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LQG/LTR CONTROL SYSTEM DESIGN FOR A LOW-PRESSURE FEEDWATER HEATER TRAIN WITH TIME DELAY*

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Abstract: An LQG/LTR control system design is formulated for a feedwater heater train with output time delay. This approach involves factoring the feedwater heater train plant into nonminimum and minimum phase components to allow the design of a robust controller for the minimum phase component of the plant using the LQG/LTR technique (minimum phase method).

The nonminimum phase component takes the form of an all-pass filter containing the rhp zeroes of the first-order approximation of the time delay component. Using this nonminimum phase all-pass filter, certain singular-value multiplicative error bounds can be established to obtain a stable control system when using the LQG/LTR design technique on the minimum phase component of the plant.

New analysis methods using singular values are integrated into the conventional singular-value performance and stability robustness analysis procedure. These new analysis methods allow computation of the maximum allowable time delay before instability occurs for both SISO and MIMO control systems.

Introduction

In the nuclear power industry reliable control of nuclear reactors is of great concern because of the possible detrimental effects of an accident on the environment and the public. Reliable control of a nuclear reactor also minimizes shutdowns, which can cost on the order of millions of dollars if they continue for a prolonged period of time. One major source of nuclear power plant shutdown is the feedwater control system. In the nuclear power industry, feedwater control systems have been responsible for three U.S. plant shutdowns per year for BWRs (boiling water reactors) [1]. The Northern States Power (NSP) Company established an overall system design goal of 10 years without a control-system-related reactor scram. Criteria such as this are resulting in a search for modern control system design strategies to apply to nuclear reactor systems.

To improve the control system design strategy of the feedwater control system, the often neglected process time delay will be considered. Using a newly developed procedure a stable control system is obtainable for a feedwater plant with time delay. The new design and stability analysis procedure for the feedwater control system is presented in this paper [2].

This design procedure uses the LQG/LTR design technique with additions and modifications to the existing design and analysis procedure. The present LQG/LTR design technique is applicable to minimum phase plants. The resulting controller guarantees good robustness properties at either the plant output or input. These robustness properties, however, are not guaranteed for a nonminimum phase plant. The nonminimum phase component of this plant is assumed to originate from time delay at the plant output.

A plant with rhp zeros (process delay) imbedded in its state equations can be factored to represent a minimum phase component and time delay at either the input or the output of the plant.

The procedure to be presented here will provide a quantitative measure of how time delay degrades the stability robustness of the control system. This procedure will also include an analysis of the maximum allowable time delay before instability occurs in the LQG/LTR control system designed for a plant with time delay. The procedure will further establish a clear connection between the frequency domain stability criteria for plants with time delay and will explain how the time domain stability results verify the frequency domain stability results.

It will be shown that by modifying the LQG/LTR design method a more intuitive and analytical approach can be taken in designing a stable control system for a nonminimum phase plant.

Linearized Model

The nonlinear mathematical model of the low-pressure feedwater heater train shown in Fig. 1 is obtained using the Modular

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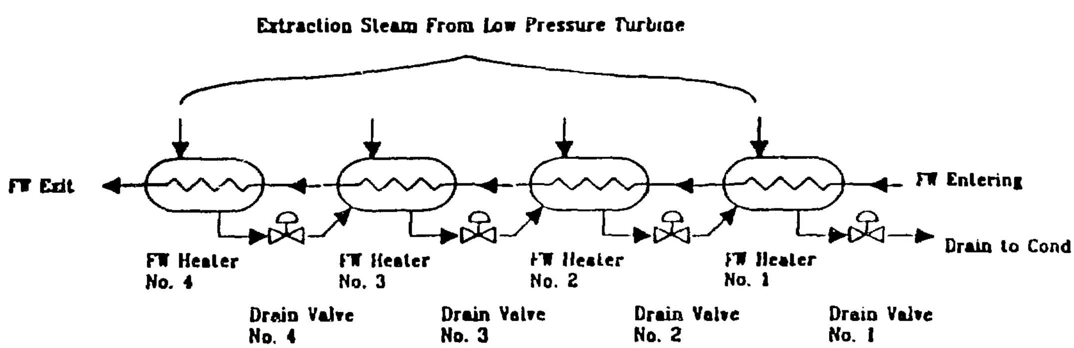

Fig. 1. Flow diagram of low-pressure feedwater heater train.

Modeling System (MMS) [3]. The nonlinear plant is linearized about a nominal operating point, resulting in a state differential equation of the form

$$\dot{x} = Ax(t) + Bu(t)$$
$$y = Cx(t) \tag{1}$$

where A, B, and C are system matrices. The linearized model consists of 24 state variables (pressures and enthalpies), 4 inputs (control valves fraction open), and 4 outputs (tank levels) [4]. The system matrices of Eq. (1) are used to define the minimum phase model excluding the time delay at the plant output.

The time delay in this system will be attributed to computational and communication delays, which are assumed to be caused by the computational and communication limitations of the simulator being used to evaluate the controller. The computational and communication delay is to be 0.5 s. The process dynamics for this system are known to have insignificant process delays. Even though no knowledge of process time delay is available during the analysis, the control system's capability to tolerate time delay (which is significantly greater than the computational time delay) will be evaluated.

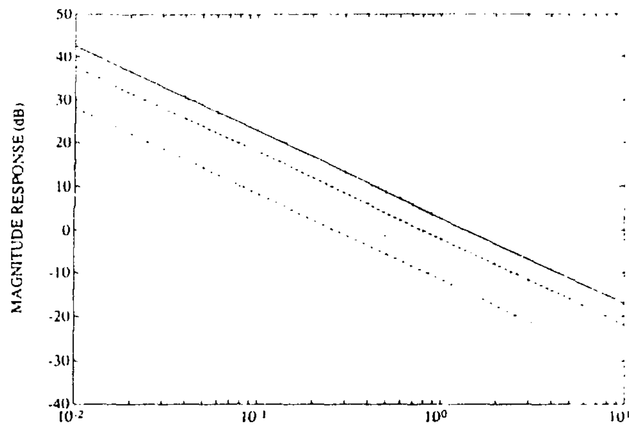

Fig. 2. SVP return ratio of nominal plant [$G(s)$].

Controller Design And Evaluation

This control system design will require a small or zero steady state output tracking error in response to a required command for a desired feedwater heater tank level. For zero steady state error it requires the control system to add integral action at the plant input if zero steady state error is desired, or the plant to inherently have pure integral action (poles at the origin of the s-plane). Taking advantage of the inherent integral action of the plant as indicated by the singular value plot (SVP) of the return ratio in Fig. 2, augmentation of the plant input with integral components can be avoided.

Step 1. The first step in this procedure of control system design requires the nonminimum phase plant to be factored into a nominal plant matrix transfer function and the time delay matrix transfer function at the plant output. The perturbed plant described in transfer function form is as follows:

$$G'(s) = B_z(s)G(s) \tag{2}$$

where $G(s)$ is a minimum phase model of the plant and $B_z(s)$ is the matrix transfer function for output time delay. Considering exact representation of time delay $B_z(s)$ consists of $e^{-\tau s}$ on the matrix transfer function diagonal. The time delay matrix $B_z(s)$ diagonal element $e^{-\tau s}$ can be represented as a low-order Padé approximation:

$$e^{-\tau s} \approx \frac{1 - s(\tau/2)}{1 + s(\tau/2)} \quad , \tag{3}$$

which is an all-pass filter contributing a nonminimum phase zero to the plant open-loop system where τ is the system time delay. This approximation of time delay is considered valid in this control system design because the errors in approximating $e^{-\tau s}$ become significant only at higher frequencies. Therefore the mathematical difficulty involved in synthesizing a controller for an exact time delay yields no significant benefits when compared to using the approximation in a low-bandwidth process control system. Figure 3 represents the perturbed feedwater control system block diagram.

Step 2. The significance of the required factorization of the plant into minimum and nonminimum phase components is that the nonminimum phase component can be used to characterize a model uncertainty $\Delta G_D(s)$. The nominal minimum phase plant component is used in the control system design. The model uncertainty $\Delta G_D(s)$ due to time delay thus can be used to ensure stability robustness of the minimum phase plant design, thus also ensuring stability robustness of the nonminimum phase perturbed plant.

The model uncertainty due to the time delay component of the perturbed plant is as follows:

$$\Delta G_D(s) = I - B_z(s) \quad , \tag{4}$$

where $B_z(s)$ is as previously defined.

Step 3. Now the necessary balancing transformation matrix L and the scalar gain parameter μ will be selected to obtain the appropriate transfer function loop shape

$$G_{FOL}(j\omega) = (1/\sqrt{\mu})[C(sI - A)^{-1}L] \tag{5}$$

to meet the desired design specifications for this control system as shown in Table 1. An additional constraint required in this design is to place a bandwidth constraint of 0.1 rad/s on the control system. This bandwidth constraint is required because in actual practice a process control system will have a limited rate of response due to plant physical limitations.

It is also desired to have the control loops respond similarly considering the bandwidth limitations. Therefore it is required that $G_{FOL}(j\omega)$ have singular values that are equal to 1 $\{\sigma_i[G_{FOL}(j\omega)] = 1\}$ at the gain crossover frequency of 0.1 rad/s. Also, it is desired to have the singular values the same at both low and high frequencies, which will assist in achieving the design specifications. In this design it is desired to balance the singular values at an intermediate frequency of 0.1 rad/s.

The balancing transformation L is obtained using a design tool called CASCADE [5]. The scaler μ (the gain parameter) is selected to be 1.0.

Step 4. Using the numerical values of L and μ obtained in Step 3 in the Kalman filter algebraic Riccati equation (ARE),

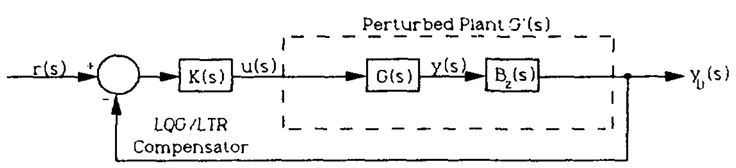

Fig. 3. Perturbed feedwater control system.

Table 1. Feedwater control system design specifications

System Requirements	Range
Good command-following	$\underline{\sigma}[G(j\omega)K(j\omega)] > 20\ dB\ \ \forall\ \omega < 0.01\ rad/s$
Good disturbance rejection	$\underline{\sigma}[G(j\omega)K(j\omega)] > 20\ dB\ \ \forall\ \omega < 0.01\ rad/s$
Good immunity to noise	$\bar{\sigma}[G(j\omega)K(j\omega)] < -5\ dB\ \ \forall\ \omega > 1.0\ rad/s$
Good system response to high-frequency modeling error*	$\underline{\sigma}\{I + [G(j\omega)K(j\omega)]^{-1}\} > \lVert\Delta G_D(j\omega)\rVert\ \ \forall\ \omega > 0\ rad/s$
Good insensitivity to parameter variations at low frequencies*	$\underline{\sigma}[G(j\omega)K(j\omega)] > 10\ dB\ \ \forall\ \omega < 0.02\ rad/s$

*Note: The $\Delta G_D(j\omega)$ indicated corresponds to the applicable multiplicative uncertainty due to time delay.

$$LL^T - (1/\mu)\Sigma C^T C\Sigma + A\Sigma + \Sigma A^T = 0 \ , \qquad (6)$$

the Σ (covariance matrix) is solved for, then using

$$F = (1/\mu)\Sigma C^T \ , \qquad (7)$$

the filter gain F is computed, then the desired Kalman filter transfer function, which meets the design specifications, is obtained. The SVP [6] of the Kalman filter transfer function $G_{KF}(s)$ is shown in Fig. 4.

Step 5. Now it is desired to obtain a numerical value of the regulator gain K such that

$$G(s)K(s) \rightarrow G_{KF}(s) \qquad (8)$$

or, simply stated, the good performance and robustness stability properties of the Kalman filter are recovered at the plant output. Using CASCADE the numerical value of the regulator gain K is obtained, thus completing the controller design.

The singular-value plots of the return ratio, return difference, and inverse return difference of $G(s)K(s)$ are shown in Figs. 5, 6, and 7 respectively. The SVP of the inverse return difference of $G(s)K(s)$ also includes a plot of the SVP of $\bar{\sigma}[\Delta G_D(s)]$ for $\tau = 0.5$ s. Upon examination of these SVPs it is seen that the desired design specifications are met.

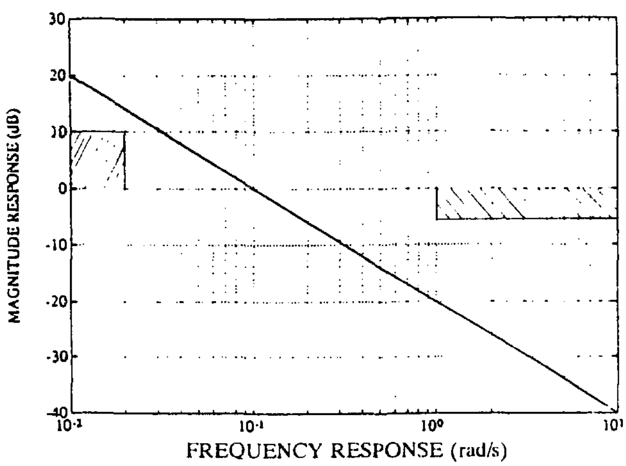

Fig. 4. Singular value plot of the Kalman filter transfer function $[G_{KF}(s)]$.

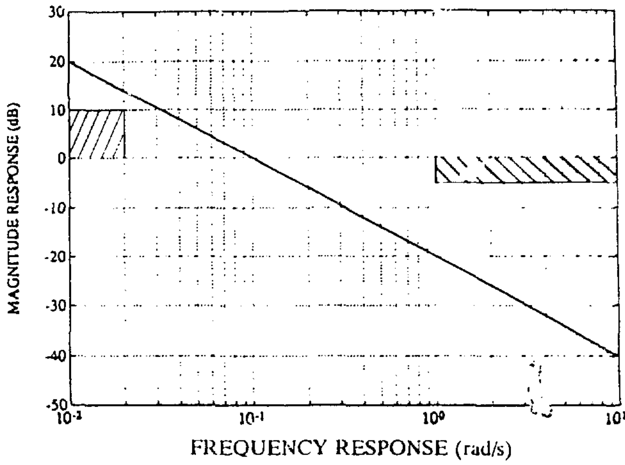

Fig. 5. SVP of the return ratio of a compensated system $[G(s)K(s)]$.

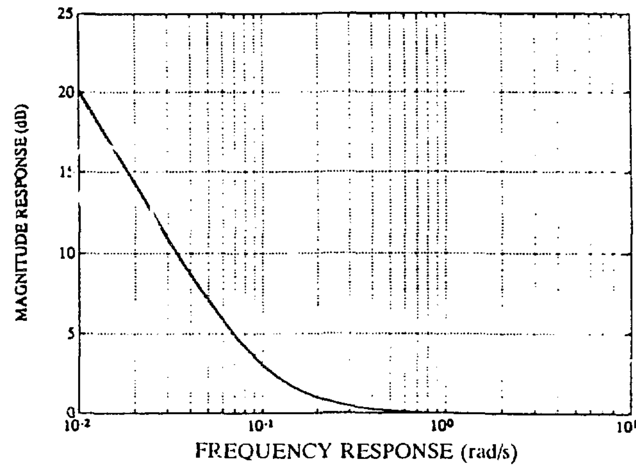

Fig. 6. SVP of the return difference of a compensated system $[I + G(s)K(s)]$.

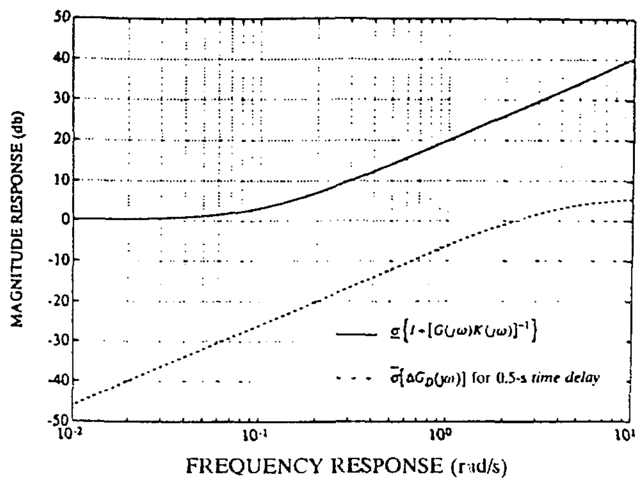

Fig. 7. SVP of the inverse return difference of a compensated system and $\Delta G_D(s)$ for a time delay of 0.5 s.

Step 6. Next the maximum value of allowable time delay that will destabilize the newly designed control system is computed using the analytical method described by [7]. Applying this analytical method yields a maximum allowable time delay (MATD) of $\tau_A = 14.7$ s. Figure 8 shows the SVP of the multiplicative stability bound $\Delta G_D(s)$ with a time delay of 14.7 s and the inverse return difference of the compensated system. It is seen that $\Delta G_D(s)$ for a time delay of 14.7 s is not significant enough to destabilize the compensated system.

As the time delay of $\Delta G_D(s)$ increases above 14.7 s it is seen that the $\bar{\sigma}[\Delta G_D(s)]$ also increases. The amount of maximum time delay, τ_G, that is graphically seen (Fig. 8) to be allowable before the stability inequality is violated is ~19.0 s. This graphical result appears to verify that the MATD of $\tau_A = 14.7$ s required to destabilize the control system is a very conservative value.

The output transient responses of the closed-loop level control system are now evaluated as the time delay increases from 0 to 20 s. The transient response of the nominal control system with no time delay is shown in Fig. 9. The results of the transient responses of Figs. 10–12 verify the conservative result of τ_A as the MATD as seen in the frequency domain. The results of Fig. 12 show that the control system becomes unstable at ~20 s, which is slightly greater than τ_G, the graphically obtained value of the MATD required to destabilize the control system.

CONCLUSIONS

In this work a systematic control design methodology has been introduced for a system with time delay. The methodology allows the synthesis of a stable control system for plants with uniformly varying time delay at the plant input or output.

In the analysis portion, graphical and analytical techniques were presented to evaluate the maximum allowable time delay (MATD) required to destabilize the control system. Through graphical analysis in the frequency domain and time domain analysis the analytical method of computing the MATD is seen to be conservative. The analysis in this methodology allows quick computation of the effects of possible time delay quantities on the stability of an existing or

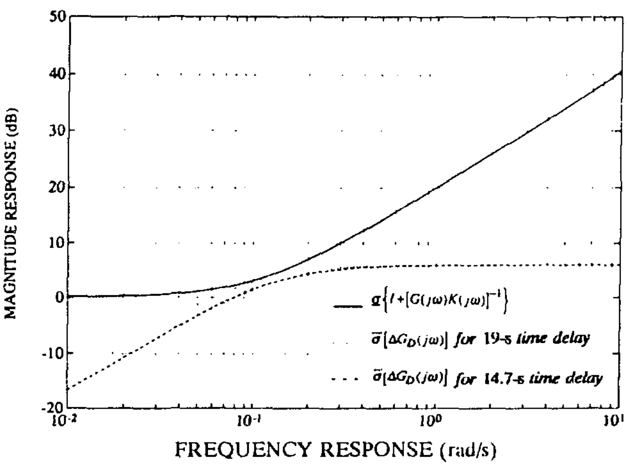

Fig. 8. SVP of the inverse return difference of a compensated system and the model uncertainty due to time delays of 14.7 and 19.0 s.

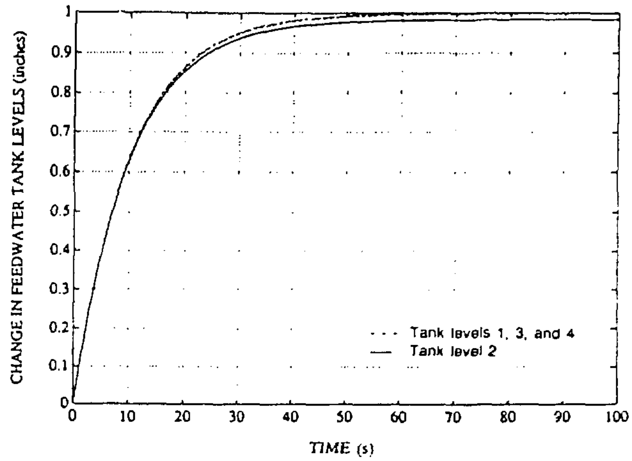

Fig. 9. Transient response for level demand with no time delay.

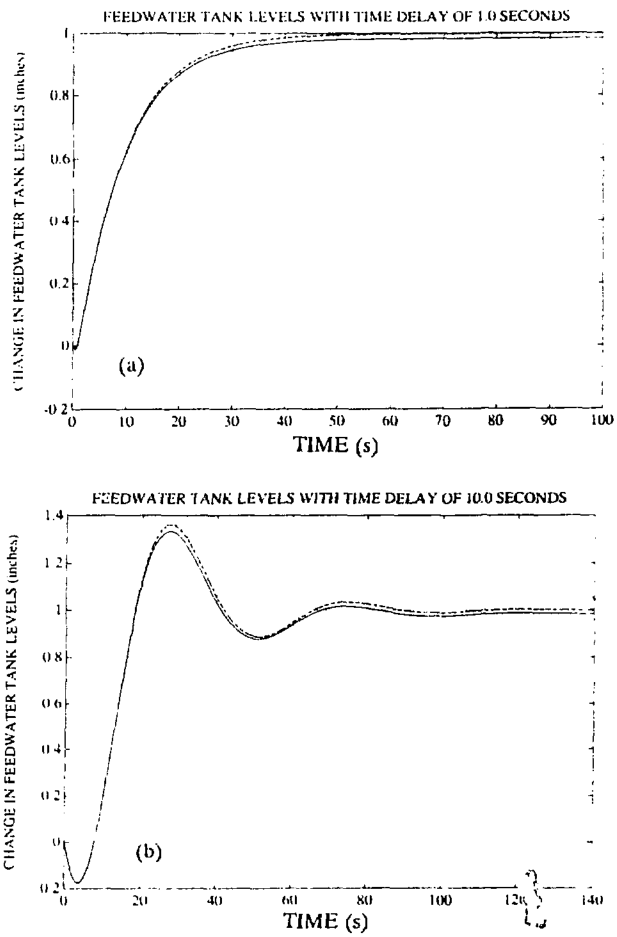

Fig. 10. Transient responses for level demand with time delays of (a) 1.0 and (b) 10.0 s.

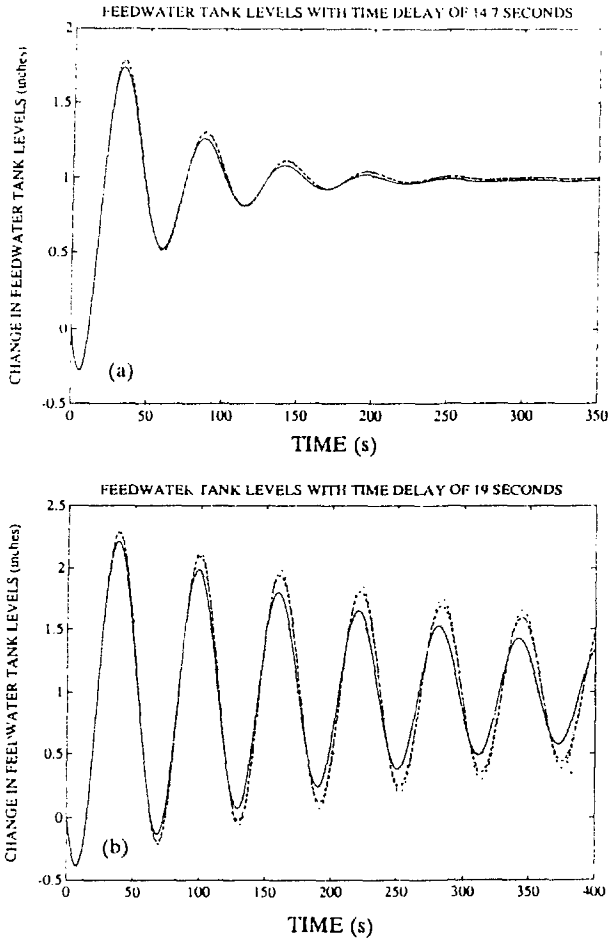

Fig. 11. Transient responses for level demand with time delays of (a) 14.7 and (b) 19.0 s.

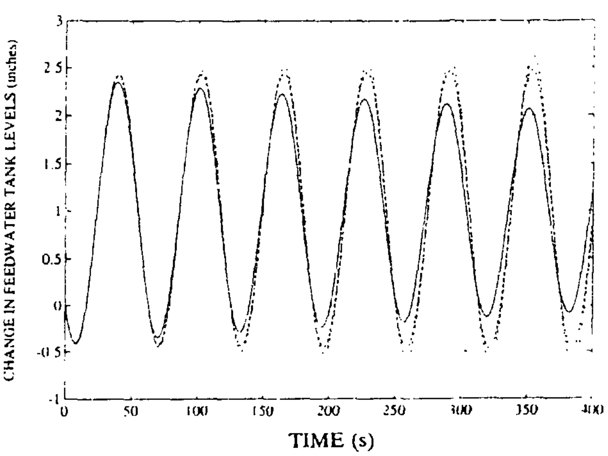

Fig. 12. Transient responses for level demand with a time delay of 20.0 s.

newly designed control system, regardless of whether it is SISO or MIMO.

In the design portion of this methodology all of the performance and stability robustness aspects can be established in the frequency domain. The performance robustness of the desired system may be limited by the stability robustness due to noise or model uncertainty, whichever is more restrictive.

The benefit of this methodology is that it allows the synthesis of a stable closed-loop control system with some limitations in achievable performance. Most of all, this methodology allows the classical control concepts for SISO plants with time delay to be used in MIMO plants with time delay, thus avoiding the loss of intuitive control concepts in a vastness of formidable mathematics.

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