1}{2}\sin 2\gamma\right]_{r=0}^{r=\infty}$ and $\phi_L = \lim_{T\to\infty} \frac{(1+\delta)}{2\pi} \cdot \left[\gamma - \frac{1}{2}\sin 2\gamma\right]_{t=-T}^{t=T}\Big|_{r=\infty}$, where $\delta = 0$ for meron and 1 for vacuum.

Table Caption

Table I: The topological charges of various transverse configurations and the corresponding gauge transformation h. (i)-(iv) correspond to meron (m) and anti-meron ($\bar{m}$) configurations in the transverse gauge defined by Eq. (25). Q(x) is the topological charge density.

Figure Caption

Fig.1: Numerical solution f(r,t) versus $\ln(r/r_0)$ with $r_0 = e^{-10}$ for various t. (a), (b), and (c) exhibit the small t behavior of f(r,t) with the slope parameter *C(t)* $f'(r=0,t) = 100/t$. Note that, as $t \to 0_+$, the bump structure *is pushed toward r=0* and f(r,t) approaches $\pi/2$ except for r=0. (d) exhibits the large t *behavior of f(r,t)* (for *C*=10). (e) The solution f(r) of the damped pendulum equation (14) *with the slope* parameter $f'(r=0) = 10$ is shown. The similarity between (d) and (e) indicates *that for a common* slope parameter, at large t, the function *f(r,t)* approaches *f(r)*.

Table I

	Transverse Field $B_\mu(\vec{x},t)$						Grauge Transformation $h(\vec{x},t)$			
	ϕ_+	ϕ_-	ϕ_L	q		Q(x)	ϕ_+	ϕ_-	ϕ_L	q
$\alpha = \frac{\pi}{2}$	0	0	0	0		0	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$
(i)	$\frac{1}{2}$	0	0	$\frac{1}{2}$	m	$\frac{1}{4\pi}\delta(t+0)\delta(r)$	$\frac{1}{4}$	$\frac{1}{4}$	0	0
(ii)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	m	$\frac{1}{4\pi}\delta(t-0)\delta(r)$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0
(iii)	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\bar{m}$	$\frac{-1}{4\pi}\delta(t+0)\delta(r)$	$-\frac{3}{4}$	$\frac{1}{4}$	0	-1
(iv)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\bar{m}$	$\frac{-1}{4\pi}\delta(t-0)\delta(r)$	$-\frac{1}{4}$	$\frac{3}{4}$	0	-1
AFF Meron	$\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	m	$\frac{1}{4\pi}\delta(t)\delta(r)$