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TITLE: **THE ANALYSIS OF TANK CALIBRATION DATA FROM SEVERAL RUNS**

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THE ANALYSIS OF TANK CALIBRATION DATA FROM SEVERAL RUNS

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Abstract

The experimentally determined relationship between the level of liquid and volume of liquid in a tank can be used to obtain volume estimates that correspond to liquid-level measurements. Several calibration experiments (runs) are made to estimate the calibration equation. The calibration equation is used to estimate the quantity of liquid transferred from a tank between measurement periods.

Difficulties can arise when run-to-run differences are large relative to the precisions of liquid level and volume measurements. This paper addresses the seldom-discussed but important problem of combining and analyzing data from two or more calibration runs. Emphasis is placed on exploratory methods such as diagnostic plots that are compatible with applicable statistical models.

Introduction

Accurate volume measurements are an important component of many nuclear materials accounting systems. Carefully measured increments of liquid are added to a nuclear process tank and compared with corresponding measured liquid levels. This procedure is repeated several times to obtain an estimated calibration equation or its inverse. The repeated experiments are referred to as "runs."

Liquid levels in a nuclear process tank are not usually measured directly. Typically, tanks are equipped with bubbler probes, and a pressure is measured from which liquid level estimates are obtained. The techniques discussed in this paper apply to either indirect or direct liquid-level measurements. Consequently, the term liquid level is used in this discussion--regardless of how the measurements are made.

In practice, it is necessary to adjust raw data from the calibration experiment to a set of reference conditions. This adjustment, defined as "data standardization," involves corrections for thermal tank expansion and liquid density changes arising from temperature changes. The assumption is made throughout this paper that data standardization has taken place. (See Jones[1] for a discussion of data standardization.) The standardized liquid-level measurements are denoted by Y, and the standardized cumulative volume measurements are denoted by X.

Many process tanks have a "heel" that retains a quantity of liquid when the tank is emptied. Because of the heel, liquid-level measurements (Y) are associated with the cumulative volume of liquid added during the calibration (X) rather than the "actual" volumes of liquid in the tank.

This paper deals with developing a calibration equation from standardized calibration data from two or more runs. The assumption is made that run-to-run differences are "large" relative to the precision of calibration data. If this is not the case, data can be treated as if from a single run.

The methodology presented here is compatible with a variety of statistical models and is illustrated with data obtained from a large nuclear process tank at the Savannah River Plant.

Development of a Calibration Model

A suggested calibration model is derived from a set of standardized calibration data by carrying out the following four steps:

(1) select the appropriate error model,
(2) align the data from calibration runs,
(3) determine change points, and
(4) estimate parameters in the calibration model.

Error Model Selection

The calibration equation for a nuclear process tank is typically a piecewise first or second degree polynomial liquid-level/volume relationship. The choice of an error model is dependent upon assumptions. The most general model assumes both variables are measured with error and uses techniques suggested by Mandel[2] for obtaining the calibration equation. If liquid levels are measured with much less relative precision than volume measurements, the classical error model is appropriate.[3] If the less precise term is in volume measurements, then the cumulative error model is used.[4]

Data Alignment

When it is impossible to completely empty a tank and the amount remaining cannot be measured, each calibration run begins with a tank with an unknown quantity of liquid (the heel). Heel volumes may vary among runs; consequently, adjusting cumulative volumes or "data alignment" is required to make the data more "coincident" for analysis.

The first step in the analysis is to determine whether data alignment is required. A linear least squares fit to the data from each run is used to estimate the coefficients in the model $Y = a + bX$, where Y is the level and X is the volume. A profile plot for each run is made by plotting the differences, $Y - \hat{Y}$, between the observed level Y and the level $\hat{Y}$ predicted by the linear model. When plotted on the same axis, profiles from each run would be nearly coincident if no realignment were necessary. Realignment is required when the lines are nearly parallel. Potentially serious problems related to run-to-run differences exist if lines are not "roughly" parallel.

When "statistical" alignment is necessary, the ad hoc method described here has been used for tank calibrations at the Rockwell-Hanford facility. The following steps are required.

(1) Select 5-10 data points near the beginning of each calibration run where the function appears to be linear.
(2) For J runs and m data points per run, fit the model $X_{ij} = a + bY_{ij}$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, J$, to the data obtained in Step (1) and compute the residuals for each data point. The fit must be adequate in the sense that the residuals exhibit reasonable patterns.
(3) Compute the average of the residuals from the jth run, $\bar{R}_j$, $j = 1, 2, \ldots, J$.
(4) For each data point (Y_{ij}, X_{ij}) from the jth run, replace X_{ij} with $X_{ij} - \bar{R}_j$.

The resulting calibration equation can only be used to measure transfer terms of liquid level unless an estimate of heel volume is available.

Determination of Change Points

Because of irregularities in construction and internal devices such as agitators and heating coils, the cross-sectional area of a tank changes as a function of liquid level. A change point is a volume (or level) at which a sudden change occurs in the calibration function. After the change points have been determined, a piecewise (segment by segment) fit to the data is obtained.

Change points can be identified from tank blueprints that identify tank irregularities. We will augment such identifications by analyzing the alignment data to determine change points.

One method for identifying change points uses a profile plot, which is a plot of the residuals computed from the data when determining if alignment is necessary. An abrupt change in the plots indicates a point where the functional relationship between the level and the volume has changed.

Another procedure for finding change points uses the slope of the calibration data. An incremental slope plot is simply a plot of the values

$$b_i = \frac{(Y_i - Y_{i-1})}{(X_i - X_{i-1})}$$

against X_i, where the X_i and Y_i are from the same run.

Changes in values of b_i correspond to abrupt changes in the profile plot. The use of profile plots and incremental slope plots allows sharply defined change points to be identified.

The general procedure in locating change points is given as follows.

(1) Tentatively identify change points from structural information about the tank and examine profile and incremental slope plots.
(2) Fit an equation to the data of each segment determined from (1).
(3) Examine the residuals for patterns.
(4) Examine the incremental slope plots.
(5) Redefine change points as required and repeat the procedure.

See Ref. 3 for an excellent discussion on what constitutes favorable residual patterns.

Parameter Estimation

It has been conveniently assumed that the calibration function is modeled piecewise by linear and/or quadratic polynomials. The estimation of parameters in these polynomials is contingent upon the error model, which in turn depends upon the measurement components of the calibration system. We will briefly discuss some of the important estimation methods.

Classical Least Squares.[3,6] The classical least squares regression model is used to estimate parameters when the assumption is made that relative errors in liquid-level measurements are large compared with volume measurement errors.

Advantages include ease of computation and minimum variance unbiased estimates of liquid level. Disadvantages include the undefined variance of predicted volume from a liquid level measurement,

Cumulative Error Model.[4,7] The cumulative error model is used when the standard deviation in volume increments is a constant times liquid level differences and the error in liquid level measurements is negligible. Estimates of parameters are obtained by maximum likelihood.

Advantages include the simplicity of computation and defined variance of predicted volumes. Disadvantages include lack of available procedures for quadratic polynomials, the use of just the first and last points when computing the slope, and the possibility of inflation in the estimated variance.

Variables in Error Model.[7] Maximum likelihood methods are used under the assumption that both variables are in error. The main advantage of this model is that it treats the general case when both variables are in error. The major disadvantage is the computation problems involved in obtaining estimates.

Methods such as spline functions,[8] Bayesian procedures,[9] and nonparametric methods[10] are of interest. Because these methods have been applied to a much lesser extent than those given above, we will not review them at this time.

Example

Data from a 20-ft high, 20 000-L tank at Savannah River Plant are used to illustrate combining calibration runs. Table I gives level (inches) and volume (liters) data from 35 incremental additions of ~570 L each. All measurements have been standardized to 25°C.

The classical model was assumed; that is, errors in volumes were negligible, whereas errors in determining levels were significant.

A profile plot using all 140 experimental data points is given in Fig. 1. The graph uses results of a least-squares fit and consists of plotting residuals, $Y_i - \hat{Y}_i$, versus X_i. The "sawtooth" shape of the profile illustrates apparent structural changes in the tank at 6000 L (increments 10 12) and 13 000 L (increments 22-23). The profiles appear to be linear in all segments.

The profiles in Fig. 1 are coincident; thus, data alignment is not required. There do not appear to be any serious run to-run differences.

A plot of incremental slopes of each data set produces almost the same results. Run 1 is depicted in Fig. 2. The sharpest decreases in slope occur at "change points" 6000 L and 13,000 L, in agreement with the profile results.

The classical error model and least-squares regression were used to fit calibration equations to the data in three segments. Results are given in Table II and Fig. 3.

Figure 3 can be used to demonstrate how the calibration equations can be used to estimate transferred volumes of liquid. Assume that liquid-level measurements of 90" and 60" have been made. Using the equations given in Table II, we obtain the following values of volumes X_1 and X_2:

$$X_1 = \frac{90.0 - 3.348}{0.0055} = 15754.91$$

and

$$X_2 = \frac{60.0 - 3.348}{0.0055} = 10300.36 \quad .$$

The amount of transferred liquid is $X_1 - X_2 = 5454.55$.

A discussion of estimating errors of transfer volumes will be given in a future paper.

Summary

The difficult but important problem of combining data from at least two calibration runs is approached by a four-step procedure. These steps include selection of an error model, data alignment (if necessary), determination of change points, and parameter estimation. This calibration equation can then be used to determine estimates of volume transferred from the tank. Estimates of volume uncertainty will be deferred to a future paper.

The four-step procedure is exemplified using data from a Savannah River Plant nuclear processing tank.

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TABLE I

TANK CALIBRATION DATA COLLECTED FROM FOUR RUNS
AT SAVANNAH RIVER PLANT

RUN 1		RUN 2		RUN 3		RUN 4	
VOLUME (L)	LEVEL (IN.)	VOLUME (L)	LEVEL (IN.)	VOLUME (L)	LEVEL (IN.)	VOLUME (L)	LEVEL (IN.)
567.01929	1.96251	567.68903	1.91269	568.55939	2.00680	570.44928	1.97912
1141.42029	5.34501	1138.99194	5.28688	1140.80347	5.38376	1144.33130	5.36992
1712.37537	8.72474	1710.39526	8.67491	1711.94360	8.77179	1718.89978	8.78010
2276.53662	12.04634	2278.58618	12.02142	2280.51880	12.12384	2290.10718	12.15429
2844.43872	15.35686	2846.77612	15.35963	2848.78052	15.47035	2861.18311	15.50634
3411.91138	18.71168	3414.09863	18.71721	3417.21094	18.85562	3432.39795	18.88330
3980.07837	22.04989	3982.32495	22.06373	3986.14331	22.21043	4003.61279	22.25195
4548.34570	25.42685	4550.75195	25.41024	4555.27637	25.55971	4574.42627	25.61507
5116.54248	28.74845	5118.71143	28.74568	5122.43408	28.90345	5144.69922	28.95328
5683.60449	32.08112	5686.40381	32.08389	5690.37451	32.24443	5715.59033	32.33301
6251.23047	35.31414	6254.98291	35.31691	6258.23145	35.49683	6286.51367	35.58541
6819.55957	38.50288	6823.56201	38.52502	6826.32227	38.68280	6856.95117	38.78798
7387.95508	41.81618	7392.04053	41.84109	7395.01514	42.00717	7427.40479	42.12896
7956.45068	45.17099	7960.57031	45.19590	7963.12207	45.36475	7998.07520	45.49762
8524.82910	48.54795	8528.36328	48.56733	8532.63477	48.71680	8569.04688	48.86074
9093.29102	51.91384	9097.05957	51.94429	9100.84180	52.07715	9140.21875	52.24323
9666.33789	55.30741	9669.67383	55.34339	9673.58398	55.47626	9715.50586	55.65341
10236.67285	58.66499	10239.95996	58.69544	10244.65234	58.83661	10287.94727	59.01653
10806.17090	62.03642	10808.53906	62.05579	10812.10645	62.18866	10858.68164	62.40179
11373.96094	65.38016	11376.71680	65.37186	11379.56055	65.54347	11429.43262	65.77322
11942.45410	68.74052	11944.89453	68.75158	11947.71777	68.88998	12000.40137	69.14188
12510.64648	72.01506	12512.80469	72.01782	12515.50586	72.17007	12571.37012	72.43025
13079.10547	75.19549	13081.40039	75.20103	13083.71289	75.34496	13142.15430	75.61623
13648.33398	78.33440	13650.28125	78.33994	13651.51855	78.49771	13713.15625	78.77451
14216.65820	81.43179	14218.59277	81.45670	14219.42383	81.60341	14284.05762	81.92173
14785.28418	84.56517	14787.25488	84.60392	14787.53027	84.74509	14854.08887	85.04680
15353.50781	87.67640	15355.61621	87.72899	15355.63672	87.87292	15423.93555	88.18295
15921.63184	90.78764	15924.36230	90.85130	15923.74316	90.99800	15994.58594	91.32186
16490.04102	93.96252	16492.80664	93.99021	16492.05078	94.13691	16565.45313	94.47461
17059.85547	97.06268	17061.25195	97.10144	17060.35742	97.22323	17136.32031	97.56923
17628.06250	100.19883	17629.77930	100.25143	17628.86523	100.37321	17707.08789	100.73582
18194.56250	103.29900	18198.10547	103.37373	18197.07227	103.49275	18277.75391	103.86920
18762.97070	106.41576	18766.63281	106.49049	18765.37891	106.61782	18848.32031	107.01642
19330.27539	109.53252	19335.06055	109.61834	19333.68555	109.74566	19419.08789	110.15257
9897.77930	112.61885	9903.38672	112.72681	9901.99219	112.84029	9989.85547	113.26379

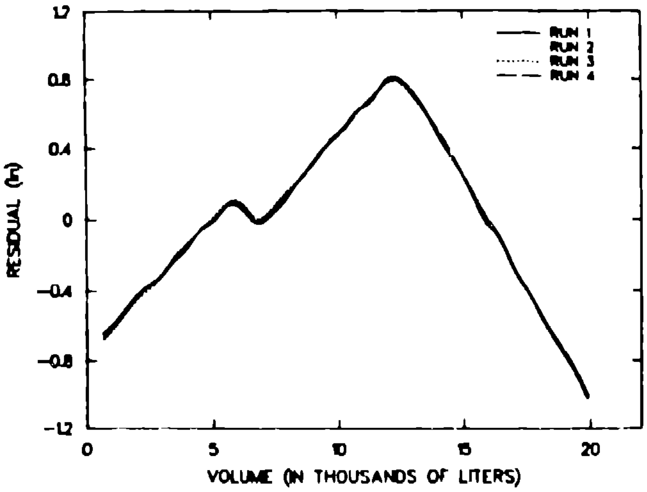

Fig. 1. A profile plot of 4 runs consisting of 35 data points each.

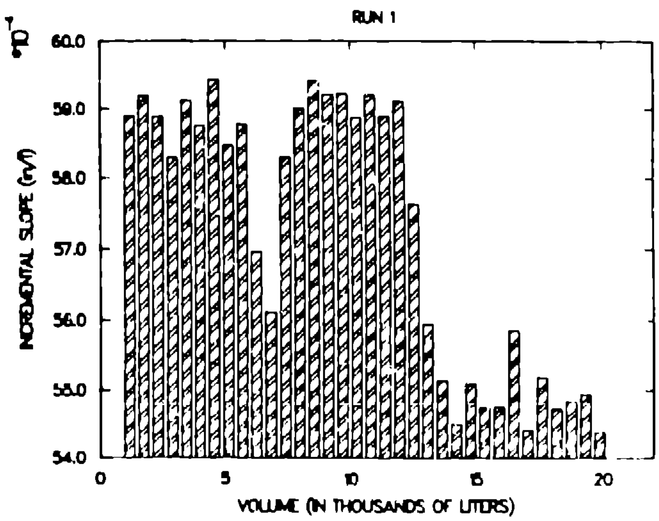

Fig. 2. A plot of incremental slopes for Run 1.

TABLE II

CALIBRATION EQUATIONS TO DETERMINE
VOLUME OF LIQUID TRANSFER

SECTION 1

$$Y = 8.25 \times 10^{-9} X^2 + 0.00594X - 1.422$$

$$0L < X \leq 8883L$$

$$X = \frac{-0.00594 + \sqrt{3.5284 \times 10^{5} - 3.3 \times 10^{-8}(1.422 + Y)}}{-1.70 \times 10^{-8}}$$

$$0" < Y \leq 50.7"$$

SECTION 2

$$Y = 0.0059X - 1.688$$

$$8883L < X \leq 12\,609L$$

$$X = \frac{Y + 1.688}{0.0059}$$

$$50.7" < Y \leq 72.7$$

SECTION 3

$$Y = 0.0055X + 3.348$$

$$12\,609L < X < 20\,000L$$

$$X = \frac{Y + 3.348}{0.0055}$$

$$72.7" < Y < 113.3"$$

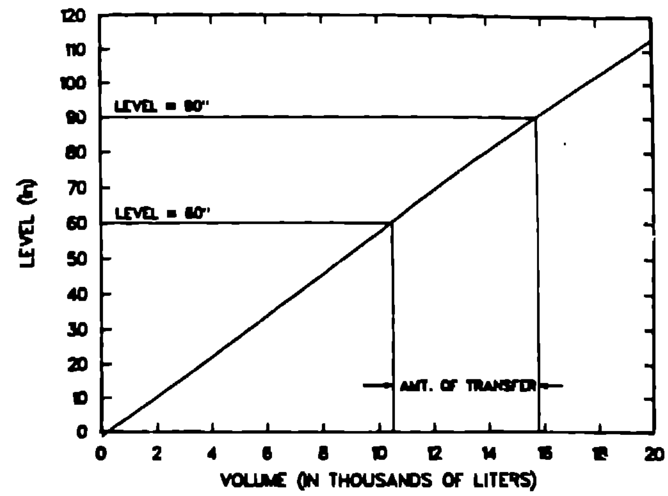

Fig. 3. A plot of the calibration equation and an illustration for obtaining the transfer of liquid volume.