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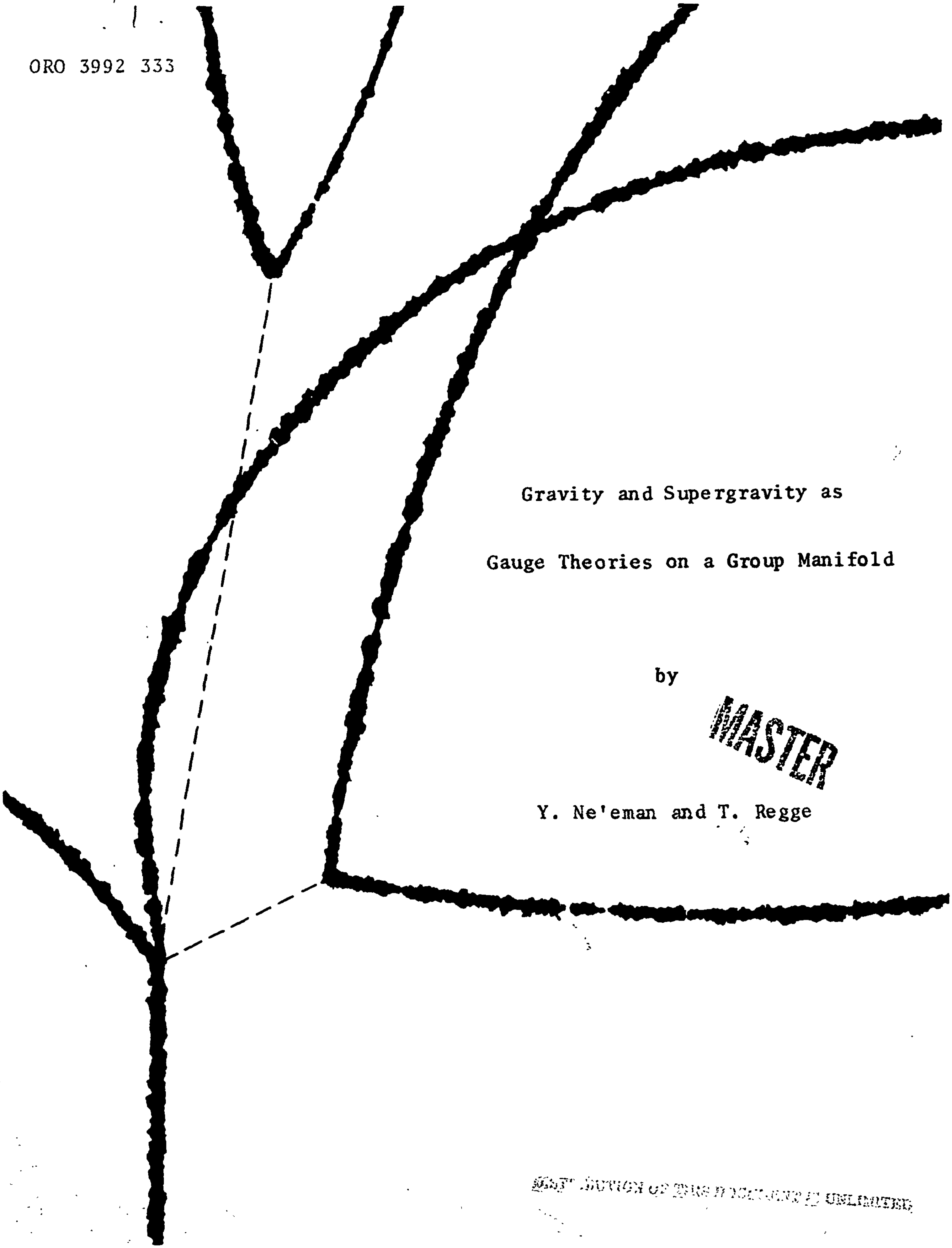

Gravity and Supergravity as Gauge Theories on a Group Manifold

by

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Gravity and Supergravity as Gauge Theories on a Group Manifold

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Abstract

We construct generalizations of Gravity, including Supergravity, by writing the theory on the Group manifold (Poincaré for Gravity, the Graded-Poincaré group for Supergravity). The action involves forms over the group, restricted to a 4-dimensional submanifold. The equations of motion produce a Lorentz-gauge in gravity and supergravity, and an additional anholonomic supersymmetric coordinate transformation which reduces to the "local supersymmetry" of supergravity.

In this letter we propose a method for the construction and discussion of generalizations of conventional Einstein gravity theory, including recent theories of supergravity.[1].

The idea is to write such a theory on a group manifold. Let G be [a non semi-simple] Lie group of dimension c [e.g. the Poincaré group P for gravity, the Graded Poincaré[2] formal-group[3] GP for supergravity]. On G we give a set of c forms ρ^A and define G-curvature as the 2-forms

$$R^A = d\rho^A - \tfrac{1}{2}\, \rho^B \wedge \rho^E\, C_{BE}^{\cdot\cdot\ A} \qquad (1)$$

where $C_{BE}^{\cdot\cdot\ A} = -(-1)^{be}\, C_{EB}^{\cdot\cdot\ A}$ are the (graded, with b,e the gradings[4] of B,E) structure constants of the Lie algebra $\mathfrak{g}$ of G. For the case $\rho^A = \omega^A$, where ω^A are the left invariant Cartan forms on G, $R^A = 0$ and (1) realizes the Cartan-Maurer equations. We refer to this case as the "flat" geometry.

We regard the ρ^A as the "objects" of the theory, to be treated as the potentials of a Yang-Mills G-gauge theory, and the R^A are the corresponding field-strengths. Given any C -multiplet η^A (η_A) we define G-covariant derivatives as,

$$\begin{aligned} (D\eta)^A &= d\eta^A - \rho^B \wedge \eta^E\, C_{BE}^{\cdot\cdot A} \\ (D\eta)_A &= d\eta_A + \rho^B \wedge \eta_E\, C_{BA}^{\cdot\cdot E} \end{aligned} \qquad (2)$$

Gravitational theories follow from an Action of the form

$$A = \int_{\mathfrak{M}^4} R^A \wedge \zeta_A \qquad (3)$$

where $\mathfrak{M}^4$ is any 4-dimensional submanifold of G, and $\zeta_A = \zeta_A(\rho)$ is a 2-form constructed as a quadratic polynomial in the ρ^A. A should be stationary with respect to all variations of ρ^A and of $\mathfrak{M}^4$. This last condition turns out, however, to be trivially satisfied by virtue of the general covariance of the theory. The theory should admit "flat" space $\rho^A = \omega^A$ as a solution. The field equations can be symbolycally written as

$$R^A \wedge \frac{\delta\zeta_A}{\delta\rho^B} - D\zeta_B = 0 \qquad (4)$$

which is satisfied by $\rho^A = \omega^A$, $R^A = 0$ only if

$$D\zeta_A(\omega) = 0 \qquad (5)$$

A form ζ_A satisfying (5) will be designated as a pseudo-curvature.

Clearly, the action (3) is not invariant under all G-gauge transformations. For example, Einstein gravity [5], which can be written as in (3),

$$\zeta_{ab} = \varepsilon_{abcd}\, \rho^c \wedge \rho^d , \qquad \zeta_a = 0$$
$$A = \frac{1}{8} \int_{\mathfrak{M}^4} R^{ab} \wedge \zeta_{ab} \qquad (6)$$

is only SO(3,1) gauge-invariant. Therefore, the assumption of a pseudo-curvature ζ_A breaks the G-symmetry. Pseudo-curvatures are tied to a non-trivial Segal-Inönu-Wigner contraction procedure from a semi-simple group G' to G; in the above case, from SO(3,2) to P. In fact, if $C_{BE}^{\cdot\cdot A}(\varepsilon)$ are the structure constants of G', and $C_{BE}^{\cdot\cdot A}(0) = C_{BE}^{\cdot\cdot A}$ those of G,

$$\left.\frac{\partial C_{BE}^{\cdot\cdot A}(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon = 0} \rho^B \wedge \rho^E = \zeta^A \tag{7}$$

is a pseudo-curvature. Pseudo-curvatures are related to the Chevalley cohomologies on groups, and to the MacDowell - Mansouri derivations [6] of gravity theories.

The action (3) for supergravity is given by selecting ζ_A as:

$$\begin{aligned} &\zeta_{ab} = \varepsilon_{abcd}\, \rho^c \wedge \rho^d \\ &\zeta_a = 0 \\ &\bar{\zeta}_\alpha = -4i\ \bar{\rho}\gamma^a\ \rho^a\gamma_5 \quad \text{or} \quad \zeta^\alpha = -4i\ \gamma_5\ \gamma^a\ \rho^a\ \rho \end{aligned} \tag{8}$$

We assume the validity of the field-equations (4) throughout G. Clearly, if A is invariant under a gauge subgroup $H \subset G$ (i.e. SO(3,1) for both P and GP), a solution on G can be derived from its boundary value on $Z = G/_H$ through the so-called factorization hypothesis,

$$\rho^A = \omega_H^A + \tau^B\ \ \mathrm{ad}(q^{-1})_B^{\cdot A} \tag{9}$$

where, locally, G has coordinates (z, q), $z\varepsilon Z$, $q\varepsilon H$, τ^B are the (factorized) forms on Z, ω_H^A the restrictions of the Cartan forms ω^A to H. We conjecture that in general ρ^A is determined by its boundary values on any $\mathfrak{M}^4$, apart from a generic coordinate transformation on G. This can be proved to hold if a solution of (4) is sufficiently close to a factorized one. Globally, one cannot exclude the possible existence of "twisted" topologically inequivalent factorized solutions sharing the same boundary conditions on $\mathfrak{M}^4$.

On P (GP), eq. (9) becomes,

$$\rho^{ab} = (\Xi^{-1} d \Xi)^{ab} + r^{cd} \Xi^{db} \Xi^{ca}$$

$$\rho^{a} = \Xi^{ca} \tau^{c} \qquad (10)$$

$$(\rho = u (\Xi^{-1}) \tau)$$

where $\Xi^{ab} \in H$, $\Xi^{ab} \Xi^{cb} = \delta^{ac}$ (we disregard the Minkwoski metric). Z is $\mathbb{R}^4$ in P, and $\mathbb{R}^{4/4}$ ("superspace", with 4 Bose and 4 Fermi dimensions [7]) in GP. For P and Einstein's gravity, Z coincides with $\mathfrak{M}^4$ so that (9) connects solutions on possible choices of $\mathfrak{M}^4$ as discussed. For GP, $\mathfrak{M}^4$ is still a submanifold of $\mathbb{R}^{4/4}$. In principle, eq. (4-5) should determine the extension of the form ρ^A to $\mathbb{R}^{4/4}$ from their restriction on $\mathfrak{M}^4$. However, this is not achieved through a gauge transformation as it is for the Ξ^{ab} variables on SO(3,1). In order to compute the change in the ρ^A while moving infinitesimally from a surface $\mathfrak{M}^4$ to $\mathfrak{M}^{4'}$, we utilize a generalization to an arbitrary group G of a set of transformations defined by Von der Heyde[8] and by Hehl and collaborators[9]. Carrying out an infinitesimal coordinate change $x^M \rightarrow x^M + \varepsilon^M$ (M, N are holonomic indices; A, B are anholonomic) we find for the change in ρ^A,

$$\delta\rho^A = D \varepsilon^A - 2 (\varepsilon, R^A) \qquad (11)$$

where (this is an "anholononized general coordinate transformation", AGCT)

$$\varepsilon^A = \varepsilon^M \rho_M{}^A \quad , \quad \rho^A = dx^M \rho_M{}^A$$

and

$$R^A = \tfrac{1}{2}\, dx^M \wedge dx^N \; R_{MN}{}^A$$

$$(\epsilon, R^A) = \tfrac{1}{2}\, dx^M \epsilon^N \; R_{MN}{}^A = \tfrac{1}{2}\, \rho^B \epsilon^C \; R_{BC}{}^A \qquad (12)$$

The ϵ^A are related to the lapse and shift functions of the standard canonical formalism of General Relativity. For the index A in the Majorana range, eq.(11) yields the "local supersymmetry" transformations of supergravity. However, the exact comparison with supergravity requires the repeated use of field equations (4-5). For GP, the field equations are,

$$R^a = 0 \qquad (13a)$$

$$R^{ab} \rho^f \epsilon_{abfe} - 2i\, \bar{R}\, \gamma_5 \gamma_e \;\rho = 0 \qquad (13b)$$

$$\gamma^a \rho^a R = 0 \qquad (13c)$$

Direct calculation of the anholonomic components $R_{BC}{}^A$ from (13) yields,

$$R_{BC}{}^a = 0 \,, \quad R_{\alpha\beta}{}^A = 0 \,, \quad R_{(ab)\,B}{}^A = 0 \qquad (14)$$

where a is a (translation) vector index, (ab) skew-symmetric tensor value (for an **SO**(3,1) direction) component, α, β are Majorana spinor indices for the odd generators. The surviving components can be seen to coincide with those of ref.[10], after some algebra involving (13b). Inserting these components in eq.(11) reproduces the conventional supergravity transformations

$$
\begin{aligned}
\delta\rho^{ab} &= D\,\varepsilon^{ab} - \rho^{c}\,\varepsilon^{d}\,R_{cd}{}^{ab} - \rho^{c}\,\bar{\varepsilon}^{\dot{\alpha}}\,R_{c\alpha}{}^{ab} \\
\delta\rho^{a} &= D\,\varepsilon^{a} \qquad\qquad (15) \\
\delta\rho &= D\,\varepsilon - \rho^{c}\,\varepsilon^{d}\,R_{cd}
\end{aligned}
$$

where

$$
\begin{aligned}
D\,\varepsilon^{ab} &= d\,\varepsilon^{ab} + \rho^{at} \wedge \varepsilon^{tb} - \rho^{tb} \wedge \varepsilon^{at} \\
D\,\varepsilon^{a} &= d\,\varepsilon^{a} + \rho^{at} \wedge \varepsilon^{t} - \rho^{t} \wedge \varepsilon^{at} + \bar{\rho}\,\gamma^{a}\,\varepsilon \qquad\qquad (16) \\
D\,\varepsilon &= d\varepsilon + \tfrac{1}{2}\,(\rho^{ab}\,\sigma^{ab}) \wedge \varepsilon - \tfrac{1}{2}\,\sigma^{ab}\,\rho \wedge \varepsilon^{ab}
\end{aligned}
$$

In this sense, the exponentiation of equations (11) starting from an arbitrary $\mathfrak{M}^4$ is "condensed" into forms ρ^A on $\mathbb{R}^{4/4}$, and the $\mathbb{R}^{4/4}$ theory can be viewed as the collection of supersymmetric-related theories on $\mathfrak{M}^4$. Formula (11) therefore solves in principle the problem of constructing supersymmetric transformations for any theory. Clearly, the field equations impose very stringent conditions on $R_{BE}{}^{A}$, and supergravity is certainly unique in yielding these restrictions. However, any covariant theory on GP or $R^{4/4}$ is compatible with (11), with the specific form of $R_{BE}{}^{A}$ depending on the field equations. The values of these curvature components may then be much more complicated and there is no a priori guarantee that $R_{BE}{}^{A}$ are functionals of the restrictions of ρ^A on $\mathfrak{M}^4$ only, as needed in order that (11) be an effective transformation. In the present formulation, the choice of an action is highly restricted by (5). In all supersymmetry theories, we expect the $\mathfrak{M}^4$ to remain an even 4-dimensional manifold. This avoids the use of the Berezin integration[3] on the variables, whose formal difficulties have impeded the development of viable 8-dimensional actions[11][12]. From our point of view, the odd variables are just a short-hand for a collection of Fermi fields needed to specify $\mathfrak{M}^4 \in G$.

An extended exposition of these ideas will be published elsewhere.

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