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EQUILIBRIUM FIELD COIL CONSIDERATIONS FOR TOKAMAK REACTORS

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EQUILIBRIUM FIELD COIL CONSIDERATIONS FOR TOKAMAK REACTORS*

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Results of studies of the equilibrium field coil system for a variety of tokamak fusion reactor configurations are presented. These results include the determination of the EF coil currents, sizes, Ampere-turns, and stored energies for various reactor sizes, toroidal fields, plasma shapes, and plasma parameters. Problems are found with highly D-shaped plasmas and with high toroidal fields for smaller reactors. A simple expression, which adequately matches the wide range of cases considered, is also given for the stored energy.

MODEL EQUILIBRIUM COIL SYSTEM

In order to have a model equilibrium field (EF) coil system suitable for comparing a large number of reactor systems with different parameters, the following, somewhat simplified system was chosen: The model consists of N coil pairs (one coil up and one coil down) equally spaced around the toroidal field (TF) coils and a distance, Δ, from them. The TF coils are assumed to be constant tension, and their shape is calculated by the approximate method of Moses and Young.[1,2] The plasma boundary is given by the form

$$R = R_0 + a\cos(\theta + d\sin\theta)$$

$$Z = \kappa a \sin\theta .$$

κ is the plasma elongation, d is related to the plasma D-shapedness, R_0 is the major radius, and a is the minor radius of the plasma. A scrape-off region, $\Delta_v = 0.2$ m, is allowed around the plasma. The blanket thickness on the inside is assumed to be $\Delta_{BS}^i = 1.0$ m, and on the outside is assumed to be $\Delta_{BS}^o = 1.5$ m. An extra distance, $\Delta_d = 0.5$ m, is allowed on the outside for access. The TF coil magnet thickness, Δ_m, depends on the major radius and the toroidal field, B_{max}^{TFC}, at the coils. Table I gives the values used in this study.[2]

TABLE I. Coil Thickness

R_0, m \ B_{max}^{TFC}, T	7	9	11	13
6	0.379	0.502	1.052	1.470
8	0.418	0.561	1.250	1.756
10	0.455	0.622	1.459	2.053

These parameters are sufficient to determine the locations of the TF coils and hence in this model the locations of the EF coils. A typical example is shown in Fig. 1. It should be noted that the model allows coils in the central area inboard of the TF coils. This may not be possible in practice, and configurations which require a substantial amount of EF current in this area should be looked at with suspicion.

With the positions of the EF coils fixed, the current in them is determined by a simultaneous least-squares fit to the desired external field inside the plasma and a minimization of the stored energy in the EF system. This procedure is described in detail in Refs. 3-4. The coil size is adjusted by iteration to maintain a current density of $J_{EF} = 17.6$ MA/m^2, so that the coil size is proportional to the current carried.

The desired external field is calculated numerically by fixed boundary MHD calculations for given plasma parameters such as the cross-sectional shape

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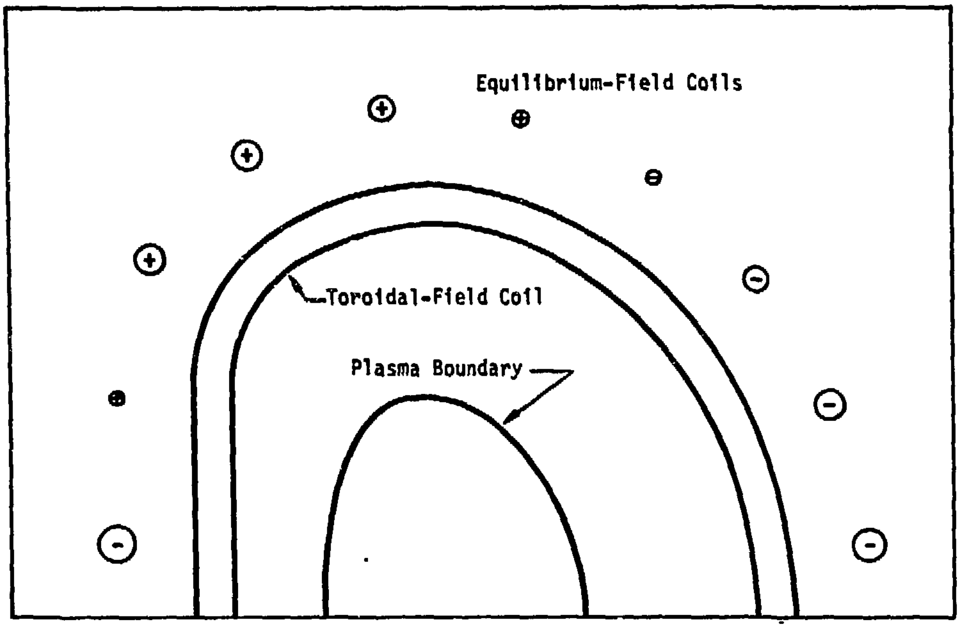

FIGURE 1. Equilibrium field coil model for a 6-m major radius reactor with 9 T toroidal coils for a somewhat elongated D-shaped plasma with toroidal beta of 8%, plasma current of 9.8 MA, and aspect ratio of 3.5. ($N = 10$ and $\Delta = 1$ m.)

(determined by κ,d); plasma beta, β_t; safety factor, q; and plasma pressure profile. All of the equilibria in this paper have $q(0) = 1$, $q(a) = 3$, and a relatively broad pressure profile. The base case is ($\kappa = 1.65$, $d = 0.25$, $A = 3.5$, $\beta_t = 0.08$, $R_0 = 6$ m, $B_{max}^{TFC} = 9$ T), similar to that of the Argonne 1977 EPR design.[4]

The external field that must be produced by the EF coils is the total field from the MHD equilibrium less that part of the field that is produced by the plasma current itself. Figure 2 shows examples of these fields for various plasma cross sections.

By varying the relative importance of the external field and the stored energy in the least-squares minimization, one can get a range of configurations, trading off accuracy in matching the equilibrium against stored energy. The EF system parameters given in this paper correspond to a 0.4% match to the desired external field. Experience has shown that if the EF coil configuration reproduces the external field from the fixed boundary MHD equilibrium to an average error of 0.4%, then the corresponding free boundary MHD equilibrium (where the EF coils are specified instead of the boundary shape) will adequately match the parameters and boundary of the fixed boundary equilibrium.

The EF coil radius is restricted to $a_{EF} < 0.33$ m. This restriction only appears in a few of the extreme cases. Only one reactor configuration ($R_0 = 6$ m, $B_{max}^{TFC} = 13$ T) calculated for this paper had no solution with this restriction.

A final caveat is that no attempt was made to decouple the EF and ohmic heating coil systems, even though this is most likely desirable in practice. Decoupling would increase the EF system stored energy, Ampere-turns, coil volume, etc. The increase is not too great, however, if there is a reasonably large number of coils.

NUMBER OF COILS

Figure 3 shows the relation between the stored energy and the number of coils. With too few coils it is difficult to match the equilibrium. For more than about 10 coil pairs little improvement is gained and the access for vacuum ports, neutral beams, and other penetrations is hampered. It

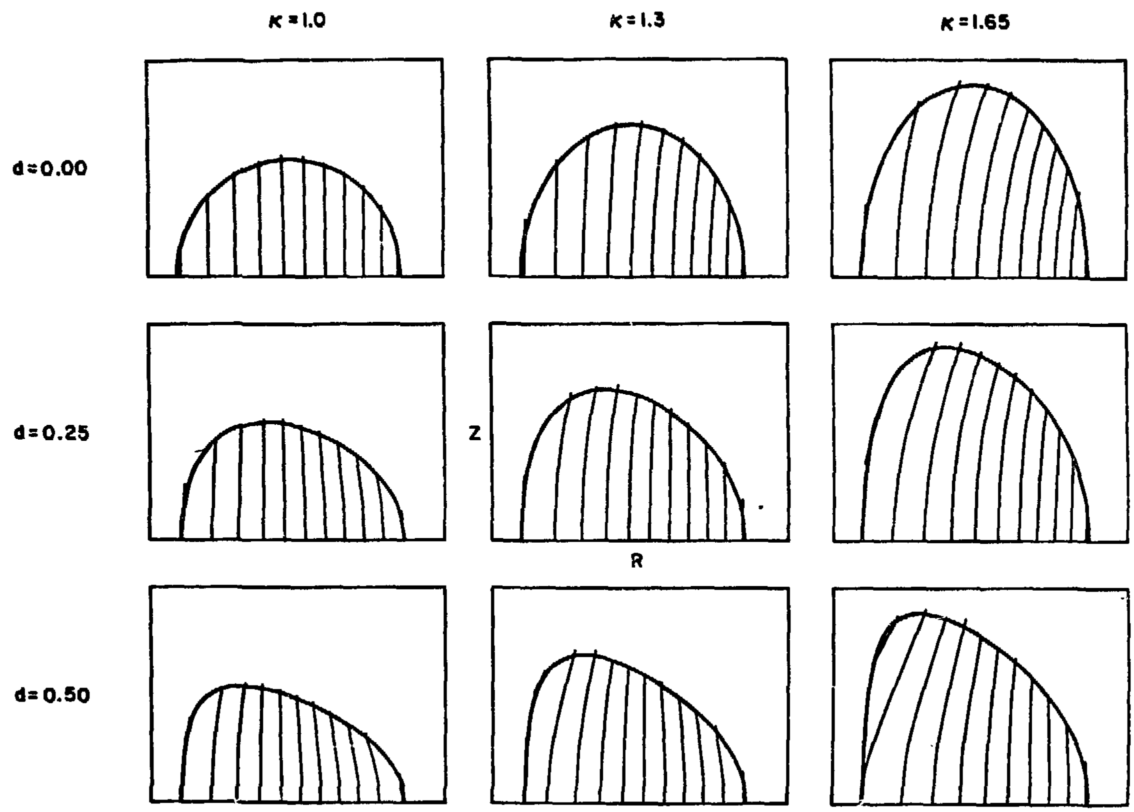

FIGURE 2. Required equilibrium fields for several plasma cross sections. The equilibria all have β_t = 0.08, q(0) = 1, q(a) = 3, and A = 3.5. (The cases with field lines concave to the right are unstable to vertical displacements.)

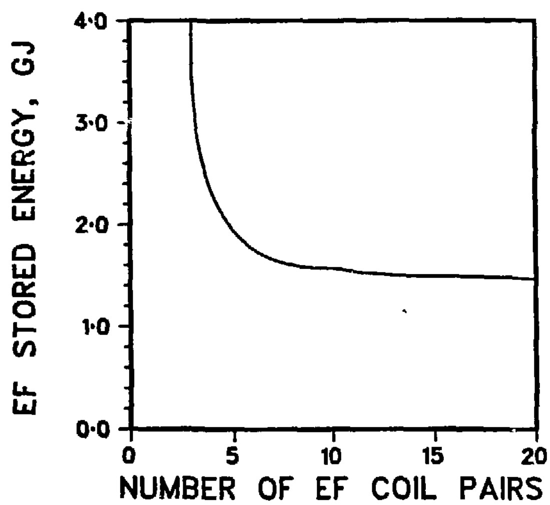

FIGURE 3. Stored energy as a function of the number of coils. Base case equilibrium.

should be noted that Fig. 3 is for a relatively difficult (κ = 1.65, d = 0.25) plasma shape. Fewer coils may be necessary for a more nearly circular shape. In addition, if some attempt is made to optimize the coil locations for a particular device, fewer coils may be necessary.[4] On the other hand, in order to maintain the flexibility to handle, for example, equilibria with different parameters, it is desirable to include a reasonable number of coils. In the rest of this study the number of coils was held constant at N = 10 coil pairs.

DISTANCE FROM TF COIL

The stored energy and other EF system parameters are very sensitive to the distance, Δ, of the EF coils from the TF coils. The relationship between the stored energy and this distance is shown in Fig. 4. The EF coils must be sufficiently far

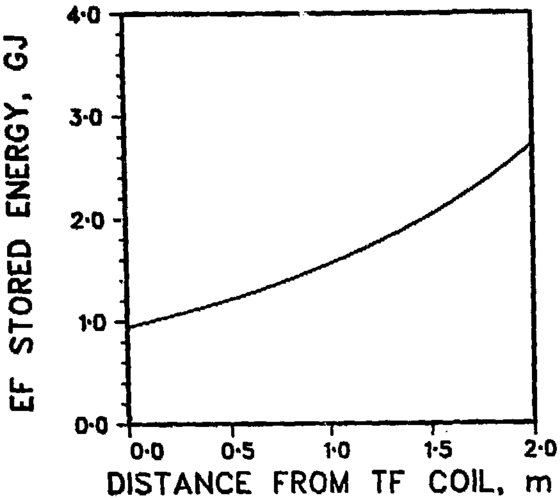

FIGURE 4. Stored energy as a function of the distance from the TF coils. Base case equilibrium.

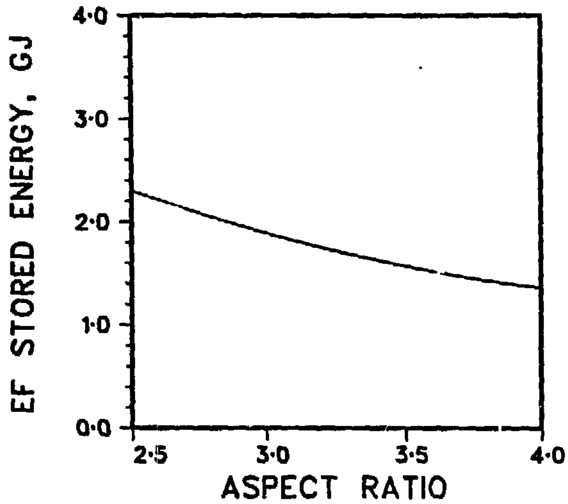

FIGURE 5. Stored energy as a function of aspect ratio. (R_0 is fixed.) Base case equilibrium.

from the TF coils that the heating due to currents induced in the TF coils by the EF coils is not excessive. In addition, the stray field from the EF coils reduces the useable toroidal field in the TF coils, so that the EF coils must also be far enough away to keep the stray field low.

Only EF coils outside the TF coils are considered in this study for the following reasons: (1) Superconducting coils inside the TF coils would be hard to wind and normal conducting coils would most likely have too great a power loss; (2) the space inside the TF coils is well occupied by other systems such as the blanket, vacuum ducts, etc.; (3) inside coils would interfere with access and remote maintenance.

The remainder of this study assumes the EF coils are $\Delta = 1.0$ m from and outside the TF coils. Further study of considerations beyond the scope of this paper would be necessary to determine a more accurate dependence of Δ on the reactor parameters such as size and toroidal field.

ASPECT RATIO AND β_t

The EF system is not especially sensitive to aspect ratio, $A = R_0/a$, nor β_t as shown in Figs. 5–6. The stored energy varies approximately as A^{-1}. Both the lower aspect ratios (with R_0 fixed) and higher betas have higher currents, which accounts for the increase in stored energy.

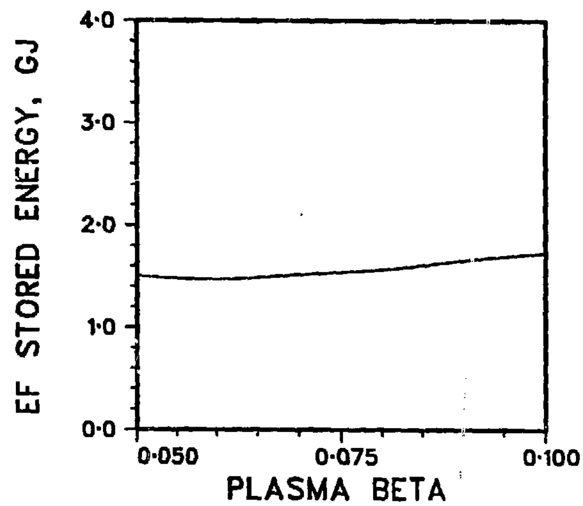

FIGURE 6. Stored energy as a function of plasma beta. Base case equilibrium.

PLASMA SHAPE

The EF system is quite sensitive to the plasma cross section as shown in Fig. 7. In particular, the more highly D-shaped plasmas are hard to produce: the natural configuration seems to be an ellipse. In particular, as can be seen in Fig. 8, the more highly D-shaped plasmas have a large amount of EF current in the central region inboard of the TF coils. It has been found previously that if EF coils are not allowed in this region, then these higher values of d cannot be produced.[4] The difficulty in producing the more D-shaped

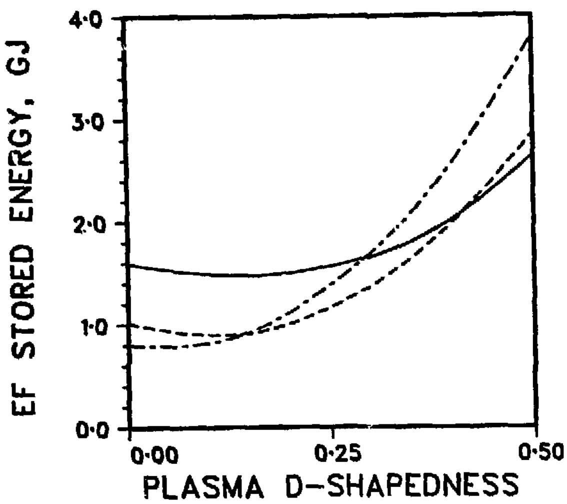

FIGURE 7. Stored energy as a function of elongation and D-shapedness. The solid line is for $\kappa = 1.65$; the dashed line is for $\kappa = 1.3$; and the dash-dot line is for $\kappa = 1.0$. The reason for the cross-overs is that a given value of d represents more distortion from the d = 0 case when the elongation is less, as can be seen in Figure 2.

plasmas must be traded off against the fact that such plasmas are predicted to have higher β_t values and hence increased performance.

It should be noted as well that the more highly elongated, D-shaped plasmas are unstable to vertical displacements; that is, the $\vec{J} \times \vec{B}$ force, which is perpendicular to the external field lines as shown in Fig. 2, has a component away from the midplane. Even if the overall position can be stabilized by feedback, locally this force may tend to distort the plasma.

MAJOR RADIUS

The EF system difficulty increases more slowly with size than the volume does. This is primarily because the plasma current increases slowly with size for a given cross section and equilibrium. The stored energy, in particular, increases a little faster than R_0^2 as seen in Fig. 9. This scaling is favorable to large reactors.

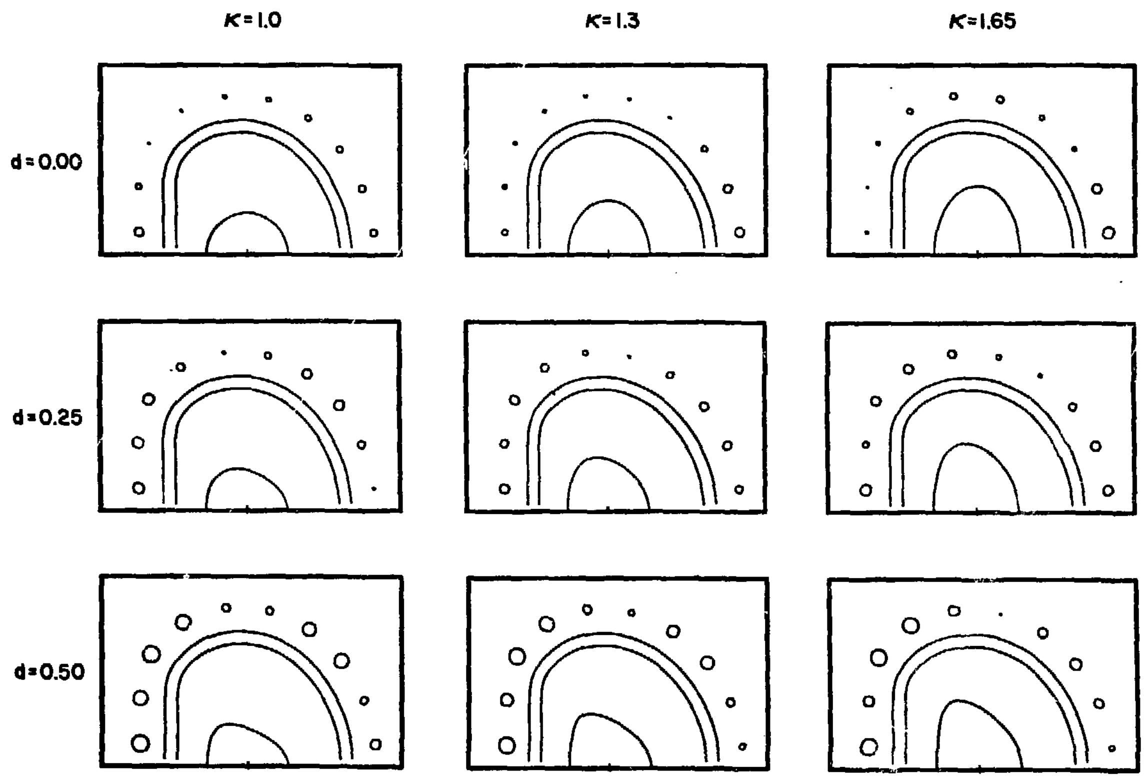

FIGURE 8. The EF system model as a function of elongation and D-shapedness.

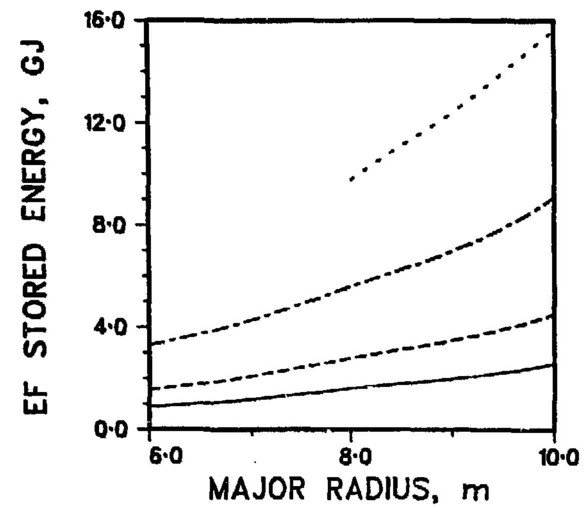

FIGURE 9. Stored energy as a function of major radius. The solid line is for B^{TFC}_{max} = 7 T; the dashed line is for 9 T; the dash-dot line is for 11 T; and the dotted line is for 13 T.

TOROIDAL FIELD

There is a strong dependence of the EF system on the toroidal field, as shown in Fig. 10. The stored energy, in particular, increases almost as $[B^{TFC}_{max}]^3$. This is faster than the square dependence one might expect, because the higher field magnets are thicker, making the EF coil further from the plasma.

Figures 11 and 12 show the EF system model for several values of toroidal field. It can be seen that for the smaller reactor, fields higher than about 9 T probably have too much current in the central region to be practical. (Less current, however, would be required for a more nearly circular plasma.) There seems to be no essential problem with high fields in the larger reactor, but the EF coils are quite large and carry a substantial amount of current.

FIT TO THE DATA

The stored energy for the range of cases treated in this paper can be fit within an error of 20% by the following formula:

$$u(GJ) = 1.3 \times 10^{-4} F(\kappa,d) A^{-1} \exp[0.53\Delta(m)] \cdot [R(m)]^{2.2} [B^{TFC}_{max}(T)]^{2.8} ,$$

where $F(\kappa,d)$ is a form factor with value unity for the base case (κ = 1.65, d = 0.25). The dependence of this form factor is quite model dependent. The data in this paper (see Fig. 7) are matched by:

$$F(\kappa,d) = 1.9 - 2.5\,\kappa + 1.4\,\kappa^2 + (11.7 - 20.0\,\kappa + 7.0\,\kappa^2)d + (17.1 + 1.4\,\kappa - 4.0\,\kappa^2)d^2 .$$

It should be kept in mind that d values greater than 0.25 have a substantial amount of current in the central region, and that a realistic system would probably have a stronger increase of $F(\kappa,d)$ with d. It should also be recalled that small, high-field reactors are more difficult than this formula would indicate.

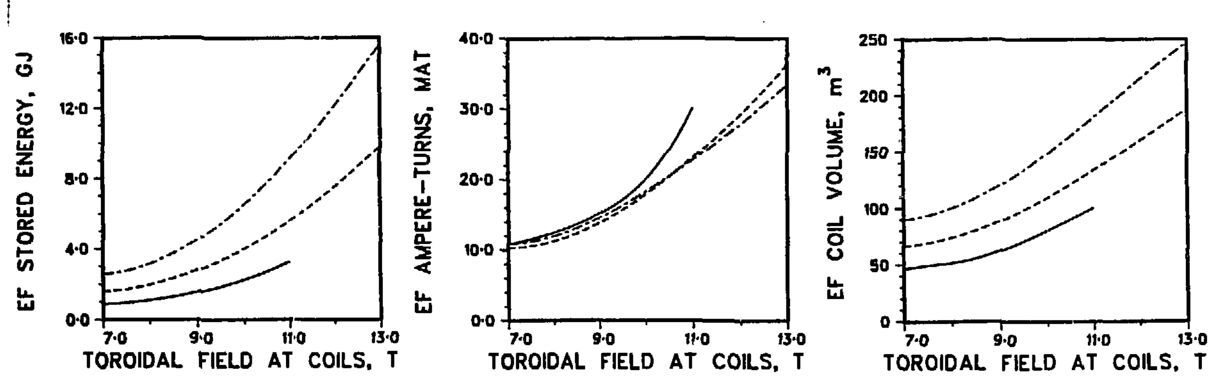

FIGURE 10. Stored energy, Ampere-turns, and coil volume as a function of toroidal field. The solid line is for R = 6 m, the dashed line is for R = 8 m, and the dash-dot line is for R = 10 m.

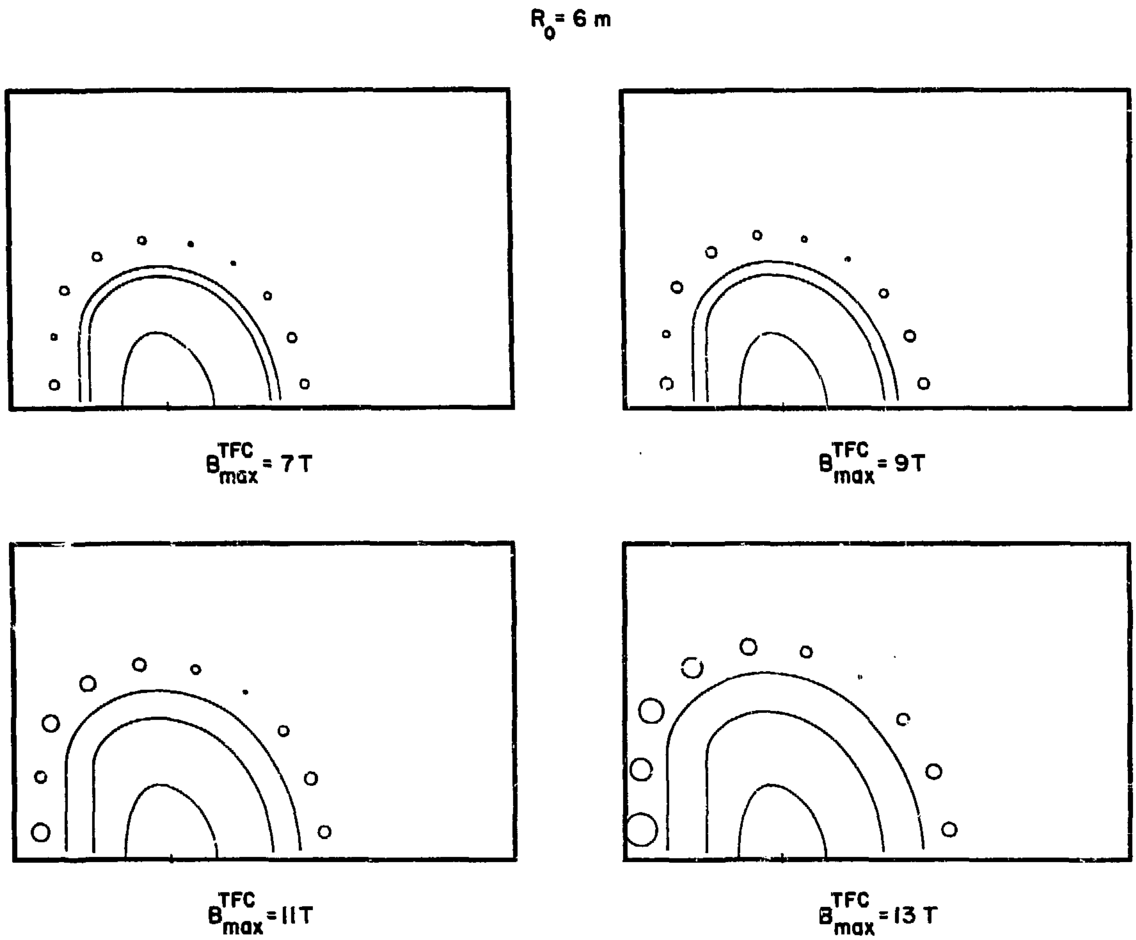

FIGURE 11. The EF system model as a function of toroidal field for a small reactor. No solution exists for the B_{max}^{TFC} = 13 T case if the coils are constrained to be less than 0.33 m, and the case shown is obviously impractical.

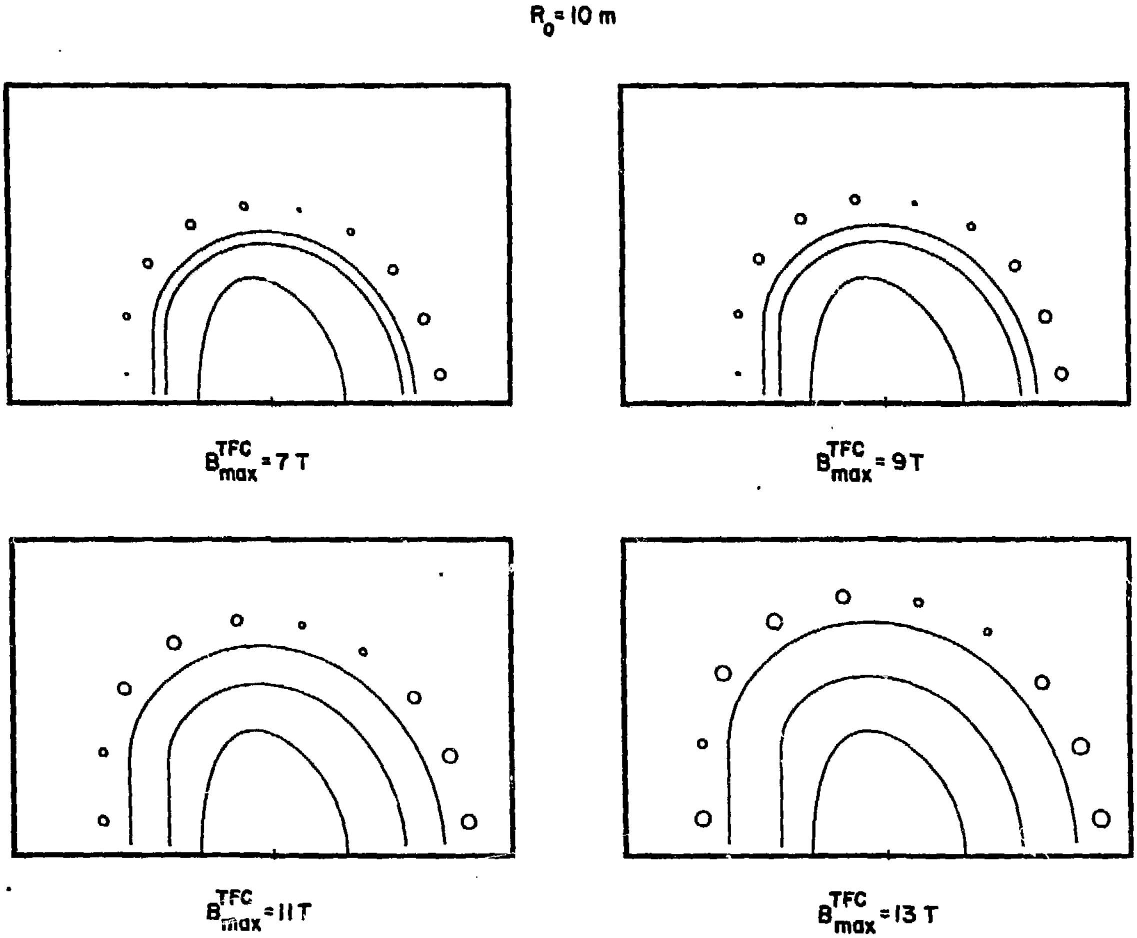

FIGURE 12. The EF system model as a function of toroidal field for a large reactor.

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