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STAGGERING TOWARDS A CALCULATION OF WEAK AMPLITUDES*

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ABSTRACT

An explanation is given of the methods required to calculate hadronic matrix elements of the weak Hamiltonian using lattice QCD with staggered fermions. New results are presented for the 1-loop perturbative mixing of the weak interaction operators. New numerical techniques designed for staggered fermions are described. A preliminary result for the kaon B parameter is presented.

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1. INTRODUCTION

In this talk I describe work done in collaboration with Rajan Gupta (Los Alamos), Gerry Guralnik and Greg Kilcup (Brown). and Apoorva Patel (CERN). This is very much a progress report, and in fact the numerical results I present below differ from those that were shown in the conference. We have found, and corrected, a simple error in our analysis program. I think that there are no further bugs, but only time will tell.

One of the excellent features of this workshop was the interaction between the different lattice groups, and between those using approximate analytic methods and the lattice groups. It became apparent to me from these conversations, and from the comments of others since, that there is not only a gulf of comprehension between those practicing analytic and approximate methods, but that there is also a gulf between those of us using staggered fermions and those (represented here by Gavela, Maiani and Martinelli (ORSAY/CERN/ROME)[1] and by Soni (UCLA)[2]) using Wilson fermions. Thus I am devoting the first half of this talk to an attempt to explain, as clearly as I can, what it is that we actually have to calculate with staggered fermions, and what are the problems with our approach. In particular, I hope to make it possible for others to read our technical papers[3] and not get lost in the maze of symbols and abbreviations. None of our work is technically difficult, but there are many details to encode in equations, and thus we resorted to a compact notation. In retrospect, this made the papers rather hard to penetrate.

After all has been made crystal clear, I will turn to some new material. First I want to explain the numerical tricks that we have found useful, and quite possibly essential, in order to complete the calculation of the amplitudes needed for both the real and imaginary parts of the kaon decay amplitudes. Second. we have

completed a calculation of the 1-loop mixing amongst the four-fermion operators whose matrix elements one wishes to calculate. This is a standard lattice perturbative calculation, which only becomes tricky because of the enormous number of operators that appear with staggered fermions. Part of this calculation has been done independently by Sheard[4]. We have not fully checked our calculation, nor checked our results against those of Sheard, so our numbers should be considered preliminary.

Finally, I will discuss some new results for matrix elements, which use the new numerical techniques. We have results for the electromagnetic penguin (EMP) operator – also known as the "electro-penguin" operator – and for the kaon B parameter. The *cognoscenti* will immediately realise that these matrix elements do not involve the so-called "eye diagrams", and thus are the simplest to compute numerically. The EMP matrix element is indeed simple to calculate, and all three groups roughly agree on its matrix elements. The B parameter for the kaon is much harder to calculate. It is here that staggered fermions have an advantage because of the exact axial symmetry. This guarantees the correct chiral behavior of the matrix element, at least for small enough kaon masses. We see an interesting pattern of deviations from the correct chiral behavior, using kaon masses as small or smaller than the other groups have used, which may mean that the calculations will be somewhat harder than one might have hoped.

In the talk at the workshop, I also discussed our somewhat old (though still perfectly valid) results for the matrix element of the penguin operators responsible for the non-zero value of ϵ'. This discussion added little to that I gave in a talk at Moriond[5] and has in any case been updated in a another talk[6]. Thus I have said little about it in this writeup. In any case, I think that it is too early to determine

the size of the systematic errors due to, among other things, the quenched approximation. Thus I do not want to advertise lattice results too enthusiastically, when I am not confident that they will stand the test of time.

Despite this, my general feeling at the moment is one of guarded optimism. The other groups have laid the foundation for the calculation of the matrix elements using Wilson fermions, and have shown that the method yields results which are just beginning to appear above the noise. This is very impressive, though for the moment one should be cautious about the precise numerical values. We have built upon the earlier work and, I claim, found a way to do the calculations with staggered fermions. Ultimately, both methods must agree on the answer, and such agreement will provide a stringent check on the unimportance of the lattice approximation.

2. STAGGERED FERMIONS

Staggered fermions were invented by Susskind[7] (in the Hamiltonian formulation) in order to have a formulation of lattice fermions with a chiral symmetry (discrete in the Hamiltonian case). Euclidean staggered fermions were studied extensively by Jan Smit and various collaborators[8]. In the Euclidean formulation there is a continuous chiral symmetry which is softly broken by the mass term. For this reason staggered fermions have been the laboratory for lattice studies of chiral symmetry breaking. And it was because of this that we decided to look into the calculation of weak decay amplitudes of kaons, since chiral symmetry plays an important role in the physics of pions and kaons.

Nothing comes for free. The advantage of having a chiral symmetry is offset by a four-fold increase in the number of flavors. This means that, at least in the way

we are doing the calculation, one is not simulating the theory one wants to, i.e. QCD. One has more flavors than one wants, and this has to be taken into account. With Wilson fermions, on the other hand, one recovers chiral symmetry only in the continuum limit, but one can work with the physical number of flavors. Thus with Wilson fermions the continuum theory is QCD. In the following, I will make a few comparisons between the Wilson and staggered fermion approaches, but my major emphasis will be on explaining what assumptions go into the staggered fermion calculation.

For my purposes here, the easiest way to discuss staggered fermions is to collect the four flavors of fermion into a 4×4 matrix, Q.[9] Lorentz transforms act on Q from the left, in the usual way for a spinor, while the $U(4)$ of flavor acts from the right. Q lives on sites y, and the lattice action without gauge fields can be written

$$S_F = \tfrac{1}{2} N_f^2 \sum_y \left[\sum_\mu \mathrm{Tr}\left(\overline{Q} \Delta_\mu^{(1)} \gamma_\mu Q\right) + 2 \sum_\mu \mathrm{Tr}\left(\overline{Q} \gamma_5 \Delta_\mu^{(3)} Q \gamma_\mu \gamma_5\right) + 2m\,\mathrm{Tr}(\overline{Q}Q)\right]$$

Here $N_f = 4$ is the number of continuum fermions that the staggered fermions represent, and the relative normalization of terms is correct, but not important for the following. μ runs over the four Euclidean directions, and $\Delta_\mu^{(i)}$ is the i-th symmetric lattice derivative in the μ-th direction. The first and third terms are the usual naive discretization of the Dirac action, while the second term is a Wilson-fermion-like term which removes the doubling problem. Note that this term is of $\mathcal{O}(a)$ relative to the other terms, a being the lattice spacing. It is also the only term in the action which contains flavor gamma matrices (those to the right of the Qs), and thus it breaks the flavor symmetries.

So far this looks very similar to four copies of the standard Wilson fermion action. The second term breaks the flavor diagonal chiral symmetry, and also

breaks all the off diagonal flavor symmetries. For Wilson fermions, the breaking of chiral symmetry means that the quark mass m gets additively renormalized, and one has to calculate non-perturbatively where the chiral symmetry is restored when one is away from the continuum limit. However, the reason for having four flavors is that the action retains a flavor non-singlet chiral symmetry generated by

$$Q \longrightarrow Q + i\theta\gamma_5 Q\gamma_5$$
$$\overline{Q} \longrightarrow \overline{Q} + i\theta\gamma_5 \overline{Q}\gamma_5$$

This symmetry is exact for $m = 0$, and guarantees that mass renormalization is multiplicative. Thus, even at finite a, the symmetry is always restored when $m \to 0$. It is only for four flavors (or multiples of four) that this trick can be played, i.e. that the Wilson term retains a chiral symmetry.

Let me immediately comment on a confusion/objection to what I have just said. The axial symmetry is a *flavor non-singlet* symmetry. Thus one can regroup left and right handed components and the symmetry will appear like a vector symmetry. Which of these is the correct grouping is determined by the way in which the chiral symmetry breaking condensate aligns. With the flavor diagonal mass term, the condensate is forced to be a flavor singlet: $\langle Tr(\overline{Q}Q)\rangle \neq 0$. This in turn forces the diagonal groupings of left and right handed components, and relative to this the symmetry displayed above is indeed axial. That chiral symmetry breaking does occur, and that it does so in this way, has been shown analytically for strong coupling, and numerically for intermediate and weak couplings.

To include gauge fields into the staggered fermion action one must spread the 16 component Q field over a 2^4 hypercube of a lattice with half the lattice spacing. That is, each lattice point to which a Q field is attached becomes a hypercube. On

each point of the fine lattice one has a single component field χ. These are related to the Q fields by

$$Q(y) \equiv \frac{1}{\sqrt{N_f^3}} \sum_h \Gamma_h \, \chi(y+h) \;=\; \frac{1}{\sqrt{N_f^3}} \sum_h \gamma_1^{h_1} \gamma_2^{h_2} \gamma_3^{h_3} \gamma_4^{h_4} \, \chi(y+h).$$

Here $h = (h_1, h_2, h_3, h_4)$ runs over the 2^4 positions in the hypercube, and the equation defines Γ_h. The original action S_F was constructed such that, when written in terms of the χs, only nearest neighbor terms survive. Terms with greater "distance" between $\overline{\chi}$ and χ cancel between the first and second terms in S_F. The mass term by itself is local ("distance 0") in terms of the χs. To make the theory into a gauge theory, one puts group matrices on links, endows $\overline{\chi}$ and χ with color indices, and simply inserts the appropriate link matrix between neighboring $\overline{\chi}$ and χ. This means that some of the gauge fields are *inside* the Q field, rather than between adjacent Qs. This peculiarity is needed to retain all the symmetries, discrete and continuous, of the free staggered action. Without these symmetries, one can dynamically generate extra mass terms, such as $\sum_\mu \mathrm{Tr}(\overline{Q} Q \gamma_\mu)$, which do not break the chiral symmetry[10]. Relative to these mass terms, the symmetry displayed above is vector.

The complete symmetries of the staggered action are important if one wants to classify operators and states into irreducible representations. This classification was first studied by Golterman and Smit[11], and then simplified and extended by Greg Kilcup and I[12]. For the calculation of weak amplitudes, the most important part of the symmetry is the axial transformation displayed above. In terms of the single component field it is

$$\chi(n) \longrightarrow \chi(n) e^{i\varepsilon(n)\theta}$$

$$\overline{\chi}(n) \longrightarrow \overline{\chi}(n) e^{i\varepsilon(n)\theta}$$

Here the points on the fine lattice are now labelled by n, and $\epsilon(n)$ is $+1$ on even points, -1 on odd points. It is also useful to know how various Q bilinears appear when expanded out in terms of the χs. A general bilinear is $\mathrm{Tr}(\overline{Q}\Gamma_s Q\Gamma_f^\dagger)$, where the $\Gamma_{s,f}$ are members of the Clifford algebra representing the spin and flavor of the bilinear, respectively. Each member of the Clifford algebra is specified by a four-vector of binary numbers. It is simple to see that the bilinear consists of a sum of the form $\sum_h [\mathrm{phase}(h,s,p)\overline{\chi}(h+s+p)\chi(h)]$. Here the arithmetic is mod-2, and the phase, which is ± 1, depends on h, s and p. Thus the displacement between the fields is $s+p$; we often refer to $|s+p|^2$ as the "distance" of the operator.

My point in going through this is to emphasize a major numerical headache one has when dealing with staggered fermions: that one must use non-local operators. Compared to Wilson fermions, staggered fermions have a computational advantage because they only have one component per site. Thus the matrices to invert are smaller than for Wilson fermions. However, unless one is careful, this gain can be wiped out by the need to use non-local operators.

It may be helpful to display some examples of the bilinears that we need in to calculation of weak amplitudes. The simplest is the singlet scalar density: $\mathrm{Tr}(\overline{Q}Q)$. This is distance 0, and is simply $\sum_h \overline{\chi}(h)\chi(h)$ with no phases. The next most simple operator is the pseudoscalar density with the flavor associated with the axial symmetry: $\mathrm{Tr}(\overline{Q}\gamma_5 Q\gamma_5)$. This is the operator that the scalar density mixes with when one makes an axial rotation. Thus it is the operator which creates the (pseudo-)Goldstone pion associated with the spontaneous breakdown of the axial symmetry. It is also distance 0: $\sum_h \epsilon(h)\overline{\chi}(h)\chi(h)$. The conserved vector current and partially conserved axial currents are both distance 1: $\mathrm{Tr}(\overline{Q}\gamma_\mu Q)$ and $\mathrm{Tr}(\overline{Q}\gamma_\mu\gamma_5 Q\gamma_5)$. The former is a flavor singlet, while the latter has the by now

familiar non-singlet flavor. We will also need the vector current with the flavor quantum numbers of the conserved vector current, $\mathrm{Tr}(\overline{Q}\gamma_\mu Q\gamma_5)$, and the flavor singlet axial vector current, $\mathrm{Tr}(\overline{Q}\gamma_\mu\gamma_5 Q)$. Both are distance 3. and both correspond to symmetries that are broken by the Wilson-like-term in the action. Finally, we need the non-singlet scalar, $\mathrm{Tr}(\overline{Q}Q\gamma_5)$, and the flavor singlet pseudoscalar, $\mathrm{Tr}(\overline{Q}\gamma_5 Q)$. These are both distance 4.

One might object to the use of the Q bilinears, since, in terms of the χs, they do not retain all of the symmetries. Including appropriate phase factors, the full action is invariant under unit translations on the fine lattice. In terms of the Qs, this symmetry is a mixture of a translation and a flavor-spin rotation. The bilinears by themselves do not form a representation of this symmetry. For example consider the action itself – the first term in S_F is not a scalar under the full lattice symmetry group, but can be made so by the addition of the $\mathcal{O}(a)$ term. This is the solution in general: one can add terms of $\mathcal{O}(a)$ and obtain good representations of the lattice group, at least for zero momentum bilinears[11,12]. This means that, if one is sufficiently close to the continuum limit, one can leave out the extra terms. I will have more to say on this in the section on perturbative mixing.

For four fermion operators we have found it simplest to write everything in terms of Qs. Our operators thus live on a single 2^4 hypercube, and, when written in terms of χs, are made up of a sum of 16×16 terms. In fact, our analysis programs first convert the propagators calculated in terms of the χs into the Q basis.

The operators we use thus extend over two timeslices. This clouds their interpretation as operators local in time. However, for staggered fermions the transfer matrix must be defined as moving one over two timeslices. The construction has

been sketched by Smit[11]. Thus there is no problem of principle, though one might be worried that the approach to the continuum limit might be slower than with Wilson fermions, since there can be momentum "within" the operators.

So far I have been discussing material well known to all practitioners of staggered fermion calculations. Let me summarize, before proceeding into the weak amplitude calculations. A single staggered species represents four degenerate flavors in the continuum limit. This is true in perturbation theory (at least to the order that detailed calculations have been done); whether it is true non-perturbatively can only be tested by numerical calculations. A simple consequence of the symmetry restoration would be the appearance of multiplets of 15 degenerate states (16 in the quenched approximation) for each spin-parity. Thus there should be 16 pions, 16 rhos, etc.. On the lattice, these multiplets are split up into several smaller ones. For example the 15 ρs, each with 3 spin components, break down into 7 three dimensional and 4 six dimensional representations. Amongst the pions, only that with flavor γ_5 should show exact Goldstone behavior. But the other 14 should also do so in the continuum limit.

These straightforward tests have only been applied in any detail to pions in SU(2) gauge theory[13]. Further tests have been hindered by the expense of calculating fermion propagators. Most calculations only use 1, or at most 2, base points for the propagators, with which one can only project onto a few of the possible states. A spin-off from our calculations is a measurement of many more masses. With the wall-source technique described below, we measure 4 pion and 12 rho masses, with the latter falling into 8 distinct representations. The resulting masses as a function of average quark mass (all masses are in lattice units) are shown in Figure 1. The smallest quark mass is roughly half the strange quark mass. This

is based on 24 measurements on $16^3 \times 40$ configurations at $\beta = 6.0$. We find the result quite encouraging. The errors on the ρs, and the heaviest 3 πs, are of the same size as the splitting between the representations. However, the Goldstone pion is significantly lighter than the others. There is a slight curvature visible in the Goldstone pion curve, and if one plots the square of its mass versus the quark mass the points fall on a straight line extrapolating to the origin. This is the expected chiral behavior, which is guaranteed by Ward Identities. Similar plots for the other pions are also consistent with straight lines, but extrapolate to non-zero values at $m_q = 0$.

Let me now turn to our staggered calculation of weak amplitudes. As I see it, there are two possible methods. The first is to break the degeneracy between the flavors explicitly, and to identify the four flavors as u, d, s and c[14]. Such explicit breaking is indeed possible[6]. The trouble with this approach is that (a) one has to adjust the symmetry breaking terms non-perturbatively; and (b) some of the currents from which one is building the weak Hamiltonian are not partially conserved. This is exactly the situation one has with Wilson fermions, and one has to perform a similar theoretical analysis of the recovery of the flavor symmetries in the continuum limit[15]. This analysis has not been done but it seems to me doubtful that one would gain anything over Wilson fermions.

The approach we have followed is based on using the exact axial symmetry to the fullest possible extent. We have to introduce one species of staggered fermion, i.e. one Q matrix, for each continuum flavor: U, D, S, ... Our external particles are the Goldstone pions constructed from the same quarks as in the continuum, e.g. our "K^+" is created by $\mathrm{Tr}(\bar{U}\gamma_5 S \gamma_5)$. This means that (if we only consider the light quarks) the theory we are simulating has 12 flavors in the continuum

limit. *I stress that this is NOT QCD.* It is a relative of QCD in which we can measure matrix elements, and from those measurements deduce something about the matrix elements in QCD.

The fact that the theory is different from QCD has been the source of some confusion. I have heard and read the following argument against the use of staggered fermions. In QCD, the vector part of the weak current is related to the electromagnetic current by an SU(3) rotation, and the triangle diagram consisting of the π^0 axial current, and two electromagnetic currents is anomalous. With staggered fermions, the weak current is represented by the conserved vector and axial currents, and the π^0 is created by the conserved axial current. But the triangle diagram involving these currents is not anomalous on the lattice. Thus the mapping from QCD to staggered fermions must be faulty somewhere.

The flaw in this argument is that, in our approach, the weak current is not represented by the conserved lattice currents. As will be spelled out below in gory detail, one maps from continuum to lattice contraction by contraction, not operator by operator. Some of the contractions of a continuum operator use the lattice conserved currents, others do not. It is true that we make essential use of the axial *symmetry*. In fact, the staggered theory we use has, for 3 continuum flavors, a set of currents satisfying an $SU(3)_V \times SU(3)_A$ algebra. But, as the argument given above shows, these currents differ from those of the continuum, and one can only use the algebra indirectly.

It might be worth mentioning that one can extract the usual anomalous terms from triangle diagrams, if one uses appropriate currents. The reason that the VAA triangle with conserved currents vanishes is that A has flavor γ_5, while V is flavor singlet. Thus the triangle has a flavor factor of $\mathrm{Tr}(\gamma_5) = 0$. If instead one uses the

flavor singlet axial current, $\mathrm{Tr}(\overline{Q}\gamma_\mu\gamma_5 Q)$, which is not conserved, one obtains the continuum anomaly multiplied by a factor of N_f.[16] The factor of N_f comes from the four flavors running around the loop. This is a nice check on the consistency of the interpretation of staggered fermions.

Having brought up the factors of N_f, let me make a further comment. If the flavor breaking terms become unimportant as $a \to 0$, then in the way we do the calculations there is a trivial overall factor of N_f for each valence fermion loop. We can simply remove this by hand. The corrections to this are suppressed by powers of a. Much larger effects of flavor breaking will be discussed in section 4. What is much more worrisome is how to remove the factors of N_f in the dynamical quark loops. This can be done formally by taking the N_fth root of the fermions determinant. This is a standard procedure for those wishing to study chiral symmetry breaking as a function of the number of flavors. If the flavor symmetry breaking is weak, then taking the root does indeed reduce the symmetries of the theory. But extensive tests will need to be done to see how fast the continuum limit is approached.

The example I want to work through is the kaon B parameter. This is defined as the ratio of the matrix element

$$\mathcal{M} = \langle \overline{K}_0 |\, \bar{s}_a \gamma_\mu (1+\gamma_5) d_a \; \bar{s}_b \gamma_\mu (1+\gamma_5) d_b \,| K_0 \rangle,$$

to its value in vacuum saturation approximation: $B = \mathcal{M}/(4/3 f_K^2 m_K^2)$. Here a and b are color indices, and in my normalization $f_\pi \approx 135\,\mathrm{MeV}$. This definition of B is not renormalization group invariant, and sometimes one defines an invariant B parameter by multiplying by appropriate powers of α_s. I prefer not to do this since there are considerable ambiguities in the value of α_s, a point I will come back

to in section 4. Lattice calculations give both the numerator and denominator of B, but it is better to compare their ratio to experiment since there will be some cancellation of errors. For example, the uncertainty in the overall scale appears only logarithmically, through the scale of the coefficient functions.

The first step towards putting the matrix elements on the lattice is to imagine that there are two extra quarks, call them d' and s', which are degenerate respectively with d and s. Then we can rewrite the matrix element as:

$$\begin{aligned}
\mathcal{M} &= \mathcal{M}^1 + \mathcal{M}^2 \\
\mathcal{M}^1 &= \langle \overline{K}_0' | \, \bar{s}'_a \gamma_\mu (1+\gamma_5) d'_b \; \bar{s}_b \gamma_\mu (1+\gamma_5) d_a \, | K_0 \rangle \\
\mathcal{M}^2 &= \langle \overline{K}_0' | \, \bar{s}'_a \gamma_\mu (1+\gamma_5) d'_a \; \bar{s}_b \gamma_\mu (1+\gamma_5) d_b \, | K_0 \rangle
\end{aligned}$$

where K_0' is composed of s' and d'. The point of including the extra quarks is to uniquely specify the contractions. The superscripts on the $\mathcal{M}$ indicate the number of color index loops that there are in the contractions. This breakdown of $\mathcal{M}$ is straightforward in the quenched approximation. For the full theory we have to stipulate that one uses only the square root of the determinant in the measure, i.e. only s and d are dynamical.

Next, let us decompose the matrix element a second time

$$\begin{aligned}
\mathcal{M}^1 &= \mathcal{M}^1_V + \mathcal{M}^1_A \\
\mathcal{M}^1_V &= \langle \overline{K}_0' | \, \bar{s}'_a \gamma_\mu d'_b \; \bar{s}_b \gamma_\mu d_a \, | K_0 \rangle \\
\mathcal{M}^1_A &= \langle \overline{K}_0' | \, \bar{s}'_a \gamma_\mu \gamma_5 d'_b \; \bar{s}_b \gamma_\mu \gamma_5 d_a \, | K_0 \rangle,
\end{aligned}$$

with a similar equation for $\mathcal{M}^2$. The subscripts indicate whether we have taken the VV or AA part of the matrix element. We have now broken $\mathcal{M}$ down into four parts. It is a very important fact that each *separately* satisfies a Ward Identity

which guarantees that it vanishes as $m_K m_{K'}$ in the chiral limit. This is clearly not true if the primed quarks are replaced by the original quarks. For then there is a second contraction in which the operator appears to have an SS+PP spin structure. This gives a matrix element which does not vanish in the chiral limit. This is the reason for the device of the primed quarks.

Let me discuss three objections to the argument that each of the pieces of $\mathcal{M}$ vanishes separately in the chiral limit. First, I have kept only the s and d quarks dynamical, so the symmetry of the dynamical quarks is smaller than that of the "valence" quarks. Thus my theory is not well defined. However, one can resort to a derivation of the chiral behavior involving quark propagators on background gauge fields. This "hands-on" method (see below) mimics term by term the continuum current algebra derivations of Ward Identities. It clearly fails if there is no chirally invariant regulator. The second objection is that the renormalization group will mix the operators with others. In fact, in the continuum, using massless quarks, the four operators mix only among themselves. The eigenvectors are LL and LR operators, with particular color structures. Thus, mixing amongst these operators is consistent with the claim that they all have the same chiral behavior. Again, one needs to respect the chiral symmetry in the perturbative calculation.

The final objection is that the chirally invariant regulator that is required to rebut the first two objections does not exist on the lattice. This is true; but this is where staggered fermions with their chiral symmetry ride in to save the day. What exactly goes wrong without the chiral symmetry? One example is that the one loop perturbative corrections mix an operator such as $\mathcal{M}_1^2$ with that appearing in

$$\langle \overline{K}'_0 | \; \bar{s}'_a \gamma_5 d'_b \; \bar{s}_b \gamma_5 d_a \; | K_0 \rangle .$$

The mixing is perturbatively calculable, and thus in principle can be subtracted away. However, the matrix element just displayed does not vanish in the chiral limit, so that 1-loop perturbative subtraction may still leave a large residue. This can make the extraction of the true matrix element difficult, as discussed here by Martinelli[1] and Soni[2].

This problem is fixed by increasing the number of flavors by a factor of N_f. If I maintain the device of keeping the primed quarks the staggered theory has 16 flavors—S, D, S' and D'. Out of the $SU(16)_L \times SU(16)_R$ symmetry only an $SU(4)_V \times SU(4)_A$ subgroup is exact on the lattice. But this residual symmetry can be used to protect certain matrix elements and thus remove the third objection. One way to do this is as follows. Create the external kaons with $\mathrm{Tr}(\overline{S}\gamma_5 D\gamma_5)$, so that they are pseudo-Goldstone pions. Replace each bilinear in the four fermion operator with the corresponding staggered bilinear *using flavor* γ_5. Thus, for example:

$$\mathcal{M}_V^2 \to \frac{1}{N_f}\langle \overline{K}'_0|\ \mathrm{Tr}(\overline{S}'_a\gamma_\mu D'_a\gamma_5)\ \mathrm{Tr}(\overline{S}_b\gamma_\mu D_b\gamma_5)\ |K_0\rangle$$

$$\mathcal{M}_A^1 \to \frac{1}{N_f}\langle \overline{K}'_0|\ \mathrm{Tr}(\overline{S}'_a\gamma_\mu\gamma_5 D'_b\gamma_5)\ \mathrm{Tr}(\overline{S}_b\gamma_\mu\gamma_5 D_a\gamma_5)\ |K_0\rangle$$

The factors of $1/N_f$ account for the four flavors flowing around the two loops (one factor of $1/N_f$ is accounted for by the normalization of the staggered kaon wave function). Notice that the V terms have become distance three operators, while the A terms are distance 1. This explains the notation we adopted to refer to these lattice matrix elements:[3]

$$\mathcal{M}_V^2 \to \mathrm{E}_3^2\ \ \mathcal{M}_V^1 \to \mathrm{E}_3^1\ \ \mathcal{M}_A^2 \to \mathrm{E}_1^2\ \ \mathcal{M}_A^1 \to \mathrm{E}_1^1\ .$$

Here the subscript is the distance, and the superscript the number of color loops.

The E in this notation refers to the fact that these are so-called "eight" diagrams, as opposed to the "eye" diagrams that probably explain the $\Delta I = 1/2$ rule.

As an aside, let me mention that there are other prescriptions for calculating the matrix elements. The above method breaks the bilinear into two separate traces over the staggered flavor indices. Apoorva Patel[17], in his strong coupling calculations, uses a single staggered flavor loop. There is no theoretical reason I know of to prefer one method over the other; in fact, it would provide a nice check if the two methods gave the same answer. The methods differ by terms that vanish in the continuum limit. But, for the moment, we proceed with the two-flavor-trace staggered operators.

A similar line of argument provides a prescription for calculating other matrix elements. The EMP matrix elements, for example, involve distance 0 (PP) and distance 4 (SS) contractions. In the notation which we have just introduced, one has to use E_0^1, E_0^2, E_4^1 and E_4^2. The full expressions are given in Ref. 3. The only new twist occurs for the eye diagrams. In these, one bilinear converts a kaon into a pion, while the other is attached to the eye loop. These bilinears must be replaced by staggered fermion bilinears with flavor matrix I, the identity. This swaps the distances: V is now distance 1, A distance 3, etc.. The notation for the eye diagrams follows logically from that given above: I_d^c, where $c = 1, 2$ is the number of color loops, and $d = 0, 1, 3, 4$ is the distance. One final notation is that for the odd distance bilinears, the distance label can be augmented with a direction to indicate the index μ. Thus

$$E_1^2 = E_{1x}^2 + E_{1y}^2 + E_{1z}^2 + E_{1t}^2.$$

In case you are longing for more details, these are provided in Ref. 3.

So, to summarize, we first have to accept that in the continuum limit a single species of staggered fermion represents four degenerate flavors. Then we take the weak matrix elements in terms of the physical quarks, quadruple the number of quark flavors, and form particular contractions in the resulting multi-flavor theory. (The introduction of the S' and D' served to pick out particular contractions, and is not strictly necessary once we go to the staggered theory where we have a chiral symmetry which allows us to separate one contraction from another.) Dividing by appropriate factors of N_f we come to a calculation using staggered fermions which gives the same answer as the original QCD calculation.

The question which must be on the tip of everyone's tongue is: What is the purpose of such acrobatics? The answer is Ward Identities. The correlation functions implicitly defined above satisfy simple Ward Identities which mimic those of the continuum matrix elements. These Ward Identities imply that the chiral behavior expected of the continuum theory holds true also on the lattice. This is to be compared to the Wilson fermion approach in which the chiral behavior has to be regained by various subtractions (at least for the B parameter).

Let me use the kaon B parameter, again, to give an example of the Ward Identities in action. Here, one calculates an "eight" correlator, in which the operators described above reside at time 0 (all operators are at zero spatial momenta), the kaon is created at time t_K, and then destroyed at $t_{\overline{K}}$. Call this correlator $E(t_K, t_{\overline{K}})$. The required matrix element is obtained by taking $t_K \to \infty$ and $t_{\overline{K}} \to -\infty$, and extracting the coefficient of the leading exponential behavior. Of course, the roles of $\pm\infty$ can be switched. The Ward Identities state (and this is true for $m_s \neq m_d$):[12]

$$\sum_{t_K} E(t_K, t_{\overline{K}}) \;=\; \sum_{t_{\overline{K}}} E(t_K, t_{\overline{K}}) \;=\; 0\,.$$

Thus the correlators vanish if either of the kaons is put to zero momentum. This is true configuration by configuration (whether generated with quenched or full measure). and for each of the matrix elements E_i^ζ separately. It is also independent of the manner in which gauge links are put in the non-local operators (see below). However, it is necessary that the pion be created by the local pion operator.

To use these identities one assumes that the pion contribution dominates the correlator, and that there is a momentum expansion for the pion matrix elements. Then it follows that the matrix elements are $\propto p_K p_K$. Thus, on shell, they are proportional to m_K^2. Let me stress that the assumptions leading to these conclusions may be questionable in present simulations, since quite large quark masses are being used. Nevertheless, for small enough quark masses the conclusions should hold.

For details of how the Ward Identities work for eye diagrams, see reference 12. Things are slightly more complicated, but again just as in the continuum. It is our claim that we can calculate all the matrix elements needed for kaon decay amplitudes, both real and imaginary parts[3].

Let me now turn to two technical issues that are crucial to actually carrying out this program: how one deals numerically with non-local operators, and how one calculates the finite parts of the connection between continuum QCD operators and our funny staggered operators.

3. NUMERICAL TECHNIQUES

We have tested and are using four sort-of-new ideas. The first is to put the lattice into a hybrid gauge, defined as follows. The lattices are generated using periodic boundary conditions in all directions. They are first fixed to Landau gauge (the lattice equivalent of $\sum_\mu \partial_\mu A^a_\mu = 0$). This makes the configurations maximally smooth, and as close to the trivial configuration as possible. There are possible problems with Gribov copies, which for the moment we ignore. The second step in the gauge fixing process is to put the "edge" time slices (labelled $t = 0,\ 39$) into Coulomb gauge. This is the lattice version of $\sum_{i=1,3} \partial_i A_i = 0$, and makes the spatial links on these timeslices even more smooth, at the expense of roughening the time-directed links sticking out of these timeslices. Again, we ignore possible problems of Gribov copies.

The point of this gauge fixing is two-fold. First, it allows us to deal with the problem of making the non-local operators gauge invariant. One has two options: either put in the links between $\overline{\chi}$-χ pairs with contracted color indices; or use a fixed gauge and ignore the links. In our previous work[3], we chose the former option, while now we are using the simpler second option. The operators differ by a finite, perturbatively calculable amount, and so eventually one should do it both ways and compare. We use Landau gauge because it flattens out the gauge field in a way consistent with the symmetries of the operators (again, modulo possible Gribov problems). We can also do the perturbative calculations in Landau gauge.

The reason for putting the edges of the lattice into Coulomb gauge is to allow the use of "wall sources". Traditionally, fermion propagators are calculated from a single base point. When one combines such propagators with their conjugates, one is creating mesons with a local operator. This causes two problems. First, the

overlap of the operator with the states is suppressed by powers of a - the operator is too small. Second, the operator couples to radial excitations quite strongly. Exactly the same problems were found in glueball calculations using small Wilson loops to create the states. Progress in glueball mass calculations has required the use of extended and smeared operators. The larger size of the operators increases the overlap with the states, and lowers the coupling to excitations. Furthermore, with gauge links one has the additional problem of ultraviolet noise, and this is reduced by the smearing.

We have attempted to mimic this approach as closely as possible with fermions. Our idea, based loosely on that of Billoire et al[18], is to solve the Dirac equation with a source consisting of δ-functions at every site on one timeslice. When one contracts the resulting propagator with its conjugate, one is creating states with an operator consisting of a sum of terms in which quark and antiquark are separated by all possible distances. This means that our operators are as extended as possible, enhancing the signal and reducing the coupling to radial excitations. The use of a fixed gauge means that the terms with separated quark and antiquark do not average to zero. The choice of a smooth gauge is to reduce the ultraviolet noise as much as possible.

A similar method has recently been used, independently, by the APE collaboration to calculate hadron masses with Wilson fermions[19]. They find spectacular success for baryon masses. Our tests of the efficacy of the wall sources are much less thorough, so we are much encouraged by the APE results. The APE method actually differs from ours in a number of ways. Their wall source does not run across the entire lattice–this may reduce the signal less than the noise, since the gauge configuration is less smooth over longer distances. We prefer to keep our

source across the entire lattice since this forces the states to be at zero spatial momentum. Secondly, we use Dirichlet, rather than periodic, boundary conditions in the time direction. Our sources actually sit right next to the boundary. This is to allows us to use the entire center of the lattice to make measurements of matrix elements. Furthermore, we have found[3] that spurious contributions in which quark lines wrap around the lattice in the time direction can make it harder to extract the signal. Such contributions are forbidden by the fixed boundary conditions.

The final difference concerns the number of δ functions used. APE do a separate calculation for each initial color and spinor index. We do a separate calculation for each initial color, but we are using simultaneously a delta function at all the spinor (and staggered flavor) positions. Let me illustrate the idea for Wilson fermions. A similar calculation would put a δ-function at all spinor positions. If two of the resulting propagators were combined one would create states of all possible spin-parities. However, far enough from the source only the lightest state, the pion, would survive. In this way a single calculation could yield a cheap, though dirty, pion.

We are actually not quite as cheap as this. After some experimentation, we settled on using two wall sources: all sites with the same phase (called 'q'), and all sites with phase $\epsilon(n)$ (called 'o'). A q source alone is unsatisfactory for the following reason. If we combine a q with its conjugate (to make a qq), then when one works out which meson states are created[20], one finds that there are two pions. One is the Goldstone pion, which we want. In the notation for bilinears introduced above, it has [s;f]=[1111; 1111]. The other pion has quantum numbers [1111; 1101]. Notice that the other pion is distance 1 in the z direction, an example of the fact that combining two wall sources gives particles of all distances. Although the other pion

is not as light as the Goldstone pion (see Figure 1), it is not sufficiently suppressed for our purposes. We can project against it by forming $qq + oo$. Because of the $\epsilon(n)$ in the o source, $qq + oo$ does not couple to any odd spatial distance states. Of course, $qq + oo$ still couples to a number of ρs, etc., but these are all sufficiently heavy as to be unimportant.

We calculate with q and o wall-sources from both ends of the lattice. Using these, we can extract the masses of many states, most of them non-local, by projecting on the open end of the quark propagators. The Goldstone pion, and the other pion, are the only particles whose signals travel entirely across the lattice. The ρs make it somewhat less than half way across, and the baryons less still. Nevertheless, most the signals are very clean, with little evidence for radial excitations, so we can extract masses quite cleanly. Our wall sources allow us to investigate new baryons too. Most exciting is that we can extract the staggered equivalent of the Δ mass for the first time.[20]

For present purposes, the most important point of the wall sources is that they produce zero momentum pions abundantly, and that if we produce them from both ends of the lattice, then we have free fermions legs to contract together in the middle of the lattice. We do this contraction using all the complicated non-local operators that are required for staggered fermions. Furthermore, we can do this across the entire spatial lattice, thereby increasing the signal. In addition to measuring all the matrix elements we want for kaon decays, this will allow us to make many checks, which I don't have time to go into here. It is also possible to use the same wall sources to create 2 or more pions: we have not yet investigated this avenue.

An example of our raw data is shown in Figure 2. This is for the $E_{1_t}^2$ matrix

element, i.e. the time component of the AA, two-color-loop part of the B parameter. Also shown is the lattice vacuum saturation result. Near the edges there are contributions from the many heavier particles that are created by the wall sources. From $t = 10 - 29$, however, the pion contribution is dominant. Notice that this contribution is flat, because, whatever the position of the operator, a pion has to propagate the length of the lattice. To extract numbers we average over the central region, and use the jackknife method to estimate errors. However, it is clear that we do not need to do any fancy fitting to extract the signal. Signals of similar quality are found in most channels.

Let me mention some disadvantages of our methods. One is that we cannot check the exponential fall off of the external pions individually. This is not essential, but would be a nice check. We can make this check by moving the wall-sources to a different time. By using extended sources we have also lost the exact Ward Identities described above. Of course, if the conclusions concerning the chiral behavior of the matrix elements are true, then they should be independent of the operators which create the pions. Nevertheless, in the event of a departure from the expected behavior, it would be nice to have correlators for which the Ward Identities apply. Fortunately, we do have such data on $12^3 \times 30$ lattices, at slightly stronger coupling. The final disadvantage concerns the extraction of actual matrix elements, rather than ratios such as the B parameter. We create the states with a very complicated operator sitting right next to the boundary. We need to know the amplitude for this creation, and the only way to calculate this is to destroy the state at the other end of the lattice with the same operator. Fortunately, for the pion, we can calculate this amplitude, though not for the more massive particles.

I promised four new ideas, and only have described two. Let me very briefly

describe the others. The first is to use extrapolation when calculating propagators at a variety of masses on the same configuration. We are using m=.03, .02 and .01. We first calculate with the heaviest mass (270 conjugate gradient iterations), then use this as a seed for m = .02 (260 iterations). This is standard practice, and saves $\sim$ 200 iterations. However, we then linearly extrapolate point by point to get a seed for the m = .01 calculation, which then takes 400 iterations instead of 700. Thus we reduce the time needed by 30%. If we want further propagators we use polynomial extrapolation to whatever order is allowed. This is important since, to do the subtractions required for the $\Delta I = 1/2$ amplitudes, we have to use a numerical derivative with respect to the quark mass.

Finally, to calculate the inner loop in the eye diagrams we need propagators from nearly all points, but only to close by neighbors. To do this we are using pseudo-fermions. These have been generated, and are presently being analysed.[20]

4. PERTURBATIVE CORRECTIONS

An essential ingredient in any lattice measurement of weak amplitudes is a calculation of the relationship between the continuum operators we want to use, and the lattice operators we are forced to use. The fact that we want to use continuum operators is forced upon us by the lattice fermion doubling problem. If we could put the standard model on the lattice without breaking the chiral symmetry, then we could simply use the lattice regularization scheme as a standard instead of, say, $\overline{MS}$. But, as discussed above, we either have to break the chiral symmetry explicitly (Wilson fermions), or do our calculations in a different theory (staggered).

One does the calculations by probing the operators using massless quark and antiquark external states at small (or zero) momenta. At tree level, the continuum and lattice operators give the same matrix elements (once appropriate factors of N_f have been removed by hand); this is by construction. At one-loop, there are logarithmic divergences which are regulated on the lattice by the finite lattice spacing, and in the continuum by some scheme such as dimensional reduction. These logarithmic divergences need to be the same for both lattice and continuum operators; that this is true for the staggered fermion operators we use is a non-trivial check of our approach. The finite parts of the one-loop corrections, on the other hand, give different corrections to the lattice and continuum operators, and in particular different mixing amongst the operators. Thus one must use a particular combination of lattice operators, with appropriate correction factors, in order to represent a given continuum operator.

To make this concrete let me first consider corrections to fermion bilinears $\mathcal{O}_{sf} = \mathrm{Tr}(\overline{Q}\Gamma_s Q\Gamma_f^\dagger)$. For the operators made gauge invariant by the addition of gauge links, these corrections were calculated by Golterman and Smit[8] and by Daniel and Sheard.[21]. We[22] have checked these results, and extended them to operators in Landau gauge with no gauge links. The general form of the answer is

$$\mathcal{O}_i^{CONT} = \mathcal{O}_i^{LATT} + \sum_j \mathcal{O}_j^{LATT} \frac{4}{3}\frac{g^2}{16\pi^2}(\delta_{ij} d_i - c_{ij})$$

$$d_i = (\sigma_s - 1)(\ln(a^2\mu^2) - 4.29201) - \sqrt{\sigma_s}$$

where both i and j serve as labels for spin and flavor, d_i are the finite residues of the cancelled logarithmic divergences, and c_{ij} are the finite mixing terms. σ_s depends only on the spin of the bilinear: it is [4,1,0,1,4] for s=[S,V,T,A,P]. The continuum subtractions have been done in the dimensional reduction scheme, which

has introduced the scale μ.

The possible mixings are constrained by a subgroup of the full staggered symmetry[21]. Thus many of the bilinears, including all the examples given above, do not mix. Furthermore, it turns out that the few off-diagonal entries are small. The diagonal elements depend sensitively on whether one uses the Landau gauge operators, or the standard ones. With the latter choice the c_{ii} range from +27, for a distance 0 operator (i=[0000; 0000]), to -50, for distance 4 (i=[0000; 1111]). We work at $g = 1$, so this corresponds to a large variation of about 0.6 in the correction. On the other hand, the correction ranges from +27 to -5 for the Landau gauge operators. A large part of this difference is due to the absence of gluon tadpole loops for the Landau gauge operators. These give a contribution of ~ -13 per link for the standard operators. Thus the Landau gauge operators are preferable not only because they are simpler to use, but also because they have smaller perturbative corrections.

The corrections to the four fermion operators at 1-loop have been calculated by Sheard[4], and independently by us[22]. Sheard does the calculation only for operators with gauge links included. This is a big mess, and we are in the process of checking our results, and comparing with those of Sheard. We have in addition calculated the penguin graphs which are needed for the $\Delta I = 1/2$ operators.

For the Landau gauge operators which we actually use, the calculation reduces to that for bilinears. This reduction is done using Fierz transforms and charge conjugation, as in the continuum. Thus I am quite confident in the numbers I will present, though they await final confirmation. The only difficulty with the calculation is one of bookkeeping. The lattice symmetries do not provide much restriction on the mixing, since many combinations of two bilinears transform as

scalars. The only restrictions come from the axial symmetry.

Consider the example of the operator

$$\sum_{\mu} \mathrm{Tr}(\overline{S}'_a \gamma_\mu \gamma_5 D'_a \gamma_5)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu \gamma_5 D_b \gamma_5).$$

This mixes itself and with

$$\mathrm{Tr}(\overline{S}'_a \gamma_\mu \gamma_5 D'_b \gamma_5)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu \gamma_5 D_a \gamma_5); \qquad \mathrm{Tr}(\overline{S}'_a \gamma_\mu D'_a \gamma_5)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu D_b \gamma_5);$$

$$\mathrm{Tr}(\overline{S}'_a \gamma_\mu D'_b \gamma_5)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu D_a \gamma_5); \qquad \mathrm{Tr}(\overline{S}'_a \gamma_\mu D'_a)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu D_b);$$

$$\mathrm{Tr}(\overline{S}'_a \gamma_\mu \gamma_5 D'_b)\ \mathrm{Tr}(\overline{S}_b \gamma_\mu \gamma_5 D_a); \qquad \mathrm{Tr}(\overline{S}'_a D'_a \gamma_\mu \gamma_5)\ \mathrm{Tr}(\overline{S}_b D_b \gamma_\mu \gamma_5); \quad \ldots$$

All the operators in the list, including those represented by the ellipses, have distance 1 or distance 3. This is required by the axial symmetry. All of these operators separately satisfy the Ward Identity described in section 2. The first three operators have the same flavor as the original operator. In the continuum, these are the only operators that would be needed, and all receive logarithmically divergent coefficients. On the lattice it is also true that these three are the only operators to get divergent coefficients, and these coefficients agree with those of the continuum. Thus the lattice anomalous dimension matrix is the same as that in the continuum. The finite corrections, however, produce all the other operators in the list, as well as contributing to the first three.

Now comes a tricky point, which is very important. The perturbative calculation has taken into account the short distance effects of the flavor breaking term in the action. A low momentum quark-antiquark pair approaching the bare lattice operator actually sees it as a sum of the operators listed above. Let the $\overline{Q}Q$ pair be projected into a particular spin and flavor state. Then only those operators in the list which have the same flavor as that of the external state will contribute.

The claim is that the non-perturbative dynamics which combines the $\overline{Q}Q$ pair into a state only breaks the flavor symmetry at $O(a)$, and is thus very small. This is checked by the restoration of flavor symmetry in the spectrum. The flavor-breaking is only logarithmically suppressed (i.e. appears in $g(a)$) in the finite parts of the perturbative corrections, and these cannot be ignored. Thus the finite corrections to operators which have the same flavor as the external states must be kept.

This argument can actually be checked by using different external states, and we are in the process of doing this.[20]

Assuming that this argument is correct, the perturbative calculation simplifies considerably. We need keep only the four operators nto which the B parameter decomposes—$\mathcal{M}^{1,2}_{V,A}$. When all the dust settles we find that the matrix of corrections (including the logarithmic part) is diagonal (to 1% accuracy) and that all the diagonal elements lie in the range $-(26-36)g^2/16\pi^2$. Thus one makes only a 2% error if one simply multiplies all matrix elements by a factor of $1-30g^2/16\pi^2 \sim .81$. This is an acceptable size for a correction. The correction term for the vacuum saturated matrix element is exactly diagonal and of size $-31.4g^2/16\pi^2$. Thus, to a very good approximation the B parameter receives no perturbative corrections.

The situation is not so clean for the EMP operators. The mixing matrix is not diagonal, and the entries are larger. The extreme case is the correction to the vacuum saturation matrix element of size $1-94g^2/16\pi^2 \approx .40$. Again, when one takes ratios to form the quantities B_7 and B_8 (see Ref. 3 for definitions), there is a large cancellation of the perturbative corrections. Nevertheless, their large size is cause for concern.

I have skimmed over an important uncertainty by naively using $g = 1$, the lattice value. At 1 loop order, one could equally well use $g(\mu)$, the value of the

continuum dimensional reduction coupling constant. It is well known that lattice coupling constants are smaller than their continuum counterparts, for $a\mu \sim 1$. This translates into the possibility that the perturbative corrections might be almost twice as large as the numbers quoted above. This would be tolerable for the B parameter, but not for the EMP matrix elements. Higher loop calculations are needed to decide which is the correct coupling to use.

5. RESULTS FOR B PARAMETERS

With no further ado I present our new results. The raw numbers are shown in figure 3. We have three quark masses, allowing us to make 6 different kaons. To very good approximation the square of the kaon mass is proportional to the average quark mass, so the x axis can be thought of as m_K^2. I show separately the "AA" contribution ($\mathcal{M}_A = \mathcal{M}_A^1 + \mathcal{M}_A^2$), the "VV" contribution ($\mathcal{M}_V = \mathcal{M}_V^1 + \mathcal{M}_V^2$), and the sum ($\mathcal{M}$). All of these are divided by the lattice vacuum saturation value for $4/3\mathcal{M}_{Ast}^2$, so that they are correctly normalised B parameters. I stress that the results are essentially raw numbers, read off plots such as figure 2. The error analysis is done using the jackknife method. As just discussed, the perturbative corrections to B are very small.

There are a number of features of the results that I want to bring out. The first concerns the chiral behavior of the matrix elements. The vacuum saturation result is $\propto f_K^2 m_K^2$ by definition. Thus if a matrix element has the correct chiral behavior, the B parameter will be independent of m_q. If a matrix element is constant, however, then the B parameter will grow as $1/m_q$. Our data seem to show that $\mathcal{M}_A$ and $\mathcal{M}_V$ have the wrong chiral behavior, while their sum displays

the correct chiral behavior. I stress that each of the parts of $\mathcal{M}$ should *separately* behave as m_K^2 in the chiral limit.

What if anything, has gone wrong? One's first thought is that it has something to do with the wall sources. This can be ruled out by looking at our old $12^3 \times 30$ data. This data is less extensive, but shows exactly the same pattern of bad chiral behavior. I showed some of the data in the 1986 lattice conference.[24] We have sat on the data because we had no clear explanation of the poor chiral behavior. We worried that "wrap-around" contributions, allowed because we used anti-periodic boundary conditions, could have introduced systematic errors. The confirmation of the bad chiral behavior suggests that the old calculation may be better than we thought. Because of this, and because the Ward Identities are exact for the old data, we have dragged it out and looked more carefully at how the arguments leading from Ward Identities to chiral behavior break down. We have come up with no simple answer, so far. But it is clear that terms of higher order than quadratic in the kaon momenta are important.

This is consistent with what I hope is the correct answer to what has gone wrong. Nothing! We are simply at too high a quark mass to see the chiral behavior. This explanation would have the $\mathcal{M}_V$ and $\mathcal{M}_A$ curves level off at not much smaller quark mass. This is not solely an idea designed to save face. Vacuum saturation is found, in a number of lattice calculations, to work very well for large quark masses, for reasons that are not fully understood. Thus the $\mathcal{M}_A$ curve in Figure 3 should be 1 at large quark masses, while that for $\mathcal{M}_V$ should be 0. For small quark masses, chiral symmetry should apply, and both $\mathcal{M}_A$ and $\mathcal{M}_V$ curves should be flat. If vacuum saturation is to fail anywhere, there must be a region where the curves differ significantly from a constant. It seems to me that the transition

region is not likely to occur much above the strange quark mass. If so, and if there are significant deviations from vacuum saturation, then the curves must look something like ours do.

It would be very nice to see a similar decomposition of the Wilson fermion data. This cannot be done for the B parameter, but it can be done in the related case of $\Delta I = 3/2$ $K \to \pi\pi$ amplitudes.

The full B parameter shows much better chiral behavior. This is due to a cancellation between the "AA" and "VV" parts. Our original program had an error in the sign of the "VV" part, leading to a very different result. Since our lightest quark has $m_q \approx m_s/2$, the mass of our lightest kaon is roughly the same as that of the physical kaon. Assuming that B_K depends only on the average quark mass, which is true within errors for our data, we have the result $B_K = 1.04(14)(??)$. I have purposely put in the question marks to emphasize that we do not know the systematic errors. In particular, we need to see whether B_K does only depend on the average quark mass when one of the masses is much lighter than the strange quark. Indeed, our own data shows us that large changes are occurring in some matrix elements. There is also the unknown systematic error of the quenched approximation.

Despite these unknown errors, it must be pointed out that the Wilson fermion result[1] is two thirds of ours. A conservative person would use this as an estimate of the systematic error. Furthermore, our result is evaluated at a scale of $\sim 2\,\mathrm{GeV}$, and so the appropriate coefficient function is *larger* than for smaller scales. So our number is definitely at the high end of the range that is usually considered.

We also have new data for the matrix elements of the electromagnetic penguin operators. Here the perturbative corrections are larger, and involve non-trivial

mixing. Our preliminary results are $B_7 = 0.8(2)(?)$ and $B_9 = 1.2(2)(?)$. In other words, our results are completely compatible with vacuum saturation approximation. They are also in agreement with our earlier data[3] within errors, and with the Wilson fermion calculations[1,2].

6. CONCLUSIONS

I think that quenched lattice calculations of weak amplitudes are entering a phase in which detailed checks of systematic effects can be made. This is very encouraging. Perhaps foremost among the tests that must be done is a study of smaller quark masses. This is expensive in computer time, but if the results of Figure 3 are any guide, it is essential. Other checks I have in mind for us include detailed tests of perturbation theory, checking current algebra by comparing single pion and two pion amplitudes, and studying the way in which Ward Identities are saturated. Tests I would like to see done with Wilson fermions include a study of the final state interaction between the two pions in the $K \to \pi\pi$ amplitudes. Current algebra guarantees that these interactions are small for small pion masses, but they need not be so for larger masses.

I hope I have convinced you that staggered fermions offer a viable alternative to Wilson fermions. For some matrix elements, they are superior, because of the Ward Identities. For others, the complications of non-local operators makes them more difficult to use. In any case, a comparison of Wilson and staggered results is a stringent check on the size of lattice artifacts. There has been good agreement on the EMP matrix elements for some time. Now that we are able to calculate B_K a much more significant comparison can be made. Unfortunately, it suggests large systematic errors. We hope in the near future to have results for

the $\Delta I = 1/2$ amplitudes, so that a comparison can be made with the results from Wilson fermions that just appearing through the noise[1,2].

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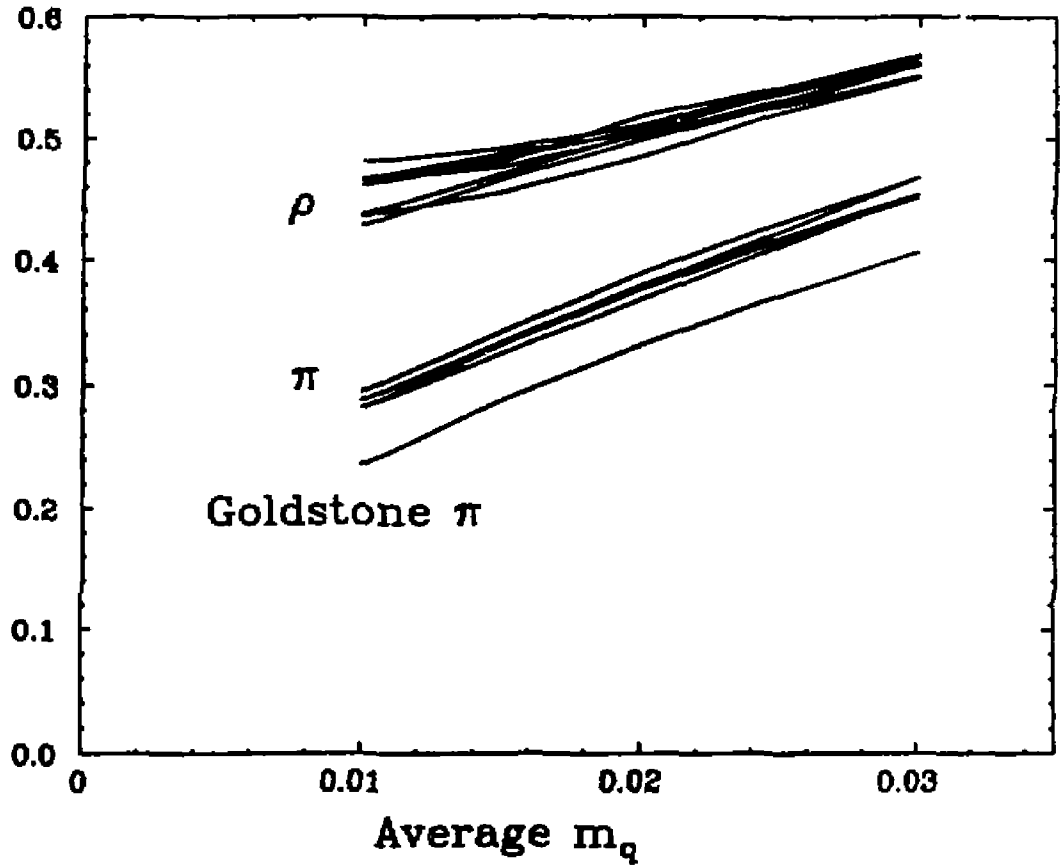

Figure 1

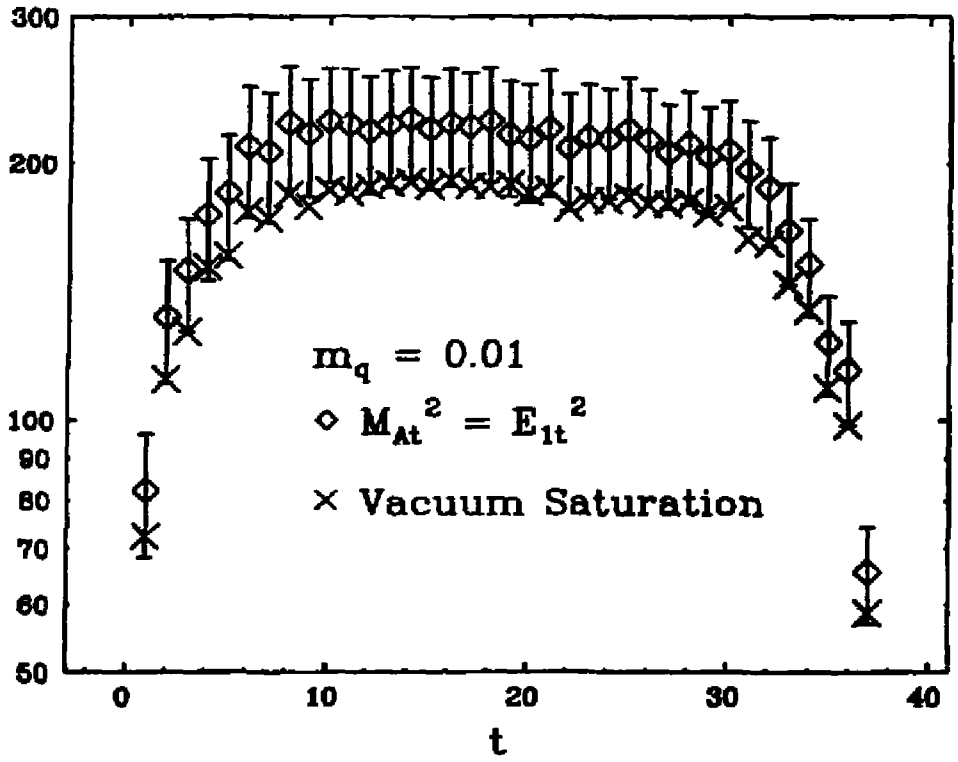

Figure 2

DISCLAIMER

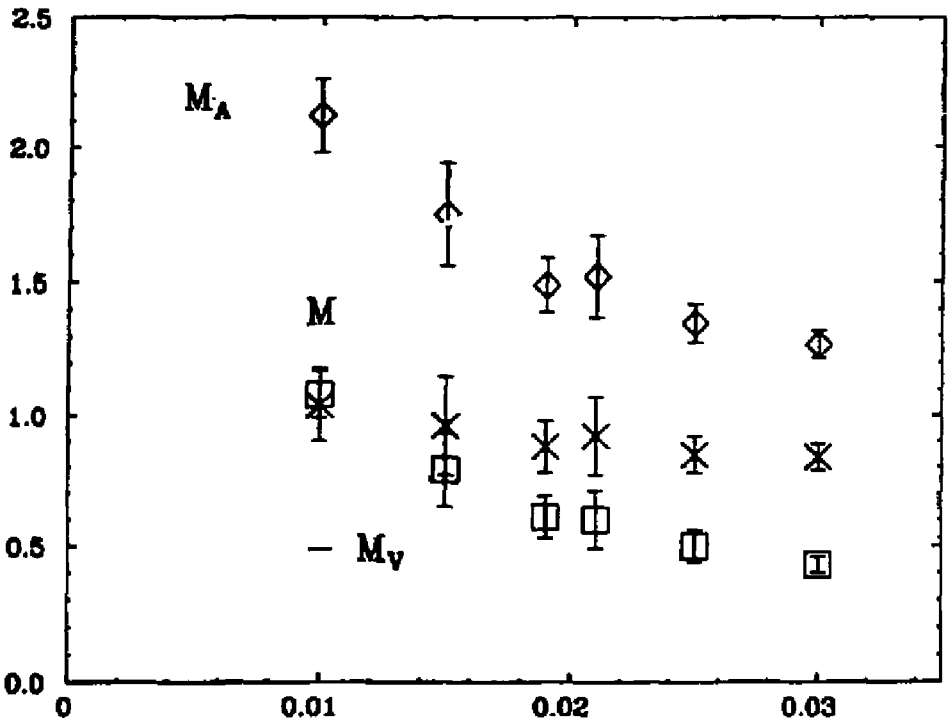

Figure 3