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SENSITIVITY TO ALERT RATES AT MODERATE FORCE LEVELS

Gregory H. Canavan

This analysis extends previous studies of sensitivity to survivability and alert rates to low force levels. Stability and first strike considerations favor reduction of both survivable and vulnerable forces for a range of conditions. Reductions in the number of weapons per vulnerable missile always increases first strike costs, and would have to be offset by non-stability considerations.

Introduction. This report discusses incentives to reduce forces at low force levels using a stability framework with elements common to U.S. and Russian analyses. It represents forces by effective vulnerable and survivable missiles forces and studies exchanges between them, using an aggregated, probabilistic treatment of their interaction and analytic optimization of strikes based on minimization of first strike costs.[1] The forces used model the missile forces and weapons at levels suggested by a recent National Academy of Sciences study to retain the core deterrent function of nuclear weapons at lower force levels.[2] For moderate forces, non-alert forces are destroyed with the expenditure of few weapons, and second strikes are composed of surviving ICBMs and SLBMs. Aircraft-borne weapons are not counted in the analysis below because they are too slow to participate in first strikes and inadequately survivable to participate in second strikes under projected conditions.

For moderate forces, stability indices, which fall rapidly for multiple vulnerable weapons, are further reduced in proportion to the number of survivable weapons in port, which enter the analysis as vulnerable missiles. The derivative of the first strike cost with respect to weapons per vulnerable missile is everywhere negative, so that any decrease in the weapons per missile would increase first strike cost, but the derivatives of first strike cost with respect to vulnerable and survivable missiles are positive for adequate ranges of weapons, damage preference, and alert rates to permit stable reductions in missiles. Reducing the number of missiles increases stability, although that result is sensitive to how target sets are assumed to scale with weapon inventories. Stability and first strike cost considerations favor reduction of survivable and vulnerable forces. Reducing the number of weapons per vulnerable missile always increases first strike costs, which would have to be offset by non-stability considerations.

Review of exchange model. Exchanges between equal missile missiles forces can be modeled in terms of the first and second strikes each side could deliver. If unprime strikes first, and a fraction f of his weapons is directed at prime's vulnerable missiles, unprime's first strike on value targets is

$$F = (1 - f)(mM + nN). \tag{1}$$

Ignoring the small number of weapons allocated to non-alert missiles, the average number of weapons delivered on each of prime's vulnerable missile is

$$r = f(mM + nN)/M'. \qquad (2)$$

For r large, the approximate average probability of survival of a vulnerable missile is[3]

$$Q' \approx q^r = e^{fW\ln q/M'}, \qquad (3)$$

where $p = 1 - q$ is the attacking missile's single shot probability of kill.

The fraction of prime survivable missiles off alert, e.g., in port, is h'. The value h' = 0 would correspond to all missiles at sea, or capable of firing from port, which is the case treated earlier. h' = 1 corresponds to all SSBNs in port, in which case they would enter the analysis as essentially a few, highly-MIRVed, vulnerable missiles. Nominal calculations below assume h = h' = 0.6, which corresponds roughly to historical and projected conditions of ~ 40% of SSBNs at sea. The ability to disperse on warning would produce values of h and h' between 0.6 and 0. Increasing h decreases the alert rate, which conceptually changes survivable missiles and weapons into vulnerable ones. With these expressions for vulnerable and non-alert missile survivability, prime's second strike becomes

$$S' = m'M'Q + (1 - h')n'N', \qquad (4)$$

all of which is delivered on value, as missiles remaining at the end of the exchange have no value. The corresponding equations for unprime's second and prime's first strikes can be derived by conjugating the above equations. This simplification of the exchange to one strike by each side emphasizes the deterrent role of the weapons over any war fighting role in extended engagements.[4] First and second strikes are converted into costs, whose ratios are interpreted as stability indices, through the process discussed in the Appendix.

First strike cost for moderate forces. At the end of the transition, each sides' posited forces are roughly equal, so it is acceptable to assume them equal, drop the unprime-prime notation, and study alert rates in a simpler context. The cost for striking first then becomes

$$(1 + L)C_1 \approx kS + L(1 - kF) \approx k[QmM + (1 - h)nN] + L[1 - k(1 - f)W] \qquad (5)$$

The derivative of cost with respect to the weapon allocation is

$$(1 + L)\partial C_1/\partial f \approx kmM\partial Qm/\partial f + LkW = kmM(W\ln q/M)Q + LkW, \qquad (6)$$

which is minimized by the choice

$$f = M/W\ln q \, \ln(-L/m\ln q), \qquad (7)$$

which is, as before, independent of h. Figure 1 shows $r = fW/M = \ln(-L/m\ln q) / \ln q$, the average number of weapons allocated to each vulnerable missile for p = 0.8 and a range of values of L. For a nominal L = 0.25, r increases from 1.2 to 1.8 as m increases from 1 to 3. For L = 0.5, a more aggressive attacker, r falls as weapons are shifted from missiles to value targets, varying from 0.8 to 1.4, falling ~ 0.4 at each value of m. For L = 1, an attacker indifferent between damage to self and other, r falls by another such increment, although there r is so low as to strain

the approximation in Eq. (3). Survivable forces do not affect allocations, but do change first strike costs by

$$(1 + L)\Delta C_1 \approx k(1 - h)\Delta(nN) - Lk\Delta(W - rM) = k(1 - h - L)\Delta(nN). \tag{8}$$

However, for typical parameter values, e.g., $h \approx 0.6$, $L \approx 0.5$, and $f \approx 0.4$, the term in brackets is ≈ 0.1 and the reduction is small.

Difference stability indices. The normalized difference stability index is

$$j = (S - F)/kM = Qm - [m - (1/\ln q) \ln(-L/m\ln q)] - hnN/M = s - f - t, \tag{9}$$

where $s = Qm$ and $f = m - r$ are the second and first strikes by vulnerable weapons in the absence of SSBNs, and $t = hnN/M$ is the correction due to alert survivable systems, which do not cancel out for $h \neq 0$, because the number of survivable weapons in first and second strikes differ. Thus, nN makes a simple additive subtraction from the j for full alert. $nN/M \sim 4\text{-}5$ at posited deep reductions, so the correction can be large. Figure 2 shows s, f, t, and j versus m for L =0.25. The top curve is $t = 1.2$, corresponding to $nN/m = 4$ and $h = 0.4$. The second curve is the first strike f, which increases to unity by $m = 3$. The third is $s \sim 0.15$, which is constant since by Eqs. (3) and (7), $Q \approx q^r = e^{fW\ln q/M} = -L/m\ln q$, so that $s = mQ = -L/\ln q$, which is independent of m. The bottom curve is $j \approx -(f + t)$, which is $\approx -t$ for m small and $\approx -f$ for m large.

Figure 3 shows the variation of j with m for various h. The top curve is for $h = 0$, i.e., all forces treated as alert. It starts at about 0.3 at $m = 1$, falls to 0 at $m = 1.6$, and further to -1.2 at m = 3. This difference stability index is referenced to 1; thus, $1 < m < 1.6$ is a region of strong stability—i.e., positive disincentive to attack— while $m > 1.6$ is a region of reduced stability, where a potential attacker might see an apparent advantage in striking first in a crisis. The middle curve's components are displayed on Fig. 2. It starts at about -1 and falls to about -2.3. It is not positive in any region. The bottom curve for $h = 0.6$, the historical and projected value, starts at about -2.2 and falls to -3.3. Overall, increasing nN/M or h shifts the stability index down proportionally. For any non-zero h and nN, this shift puts the whole curve in the unstable region. For any hnN, it is necessary to reduce m to one to maximize stability.

Connection to ratio indices. The smallest force considered by the NAS report has M = 60 and N = 240 singlet missiles, which corresponds to the nN/M = 4 used in constructing Fig. 3. For the nominal $h = 0.6$, at $m = 1$ the bottom curve gives $j \approx -2$. That gives a ratio stability index of $I \approx 1 + kMj \approx 1 + 0.001 \times 60 \times (-2) \approx 0.9$, in accord with the more detailed calculations discussed earlier.[5] For moderate forces, if k and j do not change, as M decreases, I approaches stability at unity, and if m, n, L, and p do not change, j will not change.

For k to remain constant, the number of value targets held at risk must not change, which is unlikely. At the beginning of this decade, there were about 30,000 value targets; after the cold war, the number dropped to 10,000. With START, it is apparently drops to 3,000, and there are indications that the number of key targets could fall to the 1,000 used above. There appears to be

a refinement or reduction of the number of essential targets to accommodate the weapons available. If so, stability would scale ~ kM ~ constant during deep reductions, so that stability indices would remain constant, absent changes in m, n, L, and p. Even at 1,000-300 weapons, there are many more weapons than large cities on either side, so it is appropriate to concentrate on value targets at levels down to perhaps 100 weapons—although at very low force levels it will ultimately be necessary to refine the discussion of other than military targets.

Cost variations and incentives. Consistent with the first strike metric used here, the decision to add or delete forces should depend on whether they increase or reduce the cost of first strikes. With the optimal allocation of Eq. (7), the first strike cost of Eq. (5) becomes

$$(1 + L)C_1 \approx k[QmM + (1 - h)nN] + L[1 - k(W - rM)]$$

$$= L + \{[Qm - L(m - r)] + (1 - h - L)nN/M\}kM, \tag{10}$$

which is shown in Fig. 4 for L = 0.25, p = 0.8, and kM = 1. Cost is at a maximum for h = 0 because the opponent's second strike forces are at maximum potential. C_1 falls from about 2.8 at m = 1 to 2.4 at m = 3, a slope of $\approx$ -0.4 / 2 $\approx$ -0.2. Decrements are similar for h = 0.3 and 0.6, where C_1 falls from about 0.8 at m = 1 to 0.4. The attacker minimizes C_1 with respect to each variable at his disposal: m, M and nN. The partial derivative of C_1 with respect to m is

$$(1 + L)\partial C_1/\partial m \approx \partial[Qm - L(m - r)]/\partial m = -kML(1 - \partial r/\partial m) = -kML(1 + 1/mlnq), \tag{11}$$

which is shown in Fig. 5 for kM = 1. $\partial C_1/\partial m$ is independent of h and N, varies strongly with L and m, and is negative for all L and m. For a given m, it falls roughly in proportion to L. L = 0.25, $\partial C_1/\partial m$ falls from about -0.07 at m = 1 to -0.15 at m = 3. Thus, doubling m from 1 to 2 would decrease C_1 by about 0.07, which produces the reduction from about 2.8 to 2.6 seen in the top curve of Fig. 4.

The importance of $\partial C_1/\partial m$ being everywhere negative is that any positive increment Δm will cause a negative increment $\Delta C_1 = \partial C_1/\partial m \, \Delta m < 0$, which will reduce the first strike cost. Thus, $\partial C_1/\partial m$ everywhere negative means there is never an incentive, from stability considerations, to reduce the number of weapons per vulnerable missile into the range needed for stability. The derivative becomes more negative for larger L and m, which means that greater exogenous forces would have to be applied to offset stability considerations there.

The derivative of the first strike cost of Eq. (1), with respect to vulnerable missiles M is

$$(1 + L)\partial C_1/\partial kM \approx Qm - L(m - r), \tag{12}$$

which is shown in Fig. 6. $\partial C_1/\partial kM$ decreases as L or m increase. For L = 0.25, it falls from about 0.15 to 0 as m increases from 1 to 2.25 and then to -0.1 as m increases to 3. For m < 2.25, decreasing m would decrease C_1, which would be a stabilizing move, i.e., incentivized by considerations of self interest. For larger m there would be a disincentive to reduce m. For L = 0.5, $\partial C_1/\partial kM$ decreases from about 0.12 to 0 as m increases from 1 to 1.6 and to -0.1 as m increases to 3. For L = 1, $\partial C_1/\partial kM$ is everywhere negative and falls from -0.05 to -0.7.

The derivative of the first strike cost with respect to survivable weapons nN is

$$(1 + L)\partial C_1/\partial knN \approx (1 - h - L),\tag{13}$$

which is shown as a function of h in Fig. 7. The curves are straight lines; their slopes are inversely proportional to L. The top curve for $L = 0.25$ falls from about 0.6 at $h = 0$, all SSBNs on alert, to zero at $h = 0.75$. At $h = 0.6$, $\partial C_1/\partial knN \approx 0.15$, which would incentivize the further reduction of nN. For $L = 0.5$, the crossing is at about $h = 0.5$, so that at $h = 0.6$ there would be a disincentive of ≈ -0.05 to reducing survivable missiles. For $L = 1$, the crossing is at $h = 0$ and the disincentive at $h = 0.6$ is about -0.35.

While $\partial C_1/\partial kM$ and $\partial C_1/\partial knN$ depend on different variables, it is possible to study their combined effect through the total derivative

$$dC_1 = \partial C_1/\partial m\, dm + \partial C_1/\partial kM\, dn + \partial C_1/\partial knN\, dknN.\tag{14}$$

Since $\partial C_1/\partial m$ only depends linearly on kM, it is useful to take $dm = 0$ and study the combined dependence on M and N. Posited reductions have $nN/M \approx 4$, so $dM \approx dnN$, and

$$dC_1 \approx \partial C_1/\partial kMdkM + \partial C_1/\partial knN\, dknN = (\partial C_1/\partial kM + \partial C_1/\partial knN)\, dM,\tag{15}$$

which is shown in Fig. 8 for $L = 0.25$. The top curve for $h = 0$ falls from about 0.75 to 0.5 as m increases from 1 to 3. Those for $h = 0.3$ and 0.6 are parallel and 0.24 lower at each value of m. Because dC_1/dM is everywhere positive, proportional decreases in M and N would produce a decrease in dC_1, which means that the incentives from stability and first strike costs would agree on the desirability of reductions. For each h, the contribution varies from about 0.1 to -0.1 as m increases from 1 to 3. Thus the roughly 0.6, 0.36, and 0.12 contributions from $\partial C_1/\partial knN$ are dominant. Figure 7 shows that for $L = 0.5$, those contributions would fall to 0.3, 0.15, and -0.1, for which the total derivative would be negative in places.

An explicit goal of arms control discussions is to eliminate multiple warhead weapons; thus it is useful to examine the behavior of these cost curves and their derivatives as functions of M and N for $m = 1$. Figure 9 shows C_1 as a function of h for $kM = 0.06$. The top curve for $L = 1$ falls from about 0.5 to 0.4, the lower curves for $L = 0.5$ and 1 are about 0.1 and 0.2 lower at each m. Figure 10 shows dC_1/dM as a function of h for proportional changes in M and N. The top curve for $L = 0.25$ drops from 0.7 at $h = 0$ to ≈ 0 by $h = 1$. At $h = 0.6$ it is about 0.25. The curve for $L = 0.5$ falls from 0.4 to -0.2, crossing zero at $h \approx 0.7$. At $h = 0.6$ it is ≈ 0.05. The curve for $L = 1$ is negative for all h, which indicates that any decrease in M and N would increase first strike costs, so that stability and first strike considerations would be in disagreement.

Summary and conclusions. This note discusses incentives to reduce forces at low force levels using a stability framework with elements common to U.S. and Russian analyses. It converts forces into effective vulnerable and survivable missiles forces and studies exchanges between them using an aggregated, probabilistic treatment of the interaction of the two sides' missile forces, approximations to the first and second strikes each could deliver, and missile

allocations between missile and value targets based on analytic minimization of first strike costs. In this limit, non-alert forces are destroyed with the expenditure of few weapons and second strikes are composed of surviving ICBMs and SLBMs on patrol.

For moderate forces the stability index, which falls rapidly for multiple vulnerable weapons, is reduced in proportion to the number of survivable weapons in port. The derivative of the first strike cost with respect to weapons per vulnerable missile is everywhere negative, especially for strong damage preferences, so any decrease in the weapons per missile would increase first strike cost. The derivative of first strike cost with respect to vulnerable missiles is positive for small numbers of weapons per missiles and damage preferences, where decreases in missiles would both increase stability and decrease first strike costs. The derivative of first strike cost with respect to survivable weapons is positive for low damage preferences and high alert rates. Proportional reductions in vulnerable and survivable missiles give positive derivatives for all alert rates for small damage preferences. If it is possible to reduce to singlet missiles, derivatives should be positive at high alert rates and low to moderate damage preferences.

Reducing the number of missiles increases stability, although at some point it will be necessary to reconsider how the targets held at risk scale with weapon inventories. The reduction of survivable forces is generally stabilizing and favored by first strike considerations. The reduction of vulnerable forces is stabilizing and favored by first strike considerations in a range of cases. Reductions in the number of weapons per vulnerable missile always increases first strike costs, so they would have to be offset by other, non-stability considerations.

Appendix. Costs and indices. First and second strike magnitudes are converted into first and second strike costs through exponential approximations to the fractions of military value targets destroyed. The cost of damage to self when unprime strikes first is

$$C_{1s} = (1 - e^{-kS'})/(1 + L), \qquad (A1)$$

where $k \approx 1/1000$ is the size of unprime target set prime wishes to hold at risk, and L is a weighting parameter that represents the attacker's relative preference for inflicting damage on the other and preventing damage to self. The cost of incomplete damage to prime is

$$C_{1o} = Le^{-k'F}/(1 + L), \qquad (A2)$$

where $k' \approx 1/1000$ is the inverse of the prime value unprime wishes to hold at risk. It is conventional to use their weighted sum as an approximate total cost for striking first[6]

$$C_1 = C_{1s} + C_{1o} = (1 - e^{-kS'} + Le^{-k'F})/(1 + L). \qquad (A3)$$

Second strike costs are also composed of damage to self and other, which are approximated by

$$C_{2s} = (1 - e^{-kF'})/(1 + L); \qquad (A4)$$

$$C_{2o} = Le^{-k'S}/(1 + L); \qquad (A5)$$

so the total cost for unprime striking second is

$$C_2 = C_{2s} + C_{2o} = (1 - e^{-kF'} + Le^{-k'S})/(1 + L), \qquad (A6)$$

It is conventional to use the ratio of first and second strike costs, $I = C_1/C_2$, as a stability index for unprime, and $I' = C_1'/C_2'$, as an index for prime. When they are large, the two sides see no advantage to striking first. When they are small, there is an apparent incentive to attack first in a crisis. For unequal forces, their product is used as a compound index [7]

$$\text{Index} = I \times I' = (C_1/C_2)(C_1'/C_2'), \qquad (A7)$$

in which the smaller of the two indices determines overall stability.

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1 r v m

Fig. 1. r vs m various L

Fig. 2. s, f, t, and j vs m

m
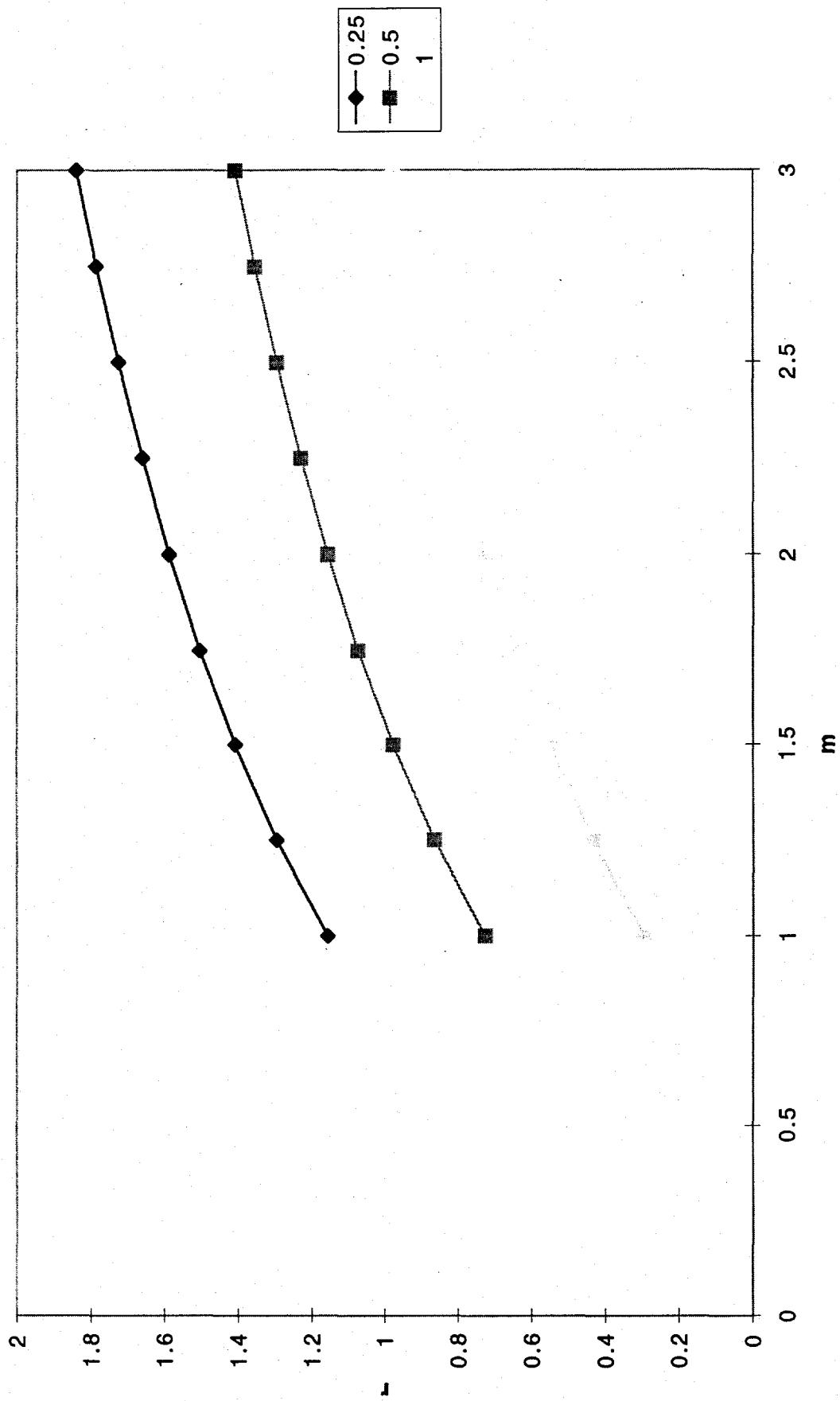
3 j v m
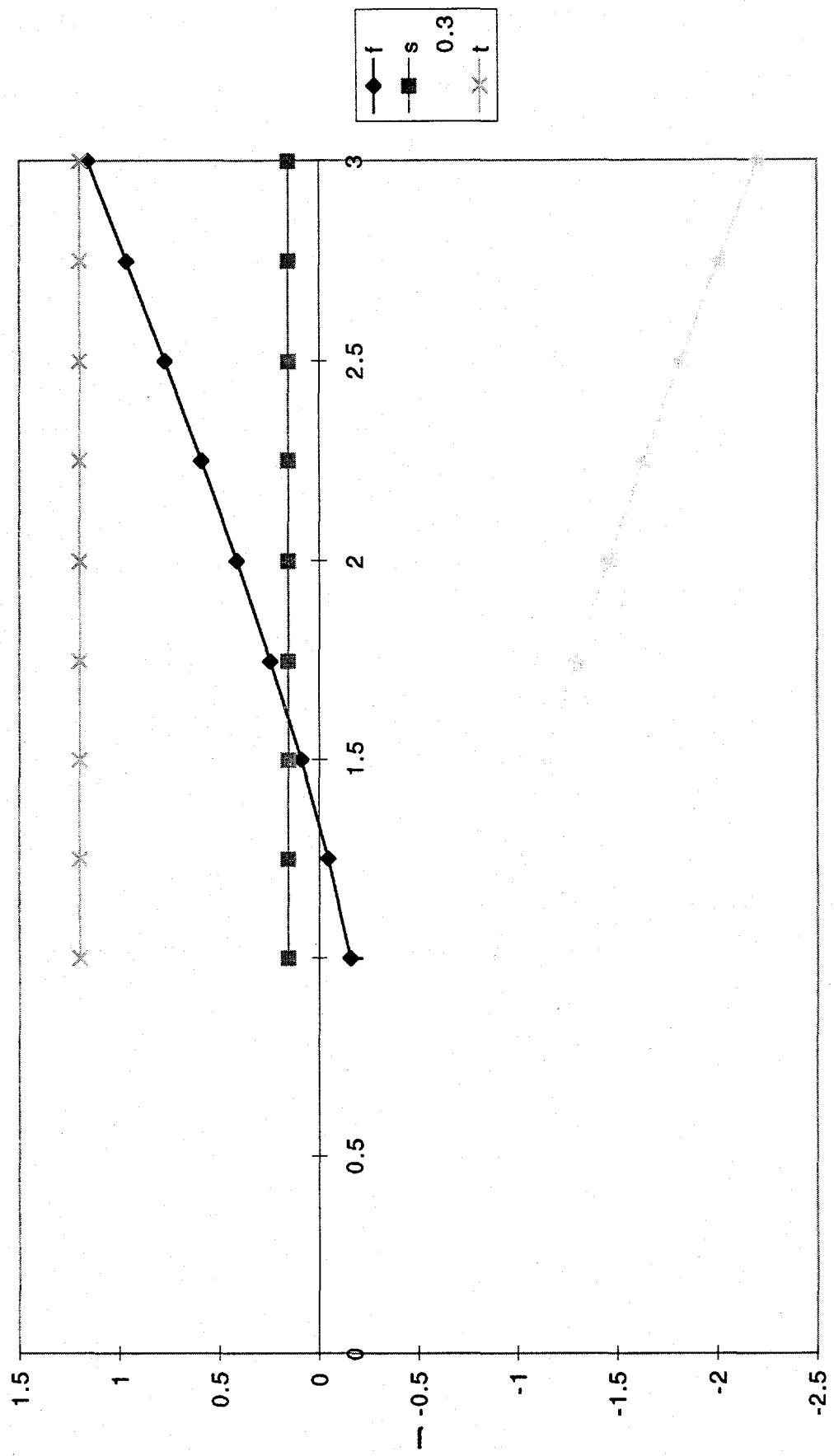
Fig. 3. j vs m for various values of h

m

4 C1 v m;L
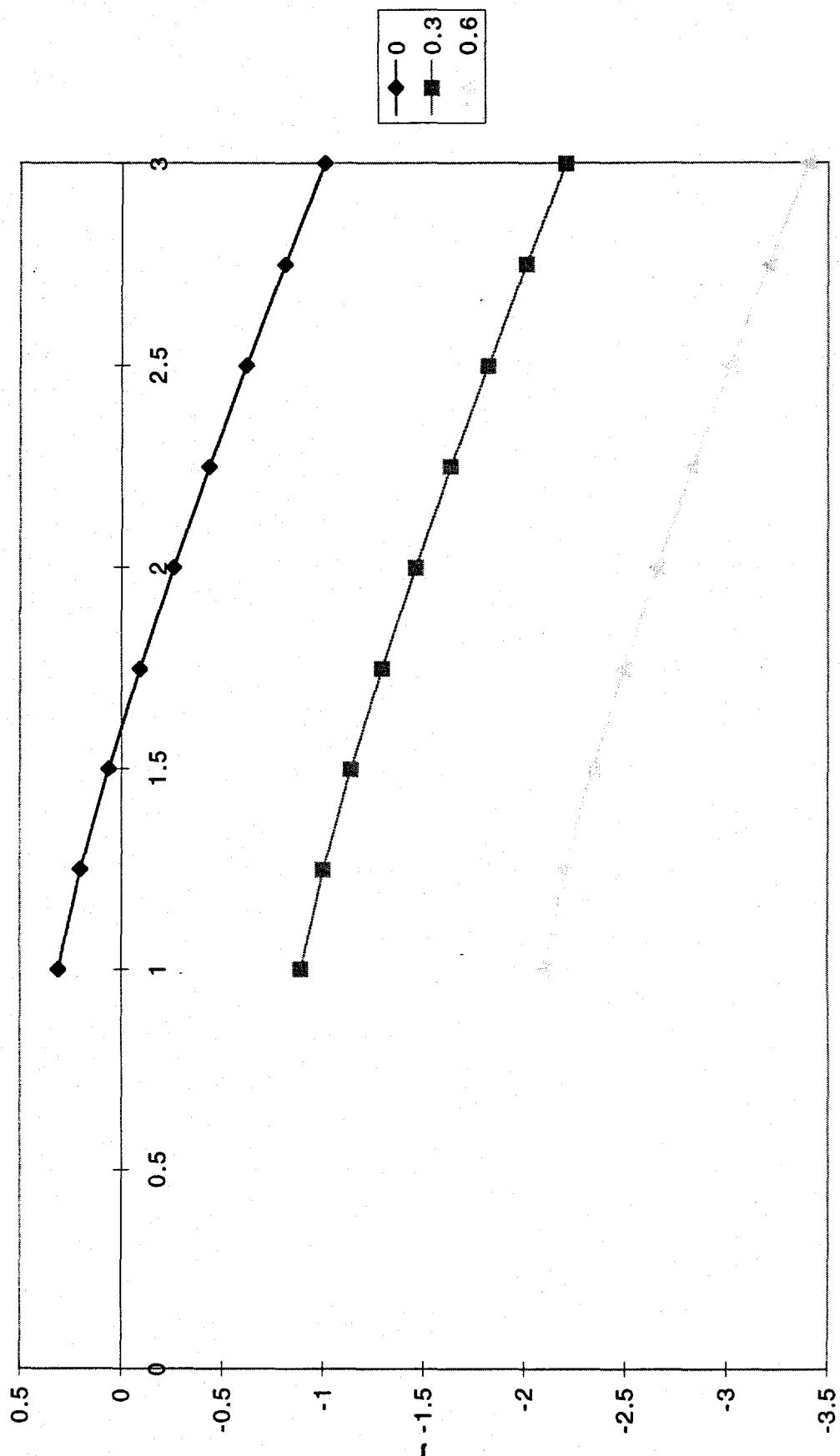
Fig. 4. C1 vs m for various alert rates h
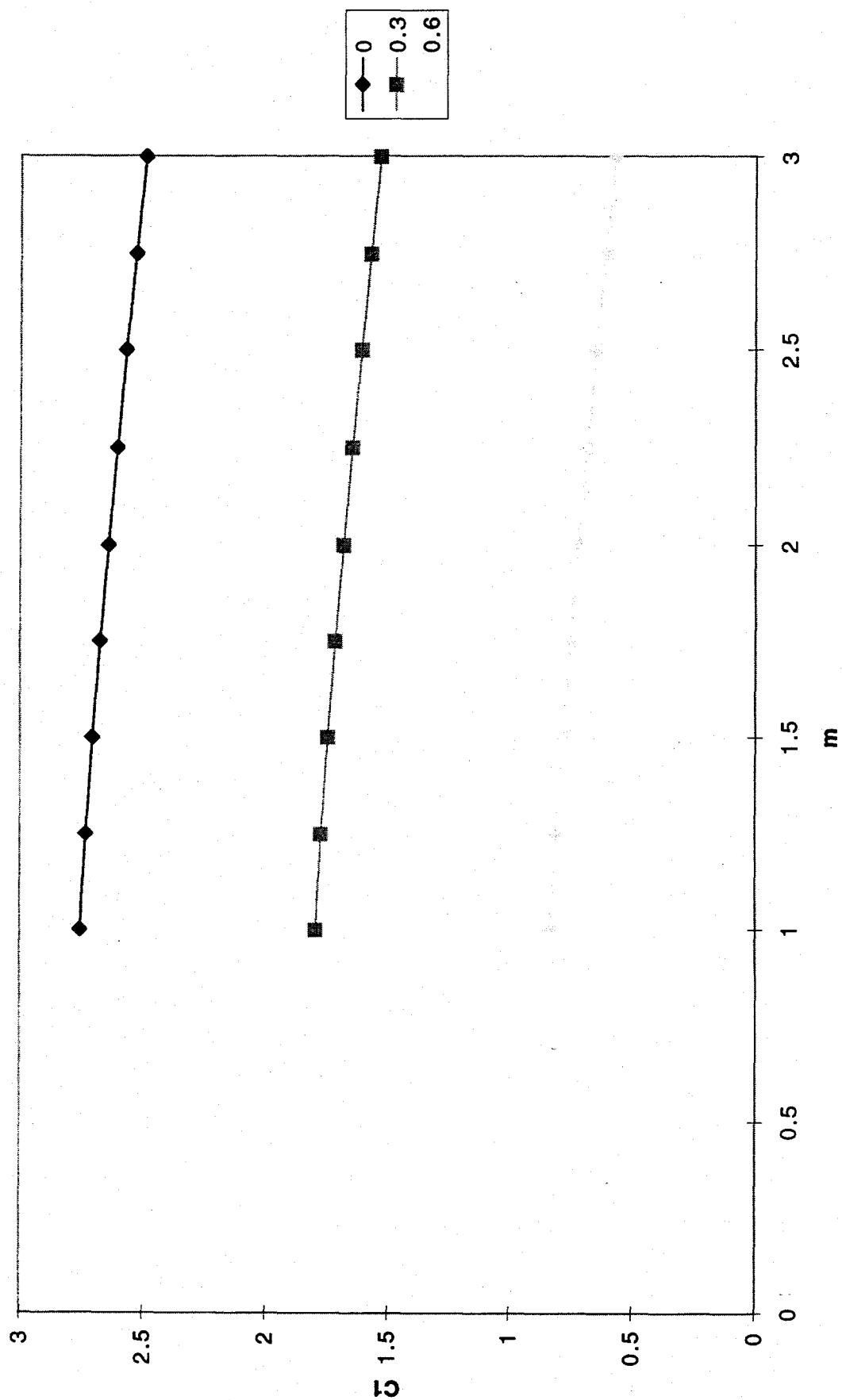
Fig. 5. dC1/dm vs m for various L

6 dC1 d M v m

# Fig. 6. dC1/dM vs m for various L

m
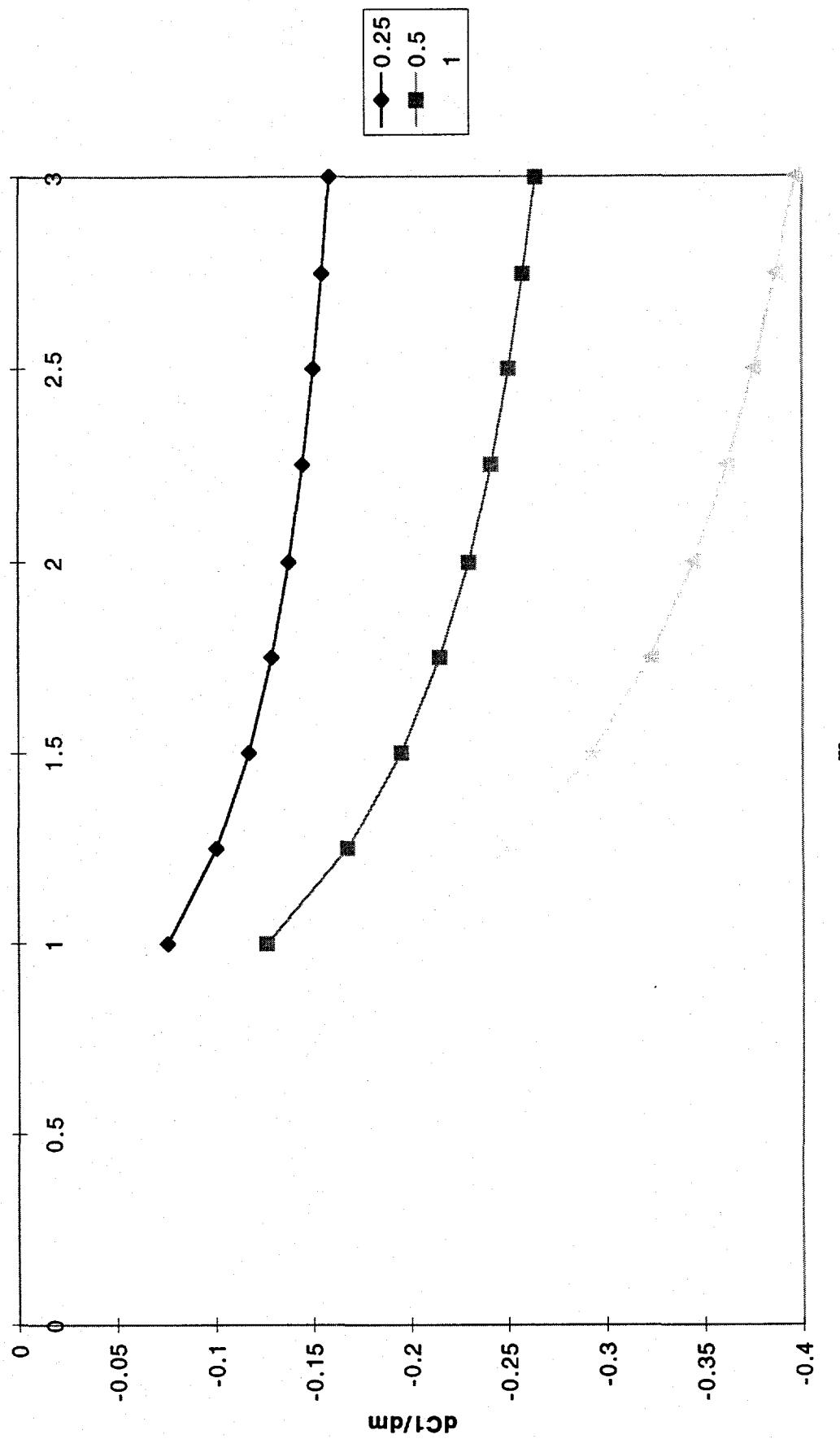
7 dC1 d N v h
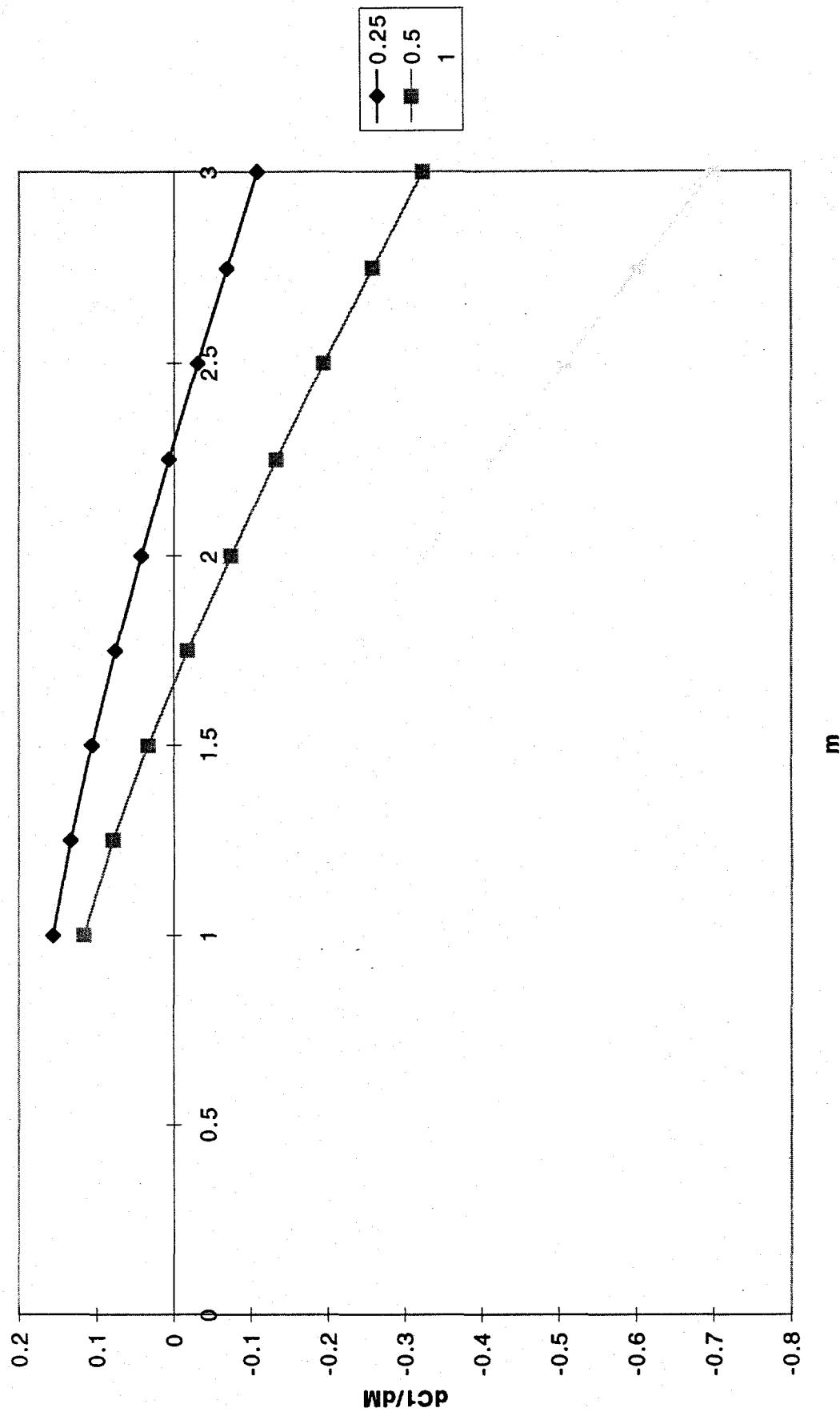
Fig. 7. dC1/dknN vs h various L
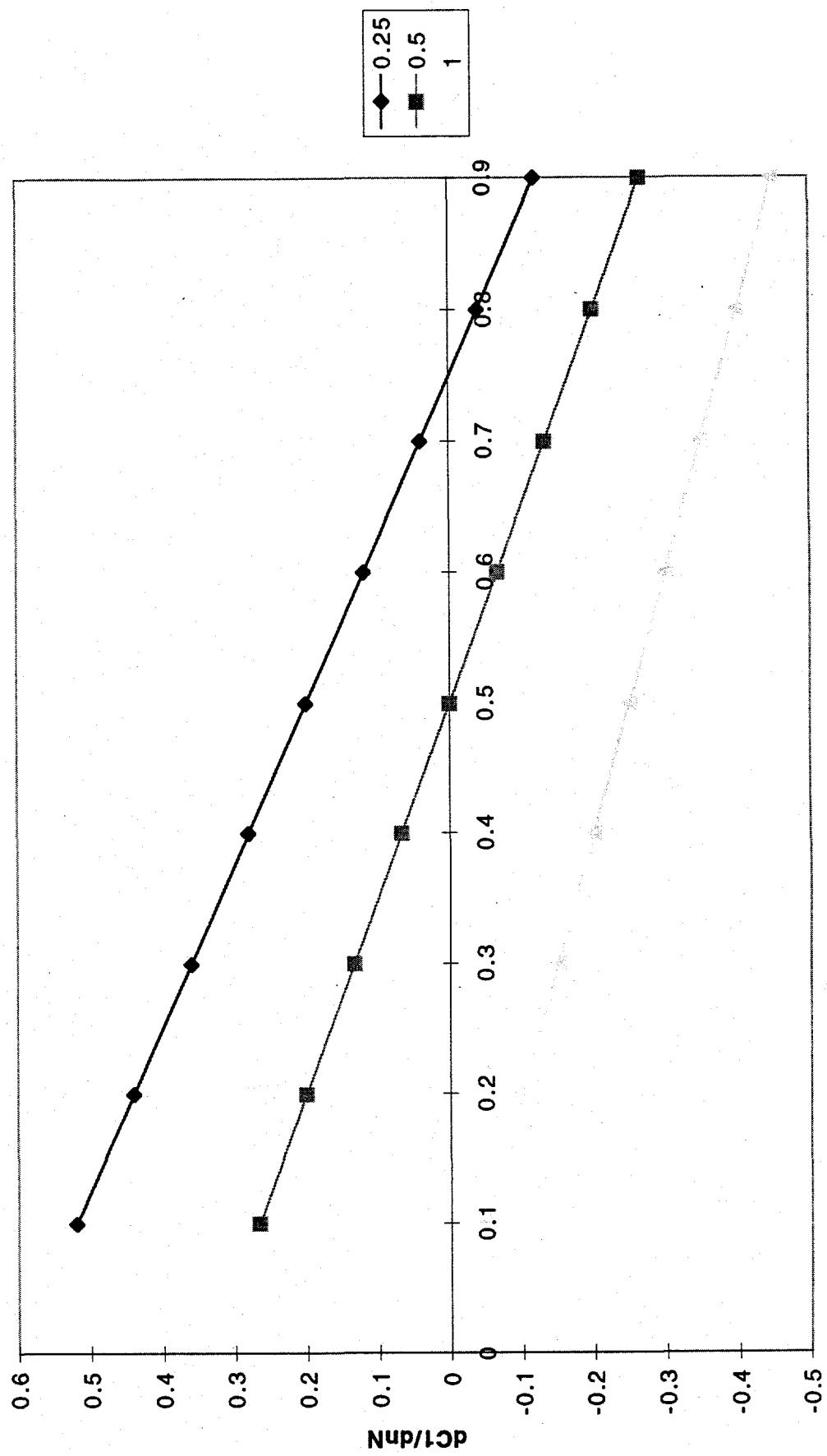
8 dC1 d M & N v m

# Fig. 8. dC1/dM&N vs m various h
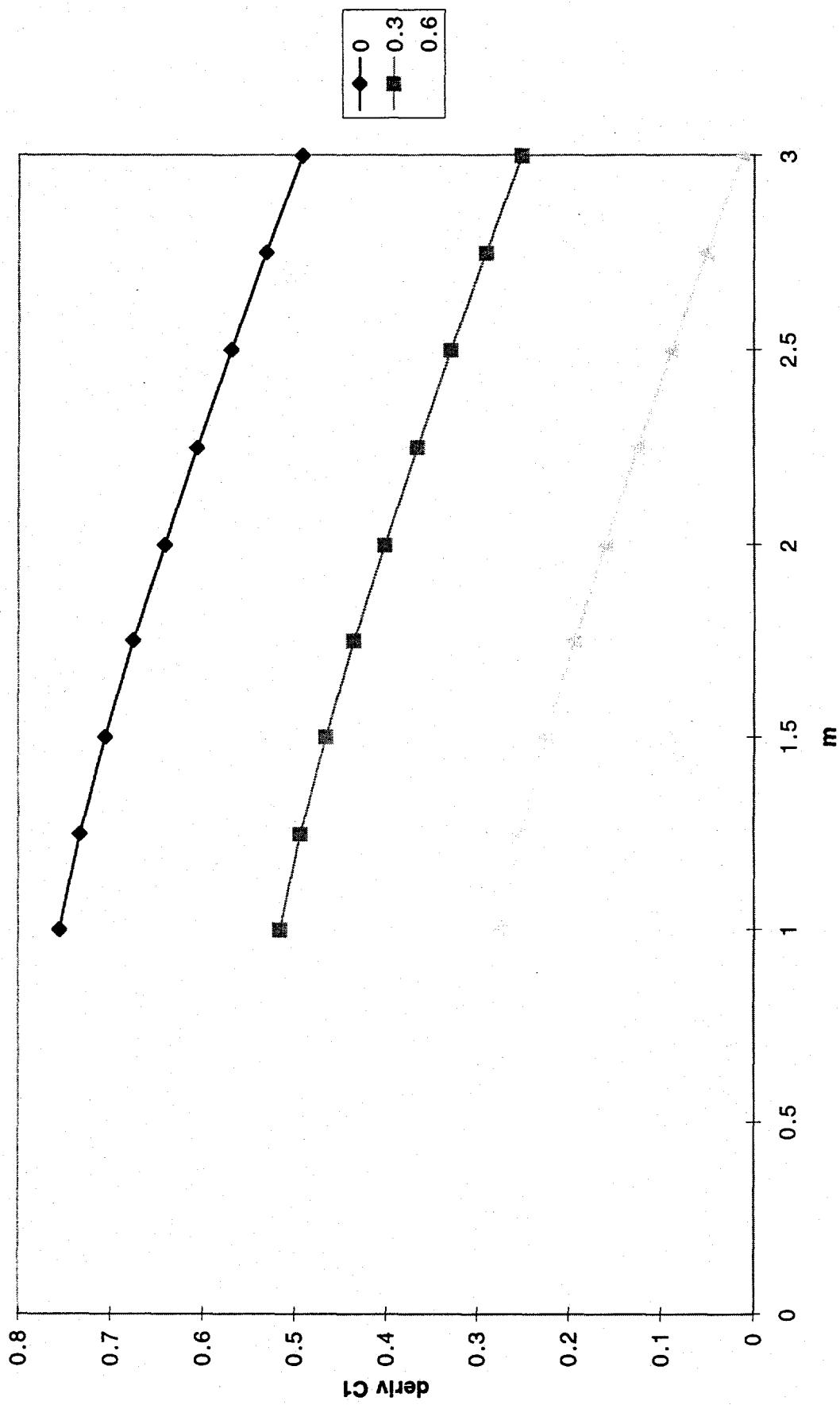
Fig. 9. C1 vs h for various L;m = 1

10 dC1 sum v h
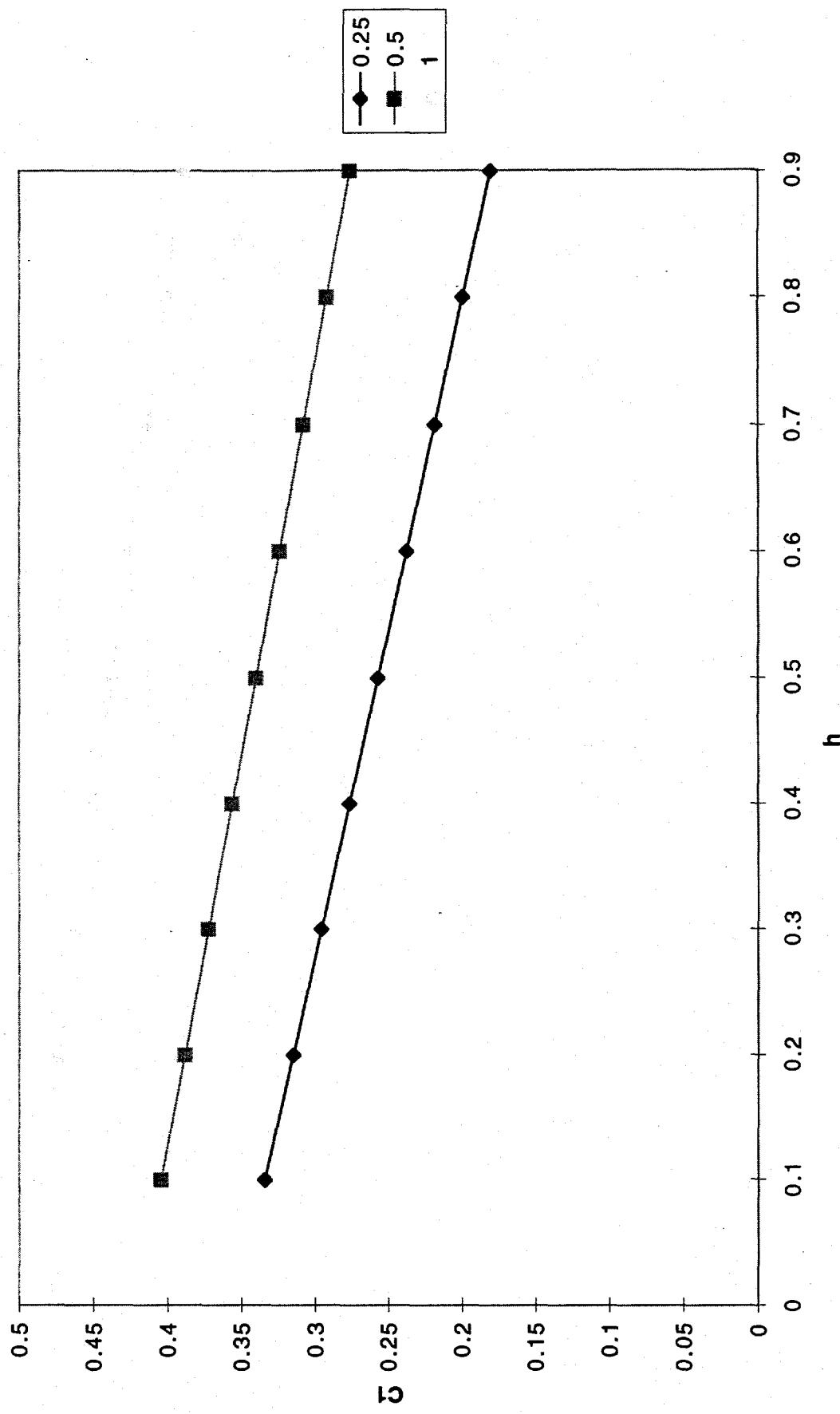
Fig. 10. dC1/dM total vs h various L; m = 0

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