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INVERSION OF QUASI-PERIODIC DEVIATIONS BETWEEN LOW-DEGREE SOLAR GRAVITY MODE EIGENFREQUENCIES AND ASYMPTOTIC THEORY EIGENFREQUENCIES

Henry A. Hill, Qiang Gao and Ross D. Rosenwald

Department of Physics
University of Arizona
Tucson, Arizona 85721

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INVERSION OF QUASI-PERIODIC DEVIATIONS BETWEEN LOW-DEGREE SOLAR GRAVITY MODE EIGENFREQUENCIES AND ASYMPTOTIC THEORY EIGENFREQUENCIES

Henry A. Hill, Qiang Gao and Ross D. Rosenwald

Department of Physics, University of Arizona, Tucson, Arizona 85721

ABSTRACT

The fine structure found by Gu, Hill and Rosenwald (Ref. 1) between asymptotic theory eigenfrequencies and the observed eigenfrequencies reported by Hill and Gu (Ref. 2) is interpreted as the result of conditions not being met for the applicability of asymptotic theory at one or more radii in the solar interior. From an inversion of the observed fine structure, reasonably good agreement is obtained between observation and theory for either a localized perturbation in internal structure at $r/R \approx 0.06$ or at $r/R \approx 0.23$. The latter solution is, however, the better one. The amplitude of the perturbation in the mean molecular weight required to produce the fine structure is also inferred.

Keywords: Gravity-mode, Internal structure, Inversion

1. INTRODUCTION

A fine structure may be present in an eigenfrequency spectrum for a number of different reasons. The more widely known example from solar seismology is the fine structure generated by rotation of the Sun. For a given set of normal modes, a fine structure may also be caused by localized changes in the background state of the Sun when those changes occur over a distance shorter than the local radial wavelengths of the respective eigenfunctions. Examples of this are found in the works of Hill and Rosenwald (Ref. 3), Kidman and Cox (Ref. 4), Gavryusev and Gavryuseva (Ref. 5), and Berthomieu and Provost (Ref. 6). Analogous effects have received much attention in the study of the normal modes of the Earth. For example, Lapwood and Usami (Ref. 7) have called this fine structure effect the "solotone" effect.

The first example of such structurally induced fine structure on solar eigenfrequencies was obtained by Hill and Rosenwald (Ref. 3) for low-degree five min oscillations. This fine structure has been identified with structural changes that occur over a relatively short distance at the base of the convection zone. A second example of observed fine structure which is presumably structurally induced has been found in the study of low-degree solar gravity modes by Rosenwald, Hill and Gu (Ref. 8) and Gu, Hill and Rosenwald (Ref. 1). This fine structure was detected as quasi-periodic deviations of the observed eigenfrequency spectrum from asymptotic theory predictions.

It was observed in (Refs. 1,8) that a perturbation in the Brunt-Väisälä frequency localized at 0.2 solar radii would give rise to the type of fine structure observed. An inversion of the observed fine structure is made in the following sections to better determine the properties of the localized structural changes that could account for the fine structure.

2. OBSERVED FINE STRUCTURE

A series of low-degree gravity mode multiplets have been classified in the SCLERA (Ref. 9) mode classification program (Ref. 2). The $m = 0$ eigenfrequencies $\nu_{n\ell}$ of the above modes lie between 60 and 220μHz with degrees $\ell = 1, \ldots, 5$. Subscript n is the radial order and m is the azimuthal order. A total of 53 multiplets are included in this set. The eigenfrequencies of these multiplets were observed to have a fine structure in addition to that due to rotation. A quantitative study of the newly discovered fine structure has been made by Gu, Hill and Rosenwald (Ref. 1) by comparing the observed $\nu_{n\ell}$ with those predicted by asymptotic theory.

The observed fine structure effects, which are of interest here, are characterized as quasi-periodic deviations of the classified spectrum from the spectrum predicted by second order asymptotic theory. The deviations are periodic in radial order n for a fixed ℓ and not dependent on m. For more specifics concerning these deviations, reference is made to Gu, Hill and Rosenwald (Ref. 1). Both the amplitudes and the periods of the deviations will be used in the inversion project.

3. INVERSION TECHNIQUE AND RESULTS

A particularly simple inversion technique may be employed to infer the internal properties of the Sun which give rise to quasi-periodic deviations of $\nu_{n\ell}$ from the values predicted by asymptotic theory. As noted in the Introduction, quasi-periodic deviations from asymptotic theory are expected for a given set of normal modes when a significant fractional change in the background state of the Sun occurs within a characteristic length shorter than the distance characterizing changes in the respective eigenfunctions. The location in radius of a significant fractional change in the background state determines the period in n of the periodic deviation; the magnitude of the fractional change in the background state determines the amplitude of the periodic deviation. It should thus be possible, by using observational results and sensitivity analysis results for a given standard solar model, to determine both the location and magnitude of the change in the background state primarily responsible for the observed quasi-periodic deviations.

The sensitivity analysis required for the inversion project was performed using the standard solar model of Saio (Ref. 10). In this analysis, the derivatives $\Delta\nu_{n\ell} \;/\; \Delta A_i^*$ were determined as a function of n, ℓ and i, where

$\Delta\nu_{n\ell}$ is the perturbation in eigenfrequency $\nu_{n\ell}$ due to a perturbation ΔA_i^* in A^* at model zone i. The parameter A^* is related to the Brunt-Väisälä frequency N by

$$A^* = \frac{rN^2}{g} \qquad (1)$$

where g is the value of the acceleration of gravity at radius r. For the determination of the derivatives $\Delta\nu_{n\ell} / \Delta A_i^*$, the $\nu_{n\ell}$ for the $\ell = 2, \ldots, 5$ gravity mode multiplets classified by Hill and Gu (Ref. 2) were calculated both for the unperturbed Saio model and the Saio model perturbed by ΔA_i^*. In these calculations, standard inner and outer boundary conditions (Ref. 11) were used. It is noted that the Saio (Ref. 10) standard solar model may furnish an adequate starting point for the inversion project because the $\nu_{n\ell}$ of this model are in reasonably good agreement with the classified eigenfrequencies of Hill and Gu (Ref. 2).

Two independent parameters appearing in the differential equations which uniquely describe linear adiabatic oscillations of the Sun are A^* and V_g, where

$$V_g = \frac{gr}{c^2} \qquad (2)$$

and c is the speed of sound. It is only necessary in the present analysis, however, to consider perturbations in A^*. This is because, first, in the solar interior below the convection zone, the magnitude of the terms $\Delta\nu_{n\ell}$ / $(\Delta V_{g,i} / V_g)$ are typically smaller than $\Delta\nu_{n\ell}$ / $(\Delta A_i^* / A^*)$ by a factor of 100. Second, A^* can exhibit rather large peaks due to the relatively high sensitivity of N^2 to gradients in the mean molecular weight μ.

The $\Delta\nu_{n\ell}$ / ΔA_i^* are found for a given ℓ and i to be a quasi-periodic function of n. The results for $\ell = 4$ are shown in Figure 1 for $\Delta A_i^*/A_i^* = 0.06$ in a zone with a mean fractional radius of $x = 0.249$ and a zone width $\Delta x = 0.0089$, where $x = r/R$ and R is the solar radius. Similar results are obtained for $\ell = 2$, 3 and 5. The location of the minima $n_{min}(\ell, x_i)$ in the set of figures are determined as functions of ℓ and x_i. For each value of ℓ, a diagnostic diagram is generated by plotting the fractional zone radii x_i as a function of the location of the minima $n_{min}(\ell, x_i)$. The diagnostic diagram obtained for $\ell = 4$ is shown in Figure 2. The minima are used in this analysis because a positive perturbation in A^* gives rise to a negative value of $\Delta\nu_{n\ell}$ (see Figure 1).

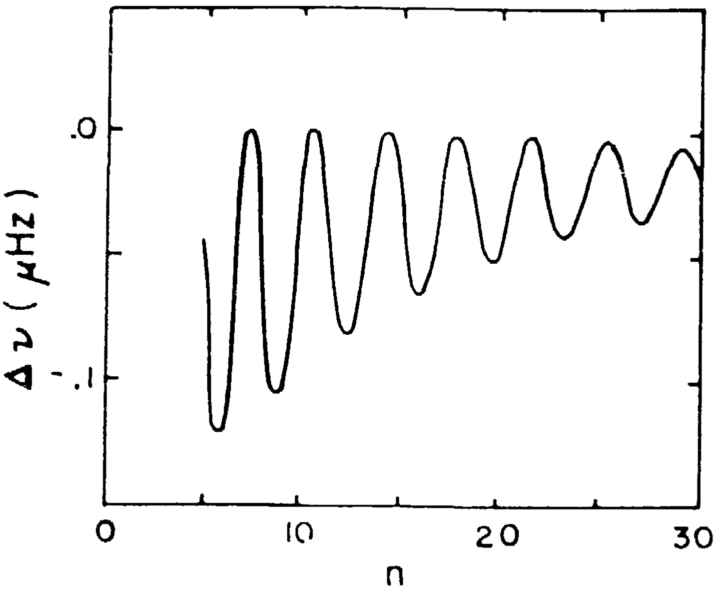

Fig. 1. The derivatives $\Delta\nu_{n\ell}/\Delta A_i^*$ displayed as a function of n for $\ell=4$ and $x_i = 0.249$. The derivatives were computed with $\Delta A_i^* = 0.06\ A_i^*$ and $\Delta x_i = 0.0089$.

The second phase of the inversion technique consisted of placing vertical lines on the diagnostic diagrams at the locations of the observed minima n_{min} and testing for a solution. The vertical lines appropriate for the $\ell = 4$ mode are also shown in Figure 2.

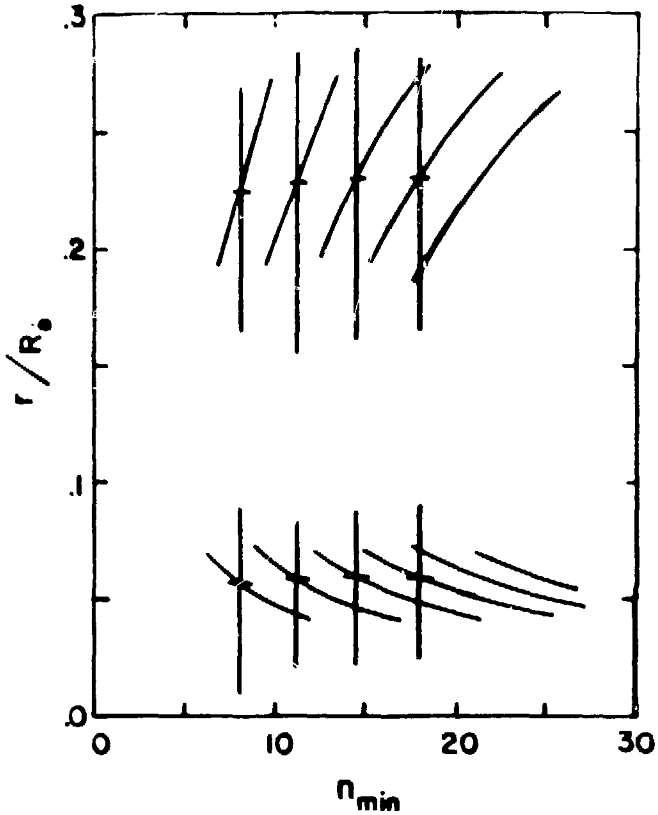

Fig. 2. The family of curves which gives the locations of the minima n_{min} for ℓ=4 multiplets in the derivatives $\Delta\nu_{n\ell}/\Delta A_i^*$ as a function of the x location of ΔA_i^*.

A solution is considered to exist if there is a set of intersections of the two families of lines in the diagnostic diagrams that lie on a horizontal line. It is apparent from inspection of Figure 2 that for ℓ = 4, there are solutions for x ≈ 0.23 and for x ≈ 0.06. Results consistent with the ℓ = 4 solutions at x ≈ 0.23 and x ≈ 0.06 are also found for ℓ = 2, 3 and 5 modes. However, the relative scatter of the locations of the intersections in the diagnostic diagrams about the $x \approx 0.06$ solution is about a factor of 5 larger than the relative scatter about the other possible solution at $x \approx 0.23$. Also, the better $x \approx 0.23$ solution is obtained when comparing the observed n_{min} with the model determined n_{min}, not the observed n_{max} with the model determined n_{min}. It is thus concluded that the primary source of the observed periodic deviations is a positive perturbation in A^* located at a radius near $x = 0.23$. The weighted average $\bar{x}$ of the solutions for $\ell = 2, 3, 4$ and 5 yields the result

$$\bar{x} = 0.235 \pm .005 \quad . \tag{3}$$

The error in Equation (3) is based on the standard deviation of the x coordinates of the intersections found in the diagnostic diagrams for the $x \approx 0.23$ solution and is a measure of the internal consistency of the findings. Certainly, the value of the absolute error depends on the correctness of the Saio (Ref. 10) solar model and no estimate of this type of error has been made.

There is evidence in the observations of a secondary perturbation. This is inferred from the lack of a significant n dependent amplitude in the observed periodic deviations, while the model generated amplitudes are approximately proportional to $1/n$ (see Figure 1). The lack of a significant n dependent amplitude could be the result of interference with a second periodic deviation due to a ΔA_i^* located at a fractional radius of

$$\bar{x} \approx 0.19 \quad . \qquad (4)$$

The strength of this secondary perturbation in A^* would be about 1/3 of the primary perturbation in A^* located at $x \approx 0.23$. It is assumed in the following sections that the sign of the secondary perturbation in A^* is also positive. However, no information about the sign has been inferred from the observations.

4. MAGNITUDE OF $\int (\Delta A^*/A^*)\,dr$

The magnitude of $\int \Delta A^*/A^* dr$ can be estimated from the observed amplitudes of the periodic deviations and the computed derivatives $\Delta\nu_{n\ell} / \Delta A_i^*$. The results are

$$\int \frac{\Delta A^*}{A^*}\,dr = \begin{cases} 0.0045R; & \bar{x} = 0.235 \\ 0.0015R; & \bar{x} = 0.19 \end{cases} \quad . \qquad (5)$$

It is assumed in obtaining the results expressed in Equation (5) that the effective radial widths of ΔA^* are less than approximately one quarter of the radial wavelength of the eigenfunctions used in the observations. This corresponds to a $\Delta x \lessapprox 0.015$.

5. INFERRED RADIAL PROPERTIES OF μ

One physical parameter that could be responsible for the localized perturbations in A^* is the mean molecular weight μ. For a sharp transition of μ from one value to a second value, the change $\Delta\mu = \mu(r_2) - \mu(r_1)$ can be expressed to a good approximation in terms of ΔA^* as

$$\frac{\Delta\mu}{\mu} = - \frac{Rc_1}{g} \cdot \frac{N^2}{r} \int_{r_1}^{r_2} \frac{\Delta A^* dr}{A^*} \qquad (6)$$

where

$$\frac{1}{c_1} = \frac{M_r}{r^3} \cdot \frac{R^3}{M} , \qquad (7)$$

$r_2 > r_1$, M_r is the solar mass interior to radius r and M is the total solar mass.

Using results for $\int(\Delta A^*/A^*)\, dr$ given by Equation (5), we find

$$\frac{\Delta\mu}{\mu} = \begin{cases} -0.009 \; ; \; \bar{x} = 0.235 \\ -0.003 \; ; \; \bar{x} \approx 0.19 \end{cases} \quad . \qquad (8)$$

5. DISCUSSION

The basic features of the fine structure found in the classified low-degree gravity mode spectrum (Ref. 1) can be reproduced by a positive perturbation in A^* at $\bar{x} = 0.235$. The effective radial width of this perturbation is $\Delta x \lesssim 0.015$. This perturbation in A^* could be produced by a negative change in the mean molecular weight occurring in the effective radial width of $\Delta x \lesssim 0.015$. The magnitude of the fractional change in μ is $|\Delta\mu/\mu| = 0.009$ at $\bar{x} = 0.235$.

Does this inferred change in μ imply a mixed core and if so, is a mixed core consistent with other observational results? If the core of the Saio model (Ref. 10) were completely and instantaneously mixed out to $x = 0.235$ (but was unmixed prior to this), there would exist at $x = 0.235$ a decrease in μ with the fractional change of

$$\left(\frac{\Delta\mu}{\mu} \right)_{mix} = -0.060 \quad . \qquad (9)$$

Therefore, the inferred $\Delta\mu/\mu$ at $\bar{x} = 0.235$ given in Equation (8) is only a fraction of $(\Delta\mu/\mu)_{mix}$. Quantitatively,

$$\frac{\left(\frac{\Delta\mu}{\mu}\right)}{\left(\frac{\Delta\mu}{\mu}\right)_{mix}} = 0.15 \qquad (10)$$

It is concluded from the result in Equation (10) that the small mixing required to yield the inferred $\Delta\mu/\mu$ given by Equation (8) is sufficiently small so as not to be very important in resolving the solar neutrino paradox. The fractional change is also sufficiently small so as not to be inconsistent with the observational based value of T_0 obtained by Gu, Hill and Rosenwald (Ref. 1) and the five min oscillations which indicate that the core is not mixed.

Clearly, the inferred $\Delta\mu/\mu$ could be the result of a partial mixing of the core in the past. One interesting question that might be raised is what would be the time of the following hypothetical complete mixing event: The core of the Sun becomes completely mixed at an early age out to $x = 0.235$, then the interior becomes permanently stable against mixing while the discontinuity in μ at $x = 0.235$ remains as a "fossil". The answer to this question is an age of the order of 0.7 Gyr. Actually, the restriction on stability can be relaxed to the situation where subsequent mixing occurs at x values significantly less that 0.235. In fact, the second inferred $\Delta\mu/\mu$ at $\bar{x} \approx 0.19$ might be a location of subsequent mixing.

In light of this discussion, the observed fine structure in the eigenfrequencies may be considered "fossil" records or "footprints" in the Sun. This is a particularly intriguing possibility.

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