# OBSERVED ASYMPTOTIC PROPERTIES OF LOW-DEGREE SOLAR GRAVITY-MODE EIGENFREQUENCIES

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## ABSTRACT

The asymptotic properties of the low-degree solar gravity modes classified by Hill and Gu (Ref. 1) are studied in the framework of first- and second-order asymptotic theory predictions (Ref. 2). The results of this analysis demonstrate the necessity of retaining the second-order term in asymptotic theory to describe the eigenfrequency spectrum. In this theory, there are two first-order parameters, $T_0$ and $\delta$, and two second-order parameters, $V_1$ and $V_2$. Values of the parameters obtained in this analysis are: $T_0 = 36.31 \pm 0.12$ min, $\delta = -0.43 \pm 0.13$, $V_1 = 0.35$, and $V_2 = 4.76$. There remain differences of $\approx 0.3\ \mu$Hz between the asymptotic theory eigenfrequencies and observed eigenfrequencies which are quasi-periodic functions of the radial order n for a given value of the degree $\ell$.

Keywords: gravity modes, asymptotic parameters

## 1. INTRODUCTION

The theoretical predictions of asymptotic theory (Ref. 2) have been tested both with eigenfrequency spectra of standard solar models (Refs. 3,4) and an eigenfrequency spectra based on observations (Refs. 5-7). The period $T_{n\ell}$ of a mode of radial order n and degree $\ell$ (azimuthal order m = 0) in second-order asymptotic theory (Ref. 2) is given by

$$T_{n\ell} = T_0\left[n + \frac{\ell}{2} + \delta\right] \frac{1}{[\ell(\ell+1)]^{1/2}} + T_0^2\,[\ell(\ell+1)\,V_1 + V_2]\,/\,[T_{n\ell}\,\ell(\ell+1)] \quad , \tag{1}$$

where $T_0$, $\delta$, $V_1$ and $V_2$ are constants. The term $T_0$ is related to the Brunt-Väisälä frequency N by the equation

$$T_0 = 2\pi^2 \left[\int_0^{r_c} N \frac{dr}{r}\right]^{-1} \quad , \tag{2}$$

where $r_c$ is the radius of the convection zone. The constant $\delta$ depends on the structure just below the base of the convection zone. The expressions for $V_1$ and $V_2$ are much more complicated than that for $T_0$ and reference is made to Tassoul (Ref. 2) for these details. In first-order asymptotic theory, $V_1 = V_2 = 0$.

The extent to which the asymptotic theory predictions are realized depends upon the degree to which the properties of the solar interior meet the conditions required for the applicability of the theory. As Christensen-Dalsgaard et al. (Ref. 8) state, "The asymptotic analysis is valid if the scale of variation of the background state is much greater than all oscillation wavelengths considered, and that condition is indeed satisfied throughout most of the interior of any traditional theoretical model of the Sun." As a corollary, any observed systematic deviations from asymptotic theory predictions may be the manifestation of conditions in the solar interior which do not meet the requirements for asymptotic theory to be applicable. Therefore, the study of the asymptotic properties of the observed eigenfrequency spectrum takes on a particular importance: in addition to possibly obtaining constraints on $T_0$, $\delta$, $V_1$ and $V_2$; it may offer a diagnostic probe to those regions of the solar interior where a significant fractional change occurs in the background state in a characteristic length of the eigenfunctions used in the study. This is suggestive of the null experiments encountered in physics and enjoys the same attractive feature. That feature in this case is that the deviations from the predictions of asymptotic theory allow us to study directly those presumably restricted regions where the conditions for the applicability of asymptotic theory are not met.

In the following sections we examine $(\nu_{n\ell})_{ob} - (\nu_{n\ell})_{th}$ for the $\ell = 2, \ldots, 5$ gravity modes obtained in the mode classifications of Hill and Gu (Ref. 1) for asymptotic properties in the framework of first- and second-order asymptotic theory predictions given by Eq. (1).

## 2. LOW-DEGREE GRAVITY-MODE CLASSIFICATIONS

A series of low-degree gravity-mode multiplets with m = 0 eigenfrequencies between 60 and 220 $\mu$Hz have been classified by Hill and Gu (Ref. 1). The work on gravity-mode classifications is part of an extensive mode classification program at SCLERA (Ref. 9) that was started in 1982. This program has been described in numerous works (Refs. 10-14) and is based on the physical properties of the eigenfunctions, on the use of combinations of observations that possess different spatial filter functions, and on the theoretical eigenfrequency spectrum of a standard solar model for determining the radial order n.

The gravity-mode spectrum, in general, is much more complex than the low-order, low-degree, acoustic-mode spectrum. The complexity of the spectrum was first reduced to a manageable level in the work of Hill (Ref. 12) by the successful combination of differential velocity observations from the Crimean Astrophysical Observatory (Ref. 15) and of the 1979 differential-radius observations made at SCLERA (Ref. 16). Because of the different spatial filter functions for these two sets of observations, it was possible to decouple $\ell$ and m in the mode classification program. The result was the classification of 31 gravity-mode multiplets with $\ell = 1, \ldots, 5$. In a subsequent analysis, Gu and Hill (Ref. 17) and Hill and Gu (Ref. 1) extended the Hill (Ref. 12) work using the mode classification techniques developed at SCLERA. The result was 53 classified gravity-mode multiplets. The 53 multiplets included the original 31 classified by Hill (Ref. 12) with several reclassifications.

## 3. TESTS OF GRAVITY-MODE CLASSIFICATIONS

There have been a series of independent tests made of the Hill (Ref. 12), Gu and Hill (Ref. 17), and Hill and Gu (Ref. 1) gravity-mode classifications. The first series by Hill and Kroll (Ref. 18); Kroll, Huang, and Hill (Ref. 19); and Kroll, Hill, and Chen (Ref. 20) is based on an analysis of the total irradiance observations obtained from the Solar Maximum Mission. The second series is based on the power spectrum of the 1985 radiation intensity observations made by Oglesby (Refs. 21,22). The third series by Hill (Ref. 23) is based on an analysis of the 1978 solar diameter observations made by Caudell et al. (Ref. 24). These series of tests fall into several categories. Some test for detection of gravity modes, some test for identification of multiplets, and others test $\ell$ and m assignments that have been made. Highly statistically significant positive results have been obtained in each of the series of tests. It is the strength of these independent tests that motivate the determination of the asymptotic properties of the classified spectrum by Hill and Gu (Ref. 1).

## 4. RESULTS

The values of $\nu_{n\ell}$ for the gravity modes classified by Hill and Gu (Ref. 1) were first examined in the framework of first-order asymptotic theory. In this case, the theoretical $\nu_{n\ell}$ are given by Eq. (1) with $V_1 = V_2 = 0$. Results similar to those reported by Rosenwald and Hill (Ref. 6) were found. In the work of Rosenwald and Hill (Ref. 6), the values obtained for $T_0$ were 31.3, 33.1, 35.3, 36.6, and 38.2 min for the $\ell = 1, \ldots, 5$ modes, respectively. Thus, the limited utility of first-order asymptotic theory in representing low-degree gravity mode eigenfrequencies is further corroborated.

With the further demonstration of the inadequacy of first-order asymptotic theory, the values of $\nu_{n\ell}$ for the gravity modes classified by Hill and Gu (Ref. 1) were examined with the second-order $V_1$ and $V_2$ terms retained in Eq. (1). In this analysis, the classified values $(\nu_{n\ell})_{ob}$ were fit in a least-squares analysis with the theoretical $(\nu_{n\ell})_{th}$ given by Eq. (1) for the best values of $T_0$, $\delta$, $V_1$ and $V_2$. The results of the analysis are:

$$\begin{aligned} T_0 &= 36.31 \pm 0.12 \text{ min} \\ \delta &= -0.43 \pm 0.13 \\ V_1 &= 0.35 \\ V_2 &= 4.76 \end{aligned} \qquad (3)$$

The least-squares analysis was done separately for each set of multiplets with $\ell$ = 2, ..., 5. The $\ell$ = 1, m = 0 classified eigenfrequencies were not used in this analysis because there are only a few data points available from this set of multiplets.

Using the values of the constants in Eq. (1), the proficiency of the second-order theory to describe the observed $\nu_{n\ell}$ is tested by examining the difference $(\nu_{n\ell})_{ob} - (\nu_{n\ell})_{th}$ as a function of n for each of the values of $\ell$. The results for $\ell$ = 4 are shown in Fig. 1, which clearly show deviations that are quasi-periodic functions of the radial order n. Similar results are found for $\ell$ = 2, 3 and 5. The values of $n_{max}$ and $n_{min}$ are listed in Table 1, where $n_{max}$ ($n_{min}$) denotes values of n for which $(\nu_{n\ell})_{ob} - (\nu_{n\ell})_{th}$ obtains a local maximum (minimum). A typical amplitude for these deviations is 0.3 μHz.

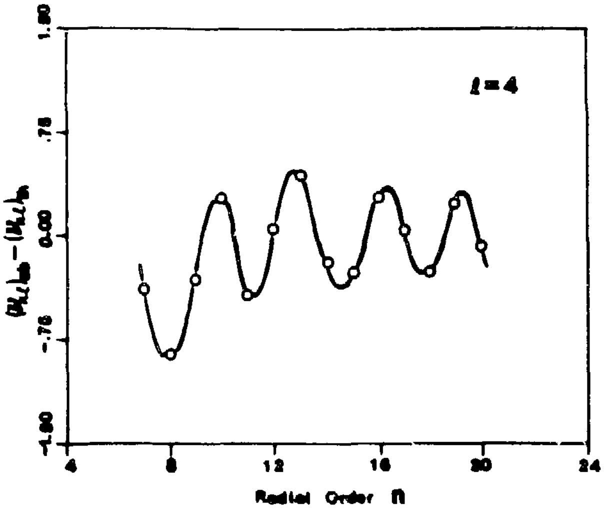


Figure 1. The observed deviation $\Delta\nu_{n\ell}$ from second-order asymptotic theory predictions as a function of n for $\ell$ = 4 modes. The $\Delta\nu_{n\ell} = (\nu_{n\ell})_{ob} - (\nu_{n\ell})_{th}$, where $(\nu_{n\ell})_{ob}$ are the classified values of $\nu_{n\ell}$ by Hill and Gu (Ref. 1) and the $(\nu_{n\ell})_{th}$ are the values of $\nu_{n\ell}$ given by the second-order asymptotic theory that has been adjusted in a least-squares analysis to fit the $(\nu_{n\ell})_{ob}$. The quasi-periodic behavior of the fine structure represented by the $\Delta\nu_{n\ell}$ is noted.

The structure in Fig. 1 and the similar structure found for the other values of $\ell$ give some hint of an interrelationship of the type found by Hill and Rosenwald (Ref. 5) for the 5-min oscillations. It is also found empirically that the radial orders $n_{max}$ and $n_{min}$ are ordered by $\ell$.

## 5. SUMMARY

It is found that the g-mode spectrum classified by Hill and Gu (Ref. 1) can be represented quite well by second-order asymptotic theory accompanied by a quasi-periodic term. The quasi-periodic term has an amplitude of approximately 0.3 μHz.

TABLE I

Values of n for maxima and minima in $(\nu_{n\ell})_{ob} - (\nu_{n\ell})_{th}$ for each given $\ell$

| $\ell$ | | | $n_{max}$ | | | |
|---|---|---|---|---|---|---|
| 2 | 9.1 | 13.5 | | | | |
| 3 | 9.2 | 13.3 | 17.0 | | | |
| 4 | 10.0 | 12.9 | 16.1 | 19.0 | | |
| 5 | | | 17.2 | 20.0 | 23.1 | |
| $\ell$ | | | $n_{min}$ | | | |
| 2 | 7.2 | 11.4 | 15.6 | | | |
| 3 | 7.4 | 11.3 | 15.3 | 18.5 | | |
| 4 | 8.0 | 11.2 | 14.6 | 17.8 | | |
| 5 | | | | 18.6 | 21.7 | 24.8 |

The set of second-order asymptotic theory parameters obtained in the fit to the classified spectrum can give us important information on the internal structure of the Sun and, in particular, can be used to reduce the number of viable solar models. The value of $T_0$ can be computed for different solar models. On comparing these theoretical predictions with the observational-based value of $T_0$ = 36.31 min given by Eq. (3), it is found that the present result is in general agreement with standard solar models, but inconsistent with WIMP models and mixed models (see Ref. 25 and references therein). The later two models were suggested as possible solutions of the solar neutrino paradox. The value of $\delta$ is predicted to have a value between $-(^1/_4)$ and $-(^5/_{12})$, depending on the structure of the temperature gradient at the base of the convection zone. The observed value of $\delta$ = -0.425 is close to $-(^5/_{12})$, the value obtained by standard solar models computed with local mixing length theory (Ref. 26). This limiting value is predicted if the subadiabatic temperature gradient falls linearly to zero as the lower boundary of the convection zone is approached from below and being essentially zero in the convection zone (Ref. 26).

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