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DIMENSIONAL ANALYSIS, SIMILARITY, ANALOGY, AND THE SIMULATION THEORY

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by

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DIMENSIONAL ANALYSIS, SIMILARITY, ANALOGY AND THE SIMULATION THEORY (ABSTRACT)

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Dimensional analysis, similarity, analogy, and cybernetics are shown to be four consecutive steps in application of the simulation theory. This paper introduces the classes of phenomena which follow the same formal mathematical equations as models of the natural laws and the interior sphere of restraints groups of phenomena in which we can introduce simplified nondimensional mathematic equations. The simulation by similarity in a specific field of physics, by analogy in two or more different fields of physics and by cybernetics in nature in two or more fields of mathematics, physics, biology, economics, politics, sociology, etc., appears as a unique theory which permits to transport the results of experiments from the models, convenable selected to meet the conditions of researches, constructions and measurements in the laboratories to the originals which are the primaries objectives of the researches. Some interesting conclusions which cannot be avoided in use of simplified nondimensional mathematical equations as models of natural laws are presented. Interesting limitations on the use of simulation theory based on assumed simplifications are recognized.

This paper shows as necessary, in scientific research, to write mathematical models of general laws which will be applied to nature in its entirety. The paper proposes the extent of the second law of thermodynamics as the generalized law of entropy to model life and its activities.

This paper shows that the physical studies and philosophical interpretations of phenomena and natural laws cannot be separated in scientific work; they are interconnected and one cannot be put above the others.

DIMENSIONAL ANALYSIS, SIMILARITY, ANALOGY, AND THE SIMULATION THEORY

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Simulation theory, which allows the study of an original phenomenon by a model, or even by computer calculations, is not new. It originated before Newton in the sixteenth century with the dimensional analysis, and was continued through similarity, analogy, and cybernetics into the present.

This article attempts to show: (1) how the basic theory can be applied in the study of a phenomenon establishing the adequate mathematical forms for the natural laws; (2) how the simulation theory should be understood and used; (3) what conditions should be introduced when using the simulation theory; and (4) the limitations and restrictions inherent in each study.

As a result of the material introduced, it appears that the experimental equations for a phenomenon, or group of phenomena, defined by similarity, analogy, or cybernetics, should be presented in dimensionless numbers (criteria) and should be completed within the area of viability in which the experiment was performed. Some examples are introduced here to facilitate following the theory and to show directly how this theory can be applied to the physical phenomena studies.

Through the use of *dimensional analysis*, it is possible to derive the physical laws through their mathematical forms (equations) knowing that the equiatons have only equidimensional components. At the same time, the dimensional analysis establishes connections between the physical quantities which are characteristics for a physical phenomenon, when we do not know the mathematical forms for the basic equations.

The dimensional analysis is based on the multiplying law, called the π theorem: *a mathematic equation which represents a physical law, and has k primary quantities, and n-k derivative, may be written as a function of n-k dimensionless numbers (criteria).*

Having A_i primary quantities and B_{k+j} derivative, with $[A_i]$, $[B_{k+j}]$ measurement units, and $a_i = A_i : [A_i]$, $b_{k+j} = B_{k+j} : [B_{k+j}]$ numerical values, the equations which represent a phenomenon and depend only on physical quantitites, being independent of the measurement units, are:

$$f\ (a_1, a_2 \ldots a_k, b_{k+1} \ldots b_n) = f\ (a_1', a_2' \ldots a_k, b_{k+1}' \ldots \ldots b_n') = 0. \quad (1)$$

The second equation of (1) has the measurement units $[A_i] : c_i$, with c_i numerical values. In that:

$$a_i = \frac{A_i}{[A_i]} c_i = a_i c_i\ , \quad (2)$$

$$b_{k+j}' = \frac{B_{k+j}}{\prod_{i=1}^{k} [A_i]^{r_i}} \prod_{i=1}^{k} c_i^{r_i} = b_{k+j} \prod_{i=1}^{k} c_i^{r_i}\ , \quad (3)$$

in which the measurement units $[B_{k+j}]$ and $[B_{k+j}']$ are being written with primary units. Because c_j may have any values, with $a_i\ c_i = 1$:

$$f\ (a_1, a_2 \ldots a_k, b_{k+j} \ldots b_n) = f\ (1, 1 \ldots 1, b_{k+1} \prod_{i=1}^{k} c_i^{m_i} \ldots \ldots b_n \prod_{i=1}^{k} c_i^{p_i}) = F\ (\frac{b_{k+1}}{\prod_{i=1}^{k} a_i^{m_i}}, \ldots \frac{b_n}{\prod_{i=1}^{k} a_i^{p_i}}) = F\ (\pi_1, \ldots \pi_{n-k}) = 0, \quad (4)$$

$$\text{with: } \pi_j = b_{k+j} \prod_{i=1}^{k} c_i^{r_i} = \frac{b_{k+j}}{\prod_{i=1}^{k} a_i^{r_i}}\ , \quad (5)$$

a dimensionless number.

The result is that an equation which represents a phenomenon having n quantities (k primaries and n-k derivatives from the primaries) may be written as a new equation with n-k dimensionless numbers.

Although the dimensional analysis can be used in the physical studies, especially when the phenomena equations are not known, it may give wrong conclusions especially when:

(a) not all quantitites that are characteristic for the studied phenomenon are known;

(b) the general equations include dimensionless quantities that are out of the dimensional analysis control, and that can be known only on an experimental basis;

(c) wrong quantities, which normally cannot be excluded on the calculation basis, are introduced into the study;

(d) the equations contain more quantities with the same dimension, which cannot be separated on a calculation basis.

Some examples of using dimensional analysis to derive an equation are given below:

Example 1 - using the dimensional analysis, derive the equations that represent the ideal gas law in thermodynamics.

Solution 1: In the equation shall be introduced n = 5 quantities (pressure = p; volume = V; temperature = T; mass = M; and the ideal gas constant = R). With these

$$f\ (p,V,T,M,R) = 0.$$

Having a total of (n = 5) quantities, the primary can be selected on a dimensional basis:

$$[p] = \frac{[F]}{[A]} = ML^{-1}T^{-2}L^{-2}\ ,$$

$$[V] = L^3\ ,$$

$$[T] = \theta\ ,$$

$$[M] = M ,$$

$$[R] = L^2 MT^{-2} \theta^{-1} ,$$

where in the SI system, M = mass; L = length; T = time, and Θ = temperature.

For the ideal gas law the resultant, k = 4 and n-k = 1:

$$F (\pi_1) = 0$$

The dimensionless number π_1, can be written:

$$\pi_1 = p V^p T^r M^s R^t = ML^{-1} T^{-2} L^{-3p} \theta^r M^s L^{2t} M^t T^{-2t} \theta^{-t}$$

and from this:

$$1 + s + t = 0$$

$$-1 + 3p + 2t = 0$$

$$-2 - 2t = 0$$

$$r - t = 0$$

having:

$$p = 1, r = -1, s = 0, t = -1,$$

and:

$$\pi_1 = \frac{pV}{RT} = \text{constant} = n,$$

or:

$$pV = nRT.$$

The constant, n, should be established on an experimental basis.

Example 2 - Derive the transient conduction's law through dimensional analysis method.

Solution 2: In the equation should be introduced n = 6 quantities (temperature = t, time = τ, length = l, thermal diffusivity = a, thermal conductivity = k, and coefficient of heat transfer through a surface = h):

$$f (t, \tau, l, a, k.h) = 0,$$

with the dimensions:

$$[t] = \theta$$

$$[\tau] = T,$$

$$[l] = L,$$

$$[a] = L^2\ T^{-1},$$

$$[k] = LMT^{-3}\ \theta^{-1},$$

$$[h] = MT^{-3}\ \theta^{-1},$$

The primary quantities are: k = 4 (t, τ, l, k) and the derivatives n-k = 2 (a,h). With that, the basic equation should be written with two dimensionless numbers:

$$F\ (\pi_1,\ \pi_2) = 0,$$

or:

$$F\ \left(\frac{a}{t^{m_1}\ \tau^{p_1}\ l^{r_1}\ k^{s_1}},\ \frac{h}{t^{m_2}\ \tau^{p_2}\ l^{r_2}\ k^{s_2}}\right) = 0\ .$$

From the dimensional analysis:

$$\pi_1 = L^2\ T^{-1}\ \theta^{-m_1}\ T^{-p_1}\ L^{-r_1}\ L^{-s_1}\ M^{-s_1}\ T^{3s_1}\ \theta^{s_1},$$

and with:

$$2 - r_1 - s_1 = 0,$$

$$- s_1 = 0\ ,$$

$$-1 - p_1 + 3s_1 = 0\ ,$$

$$- m_1 + S_1 = 0,$$

it results:

$$m_1 = 0\ ,\ p_1 = 1,\ r_1 = 2,\ s_1 = 0.$$

The dimensionless number calculated is the Fourier number:

$$\pi_1 = Fo = \frac{a\tau}{l^2}\ .$$

Also from dimensional analysis:

$$\pi_2 = MT^{-3}\,\theta^{-1}\,\theta^{-m_2}\,T^{-p_2}\,L^{-r_2}\,L^{-s_2}\,M^{-s_2}\,T^{3s_2}\,\theta^{s_2} \quad ,$$

$$-r_2 - s_2 = 0$$

$$1 - s_2 = 0$$

$$-3 - p_2 + 3s_2 = 0$$

$$-1 - m_2 + s_2 = 0$$

$$m_2 = 0,\ p_2 = 0,\ r_2 = -1,\ s_2 = 1.$$

The dimensionless number calculated is the Biot number:

$$\pi_2 = Bi = \frac{h\,l}{k} .$$

The basic equation is:

$$F\ (Fo,\ Bi) = 0.$$

Because of the restrictions of the dimensional analysis, the dimensionless number $\frac{t}{t_o}$ was lost in our calculations.

Thus:

$$\phi\ (Fo,\ Bi,\ \frac{t}{t_o}) = 0 \ ,$$

or:

$$t = t_o\ \phi\ (Fo,\ Bi) \ .$$

We have the correct equation which can be completely established on a similarity basis as shown below.

Physical similarity is the next step — starting with dimensional analysis — in establishing the physical laws for the phenomena by starting with the general equations of the phenomenon, for which we apply the dimensional analysis. A physical law is written as an equation in which, of course, all terms are equidimentional. All phenomena that follow a law written as a single equation, are in a *similarity class* if they are included in same physical field, but they are in a *analogy class* if they are included in different physical fields.

A physical law may be written through an equation or equation system algebraical, differential, integral, or integro-differential. Except for the algebraical case, the general solutions are multiple, i.e., value of the constant may vary from $-\infty$ to $+\infty$, and the particular single solution is established by singularity conditions that separate the studied phenomenon from others in the similarity or analogy class. The singularity conditions can be as forms algebraical, differential, integral or integro-differential. To establish the single algebraic equations that represent the physical law for a studied phenomenon, the basic equations and singularity conditions may be used with one of the following methods:

(a) a direct settlement of the equations;

(b) an approximate settlement of the equations, using some admissible simplifications; or

(c) an experimental settlement based on physical similarity or analogy.

The physical similarity is the connection between the theory and the experiment and can be used successfully where the first two methods are not applicable. The similarity is the result of application of the dimensional analysis to the basic equations for the similarity class of the physical phenomena. The compatibility conditions that result from changing the measurement units (which mathematically correspond to an affine transformation) establish the dimensionless numbers.

Use of similarity is accomplished on the basis of its laws. There are two postulates and three laws for the similarity.

The basic postulates are:

1. *All natural phenomena have laws that can be written as mathematical equations;*

2. *The dimensionless numbers that correspond to a system of differential, integral, or integro-differential equations, written for an element of their integration domain, are valid for all domains.*

The first law of the similarity is: *If a multiple phenomenon which corresponds to an equation system forms a similarity group, each of the dimensionless numbers has a unique numeric value for the entire group's phenomena* (Newton 1686).

For two phenomena A and B:

$$f\,(a_1^A,\ a_2^A \ldots a_k^A,\ b_{k+1}^A \ldots b_n^A) = f\,(a_1^B,\ a_2^B \ldots a_k^B,\ b_{k+1}^B \ldots \ldots b_n^B) = 0 \qquad (6)$$

The dimensionless forms are:

$$F\left[\frac{b_{k+1}^A}{\prod_{i=1}^{k} (a_i^A)^{m_i}} + \ldots \frac{b_n^A}{\prod_{i=1}^{k} (a_i^A)^{p_i}}\right] = 0, \qquad (7)$$

$$F\left[\frac{b_{k+1}^B}{\prod_{i=1}^{k} (a_i^B)^{m_i}} + \ldots \frac{b_n^B}{\prod_{i=1}^{k} (a_i^B)^{p_i}}\right] = 0. \qquad (8)$$

Because between quantities A and B are the connections $a_i^A = c_i a_i^B$ and $b_{k+j}^A = c_{k+j}\, b_{k+j}^B$ the equation (7) in the following form:

$$F\left[\frac{c_{k+1}}{\prod_{i=1}^{k} c_i^{m_i}} \; \frac{b_{k+1}^B}{\prod_{i=1}^{k} (a_i^B)^{m_i}} \ldots \frac{C_n}{\prod_{i=1}^{k} c_i^{p_i}} \; \frac{b_n^B}{\prod_{i=1}^{k} (a_i^B)^{p_i}}\right] = 0\ , \qquad (9)$$

should be identical with (8). With $c_{kj} : \prod_{j=1}^{k} c_i^{r_i} = 1$, with $c_i = a_i^A : a_i^B$ and $c_{k+j} = b_{k+j}^A : b_{k+j}^B$ it results:

$$\frac{b_{k+j}^A}{\prod_{i=1}^{k} (a_i^A)^{r_j}} = \frac{b_{k+j}^B}{\prod_{i=1}^{k} (a_i^B)^{r_j}} \quad \text{or:} \quad \pi_j^A = \pi_j^B \ . \tag{10}$$

The second law of similarity is: *The general solution for an equation system (algebraic, differential, integral, integro-differential) which corresponds to a similarity phenomena group, may be written with similarity dimensionless numbers obtained from the equation's system. The singular solutions which correspond to known singularity conditions, may be written with the same dimensionless numbers or with the dimensionless numbers that result from the singularity conditions,* and with dimensionless ratios between quantities which are characteristic of the singularity conditions and some special values for these quantities (Buckingham and Federman, 1911). That law is known as π theorem.

Having:

$$f\ (a_1,\ a_2\ .\ .\ .\ a_k,\ b_{k+1},\ .\ .\ .\ b_n) = 0 \tag{11}$$

this differential or integral equation can be written as:

$$F\ (\pi_1,\ .\ .\ .\ \pi_{n-k}) = 0 \tag{12}$$

with: $\pi_j = b_{k+j} : \prod_{i=1}^{k} (a_i)^{r_i}$, and the solution will be:

$$\phi\ (\pi_1,\ .\ .\ .\ \pi_{n-k}) = 0 \tag{13}$$

A singular solution can be written with (n-k) dimensionless numbers from (13) and [s - (n-k)] dimensionless numbers from singularity conditions, and (k-s) dimensionless ratios:

$$\phi\ [\ \pi_1,\ \ldots\ \pi_{n-k},\ \pi_{n-k+1},\ \ldots\ \pi_s,\ \left(\frac{x}{x_o}\right)_{s+1}\ \ldots\ \left(\frac{z}{z_o}\right)_p\ ] = 0\ . \qquad (14)$$

The third law of similarity is: *the multitude of the phenomena which correspond with an equation's system and known singularity conditions, are into a similarity phenomena group only if the dimensionless numbers which result from the equation's systems and singularity conditions have unique values (M.V. Kirpicev and A. A. Guhman, 1933).*

The third law can be enunicate with less rigorism as follows: *Are similarity phenomena, those for which the singularity conditions are same and the dimensionless numbers have some unique values.*

Two similarity phenomena A and B may be written as follows:

$$F\left[\frac{b_{k+1}^{A}}{\prod_{i=1}^{k}(a_i^{A})^{m_i}},\ \ldots\ \frac{b_n^{A}}{\prod_{i=1}^{k}(a_i^{A})^{p_i}}\right] = F\ (\pi_1^{A},\ \ldots\ \pi_{n-k}^{A}) = 0\ , \qquad (15)$$

$$F\left[\frac{b_{k+1}^{B}}{\prod_{i-1}^{k}(a_i^{B})^{m_i}},\ \ldots\ \frac{b_n^{B}}{\prod_{i=1}^{k}(a_i^{B})^{p_i}}\right] = F\ (\pi_1^{B},\ \ldots\ \pi_{n-k}^{B}) = 0\ , \qquad (16)$$

and from the first similarity law having $\pi_j^{A} = \pi_j^{B}$:

$$\frac{b_{k+j}^{A}}{b_{k+j}^{B}} = \frac{\prod_{i=1}^{k}(a_i^{A})^{r_i}}{\prod_{i=1}^{k}(a_i^{B})^{r_i}} = \prod_{i=1}^{k}\left(\frac{a_i^{A}}{a_i^{B}}\right)^{r_i} = \prod_{i=1}^{k}(c_i)^{r_i} = c_{k+j} = \text{constant.} \qquad (17)$$

c_{k+j} are similarity constants with different values. From the equation (17) b_{k+j}^{A} and b_{k+j}^{B} can be univocal determined.

For the third law of similarity was not possible to have a mathematical demonstration, because the singularity conditions are not known in a general form. A demonstration was done in base of a logistic mathematics. The similarity laws permit:

1. *To generalize the results from an experience to the phenomena in entire similarity group, which is characterized by known values for dimensionless numbers.*
2. *To write the mathematical equations for the physical laws using the dimensionless numbers, less complex and more accurate.*
3. *To study a similarity group through experimental methods more simple and more accurate.*

The similarity use as method and experience, because the mathematical solution for the equations system with the singularity conditions is not always possible because the similarity solutions are approximatve we restrict voluntary the area of viability of these solutions, with the intention to simplifying the results.

The general equations system for the system in a group including the singularity conditions may be written:

$$\sum_{j=1}^{n} A_{ij} = 0 \text{ ,} \tag{18}$$

with i = 1,2 . . . n, and:

$$A_{ij} = M_{ij} \prod_{r=1}^{p} a_r^{\,m_{ijr}}, \quad A_{ik} = M_{ik} \prod_{r=1}^{p} a_r^{\,m_{ikr}} \text{ ,} \tag{19}$$

where M is a numeric value and a_1 (r = 1,2 . . . p) are the primary quantities.

The terms (19) are equidimensional and they are independent to the changes of measurement units. Because $a'_r = c_i a_r$, it results the compatibility condition:

$$\prod_{r=1}^{p} c_r^{m_{ijr}} = \prod_{r=1}^{p} c_r^{m_{ikr}} \quad , \tag{20}$$

and with $c_r = a_r : a'_r$ the dimensionless number is:

$$\pi_1 = \frac{\prod_{r=1}^{p} a_r^{m_{ijr}}}{\prod_{r=1}^{p} a_r^{m_{ikr}}} = \prod_{r=1}^{p} a_r^{m_{ijr} - m_{ikr}} = \text{idem} . \tag{21}$$

Having the dimensionless numbers which they are not all independent, the solution of (18) in a finite form, on a second law of similarity base is:

$$\phi\, [\pi_1, \ldots \pi_{n-k}, \pi_{n-k+1}, \ldots \pi_s , \left(\frac{x}{x_o}\right)_{s+1}, \ldots \left(\frac{z}{z_o}\right)_p] = 0 , \tag{22}$$

and the dimensionless number which includes the value which is intended to be known:

$$\pi_j = e\, [\pi_1, \ldots \pi_{j-1}, \pi_{j+1}, \ldots \pi_s, \left(\frac{x}{x_o}\right)_{s+1}, \ldots \left(\frac{z}{z_o}\right)_p] . \tag{23}$$

This equation (23) which usually is the solution of a complex equation system with partial derivative, may be written through the variable separation method:

$$\pi_j = c_j\, \pi_1^{m_1} \ldots \pi_{j-1}^{m_{j-1}} \cdot \pi_{j+1}^{m_{j+1}} \ldots \pi_s^{m_s} \cdot \left(\frac{x}{x_o}\right)_{s+1}^{m_{s+1}} \ldots$$

$$\left(\frac{z}{z_o}\right)_p^{m_p} . \tag{24}$$

Restricting voluntarily the area of valability (applicability) we may use only a term replacing Eq. 24 but with the adequate coefficient and exponents:

$$\pi_j = c\, \pi_1^{m_1} \cdots \pi_{j-1}^{m_{j-1}} \cdots \pi_{j+1}^{m_{j+1}} \cdots \pi_s^{m_s} \cdot \left(\frac{x}{x_o}\right)_{s+1}^{m_{s+1}} \cdot\cdot \left(\frac{z}{z_o}\right)_p^{m_p} \quad . \quad (25)$$

So in a restricted area of the surface represented in u dimensional space by Eq. 24, we superpose a simplified surface Eq. 25. However, the area of valability is restricted only in researched area. In Eq. 25, we establish theoretically the components of the equation, and, if possible, some of the exponents. Next, we determine experimentally the constant c and the other exponents.

If we use all dimensionless numbers in Eq. 25, we introduce *complete similarity*. In Eq. 25 some dimensionless numbers are important (if the exponents m_j are far from zero); some are not as important (if the exponents m_i are very close to zero). If we use only the determinative dimensionless numbers in the solution, we introduce *incomplete similarity,* which is more restricted than the complete similarity, easier to apply. The use of incomplete similarity should be made with care. The determinative dimensionless numbers being done by the careful made experiences.

In general, some dimensionless numbers are not determinative in a restricted area of validity. For these, we may use two more similarity of equations, each having restricted area of validity in the experimental area.

Example 3 – Having the simplest solution (25) in form $\pi_1 = c\,\pi_2^m$, we will determine the constant, c, and the exponent m.

Solution 3: On simulation theory base from the similarity group of phenomena will select one which can be easier investigated in laboratory, and for this phenomenon we will establish simulaneous couples of values (π_{1i}, π_{2i}).

Using logarithmic values for these couples ($\ln\pi_{1i}$, $\ln\pi_{2i}$) a series of points can be plotted as shown in Fig. 1.

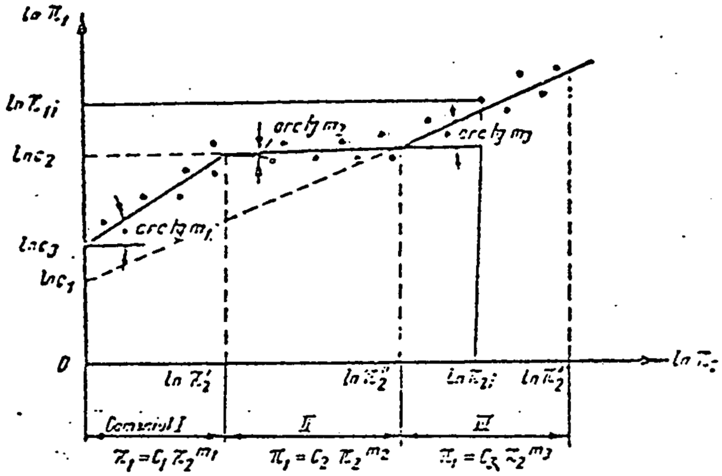

Fig. I

Through these points should be drawn a straight line:

$$\ln \pi_1 = \ln c + m \ln \pi_2 .$$

This is not possible, therefore, we should restrict the area of viability as shown in the figure (1 = I, II, and III) having three formulas for each with c_i the ordinate at the origin and m_i the lines' slope. Each straight line is drawn under the Gauss error conditions. Using Gauss equations for deviations in the ordinates and abscissas, we can determine two regression lines and select the bisection for the solution. *The result is an equation in which the constant and the exponents are known experimentally but do not have viability outside the area in which the experiment was done. No experimental formula can be used unless it is completed within the area of viability in which the experiment was performed.*

Example 4 - Using the differential equations for heat conduction, we will determine the solution for transient flow without internal sources, established through dimensional analysis in Example 2.

Solution 4: The general equations for heat conduction are: Fourier equation for the temperature field, without internal heat sources:

$$\frac{\partial t}{\partial \tau} = a \text{ div grad } t,$$

Singularity condition at time origin:

$$t\ (\overline{r}, \tau = 0) = t_o\ (\overline{r}).$$

Singularity condition of heat exchange in surface:

$$-k \text{ grad } t = f\ (t_s - t_f)\ \overline{n}^\circ$$

in which t is the temperature field, τ the time; a the thermal diffusivity; $\overline{r}$ the position vector in space; k the thermal conductivity; f the coefficient of heat transfer through surface S; t_s the surface temperature; t_f the fluid average temperature; and $\overline{n}^\circ$ the unit vector for the normal of the surface, S.

With these, from first similarity law, changing the dimension units:

$$t' = c_t \cdot t \ ,$$

$$t'_o = c_{to}\ t_o \ ,$$

$$\tau' = c_\tau \cdot \tau \ ,$$

$$a' = c_a \cdot a \ ,$$

$$l' = c_l \cdot l \ ,$$

$$k' = c_k \cdot k \ ,$$

$$h' = c_h \cdot h \ .$$

it results:

$$\frac{\partial t'}{\partial \tau'} = \frac{c_t}{c_\tau} \frac{\partial t}{\partial \tau} = a' \text{ div grad } t' = c_a \, a \frac{c_t}{c_1 2} \text{ div grad } t,$$

$$t' (\overline{r}, \tau = 0) = c_t : t (\overline{r}, \tau = 0) = t'_o(\overline{r}) = c_{to} \; t_o(\overline{r}) \; ,$$

$$- k' \text{ grad } t' = -c_k \, . \, k \frac{c_t}{c_l} \text{ grad } t = f' \; (t'_s - t'_f) \; \overline{n}^\circ =$$

$$c_f \, . \, f \, c_t \; (t_s - t_f) \, . \, \overline{n}^\circ$$

Because the last system represents a phenomenon in the same group of similarity as the first one:

$$\frac{c_t}{c_\tau} = c_a \frac{c_t}{c_l^2} \; , \; c_t = c_{to} \; , \; c_k \frac{c_t}{c_l} = c_f . \; c_t \; ,$$

and with $c_j = a_j : a'_j$ the following dimensionless numbers result:

$$\frac{a\tau}{l^2} = \frac{a'\tau'}{l'^2} = \text{Fo (Fourier)}, \; \frac{t}{t_o} = \frac{t'}{t'_o} \; , \; \frac{fl}{k} = \frac{f'l'}{k'} = \text{Bi (Biot)}.$$

Using the second similarity law, the solution is:

$$\phi \; (\text{Fo, Bi}, \frac{t}{t_o}) = 0$$

or:

$$t = t_o \, \phi \; (\text{Fo, Bi}) \; ,$$

as we found in Example 2. This equation, with a restricted viability area may be written:

$$t = t_o \; c \; \text{Fo}^m \; \text{Bi}^n$$

$$(\text{Fo.Bi})_1 < (\text{Fo.Bi}) < (\text{Fo.Bi})_2.$$

Physical analogy represents the next step after the similarity, and its application is the same as the last one, but within larger analogy class. An analogy group is a group of physical phenomena from different physical fields, and is content inside an analogy class for which the definition has been established.

For example, analogy study starts from the common equations system which represents phenomena laws and common singularity conditions which separate the group from the class. Because we used mathematical equations and not direct physical phenomena in past demonstrations, the analogy has the same laws as the similarity:

- First law of the analogy: *For the analogy phenomena the same dimensionless numbers that correspond to various physical fields have the same values.*
- Second analogy law: *The general solution for an analogy group may be written with the dimensionless numbers determinated from the basic equations. The solutions that correspond to the singularity conditions may be written with the same dimensionless numbers and dimensionless ratios determined by the singularity conditions.*
- Third analogy law: *Analogy phenomena are those for which the singularity conditions correspond and which have the dimensionless numbers with the same values.*

The experiment for analogy differs from that for similarity because the experimental phenomenon (model) which can be investigated easier in the laboratory is from a different physical field than the original studied.

Example 4 – Determine the coefficient of mass diffusion on the basis of analogy convection-diffusion, if the coefficient of heat transfer through the surface is known from the equation:

$$Nu = \frac{f\ l}{k} = c'\ Re^m\ P_r^{\ n} \quad ,$$

with c, m, and n determined from a viability area $Re_1 < Re < Re_2$.

Solution 4 – The equations that represent the analogy class for convection and diffusion are:

1. the thermal and mass diffusion, with the constants in uniform fields, and without internal sources:

$$\frac{\partial A}{\partial \tau} + \overline{w}\ \text{grad}\ A = B\ \text{div grad}\ A \ ;$$

2. the continuity equation for the incomprissible fluids:

$$\text{div}\ \overline{w} = 0 \ ,$$

3. the Navier-Stokes equation for viscous fluid flow:

$$\frac{\partial \overline{w}}{\partial \tau} + \overline{w}\ \text{div}\ \overline{w} = \overline{g}\ (1 - \sigma \Delta t) - \frac{1}{\rho}\ \text{grad}\ p + \nu \nabla^2\ \overline{w} \ ,$$

4. the singularity condition for the repartition in field at origin time:

$$A\ (\overline{r},\ \tau = o\ ) = A_o\ (\overline{r}),$$

5. the singularity condition for the heat and mass transfer in surface, S:

$$C\ \text{grad}\ A = D\ (A_s - A_f)\ \overline{n}^{\circ},$$

where:

τ = time ,

$\overline{w}$ = the velocity field ,

$\overline{g}$ = the gravity field,

σ = volume dilatation coefficient,

t = temperature field,

ρ = density field,

p = pression field,

ν = cinematic viscosity field,

$\bar{r}$ = position vector in space,

$\bar{n}^{\circ}$ = the unit vector for the normal separation surface, S.

The convection class has the characteristic quantities:

$A = t,\ B = a,\ C = k,\ D = f,$

with:

t = temperature field ,

a = thermal diffusivity field ,

k = thermal conductivity field ,

f = the coefficient of heat transfer in the surface, S.

The diffusion class has the characteristic quantities:

$$A = c\ \gamma,\ B = C = k_d,\ D = \frac{\beta}{\gamma}\ ,$$

with:

c = concentrations field,

γ = specific weight field,

k_d = diffusion coefficients field ,

β = the coefficient of mass-diffusion in the surface, S.

From these, the solution for the anology group is:

$$Nu_j = c\ Re^m\ Pr_j^{\ n}\ .$$

with:

$$Nu_j = \frac{D\ l}{c}\ ,\ Re = \frac{w\ l}{\nu}\ ,\ Pr_j = \frac{\nu}{\beta}\ .$$

The mass diffusion coefficient, β, in the viability area $Re_1 < Re < Re_2$:

$$Nu_D = \frac{\beta\ l}{\gamma\ k_d} = c'\ Re^m\ Pr_D^{\ n} = c'\ (\frac{w\ l}{\nu})^m\ (\frac{\nu}{k_d})^n\ ,$$

the convection equation is:

$$Nu = \frac{f\ l}{k} = c'\ Re^m\ Pr^n = c'\ (\frac{w\ l}{\nu})^m\ (\frac{\nu}{a})^n\ ,$$

and

$$\beta = \gamma f\ \frac{k_d}{k}\ (\frac{a}{k_d})^n = \gamma f\ \frac{a}{k}\ Le^{1-n}$$

with $Le = k_d$: a Lewis dimensionless number.

Cybernetics represents the next step from analogy, when into the cybernetic class phenomena from different sciences -- physics, biology, psychology, politics, etc., are introduced.

The same laws of similarity and analogy govern the cybernetics and separate the groups from the class. Use of cybernetic models for the phenomena from completely different fields of science should be preceeded by writing the general laws, with unique mathamatical equation forms in science. The general laws in science can be written starting with the Second Law of Thermodynamics, by introducing that as an enthropy variation law (Albert Einstein started research to write general laws in science).

The simulation theory, a result of the similarity and analogy applications, is based on the fact that experiments may use any phenomenon from the similarity or analogy group, under conditions of the group established by similarity or analogy laws.

The phenomenon investigated is the *model*, and the phenomenon for which we want to transfer the results of the experiment is the *original*. The selection of the model should be made by using a *scale*.

We establish the scale such that the model will be within possible measurements. The scale is selected for one of the quantities -- usually for lengths, but this is not obligatory. Other unit quantities will be a result of the constant value for dimensionless numbers and ratios established by similarity and analogy laws.

The scale should be selected such that the average values for the model and original will not differ greatly. Moreover, the secondary effects (usually in discontinuity areas) which change the phenomenon aspect when some quantities take critical values, should be the same for the model and original.

A complete similarity or analogy cannot be used in the experimental work because it makes the work very difficult and often impossible. In the experimental work, some dimensionless numbers are incompatible for a phenomenon; these cannot be used simultaneously in an experiment. This is not contradictory with similarity or analogy theory; it has happened because for the model some quantities cannot be varied in such large limits to ensure unique values for the dimensionless numbers.

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