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AUTHOR(S): J. L. Friar, T-5

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Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

THREE-BODY FORCES

J. L. Friar
Los Alamos National Laboratory, Los Alamos, NM 87545

ABSTRACT

Three-body forces are defined and their properties discussed. Evidence for such forces in the trinucleon bound states and scattering reactions is reviewed. The binding energy defects of the trinucleon bound states, the ^{3}He charge density, the Phillips line for doublet n-d scattering lengths, and three-nucleon breakup reactions are discussed, together with the possible influence of three-body forces on these observables.

INTRODUCTION AND DEFINITIONS

Traditionally, nuclear physics has attempted to describe the nucleus as a collection of nonrelativistic nucleons interacting via two-nucleon forces.[1] These forces depend on the coordinates (and possibly momenta), as well as spins and isospins, of only two nucleons. This is a tremendous simplification which has no theoretical justification, other than a rough consistency between predictions of the theoretical (two-body) models and experiments. I use the word "rough" purposefully, because a serious impediment to the advancement of nuclear theory has been the general inability to calculate accurate nuclear wave functions for realistic nucleon-nucleon potential models. Any lack of consistency between theory and experiment is usually blamed on "poor structure calculations".

The few-nucleon systems, on the other hand, have traditionally been a testing ground for new ideas because our ability to solve the Schrödinger equation is greatest for these simple cases. Although variational techniques have long been used to obtain wave functions and energy eigenvalue bounds, the real impetus in this field was the seminal work of Faddeev.[2] Originally designed to handle the boundary conditions in scattering problems, Faddeev's approach to solving the Schrödinger equation has been very successful in treating bound states as well. Computational sophistication has improved dramatically in the past decade, to the point where we can experimentally challenge predictions of the standard two-body models.[3-8]

If this challenge proves unsuccessful, to what cause can the blame be laid? This is always a difficult question to answer, because there are many uncertainties in the traditional nuclear physics approach to calculating observables. The major possible uncertainties are threefold:

1. Three-body forces exist which depend on the simultaneous positions, momenta, spins, and isospins of three-nucleons;
2. The effect of relativity is non-negligible, and relativistic corrections are important;

3. The meson degrees of freedom and nucleon substructure make important contributions to observables.

It should be borne in mind that these 3 categories are not distinct; there is considerable overlap and in some cases the boundaries between them are completely blurred.

We will concentrate our attention on the first category. The definition of a three-body force given above seems obvious, but is incomplete. We must have some way of distinguishing 2 successive two-body forces between 3 objects and a real three-body force. Our working definition for the purpose of this talk will be: Forces which depend on the simultaneous coordinates of 3 nucleons, when only nucleon degrees of freedom are taken into account. The latter caveat is required, because there are few three-body force components which are in any sense "fundamental"; that is, it is possible and proper to disagree whether certain forces are three-body in nature or a complication two-body force situation.

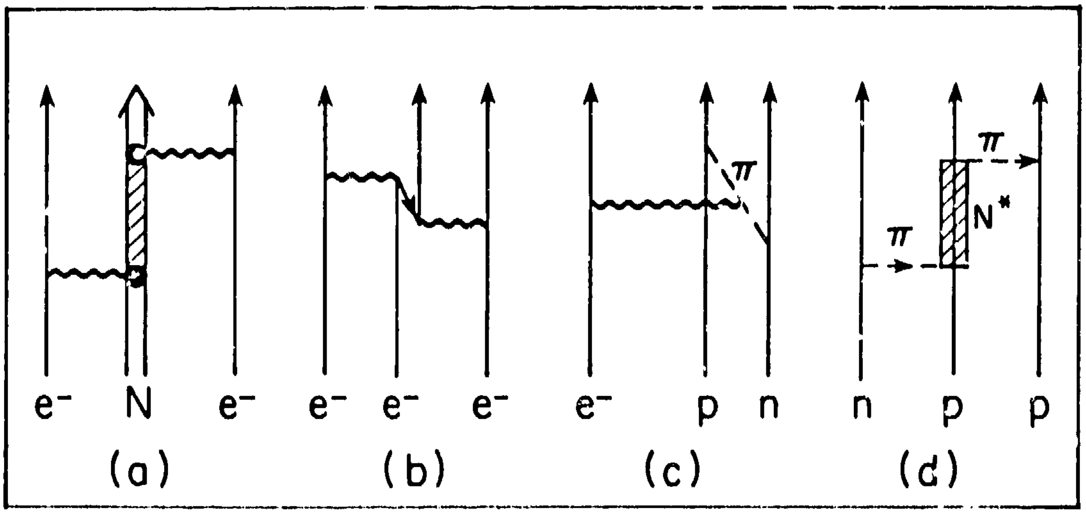

Figure 1. Various physical situations which lead to three-body forces.

The illustration above depicts 4 situations which can be understood as mediated by three-body forces. Figure (1a) illustrates 2 electrons interacting with a polarizable nucleus by means of Coulomb forces. If one chooses to treat the nucleus as an elementary particle, the force is a three-body force. If we agree to treat the nucleon degrees of freedom in the nucleus, a microscope pointed at that nucleus would reveal that the electrons were interacting with the nucleons via purely two-body forces. Thus this atomic physics conundrum is resolved by the answer to the question: Do we treat the nucleus as a "frozen" structureless object or as a collection of nucleons? Figure (1b) illustrates a classic situation, the three-electron force.[9] The middle electron is interacting via photon exchange with the outer pair of electrons. The central electron accomplishes this by emitting a photon, creating an electron-positron pair, and then annihilating the positron on the remaining electron line, which emits another photon. Everyone would agree that the lepton pair part of the diagram is special, and that without it we would be dealing with repeated two-body forces. On the other hand, a relativistic calculation which uses Dirac wave functions for the electrons and two-body forces would include this process in a natural way. In a nonrelativistic treatment, Figure (1b) depicts a three-body force; in a relativistic treatment, it is simply two two-body forces. Figure (1c) depicts a meson exchange current in a deuteron, "seen" by a photon from a passing electron.[10] This is the closest thing to a "fundamental" three-body force that I know. Figure (1d) is relevant to our real interest. A neutron emits a pion which strikes a proton, turning it into an excited state (N*), which decays by emitting a pion, later absorbed by another proton. Is this a three-body force? It is according to our definition. It is not, however, considered as such in calculations[12] which include N*'s in the nuclear wave function as identifiable components.

Thus, "freezing out" degrees of freedom always leads to three-body forces: in Figure 1(a), the nucleus; in (b), the negative-energy wave function components of the electron; in (c), the pion; in (d), the nucleon substructure (N*). I have spent too much time defining three-body forces, but I have been at meetings where the experts have argued over such matters, leaving everyone else confused. We see, however, that the litmus test is simple: degrees of freedom other than nucleons which are included in the nuclear wave function don't lead to explicit three-body forces; freezing them out produces such forces. Figure (1d) is a major (the major?) component of the nuclear three-body force.

PROPERTIES OF THREE-BODY FORCES

However we choose to define our three-body force, it will depend on the coordinates of three nucleons, including spins, isospins, etc. Thus, an obvious restriction is that one needs at least 3 nucleons for such a force to manifest itself. We will concentrate on the three-nucleon system, for reasons stated earlier. The coordinates of such a system are depicted in Figure (2); two distances, x and y, and one angle, θ, are required to completely specify the relative orientation of the three nucleons. The most important of these coordinates for our purposes is θ. It is entirely reasonable, even probable, that a three-body force would depend on this coordinate. It could, for example, be repulsive when the nucleons are in a linear configuration (θ=0 or π) and attractive when the configuration is isosceles ($\theta=\pi/2$). Thus, a three-body force can select particular three-body configurations, diminishing some and enhancing others. The force will obviously depend on x and y, and the example shown in Figure (1d) depends in an essential way on spin and isospin. This means that the manifestations of such a force in the three-nucleon system and in nuclear matter could be very different, indeed, oppcsite!

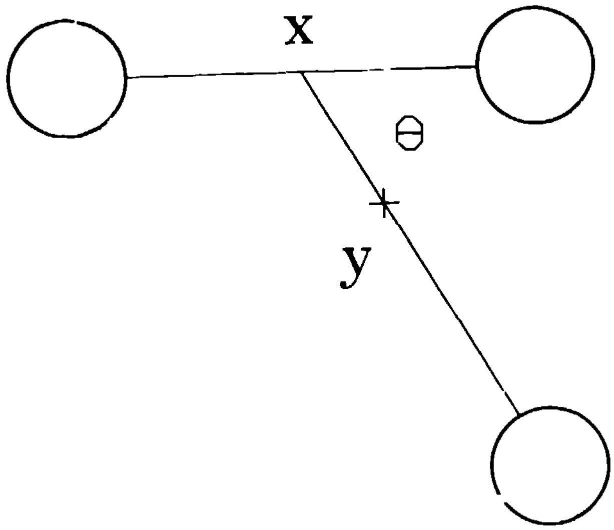

Figure 2. Coordinates of the three-body problem.

The scales of three-body forces are best illustrated by two examples. The three-electron force (and possibly the first three-nucleon force) was developed many years ago by Primakoff and Holstein,[9] and corresponds to the diagram (1b). It is relatively easy to calculate the force if one remembers that the electromagnetic $\vec{A}^2$ term in the nonrelativistic Hamiltonian, $(\vec{p}-e\vec{A}/c)^2/2m = \vec{p}^2/2m - e\vec{A}\cdot\vec{p}/mc + e^2\vec{A}^2/2mc^2$, is due to the virtual electron-positron "pair." Thus the force we wish is obtained by connecting the $\vec{A}^2$ - vertex to two $\vec{A}\cdot\vec{p}$ vertices with photons. Schematically, this force behaves as $V_c^2\,\vec{p}^2/m^3c^4$, where V_c is the ordinary Coulomb potential between two electrons. The force is momentum-dependent, and very weak in most circumstances because of the $1/c^4$ factor; it is a second-order relativistic correction and most atoms are basically nonrelativistic. To the best of my knowledge, it has never been seen experimentally.

The most popular of the three-nucleon forces is the Tucson force,[13] which continues a tradition of naming recent forces after cities (Paris, Bonn, Graz, etc.). This force is purely two-pion-exchange, the longest-ranged component of the total force. Its construction was motivated by the following argument: (1) As illustrated in Figure (1d), the primary ingredient of the two-pion-exchange-three-body-force (2PE3BF) is the off-shell pion-nucleon scattering amplitude; (2) Considerable theoretical and experimental information exists on this quantity from particle physics; (3) We need to consider only s- and p- wave π-N scattering, because the angular momentum barrier suppresses higher partial waves. With these ingredients and assumptions the Tucson force was constructed.

We wish to emphasize several aspects of this force, and similar forces. Firstly, it is very spin dependent and isospin dependent because the basic pion-nucleon ingredients are so dependent. Secondly, it is momentum dependent, although there is some evidence that the momentum-dependent terms are relatively unimportant. Thirdly, although of one-pion-range in each pair of nucleons, it has very strong short-range behavior which seems to dominate the physics. Fourthly, it does not include the exchanges of other mesons.

Schematically, the Tucson force has the form V_π^2/Mc^2, where V_π is the two-body one-pion-exchange potential and M is a mass which we can take to be the nucleon mass. The form suggests a size corresponding to a relativistic correction. Unlike the atomic case, the nuclear three-body force should be appreciable.

It should be borne in mind that the construction of such forces is a theoretical exercise with limited experimental input. How much progress on the two-nucleon problem would have been made without experimental input? A certain amount of caution should be exercised by all!

EVIDENCE FOR THREE-BODY FORCES IN BOUND STATES

One can argue that relativistic corrections in a nucleus should be a few percent on the average.[14] In the trinucleon system, the total potential energy is estimated to be 50-60 MeV.[3] A few percent of this would be on the order of 1 MeV. Of course, the kinetic energy largely cancels the potential energy, leaving a residue of 8.5 MeV binding for ^{3}H. Thus, a change of only a few percent in potential energy would have an appreciable effect on the binding energy. Table I shows the results of recent calculations[3,7,8,15] of the properties of ^{3}He and ^{3}H using only two-body potentials and their comparison with experiment. The binding energy refers to ^{3}H only; the binding energy difference of ^{3}He and ^{3}H is an interesting and partly unresolved problem. The Reid Soft Core, Super-Soft Core, Paris, and Argonne potentials all underbind the triton by 1.0 - 1.3 MeV and have a correspondingly too large radius. While one cannot be certain that the underbinding would not be eliminated by a judiciously rearranged two-body potential, it is *a priori* reasonable to investigate other possibilities.

Another possibility is relativistic corrections[14] to our non-relativistic two-body potential model. We have already seen that the three-body forces can be viewed in this light. Most calculations of the former effects find a small residue resulting from a cancellation of the relativistic corrections to the kinetic and potential energies. None of these calculations are sufficiently definitive to rule out the possibility that relativistic corrections to the two-body model are unimportant.

TABLE I. Comparison of calculated trinucleon properties vs. experiment. The four models correspond to the Reid Soft Core, Supersoft Core (C), Paris and Argonne potentials. The radii include the intrinsic sizes of the nucleons.

Model	$\langle r^2\rangle^{1/2}_{^3He}$	$\langle r^2\rangle^{1/2}_{^3H}$	E_B
RSC	2.09 fm	1.83 fm	7.2 MeV
SSC(C)	2.00 fm	1.76 fm	7.6 MeV
Paris	2.02 fm	-	7.4 MeV
Argonne (V14)	2.06 fm	1.81 fm	7.4 MeV
Expt.	1.86(3) fm	1.69(5) fm	8.5 MeV

Another possibility is that three-body forces are important and account for the discrepancies in the trinucleon bound states. At this time there exist five calculations[5,6,15,16,17] using part or all of the Tucson force and realistic two-body forces. Unfortunately, none of the calculations are identical and none get the same result. There is rough agreement that the attractive p-wave part of the Tucson force gives 1.0 - 1.5 MeV additional binding when recommended values of the coupling constants are used. The s-wave part of the interaction, however, is repulsive and estimated to decrease binding by 0.4-1.0 MeV. The net result varies from 0 to 1.3 MeV, according to the calculations. Since the two-body force models, calculational techniques, and approximations used to obtain these numbers were quite different, it is not clear what the problem is. I personally doubt that the two-body model makes that much difference, but I have no proof of this assertion. These calculations are very different and resolution of the problem will come with time.

Another interesting and controversial comparison is shown in Figure (3), which depicts the charge density of ^{3}He, with "experimental" data, and densities calculated at Los Alamos[18] using the Reid Soft Core two-body potential with and without a Coulomb interaction between the two protons. I use the word experimental advisedly, since considerable theoretical assumptions and extrapolations are required to extract this density from the experimental data. The error bars are statistical only and are considerably smaller than the sum of the theoretical uncertainties. Nevertheless, the hole reflects a longstanding problem: the lack of theoretical strength in the ^{3}He charge form factor in the region of the secondary maximum. The form factor $F(q^2)$ is simply the Fourier transform of the charge density $\rho(r)$, normalized to 1. Inverting this relationship provides a way of calculating $\rho(r)$ at any point.

In particular

$$\rho(o) = \frac{2}{\pi} \int_0^\infty F(q^2)\, q^2 dq \qquad (1)$$

The form factor is 1 if the momentum transfer q vanishes, decreases to 0 as q^2 increases, becomes negative, turns around and approaches 0 again. It is in the region of large negative values that theoretical calculations have always been deficient. This region of q^2 accounts at least qualitatively for the hole in ρ at the origin, which follows from the structure of the integral in (1). Various reasons have been advocated for this deficiency, including meson-exchange currents and three-body forces. The former is an exceptionally complicated business involving relativistic corrections, which we will ignore.[14] The second explanation has a particularly simple geometrical aspect, which has galvanized the theoretical community. We will assume it is correct for the purpose of exposition.

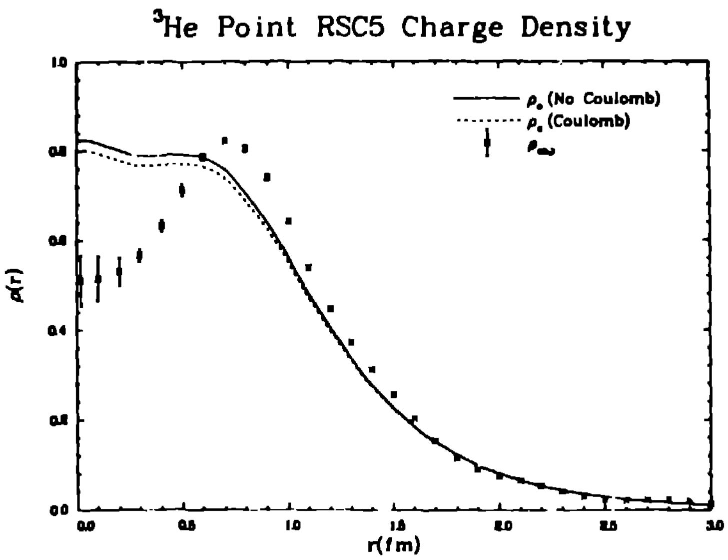

Figure 3. ^{3}He charge density calculations vs. experiment.

Consider Figure (2) again. The coordinate r in ρ(r) is the distance from one of the protons (taken to be the bottom nucleon on the figure) and the center-of-mass of the system (the cross). Thus we have the geometrical relationship: r=2y/3. Clearly, vanishing r implies vanishing y, and this further implies a linear configuration of nucleons. Our theoretical models have too much attraction for such a configuration, implying two large a value of ρ(r). We need to decrease this configuration, while increasing the binding, which depends more on the isosceles configurations. A three-nucleon force can accomplish both these goals if it behaves as follows:

We need to	Three-body force should be
A. Deplete linear configurations;	Repulsive at $\theta=0,\pi$;
B. Enhance isosceles configurations;	Attractive near $\theta=\pi/2$;
C. Increase binding.	Net attractive.

This scenario is possible, and attractive from a theoretical viewpoint. Time will tell if it is correct or merely a chimera.

Three further aspects of the "hole" need to be discussed before abandoning the topic. Although the hole looks very large in Figure (3), it requires less than 1% of the total charge in ^{3}He to fill it in completely. This is consistent with our earlier estimate of the size of the three-body force, and follows from the presence of r^2 in the volume element of integration. In addition, the hole has nothing to do with repulsion in the nucleon-nucleon force. The charge density is actually an integral over the wave function, which has the structure

$$\rho(r) \sim \int \Psi^2(x,3r/2,\theta)d^3x, \qquad (2)$$

where we have replaced y by 3r/2 as indicated earlier. Moreover, it is easy to visualize that for y=r=0, the wave function is independent of the variable θ. Figure (4) depicts the major component (u) of the ^{3}He wave function for θ=0. The deep trough I call "death valley" is due to the strong short-range repulsion when two nucleons try to overlap (y=x/2). The charge density ρ(0) is given by an integral along the y=0 plane and is most strongly influenced by the large structure in the vicinity of x=1.5 fm, which is near where the nucleon-nucleon force is most attractive. Thus, "death valley" has relatively little effect on ρ(o). Finally, the previously discussed inconclusive calculations using the Tucson force find a small effect on ρ(o).

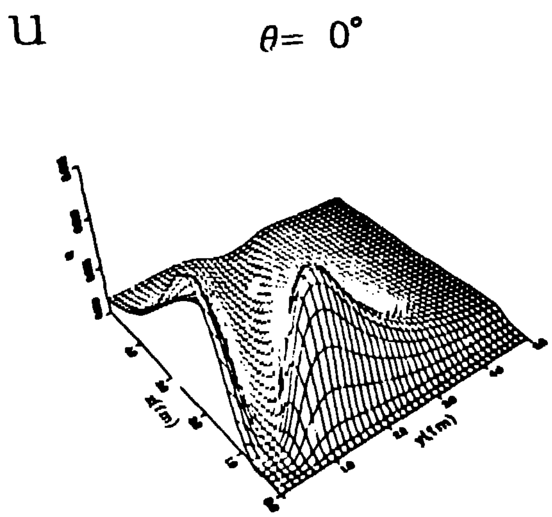

Figure 4. Principal component of the ^{3}He wave function calculated using the Reid Soft Core potential model.

EVIDENCE FOR THREE-BODY FORCES FROM SCATTERING

In principle, scattering processes should provide much more information than bound states in the search for evidence of three-body forces. In fact such evidence is lamentably scant, and is tied in a curious way to the bound state evidence.

If neutrons at vanishing energy are scattered from deuterons, the scattering lengths are measured, of which there are two. The deuteron spin of 1 couples to the neutron's spin 1/2 to form two independent total spin combinations: quartet (3/2) and doublet (1/2). The quartet scattering length is large and influenced primarily by the value of the deuteron binding energy. The doublet scattering length $a^{[2]}$ is small and strongly coupled in a somewhat mysterious way with the corresponding values of the triton binding energy.[9] This feature is known as the Phillips line[20] and is depicted in Figure (5), with 4 theoretical points[21,22] used to generate the fitted line, and the one (unfitted) experimental point. The triton binding energy defect that seems to be a universal feature of "realistic" two-body potential calculations means that $a^{[2]}$ is too large, since E_B is too small. Whatever physics accounts for the binding defect would also move $a^{[2]}$ closer to the experimental point, assuming that the Phillips line is indeed a universal feature of doublet scattering lengths. Calculations by Torre, Benaycun, and Chauvin[6] show that this general trend is followed when three-nucleon forces are included.

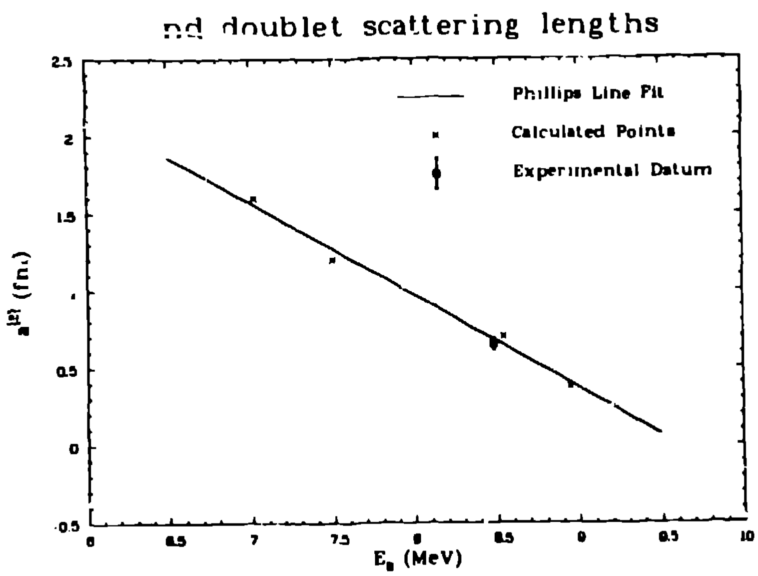

Figure 5. Calculated doublet nd scattering lengths vs. triton binding energies, and the experimental datum.

In order to exploit the additional freedom one has in scattering, one should work at slightly higher energy. Frankly, I'm not sure what types of experiments should be performed. If the geometrical feature of three-body forces we discussed earlier is correct, perhaps experiments of the the type shown in Figure (6) will show sensitivity to three-body forces. Figure (6a) depicts a bound neutron (open circle) and proton (shaded circle) colliding with a proton in the center-of-mass. Two possible final state configurations are shown, with an isosceles scheme in (b) and a collinear one in (c). These processes should show some sensitivity to three-body forces. Detailed calculations would be necessary to make this argument convincing.

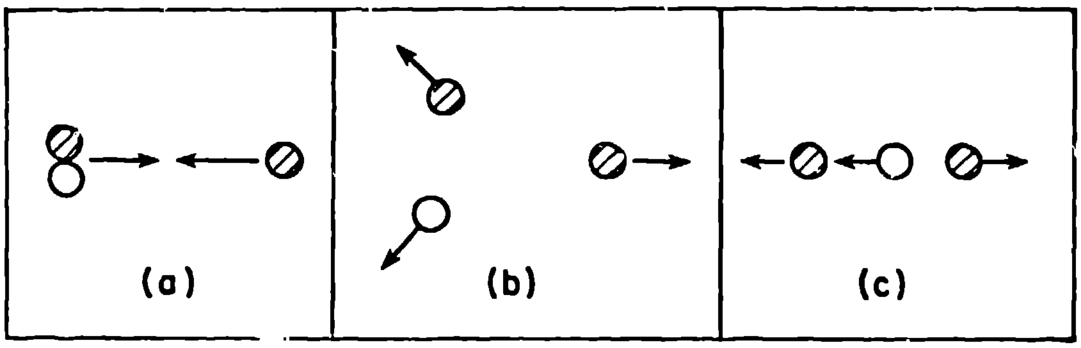

Figure 6. Various final state configurations from p+d scattering.

Finally, it has been suggested[23] that the small discrepancy between the neutron-neutron scattering lengths from the $d(\pi^-,\gamma)2n$ and $d(n,2n)p$ reactions may result from three-nucleon force effects in the final state of the latter reaction. This is an intriguing speculation which also requires more detailed calculations.

CONCLUSIONS AND SUMMARY

It is obvious that much more work needs to be performed on this rapidly developing area of investigation. Because of the inherent limitations of purely theoretical efforts to understand hadronic physics, our approaches to the three-nucleon force should be diverse; there will be many revisions of the forces. Currently there is no concrete evidence for three-nucleon forces, but several intriguing possibilities are suggestive. In recent calculations of the additional triton binding due to the Tucson force, all is chaotic. This is quite understandable in view of the complexity of the calculational problem; order will appear in time. The few-body field remains a very active area for the investigation of new ideas and fundamental nuclear mechanisms: in this case, the effect of three-nucleon forces.

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