DOE/ER/25117--T1

DE93 010134

Progress report to
THE DEPARTMENT OF ENERGY
Contract No. DE-FG03-92ER25117

NEW APPROACHES TO LINEAR AND NONLINEAR PROGRAMMING

April 15, 1992–January 31, 1993

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February 1993

1 Project Description

This report describes technical progress during the past eight months on DOE Contract DE-FG03-92ER25117. The project involves study of the theoretical properties and computational performance of algorithms that solve linear and nonlinear programs. Particular emphasis is placed on algorithms to solve large problems. Such problems are especially important in the energy area. For example, the safe and efficient distribution of electricity and the identification of the state of an electrical network are both large-scale nonlinearly constrained optimization problems. Other application areas in which we are actively involved include optimal trajectory calculations and optimal structural design.

2 Review of Progress

2.1 Summary

During the last eight months a number of important milestones were reached. These were in part built on earlier research work. Several technical talks were also presented at international meetings.

2.2 An SQP method for large-scale optimization

The doctoral thesis of Eldersveld appeared (SOL 92-4), describing a sequential quadratic programming (SQP) algorithm for large-scale nonlinearly constrained optimization problems. This is the first adaptation of an SQP method to the large-scale case for general problems. A presentation of the work was made at the SIAM Conference on Optimization.

Much work remains to make the method both robust and efficient. A target in both these categories can be obtained from the performance of our dense SQP code, NPSOL. An implementation named SNOPT is currently being developed and tested on real-world problems arising in optimal control (as well as on a suite of 90 small but general test problems).

The approach is based on making a quasi-Newton approximation to a *projected* or *reduced* Hessian matrix. The projection is chosen so that an accurate approximation is required only in a certain subspace that is much smaller than the full space. A simple and sparse representation can be used in the remaining space, making operations with the projected Hessian relatively cheap when compared to the unprojected Hessian. Our earlier work showed how the projection may be chosen to make operations with them cheap. Current research is directed at how a reduced Hessian can be *accurately* approximated.

A particularly vexing problem is how to model the slack variables. Since it is known that slacks occur linearly in the Lagrangian function, we require the projected Hessian to reflect this property. A description of SNOPT will be presented at the conference on Large-scale Optimization, Florida, February 1993.

2.3 An efficient line-search for barrier functions

Barrier function methods have soared in popularity since we showed that Karmarkar's method for linear programming was a specific case of a barrier-function method (see Gill *et al.*, 1986). Much work is now directed at using such methods to solve nonlinear optimization problems.

In barrier-type methods it is necessary to perform a one-dimensional search to obtain an improved estimate to the solution. Such a search is usually based on a low-degree polynomial approximation to the function along the search direction. It is known that polynomials do not make good approximations to a barrier function, since the latter has singularities. In Murray and Wright (1993) we show how to perform efficient line-searches for barrier functions. The approach has been adopted by Nash and Sofer in their implementation of a barrier method (Nash and Sofer, 1991) and has improved the efficiency of their software by 30%.

2.4 AMPL

A new modeling language for mathematical programming has been produced by Fourer *et al.* (1993). Much effort was made during this period to incorporate our large-scale optimization code MINOS within this language. We anticipate that AMPL, like the earlier modeling language GAMS, will prove very popular with practitioners. MINOS is the only method for nonlinear optimization current being distributed with the AMPL package. It is the feedback from practitioners that drives our new research work.

2.5 A modified Newton method

A new modified Newton method has been developed for unconstrained and contrained optimization. It is particularly attractive in the large-scale case (SOL 93–1) and is based on the partial Cholesky factorization of the Hessian of the Lagrangian. For the first time it is shown how a direction of "sufficient" negative curvature may be obtained without the need for some form of complete pivoting. Such a pivoting strategy would be untenable for large problems.

The best direction of negative curvature is the eigenvector corresponding to the most negative eigenvalue of the Hessian. A sufficient direction of negative curvature is one that is not arbitrarily poor when compared to the best. It can be shown that all previously known methods that are adaptable to the large-scale case (such as that based on the Bunch-Kaufman factorization) are *not* sufficient in this sense. The ability to compute directions of sufficient negative curvature is essential if points satisfying the second-order necessary conditions for a minimizer are to be obtained. The work will be presented at the conference on Large-scale Optimization, Florida, February 1993.

3 Presentations

The following technical presentations have been given at meetings by the research personnel during April 15, 1992–January 31, 1993.

1. J. G. Braunstein and P. E. Gill, ℓ-infinity algorithms for linear programming, Fourth SIAM Conference on Optimization, Chicago, IL, May 11–13, 1992.
2. P. E. Gill, Organized minisymposium on Large-Scale Nonlinear Optimization, Fourth SIAM Conference on Optimization, Chicago, IL, May 11–13, 1992.
3. P. E. Gill, W. Murray and M. A. Saunders, SQP methods and their application to trajectory optimization, Fourth SIAM Conference on Optimization, Chicago, IL, May 11–13, 1992.
4. S. K. Eldersveld, SQP algorithms for large-scale constrained optimization, Fourth SIAM Conference on Optimization, Chicago, IL, May 11–13, 1992.
5. A. Forsgren and W. Murray, Large-scale issues in Newton methods for linearly constrained optimization, Fourth SIAM Conference on Optimization, Chicago, IL, May 11–13, 1992.
6. P. E. Gill, Large-scale quadratic programming, Third Optimization Days, Royal Institute of Technology, Stockholm, June 23–25, 1992.
7. W. Murray, The choice of merit function for SQP algorithms, Third Optimization Days, Royal Institute of Technology, Stockholm, June 23–25, 1992.
8. P. E. Gill, An SQP algorithm for large optimization problems arising in trajectory calculations, SIAM 40th Anniversary Meeting, Los Angeles, CA, July 20–24, 1992.
9. P. E. Gill, Large-scale SQP methods and their application in trajectory optimization, 9th IFAC Workshop on Control Applications in Optimization, Fachhochschule, Munich, Germany, September 2, 1992.
10. P. E. Gill, An SQP method for large problems, DVLR, Oberphaffenhofen, Germany, September 8, 1992.
11. M. A. Saunders, Organized session on Large-scale Constrained Optimization, ORSA/TIMS, San Francisco, CA, November 1–4, 1992.
12. M. A. Saunders, Optimizers to go, ORSA/TIMS, San Francisco, CA, November 1–4, 1992.

4 References

S. K. Eldersveld (1992). Large-scale sequential quadratic programming, SOL 92–4, Department of Operations Research, Stanford University, Stanford, CA.

A. Forsgren, P. E. Gill and W. Murray (1993). Computing modified Newton directions using a partial Cholesky factorization, SOL 93–1, Department of Operations Research, Stanford University, Stanford, CA.

R. Fourer, D. M. Gay and B. W. Kernighan (1993). *AMPL: A Modeling Language for Mathematical Programming*, The Scientific Press, South San Francisco, CA.

P. E. Gill, W. Murray, M. A. Saunders, J. A. Tomlin and M. H. Wright (1986). On projected Newton barrier methods for linear programming and an equivalence to Karmarkar's projective method, *Mathematical Programming* 36, 183–209.

W. Murray and M. H. Wright (to appear). An efficient line-search for logarithmic barrier functions, *SIAM J. on Optimization.*

S. G. Nash and A. Sofer (1991). A barrier method for large-scale contrained optimization, Report 91-10, Department of Operations Research and Applied Statistics, George Mason University, Fairfax, VA.

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5/12/93