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Optimal Routing of Hazardous Substances in Time-Varying, Stochastic Transportation Networks

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A Report on

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Abstract

This report is concerned with the selection of routes in a network along which to transport hazardous substances, taking into consideration several key factors pertaining to the cost of transport and the risk of population exposure in the event of an accident. Furthermore, the fact that travel time and the risk measures are not constant over time is explicitly recognized in the routing decisions. Existing approaches typically assume static conditions, possibly resulting in inefficient route selection and unnecessary risk exposure.

The report describes the application of recent advances in network analysis methodologies to the problem of routing hazardous substances. Several specific problem formulations are presented, reflecting different degrees of risk aversion on the part of the decision-maker, as well as different possible operational scenarios. All procedures explicitly consider travel times and travel costs (including risk measures) to be stochastic time-varying quantities. The procedures include both exact algorithms, which may require extensive computational effort in some situations, as well as more efficient heuristics that may not guarantee a Pareto-optimal solution.

All procedures are systematically illustrated for an example application using the Texas highway network, for both "normal" and "incident" condition scenarios. The application illustrates the trade-offs between the information obtained in the solution and computational efficiency, and highlights the benefits of incorporating these procedures in a decision-support system for hazardous substance shipment routing decisions.

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1. INTRODUCTION

In this report, the problem of determining "superior" routes on which to transport hazardous substances is addressed. Two primary criteria are considered: travel time and population (as an indicator of exposure) along a route. Both criteria can at best be known *a priori* with uncertainty. In addition, both criteria dynamically change over time. Thus, both risk (potential population exposure) and travel time along each link of a transportation network are assumed to be random variables whose probability distribution functions vary over time. Therefore, the problem of selecting a "superior" route for the transport of hazardous substances requires consideration of multiple conflicting objectives involving multiple time-varying stochastic attributes.

Procedures that address several problem formulations are presented in this report. The problems addressed include: (1) the generation of all *a priori* Pareto-optimal least time or least risk paths, (2) the determination of the *a priori* least expected time or least expected risk paths, (3) the determination of the least possible risk or least possible travel time paths, and (4) the generation of all *a priori* Pareto-optimal paths where travel time and risk are simultaneously minimized. An actual transportation network, the Texas highway system, is used to illustrate the procedures.

In Section 2, background on the problem of determining routes for the transport of hazardous materials is presented. In Section 3, the Texas network is described. An overview of the computational steps are given in Section 4 for determining "least time paths" in stochastic, time-varying networks, including the STDLT(DD), STDLT(SD), FOSD-EX Limited STDLT, EV, LEAST, and ELB procedures. In Section 5, modifications to the procedures of Section 4, required for determining Pareto-optimal least risk paths, least expected risk paths, least possible risk paths, as well as lower bounds on the expected risk of the least expected risk paths are presented. These extensions are required because the optimal, or Pareto-optimal least risk paths cannot be determined by simply replacing the time-varying travel time random variables by attributes other than the travel time. This point is raised by Ziliaskopoulos (1994) in the context of least cost paths in deterministic, time-varying networks. Ziliaskopoulos points out that, while cost (any attribute that is not a travel time) can simply replace time in solving the deterministic, time-invariant least cost path problem, both time and cost are required for solving the deterministic, time-varying least cost path problem. In Section 6, extensions of the STDLT(SD) and EV algorithms for generating Pareto-optimal paths for multio-bjective problems are given. In Section 7, "superior" paths in terms of "minimizing" travel time, "minimizing" cost (population) and simultaneously minimizing travel time and cost are determined for the example application. A "best compromise" path is selected. In

Section 8, the procedures are illustrated for a scenario under which two incidents occur on the morning of the shipment. The procedures are reapplied to determine a new "best compromise" path based on this new information. Finally, concluding remarks are given in Section 9.

2. BACKGROUND REVIEW

The transportation of hazardous materials and wastes concerns most communities worldwide. In 1982, in the United States alone, it was estimated that 1.5 billion tons or 784 billion ton-miles of hazardous materials were shipped by water, truck, rail and air (List and Abkowitz, 1986). Several hundred thousand shipments of hazardous materials are moved daily through the United States' transportation system (Gopalan et al., 1990; Turnquist, 1993). The routes that are selected for transporting such substances can affect the amount of risk exposure to the surrounding population and environment. In selecting a route, one implicitly selects a certain segment of the population that will be most affected. In addition to imposing risk on a select group of people and geographic region, there may be a higher probability of an accident, and hence of a release, along certain routes due to geometric configuration, weather conditions or other characteristics. A release, whether a spill, an explosion, or a fire, may entail both damages and costs to the environment, the surrounding populace, as well as the party responsible for the shipment. Although the location of the storage, or processing, facilities directly impact the path selection process and thus, the communities that will be affected, it remains outside the scope of this discussion.

Many studies in the literature have addressed the problem of determining best routes for the shipment of hazardous substances. In general, the main objective of these studies is to select the route or set of routes that impose the least risk to the environment and to the general population while not imposing large costs to the shippers. Risk is a surrogate for the amount of damage imposed on society as a consequence of a release that results from an accident. The amount of this damage is difficult to quantify as the consequences depend on many factors, such as the type of substance being transported, the type of incident and the effects of weather and other external factors that may influence the dispersion and nature of the incident. For example, in the event that a release occurs and a plume forms, the direction and distance that the plume travels depends on wind velocity. Some studies represent risk by a single measure; often, the risk of a certain route is assumed to be an additive function of the risk of the constituent links. Sometimes a risk profile is used and thus, the uncertain nature of this criterion is recognized. Additional detail on the quantification and assessment of risk and the estimation of release rates is given in several references (Abkowitz et al., 1984; Pijawka et al., 1985; Scanlon, 1985; Radwan et al., 1986; Abkowitz and Cheng, 1988; Glickman, 1989; Harwood et al., 1990; Glickman, 1991; Saccomanno and Shortreed, 1993; and Alp, 1995).

Many studies recognize the difficulties of evaluating risk and the multi-objective nature of the hazardous materials shipment problem. As with many multi-objective problems, this, too, has as many formulations as there are perspectives. Perhaps the most straightforward of these can be found in Batta and Chiu (1988) where two models, each with a single objective function, for determining the best route on which to transport hazardous substances is presented. The objective of the first model is to find the path that minimizes the proximity to areas of at least some given population size. The second model incorporates the probability of and consequences of an accident into the first model and thus, seeks the path that minimizes "expected damage."

Several studies formulate the problem with multiple conflicting objectives and present procedures for determining the Pareto-optimal paths. Kalelkar and Brooks (1978) present a decision-theoretic approach for selecting the best path from a set of Pareto-optimal paths given multiple conflicting objectives. McCord and Leu (1995) formulate a multi-objective problem of determining the path or set of paths that minimizes the travel cost and population exposure in the event of an incident. Multi-attribute utility theory is used to transform the problem to a single objective problem and a shortest path algorithm is used to determine the best path with respect to the additive utility function provided. The work recognizes that the paths so determined are highly sensitive to variability in the parameters of the utility function. Thus, by slightly varying the parameters within specified ranges, a set of routes is generated, each of which has the least expected utility for the origin-destination pair and for the given utility function. Saccomanno and Chan (1985) evaluate the cost-effectiveness of paths that either minimize the probability of an accident, minimize the operating costs, or minimize exposure to risk. The effects of variability of road conditions on each of these objectives is examined. Current et al. (1988) formulate the problem of determining the Pareto-optimal paths that minimize path length and minimize the total population exposed to the risks involved in the transport as a minimum covering/shortest path problem.

When multiple shipments are required, a single population or geographic region may incur greater risk if all shipments are made over the same path. Therefore, it is necessary to consider the equity of risk when selecting routes for multiple shipments. Equity of risk is addressed in a decision theoretic framework by Keeney (1980). Numerous studies address the problem of equity of risk through a variety of approaches. For example, Zografos and Davis (1989) use a pre-emptive goal programming approach to solve the multi-objective problem of minimizing overall risk, risk to special population categories, travel time, and property damage. By constraining the capacity of the links, one can determine paths that are more equitable in terms of risk over multiple shipments. Both capacitated and incapacitated models are described. Gopalan et al. (1990) present an integer programming formulation of the problem of minimizing the total risk but maintaining equity in distribution of risk across geographic regions by constraining the difference in risk of pairs of regions to be within a given threshold. They recognize that in reality multiple shipments are required and present a heuristic solution to the multiple trip scenario where repeated solutions to the single trip problem are solved to optimality by a Lagrangian dual approach. Similarly, Lindner-Dutton et al. (1991) address the problem of determining the set of routes that equitably distribute the risk associated with transporting hazardous materials between every pair of regions over time. They consider a sequence of shipments that are to occur over a period of time. Using the model for measuring risk along a route described by Batta and Chiu (1988), they formulate the problem first as an integer programming problem and second as a dynamic programming problem.

List and Mirchandani (1991) formulate a multi-objective problem of minimizing risk, minimizing cost, and distributing risk equitably for which the set of Pareto-optimal solutions is determined. Sivakumar et al. (1995) introduce a column generation procedure for determining the set of routes that collectively minimize the expected risk until the occurrence of the first accident, where the accident probabilities, expected *a priori* risk, transportation cost, and equity of risk are all constrained. The mathematical formulation is transformed into a maximization formulation with a constrained shortest path sub-problem. As the transformation does not guarantee that the arc costs in this sub-problem are all non-negative, the final results are not guaranteed to be optimal. Similarly, Jin et al. (1996) introduce Lagrangian relaxation based algorithms to address the problems of minimizing either total risk or risk per trip over multiple shipments given that shipments stop after the occurrence of a pre-specified number of accidents.

While equity-of-exposure-to-risk is a very important issue, achieving it does not come without a price. That is, the set of paths that most equitably distributes risk may have a higher total risk than a less equitable solution. Many studies do not take the equity-of-risk-into account. Some studies recognize that risk cannot be known *a priori* with certainty and thus, solve a stochastic multi-objective routing problem. For example, Turnquist (1993) determines the set of Pareto-optimal paths in terms of the following multiple objectives: minimize accident rates resulting in releases, minimize the population exposed within a certain distance of the roadway, and minimize distance, where the accident probabilities and the population exposure are known only probabilistically. Using the distribution functions of each Pareto-optimal path on each criterion, Turnquist shows the trade-offs among the non-dominated solutions and the decision-making issues that arise as a result of the overlap in the distribution functions. Similarly, Wijeratne et al. (1993) address the multi-objective problem with uncertain attributes. They present two methods based on an approximation of stochastic dominance for comparing path distribution functions for a single stochastic criterion. The problem is extended to multiple criteria but the criteria are reduced to two deterministic factors; hence, the final problem is a deterministic multi-objective problem.

Several other factors affect the selection of a best route. Cox and Turnquist (1986) present optimal strategies for determining the best departure time for transporting hazardous substances on fixed routes that contain cities with curfews (a city may impose restrictions on the time of the day during which the vehicle is permitted to travel) where travel times are either deterministic or stochastic in nature. Since en-route delay due to curfews can only increase the time that a population will be exposed to the risk of the occurrence of a release, the best departure time is the one that results in a path with the least delay.

A review of the literature preceding 1990 on the transportation of hazardous materials, as well as siting of hazardous substance facilities and joint routing and facility location is found in List et al. (1991). For the interested reader, other studies that address the simultaneous siting and routing problem include List and Mirchandani (1991) and ReVelle et al. (1991).

The most recent advances in this area have been in terms of evaluating or demonstrating the advantages of using advanced technologies in routing hazardous substances. Many available technologies can be applied to either aid in the decision-making process of selecting a route, such as GIS (Geographical Information Systems) and other decision support systems, or in increasing the safety of travel, through devices used in Automatic Vehicle Identification (AVI), Automatic Vehicle Classification (AVC), Weigh-in-Motion (WIM), Automatic Toll Collection (ATC), On-Board Computing (OBC), Two-Way Communication (TWC), and Automatic Vehicle Location (AVL).

GIS can incorporate many types and sources of data, such as the network structure, demographics, and geographic and roadway characteristics, into a single unified representation, (Abkowitz et al., 1990). Lepofsky and Abkowitz (1993) discuss the benefits of using GIST (Geographic Information Systems for Transportation) for routing vehicles carrying hazardous substances as GIST can be used to provide decision support for risk assessment, routing and scheduling, emergency preparedness, evacuation planning and incident management. The routes are determined using a single objective function consisting of a weighted additive combination of the mentioned multiple objectives. By varying these weights the relative trade-offs among objectives offered by the various paths can be examined. Thus, the best routes can be determined by combining a great deal of information into a single integrated environment.

Beroggi and Wallace (1995) develop several decision support models for selecting alternate routes for vehicles transporting hazardous substances given that an incident occurs on their route within a time-frame that will affect their safety. The work shows that advanced communication technologies integrated with a good decision support system can lead to better decisions by the dispatchers and thus, safer shipments. Other works of interest include Saccomanno et al. (1988) and Weigkricht and Fedra (1995).

Technologies used in AVI, AVC, WIM, ATC, OBC, TWC and AVL permit real-time monitoring of the vehicle's location, continuous motion of the vehicle through state borders and toll collection booths, continuous communication with a dispatcher which leads to better operating performance, greater compliance with regulations, and decreased response time of emergency response teams in the event of an incident with a more accurate account of the situation and the materials involved. Thus, incorporating these advanced technologies can lead to safer transportation of hazardous substances. See Allen (1991) and Boghani (1990) for more detail.

The primary focus of this report is in the development of a methodology for selecting a path that is best in a multi-objective framework where the criteria are both time-varying and stochastic in nature. The use of such advanced technologies is not explicitly discussed. Even if advanced technologies are employed, both travel time and risk can at best be known *a priori* with uncertainty. In addition, travel times and the amount of risk vary over time because the characteristics of the network, in terms of congestion, weather conditions, etc., vary with time. Thus, travel times and risk are random variables whose distribution functions vary with time. The problem of determining the paths that simultaneously "minimize" both travel time and risk imposed on the surrounding population is considered.

3. TEXAS NETWORK DESCRIPTION

In this section, a hypothetical scenario requiring the specification of a "best compromise" path for the transport of a hazardous substance through a test network is described. This is followed by a description of the test network.

Plans for the construction of a new commercial airport in Austin, Texas, on the grounds of the former Bergstrom Air Force Base, are currently under way. Suppose during the construction a repository of nuclear weapons is discovered. The weapons must be shipped to the Pantex plant, a nuclear facility in Amarillo, Texas. In Section 7.4, a "best compromise" route is determined that simultaneously minimizes travel time and the potential population that could be exposed to this dangerous substance in the event of an incident. This second criterion is measured by the number of people who are assumed to be within an effective distance of the roadway. Future travel time and potential population exposure (measured by population) are assumed to be independent, time-varying quantities that are at best known *a priori* probabilistically. The portion of the Texas highway system applicable for this network is depicted in Figure 3.1. This highway system is represented by a graph with 183 nodes (representing cities in Texas) and 549 arcs (representing highway links between the cities). The travel times and population along a route are random variables with time-varying probability distribution functions, represented by the weights of the arcs.

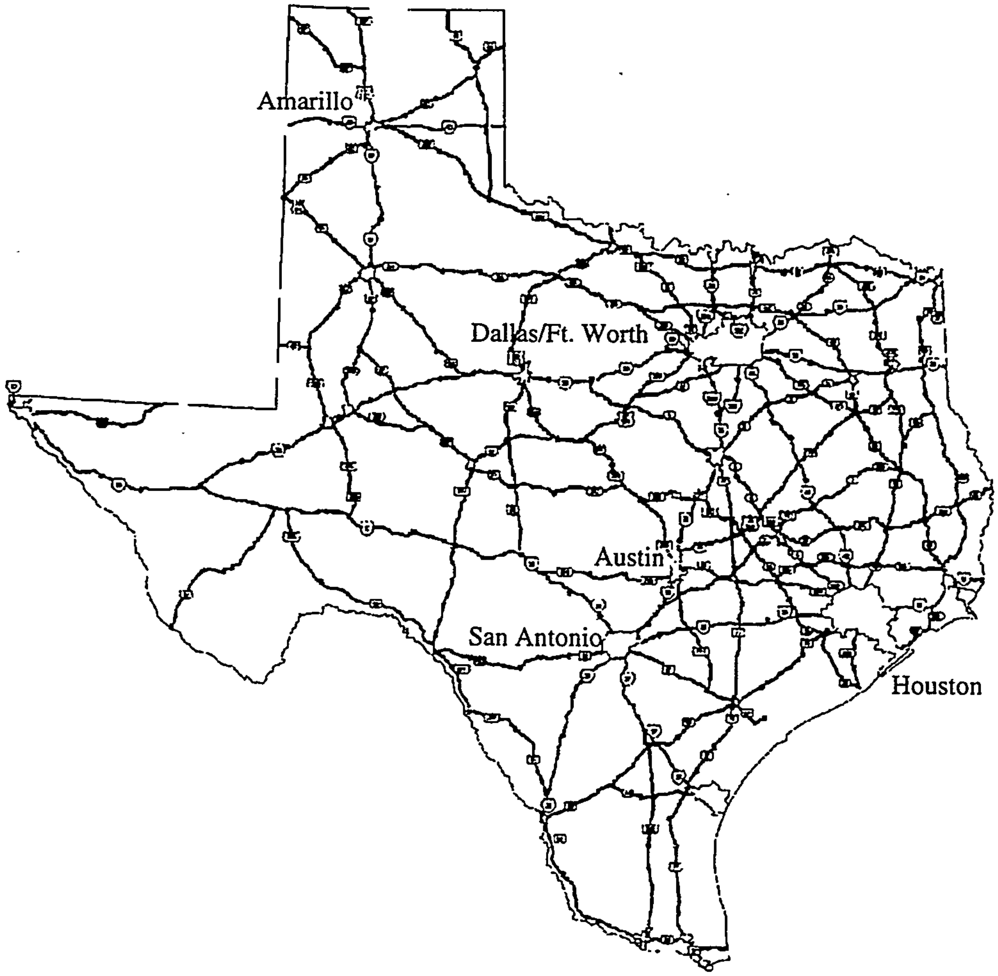

Figure 3.1: The Texas Network

The trip is scheduled to start at 7:30 a.m., which is close to the start of rush hour. It is assumed that the morning peak period lasts for approximately one hour on the arcs emanating from each of the following four cities: Houston, Dallas/Fort Worth, Austin, and San Antonio. Loosely based on the guidelines given in several traffic engineering books (for example, May (1990)), the average speeds are assumed to decrease from 60 mph to 30 mph in the first half of the peak hour and increase again to 60 mph by the end of the peak hour. The average speed over the remainder of the day is assumed to be constant at 60 mph. A second peak period is not considered.

Both travel time and population along links are assumed to have underlying normal distributions. For each time interval, the mean travel time for each link is determined from the distance along the link (based on the Texas Comptroller of Public Accounts Mileage Guide) divided by the average speed. Based on the results of a study conducted in Austin, Texas, for expressways, described in Herman et al. (1989), the standard deviation of the travel time can be estimated from the following equation:

$$STD = -54.438 + 0.989 \text{ Mean}$$

where STD, the standard deviation of travel time, and Mean, the mean travel time, are in units of seconds per mile. Normal random variates for each time interval are generated using the mean and standard deviation of travel time. That is, it is assumed that the underlying distribution function of the random variables is normal.

Similarly, the population along each link that is potentially exposed to the risk of a shipment of a hazardous substance is determined from the population of the two cities connected to the link. The population estimates for each of the cities in this network are based on 1992 U. S. Census Bureau data. The population in each city is divided between the incident links and is assumed to be distributed uniformly over each link, providing the mean population along a link. As suggested by Turnquist (1993), the coefficient of variation of the population that is exposed is taken to be approximately 0.25. Again, for each time interval, the random variates for the population are generated assuming the underlying distribution function is normal.

The distributions for each link at each departure time are approximated by probability mass functions. The travel time and population pmf's for the peak period for the link from Austin to Lampasas are presented in Appendix B. Note that a time interval of 5 minutes is used for these tests. The mean and variance for each arc random variable (travel time and population) are obtained as described above. Given the number of elements in the pmf's, P (assumed nearly constant across arcs and departure times), the pmf's are generated as follows:

Step 1. Generate P pairs of random variates. The first random variate of each pair is a normal random variate with given mean and standard deviation. It will represent a possible travel time (population along the link). The second random variate of each pair is a uniform random variate. It will represent the corresponding probability of the occurrence of such a travel time (population).

Step 2. Normalize the second random variates of the P pairs so that their sum is equal to 1.

Step 3. Sort the pairs of random variates in ascending order of the first element of the pair (corresponding to increasing travel time).

Step 4. Collect similar travel times and sum their probabilities of occurrence.

In the remainder of this report, the problem of determining the optimal route or set of Pareto-optimal routes from Austin to Amarillo along the Texas highway system is addressed. First, the path or set of paths that are best with respect to travel time are determined. Next, the best path(s) with respect to the number of people within a certain distance from the route, generically referred to as a cost, are determined. Finally, both travel time and population en route are considered simultaneously, and two of the algorithms are extended to determine the Pareto-optimal paths with respect to both criteria.

The Texas network can be represented by a graph defined in the following way: Let $G = (\mathcal{V}, \mathcal{A}, \mathcal{S}, \mathcal{T}, \mathcal{P}_{\mathcal{T}}, C, \mathcal{P}_C)$ be a directed graph where $\mathcal{V}$ is the set of nodes, $|\mathcal{V}| = v$, and $\mathcal{A}$ is the set of arcs, $|\mathcal{A}| = m$. The travel times along the arcs are assumed to have stationary discrete distributions (possibly with underlying continuous distributions) except during the peak period $t_0 \leq t \leq t_0 + (I)\delta$. The network is considered at a set $\mathcal{S}$ of discrete times $\{t_0 + n\delta\}$, where n is an integer, $0 \leq n \leq I$, and δ is the smallest increment of time over which a perceptible change in the travel time distributions will occur for $t \in \mathcal{S}$. For each departure time and each arc (i,j), the set $\mathcal{T}(t)$ of non-negative real-valued possible travel times $\tau^k_{i,j}(t)$ for traversing the arc at the time t is given, $k=1,\ldots, K_{i,j}(t)$. Travel time $\tau^k_{i,j}(t)$ occurs with probability $\rho^k_{i,j}(t)$, where $\rho^k_{i,j}(t)$ is an element of $\mathcal{P}_{\mathcal{T}}(t)$ and

$$\sum_{k=1}^{K_{i,j}(t)} \rho^k_{i,j}(t) = 1 \quad \forall\ t \in \mathcal{S}.$$

It is assumed that $\tau^k_{i,j}(t) = \tau^k_{i,j}(t_0 + I\delta)$ and $\rho^k_{i,j}(t) = \rho^k_{i,j}(t_0 + I\delta)$ $\forall$ t occurring after the peak period, i.e. $\forall\ t > t_0 + I\delta$. Thus, the set of travel times and the corresponding set of probabilities with which each travel time will occur, $(\mathcal{T}, \mathcal{P}_{\mathcal{T}})$, is assumed to be given.

Also associated with each arc (i,j) is a set of non-negative real valued possible costs, or estimates of risk, $C^x_{i,j}(t)$, $x=1,\ldots, X_{i,j}(t)$, associated with traversing the arc at time t. Cost (population) $C^x_{i,j}(t)$ occurs with probability $q^x_{i,j}(t)$, where $q^x_{i,j}(t)$ is an element of $\mathcal{P}_C(t)$ and

$$\sum_{x=1}^{X_{i,j}(t)} q^x_{i,j}(t) = 1 \quad \forall\ t \in \mathcal{S}.$$

Similarly, it is assumed that $C^x_{i,j}(t) = C^x_{i,j}(t_0 + I\delta)$ and $q^x_{i,j}(t) = q^x_{i,j}(t_0 + I\delta)$ $\forall$ t occurring after the peak period, i.e. $\forall\ t > t_0 + I\delta$. Thus, the set of travel costs (population along the links) and the corresponding set of probabilities with which each cost (population) will occur, $(C, \mathcal{P}_C)$, is assumed to be given. In order to preserve the generality of this extension and hence, applicability to other types of costs, population is referred to as a cost.

4. DETERMINING "SUPERIOR" LEAST TIME PATHS

In this section, several procedures that have been developed as part of this work are provided for determining "least time paths" in stochastic, time-varying networks. These procedures are organized in two sets. The first set addresses the problem of generating all *a priori* Pareto-optimal least time paths and their time-varying probability distribution functions, or expected values. Since these procedures have non-polynomial worst-case computational complexity, less computationally intensive procedures are developed to determine a set of "superior" paths. An additional procedure for determining lower bounds on the expected times of the least expected time paths is presented. This procedure is also useful in illustrating the trade-offs between computational efficiency and information. These procedures comprise the second set.

4.1 DESCRIPTION OF PROCEDURES

4.1.1 Procedures for Generating All Pareto-Optimal Paths

Three dominance criteria are considered in the determination of Pareto-optimal paths, described in detail in Chapter 3 of Miller-Hooks (1997). In this section, a brief description of the rationale behind each dominance criterion is given.

The first dominance criterion, referred to as deterministic dominance, is the most "prudent." By this criterion, if for a given departure time the highest travel time on the first path is lower than the lowest travel time on the second path, then the second path has zero-probability of having a lower travel time than the first. The first path is said to dominate the second for the given departure time. For this departure time, one can choose the first path with certainty that the second path will not be better. Paths that are not dominated by any other path for all departure time intervals in the peak period with respect to deterministic dominance are referred to as **deterministically Pareto-optimal.**

The second dominance criterion, first-order stochastic dominance, is less conservative than deterministic dominance, possibly resulting in fewer Pareto-optimal paths. Here, for a given departure time, the first path dominates the second, if for all possible travel time values, the probability that the first path's travel time is less than or equal to that value is always greater than the probability that the second path's travel time is less than or equal to this same value. This criterion does not preclude the possibility of a dominated path having less time than a non-dominated path for some realizations of the arc travel time random variables. Paths that are not dominated by any other path for all departure time intervals in the peak period with respect to first-order stochastic dominance are referred to as **stochastically Pareto-optimal.**

The third criterion considered uses the expected value to establish dominance. If, for all departure time intervals in the peak period, the first path has lower expected time than the second, the first path **EV-dominates** the second.

In this research, procedures for generating all *a priori* Pareto-optimal least time paths, or least expected time paths, are developed using these criteria. The theory and methodology for determining dominance over a time period are given in Chapter 3 of Miller-Hooks (1997). In this section, brief overviews of these procedures for determining "least time paths" are presented, with detailed descriptions provided in Chapter 4 of Miller-Hooks (1997). The necessary modifications for determining "least cost paths" are given next in Section 5. In Section 6, the extensions required to determine the paths that simultaneously "minimize" both travel time and cost are described.

(a) STDLT(DD) Algorithm

A procedure based on deterministic dominance, called the STDLT (DD) (stochastic, time-dependent least time (deterministic dominance)) algorithm is presented for generating all *a priori* deterministically Pareto-optimal least time paths and their corresponding cdf's, $\forall\ i \in \mathcal{V}$ to a given destination, N, $\forall\ t \in \mathcal{S}$. Even if the arc travel time random variables are discrete, the number of path travel time realizations with positive probability may increase exponentially with the number of constituent arcs. Thus, a methodology whereby the distribution functions are "aggregated," or approximated at iso-probability levels, described in a similar context in Ziliaskopoulos (1994), is presented and implemented. The complications associated with the comparison of non-disjoint paths are taken into consideration.

(b) Range Algorithm

The Range algorithm exploits the fact that comparison via deterministic dominance requires knowledge of only the lowest and highest values of path travel times. This algorithm generates all *a priori* deterministically Pareto-optimal least time paths and the associated travel time ranges, $\forall\ i \in \mathcal{V}$ to a given destination, N, $\forall\ t \in \mathcal{S}$. The Range algorithm is very similar to the STDLT(DD) algorithm, but avoids the possibility of making erroneous dominance conclusions that may occur as a consequence of approximating the distribution functions. However, instead of determining the cdf's of the Pareto-optimal paths, only the ranges of the travel times are determined.

(c) STDLT(SD) Algorithm

An adaptation of the STDLT(DD) algorithm is presented where first-order stochastic dominance is used in place of deterministic dominance to establish Pareto-optimality. This procedure, referred to as STDLT(SD) algorithm, determines all *a priori* stochastically Pareto-optimal paths and their associated cdf's, $\forall\ i \in \mathcal{V}$ to a given destination, N; $\forall\ t \in \mathcal{S}$. Since first-order stochastic dominance is established using full information from the path travel time cdf's, the comparison of non-disjoint paths presents no added complications.

(d) EV Algorithm

The EV (expected value) algorithm is presented for generating all *a priori* least expected time paths with their associated expected times, $\forall$ i $\in \mathcal{V}$ to a given destination, N, $\forall$ t $\in \mathcal{S}$.

(e) Extensions

The STDLT(DD), Range, STDLT(SD) and the EV algorithms are developed for determining *a priori* Pareto-optimal and optimal least time paths. Adaptations of these algorithms are presented for determining the Pareto-optimal paths for applications where decisions at intermediate nodes, given the additional information on the actual (revealed) arrival times at the intermediate nodes, are allowed (i.e., in a time-adaptive route choice framework). In addition, these algorithms are extended for determining all *a priori* Pareto-optimal least cost paths, where the arc costs are stochastic and time-varying, but only indirectly dependent on arc travel times. The procedures are also extended for solving the bi-criterion shortest path problem of simultaneously minimizing travel time and cost, where both criteria are stochastic and time-varying quantities.

4.1.2 Efficient Procedures

The procedures described in Section 4.1.1 for generating all *a priori* Pareto-optimal paths are non-polynomial in worst-case computational complexity because the number of labels required to determine the non-dominated paths may grow exponentially with the size of the network. Less computationally intensive procedures are developed to determine a set of "superior" solutions. In addition, an efficient procedure for determining a lower bound on the expected time of the least expected time paths is developed. In this section, syntheses of these procedures are given. The procedures are presented in detail in Chapter 5 of Miller-Hooks (1997).

(a) Limited STDLT Procedures

The Limited STDLT procedure is a heuristic for determining "superior" paths. By limiting the number of paths that can be maintained from each node, a procedure for determining a set of "superior" paths and their cdf's (or expected values), $\forall$ i $\in \mathcal{V}$ to a given destination, N, $\forall$ t $\in \mathcal{S}$, is created that prevents the number of labels required to determine the non-dominated paths from each node from growing exponentially. If the number of labels is artificially limited, it is possible that the final set of paths for some of the origin nodes will not contain any Pareto-optimal (or least expected time) paths. Thus, it is important that the labels maintained at each node be chosen carefully. Although such a procedure does not guarantee that the solution set contains at least one Pareto-optimal path from every origin, results of numerous tests (described in Chapter 6 of Miller-

Hooks (1997)) show that such paths are still obtained even where only a very limited number of labels are maintained from each node.

(b) Absolute Least Possible Time Paths

Two computationally efficient procedures are developed for determining least possible time paths for all origins to a single destination for each departure time interval. The first procedure, referred to as the LEAST algorithm, produces a path $\forall\ i \in \mathcal{V}$ to a given destination, N, $\forall\ t \in \mathcal{S}$, that has the least possible travel time (or cost). Both the least possible travel time of this path and a lower bound on the associated probability of the occurrence of this time are determined. The second procedure, referred to as the TOP algorithm, is an extension of the LEAST algorithm and determines up to k least possible time paths, the associated travel times and the corresponding probabilities of occurrence of the travel times (or a lower bound on this probability). Such paths are guaranteed to be deterministically Pareto-optimal; therefore, these algorithms are guaranteed to produce at least one deterministically Pareto-optimal path from each origin for each departure time.

(c) Lower Bounds on Least Expected Time Paths

Finally, an efficient procedure, referred to as the ELB procedure, is presented for determining lower bounds on the expected travel times (or costs) of the least expected time paths $\forall$ $i \in \mathcal{V}$ to a given destination, N, $\forall\ t \in \mathcal{S}$. This procedure has excellent worst-case computational complexity and is instrumental in showing the trade-offs between information, such as the identification of actual paths, and computational complexity.

4.2 Procedural Steps

A detailed description of each of these procedures is beyond the scope of this report. Instead, two generic algorithms are presented to give an overview of the two sets of procedures described in Section 4.1. The reader is referred to Chapters 4 and 5 of Miller-Hooks (1997) for implementation details.

4.2.1 Procedural Steps for Determining Pareto-Optimal Paths

In this section, a single procedure is presented to describe the computational steps of the STDLT(DD), Range, STDLT(SD) and EV algorithms, described in Section 4.1.1. In order to represent each of these procedures by one set of instructions, the definition of the labels, a description of the construction of new labels, and the rules used to compare and update labels must be omitted from the description. These are defined after the algorithmic steps are presented.

PROCEDURE 1 {for determining deterministically and stochastically Pareto-optimal and least expected time paths}

(1) Initialize the labels and create the SE list.

(2) Select node-label pair from SE list.

- **IF** SE list is empty,
  THEN go to Step 4.

(3) For each predecessor node of the node-label pair:

[a] Construct new label.

[b] Compare new label to set of p-efficient labels.

- **IF** dominated by p-efficient label,
  THEN discard new label.
- **IF** not dominated,
  THEN new label is p-efficient. Add node-label pair to SE list. If it dominates a p-efficient label, discard dominated label.

[c] Repeat Step 3 for each predecessor node. If completed, go to Step 2.

(4) Stop. P-efficient paths are Pareto-optimal.

Procedure 1 is a very general description of the computational steps of the STDLT(DD), Range, STDLT(SD) and EV algorithms. In order to generate the deterministically and stochastically Pareto-optimal paths with their associated travel time cdf's or ranges as well as the least expected time paths with the corresponding expected travel times, definitions of the labels, and detailed description of label computation and comparison are required. A brief description of the labels required for each of these procedures is given next.

(1) **STDLT(DD) and STDLT(SD) algorithms:** In these algorithms, multiple vector labels are associated with each node, maintaining the travel time cdf's for all departure time intervals in the peak period of each p-efficient path. More detail on maintaining and constructing the cdf's can be found in Chapter 4 of Miller-Hooks (1997). Note that additional steps are required to generate the deterministically non-dominated paths resulting from complications due to the existence of shared arcs.

(2) **Range algorithm**: Similar to the STDLT(DD) algorithm, multiple vector labels are required from each node, one for each potentially Pareto-optimal (p-efficient) path. However, each vector label contains only the minimum and maximum path travel time for each departure time.

(3) **EV algorithm**: Again multiple vector labels are required, each containing the expected path travel time for each departure time.

4.2.2 *Procedural Steps for Determining Least Possible Time Paths and Least Expected Travel Times*

In this section, a description of the Limited STDLT procedure is given. This is followed by an overall description of the computational steps of the LEAST and ELB procedures, described in Section 4.1. Again the definition of the labels, a description of the construction of new labels, and the rules used to compare and update labels must be omitted from the description in order to maintain the generality of the procedural steps. These are defined after the algorithmic steps are presented. First, a description of the Limited STDLT procedure is given:

Limited Form of Procedure 1 {overview of Limited STDLT procedure}

(1) Initialize the labels, Q (the maximum number of labels permitted from any node), and create the SE list.

(2) Select node-label pair from SE list. If SE list is empty, go to step 4.

(3) For each predecessor node of the node-label pair:

[a] Construct new label.

[b] Compare new label to set of p-efficient labels.

•**IF** new label is dominated by a p-efficient path,
THEN discard new label.

•**IF** new label dominates a p-efficient path,
THEN replace the p-efficient label with new label. Discard other p-efficient paths that are dominated and reduce q(i) (number of p-efficient paths from node i) accordingly. Add node-label pair to SE list.

•**IF** the new label is not dominated and does not dominate any other p-efficient label:

-**IF** q(i) < Q, make new label p-efficient. Add node-label pair to SE list.
q(i) --> q(i)+1.

-**IF** q(i)=Q, then use label selection criteria to determine if new label should replace one of the other Q labels. If it does, add node-label pair of new label to SE list.

[c] Repeat for each predecessor node. If completed, go to Step 2.

(4) Stop. P-efficient labels are permanent. The algorithm terminates with a subset of the Pareto-optimal least time paths (or a set of "superior" paths) and their associated cdf's or expected values $\forall\ t \in \mathcal{S}$ and $\forall\ i \in \mathcal{V}$ to the destination node.

Two dominance criteria are considered for use in Step 3: stochastic pairwise dominance and expected value pairwise dominance, described in Section 3.5 of Miller-Hooks (1997). When the number of labels maintained has reached the limit Q and the new label is p-efficient, the path labels to remain can be determined in a variety of ways. The simplest selection procedure is to randomly select which label to discard. A slightly more complicated procedure might incorporate the number of time intervals for which a label is non-dominated. In such a procedure, the Q labels with the largest number of time intervals for which each path is non-dominated are maintained and the remaining label is discarded. Similarly, the number of time intervals for which a path has the least expected travel time can be used.

In Chapter 6 of Miller-Hooks (1997), four procedures are tested. Dominance criteria for two procedures are based on expected value pairwise dominance, referred to by "EV." The remaining two are based on stochastic pairwise dominance referred to by "FOSD." The selection criteria used in one of each pair is based on the number of time intervals for which a path has the least expected time, referred to as "EX." The remaining procedure in each pair arbitrarily selects the paths to keep, and is referred to as "ARB." These combinations are summarized in Table 4.1. The results of these tests, in terms of performance and accuracy, are discussed in Miller-Hooks (1997).

Table 4.1: Procedures for the Limited STDLT Procedure

Procedure	Dominance Criterion	Selection Criterion
1	FOSD	EX
2	FOSD	ARB
3	EV	EX
4	EV	ARB

Legend:
FOSD Stochastic pairwise dominance
EV Expected value pairwise dominance
EX Selection criterion based on expected travel time
ARB Arbitrary selection criterion

Finally, the LEAST and ELB procedures can be given through a general procedural description as follows:

PROCEDURE 2 {for determining least possible time paths and least expected times}

(1) Initialize the labels and create the SE list.

(2) Select node from SE list. If SE list is empty, go to step 4.

(3) For each predecessor node:

[a] Construct new label.

[b] Compare new label to current label and update label accordingly.

IF new label replaces current label,

THEN add node to SE list.

[c] Repeat for each predecessor node. If completed, go to Step 2.

(4) Stop.

Procedure 2 is a very general description of the computational steps of the LEAST and ELB procedures. Again, definitions of the labels, and detailed description of label computation and comparison are required. A brief description of the labels required for each of these procedures is given next. The reader is referred to Chapter 5 of Miller-Hooks (1997) for a detailed description of the steps of these procedures.

(1) **LEAST algorithm**: Only one vector label is permitted from each node, where a single deterministic quantity is maintained for each departure time. Upon termination, this quantity gives the least possible time to the destination node. The corresponding path and a lower bound on the probability of the occurrence of this travel time are also maintained. By allowing multiple vector labels at each node for each departure time, the LEAST algorithm is expanded to determine up to k absolute least possible time paths, the associated least possible travel times and corresponding probabilities of occurrence of the travel times (or a lower bound on the probabilities) and is referred to as the **TOP algorithm.**

(2) **ELB Procedure**: Only one vector label is maintained from each node. Upon termination, each entry in the vector label corresponds to a lower bound on the expected time of the least expected time paths for the given departure time.

Unlike the vector labels used in the STDLT(DD), Range, STDLT(SD) and EV algorithms, each entry in each vector label for different departure times for the LEAST, TOP and ELB procedures may correspond to different paths to the destination node.

Flow charts for these procedures are given in Appendix A.

5. DETERMINING "SUPERIOR" LEAST COST PATHS

The primary concern in adapting the algorithms for minimizing time to algorithms for minimizing cost is in computing the labels. If the objective is to minimize cost, then the labels for the cdf's of path travel time are replaced with the cdf's of the cost of traveling between that origin and the final destination. Although the costs are time-dependent, the computation of the cost label is slightly different from that of the travel time label. In this section, the required changes for some of the algorithms of Section 4 are presented. From these descriptions, the adaptation of the procedures that are not described should be apparent.

All of the assumptions concerning the travel times, and the network in general, should be assumed about the costs. Thus, we assume that the arc costs are independent and additive. In addition, arc costs are assumed to be independent of arc travel times. As the costs are time-varying quantities, the cdf of the cost of a path, for a given departure time, is determined from the convolution of the conditional pdf's or pmf's of the arc costs as described in this section.

5.1 DETERMINISTIC AND STOCHASTIC DOMINANCE

The implementation of the STDLT(DD) algorithm, presented in Section 4.2 of Miller-Hooks (1997) involving the aggregation of the distribution functions into iso-probability intervals, α, is used to generate the stochastically and deterministically non-dominated paths for the example problem. Just as $\tau^k_{i,j}(t)$ are converted to cumulative distribution functions, aggregated into intervals of α, $\Omega^p_{i,j}(t)$, $C^x_{i,j}(t)$ can be converted similarly.. Assuming the cost of an arc depends on the departure time from the arc's origin and not on the revealed (actual) travel time of the arc, the number of possible combinations of travel times on arc (i,j) with travel times on the subpath from node j at the corresponding arrival times is α^{-3}, a constant. Let $\xi^p_{i,j}(t)$ be the cumulative distribution function of the cost of travel on arc (i,j) aggregated into intervals of size α.

Let the cth label of node i at time t, be given by the vector $[\Theta^{p,c}_i(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$, where $\Theta^{p,c}_i(t)$ is the aggregated cdf of the cost of traveling on the cth Pareto-optimal path from node i to the selected destination node, N, at time t, where $c \in \aleph_i$ identifies a path from among the set of Pareto-optimal paths, and where $p\in\{\alpha, 2\alpha, \ldots, 1\}$ identifies the probability interval. Thus, $\Theta^{p,c}_i(t)$ are defined similarly to $\lambda^{p,c}_i(t)$. Also, let $[\phi^p_i(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$ be the aggregated distribution function of the cost to travel from node i at time t on a newly constructed path, similar in definition to the cdf of the new label for time, $[\kappa_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$. The set of node labels associated with path c from node i for all departure times $t \in \mathcal{S}$, $\{[\Theta_i^{p,c}(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}\}_{t\in\mathcal{S}}$ is denoted Ξ_i^c.

$[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$ are determined by a similar procedure to that of the $[\kappa_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$, presented in the description of the implementation of Step 5 of the STDLT(DD) algorithm in Section 4.2.2 in Miller-Hooks (1997). The temporary label of a new path, $[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$, can be constructed using the aggregated arc (i,j) costs and the cdf(α)'s of the subpath from node j. For each departure time, there are α^{-3} possible combinations of costs of arc (i,j) with costs of the subpath from node j at the corresponding arrival times. For each combination, $q = 1, 2, \ldots \alpha^{-3}$, the path cost at time $t \in \mathcal{S}$, $\eta_i^q(t)$, is calculated, from which the temporary label of the new path, $\phi_i^p(t)$, is calculated.

$$\eta_i^q(t) = \xi_{i,j}^x(t) + \Theta_j^{r,\mu}(t + \Omega_{i,j}^p(t)),$$

where $p = \left[\left\lfloor \frac{q-1}{1/\alpha} \right\rfloor + 1\right] \cdot \alpha$, $x = \left[\alpha \cdot \left(\left\lfloor \frac{q-1}{1/\alpha} \right\rfloor + 1\right) \bmod \frac{1}{\alpha}\right]$ and

$r = \alpha(((q-1) \bmod 1/\alpha)+1)$; $p, x, r \in \{\alpha, 2\alpha, \ldots, 1\}$. Then

$$[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}} = \eta_i^{(p/\alpha^3)}(t)$$

where $\{\eta_i^{(q)}(t)\}$ are the $\{\eta_i^q(t)\}$ sorted in ascending order. Repeat for all $t \in \mathcal{S}$. Thus, for any $i \in \mathcal{V}$ and $t \in \mathcal{S}$, the probability of any element of $\eta_i^q(t)$ is α^3 and thus, the probability of any element of $\phi_i^p(t)$ and of $\Theta_i^{p,c}(t)$ is α.

Every time label is built from two sets of distributions: the distribution of time on an arc (i,j) for each departure time from node i and the distributions of the times on the subpath from node j to the destination node for all possible arrival times at node j via arc (i,j). Each cost label is built from the cost distribution for the given departure time on arc (i,j) and the distributions of the costs on the subpath from node j for all possible arrival times at node j via arc (i,j). Thus, one must condition on the arrival times in order to determine which cost functions to use for the subpath.

The main difference in the calculation of the distribution function of the path cost and the path travel time is that the calculation of the cost requires an additional condition. That is, the probability of a given cost of the path from i is the probability that travel time on arc (i,j) is $\tau_{i,j}^k(t)$ (or, $\Omega_{i,j}^p(t)$) times the probability that the cost of that arc is $C_{i,j}^x(t)$ (or, $\xi_{i,j}^x(t)$) times the probability that the cost on the subpath from j given the arrival time is $t + \tau_{i,j}^k(t)$ (or, $t + \Omega_{i,j}^p(t)$). Thus, there is one more component in the calculations of the path cost cdf's than there is for time.

Once the cdf's of the cost of a path from node i, $[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}$, is calculated, it is compared to the p-efficient paths at node i.

Deterministic Dominance

$\{[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}\}_{t\in S}$ is deterministically p-efficient iff $\exists$ no Ξ_i^c such that

$$\Theta_i^{1,c}(t) \le \phi_i^{\alpha}(t)\ \forall\ t \in S \textit{ and } \exists\ t \in S \mid \Theta_i^{1,c}(t) < \phi_i^{\alpha}(t).$$

First-Order Stochastic Dominance

Similar to the definition of deterministically p-efficient paths, $\{[\phi_i^p(t)]_{p\in\{\alpha,2\alpha,\ldots,1\}}\}_{t\in S}$ is stochastically p-efficient iff $\exists$ no $\Xi_i^c \in \aleph_i$ such that

$$\Theta_i^{p,c}(t) \le \phi_i^p(t)\ \forall\ p\in\{\alpha, 2\alpha, \ldots, 1\} \text{ and } \forall t \in S \textit{ and}$$

$$\exists\ p \text{ and } \exists\ t \in S \mid \Theta_i^{p,c}(t) < \phi_i^p(t).$$

5.2 Expectation

EV Algorithm

The adaptation of the EV algorithm to minimization of cost is simpler than the extension of the STDLT (DD) and STDLT (SD) algorithms to cost. Let $\Theta_i^c(t)$ be the expected cost of traveling on path c from node i to the destination at departure time t. Let $\phi_i(t)$ be the label of the cost of a newly constructed path from node i to the destination node at departure time t. Equation 4.1 of Step 5 in the description of the EV algorithm presented in Section 4.4.3 in Miller-Hooks (1997) for computing least expected time paths is replaced by Equation 5.1 for determining least expected cost paths, as follows:

$$\phi_i(t)=\sum_k\sum_x\left(\left[c_{i,j}^x(t)+\Theta_j^{\mu}(t+\tau_{i,j}^k(t))\right]\cdot q_{i,j}^x(t)\cdot\rho_{i,j}^k(t)\right) \qquad (5.1)$$

where $k=1,2,\ldots,K_{i,j}(t)$ and $x=1,\ldots, X_{i,j}(t)$ (the indices of possible travel times and costs, respectively, on arc (i,j) at time t). Changes in the optimality conditions described for the extension of the STDLT(DD) algorithm to cost are obvious for the EV algorithm, and thus, will not be presented.

ELB Procedure

Likewise, the adaptation of the ELB algorithm to minimization of cost can be given as follows: Let $\Theta_i(t)$ be the label maintaining a lower bound on the cost of the least expected cost path from node i to the destination at departure time t and $\phi_i(t)$ be the label of the lower bound on the cost of a newly constructed path from node i to the destination node at departure time t. The procedure for computing the node labels to determine a lower bound on the times of the least expected time paths, given in equation (5.3) in step 4 of the ELB algorithm presented in Section 5.4.1 of Miller-Hooks (1997), is extended for determining a lower bound on the costs of the least expected cost paths as follows:

For a given $t \in \mathcal{S}$:

$$\phi_i(t)=\sum_k \sum_x \left[\left(c^x_{i,j}(t)+(\Theta_j(t+\tau^k_{i,j}(t))\right) \cdot q^x_{i,j}(t) \cdot \rho^k_{i,j}(t)\right] \qquad (5.2)$$

where $k=1,2,\ldots,K_{i,j}(t)$ and $x=1,\ldots, X_{i,j}(t)$.

5.3 Least Possible Cost

Similarly, the LEAST algorithm can be extended as follows: A single vector label is associated with each node $[\Theta^m_i(t)]_{m\in\{1,2\}}$, where $\Theta^1_i(t)$ maintains the cost of the path with the least possible value from node i to the destination at departure time t and $\Theta^2_i(t)$ maintains the lower bound on the probability of the occurrence of $\Theta^1_i(t)$. At each iteration of the algorithm, for each departure time interval t, a temporary label $[\phi^m_i(t)]_{m\in\{1,2\}}$ is constructed, where $\phi^1_i(t)$ is the minimum cost determined in this iteration from node i to the destination node with its associated lower bound on the probability of its occurrence, ϕ^2_i. To determine the least possible time path with its time and associated lower bound on the probability of the occurrence of this time, Step 4 of LEAST algorithm, presented in Section 5.3.1.1 of Miller-Hooks (1997), is replaced by the following Step 4':

Step 4'. Update the Node Labels (Cost)

For each $t \in \mathcal{S}$, update the vector $[\Theta_i^m(t)]_{m \in \{1,2\}}$ as follows:

For every possible travel time from node i at departure time t, indexed by p,

for every possible cost from i at time t, indexed by x,

Find the minimum label, $\phi_i^1(t)$ with maximum $\phi_i^2(t)$ in the case of ties where:

$$\phi_i^1(t) = \{ C_{i,j}^x(t) + \Theta_j^1(t + \tau_{i,j}^p(t)) \},$$

where p and x are the sets of indices of possible travel times and possible costs, respectively, on arc (i,j) at time t.

Let (s,r) be the pair of values of p and x for which $\phi_i^1(t)$ is minimum and in the case of a tie, where $\phi_i^2(t)$ is maximum. Then,

$$\phi_i^2(t) = \{ q_{i,j}^r \cdot \rho_{i,j}^s(t) \cdot \Theta_j^2(t + \tau_{i,j}^s(t)) \}.$$

If $\{ \phi_i^1(t) < \Theta_i^1(t)$ or $(\phi_i^1(t) = \Theta_i^1(t)$ and $\phi_i^2(t) > \Theta_i^2(t)) \}$ then

$\Theta_i^m(t) = \phi_i^m(t) \ \forall \ m \in \{1,2\}$

$\pi_i^1(t) = j,$

$\pi_i^2(t) = \min \{ t + \tau_{i,j}^s(t), t_0 + I\delta \},$

flag = 1 {flag indicates that node i should be put in the SE list}.

If i $\notin$ SE and flag = 1: flag = 0, put i in SE list.

Go to Step 3.

5.4 LIMITED STDLT

The extension to cost of the STDLT(DD) and STDLT(SD) algorithms apply to the FOSD-EX Limited STDLT procedure.

6. "SUPERIOR" PATHS FOR MULTIPLE OBJECTIVES

In Section 4, an overview of the specific algorithmic steps for determining optimal and Pareto-optimal least time paths are presented. In Section 5, the required modifications to these procedures for determining the Pareto-optimal and other "superior" paths for the single objective problem of "minimizing" population (potential population exposure) are given. In this section, the problem of generating the Pareto-optimal paths for the multi-objective problem of "minimizing" both travel time and population is addressed. Here, the concept of Pareto-optimality arises as a consequence of multiple conflicting objectives of stochastic, time-varying attributes. In order to solve this multi-objective problem, the STDLT(DD), STDLT(SD) and EV algorithms are extended for generating Pareto-optimal paths for the multiple stochastic, time-varying criteria problem.

6.1 Deterministic Dominance

Two quantities are identified in the definition of deterministic dominance from Section 3.4 of Miller-Hooks (1997), $x_{\mathcal{B}}^{t,max} | M^t$ and $x_{\mathcal{A}}^{t,min} | M^t$, representing the maximum and minimum travel times on paths $\mathcal{B}$ and $\mathcal{A}$, respectively, given M^t for departure time t from the origin of the path. Let $c_{\mathcal{B}}^{t,max} | M^t$ and $c_{\mathcal{A}}^{t,min} | M^t$ be the maximum and minimum costs on paths $\mathcal{B}$ and $\mathcal{A}$, respectively, given M^t for departure time t from the origin of the path. These quantities are defined identically to travel times $x_{\mathcal{B}}^{t,max} | M^t$ and $x_{\mathcal{A}}^{t,min} | M^t$ where the notion of travel time is replaced with the notion of cost. Pareto-optimality conditions in terms of deterministic pairwise dominance for the bi-criterion problem are given as follows:

A path $\mathcal{A}$ is efficient iff $\exists$ no path $\mathcal{B}$ such that $\forall$ realizations of M^t,

$$x_{\mathcal{B}}^{t,max} | M^t \le x_{\mathcal{A}}^{t,min} | M^t \;\; \forall \; t \in \mathcal{S} \text{ and } \exists \; t \in \mathcal{S} \,|\, x_{\mathcal{B}}^{t,max} | M^t < x_{\mathcal{A}}^{t,min} | M^t$$

and

$$c_{\mathcal{B}}^{t,max} | M^t \le c_{\mathcal{A}}^{t,min} | M^t \;\; \forall \; t \in \mathcal{S} \text{ and } \exists \; t \in \mathcal{S} \,|\, c_{\mathcal{B}}^{t,max} | M^t < c_{\mathcal{A}}^{t,min} | M^t;$$

otherwise, the path is dominated.

This implies that if a path $\mathcal{B}$ exists that is better on both criteria than $\mathcal{A}$ for all departure times, then path $\mathcal{A}$ is dominated. Conversely, if a path $\mathcal{A}$ exists that is dominated, then a path $\mathcal{B}$ must exist that is better on both criteria. There are two exceptions: if a path $\mathcal{B}$ exists for which the travel time (cost) pdf's are identical to those of path $\mathcal{A}$, then the cost (travel time) pdf's can be used alone to determine if either path dominates the other. If both the travel time pdf's and cost pdf's of path $\mathcal{B}$ are identical to those of path $\mathcal{A}$, then these paths are equivalent.

6.2 STOCHASTIC DOMINANCE

Likewise, stochastic pairwise dominance is defined in Section 3.4 of Miller-Hooks (1997). If x and c are the travel time and cost (population) random variables of path $\mathcal{A}$, with distribution functions $F^t_{\mathcal{A}}(x)$ and $F^t_{\mathcal{A}}(c)$ at departure time t, the Pareto-optimality conditions for stochastic pairwise dominance for the bi-criterion problem can be given as follows:

A path $\mathcal{A}$ is efficient iff $\exists$ no path $\mathcal{B}$ such that,

$$F^t_{\mathcal{B}}(x) \geq F^t_{\mathcal{A}}(x) \ \forall \ x \text{ and } \forall \ t \in \mathcal{S} \text{ and } \exists \ x \text{ and } \exists \ t \in \mathcal{S} \mid F^t_{\mathcal{B}}(x) > F^t_{\mathcal{A}}(x);$$

and

$$F^t_{\mathcal{B}}(c) \geq F^t_{\mathcal{A}}(c) \ \forall \ x \text{ and } \forall \ t \in \mathcal{S} \text{ and } \exists \ x \text{ and } \exists \ t \in \mathcal{S} \mid F^t_{\mathcal{B}}(c) > F^t_{\mathcal{A}}(c);$$

otherwise, the path is dominated.

6.3 EXPECTATION

Similarly, $E[c_{\mathcal{A}}]_t$ and $E[c_{\mathcal{B}}]_t$ are the expected cost of paths $\mathcal{A}$ and $\mathcal{B}$, respectively, at time interval t. These definitions correspond directly to the definitions of the expected travel times, $E[x_{\mathcal{B}}]_t$ and $E[x_{\mathcal{A}}]_t$, defined in Chapter 3 of Miller-Hooks (1997). Efficiency conditions in terms of expected value pairwise dominance are given as follows:

A path $\mathcal{A}$ is efficient iff $\exists$ no path $\mathcal{B}$ such that,

$$E[x_{\mathcal{B}}]_t \leq E[x_{\mathcal{A}}]_t \ \forall \ t \in \mathcal{S} \text{ and } \exists \ t \in \mathcal{S} \mid E[x_{\mathcal{B}}]_t < E[x_{\mathcal{A}}]_t$$

and

$$E[c_{\mathcal{B}}]_t \leq E[c_{\mathcal{A}}]_t \ \forall \ t \in \mathcal{S} \text{ and } \exists \ t \in \mathcal{S} \mid E[c_{\mathcal{B}}]_t < E[c_{\mathcal{A}}]_t$$

otherwise, the path is dominated.

As the expectation of a function is a single quantity, and not a function, equivalence is implicitly taken into account.

By replacing the efficiency conditions for a single criterion of the STDLT(SD) and EV algorithms with those of the bi-criterion problem, the STDLT(SD) and EV algorithms can be used to generate the Pareto-optimal solutions for the multiobjective problem.

7. APPLICATION TO TEXAS NETWORK

7.1 LEAST TIME PATHS

In Section 7.7, Figure 7.1 is provided to aid in choosing a single "best compromise" path. This figure may be useful to the reader throughout this section.

7.1.1 Deterministic Dominance

Seven paths are generated by the STDLT(DD) algorithm using cdf(α)'s with α of 0.1, described in Section 4.2 of Miller-Hooks (1997); these paths are deterministically non-dominated for at least one time interval in the period of interest. Of these paths, only 3 are non-dominated at the first time interval. These three paths are:

Path 1• 8-102-77-25-153-1-166-160-140-108-137-171-4
Austin-Lampasas-Goldthwaite-Brownwood-Santa Anna-Abilene-Sweetwater-Snyder-Post-Lubbock-Plainview-Tulia-Amarillo
Path 2• 8-93-68-94-161-128-126-154-23-108-137-171-4
Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Seminole-Brownfield-Lubbock-Plainview-Tulia-Amarillo
Path 3• 8-93-68-94-161-128-126-117-101-167-108-137-171-4
Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Midland-Lamesa-Tahoka-Lubbock-Plainview-Tulia-Amarillo

The approximations of the path travel time cdf's for the first time interval are given in Table 7.1. The probability that the path is less than a given travel time is given in the left-most column. For example, the probability that Path 2 takes less than 544.1 minutes is 0.3.

Table 7.1: Cdf(α)'s of Travel Time on Paths 1, 2 and 3 for Time Interval 1

Cumulative Probability	Travel time (minutes) Path 1	Path 2	Path 3
0.1	525.1	539.2	524.0
0.2	527.5	541.8	526.8
0.3	531.0	544.1	529.4
0.4	533.0	546.8	531.6
0.5	535.5	548.8	533.6
0.6	538.4	551.1	535.9
0.7	542.7	554.2	538.3
0.8	545.8	561.1	542.8
0.9	551.3	568.6	550.3
1	562.3	577.2	558.8

7.1.2 Stochastic Dominance

The STDLT(SD) algorithm using cdf(α)'s with α of 0.1, described in Section 4.3 of Miller-Hooks (1997), is used to generate the stochastically Pareto-optimal paths. Two stochastically non-dominated paths are determined, only one of which is non-dominated for the first departure time interval. This path is Path 3, described in the results for deterministic dominance. The other path that is dominated for the first time interval but is non-dominated for at least one other time interval is Path 1.

7.1.3 Expectation

Two paths are EV-non-dominated over the time period, as determined by the EV algorithm, described in Section 4.4 of Miller-Hooks (1997). Again, only one of these paths is non-dominated for the first departure time interval. This path is also Path 3 described in the results for deterministic dominance. This path's expected travel time is 520.2 minutes or 8.67 hours. Again the other path that is dominated for the first time interval but non-dominated for at least one other time interval is Path 1 from the results for deterministic dominance. The expected travel time of this path is 522.2 minutes or 8.70 hours.

The results of the ELB algorithm (of Section 4) are exact as the solution is 520.2 minutes for the least expected time from Austin to Amarillo for the first departure time interval.

7.1.4 Least Possible Time

As determined by the LEAST algorithm (of Section 4), the path with the least possible time from Austin for the first departure time interval is also Path 3 described in the results of deterministic dominance. The least possible time on this path is 479.1 minutes.

7.1.5 Limited STDLT

Four procedures based on the Limited STDLT procedure are described in Section 5.2 of Miller-Hooks (1997). For this problem, these procedures determine the same set of paths. As this application does not require real-time decision-making, only the version based on first-order stochastic dominance, referred to as Procedure 1 in Table 4.1 (FOSD-EX) of Section 4, is used in this example problem.

The results of the tests are as follows: When the number of labels maintained at each node is limited to one label, the path determined is Path 1 of the results for deterministic dominance. When the number of labels maintained at each node is increased to two or more, two paths are non-dominated from Austin: Paths 1 and 3; however, Path 3 is dominated for the first time interval.

Note that all four Limited STDLT procedures depicted in Table 4.1 result in identical paths for the same maximum number of permitted labels.

7.2 Least Cost Paths

In this section, the results of applying the algorithms for determining "least cost" paths for the Austin-Amarillo shipment are described.

7.2.1 Deterministic Dominance

The problem of determining the Pareto-optimal least cost paths cannot be solved using a reasonable amount of memory on the computer as there are more than 375 simple paths from Houston (node 87) to Amarillo that are non-dominated for at least one time interval, even with only 4 time intervals of 15 minutes each. There are more than 98 non-dominated simple paths from Austin when the program is halted. It appears that many paths are deterministically Pareto-optimal when the comparison is based on the number of people potentially exposed. Thus, first-order stochastic dominance should provide a more reasonably sized solution set.

7.2.2 Stochastic Dominance

Two paths are non-dominated with respect to cost, only the first of which is non-dominated for the first departure time from Austin.

Path 4• 8-102-21-59-10-1-6-80-156-16-54-47-108-137-171-4
Austin-Lampasas-Brady-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Benjamin-Dickens-Crosbyton-Lubbock-Plainview-Tulia-Amarillo
Path 5• 8-93-68-94-59-10-1-6-80-156-16-54-47-108-137-171-4
Austin-Johnson City-Fredericksburg-Junction-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Benjamin-Dickens-Crosbyton-Lubbock-Plainview-Tulia-Amarillo

The population probability distribution functions for the first departure time interval are given in Table 7.2.

Table 7.2: Cdf(α)'s of Population on Paths 4 and 5 for Time Interval 1

Cumulative Probability	Population (persons) Path 4	Path 5
0.1	83902	84422
0.2	86369	86490
0.3	87695	88215
0.4	90145	90266
0.5	91471	91592
0.6	93114	93634
0.7	95569	96043
0.8	97679	98199
0.9	100618	101138
1	112005	112525

7.2.3 Expectation

EV Algorithm

Two non-dominated paths are determined that are identical to those found using first-order stochastic dominance. Again, only Path 4 is non-dominated for the first departure time interval. Its expected population is 83,591 people. Path 5 is non-dominated for at least one other time interval, but not the first. Its expected population for the first time interval is 84,259 people.

ELB Procedure

The lower bound for the expected population on the least expected population path between Austin and Amarillo for the first departure time, determined by the extension of the ELB procedure, is 83,591. This is the exact value of the least expected cost path found using the extension of the EV algorithm.

7.2.4 Least Possible Cost

The path with the absolute least possible number of people for the first departure time is Path 5, described in the results for stochastic dominance, with a least possible number of 58,003 people.

7.2.5 Limited STDLT

For the first test, the number of labels is limited to one label from each node. The following path is determined from Austin:

> **Path 6•** 8-93-68-94-161-128-126-154-23-108-137-171-4
> Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Seminole-Brownfield-Lubbock-Plainview-Tulia-Amarillo

The cdf of this path for the first departure time is given in Table 7.3.

Table 7.3: Cdf(α) of Population on Path 6 for Time Interval 1

Cumulative Probability	Population (persons) Path 6
0.1	87848
0.2	90724
0.3	92250
0.4	93937
0.5	95300
0.6	97386
0.7	99835
0.8	102078
0.9	105078
1	112927

When the number of labels at each node is limited to 2 or more, Paths 4 and 5, described in the results for stochastic dominance, are identified as non-dominated. However, only Path 4 is non-dominated for the first departure time interval.

7.3 MULTIPLE OBJECTIVES

In Section 7.1, optimal and Pareto-optimal least time paths are determined for the example problem. Similarly in Section 7.2, Pareto-optimal and other "superior" paths are generated for the single objective problem of "minimizing" population (potential population exposure). In this section, Pareto-optimal paths are determined for the multiobjective problem of "minimizing" both travel time and population.

7.3.1 Deterministic Dominance

Results for deterministic dominance are not obtained due to the explosive nature of the solution set.

7.3.2 Stochastic Dominance

By the extension of the STDLT(SD) algorithm, five paths are determined to be non-dominated, four of which are non-dominated for the first departure time interval. These paths include Paths 1, 2, 3, and 4, described previously. The fifth non-dominated path that is dominated at the first time interval is Path 5, described previously. Although some of these have been previously shown, the cdf(α)'s of Paths 1 through 4 with respect to both travel time and potential population exposure are shown in Tables 7.4 and 7.5 as follows:

Table 7.4: Cdf(α)'s of Travel Time on Paths 1, 2, 3 and 4 for Time Interval 1

Cumulative Probability	Travel Time (minutes) Path 1	Path 2	Path 3	Path 4
0.1	525.1	539.2	524.0	675.3
0.2	527.5	541.9	526.8	679.0
0.3	531.0	544.1	529.4	682.2
0.4	533.3	546.8	531.6	684.7
0.5	535.5	548.8	533.6	687.4
0.6	538.5	551.1	535.9	691.1
0.7	542.7	554.2	538.3	694.9
0.8	545.8	561.1	542.8	700.3
0.9	551.3	568.6	550.3	709.5
1	562.3	577.2	558.8	720.5

Table 7.5: Cdf(α)'s of Population on Paths 1, 2, 3 and 4 for Time Interval 1

Cumulative Probability	Population (persons) Path 1	Path 2	Path 3	Path 4
0.1	109938	87848	119293	83902
0.2	112255	90724	122522	86369
0.3	114930	92250	124678	87695
0.4	117166	93937	127735	90145
0.5	119570	95300	130556	91471
0.6	121391	97386	134782	93114
0.7	124625	99835	138243	95569
0.8	128282	102078	143292	97679
0.9	132922	105185	149516	100618
1	137971	112927	157258	112005

7.3.3 Expectation

Six non-dominated paths are determined using the multi-objective extension of the EV algorithm. Two of these paths are dominated at the first time interval. The four paths that are non-dominated at the first time interval are Paths 1, 2, 3, and 4 described previously. The remaining two paths are Path 5 and the following path:

Path 7• 8-102-21-59-10-1-166-160-140-108-137-171-4
Austin-Lampasas-Brady-Eden-Ballinger-Abilene-Sweetwater-Snyder-Post-Lubbock-Plainview-Tulia-Amarillo .

The expected travel time and number of people potentially exposed on Paths 1 through 6 for the first time interval are given in Table 7.6.

Table 7.6: Expected Travel Time and Population on Paths 1 to 6 for Time Interval 1

	Path 1	Path 2	Path 3	Path 4	Path 5*	Path 6*
Travel Time (minutes)	522.2	535.2	520.2	667.8	676.3	571.6
Population (persons)	110206	88861	122613	83591	84259	101950

*These paths are dominated in the first departure time interval.

7.4 Final Path Selection

In this section, the "best compromise" path is selected for the example problem of determining a route for the transport of a hazardous substance from Austin to Amarillo, considering both travel time and population. The pertinent cities of the paths that are considered are indicated by the first three (or four) letters of their names in the map shown in Figure 7.1.

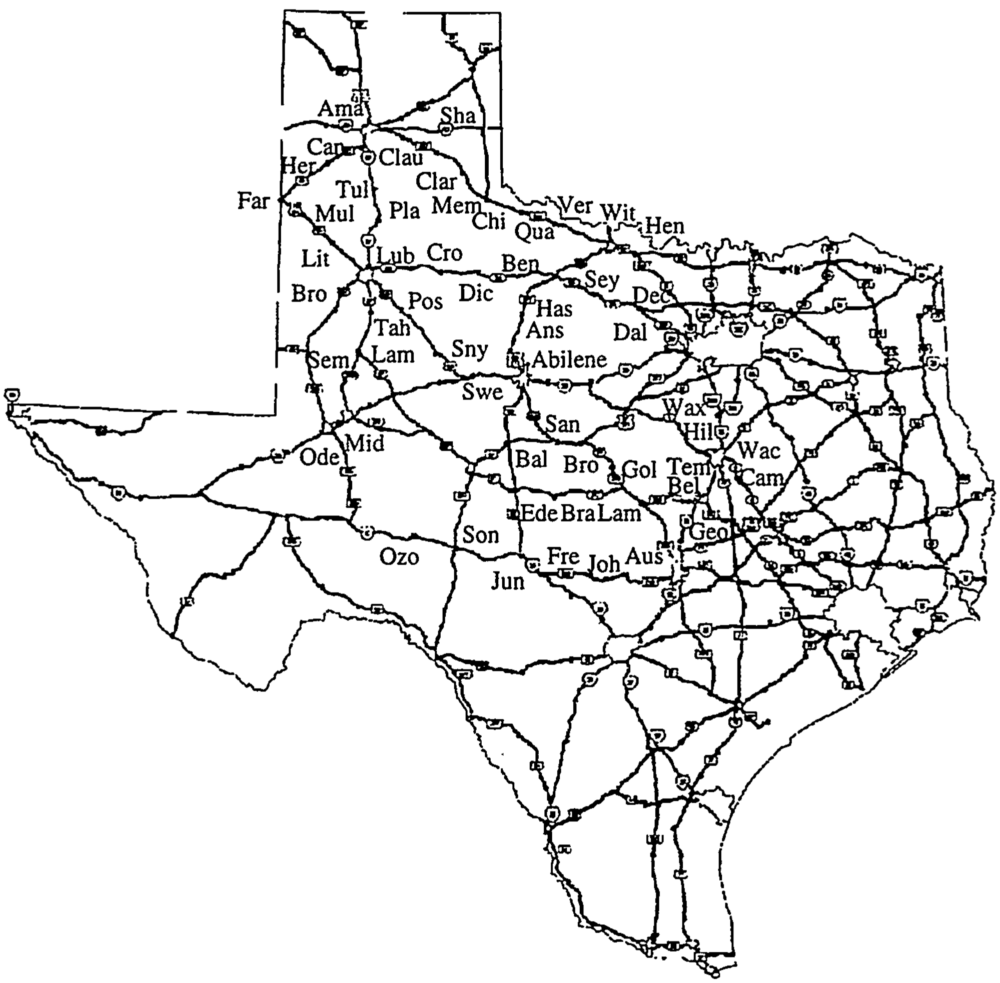

Figure 7.1: Map of Texas with Location of Pertinent Cities Along Select Paths

Path 3 is the path that would most likely be selected to ship the hazardous substances if only travel time was considered. This path has the least expected travel time for the first departure time and therefore is deterministically and stochastically Pareto-optimal for that departure time. In addition, it is the path with the least possible time and the lowest maximum possible travel time of the paths. On the other hand, Path 4 would be chosen if only the population along the route was considered. This path is the only path that is stochastically non-dominated with respect to population at the first time interval and is the least expected population path.

For the first departure time, the Pareto-optimal paths determined by the multi-objective extensions of the STDLT (SD) and EV algorithms are Paths 1, 2, 3 and 4. To aid in choosing a "best compromise" path, the efficient frontier between the expected travel time and expected population for time interval 1 is plotted, as shown in Figure 7.2.

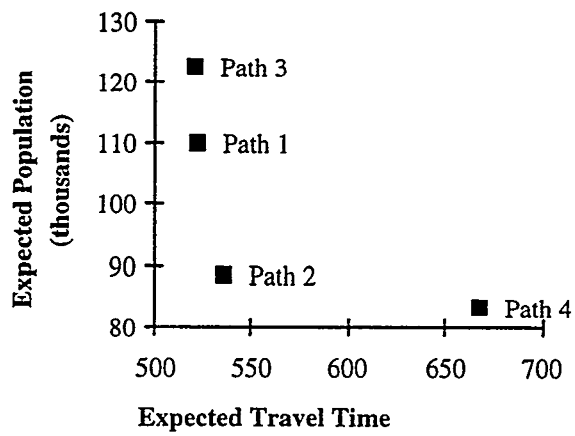

Figure 7.2: Efficient Frontier for the First Departure Time Interval

Neither Path 3 nor Path 4 would be the "best compromise" solution if one simultaneously considers both travel time and population because Path 3 has the highest expected population of all four paths and Path 4 has the highest expected travel time of the paths. Either Path 1 or 2 would make a better compromise. Since the difference in the population between the paths is much more significant than is the difference in travel time and since the highest possible population on Path 1 is significantly higher than the highest possible on Path 2, Path 2 is recommended. Of course, ultimate selection would depend on the decision-maker's preferences. The purpose of this analysis is to inform the decision process regarding the implications of alternative path selections.

8. EFFECT OF AN INCIDENT

Suppose on the day of the shipment two unforeseen events occur: On the link between Tulia and Amarillo (171,4) pipes burst somewhere under the highway, requiring road closure until the problem is fixed -- essentially closing the road for the day. Second, at 7:30 a.m., an incident occurs on the road between Austin and Lampasas (8,322) which takes 30 minutes to clear. At the last minute a new path that minimizes both criteria (travel time and population) is required. The results are given in Sections 8.1 through 8.4.

8.1 LEAST TIME PATHS

Using the STDLT(SD) algorithm, two nearly identical non-dominated paths are determined:

Path 8• 8-74-15-168-178-85-179-50-51-83-182-176-142-35-116-36-38-4
Austin-Georgetown-Belton-Temple-Waco-Hillsboro-Waxahachie-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Memphis-Clarendon-Claude-Amarillo
Path 9• 8-74-15-168-178-85-50-51-83-182-176-142-35-116-36-38-4
Austin-Georgetown-Belton-Temple-Waco-Hillsboro-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Memphis-Clarendon-Claude-Amarillo

The corresponding cdf's of these paths for time interval 1 are given in Table 8.1.

Table 8.1: Cdf(α)'s of Travel Time on Paths 8 and 9 for Time Interval 1

Cumulative Probability	Population (persons) Path 8	Path 9
0.1	575.8	575.9
0.2	578.3	578.4
0.3	581.0	581.1
0.4	583.2	583.6
0.5	585.7	586.5
0.6	588.1	589.0
0.7	592.3	593.1
0.8	597.4	597.5
0.9	603.4	603.3
1	612.0	615.2

With respect to expected value, Path 9 is the only non-dominated path with an expected travel time of 565.2 minutes or 9.42 hours. Using the FOSD-EX Limited STDLT procedure with one label permitted for each node, Path 8 is determined. When the number of labels is increased to 2 or higher, two paths are determined: Paths 8 and 9. Using the LEAST algorithm, Path 9 is found to have the least possible time. By the ELB algorithm, the lower bound is equivalent to the expected time on the least expected time path found using the EV algorithm: 565.2 minutes.

8.2 LEAST COST PATHS

Applying the extension of the STDLT (SD) algorithm to cost, two non-dominated paths, Paths 10 and 11, are determined, only the first of which is non-dominated for the first departure time interval. The corresponding cdf's of these paths are given in Table 8.2.

Path 10• 8-102-21-59-10-1-6-80-156-16-54-47-108-137-171-4
Austin-Lampasas-Brady-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Benjamin-Dickens-Crosbyton-Lubbock-Plainview-Tulia-Amarillo
Path 11• 8-93-68-94-59-10-1-6-80-156-16-54-47-108-137-171-4
Austin-Johnson City-Fredericksburg-Junction-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Benjamin-Dickens-Crosbyton-Lubbock-Plainview-Tulia-Amarillo

Table 8.2: Cdf(α)'s of Population on Paths 10 and 11 for Time Interval 1

Cumulative Probability	Population (persons) Path 10	Path 11
0.1	83902	84422
0.2	86369	86490
0.3	87695	88215
0.4	90145	90266
0.5	91471	91592
0.6	93114	93634
0.7	95569	96043
0.8	97679	98199
0.9	100618	101138
1	112005	112525

Through the extension of the EV algorithm for determining least expected cost paths, the same two paths, Path 10 and 11, are determined. Again, only Path 10 is non-dominated for the first time interval. The expected population on Path 10 is 83,591 people and on Path 11 is 84,259 people.

Applying the least cost version of the FOSD-EX procedure, Paths 10 and 11 are generated, given at least 2 labels at each node are permitted. Again only Path 10 is non-dominated for the first departure time interval. If only 1 label is permitted at each node, the following path is determined, with corresponding population cdf given in Table 8.3:

Path 12• 8-93-68-94-161-128-126-154-23-108-137-171-4
Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Seminole-Brownfield-Lubbock-Plainview-Tulia-Amarillo

Table 8.3: Cdf(α) of Population on Path 12 for Time Interval 1

Cumulative Probability	Population (persons) Path 12
0.1	87848
0.2	90724
0.3	92250
0.4	93937
0.5	95300
0.6	97386
0.7	99835
0.8	102078
0.9	105185
1	112927

By the extension of the LEAST algorithm for determining the path with the least possible cost, Path 11 is determined. By the extension of the ELB algorithm for determining a lower bound on the cost of the least expected cost path, the value is 83591 people which is identical to the expected population on the least expected cost path, Path 10, determined by the extension of the EV algorithm.

8.3 MULTIPLE OBJECTIVES

Two algorithms are run to determine the Pareto-optimal paths with respect to both travel time and population: the extension of the EV and STDLT(SD) algorithms, described in Section 6. By the extension of the STDLT(SD) algorithm, eleven paths are determined, ten of which are non-dominated for the first time interval. These ten paths include Paths 8 and 9, described previously. Due to extremely high travel times on the links between Austin and Lampasas and Tulia and Amarillo, paths that include these links are not considered, despite the fact that such a path may be best in terms of the number of people potentially exposed. The remaining 8 paths are as follows:

Path 13• 8-93-68-94-161-128-126-117-101-167-108-105-122-65-84-31-4
Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Midland-Lamesa-Tahoka-Lubbock-Littlefield-Muleshoe-Farwell-Hereford-Canyon-Amarillo

Path 14• 8-93-68-94-161-128-126-154-23-108-105-122-65-84-31-4
Austin-Johnson City-Fredericksburg-Junction-Sonora-Ozona-Odessa-Seminole-Brownfield-Lubbock-Littlefield-Muleshoe-Farwell-Hereford-Canyon-Amarillo

Path 15• 8-93-68-94-59-10-1-6-80-156-16-54-47-108-105-122-65-84-31-4
Austin-Johnson City-Fredericksburg-Junction-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Benjamin-Dickens-Crosbyton-Lubbock-Littlefield-Muleshoe-Farwell-Hereford-Canyon-Amarillo

Path 16• 8-93-68-94-59-10-1-6-80-156-182-176-142-35-157-4

Austin-Johnson City-Fredericksburg-Junction-Eden-Ballinger-Abilene-Anson-Haskell-Seymour-Wichita Falls-Vernon-Quanah-Childress-Shamrock-Amarillo

Path 17• 8-74-15-168-178-85-50-51-83-182-176-142-35-157-4

Austin-Georgetown-Belton-Temple-Waco-Hillsboro-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Shamrock-Amarillo

Path 18• 8-74-15-168-178-85-179-50-51-83-182-176-142-35-157-4

Austin-Georgetown-Belton-Temple-Waco-Hillsboro-Waxahachie-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Shamrock-Amarillo

Path 19• 8-74-28-178-85-179-50-51-83-182-176-142-35-116-36-38-4

Austin-Georgetown-Cameron-Waco-Hillsboro-Waxahachie-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Memphis-Clarendon-Claude-Amarillo

Path 20• 8-74-28-178-85-50-51-83-182-176-142-35-116-36-38-4

Austin-Georgetown-Cameron-Waco-Hillsboro-Dallas/Fort Worth-Decatur-Henrietta-Wichita Falls-Vernon-Quanah-Childress-Memphis-Clarendon-Claude-Amarillo

Note that many of these paths differ from one another only slightly. The travel time cdf's are given in Table 8.4, followed by the population cdf's in Table 8.5.

Table 8.4.a: Cdf(α)'s of Travel Time on Paths 13 through 16 for Time Interval 1

Cumulative Probability	Travel Time (minutes) Path 13	Path 14	Path 15	Path 16
0.1	589.2	604.1	751.2	709.7
0.2	591.9	606.8	754.6	713.3
0.3	594.6	609.1	757.4	716.4
0.4	597.0	612.0	760.3	719.6
0.5	598.8	614.2	762.8	724.2
0.6	601.3	616.6	767.1	729.2
0.7	604.5	620.4	772.4	736.7
0.8	609.6	627.9	779.9	744.9
0.9	617.1	635.4	788.4	753.4
1	625.6	644.0	797.7	764.4

Table 8.4.b: Cdf(α)'s of Travel Time on Paths 17 through 20 for Time Interval 1

Cumulative Probability	Travel Time (minutes) Path 17	Path 18	Path 19	Path 20
0.1	601.8	601.3	602.6	602.2
0.2	604.7	604.2	605.8	605.8
0.3	608.1	607.6	608.3	608.3
0.4	611.9	610.5	611.5	611.4
0.5	615.4	614.7	614.0	613.9
0.6	619.6	619.3	617.0	617.0
0.7	624.3	624.0	622.7	622.7
0.8	629.9	630.1	628.7	628.6
0.9	639.3	638.5	636.6	636.4
1	651.2	647.9	645.0	648.3

Table 8.5.a: Cdf(α)'s of Population on Paths 13 through 16 for Time Interval 1

Cumulative Probability	Population (persons) Path 13	Path 14	Path 15	Path 16
0.1	124114	92631	89033	88792
0.2	127353	95232	91873	95931
0.3	129990	97120	93199	98444
0.4	132354	98431	94740	100874
0.5	135752	100188	96066	106290
0.6	139889	102087	97817	108986
0.7	142815	104243	99815	115037
0.8	147364	106479	101971	126425
0.9	153588	109257	105210	133318
1	161330	116999	116597	140588

Table 8.5.b: Cdf(α)'s of Population on Paths 17 through 20 for Time Interval 1

Cumulative Probability	Travel Time (minutes) Path 17	Path 18	Path 19	Path 20
0.1	198283	201648	193712	185410
0.2	217751	221676	207682	209276
0.3	228931	229986	218487	216849
0.4	236432	238485	228040	227920
0.5	244047	249326	240611	236211
0.6	252310	262930	250729	243032
0.7	261514	271008	258302	250481
0.8	268800	279588	270873	262295
0.9	279279	295039	281039	271291
1	304263	311842	306023	298444

By the multiple objective extension of the EV algorithm (Section 6.3), 9 paths are determined, 4 of which are non-dominated for the first departure time. These four paths are Paths 9, 13, 14, and 15, described above. The expected values of time and population associated with these paths are presented in Table 8.6.

Table 8.6: Expected Population on Paths 9, 13, 14, and 15 for Time Interval 1

	Path 9	Path 13	Path 14	Path 15
Travel Time (minutes)	565.2	582.0	597.1	738.0
Population (persons)	224452	125687	91937	87314

8.4 FINAL PATH SELECTION

In this section, the "best compromise" path is selected, considering both travel time and population along the route, for the example problem of determining a path for the transport of a hazardous substance from Austin to Amarillo in the presence of incidents. The efficient frontier between expected travel times and expected population for the first departure time is plotted in Figure 8.1 for the Pareto-optimal paths that are generated by the multi-objective extension of the EV algorithm.

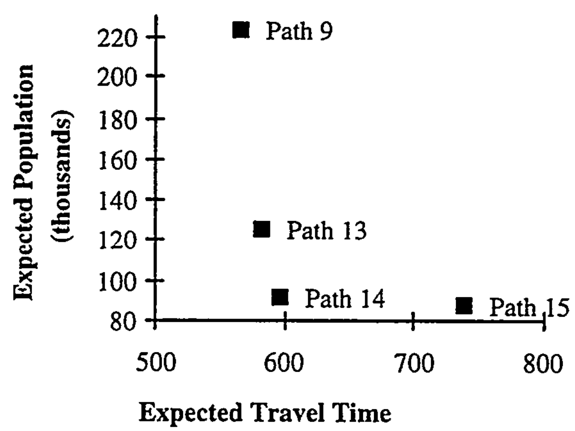

Figure 8.1: Efficient Frontier between Expected Time and Expected Population for Time Interval 1 of Paths Generated by the EV Extension

Similar to the reasoning used to determine the "best compromise" path given no known road closures or accidents prior to path selection, the paths can be narrowed down to Paths 13 and 14. Since Path 13 has a higher probability of exposing more people to the hazardous substance, and the difference between the cdf's of travel time of the two paths is not as extreme as the difference in the cdf's of the potential population exposure, Path 14 is selected. Note that both Paths 13 and 14 are dominated with respect to the single criteria of travel time and potential population exposure.

9. CONCLUSIONS

In this report, the methodology required to extend the algorithms presented in Section 4 for determining the least time paths in networks with stochastic, time-varying arc times to the problem of determining least cost paths in networks with stochastic, time-varying arc times and costs is presented. By including labels for both travel time and cost and expanding the Pareto-optimality conditions to include the simultaneous comparison of paths based on both travel time and cost, the procedures of Section 4 can be extended for use in solving multi-objective problems.

Some of the advantages and disadvantages of these procedures become evident when the problem of selecting a "best compromise" solution for the shipment of hazardous materials is addressed. One of the key points is that deterministic Pareto-optimality may result in a combinatorially explosive number of paths, as is seen in applying this methodology to determining the deterministically Pareto-optimal least cost paths for this problem. First-order stochastic dominance results in far fewer paths and comparisons based on expected value, even fewer. When multiple objectives are considered, the number of Pareto-optimal paths may preclude the use of Pareto-optimality conditions based on deterministic dominance since for many applications, generating the entire set of deterministically non-dominated solutions may be impractical.

A benefit of the EV algorithm is that the original cdf's of the random variables are used. Decisions for optimality are based on true expected values. This differs from the STDLT (DD) and STDLT (SD) algorithms where the original cdf's are aggregated and decisions are based on the resulting path cdf(α)'s. A consequence of aggregating the arc and path cdf(α)'s is that the cdf(α)'s tend to over-estimate the magnitude of the criterion, with the exception of the worst possible value, which is the true worst possible value for the path.

As expected, all least expected time paths are included in the set of stochastically Pareto-optimal paths, which in turn are all included in the set of deterministically Pareto-optimal paths. Exceptions may occur when the solution values are similar as a consequence of aggregating the cdf's.

The efficient procedures were found to perform quickly and accurately, as hoped. For more detail on algorithm performance, see Chapter 6 of Miller-Hooks (1997). The LEAST algorithm for determining the path with the least possible time or cost is useful in quickly determining a deterministically non-dominated path. An additional benefit of the LEAST algorithm is that it determines a lower bound on the probability of the occurrence of this least possible time or cost. Unfortunately, for paths of many arcs, this lower bound is essentially zero.

For both travel time and cost, the lower bound on the expected time of the least expected time path determined by the ELB algorithm is identical to the true expected time on the least expected time path. Maintaining at most 2 labels for each node, the paths determined by the Limited STDLT procedure are identical to those determined by the EV algorithm and STDLT(SD) algorithm. When only one label is maintained at each node, a dominated least cost path is determined. However, the values of the cdf's for this path are not far from those of the non-dominated paths.

To check the accuracy of the cdf(α)'s determined by the STDLT(DD) and STDLT(SD) algorithms, the Range algorithm for determining the range of the cdfs of the deterministically Pareto-optimal least time paths is employed. The non-dominated paths determined using the aggregated version of the STDLT (DD) algorithm are only a subset of the true Pareto-optimal set, as can be shown through either the exact implementation or through the Range algorithm. This is a consequence of the fact that the lower most values of the cdf's are maintained throughout the range, or exact, algorithms and therefore, a path is less likely to be dominated. The upper-most values of the cdf(α)'s are the true maximum values of the cdf's.

The hazardous material shipment problem is only one of numerous applications for which the procedures of Section 4 may prove to be useful. Other applications include: routing of emergency vehicles to and from the scene of a medical emergency, fire fighters to a fire, police officers to a request for service or scene of a crime, commercial trucks to pickups and deliveries, service vehicles to downed power lines, or wreckage from a natural disaster, as well as military applications, and other applications where response time is of the essence.

APPENDICES

APPENDIX A FLOW CHARTS OF PROCEDURES

A.1 Flow Chart of STDLT(DD) Algorithm

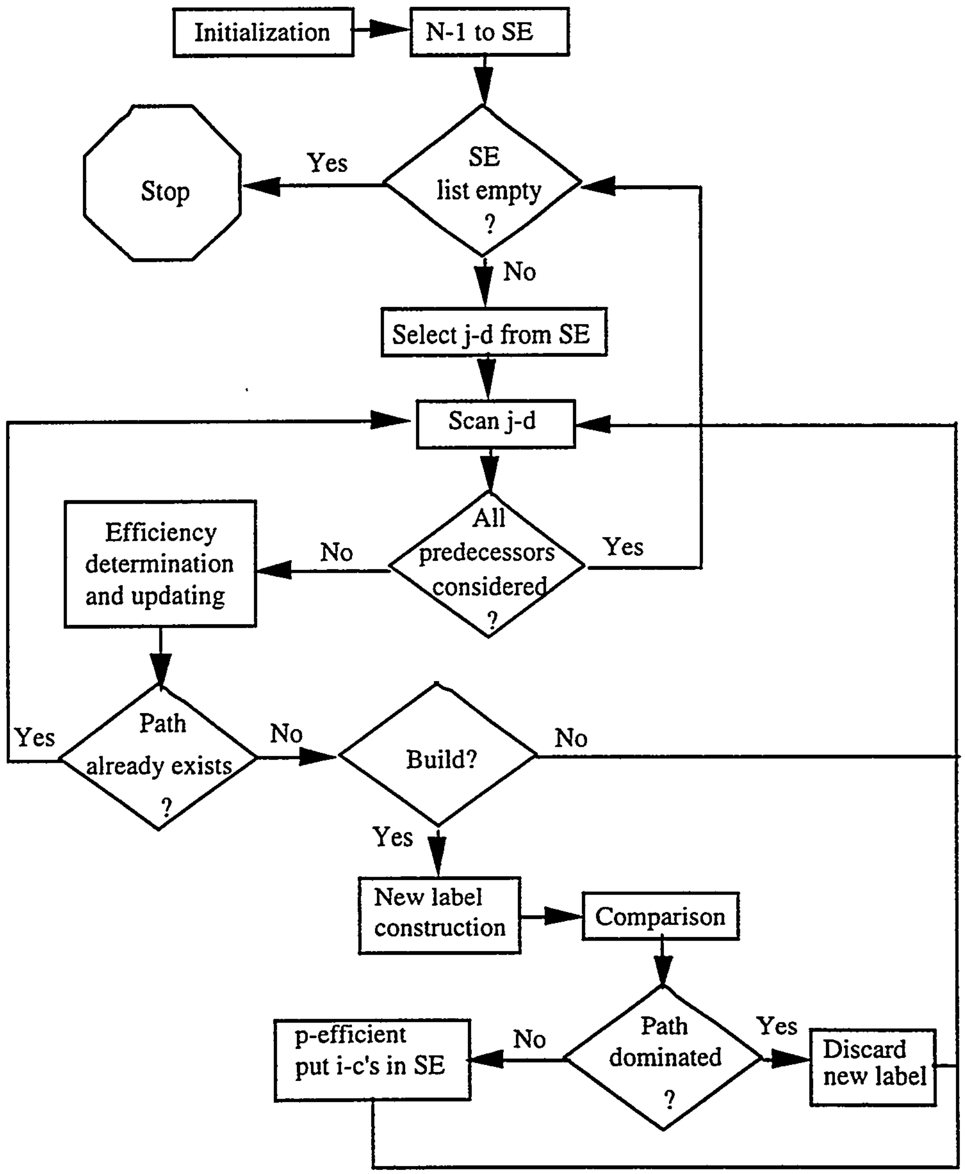

Figure A.1 Flow Chart of Basic Procedural Steps of STDLT(DD) Algorithm

A.2 Flow Chart of STDLT(SD) or EV Algorithms

A flow chart of the basic procedural steps of the STDLT (SD) algorithm is presented. The same flow chart can be used to represent the basic procedural steps of the EV algorithm because specific details concerning the computation and comparison of labels are not included.

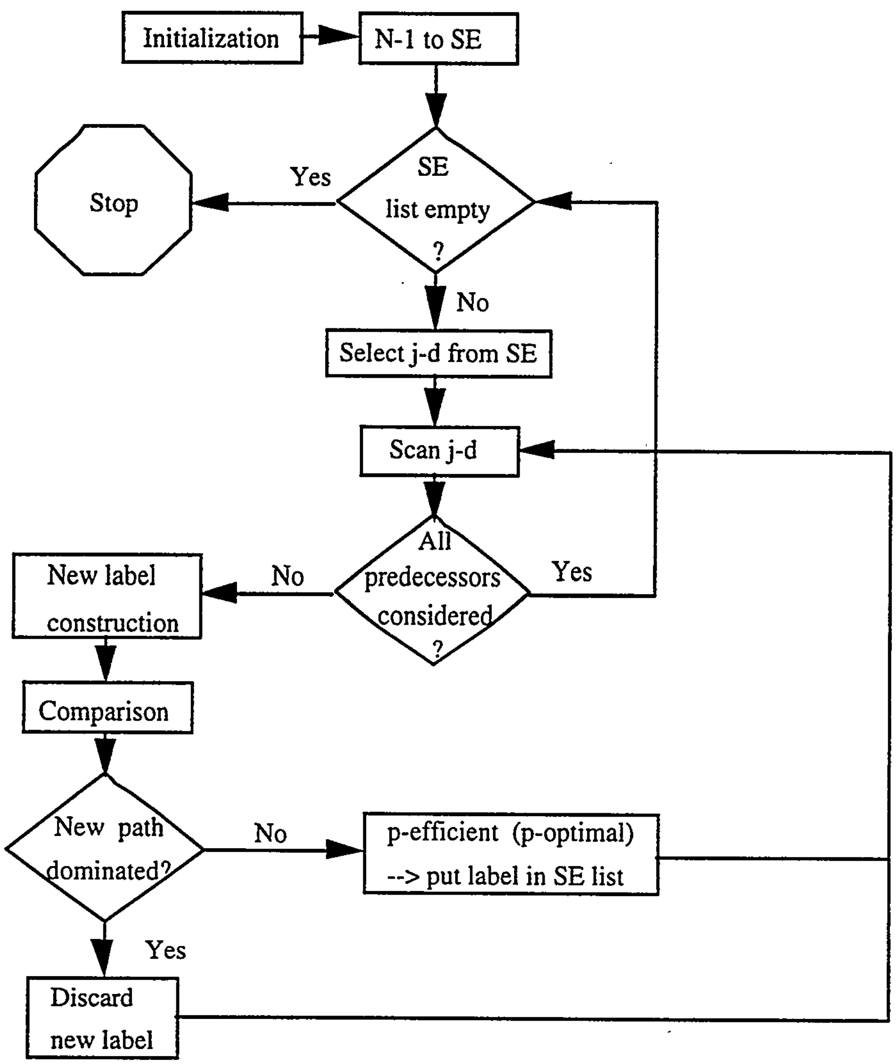

Figure A.2 Flow Chart of Basic Procedural Steps of STDLT (SD) and EV Algorithms

A.3 Flow Chart of LEAST or ELB Algorithm

A flow chart of the basic procedural steps of the LEAST algorithm is presented. The same flow chart can be used to represent the basic procedural steps of the ELB procedure because specific details concerning the computation and comparison of labels are not included.

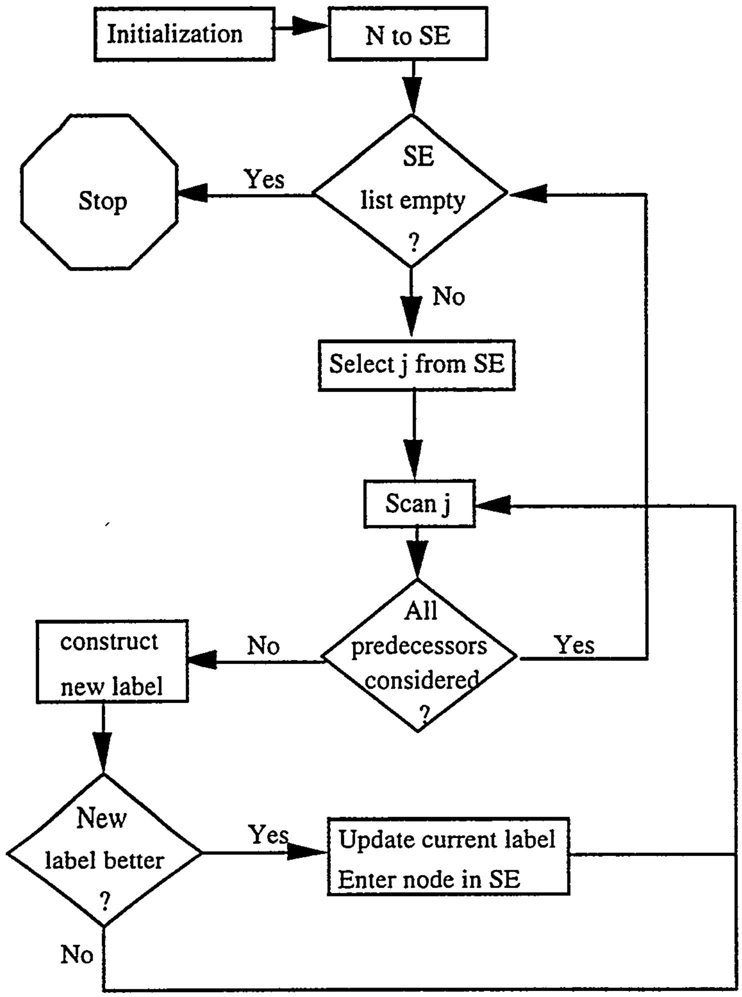

Figure A.3 Flow Chart of Basic Procedural Steps of LEAST and ELB Algorithms

A.4 Flow Chart of TOP Algorithm

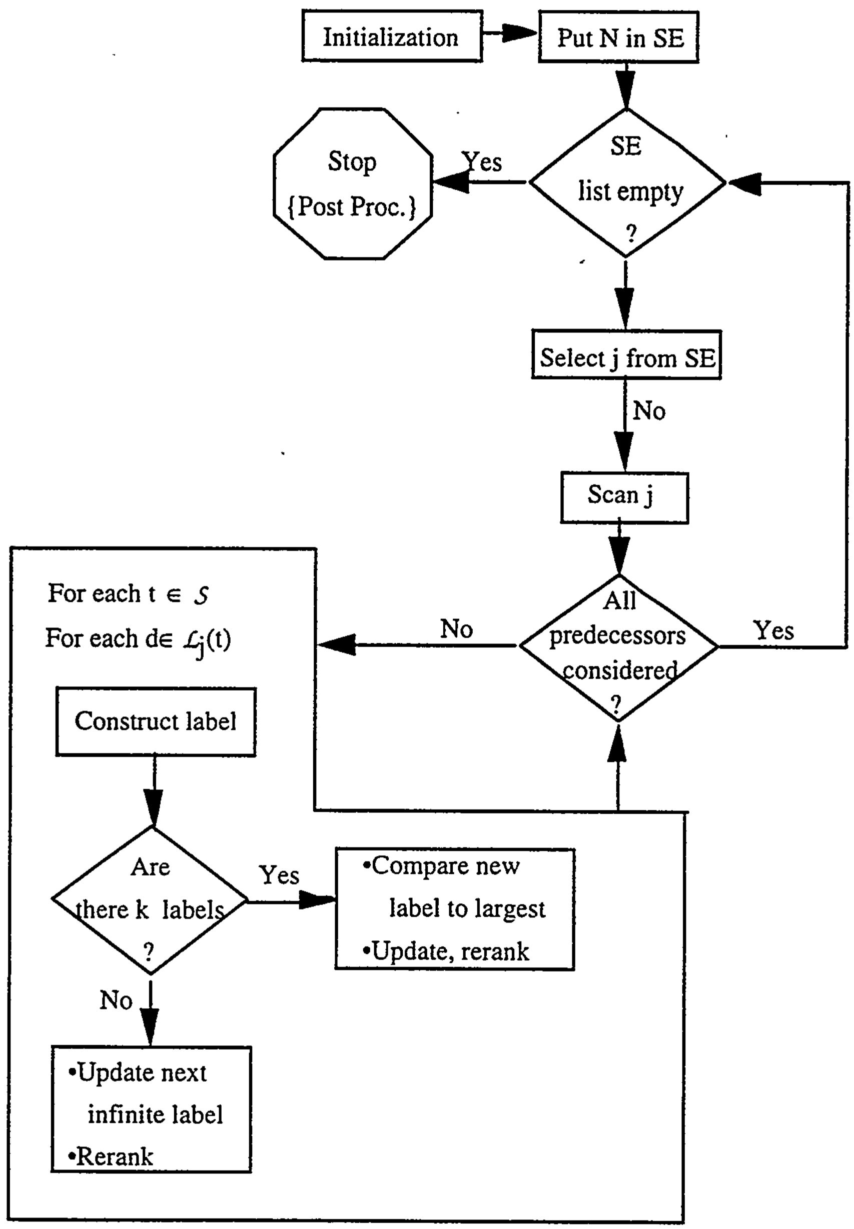

Figure A.4 Flow Chart of Basic Procedural Steps of TOP Algorithm

APPENDIX B: SOME DATA FOR THE TEXAS EXAMPLE NETWORK

The data set for the example networks, TX-HWY, used to illustrate how the solutions differ for each of the procedures, is too large to include in this appendix, and therefore, the travel times on only a single arc, the link from Austin to Lampasas, are shown here. Recall that for this example problem the time interval is assumed to be 5 minutes and the peak period is 60 minutes, resulting in 12 departure time intervals. The travel time pmf's for this arc for each departure time interval in the peak period are given as pairs of possible travel times (in minutes) with the associated probability of occurrence, respectively. Note that the length of this link is 67.5 miles.

Time Interval 1

71.627	0.486
72.191	0.086
72.598	0.101
77.079	0.161
78.037	0.165

Time Interval 2

76.559	0.326
77.923	0.402
79.841	0.039
79.980	0.067
87.384	0.167

Time Interval 3

83.205	0.335
89.309	0.067
91.722	0.328
93.791	0.125
96.792	0.144

Time Interval 4

99.009	0.061
101.729	0.402
102.525	0.124
102.719	0.061
103.890	0.353

Time Interval 5

110.181	0.187
113.962	0.043
118.916	0.358
127.391	0.185
136.131	0.226

Time Interval 6

111.865	0.213
114.502	0.221
130.205	0.145
140.738	0.283
147.123	0.138

Time Interval 7

92.824	0.255
111.482	0.109
118.869	0.261
119.470	0.073
120.647	0.301

Time Interval 8

87.722	0.234
88.949	0.214
93.514	0.185
101.315	0.124
106.600	0.243

Time Interval 9

82.663	0.178
84.711	0.189
88.008	0.172
88.088	0.233
89.807	0.228

Time Interval 10

68.072	0.204
77.113	0.091
78.040	0.252
83.006	0.354
85.376	0.100

Time Interval 11

69.088	0.126
69.566	0.242
70.080	0.242
76.003	0.223
79.029	0.166

Time Interval 12

64.550	0.090
66.598	0.062
69.644	0.181
70.026	0.362
71.698	0.305

Similarly, the population pmf's for this arc for each departure time interval in the peak period are given as pairs of possible population values with the associated probability of occurrence, respectively.

Time Interval 1

66522	0.486
68506	0.086
69940	0.101
85710	0.161
89081	0.165

Time Interval 2

62214	0.326
66788	0.402
73216	0.039
73680	0.067
98495	0.167

Time Interval 3

59176	0.335
78423	0.067
86033	0.328
92560	0.125
102023	0.144

Time Interval 4

77554	0.061
85511	0.402
87839	0.124
88404	0.061
91830	0.353

Time Interval 5

72860	0.187
82940	0.043
96147	0.358
118743	0.185
142044	0.226

Time Interval 6

36135	0.213
42402	0.221
79722	0.145
104753	0.283
119929	0.138

Time Interval 7

26587	0.255
76329	0.109
96023	0.261
97625	0.073
100765	0.301

Time Interval 8

44538	0.234
48126	0.214
61480	0.185
84300	0.124
99759	0.243

Time Interval 9

57467	0.178
63924	0.189
74321	0.172
74572	0.233
79996	0.228

Time Interval 10

33771	0.204
64071	0.091
67178	0.252
83824	0.354
91767	0.100

Time Interval 11

57588	0.126
59269	0.242
61078	0.242
81922	0.223
92572	0.166

Time Interval 12

59303	0.090
66791	0.062
77929	0.181
79327	0.362
85438	0.305

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