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NONPERTURBATIVE EFFECTS OF FERMIONS IN THE SU(2) SECTOR OF THE STANDARD MODEL*

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ABSTRACT

Some nonperturbative effects of fermions in the SU(2) sector of the standard model are studied on the lattice. The results from both analytic studies and numerical simulations with dynamical fermions are presented. Implications for the strongly coupled standard model are discussed.

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There are two main reasons why one would like to study the standard electroweak theory nonperturbatively. First, the theory is, in general, not asymptotically free. Although perturbative calculations have proved to be adequate in describing the low-energy phenomena up to about 100 GeV, it is possible that the theory becomes strongly coupled, and, therefore, nonperturbative, at higher energy. Second, it is well known that certain interesting phenomena can occur at weak couplings in a manner which is not evident in any order of perturbation theory. It is desirable to see if any such phenomena occur in the electroweak theory.

A full nonperturbative study of the electroweak theory is a formidable task. We have first studied a simplified model, namely, the $SU(2)_L$ sector of the electroweak theory with two generations and with no Yukawa couplings.[1-5] (The case with non-zero Yukawa couplings is discussed in Ref. 6.) This model can be regarded as an approximation to the standard electroweak theory in the limit where the U(1) gauge coupling and the Yukawa couplings are weak. In this limit, the right-handed fermions are free fields which decouple from the rest of the theory and hence can be neglected. With two generations, there are eight left-handed fermion doublets in the theory. Since the representations of SU(2) are real, a chiral $SU(2)_L$ theory with eight fermion doublets can be written equivalently as a vectorlike SU(2) theory with four doublets,[6-9] which can be studied nonperturbatively in a well-defined gauge-invariant manner with a lattice formulation.

The discretized Euclidean action of the model under consideration can be written in usual notation[1-5,11] as

$$S = \beta_g \sum_{plaq.} [1 - P] + \frac{1}{2} \sum_{n,\mu} \eta_{n,\mu} (\bar{\chi}_n U_{n,\mu} \chi_{n+e_\mu} - \bar{\chi}_{n+e_\mu} U^\dagger_{n,\mu} \chi_n)$$
$$+ 2\beta_h \sum_{n,\mu} Re(\phi^\dagger_n \phi_n - \phi^\dagger_n U_{n,\mu} \phi_{n+e_\mu}) + \lambda \beta_h^2 \sum_n (\phi^\dagger_n \phi_n - 1)^2 \qquad (1)$$

with $\beta_g = 4/g^2$, $\beta_h = m^2/2\lambda$, and $P = (1/2)Tr(U_{plaq.})$. (The lattice spacing $a \equiv 1$.) We use staggered fermions, which are advantageous for studies of chiral symmetry. The $\eta_{n,\mu}$ are factors from the Dirac matrices.

The purely bosonic part of the model has been studied extensively. The most notable feature of its phase diagram[12,13] is that the confinement region and the Higgs region are analytically connected. This has inspired ideas such as complementarity[14] and the Abbott-Farhi model.[15] The phase diagram in the $\beta_g - \beta_h$ plane for a large range of λ is known to be qualitatively similar to that for the limiting case $\lambda \to \infty$, on which we shall focus our present discussions for simplicity. In this limit, the scalar fields satisfy $\phi_n^\dagger \phi_n = 1$.

Knowing the properties of the model in the absence of fermions, one is led to ask: what happens when fermions are present? This question was first addressed in Ref. 1, where a numerical simulation was performed using the quenched approximation. We chose to work at $\beta_g = 0.5$ where all local observables are analytic functions of β_h in the absence of fermions. We measured the chiral condensate at several values of β_h. The data show evidence for the existence of a chiral transition at a finite β_h. We also found that, in the small-β_h region where the chiral condensate is nonzero, a massless Goldstone boson appears in the spectrum, while in the large-β_h region where the chiral condensate vanishes, there exist massless fermions. Therefore, the simulation with the quenched approximation suggests that there exists a new transition associated with the fermions.

This finding motivates one to look for a way to demonstrate the existence of the chiral transition analytically. If we can show that, in the presence of fermions, a chiral transition exists at $\beta_g = 0$, then the fact that a small-β_g expansion has a finite radius of convergence implies that the critical point at $\beta_g = 0$ is the endpoint of a line of chiral phase transitions which extends into the interior of the phase diagram at $\beta_g > 0$. However, as was noted in Ref. 16, this line of chiral transitions cannot stop in the interior of the phase diagram, since it separates a region where the chiral condensate is nonzero from another where the condensate vanishes identically, and these two regions cannot be analytically connected to each other. Therefore, if there is a chiral transition at $\beta_g = 0$, there must exist a line of chiral transitions which completely separates the phase diagram into two disjoint regions.

We have carried out a series of analytic studies[2,4–7,16–20] of various models in the strong gauge coupling limit (i.e. $\beta_g = 0$), using a mean-field type of approximation. One of the main results of these studies is that the existence of chiral transition is a very general feature of lattice gauge theories with scalar and fermion fields. Some of these works are discussed in R. Shrock's contribution at this meeting. I will only mention the results for the model which we are considering here, namely, the SU(2) gauge theory with a scalar field and a staggered fermion field, both in the fundamental representation. With the mean field approximation, we found that[2] the chiral condensate decreases monotonically and continuously as β_h increases from zero to $\beta_h \approx 2.76$, beyond which the condensate vanishes identically. Thus there is a second-order chiral transition at $\beta_g = 0$, and the critical point is at $\beta_{h,c} \approx 2.76$.

For comparison, we performed a quenched simulation[2], and the data suggests that a chiral transition occurs at approximately $\beta_{h,c} \approx 2.7$. This is in good agreement with the analytic result.

Although our analytic studies were carried out at the strong gauge coupling limit, the result is actually more general. The chiral transition found at $\beta_g = 0$ extends into the interior of the phase diagram all the way until it reaches one of the two boundaries, $\beta_h = 0$ or $\beta_g = \infty$. Furthermore, it is worth noting that the chiral transition is determined to be of second-order at $\beta_g = 0$. One expects from a small-β_g expansion that the transition will remain second-order for at least a finite range of β_g.

To get a better feeling of how fermions affect the bosonic system, especially for nonzero β_g, we performed a simulation with dynamical fermions, using a Langevin algorithm.[3] The results of this simulation are summarized as follows. At $\beta_g = 0$, the data shows that the chiral condensate decreases as β_h increases, and vanishes for sufficiently large β_h. The critical point of the chiral transition is estimated to be $\beta_h = 2.5 \pm 0.3$, which is close to the results from the analytic study and the quenched simulation. As was mentioned earlier, it is expected that the qualitative features established at $\beta_g = 0$ remain the same for a finite strip adjacent to $\beta_g = 0$. This expectation is borne out by our data at $\beta_g = 0.5$. According to our data, the chiral transitions at $\beta_g = 0$ and 0.5 are both consistent with being of second-order, in agreement with the result of our analytic study. At $\beta_g = 1.9$, all three quantities that are measured, namely, the chiral condensate $< \bar{\chi}\chi >$, the average plaquette $< P >$ and $< L > \equiv < \sum_{n,\mu} Re(\phi_n^\dagger U_{n,\mu}\phi_{n+e_\mu})/N_\ell >$ (N_ℓ is the number of links in the lattice), indicate a transition, at $\beta_h \approx 0.4$, and no other transition was found. This result suggests that the chiral transition coincides with the confinement-Higgs transition, which, in the absence of fermions, exists for[21] $\beta_g \geq 1.6$. The data provides some suggestion that the transition at $\beta_g = 1.9$ is weakly first-order.

From our simulation with dynamical fermions, we obtain the phase diagram in the presence of fermions, which is shown in fig. 6 of Ref. 3. Our dynamical fermion simulation is exploratory in nature. While the quantitative details of the phase diagram can be improved by performing a larger scale simulation with an exact algorithm, the qualitative features are expected to be unchanged. Due to the increasingly severe finite size effect as β_g increases, it is difficult to determine where the chiral phase boundary ends by numerical simulations. We do know, however, that (a) the endpoint cannot be in the interior of the phase diagram, and (b) the region with small β_g and small β_h (customarily labeled as the "confinement phase") is no longer analytically connected to the region with large β_g and large β_h (the "Higgs phase") when fermions are included. The chiral phase boundary completely separates the phase diagram into two disjoint regions characterized by whether or not chiral symmetry is spontaneously broken.

Thus, we have shown that fermions affect the phase diagram of a theory in a fundamental way. In particular, for the SU(2) sector of the standard model, the existence of the chiral phase boundary in the phase diagram implies that it is necessary to approach the continuum limit of the lattice model from within the chirally symmetric phase in order to obtain a spectrum with light fermions. The fact that the chiral transition is continuous for at least a finite range of β_g opens up new possibilities for the continuum limit of the lattice theory to be taken. It will be interesting to examine the properties of the continuum theories defined along the second-order chiral transition phase boundary.

The above discussions apply to the SU(2) sector of the strongly coupled standard model[10] (SCSM) also, since its Lagrangian has the same form as the usual standard model. It remains to be seen whether a continuum limit of the above lattice model exists which (a) can be approached from within the chirally symmetric phase, and (b) yields a spectrum of strongly coupled bound states and resonances at the weak scale in addition to the usual low-energy spectrum, as suggested by SCSM. It is worth noting that the natural place for stronly coupled bound state structure to occur is in the region which is analytically connected to the corner with small β_g and small β_h, where chiral symmetry, however, is spontaneously broken.

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